Energy formulae for reconstruction

May 30, 2017

The RD52 hadron paper (May 2017) has the formula (Equ. 2) we have used to estimate the energy of a hadronic shower:

$$E = \frac{S - \chi C}{1 - \chi},$$

where χ is purely a property of the calorimeter construction:

$$\chi = \frac{1 - (h/e)_S}{1 - (h/e)_C} \approx 0.3$$

In this paper, the "rotation method" was used to estimate the energies of pions in the Pavia Pb modules (plus the leakage counters). The method depended upon having a large ensemble of pions at the same energy, from which the intersection of the locus of points in (C, S) could be extrapolated to the $C = S$ line which thereby defined the beam energy. This is illustrated in Fig. 1.

Richard's email (below) is another method to estimate E: Start from a measurement (C_0, S_0) for one event, for example, shown as the black dot in Fig. 2, then estimate its energy from the parallel red lines which one-to-one intersect the $C = S$ line, and this intersection point is the energy of the particle. The main fact is that the red lines are all parallel and all at angle θ. i.e., all at the same slope $tan\ \theta$.

Richard's way of thinking on this is far superior to our earlier use of two linear equations in two unknowns arguments. The plot in Fig. 1 is from Don Groom in the PDG and is not only a powerful way to display, in one figure, dual-readout but also easily shows that pion showers (with more EM activity) and proton showers (with less EM activity) simply populate the red line differently, but their dual-readout energies are the same.

Start with the point at (C_0, S_0). The equation of the line through this point that has a slope m is

$$(C - C_0) = m \times (S - S_0). \tag{1}$$

The slope m is directly tan θ in Fig. 1. I will use Don Groom's notation that η is the mean value of h/e, $\eta = h/e$. The slope of the line $tan\ \theta$ is

$$tan\ \theta = m = \frac{1 - \eta_C}{1 - \eta_S}. \tag{2}$$

The intersection of this line with the line $C = S$ occurs at the point S on the x-axis given by the substitution of $C = S$ into the equation (1) of the first line:

$$S - C_0 = m\ (S - S_0). \tag{3}$$

The solution for S is the same as the shower energy E, and is

$$S = C_0 + m(E - S_0), \tag{4}$$

$$S(1 - m) = C_0 - mS_0 \tag{5}$$

$$S = \frac{C_0 - mS_0}{1 - m}. \tag{6}$$

Dividing through by m gives

$$S = \frac{S_0 - C_0/m}{1 - 1/m}. \tag{7}$$

The definition of the slope m above is $m = tan\ \theta = (1 - \eta_C)/(1 - \eta_S)$. The inverse of this, $1/m$, is the χ we use in the paper,

$$1/m = \frac{1 - \eta_S}{1 - \eta_C} = \chi, \tag{8}$$

so finally the energy estimate is

$$S \to E = \frac{S_0 - \chi C_0}{1 - \chi}, \tag{9}$$

the same formula we already use, unfortunately. No gain.

1 Richard's email: 4 April 2017

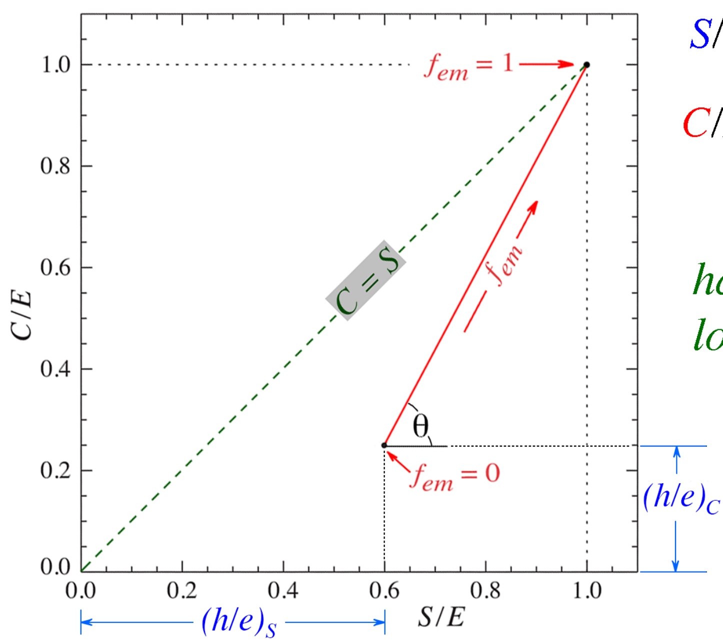
Guys,

After having given it some more thought, I think I might have found a way to optimize the precision with which we can measure the energy of individual hadron showers, i.e. for one particle as opposed to an ensemble of mono energetic particles. I attach two graphs to illustrate my point. The idea is the following. As shown in Figure 1, all hadronic data points for a given energy are located on the red line. The em fraction determines the point on this line where the data point is located. Once the em fraction is known, the energy resolution is determined by fluctuations in a plane perpendicular to this line. I think this is the best we can hope to do. The second figure shows a scatter plot where the scintillation and Cherenkov signals, measured in em GeV units are plotted on the horizontal and vertical axis, respectively. The calorimeter is, as usual, calibrated with electrons, so data points for electrons are located on the green diagonal in this plot, and the energy scale is also indicated along this line. Now the hadronic data points. For a given energy, say 80 GeV, all hadronic data points are located on a red line that makes an angle theta with the x-axis and intersects the green line at 80 GeV. For 60 GeV, the data points are located on another red line, parallel to the first one (since theta is constant for our calorimeter), which intersects the green line at 60 GeV. And so on. We can thus determine the hadron energy of a given point (the black point in this figure) by determining the distance to the nearest red lines drawn in this figure. In the example, the energy of the hadron point is probably something like 83 GeV.

Let me know your thoughts about this idea. If you think it is worth pursuing, I would like to ask John to use his considerable math skills to parameterize the red lines in terms of S,C and come up with an expression that allows us to calculate the energy of the hadron showers. Hopefully, it is not again the standard DREAM formula.

Best regards,
Richard

Principles of dual-readout calorimetry (1)



$S/E = (h/e)_S + f_{em}[1 - (h/e)_S]$

$C/E = (h/e)_C + f_{em}[1 - (h/e)_C]$

hadronic data points (S,C) located on straight (red) line

Figure 1: C/E vs. S/E plot showing the positions of $\eta_S = (h/e)_S$ and $\eta_C = (h/e)_C$. The red line with slope $tan\ \theta$ is a straight line independent of particle energy and particle type. Particles and jets with differing EM fractions simply populate the red line differently but, being on this red line, they all have the same dual-readout energy. From this figure, $tan\ \theta = (1 - \eta_C)/(1 - \eta_S)$.

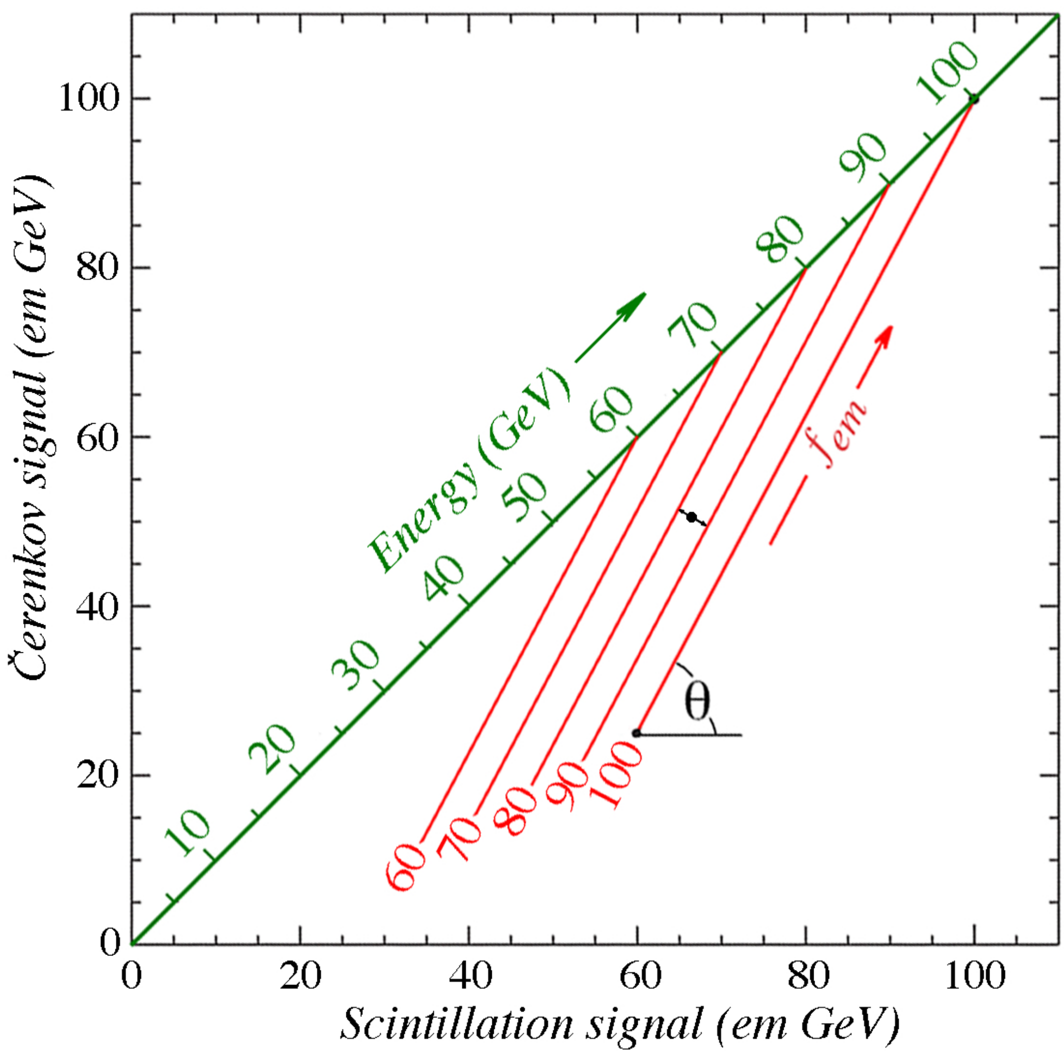


Figure 2: Richard's plot; C_0, S_0) point at approximately 84 GeV.