Constraining Goldstinos with Constrained Superfields

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Introduction

- SUSY should be broken – there is a massless Goldstino which is eaten by the gravitino.
- If the scale of SUSY breaking is low, the gravitino is light and its couplings are dominated by the couplings to the Goldstino.
- By analogy to pions we can try to understand Goldstino couplings:
  - Current algebra
  - Effective Lagrangian with a nonlinearly realized symmetry

Questions

- What are the most general interactions of Goldstinos?
  - Derive the Akulov-Volkov Lagrangian.
  - What are the corrections?

- Goldstino couplings to other particles:
  - Coupling to superfields
  - Coupling to fermions
  - Coupling to gauge fields
  - Coupling to scalars including Goldstone bosons (e.g. R-axions)

The Ferrara-Zumino (FZ) multiplet

The SUSY current and the energy momentum tensor $S_{\mu\alpha}, \; T_{\mu\nu}$ reside in a real superfield $\mathcal{J}_\mu \sim \mathcal{J}_{\alpha\dot{\alpha}}$ which satisfies the conservation equation

$$\bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

with X a chiral superfield:

$$X = x + \theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} S^{\dagger\dot{\alpha}} + \theta^2 (T^\mu_\mu + i\partial \cdot j)$$

Supersymmetry breaking

- When SUSY is broken the supercharge does not exist.
- However, the supersymmetry current does exist.
- (Anti)commutators with the SUSY charge exist.
- Therefore, we can construct a superspace.
- Hence the FZ-multiplet $\mathcal{J}_{\alpha\dot{\alpha}}$ and the chiral operator X exist.
- Everything which follows from this is true in any theory which breaks SUSY including strongly coupled (incalculable) theories.

The order parameter

- The order parameter for chiral symmetry breaking which is also an interpolating field for the pions is $\bar{\psi}_q \psi_q$. It does not have a natural normalization; it is different from f_π .
- SUSY is different.
- Our starting points is $\bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$.
- The order parameter for SUSY breaking is the F-component of $X = \theta^2 f^2 + f\theta\psi + \ldots$ (f^2 is the vacuum energy, f is the Goldstino decay constant, and ψ is the Goldstino).
- It has a natural normalization – it is well defined both in UV and in IR.

The Goldstino superfield

$$\bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

- The chiral superfield X includes an interpolating field for the Goldstino with a well defined normalization.
- We try to write a Lagrangian for the field X .
  - It does not have dimension one.
  - The scalar in its first component should not create a massless scalar.
- Define a dimension one chiral superfield X_{NL}. through $X = f X_{NL}$ and impose a constraint to eliminate its lowest component.

The constraint

We want to eliminate the first component of X_{NL} using a supersymmetric constraint.

Analog of $UU^\dagger = 1$ for pions.

The first guess is $$X_{NL}^2 = 0$$

- A nontrivial solution exists:

$$X_{NL} = \frac{\psi\psi}{F} + \theta\psi + \theta^2 F$$

  with arbitrary ψ, F .
- This expression uniquely follows from the SUSY transformations of the Goldstino ψ .

Summary so far

We consider a chiral superfield satisfying

$$X_{NL}^2 = 0$$

$$X_{NL} = \frac{\psi\psi}{F} + \theta\psi + \theta^2 F$$

Other Goldstino superfields were studied by Rocek, Lindstrom, Samuel, Wess.

Component nonlinear realizations were studied by many authors starting with Akulov and Volkov, including Brignole, Feruglio, Zwirner, Luty, Ponton, Clark, Love….

A Lagrangian for X_{NL}

$$\int d^4\theta \; X^\dagger_{NL} X_{NL} + \int d^2\theta \; f X_{NL} + h.c.$$

$$X^2_{NL} = 0$$

- Higher order terms without derivatives vanish.
- Ignoring the constraint, this is the simplest model which exhibits spontaneous SUSY breaking (it is free). It has a massless fermion and a massless scalar.
- The constraint removes the scalar…

A Lagrangian for X_{NL}

Substituting the solution

$$X_{NL} = \frac{\psi\psi}{F} + \theta\psi + \theta^2 F$$

in the free Lagrangian we find

$$\left|\partial\left(\frac{\psi\psi}{F}\right)\right|^2 + \psi^\dagger\sigma\partial\psi + |F|^2 + fF + fF^\dagger$$

Integrating out the auxiliary field using its equation of motion

$$F = -f + ...$$

we find the Akulov-Volkov Lagrangian.

Example

$$K = |\Phi|^2 - \frac{1}{M^2}|\Phi|^4 + \dots \qquad\qquad W = f\Phi$$

It breaks SUSY and the complex scalar in Φ has a mass $m \sim f/M$.

We can integrate it out. Its low momentum classical equation of motion arises from the quartic term in K with the solution $\phi = \frac{\psi\psi}{F}$

Substituting this in Φ leads to our non-linear field

$$\Phi = X_{NL} = \frac{\psi\psi}{F} + \theta\psi + \theta^2 F$$

Example

$$K = |\Phi|^2 - \frac{1}{M^2}|\Phi|^4 + \ldots \qquad W = f\Phi$$

Substituting the solution $\Phi = X_{NL}$, the quartic term vanishes. We find the quadratic Lagrangian but with a constrained field.

The low energy theory below m is universal. It is independent of higher energy physics and the values of M, m .

It is the Akulov-Volkov Lagrangian.

Corrections to the AV Lagrangian

For pions the terms in the effective Lagrangian are controlled by the number of derivatives – the pion Field is dimensionless.

In SUSY it is natural to assign weight $-1/2$ to the Goldstino (like the dimension of θ). Equivalently, we assign weight -1 to our chiral superfield X_{NL} .

The leading order terms we considered have weight zero.

Corrections to the AV Lagrangian

The leading order terms lead to terms with: 2 fermions + 1 derivative, 4 fermions + 2 derivatives, 8 fermions + 4 derivatives. All have weight zero.

Correction terms have higher weight; e.g.

$$\int d^4\theta \, |\partial X_{NL}|^2$$

For fixed number of fermions they have more derivatives.

The leading order Lagrangian has an accidental R-symmetry. It can be broken by the corrections.

Coupling X_{NL} to superfields

Consider an effective Lagrangian valid above the mass of the superpartners but below $\sqrt{f}$ (e.g. the MSSM).

This Lagrangian includes explicit soft SUSY breaking terms. These can be written in terms of a spurion.

The Lagrangian is made supersymmetric by replacing the spurion with X_{NL} .

Coupling X_{NL} to superfields

The Goldstino couplings are determined by the terms in the Lagrangian (the MSSM) and depend only on its parameters and on f .

Higher order corrections to the MSSM and the Goldstino couplings can depend on shorter distance physics.

Note the many roles of X :

- It appears in $\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$.
- It leads to the Goldstino superfield X_{NL} .
- It is the spurion.

Low energy processes

We are interested in low energy processes involving Goldstinos below the scale of the superpartners. We need to include explicit couplings of X_{NL} to the light particles and compute diagrams with intermediate superpartners.

Such computations exhibit "miraculous cancelations" and have led to some controversies in the literature.

We will look for a formalism which makes these computations obvious and the cancelations manifest.

Goldstino couplings to light fermions

We look for an effective Lagrangian below the mass of the superpartners.

Consider fermions (quarks) from chiral superfields Q .

The scalars (squarks) can be removed by considering a constrained superfield Q_{NL} satisfying

$$X_{NL} Q_{NL} = 0$$

Now we can write the most general SUSY Lagrangian with these superfields subject to their constraints.

Goldstino couplings to light fermions

- Some of the terms in this Lagrangian are universal. They depend only on f.
- Some of the terms arise from the coupling to superfields and hence they depend only on the soft terms and on f.
- In addition, there are higher order non-universal terms which arise from higher order corrections to the MSSM.
- The previously mentioned "miraculous cancelations" arise trivially as a result of the constraint (because of lack of time we will not explicitly do it here).

Goldstino coupling to gauge fields

We can extend this analysis to gauge fields:

- We need to restrict the gauge symmetry (analog of Wess-Zumino gauge):

$$X_{NL} V_{NL} = 0$$

- We need to remove gauginos

$$X_{NL} W_{\alpha NL} = 0$$

Again, the Lagrangian includes universal terms which depend only on f , terms which depend only on the soft terms, as well as higher order, non-universal terms.

Goldstino coupling to scalars and axions

It is easy to extend this formalism to other kinds of matter fields.

- Complex massless scalars from chiral superfields H are present only with fine tuning. They satisfy

$$X_{NL} H^{\dagger}_{NL} = \text{chiral}$$

- Real (Goldstone) bosons and R-axions satisfy

$$X_{NL}(A_{NL} - A^{\dagger}_{NL}) = 0$$

Conclusions

- SUSY breaking is characterized by a natural order parameter which is in a chiral superfield X .
- X includes an interpolating field for the Goldstino.
- Imposing a simple constraint $X_{NL}^2 = 0$ we easily construct the low energy Lagrangian. The corrections are easily controlled.
- The leading order Lagrangian is universal, depending only on f , but the corrections are not.
- Coupling X_{NL} to superfields, we identify it with the spurion.
- The couplings below the mass of the superpartners are easily found by using constrained superfields.

Conclusions

- The constrained superfields:
  - Fermions from chiral superfields $X_{NL}Q_{NL} = 0$
  - Vectors from vector superfields $X_{NL}V_{NL} = 0$
    $$X_{NL}W_{\alpha NL} = 0$$
  - Complex scalars from chiral fields
    $$X_{NL}H^{\dagger}_{NL} = \mathrm{chiral}$$
  - Real scalars from chiral superfields (Goldstone bosons)
    $$X_{NL}(A_{NL} - A^{\dagger}_{NL}) = 0$$
- Alternative formalisms exhibit “miraculous cancelations.”

Conclusions

- Some terms are universal and depend only on f .
- Some terms depend on f and the MSSM parameters.
- Higher order corrections to the MSSM lead to higher order corrections to the Goldstino couplings.
- This formalism has allowed us to re-derive, clarify, and significantly extend the existing literature about the coupling to light gravitinos.
- There are many potential phenomenological consequences (in progress).