$R(D^{(*)}_{(s)})$ and $R(J/\psi)$ from the lattice

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Intro & Motivation

- There are persistent anomalies in the ratios $R(B \to D^*)$ and $R(B \to D)$ (and now $R(B_c \to J/\psi)$ involving the same $b \to c$ transition.
- Reliable theory predictions needed. LQCD provides a from-first-principles approach to calculating hadronic matrix elements $\to$ form factors parameterising semileptonic decays.
- Provide reliable SM determination for $R(B_c \to J/\psi)$, to be compared with recent measurement by LHCb.
  lhcb-public.web.cern.ch/lhcb-public/

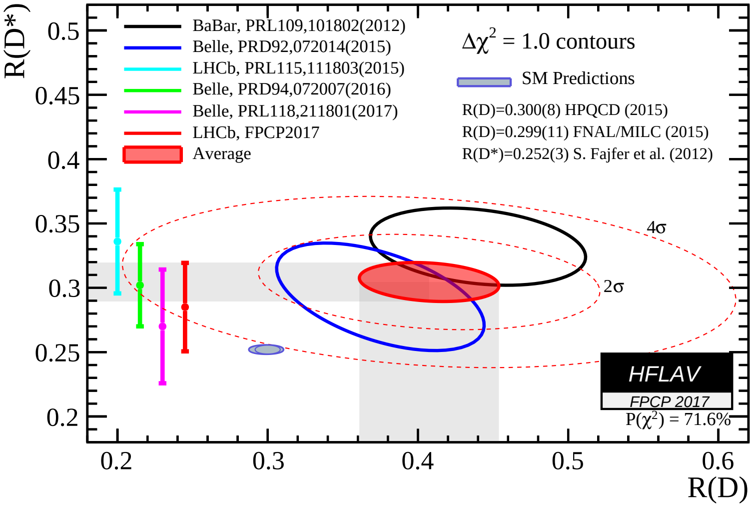
Outline

1. Intro & Motivation.
2. Summary of $B_{(s)} \to D_{(s)}^{(*)} l\nu$ lattice calculations.
3. B_c semileptonic decays.
    - Calculation details.
    - Results.
4. Summary & Future Work.

Brief reminder on lattice calculations

- Monte Carlo integration of discretised path integral, with lattice spacing a.
- Calculations must be done at multiple lattice spacings (at least two, preferably more) to assess (and remove) discretisation errors.
- Calculations often done with unphysically heavy pions in the sea (though physical mass ensembles are increasingly common). Extrapolation using multiple heavy pion masses is then needed.
- Heavy quarks require some sort of effective theory treatment, which becomes a source of systematic error. It is now (just) becoming possible to treat the b-quark relativistically (more on this later).

$R(D^{(*)})$ – Expt'l status

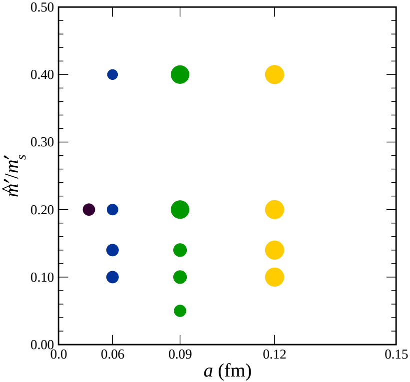


Summary of lattice $B_{(s)} \to D_{(s)}^{(*)}$

Currently results exist for

- $B_{(s)} \to D_{(s)}$ at all recoil (f_+ and f_0).
- $B \to D^*$ at zero recoil (non-zero recoil in progress).
- $B_s \to D_s^*$ at zero recoil (in progress).

$B \to D$ form factors FNAL/MILC [1503.07237]



- 14 MILC ensembles, 4 lattice spacings
- $\sim 600 - 2000$ configurations per ensemble ($\times 4$ time sources per configuration)
- Asqtad light quarks
- Fermilab b quarks
- $\mathcal{O}(a)$ improved current

$B \to D$ form factors

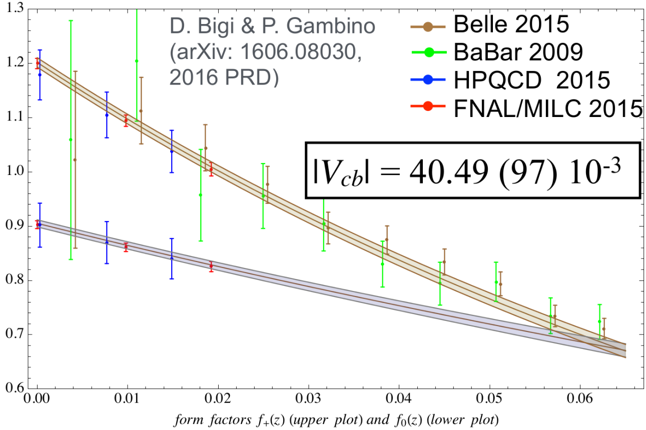
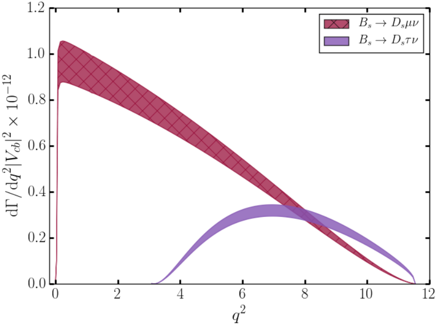


Figure courtesy A. El-Khadra

Combining data gives improved precision $R(D) = 0.299(3)$

$B_s \to D_s$ [1703.09728]

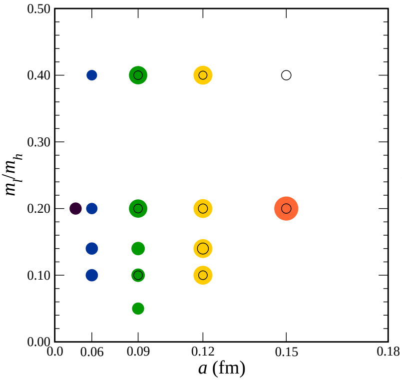
Recent result from HPQCD used NRQCD + improved staggered (asqtad) quarks to obtain $\frac{d\Gamma}{dq^2}$ over full q^2 range.

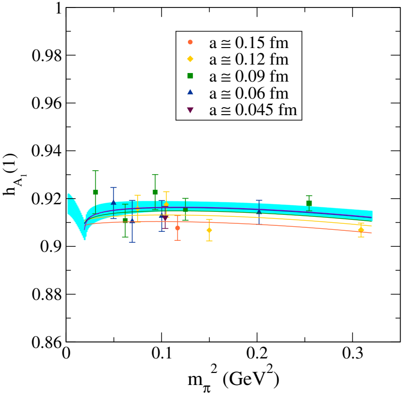


- $R(D_s) = 0.314(6)$
- Rel. q model result $R(D_s) = 0.274(20)$ [A. Bhol (2014)]

See also ETMC [1310.5238], FNAL/MILC [1202.6346], and HPQCD (HISQ) in progress [E. McLean]

$B \to D^*$ at zero recoil FNAL/MILC [1403.0635]





$B_{(s)} \to D^*_{(s)}$

Results at zero recoil from NRQCD.

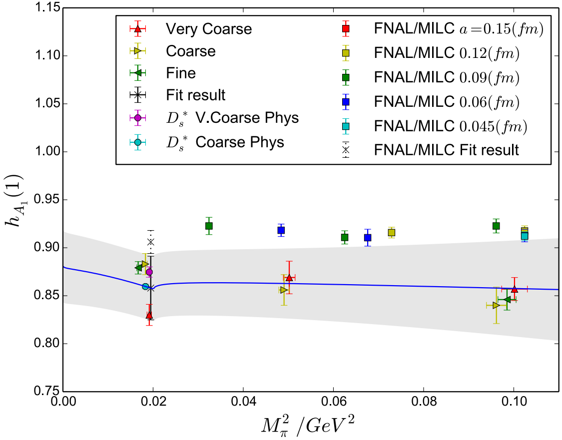
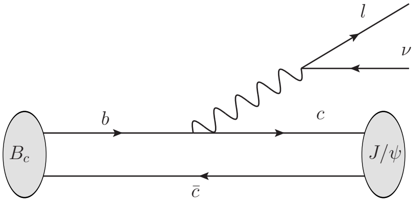


Figure courtesy Judd Harrison [1612.06716]

Calculations in progress

- $B_{(s)} \to D_{(s)}$
  - McLean (HPQCD) Lattice 2017 [1711.03487]
  - Witzel (RBC-/UKQCD) Lattice 2016 [1612.05112])
  - LANL/SNU
- $B \to D^*$
  - Vaquero Avilés-Casca (MILC) Lattice 2017 [1710.09817]
  - Park (LANL/SNU) Lattice 2017 [1711.01786]
  - Harrison (HPQCD) Lattice 2016 [1612.06716]
  - RBC/UKQCD
- $B_s \to D_s^*$
  - Harrison (HPQCD)

B_c semileptonic decays



B_c decays @ LHCb

- B_c mesons are being produced in sufficient quantity for detailed measurements at LHCb, yet there are no lattice simulations of B_c semileptonic decays needed for extracting $|V_{cb}|$ and for reliable SM prediction of $R(B_c \to J/\psi)$.

- Progress in simulating $B_c \to \eta_c \, l\nu$ and $B_c \to J/\psi \, l\nu$ semileptonic decays.
  - Technical progress in simulation of heavy quarks.
  - Results and discussion on impact for phenomenology.

c and b quarks on the lattice

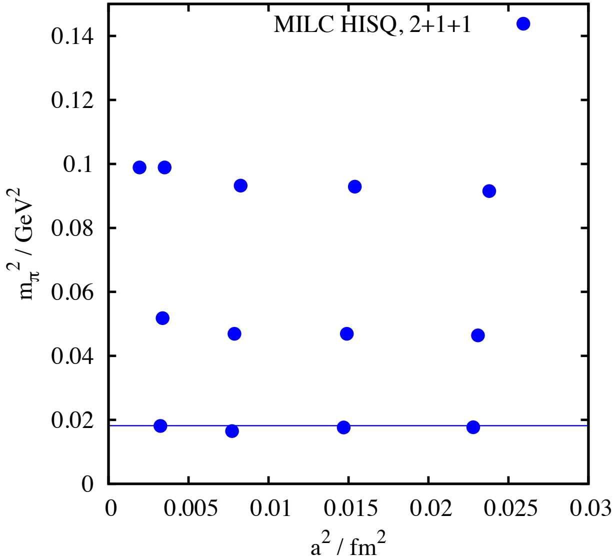
- Treatment of c and especially b quarks challenging in lattice simulations due to lattice artifacts which grow as $(am_q)^n$.

- HPQCD treats the charm quark relativistically using a highly improved staggered quark (HISQ) action.

- Generally one uses an effective theory framework to handle the b quark, here focus on NRQCD.

- HPQCD use two complementary approaches for b quarks:
  - Improved non-relativistic formalism (NRQCD) at m_b.
  - Highly improved relativistic action at small a, extrapolate $m_h \to m_b$.

MILC ensemble parameters [1004.0342,1212.4768]



DiRAC II computing

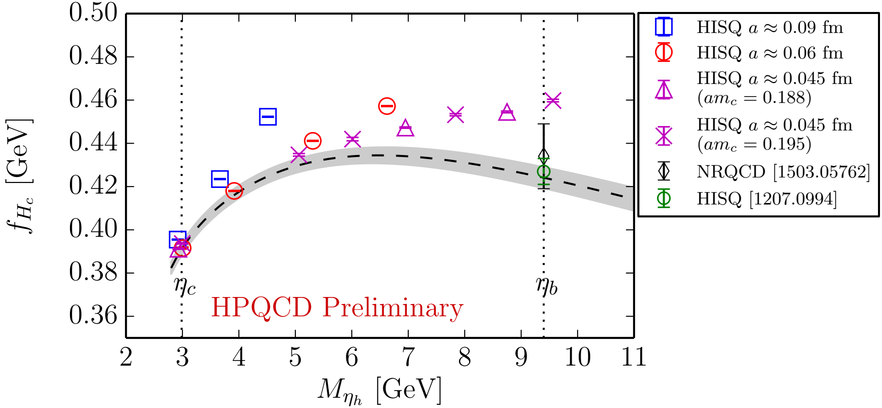
Quark propagator inversions carried out on the Darwin cluster at Cambridge.

Includes:

- 9600 Intel Sandy Bridge cores
- 2.6 GHz, 4 GB RAM/core
- 2 PB storage

Part of STFC's HPC facility for theoretical particle physics and astronomy.

Spanning c to b with HISQ



Semileptonic decays

- Study of $B_c \to \eta_c$, $B_c \to J/\psi$ decay matrix elements.
- We work in the frame where the B_c is at rest.
- The form factors which parametrise the matrix elements are functions of q^2, where q is the four-momentum transferred to the leptons.
  - $q^2_{\mathrm{max}} = (M - m)^2$, zero recoil of decay hadron.
  - $q^2 = 0$, maximum recoil of decay hadron.
- Matrix elements are determined by simultaneous fitting of three-point and two-point functions.

$B_c \to \eta_c$ and J/ψ

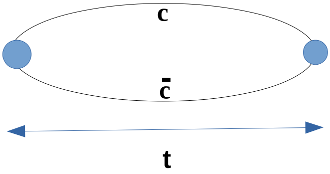
- All quarks are heavy (m_c or heavier) $\to$ good signal, can control calculation across full q^2 range, both in NRQCD and full relativistic calculation.

- NRQCD $b \to c$ transition currents of the form: $(1 + \alpha_s z + \cdots)(J_\mu^{(0)} + J_\mu^{(1)} + \cdots)$
  - Syst. error from the matching uncertainty appears in all $b \to c$ semileptonic transition calculations $B_{(s)} \to D_{(s)}^{(*)}$.
  - Can cross-check NRQCD systematics using the fully relativistic formulation.

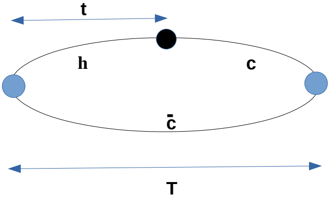
- $B_c \to \eta_c$ (pseudoscalar): $f_0(q^2)$, $f_+(q^2)$
  $B_c \to J/\psi$ (vector): $A_{0,1,2,3}(q^2)$ (three independent), $V(q^2)$

Semileptonic decays – meson correlators

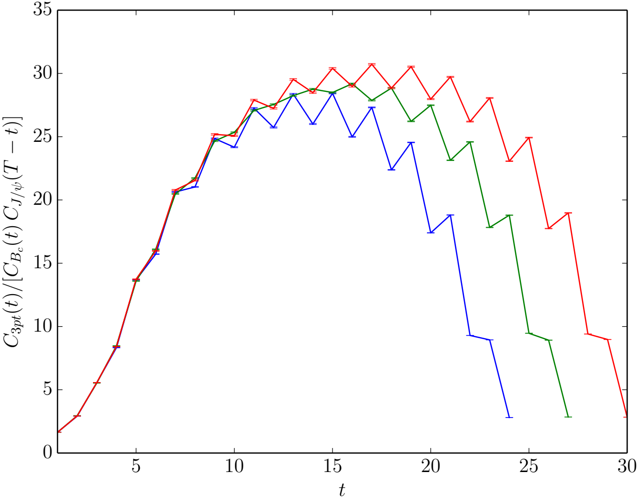
Two-point functions:



Three-point functions:



Semileptonic decays



$B_c \to \eta_c$ form factors

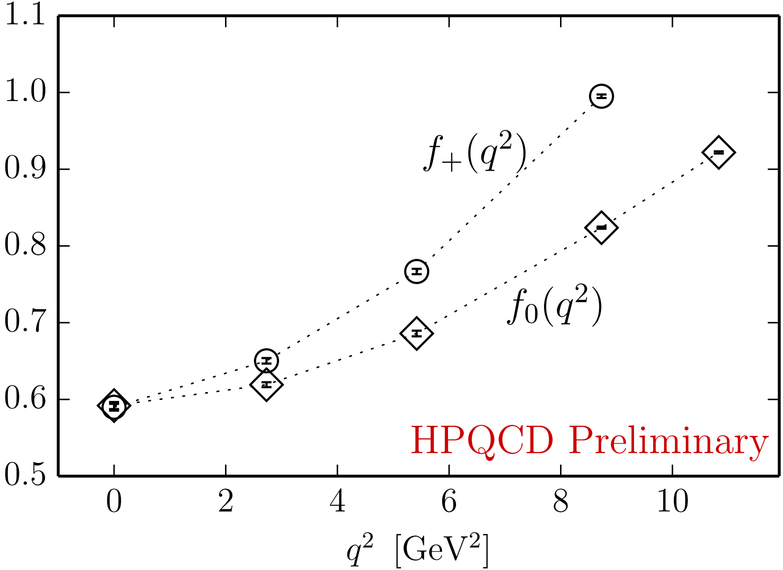
$$Z\langle\eta_c(p)|V^\mu|B_c(P)\rangle = f_+(q^2)\left[P^\mu + p^\mu - \frac{M^2-m^2}{q^2}q^\mu\right] + f_0(q^2)\,\frac{M^2-m^2}{q^2}\,q^\mu\,,$$

From PCVC,

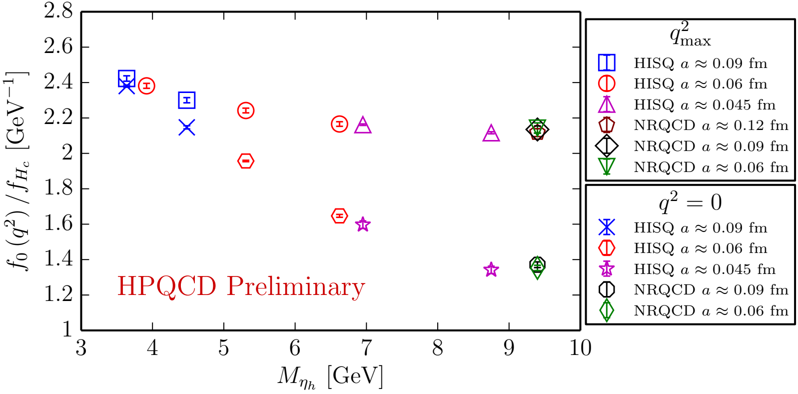
$$\langle\eta_c(p)|S|B_c(P)\rangle = \frac{M^2-m^2}{m_{b0}-m_{c0}}f_0(q^2)$$

Find Z by calculating both matrix elements at $q^2_{\rm max}$.

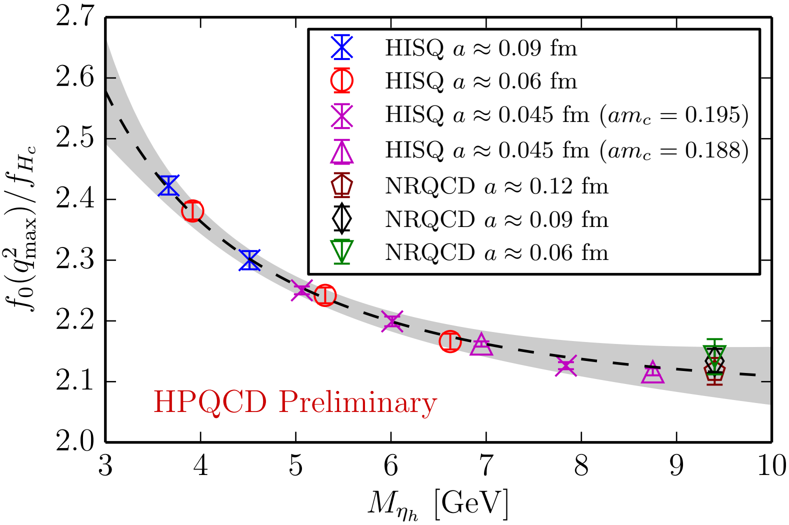
NRQCD $B_c \to \eta_c$ form factors



$f_0(q^2 = 0, \text{max})/f_{H_c}$ compared to NRQCD



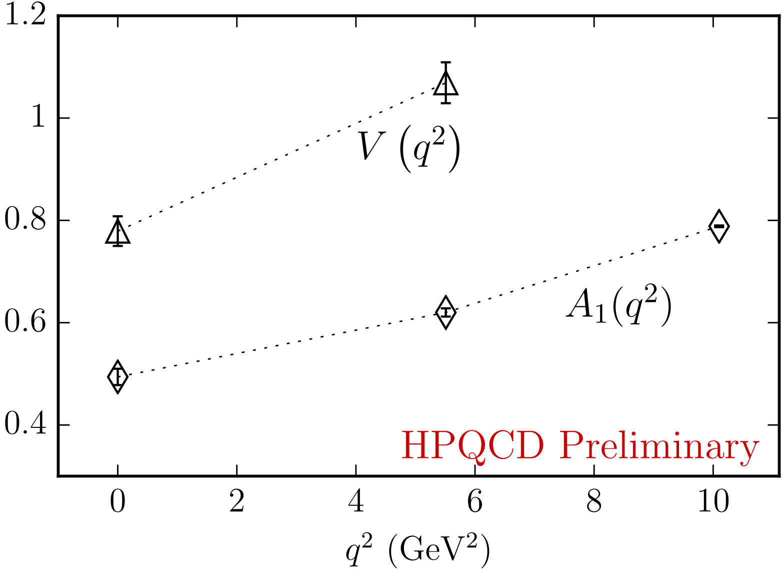
$f_0(q^2_{\mathsf{max}})/f_{H_c}$



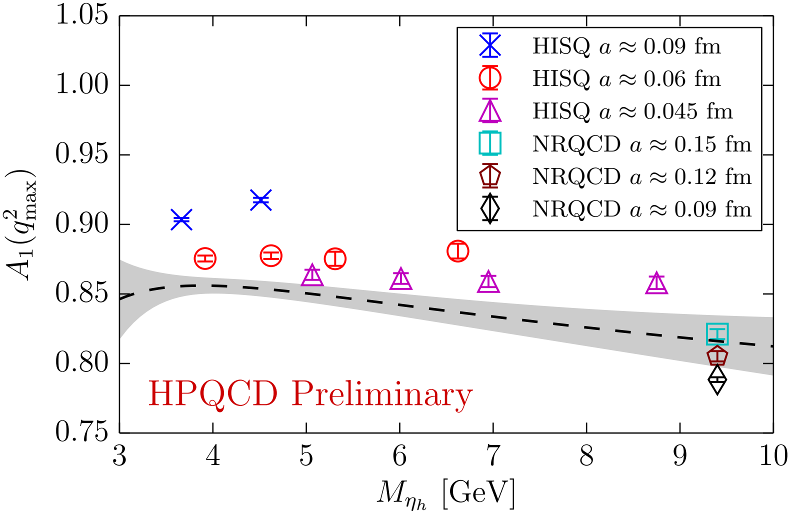
$B_c \to J/\psi$ form factors

$$\langle J/\psi(p,\varepsilon)|V^\mu - A^\mu|B_c(P)\rangle = \frac{2i\epsilon^{\mu\nu\rho\sigma}}{M+m}\varepsilon^*_\nu p_\rho P_\sigma\, V(q^2) - (M+m)\varepsilon^{*\mu}\, A_1(q^2) + \frac{\varepsilon^*\cdot q}{M+m}\,(p+P)^\mu\, A_2(q^2) + 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu\, A_3(q^2) - 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu\, A_0(q^2)$$

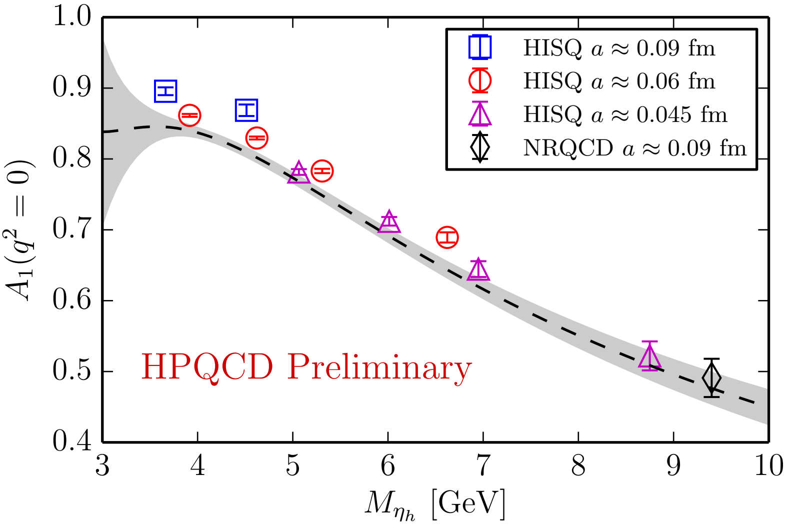
NRQCD $B_c \to J/\psi$ form factors



$A_1(q^2_{\mathrm{max}})$

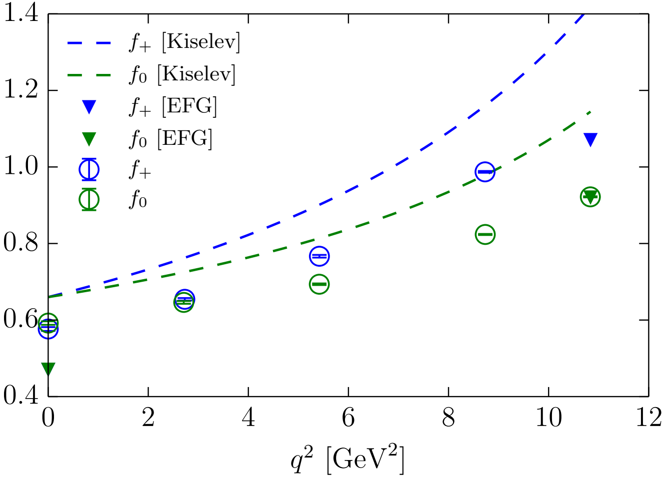


$A_1(q^2 = 0)$

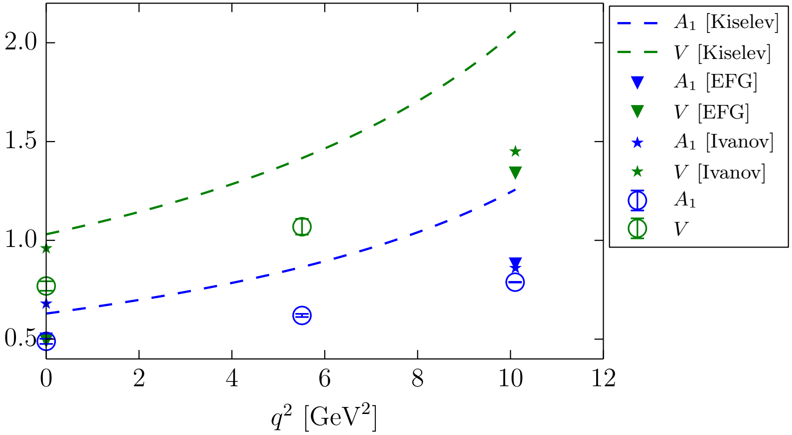


1.0
0.9
0.8
0.7
0.6
0.5
0.4
$A_1(q^2 = 0)$
HISQ $a \approx 0.09$ fm
HISQ $a \approx 0.06$ fm
HISQ $a \approx 0.045$ fm
NRQCD $a \approx 0.09$ fm
HPQCD Preliminary
3
4
5
6
7
8
9
10
M_{η_h} [GeV]

Comparisons $B_c \to \eta_c$ [hep-ph/0211021,0306306]



Comparisons $B_c \to J/\psi$ [hep-ph/0007169,0211021,0306306]



Summary & Future work

- Brief review of lattice results for $B_{(s)} \to D^{(*)}_{(s)}$.
- B_c semileptonic decay with relativistic b-quark.
  - Controlled calculation over full q^2 range.
  - Good agreement seen with NRQCD results.
- Outputs:
  - B_c to $J/\Psi \to$ new possible determination of $|V_{cb}|$.
  - Reliable SM prediction for $R(B_c \to J/\psi)$.
  - Improved understanding of NRQCD currents feeds into additional calculations (B to D, B to D^*, ...).
  - Expand relativistic calculations, e.g. to $B_s \to D^*_s$ at zero recoil.

Thank you!

Form factor extrapolation

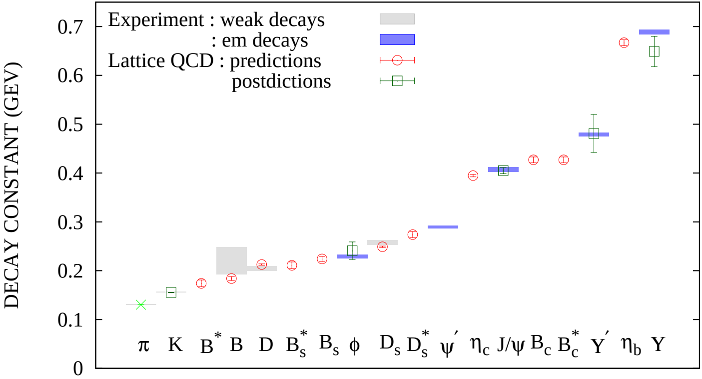
Generic HQET-inspired fit form given by

$$F(q^2, M_{\eta_h}, a^2) = A(q^2) \left(\frac{M_{\eta_h}}{M_0}\right)^b \times$$
$$\left[\sum_{ijkl} c_{ijkl}(q^2) \left(\frac{M_0}{M_{\eta_h}}\right)^i \left(\frac{am_c}{\pi}\right)^{2j} \left(\frac{am_h}{\pi}\right)^{2k} \left(\frac{a\Lambda_{\rm QCD}}{\pi}\right)^{2l}\right].$$

Continuum result evaluated at $m_h = m_b$:

$$F(q^2, M_{\eta_b}, 0) = A(q^2) \left(\frac{M_{\eta_b}}{M_0}\right)^b \sum_i c_{i000}(q^2) \left(\frac{M_0}{M_{\eta_h}}\right)^i$$

Decay constants – summary plot.



Tuning to the physical point

Bare quark masses are input parameters to lattice simulations. These parameters are tuned to reproduce physical quantities, e.g.

- $m_{ud0} \to m_\pi^2$
- $m_{s0} \to m_K^2$
- $m_{c0} \to m_{\eta_c}$

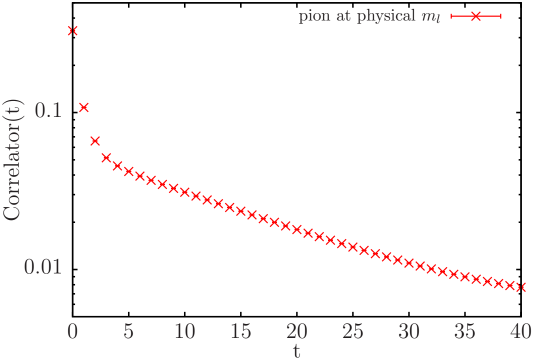
Tuning performed at multiple lattice spacings, defining a continuum trajectory for which $a^2 \to 0$ limit can be taken.

- Rest of physics is then prediction of QCD.
- Parameters can be varied away from physical values.. understand effect of quark mass, quantify systematics, etc.

Lattice QCD simulations - III

Energies and matrix elements are determined by fitting (sums of) exponentials.

$$\langle \pi(t)\ \pi^\dagger(0) \rangle \underset{\text{large } t}{\longrightarrow} \frac{|\langle 0|\pi|\pi\rangle|^2}{2m_\pi} e^{-m_\pi t} \propto f_\pi^2\, e^{-m_\pi t}$$



Simulating charm

Heavy quarks are challenging to simulate.

- Requires $am_0 < 1$ to keep discretization effects under control.
- Need large enough box to minimize finite-volume effects $\rightarrow N_{\text{site}}$ large.

These conditions can be satisfied by using a highly improved action (e.g. HISQ).

Staggered Quarks

Staggered quark action is defined in terms of one-component Grassman fields.

$$\mathcal{L}_{\text{naive}} = \bar{\chi}(x) \left[\eta^\mu(x) \Delta_\mu(U(x)) + m \right] \chi(x)$$

where

$$\eta^\mu(x) = (-1)^{\sum_{\nu<\mu} x^\nu} \, .$$

- $2a$ translation symmetry
- Represents four identical "tastes" of quark in the continuum.
- Reduce to one dynamical quark via "rooting" procedure.

HISQ Action

[hep-lat/0610092]

$$\Delta_\mu^{\mathrm{HISQ}} = \Delta_\mu(W) - \frac{a^2}{6}(1+\epsilon)\Delta_\mu^3(X)$$

- Removes all $\mathcal{O}(a^2)$ discretisation effects.
- Highly suppressed one-loop taste exchange errors.
- No tree-level $\mathcal{O}((am)^4)$ errors to leading order in v/c.