Naturalness

Two mass terms used in the literature

$$V(H) = -\frac{m_h^2}{2}|H|^2 + \frac{\lambda}{4}|H|^4$$

$$m_h^2 = \lambda v^2$$

Higgs mass prediction is obtained from it as v=174 GeV is fixed from W,Z mass

This is the source of fine tuning to keep the weak scale

Supersymmetry

stop mass

$$-\frac{m_h^2}{2} = m_{H_u}^2 + \mu^2 \longrightarrow m_{H_u}^2|_\Lambda - c\frac{m_{\rm SUSY}^2}{16\pi^2}\log\left(\frac{\Lambda}{m_{\rm SUSY}}\right)$$

$$\lambda v^2 = M_Z^2 \qquad \lambda v^2 = M_Z^2 + c y_t^2 \frac{m_t^2}{16\pi^2}\log\left(\frac{m_{\rm SUSY}}{m_t}\right)$$

Composite Higgs

top partner

$$-\frac{m_h^2}{2} = -a\frac{M^2}{16\pi^2}\log\left(\frac{\Lambda}{M}\right)$$

spontaneous symmetry breaking scale

$$\lambda v^2 = b\left(\frac{v}{f}\right)^2\frac{M^2}{16\pi^2}\log\left(\frac{\Lambda}{M}\right) \qquad M = g_* f$$

$$\lambda v^2 = b\frac{g_*^2}{16\pi^2}v^2\log\left(\frac{\Lambda}{M}\right)$$

Supersymmetry with no fine tuning predicts

$$v \sim \frac{m_{\rm SUSY}}{4\pi\lambda^{\frac{1}{2}}}$$

$$\text{fine tuning} \sim \frac{m_h^2}{m_{\rm SUSY}^2}\frac{5}{L}$$

$m_{\rm SUSY} = 1$ TeV → $10^{-2} \sim 10^{-4}$ ← $m_{\rm SUSY} = 10$ TeV

Composite Higgs with no fine tuning predicts

$$v \sim \frac{M}{4\pi\lambda^{\frac{1}{2}}}$$

$$\text{fine tuning} \sim \frac{m_h^2}{M^2}\frac{5}{L}$$

$M \geq 1$ TeV
fine tuning : a few %

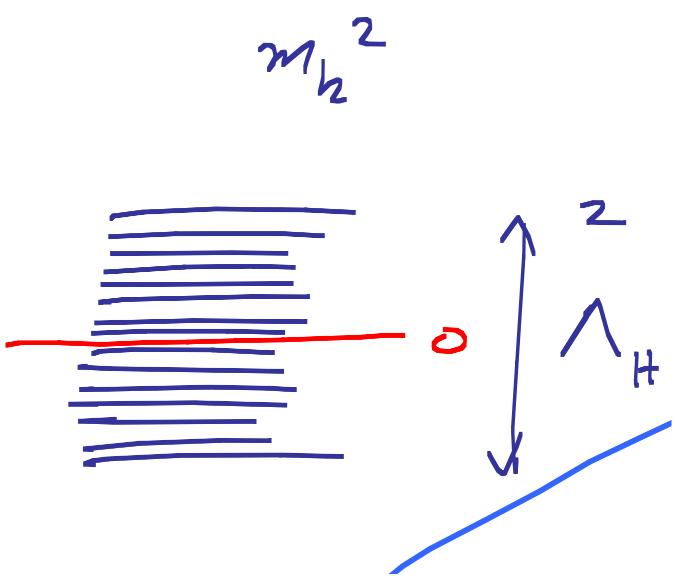
MSSM : stop at 5~10 TeV for H(125)

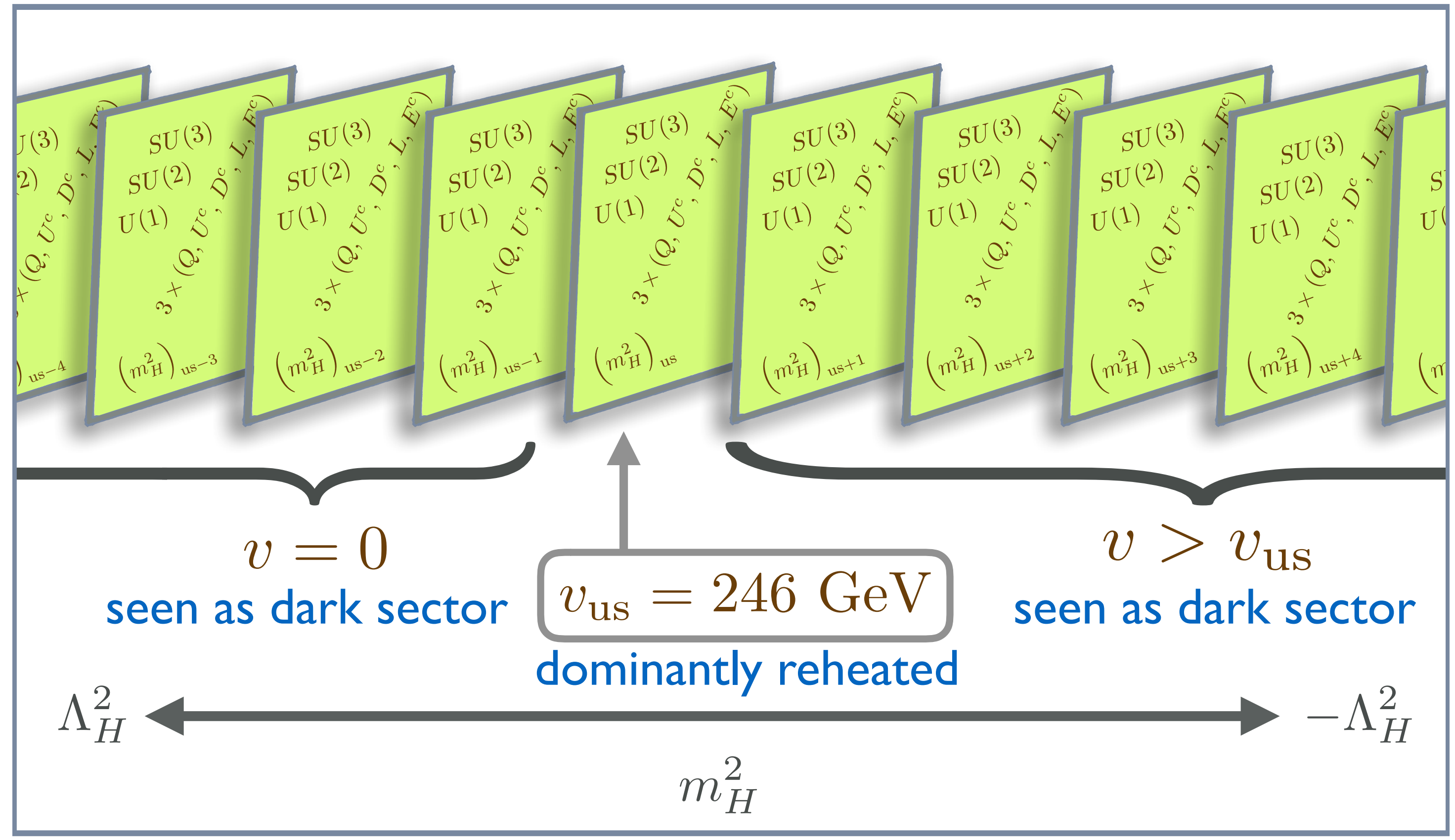
$m_h \sim m_{\rm SUSY}$ is violated

Composite Higgs : v ~ f relation is violated

No motivated models are in good shape now.

N copy of SM





$$(m_H^2)_i = -\frac{\Lambda_H^2}{N}(2i + r), \qquad -\frac{N}{2} \leq i \leq \frac{N}{2}$$

Arkani-Hamed Cohen D'agnolo Hook HDK Pinner, PRL (2016)

$m_H^2 < 0$	$m_H^2 > 0$
massless photons	massless photons
W/Z mass ~ v	W/Z mass ~ QCD ~ 100 MeV
fermion mass ~ v	fermion mass ~ extremely light
neutrino mass ~ v or v^2	sphaleron process is active at or below baryon mass :baryons converted to leptons
dark baryon to dark atom :double disc dark matter	

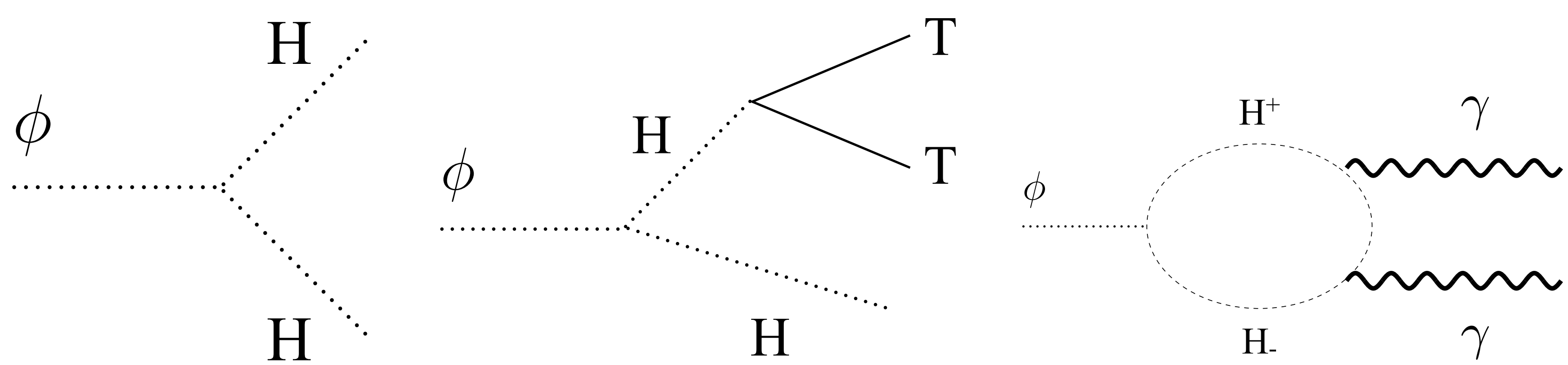
scalar reheat
$A\phi H^\dagger H$

fermion reheat
$\lambda S L H$

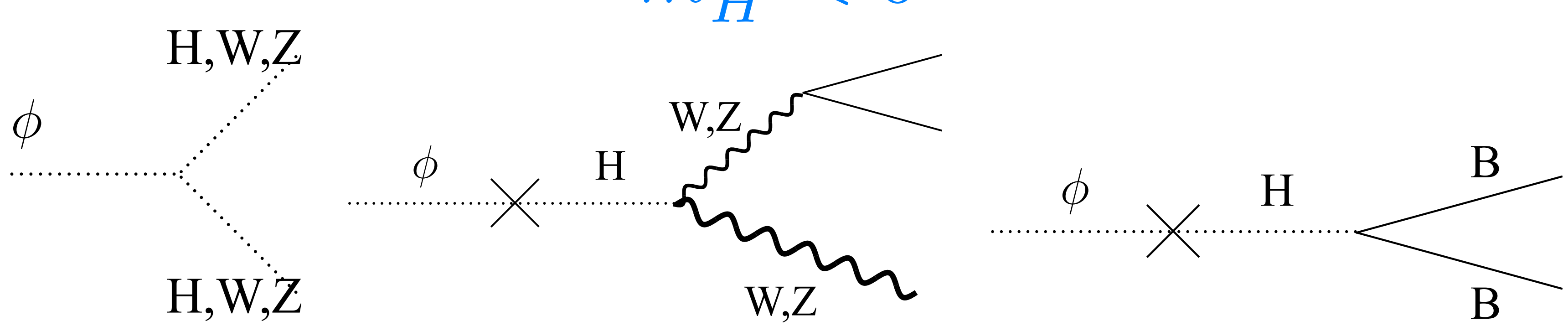
Arkani-Hamed Cohen D'agnolo
Hook HDK Pinner, PRL (2016)

population of the sectors

$$m_H^2 > 0$$



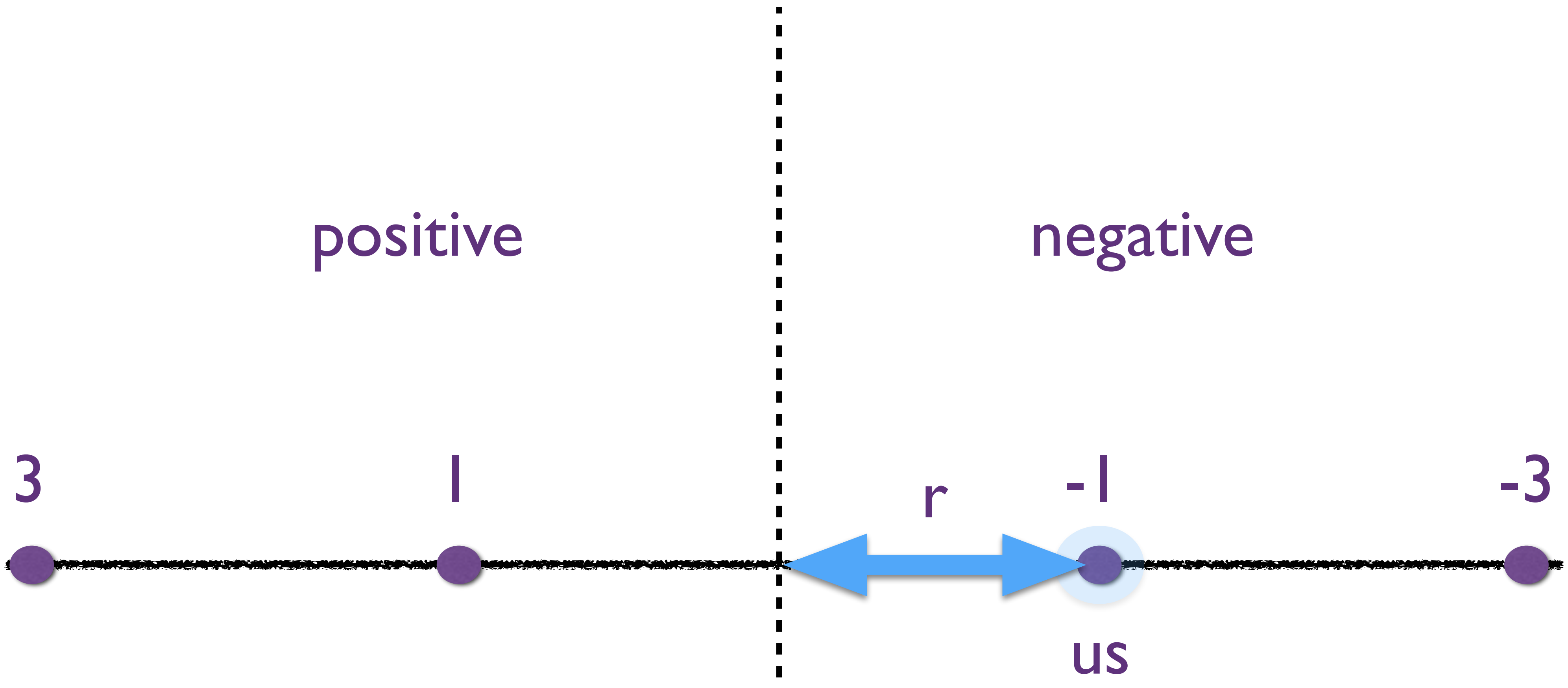
$$m_H^2 < 0$$



$$\mathcal{L}_\phi^{\langle H\rangle\neq 0} \supset \mathcal{C}_1^\phi \, a \, y_q \frac{v}{m_h^2} \phi \, q \, q^c \, ; \longrightarrow \frac{1}{i} \quad \text{for ith sector}$$

$$\mathcal{L}_\phi^{\langle H\rangle = 0} \supset \mathcal{C}_3^\phi \, a \, \frac{g^2}{16\,\pi^2} \frac{1}{m_H^2} \, \phi \, W_{\mu\nu} W^{\mu\nu} \, , \longrightarrow \frac{1}{i^2}$$

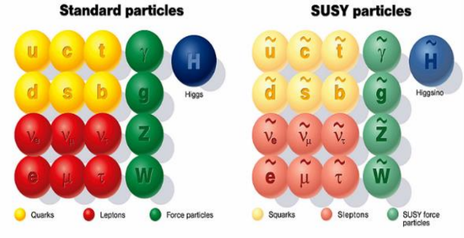
If we are accidentally close to zero by r,



$$(m_H^2)_i = -\frac{\Lambda_H^2}{N}(2\,i + r), \qquad -\frac{N}{2} \le i \le \frac{N}{2}$$

phi more preferentially decays to us.

SUSY as a solution to the hierarchy problem



$$: \quad \delta m_H^2 \sim M_{\rm Planck}^2 \quad \Rightarrow \quad \delta m_H^2 \sim m_{\rm SUSY}^2$$

$$16\pi^2 \frac{d}{d\ln Q} m_{H_u}^2 = 6 y_t^2 \left(m_{H_u}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 \right) - 6 g_2^2 M_{\tilde{W}}^2 + \ldots$$

$$16\pi^2 \frac{d}{d\ln Q} m_{\tilde{t}_L}^2 = 2 y_t^2 \left(m_{H_u}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 \right) - \frac{32}{3} g_3^2 M_{\tilde{g}}^2 - 6 g_2^2 M_{\tilde{W}}^2 + \ldots$$

$$16\pi^2 \frac{d}{d\ln Q} m_{\tilde{t}_R}^2 = 4 y_t^2 \left(m_{H_u}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 \right) - \frac{32}{3} g_3^2 M_{\tilde{g}}^2 + \ldots$$

Weak scale vs SUSY scale:

$$\frac{1}{2} M_Z^2 \simeq -|\mu|^2 - m_{H_u}^2 + \frac{m_{H_d}^2}{\tan^2\beta}$$

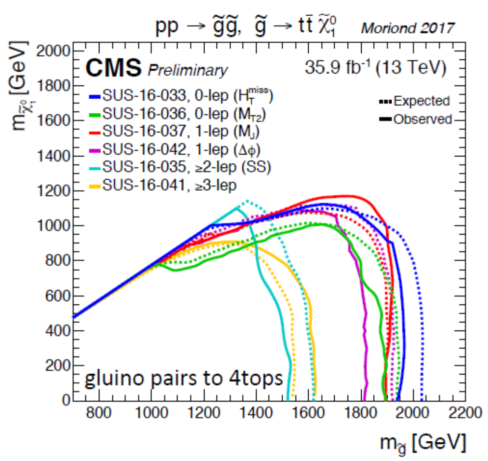
$$\simeq -0.86\bar{\mu}^2 - 0.65\bar{m}_{H_u}^2 + 0.35\bar{m}_{\tilde{t}_L}^2 + 0.3\bar{m}_{\tilde{t}_R}^2 + 1.5\bar{M}_{\tilde{g}}^2 + 0.1\bar{M}_{\tilde{g}}\bar{M}_{\tilde{W}} - 0.2\bar{M}_{\tilde{W}}^2 + \ldots$$

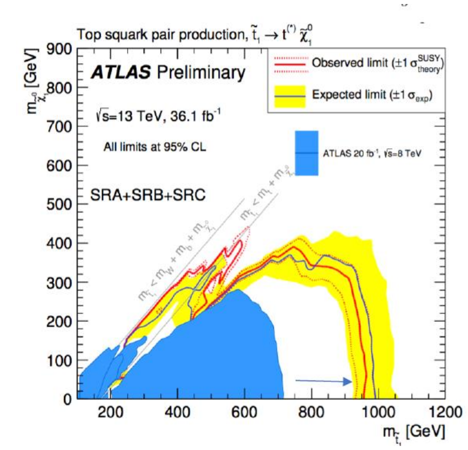
$$\left(\bar{m}_I = \text{superpartner masses at } M_{\rm GUT} \right)$$

Natural SUSY scenario:

Higgsino, stops, and gluinos are the closest friends of the Higgs boson, so the their masses should be near the weak scale to avoid fine-tuning.

Light Higgsino near the weak scale is still on open possibility, howev





Fine-tuning measure for naturalness: Ellis, Enquist, Nanopoulos, Zwirner; Barbieri, Giudice

$$\Delta = {\rm Max}\left(\left| \frac{\partial \ln M_Z^2}{\partial \ln \bar{m}_I^2} \right| \right) \gtrsim 200$$

Natural SUSY appears to be in trouble, however we should keep in mind that there is no clear-cut criterion for naturalness.

We may see the problem in different perspective:

What would be the pattern (or correlation) of SUSY-breaking masses at M_{GUT}, for which the weak scale is least sensitive to the SUSY scale?

This approach is something similar to the old Veltman condition on the SM, and relies on the assumption that underlying SUSY breaking dynamics provides such correlation among the SUSY-breaking masses.

cf: In string-motivated SUSY-breaking models, quite often the ratios among SUSY-breaking masses are determined by discrete parameters such as moduli weights, quantized fluxes, and group theory coefficients.

Naturalness might be saved with a specific pattern of SUSY mases, leading to a cancellation among the contributions to the weak scale:

$$\frac{1}{2}M_Z^2 = -0.86\bar{\mu}^2 - 0.65\bar{m}_{H_u}^2 + 0.35\bar{m}_{\tilde{t}_L}^2 + 0.3\bar{m}_{\tilde{t}_R}^2 + 1.5\bar{M}_{\tilde{g}}^2 + 0.1\bar{M}_{\tilde{g}}\bar{M}_{\tilde{W}} - 0.2\bar{M}_{\tilde{W}}^2 + \ldots$$
$$= -0.86\bar{\mu}^2 + \mathcal{O}\left(\frac{1}{16\pi^2}\bar{m}_{\rm SUSY}^2\right)$$

➔ light Higgsinos, heavy stops and gluinos near the current experimental bound, and possibly some other testable predictions

Some examples:

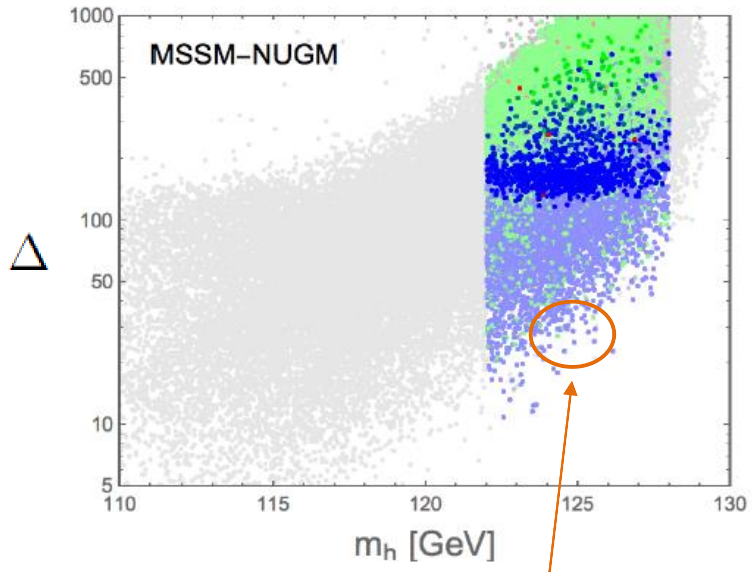
* Scalar & gaugino focus points: Feng, Matchev, Moroi; Abe, Kobayashi, Omura; Horton, Ross

$$\bar{m}_{H_u}^2 = \bar{m}_{\tilde{t}_L}^2 = \bar{m}_{\tilde{t}_R}^2 = \ldots, \quad \bar{M}_{\tilde{W}} \simeq 3\bar{M}_{\tilde{g}} \quad (\Leftrightarrow \quad M_{\tilde{g}} \simeq M_{\tilde{W}} \text{ at TeV scales})$$

* TeV scale mirage mediation (= mixed string-moduli & anomaly mediation)

$$M_{\tilde{g}} \simeq M_{\tilde{W}} \simeq M_{\tilde{B}} \simeq \sqrt{m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2} \quad \text{at TeV scales}$$

KC, Jeong, Kobayashi, Okumura; Kitano, Nomura



Ross, Schmidt-Hoberg, Staub '17

Reasonably natural SUSY $(\Delta \lesssim \text{few} \times 10)$ is possible with

* Higgsino near the weak scale,
* stops and gluinos near the present experimental bound, with $M_{\tilde{g}} \simeq M_{\tilde{W}}$ at TeV scales

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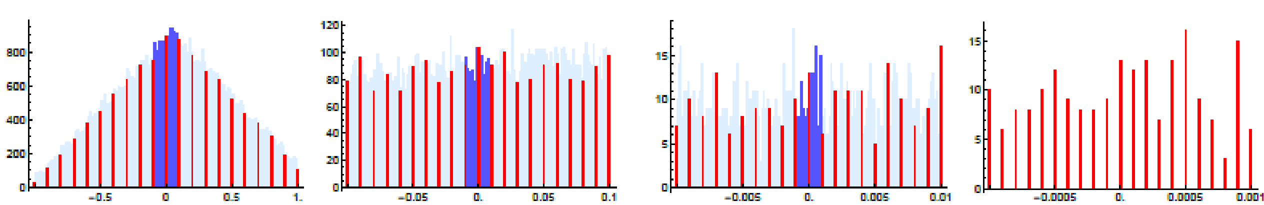
Naturalness might be saved with the complexity of SUSY models which involve many parameters contributing to the weak scale with different strength and different sign: Dermisek '16

$$\frac{1}{2}M_Z^2 = -0.86\bar{\mu}^2 - 0.65\bar{m}_{H_u}^2 + 0.35\bar{m}_{\tilde{t}_L}^2 + 0.3\bar{m}_{\tilde{t}_R}^2 + 1.5\bar{M}_{\tilde{g}}^2 + 0.1\bar{M}_{\tilde{g}}\bar{M}_{\tilde{W}} - 0.2\bar{M}_{\tilde{W}}^2 + \mathcal{O}(10^{-2}\bar{M}_{\tilde{B}}^2) + \mathcal{O}(10^{-3}\bar{m}_{\tilde{f}_L}^2) + \mathcal{O}(10^{-4}\text{Tr}(Y\bar{m}_{\tilde{f}}^2)) + \mathcal{O}((10^{-3}-10^{-5})\bar{m}_{\tilde{b},H_d}^2) + \mathcal{O}(10^{-6}m_{\tilde{c}}^2) + \mathcal{O}(10^{-11}\bar{m}_{\tilde{s}}^2)$$

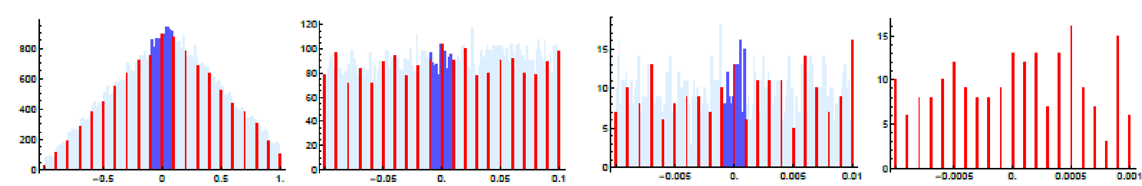
Assume all SUSY mass parameters have random distribution around a similar central value $\bar{m}$, within the range of $\mathcal{O}(\bar{m})$, and with a spacing of $\mathcal{O}(0.1\bar{m})$:

$$R[x] = x\Big(1 + \{0, \pm 0.1, \pm 0.2, \pm 0.3, \pm 0.4, \pm 0.5\}\Big)$$

Distribution of $X = -R[1] + R[1] + R[10^{-1}] + R[10^{-2}] + R[10^{-3}]$:



$$\frac{1}{2}M_Z^2 = -0.86\bar{\mu}^2 - 0.65\bar{m}_{H_u}^2 + 0.35\bar{m}_{\tilde{t}_L}^2 + 0.3\bar{m}_{\tilde{t}_R}^2 + 1.5\bar{M}_{\tilde{g}}^2 + 0.1\bar{M}_{\tilde{g}}\bar{M}_{\tilde{W}} - 0.2\bar{M}_{\tilde{W}}^2 + \mathcal{O}(10^{-2}\bar{M}_{\tilde{B}}^2) + \mathcal{O}(10^{-3}\bar{m}_{\tilde{f}_L}^2) + \mathcal{O}(10^{-4}\text{Tr}(Y\bar{m}_{\tilde{f}}^2)) + \mathcal{O}((10^{-3}-10^{-5})\bar{m}_{\tilde{b},H_d}^2) + \mathcal{O}(10^{-6}m_{\tilde{c}}^2) + \mathcal{O}(10^{-11}\bar{m}_{\tilde{s}}^2)$$



Any value of M_Z in the range $\left[10^{-3}M_{\text{SUSY}}, M_{\text{SUSY}}\right]$ is equally probable.

SUSY anywhere in the range $[M_Z, 100\,\text{TeV}]$ is equally natural.

Dermisek '16