Calculating Pull

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with Simone Marzani, Chang Wu
to appear soon

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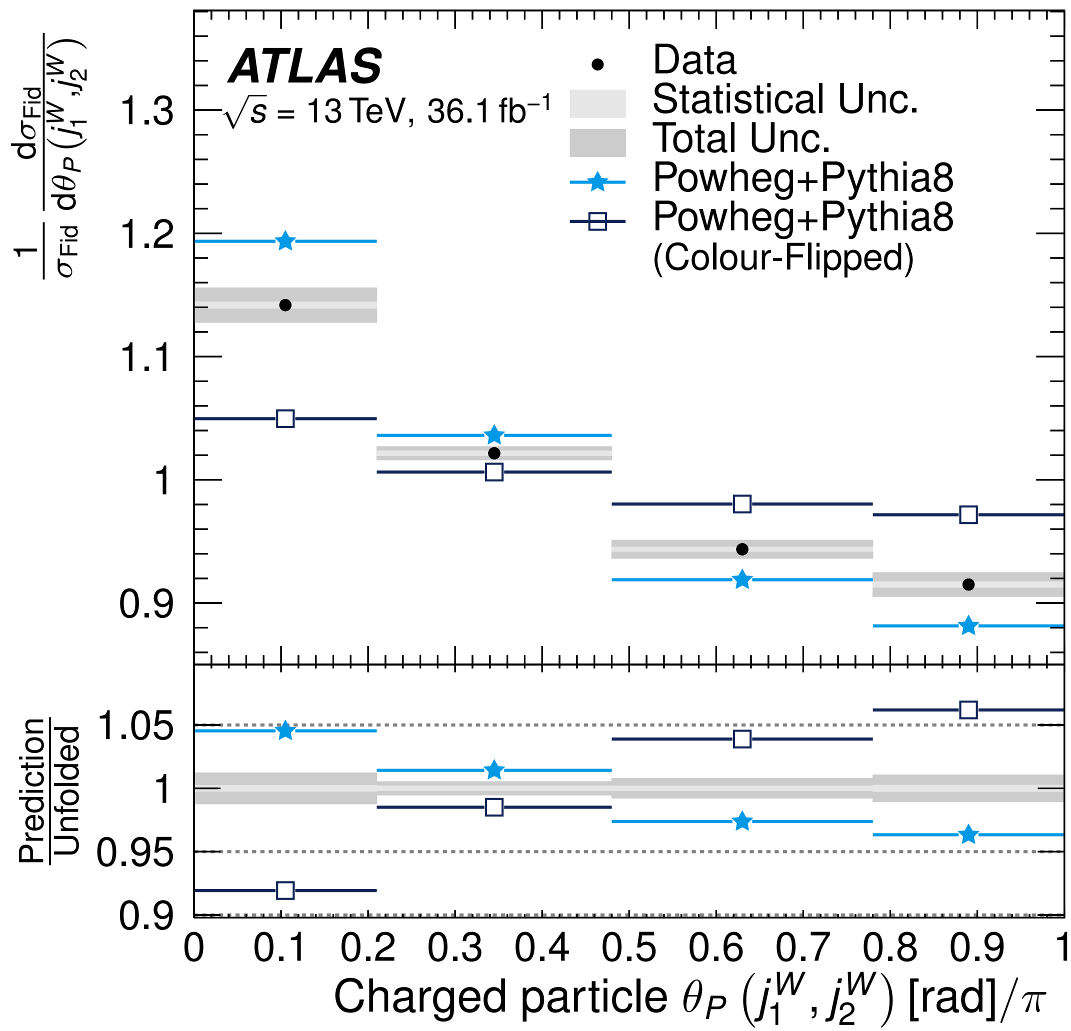
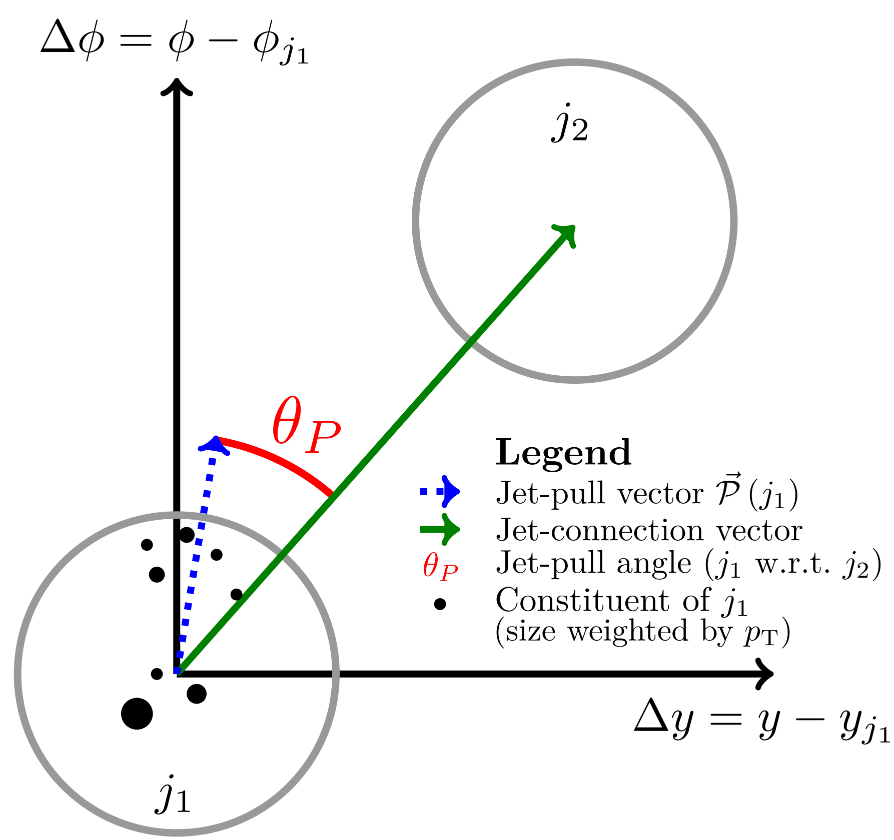
Why Pull?

Sensitive to Color Flow

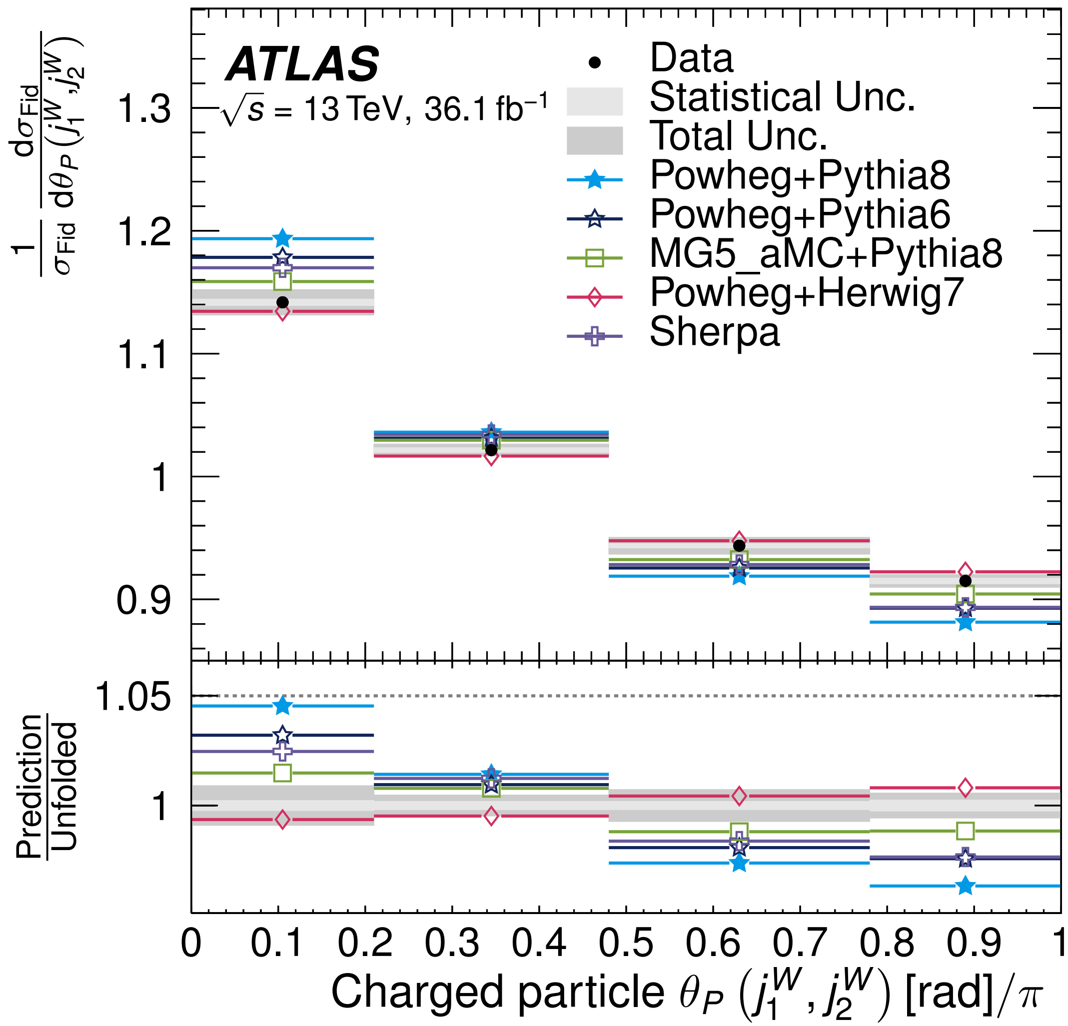


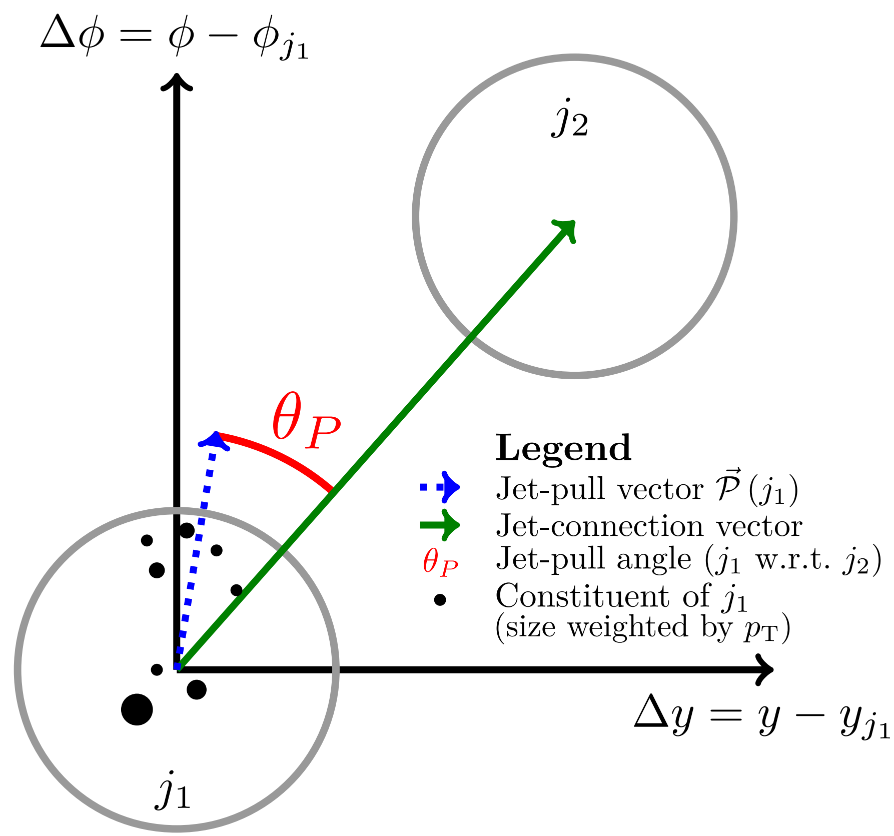


arXiv: 1001.5027, 1805.02935
see also: 1506.05629

Why Pull?

Range of Monte Carlo predictions





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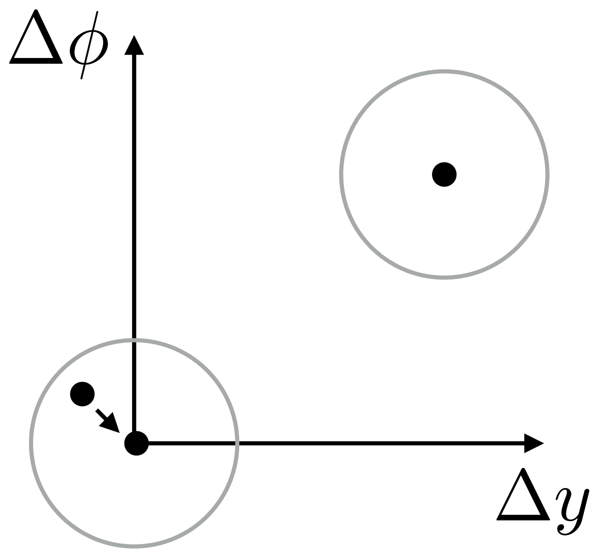
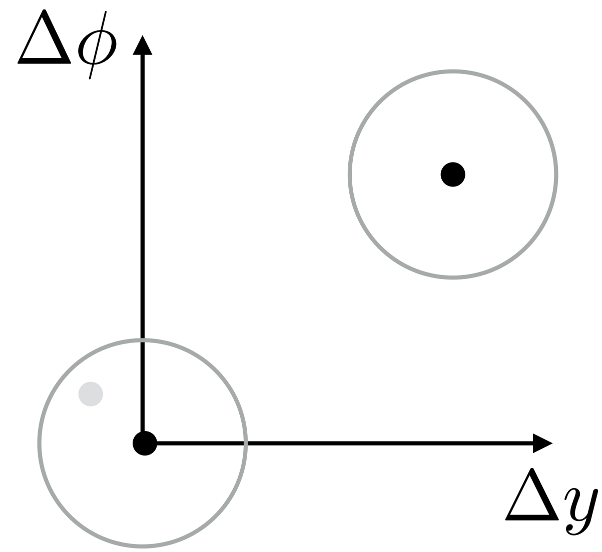
Challenges with Pull Angle

Not infrared or collinear safe



Collinear limit: uniform θ_P

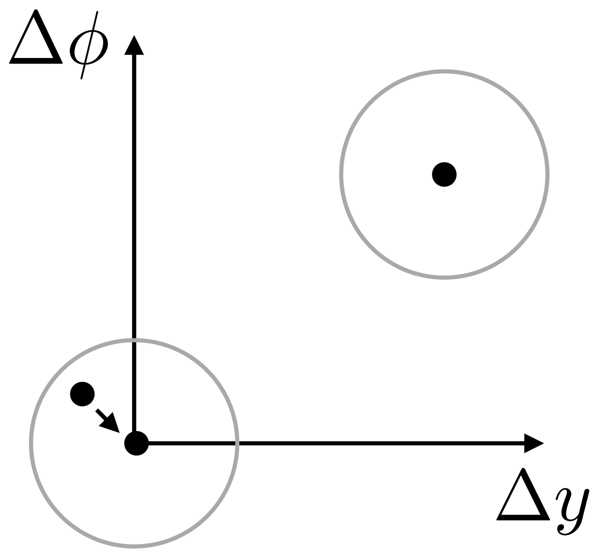


Soft limit: set by dipole

Nevertheless, it is Sudakov safe

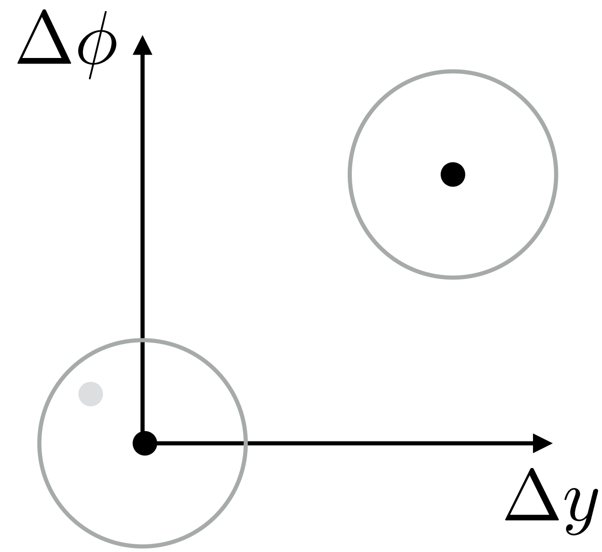
Challenges with Pull Angle

Not infrared or collinear safe



Collinear limit: uniform θ_P

Magnitude of pull vanishes



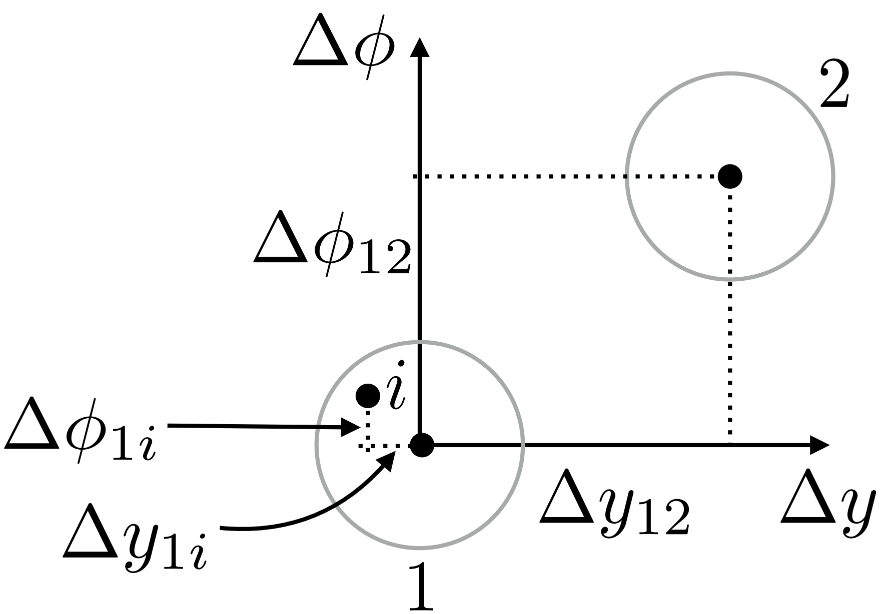
Soft limit: set by dipole

Magnitude of pull vanishes

Resolution: Measure pull angle **and** magnitude

Defining Pull

Original/ATLAS Definition



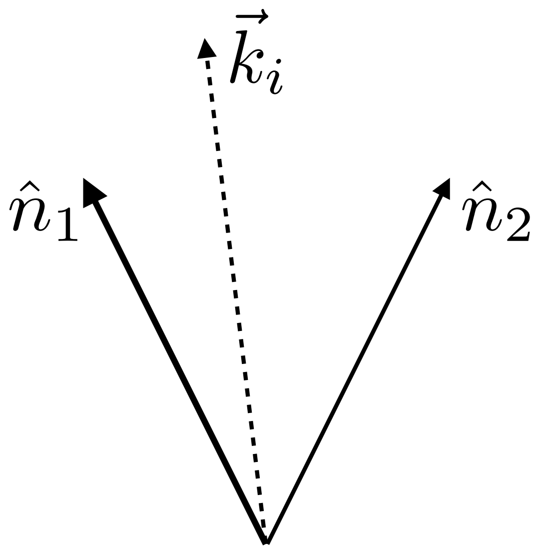
Hadron collider coordinates

Defined from 2D detector projection

$$\cos\theta_P = \frac{(\Delta y_{12}, \Delta\phi_{12})}{\sqrt{\Delta y_{12}^2 + \Delta\phi_{12}^2}} \cdot \frac{\sum_{i\in J} \frac{p_{Ti}\sqrt{\Delta y_{1i}^2 + \Delta\phi_{1i}^2}}{p_{TJ}} (\Delta y_{1i}, \Delta\phi_{1i})}{\left| \sum_{i\in J} \frac{p_{Ti}\sqrt{\Delta y_{1i}^2 + \Delta\phi_{1i}^2}}{p_{TJ}} (\Delta y_{1i}, \Delta\phi_{1i}) \right|}$$

Defining Pull

Our Calculational Definition



e+e- Coordinates (spherical)

Equivalent to ATLAS definition in the collinear limit

$$\cos\theta_P = \frac{(\hat{n}_1 \times \hat{n}_2)}{|\hat{n}_1 \times \hat{n}_2|} \cdot \frac{\sum_{i\in J} \frac{E_i}{E_J}\left[1-\left(\frac{\hat{n}_1\cdot\vec{k}_i}{E_i}\right)^2\right]\frac{(\hat{n}_1\times\vec{k}_i)}{|\hat{n}_1\times\vec{k}_i|}}{\left|\sum_{i\in J} \frac{E_i}{E_J}\left[1-\left(\frac{\hat{n}_1\cdot\vec{k}_i}{E_i}\right)^2\right]\frac{(\hat{n}_1\times\vec{k}_i)}{|\hat{n}_1\times\vec{k}_i|}\right|}$$

Calculation Technique

Pair IRC safe (magnitude) with IRC unsafe (angle)

arXiv:1502.01719

IRC unsafe ⟶ $$\frac{d\sigma}{d\theta_P} \equiv \int dt \, \frac{d^2\sigma}{dt\, d\theta_P}$$ ⟵ IRC safe

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$$= \int dt\, \frac{d\sigma}{dt}\, \frac{d^2\sigma(t)}{d\theta_P}$$

Conditional xsec (safe for $t > 0$)

Calculation Technique

Pair IRC safe (magnitude) with IRC unsafe (angle)

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IRC unsafe $\longrightarrow$ $$\frac{d\sigma}{d\theta_P} \equiv \int dt\, \frac{d^2\sigma}{dt\, d\theta_P}$$ $\longleftarrow$ IRC safe

$$= \int dt\, \frac{d\sigma}{dt}\, \frac{d^2\sigma(t)}{d\theta_P}$$

Conditional xsec
(safe for $t > 0$)

$$\approx \int dt\, \frac{d\sigma^{\mathrm{resum}}}{dt}\, \frac{d^2\sigma(t)^{\mathrm{fo}}}{d\theta_P}$$

Completely finite
(exponential suppression at $t = 0$)

Calculation Technique

Pair IRC safe (magnitude) with IRC unsafe (angle)

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$$\frac{d\sigma}{d\theta_P} \approx \int dt \, \frac{d\sigma^{\text{resum}}}{dt} \, \frac{d^2\sigma(t)^{\text{fo}}}{d\theta_P}$$

$$= \int dt \, \frac{\frac{d\sigma^{\text{resum}}}{dt}}{\frac{d\sigma^{\text{fo}}}{dt}} \, \frac{d^2\sigma^{\text{fo}}}{dt\, d\theta_P}$$

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$$\frac{d\sigma}{d\theta_P} \approx \int dt \, \frac{d\sigma^{\text{resum}}}{dt} \, \frac{d^2\sigma(t)^{\text{fo}}}{d\theta_P}$$

$$= \int dt \, \frac{\frac{d\sigma^{\text{resum}}}{dt}}{\frac{d\sigma^{\text{fo}}}{dt}} \, \frac{d^2\sigma^{\text{fo}}}{dt\, d\theta_P}$$

$$= \int dt \, \Delta_{\text{Sud}}(t) \, \frac{d^2\sigma^{\text{fo}}}{dt\, d\theta_P}$$

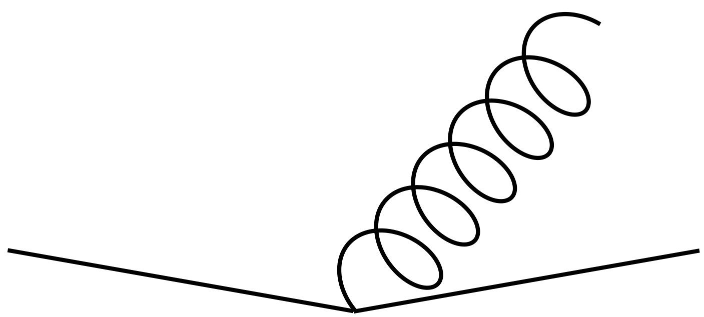
Need to calculate the fixed-order double diff'l cross section

Leading-Order Calculation

How to do the calculation (briefly)

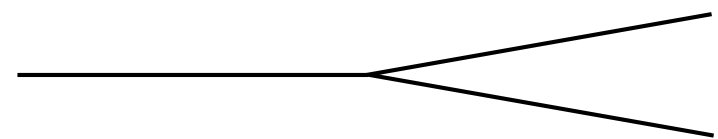
$$\frac{d^2\sigma^{t\ll 1}}{dt\,d\theta_P} = S(t,\theta_P,\mu) + J(t,\theta_P,\mu)$$

$S(t,\theta_P,\mu)$



Eikonal matrix element

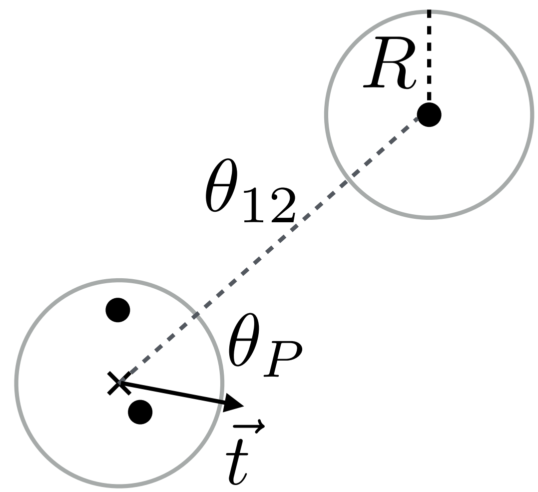
$J(t,\theta_P,\mu)$



Collinear splitting functions

Divergences regulated by μ

Leading-Order Calculation

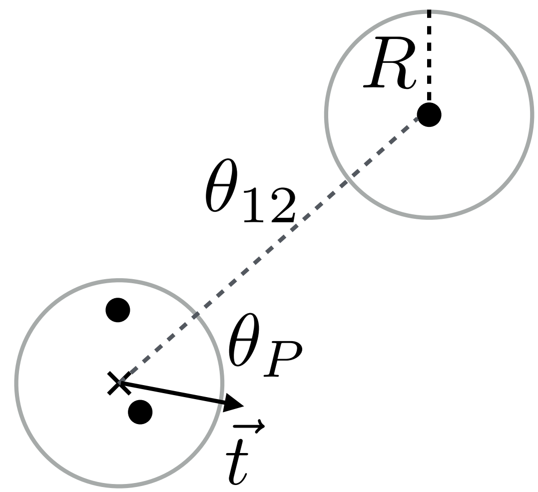


Result:

$$\frac{d^2\sigma^{t\ll 1}}{dt\,d\theta_P} = \frac{\alpha_s}{\pi^2}\frac{C_F}{t}\left[\log\frac{4\tan^2\frac{R}{2}}{t} - \frac{3}{4}\right.$$

$$\left. +2\cot\theta_P\;\;\tan^{-1}\frac{\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\sin\theta_P}{1-\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\cos\theta_P} - \log\left(1+\frac{\tan^2\frac{R}{2}}{\tan^2\frac{\theta_{12}}{2}} - 2\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\cos\theta_P\right)\right]$$

Leading-Order Calculation



Independent of pull angle

$$\frac{d^2\sigma^{t\ll 1}}{dt\, d\theta_P} = \frac{\alpha_s}{\pi^2}\frac{C_F}{t}\left[\log\frac{4\tan^2\frac{R}{2}}{t} - \frac{3}{4} + 2\cot\theta_P\;\tan^{-1}\frac{\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\sin\theta_P}{1-\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\cos\theta_P} - \log\left(1+\frac{\tan^2\frac{R}{2}}{\tan^2\frac{\theta_{12}}{2}} - 2\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\cos\theta_P\right)\right]$$

Depends on pull angle

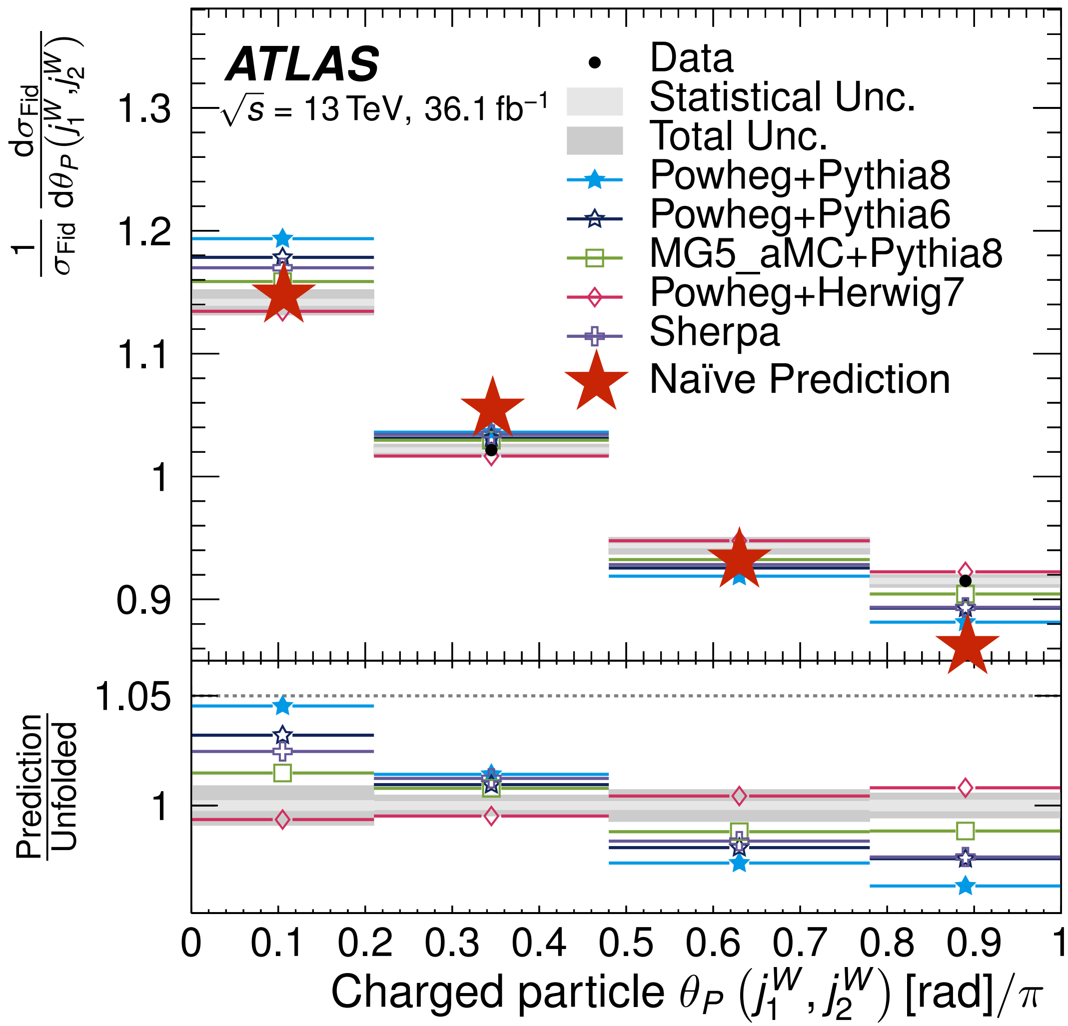
Sudakov factor only depends on t

Can only affect the relative size of these two terms

Shape is determined by the second term!

Comparison to our (first) prediction

Inclusive hadronic tops, $R = 0.4$, $\theta_{12} \approx 2$



Steeper than data

Shape will change with:

Running coupling

Soft wide angle is more likely than hard collinear

Convolving with W boost

θ_{12} isn't fixed
Higher boosts = smaller θ_{12}
= closer subjets

So, why isn't our paper published?

Things left to do:

Full double differential resummation:

$$\frac{d^2\sigma^{\text{resum}}}{d\vec{t}} = H \int d^2\vec{t}_s \, d^2\vec{t}_s \, \delta^{(2)}(\vec{t} - \vec{t}_s - \vec{t}_c) S(\vec{t}_s, \mu) J(\vec{t}_c, \mu)$$

Accomplishes formal logarithmic accuracy

Can study any function of pull vector

Honest accounting for W p_T distribution

Required for fixed-order matching

Analogies with $\vec{Q}_T$ resummation?

Natural observables to study?