Discrete Symmetries, Neutrino Mixing and Leptonic CP Violation

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Understanding the origin of the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years is one of the most challenging problems in neutrino physics. It is part of the more general fundamental problem in particle physics of understanding the origins of flavour in the quark and lepton sectors, i.e., of the patterns of quark masses and mixing, and of the charged lepton and neutrino masses and of neutrino mixing.

Of fundamental importance are also

- **the determination of the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos (which is one of the most challenging and pressing problems in present day elementary particle physics) (GERDA, CUORE, KamLAND-Zen, EXO, LEGEND, nEXO,...);**

- **determining the status of CP symmetry in the lepton sector (T2K, NOνA; T2HK, DUNE);**

- **determination of the type of spectrum neutrino masses possess, or neutrino mass ordering (T2K + NOνA; JUNO; PINGU, ORCA; T2HKK, DUNE);**

- **determination of the absolute neutrino mass scale, or** $\min(m_j)$ **(KATRIN, new ideas; cosmology).**

The program of research extends beyond 2030.

All compelling data compatible with 3-ν mixing:

$$\nu_{l\mathsf{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathsf{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (al least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \to \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \to \nu_\mu) < 1$

$P(\nu_l \to \nu_{l'}) = P(\nu_l \to \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$

Three Neutrino Mixing

$$\nu_{l\mathsf{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathsf{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}\,,$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.37 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.297$, $\cos 2\theta_{12} \gtrsim 0.29$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.53$ (2.43) [2.56 (2.54)] $\times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong$ 0.437 (0.569) [0.425 (0.589)], NO (IO) ,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0214$ (0.0218) [0.0215 (0.0216)], NO (IO).

F. Capozzi et al. (Bari Group), arXiv:1601.07777 [arXiv:1703.04471].

NuFit 3.2 (2018)

www.nu-fit.org

- data available till Jan 2018
- latest T2K data as of Aug 2017
- NOvA 8.85e20 pot disapp and appearance data, Jan 2018

I. Esteban, C. Gonzalez-Garcia, A. Hernandez, M. Maltoni, I. Martinez, T. Schwetz

NuFIT 3.2 (2018)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.14$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.272 \to 0.346$
$\theta_{12}/^\circ$	$33.62^{+0.78}_{-0.76}$	$31.42 \to 36.05$	$33.62^{+0.78}_{-0.76}$	$31.43 \to 36.06$	$31.42 \to 36.05$
$\sin^2\theta_{23}$	$0.538^{+0.033}_{-0.069}$	$0.418 \to 0.613$	$0.554^{+0.023}_{-0.033}$	$0.435 \to 0.616$	$0.418 \to 0.613$
$\theta_{23}/^\circ$	$47.2^{+1.9}_{-3.9}$	$40.3 \to 51.5$	$48.1^{+1.4}_{-1.9}$	$41.3 \to 51.7$	$40.3 \to 51.5$
$\sin^2\theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.01981 \to 0.02436$	$0.02227^{+0.00074}_{-0.00074}$	$0.02006 \to 0.02452$	$0.01981 \to 0.02436$
$\theta_{13}/^\circ$	$8.54^{+0.15}_{-0.15}$	$8.09 \to 8.98$	$8.58^{+0.14}_{-0.14}$	$8.14 \to 9.01$	$8.09 \to 8.98$
$\delta_{CP}/^\circ$	234^{+43}_{-31}	$144 \to 374$	278^{+26}_{-29}	$192 \to 354$	$144 \to 374$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$6.80 \to 8.02$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.494^{+0.033}_{-0.031}$	$+2.399 \to +2.593$	$-2.465^{+0.032}_{-0.031}$	$-2.562 \to -2.369$	$\begin{bmatrix} +2.399 \to +2.593 \\ -2.536 \to -2.395 \end{bmatrix}$

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.47$ $(2.42) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234$ (0.0240), NH (IH).
- $1\sigma(\Delta m^2_{21}) = 2.6\%$; $1\sigma(\sin^2\theta_{12}) = 5.4\%$, 2018: 4.0%;
- $1\sigma(|\Delta m^2_{31(23)}|) = 2.6\%$, 2018: 1.3%; $1\sigma(\sin^2\theta_{23}) = 9.6\%$; 2018: 6.0%;
- $1\sigma(\sin^2\theta_{13}) = 8.5\%$; 2018: 3.4%.
- $3\sigma(\Delta m^2_{21})$: $(6.99 - 8.18) \times 10^{-5}$ eV^2; $3\sigma(\sin^2\theta_{12})$: $(0.259 - 0.359)$;
$(3\sigma(\Delta m^2_{21})$: $(6.93 - 7.97) \times 10^{-5}$ eV^2; $3\sigma(\sin^2\theta_{12})$: $(0.250 - 0.354)$;)
- $3\sigma(|\Delta m^2_{31(23)}|)$: $2.27(2.23) - 2.65(2.60) \times 10^{-3}$ eV^2;
$(2.40(2.30) - 2.66(2.57) \times 10^{-3}$ eV^2;
$3\sigma(\sin^2\theta_{23})$: $0.374(0.380) - 0.628(0.641)$;
$(3\sigma(\sin^2\theta_{23})$: $0.379(0.383) - 0.616(0.637))$
- $3\sigma(\sin^2\theta_{13})$: $0.0176(0.0178) - 0.0296(0.0298)$
$(3\sigma(\sin^2\theta_{13})$: $0.0185(0.0186) - 0.0246(0.0248)$.)

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)
(F. Capozzi et al. (Bari Group), arXiv:1601.07777v1.)

- $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31(32)})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering (NO)}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering (IO)}$$

Convention: $m_1 < m_2 < m_3$ - NO, $m_3 < m_1 < m_2$ - IO

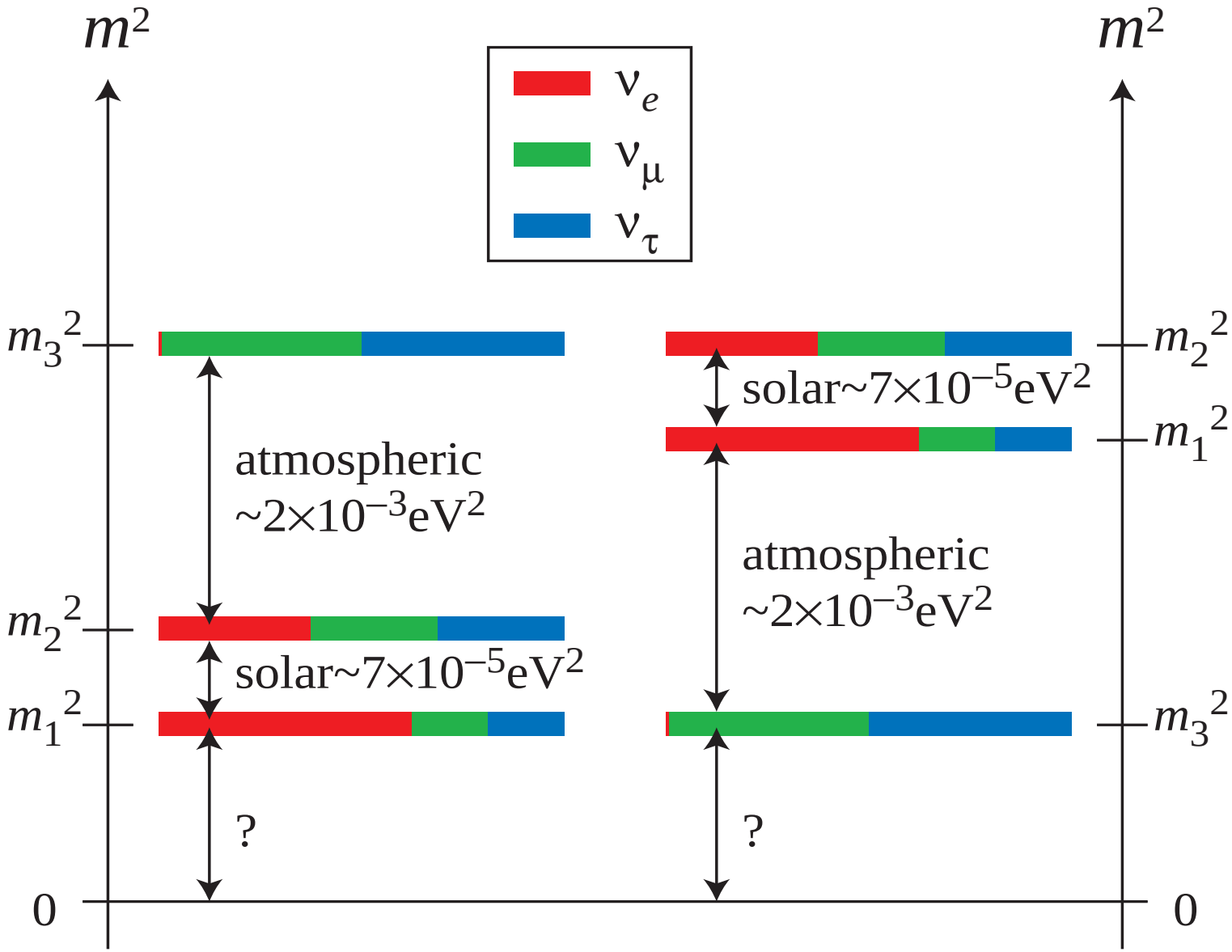
$$\Delta m^2_{31}(NO) = -\,\Delta m^2_{32}(IO)$$

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \; m^2_{1,2,3} >> |\Delta m^2_{31(32)}|, \; \text{QD}; \; m_j \gtrsim 0.10 \text{ eV}.$$

- $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}, \;\; m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$ - NO;
- $m_1 = \sqrt{m_3^2 + \Delta m^2_{23} - \Delta m^2_{21}}, \;\; m_2 = \sqrt{m_3^2 + \Delta m^2_{23}}$ - IO;


m^2
m^2
ν_e
ν_μ
ν_τ
m_3^2
m_2^2
m_1^2
0
atmospheric
~2×10^{-3}eV^2
solar~7×10^{-5}eV^2
?
m_2^2
m_1^2
m_3^2
0
solar~7×10^{-5}eV^2
atmospheric
~2×10^{-3}eV^2
?

Latest global analysis: data favors NO

IO disfavored at 3.1σ.

F. Capozzi et al., 1804.09678.

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{\rm CP}^{(l,l')} \propto J_{\rm CP} \propto \sin\theta_{13} \sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U_{e2}^{*}\, U_{\mu 1}^{*}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

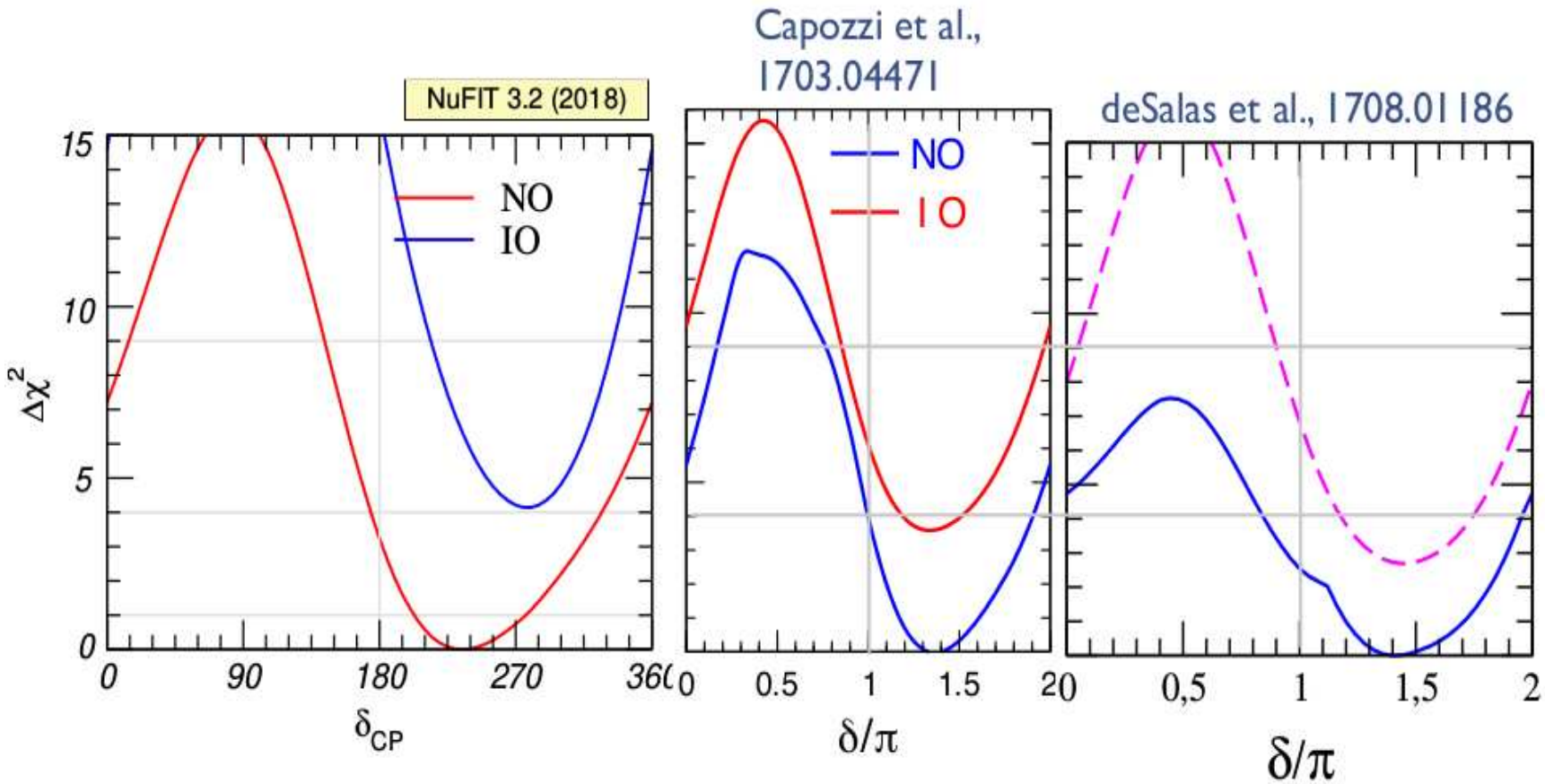
  - $|<m>|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

$$\delta \cong 3\pi/2?$$

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U_{e2}^{*}\, U_{\mu 1}^{*}\right\} \\ &= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta \end{aligned}$$

Determination of the CP phase

be aware of different data sets! (and different color coding)



T. Schwetz, Talk at CERN Neutrino Platform week, 01/02/2018

- **Best fit value:** $\delta = 1.38\,(1.31)\pi$ **[**$1.30\,(1.54)\pi$**];**
- $\delta = 0$ **or** 2π **are disfavored at** $2.4\,(3.2)\sigma$ **[**$2.6\,(3.0)\sigma$**];**
- $\delta = \pi$ **is disfavored at** $2.0\,(2.5)\sigma$ **[**$1.7\,(3.3)\sigma$**];**
- $\delta = \pi/2$ **is strongly disfavored at** $3.4\,(3.9)\sigma$ **[**$4.3\,(5.0)\sigma$**].**
- **At** 3σ**:** δ/π **is found to lie in** $(0.00 - 0.17(0.16)) \oplus (0.76(0.69) - 2.00))$ **[1.07-1.97 (0.80-2.08)].**

F. Capozzi, E. Lisi *et al.*, arXiv:1703.04471 [E. Esteban *et al.*, NuFit 3.2 (Jan., 2018)]

The Quest for Nature's Message

With the observed pattern of neutrino mixing Nature is sending us a message. The message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. The message can have two completely different contents: it can read

ANARCHY or SYMMETRY.

ANARCHY:

A. De Gouvea, H. Murayama, hep-ph/0301050; PLB, 2015.

L. Hall, H. Murayama, N. Weiner, hep-ph/9911341.

Understanding the Pattern of Neutrino Mixing: Symmetry Approach.

Examples of Predictions and Correlations.

- $\sin^2\theta_{23} = \frac{1}{2}$.
- $\sin^2\theta_{23} \cong \frac{1}{2}(1 \mp \sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong \frac{1}{2}(1 \mp 0.022)$.
- $\sin^2\theta_{23} = 0.455;\ 0.463;\ 0.537;\ 0.545;\ 0.604$ **(small uncert.).**
- $\sin^2\theta_{12} \cong \frac{1}{3}(1 + \sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong 0.340$.
- $\sin^2\theta_{12} \cong \frac{1}{3}(1 - 2\sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong 0.319$.
- **and/or** $\cos\delta = \cos\delta(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

$J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

θ^{ν}_{12},... - **known (fixed) parameters, depend on the underlying symmetry.**

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Understanding the Pattern of Neutrino Mixing: Predictions for the CPV Phase δ.

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{\rm atm} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{\rm PMNS} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \sin^{-1}\frac{1}{\sqrt{3}} - 0.020$; $\theta_{12} \cong \pi/4 - 0.20$, $\theta_{13} \cong 0 + \pi/20$, $\theta_{23} \cong \pi/4 \mp 0.10$.
- $U_{\rm PMNS}$ due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U^{\dagger}_{\rm lep}(\theta^{\ell}_{ij}, \delta^{\ell})\; Q(\psi,\omega) U_{\rm TBM,BM,LC,...}\; \bar{P}(\xi_1,\xi_2)\,,$$

with

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\rm lep}(\theta^{\ell}_{ij}, \delta^{\ell})$ - from diagonalization of the l^- mass matrix;
- $U_{\rm TBM,BM,LC,...}\; \bar{P}(\xi_1,\xi_2)$ - from diagonalization of the ν mass matrix;
- $Q(\psi,\omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\rm LC}$, $U_{\rm GRAM}$, $U_{\rm GRBM}$, $U_{\rm HGM}$:

$$U_{\rm LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c^{\nu}_{23}}{\sqrt{2}} & \frac{c^{\nu}_{23}}{\sqrt{2}} & s^{\nu}_{23} \\ \frac{s^{\nu}_{23}}{\sqrt{2}} & -\frac{s^{\nu}_{23}}{\sqrt{2}} & c^{\nu}_{23} \end{pmatrix} ; \quad \mu - \tau \text{ symmetry}: \quad \theta^{\nu}_{23} = \mp \pi/4 ;$$

$$U_{\rm GR} = \begin{pmatrix} c^{\nu}_{12} & s^{\nu}_{12} & 0 \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} ; \quad U_{\rm HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \quad \theta^{\nu}_{12} = \pi/6 .$$

$U_{\rm GRAM}$: $\sin^2\theta^{\nu}_{12} = (2+r)^{-1} \cong 0.276$, $r = (1+\sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$**)**

$U_{\rm GRBM}$: $\sin^2\theta^{\nu}_{12} = (3-r)/4 \cong 0.345$.

GRB and HG mixing: W. Rodejohann et al., 2009.

• $U_{\rm TBM}$: $s^2_{12} = 1/3$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$ must be corrected; if $\theta_{23} \neq \pi/4$, $s^2_{23} = 0.5$ must be corrected .

• $U_{\rm BM}$: $s^2_{12} = 1/2$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$, $s^2_{12} = 1/2$ and possibly $s^2_{23} = 1/2$ must be corrected.

$U_{\rm TBM(BM)}$: Groups S_4, A_4, T' (S_4),... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

• $U_{\rm GRA}$: Group A_5,...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.276$ and $s^2_{23} = 1/2$ must be corrected.

L. Everett, A. Stuart, arXiv:0812.1057;...

• $U_{\rm LC}$: alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

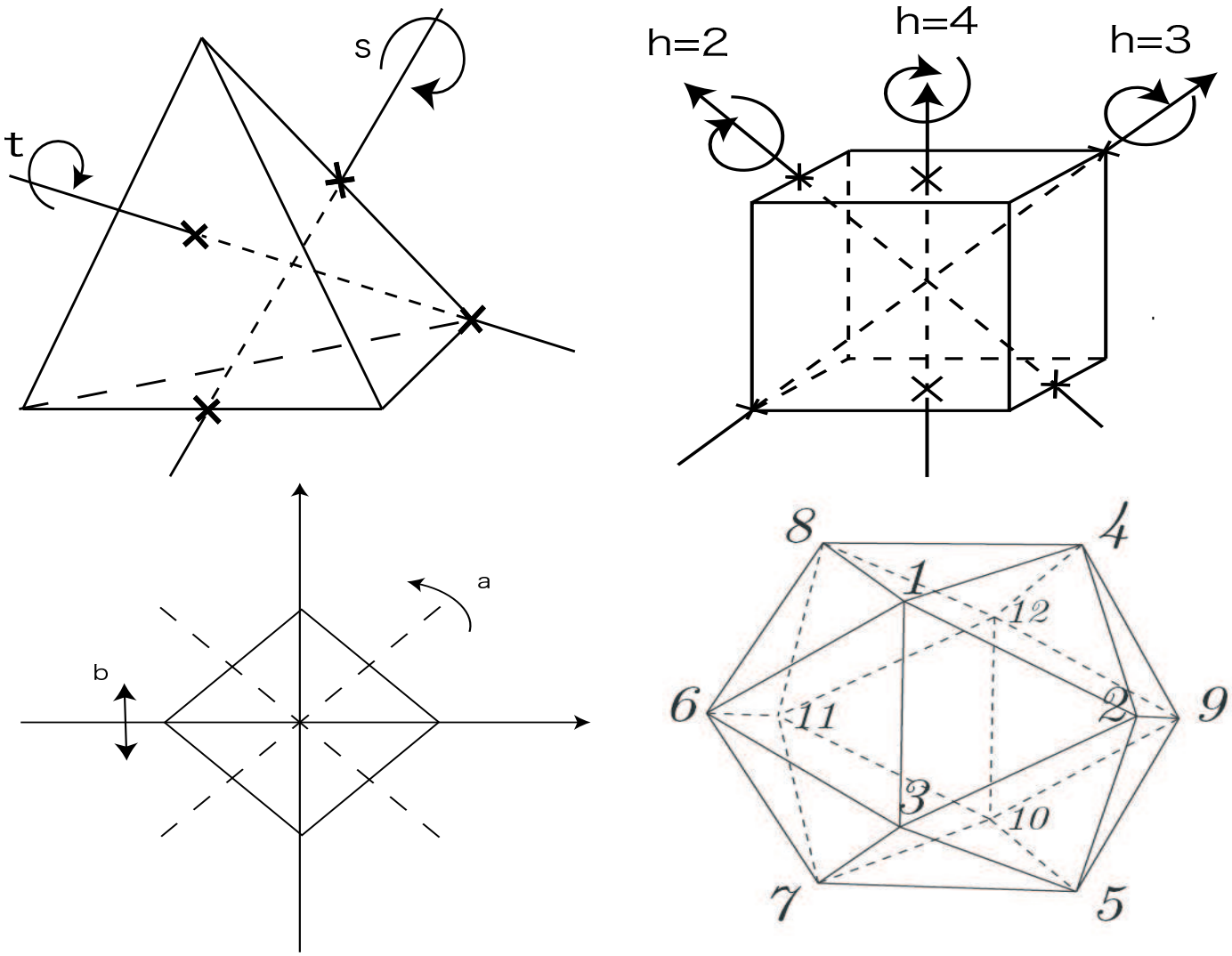
S.T.P., 1982

• $U_{\rm LC}$: $s^2_{12} = 1/2$, $s^2_{13} = 0$, s^ν_{23} - free parameter; $s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

• $U_{\rm GRB}$: Group D_{10},...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.345$ and $s^2_{23} = 1/2$ must be corrected.

• $U_{\rm HG}$: Group D_{12},...; $s^2_{13} = 0$, $s^2_{12} = 0.25$ and possibly $s^2_{23} = 1/2$ must be corrected.

For all symmetry forms considered we have: $\theta^\nu_{13} = 0$, $\theta^\nu_{23} = \mp\pi/4$.
They differ by the value of θ^ν_{12}:
TBM, BM, GRA, GRB and HG forms correspond to $\sin^2\theta^\nu_{12} = 1/3;\ 0.5;\ 0.276;\ 0.345;\ 0.25$.



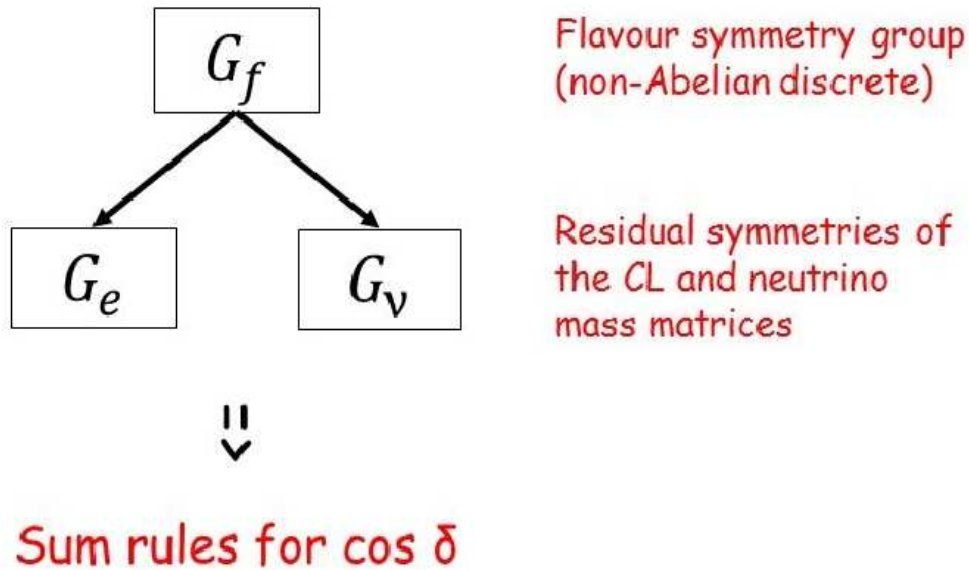
Examples of symmetries: A_4, S_4, D_4, A_5

From M. Tanimoto et al., arXiv:1003.3552

Group	Number of elements	Generators	Irreducible representations
S_4	24	S, T (U)	$\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$, $\mathbf{3}'$
A_4	12	S, T	$\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{3}$
T'	24	S, T (R)	$\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{2}$, $\mathbf{2}'$, $\mathbf{2}''$, $\mathbf{3}$
A_5	60	$\tilde{S}$, $\tilde{T}$	$\mathbf{1}$, $\mathbf{3}$, $\mathbf{3}'$, $\mathbf{4}$, $\mathbf{5}$
D_{10}	20	A, B	$\mathbf{1}_1$, $\mathbf{1}_2$, $\mathbf{1}_3$, $\mathbf{1}_4$, $\mathbf{2}_1$, $\mathbf{2}_2$, $\mathbf{2}_3$, $\mathbf{2}_4$
D_{12}	24	$\tilde{A}$, $\tilde{B}$	$\mathbf{1}_1$, $\mathbf{1}_2$, $\mathbf{1}_3$, $\mathbf{1}_4$, $\mathbf{2}_1$, $\mathbf{2}_2$, $\mathbf{2}_3$, $\mathbf{2}_4$, $\mathbf{2}_5$

Number of elements, generators and irreducible representations of some discrete groups.

How does it Work.



G_f: non-Abelian discrete (finite) flavour symmetry.

Typically (but not uniquely) $l_L(x)$, $\nu_{lL}(x)$, $l = e, \mu, \tau$ **- triplets of** G_f**.**

$G_\nu-$ **symmetry of the neutrino mass term;**

G_e- **symmetry of the charged lepton mass term.**

Majorana Mass Term for $\nu_{lL}(x)$, $l = e, \mu, \tau$ **:**

$$\mathcal{L}_M^\nu(x) = \frac{1}{2}\, \nu_{l'L}^{\mathsf{T}}(x)\, C^{-1}\, M_{l'l}\, \nu_{lL}(x) + h.c.\ ,\ C^{-1}\gamma_\alpha C = -\gamma_\alpha^{\mathsf{T}}$$

Charged lepton mass term:

$$\mathcal{L}_\ell(x) = -\ \overline{l'_L}(x)\ M_{el'l}\ l_R(x) + h.c.$$

U_e: $U_e^\dagger M_e M_e^\dagger U_e = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)$.

G_e - **residual symmetry group of** $M_e M_e^\dagger$

U_ν: $U_\nu^T M_\nu U_\nu = \text{diag}(m_1, m_2, m_3)$.
G_ν - **residual symmetry group of** M_ν

$U_{\text{PMNS}} = U_e^\dagger U_\nu$

$G_e = Z_2$; Z_n, $n > 2$; $Z_n \times Z_m$, $n, m \geq 2$
(max. $G_e = U(1) \times U(1) \times U(1)$ **+** G_e **subgroup of** $SU(3)$**: max.** $G_e = U(1) \times U(1)$**)**

ν_j, **Majorana mass term,** $m_j \neq m_k$, $j \neq k = 1, 2, 3$: $G_\nu = Z_2 \times Z_2$, Z_2
(max. $G_\nu = Z_2 \times Z_2 \times Z_2$ **+** G_ν **subgroup of** $SU(3)$**: max.** $G_\nu = Z_2 \times Z_2$**)**

ρ: **the unitary irrep. of** G_f **acting on** $l_L(x)$**:**
$l_L(x) \to \rho(g_f)\, l_L(x)$, $g_f \in G_f$**;**
g_e**: an element of residual symmetry** G_e**;**
$\rho(g_e)$**: action of** G_e **on** $l_L(x)$**,** $l = e, \mu, \tau$**:**
$l_L(x) \to \rho(g_e)\, l_L(x)$**.**

G_e **- residual symmetry group of** $M_e M_e^\dagger$**:**

$$\rho(g_e)^\dagger M_e M_e^\dagger \rho(g_e) = M_e M_e^\dagger,$$

$\rho(g_e)$ **and** $M_e M_e^\dagger$ **commute: both are diagonalised by** U_e**.**
$\rho(g_e)-$ **known:** U_e **constrained or determined.**

ρ: **the unitary irrep. of** G_f **acting on** $\nu_L(x)$**:**
$\nu_L(x) \rightarrow \rho(g_f)\,\nu_L(x)$**,** $g_f \in G_f$**;**
g_ν**: an element of residual symmetry** G_ν**;**
$\rho(g_\nu)$**: action of** G_ν **on** $\nu_{lL}(x)$**,** $l = e, \mu, \tau$**:**
$\nu_L(x) \rightarrow \rho(g_\nu)\,\nu_L(x)$**.**

M_ν **- neutrino Majorana mass matrix.**
U_ν**:** $U_\nu^T M_\nu U_\nu = \text{diag}(m_1, m_2, m_3)$**.**

G_ν **- residual symmetry group of** M_ν**:**

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu,$$

$\rho(g_\nu)$ **and** $M_\nu^\dagger M_\nu$ **commute: both are diagonalised by** U_ν**.**
$\rho(g_\nu)-$ **known:** U_ν **constrained or determined (up to Majorana phases).**

U_e and U_ν constrained or determined (U_ν up to Majorana phases):

$U_{\rm PMNS} = U_e^\dagger U_\nu$ constrained or determined (up to the Majorana phases)

The constraints depend on: G_f, $\rho(g_f)$, G_e and G_ν.

$G_f = A_4$, S_4, T', A_5.

A_4: **3 Z_2, 4 Z_3, 1 $Z_2 \times Z_2$ subgroups (total 8).**

T': **similar to A_4.**

S_4: **9 Z_2, 4 Z_3, 3 Z_4, 4 $Z_2 \times Z_2$ subgroups (total 20).**

A_5: **has 15 Z_2, 10 Z_3, 6 Z_5, 5 $Z_2 \times Z_2$ subgroups (36).**

In models with $G_\nu = Z_2 \times Z_2$:

U_ν **- determined up to re-phasing on the right and permutations of columns; the latter can be fixed within a specific model.**

In models with $G_\nu = Z_2$:

U_ν **- two free parameters, one angle and a phase, as long as the neutrino Majorana mass term does not have additional "accidental" symmetries, e.g., the $\mu - \tau$ symmetry; otherwise, determined up to re-phasing on the right and permutations of columns.**

- TBM form of $\tilde{U}_\nu$:

a) from $G_f = S_4$, $G_\nu = Z_2^S \times Z_2^U$ (S, U generators of S_4 are unbroken).

b) from $G_f = A_4$, $G_\nu = Z_2^S$ (S generator of A_4 is unbroken) $+ \mu - \tau$ accidental symmetry.

G. Altarelli, F. Feruglio, arXiv:1002.0211; I. Girardi et al., arXiv:1509.02502

c) from $G_f = T'$, $G_\nu = Z_2$ ($+ TST^2$ element of T' - unbroken).

- BM form of $\tilde{U}_\nu$:

from $G_f = S_4$, $G_\nu = Z_2 + \mu - \tau$ accidental symmetry.

G. Altarelli et al., arXiv:0903.1940; I. Girardi et al., arXiv:1509.02502

In all these cases: $l_L(x)$, $\nu_{lL}(x)$, $l = e, \mu, \tau$ - triplets of G_f.

Examples

TBM form of $\tilde{U}_\nu$:

$G_f = S_4$, $G_\nu = Z_2^S \times Z_2^U$ **(S, U generators of S_4 are unbroken).**

S_4 **generators S, T, U, presentation rules:**

$$S^2 = T^3 = (ST)^3 = U^2 = (TU)^2 = (SU)^2 = (STU)^4 = 1$$

In the $\mathbf{3}$ and $\mathbf{3}'$ representations and convenient basis,

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \; T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \; U = \mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

where $\omega = e^{i\frac{2\pi}{3}}$.

Results for U_{PMNS} are basis-independent.

Choose: $G_e = Z_3^T = \{1, T, T^2\}$;

$\rho(g_e) \equiv T$, $T-$ **diagonal:** $U_e = \text{diag}(1, 1, 1)$.

$G_\nu = Z_2^S \times Z_2^U$, $Z_2^U = \{1, U\}$, $Z_2^S = \{1, S\}$;

$\rho(g_\nu) \equiv U, S$: U, S **diagonalised by** $U_\nu = V_{\text{TBM}}$.

$U_{\text{PMNS}} = U_e^\dagger U_\nu = V_{\text{TBM}}$

Problem: $\theta_{13} = 0$ **(!)**

Solution: $G_\nu = Z_2^S$, $Z_2^S = \{1, S\}$; $\rho(g_\nu) = S$;

$V_{\text{TBM}}^\dagger S V_{\text{TBM}} = \text{diag}(-1, 1, -1)$.

Thus, $U_\nu = V_{\text{TBM}} U_{13}(\theta_{13}^\nu, \alpha) P(\xi_{21}, \xi_{31})$,

$$U_{13}(\theta^{\nu}_{13}, \alpha) = \begin{pmatrix} \cos\theta^{\nu}_{13} & 0 & \sin\theta^{\nu}_{13}\, e^{i\alpha} \\ 0 & 1 & 0 \\ -\sin\theta^{\nu}_{13}\, e^{-i\alpha} & 0 & \cos\theta^{\nu}_{13} \end{pmatrix} .$$

Predictions: $\sin^2\theta_{12} = 1/(3(1-\sin^2\theta_{13})) \cong 0.340$, $\cos\delta = \cos\delta(\theta_{13}, \theta_{23})$;

Best fit values of $\sin^2\theta_{13}$, $\sin^2\theta_{23}$**:**
$\cos\delta = 0.73$ **(**$\delta = \pm 43.3^\circ$**), NO;**
$\cos\delta = -0.86$ **(**$\delta = 180^\circ \pm 30.1^\circ$**), IO.**

The predictions are testable experimentally: will be tested at JUNO, T2K, NOνA, T2HK, DUNE experiments.

Generalised CP Invariance

G. Branco et al., 1986; F. Feruglio et al., arXiv:1211.5568; M. Holthausen et al., arXiv:1211.6953.

$H^{\nu}_{\mathrm{CP}} = \{X_\nu\}$ **GCP unitary transformations** X_ν **forming a (residual) CP symmetry of** M_ν: $X_\nu^T M_\nu X_\nu = M_\nu^*$.

Consistency condition between G_ν **(i.e.,** $\rho(g_\nu)$**) and** H^{ν}_{CP} **has to be respected:**

$$X_\nu \rho^*(g_\nu) X_\nu^{-1} = \rho(g'_\nu).$$

$g_\nu^{(\prime)}$: **an element of** G_ν;

ρ: **the unitary representation of** G_f **acting on** $\nu_L(x)$;

$\rho(g_\nu^{(\prime)})$: **action of** G_ν **on** $\nu_L(x)$, $l = e, \mu, \tau$.

GCP invariance of M_ν **determines the Majorana phases in** U_{PMNS}.

None of the symmetries leading to $U_{\rm TBM}$, $U_{\rm BM}$ or other approximate forms of $U_{\rm PMNS}$ can be exact.

Which is the correct approximate symmetry, i.e., approximate form of $U_{\rm PMNS}$ (if any)?

In the cases of U_ν given by $U_{\rm TBM}$, $U_{\rm BM}$, etc. the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.

Depending on the symmetry leading to $U_{\rm TBM,BM}$, etc. and on the form of $U_{\rm lep}$, one obtaines diifferent experimentally testable predictions for the sum of the neutrino masses, the neutrino mass spectrum, the nature (Dirac or Majorana) of ν_j and the CP violating phases in the neutrino mixing matrix. Future data will help us understand whether there is some new fundamental symmetry behind the observed patterns of neutrino mixing and Δm^2_{ij}.

Predictions and Correlations I

$U_\nu = U_{\text{TBM,BM,GRA,GRB,HG}}\,\bar{P}(\xi_1,\xi_2)$; θ^ν_{12};

$U^\dagger_\ell = R_{12}(\theta^\ell_{12})\,Q$, $Q = \text{diag}(e^{i\varphi},1,1)$
(the "minimal" = simplest case ($SU(5)\times T'$,...)

$U^\dagger_\ell = R_{12}(\theta^\ell_{12})\,R_{23}(\theta^\ell_{23})Q$, $Q = \text{diag}(1,e^{-i\psi},e^{-i\omega})$
(next-to-minimal case)

$\cos\delta = \cos\delta(\theta_{12},\theta_{23},\theta_{13};\theta^\nu_{12},...)$,

$J_{CP} = J_{CP}(\theta_{12},\theta_{23},\theta_{13},\delta) = J_{CP}(\theta_{12},\theta_{23},\theta_{13};\theta^\nu_{12},...)$,

θ^ν_{12},... - **known (fixed) parameters, depend on the underlying symmetry.**

For arbitrary fixed θ_{12}^{ν} and any θ_{23} ("minimal" and "next-to-minimal" cases):

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta_{12}^{\nu}\right.$$

$$\left.+\left(\sin^2\theta_{12}-\cos^2\theta_{12}^{\nu}\right)\left(1-\cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right]\,.$$

S.T.P., arXiv:1405.6006

This results is exact.

"Minimal" case: $\sin^2\theta_{23} = \frac{1}{2}\,\frac{1-2\sin^2\theta_{13}}{1-\sin^2\theta_{13}}$.

In all cases TBM, BM (LC), GRA, GRB, HG:

- **New sum rules relating θ_{12},θ_{13}, θ_{23} and δ;**
- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.

S.T.P., arXiv:1405.6006

- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.
- TBM case: $\delta \cong 3\pi/2$ or $\pi/2$; b.f.v. of θ_{ij}:
$\delta \cong 263.5^{\circ}$ or 96.5°, $\cos\delta = -0.114$, $J_{CP} \cong \mp 0.034$.
- GRAM case, b.f.v. of θ_{ij}: $\delta \cong 286.8^{\circ}$ or 73.2°;
$\cos\delta = 0.289$, $J_{CP} \cong \mp 0.0327$.
- GRBM case, b.f.v. of θ_{ij}: $\delta \cong 258.5^{\circ}$ or 101.5°;
$\cos\delta = -0.200$, $J_{CP} \mp 0.0333$.
- HGM case, b.f.v. of θ_{ij}: $\delta \cong 298.4^{\circ}$ or 61.6°;
$\cos\delta = 0.476$, $J_{CP} \cong \mp 0.0299$.
- BM, LC cases: $\delta \cong \pi$, $\cos\delta \cong -0.978$, $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of θ_{ij}: F. Capozzi et al., arXiv:1312.2878v1.)

S.T.P., arXiv:1405.6006

By measuring $\cos\delta$ or δ and using high precision data on θ_{12}, θ_{23} and θ_{13}, one can distiguish between different symmetry forms of $\tilde{U}_\nu$!

Relatively high precision measurement of δ will be performed at the future planned neutrino oscillation experiments, (DUNE, T2HK) see, e.g., R. Acciarri *et al.* [DUNE Collab.], arXiv:1512.06148, 1601.05471 and 1601.02984; K. Abe *et al.* [T2HK Proto-Collab.], arXiv:1502.05199 (PTEP 2015 (2015) 053C02).

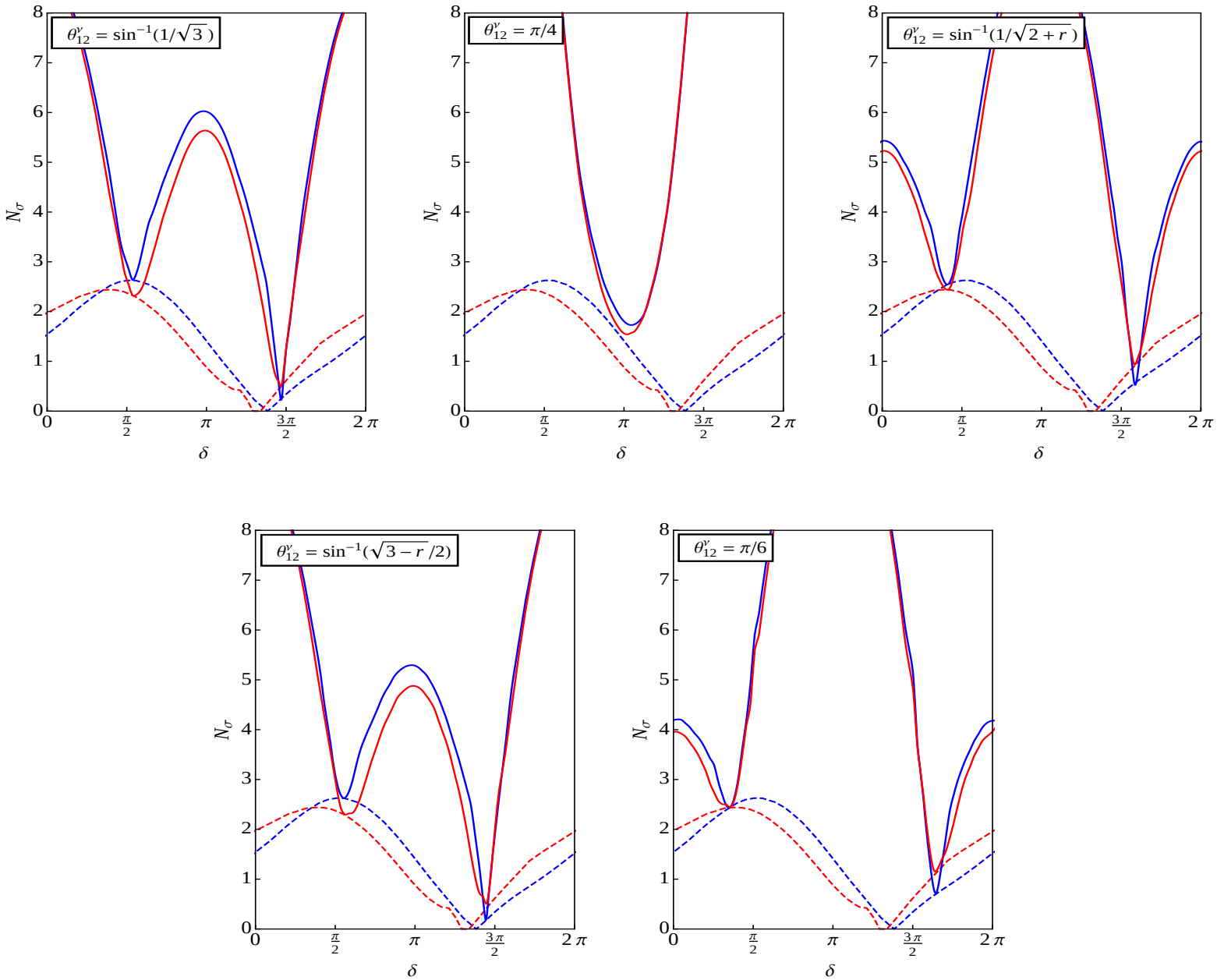
Statistical analysis, likelihood method; input "data": $\sin^2\theta_{13}$, $\sin^2\theta_{12}$, $\sin^2\theta_{12}$, δ from F. Capozzi et al., arXiv:1312.2878v2 (May 5, 2014).

$$L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$$

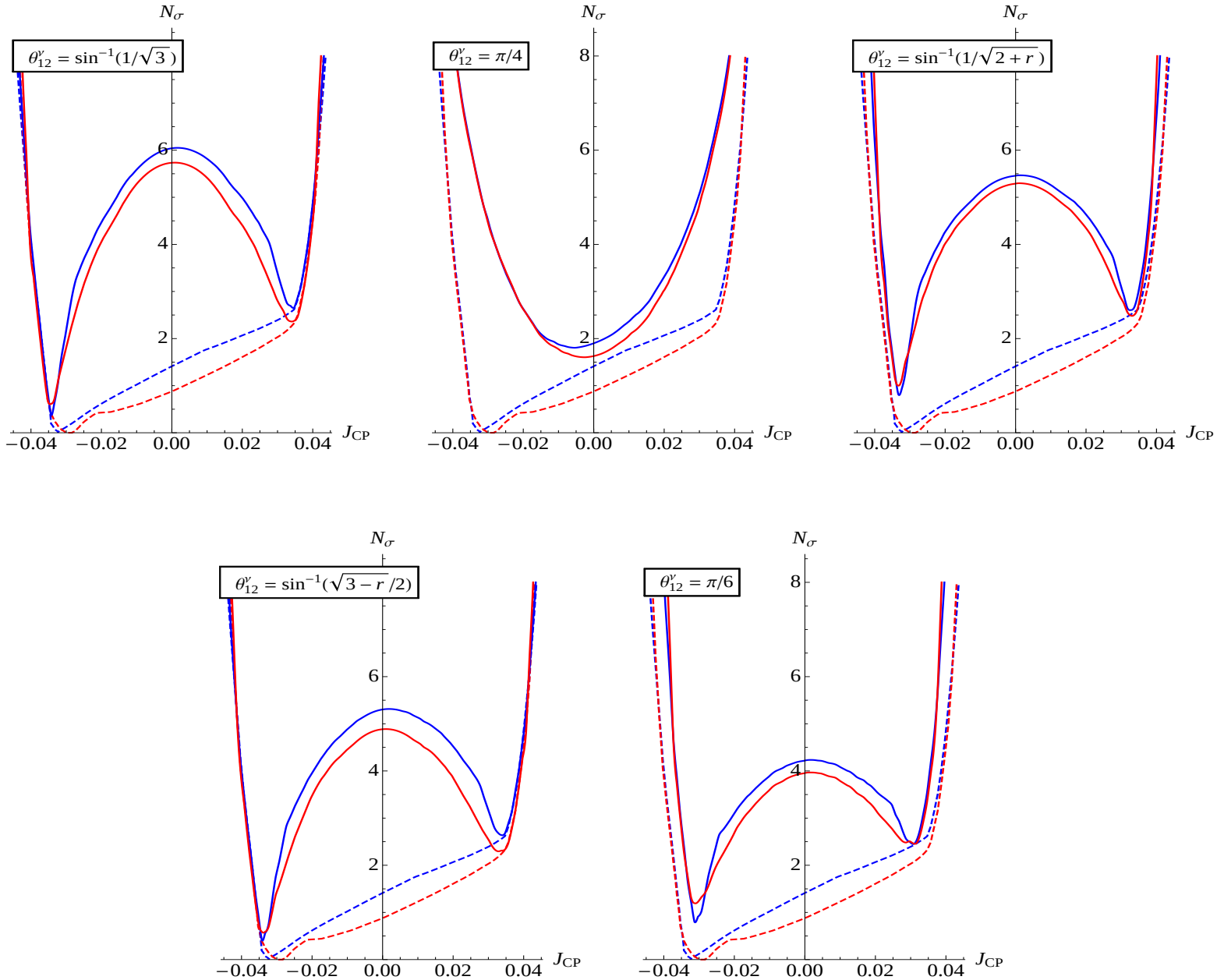
nσ confidence level interval of values of $\cos\delta$:

$$L(\cos\delta) \geq L(\chi^2_{\rm min}) \cdot L(\chi^2 = n^2)$$

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

TBM, GRA, GRB, HG: $J_{\rm CP} = 0$ **excluded at** 5σ**,** 4σ**,** 4σ**,** 3σ **confidence level.**

At 3σ**:** $0.020 \leq |J_{\rm CP}| \leq 0.039$**.**

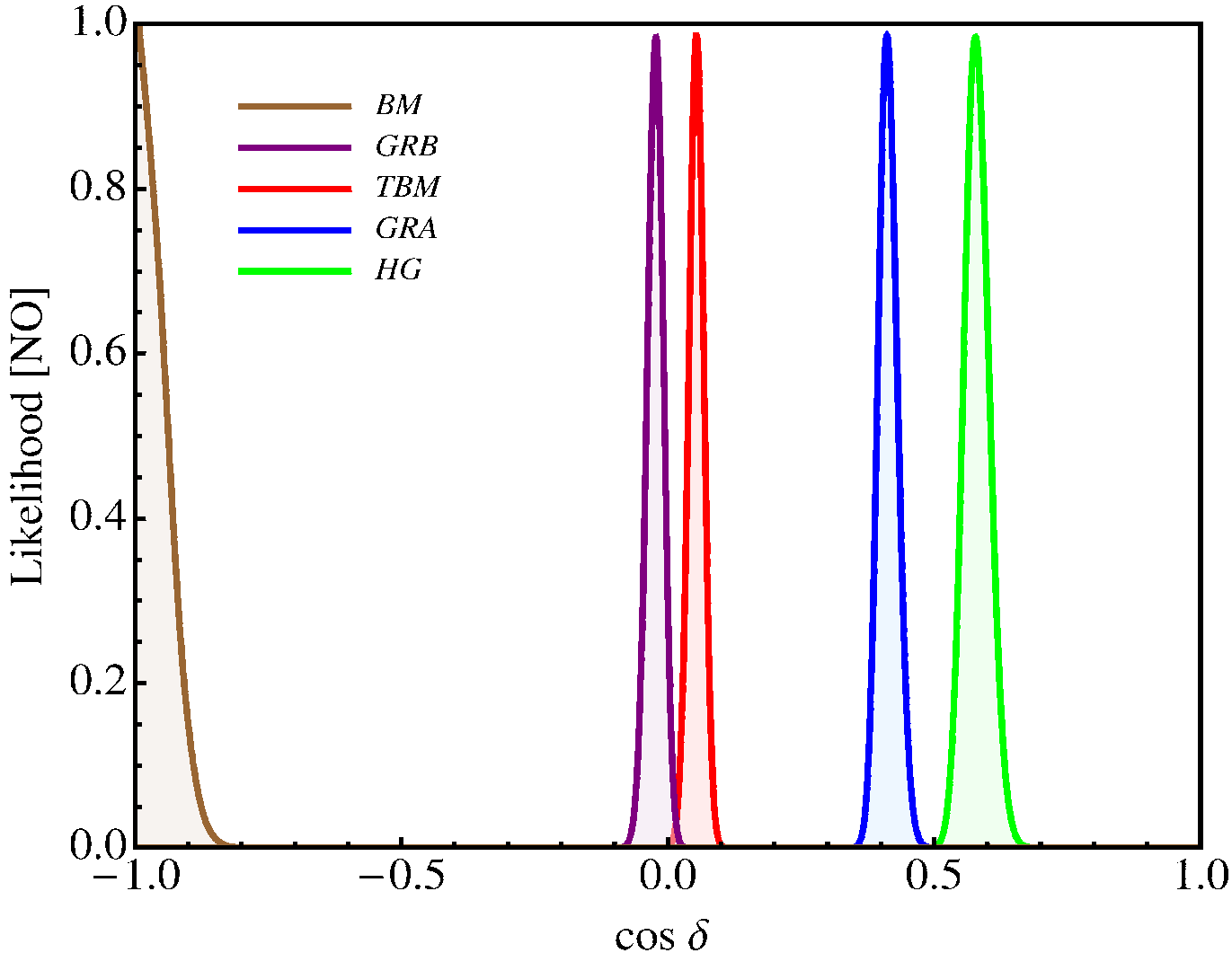
BM (LC), b.f.v.: $J_{\rm CP} = 0$**;**
at 3σ**:** -0.026 $(-0.025) \leq J_{\rm CP} \leq 0.021$ (0.023) **for NO (IO) neutrino mass spectrum.**

Prospective precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

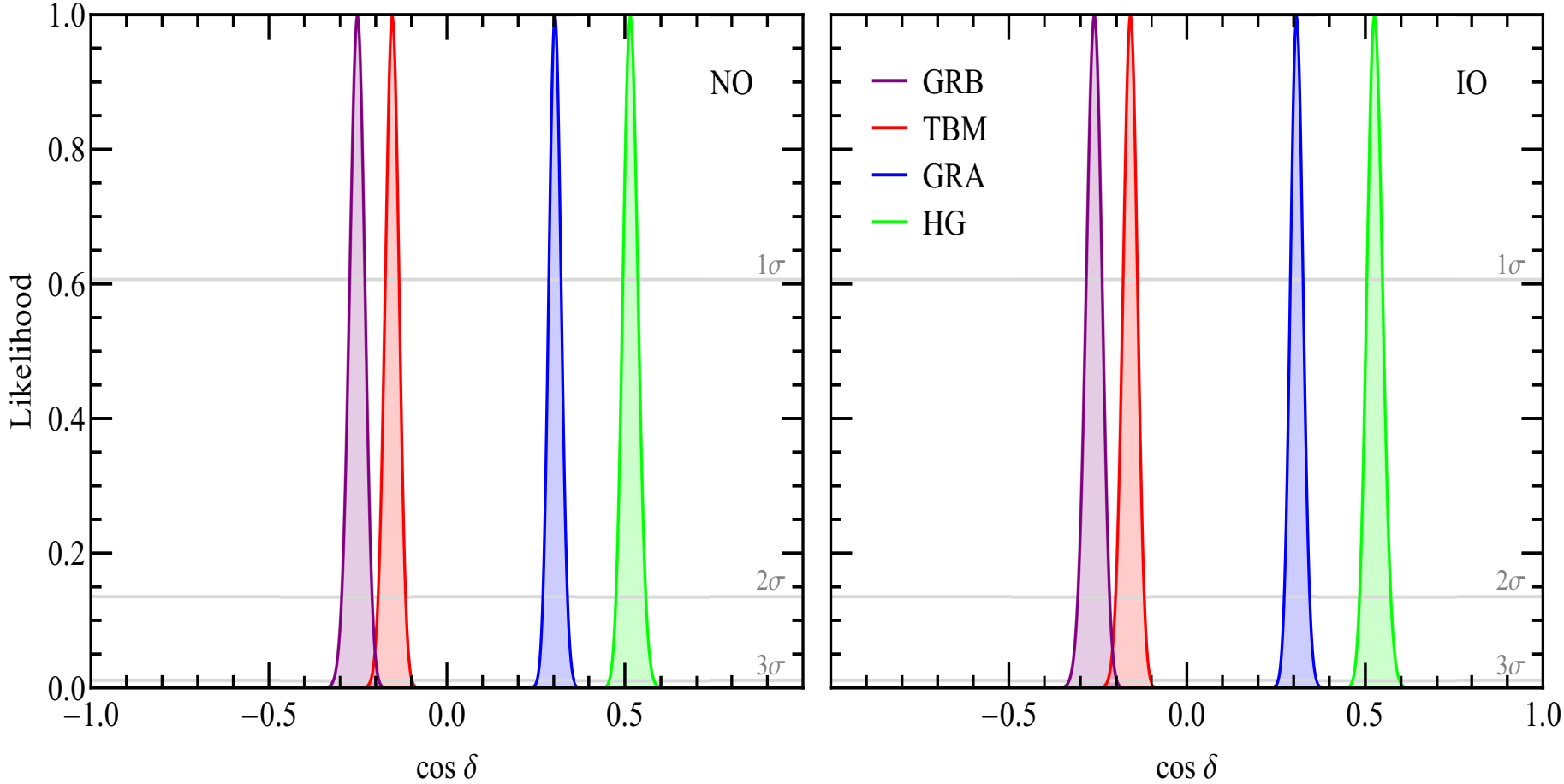
$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

$\delta(\sin^2\theta_{23}) = 5\%$ **(T2K, NOνA combined).**



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

b.f.v. of $\sin^2\theta_{12,23}$ **(Capozzi et al., 2014),** $\sin^2\theta_{12} = 0.33$**, + the prospective precision used.**

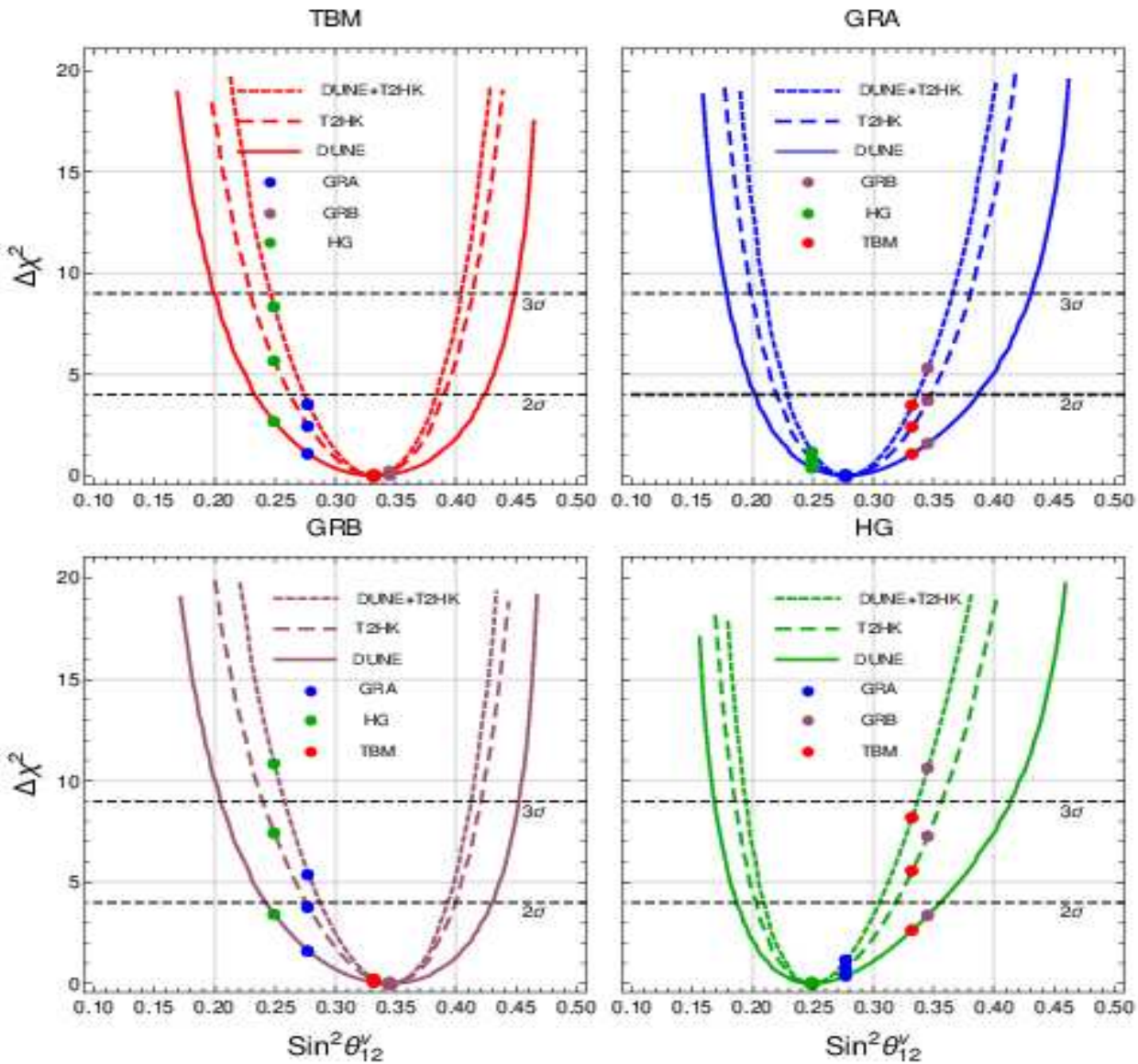


I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

b.f.v. of $\sin^2\theta_{ij}$ (Esteban et al., Jan., 2018) + the prospective precision used.

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right)\right] .$$

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE).**



Agarwalla, Chatterjee, STP, Titov, arXiv:1711.02107

GRB - HG $> 3\sigma$; **GRA - GRB** $\geq 2\sigma$; **TMB - HG** $\cong 3\sigma$; **TMB - GRA** $\cong 2\sigma$.

The same method can be used for obtaining predictions for leptonic Dirac and Majorana CP violation in a large number of cases, see S.T.P., arXiv:1405.6006; I. Girardi *et al.*, arXiv:1410.8056, arXiv:1504.00658, arXiv:1509.02502, and arXiv:1605.04172.

In I. Girardi *et al.*, arXiv:1509.02502, all symmetry breaking patterns, i.e., all possible combinations of residual symmetries G_e and G_ν of the lepton flavour symmetry groups $G_f = S_4$, A_4, T' and A_5, which could lead to correlations between some of the three neutrino mixing angles and/or between the neutrino mixing angles and the Dirac CPV phase δ, were considered.

(A) $G_e = Z_2$ **and** $G_\nu = Z_k$, $k > 2$ **or** $Z_m \times Z_n$, $m, n \geq 2$;

(B) $G_e = Z_k$, $k > 2$ **or** $Z_m \times Z_n$, $m, n \geq 2$ **and** $G_\nu = Z_2$;

(C) $G_e = Z_2$ **and** $G_\nu = Z_2$.

In these cases $U_e^\dagger$ **and/or** U_ν **of** $U = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \Psi \tilde{U}_\nu Q_0$, **are partially (or fully) determined by residual discrete symmetries of** $G_f = S_4$, A_4, T' **and** A_5.

More specifically:

A. $G_e = Z_2$**,** $G_\nu = Z_n$**,** $n > 2$ **or** $Z_n \times Z_m$**,** $n, m \geq 2$**;**
U_ν **fixed;** $U_e = U_{ij}(\theta^e_{ij}, \delta_{ij})$**,** $ij = 12(A1); 13(A2), 23(A3)$**;**
A1, A2 (A3): $\sin^2\theta_{23}$**,** $\cos\delta$ **(**θ_{12}**,** θ_{13}**) predicted.**

B. $G_e = Z_n$**,** $n > 2$ **or** $G_e = Z_n \times Z_m$**,** $n, m \geq 2$**,** $G_\nu = Z_2$**;**
$U_\nu = U_{\rm sym} U_{ij}(\theta^\nu_{ij}, \delta_{ij}) Q_0$**,** $ij = 13, 12, 23$**(B1,B2,B3);**
U_e **fixed; B1,B2 (B3):** θ_{12}**,** $\cos\delta$ **(**θ_{23}**,** θ_{13}**) predicted.**

C. $G_e = Z_2$ **and** $G_\nu = Z_2$**:** U_e **- up to** $U_{ij}(\theta^e_{ij}, \delta^{ij})$**,** U_ν **- up to** $U_{ij}(\theta^\nu_{ij}, \delta^\nu_{ij})$**,** $ij = 12; 13, 23$ **(C1 - C9);**
$\sin^2\theta_{12}$ **or** $\sin^2\theta_{23}$ **or** $\cos\delta$ **predicted.**

In the case of $G_e = Z_2$ ($G_\nu = Z_2$), U_e (U_ν) is determined up to a $U(2)$ transformation in the degenerate subspace.
When the residual symmetry is large enough, namely, $G_e = Z_n$, $n > 2$ or $G_e = Z_n \times Z_m$, $n, m \geq 2$ and/or $G_\nu = Z_2 \times Z_2$ ($G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$) for Majorana (Dirac) neutrinos, U_e and/or U_ν are/is fixed (up to diagonal phase matrices on the right, which are either unphysical for Dirac neutrinos, or contribute to the Majorana phases otherwise, and permutations of columns) by the residual symmetries of the charged lepton and neutrino mass matrices.
In the case when the discrete symmetry G_f is fully broken in one of the two sectors, the corresponding mixing matrix U_e or U_ν is unconstrained and contains in general three angles and six phases.

$G_f = A_4$, S_4, T', A_5.

A_4: **3** Z_2, **4** Z_3, **1** $Z_2 \times Z_2$ **subgroups (total 8).**

T': **similar to** A_4.

S_4: **9** Z_2, **4** Z_3, **3** Z_4, **4** $Z_2 \times Z_2$ **subgroups (total 20).**

A_5: **has 15** Z_2, **10** Z_3, **6** Z_5, **5** $Z_2 \times Z_2$ **subgroups (36).**

In the case of A_4 (T') symmetry only there are 64 models (up to permutation of rows and columns).

A_4:
$(G_e, G_\nu) = (Z_2, Z_3)$, **A1 - A3;**
$(G_e, G_\nu) = (Z_2, Z_2)$, **A1 - A3;**
$(G_e, G_\nu) = (Z_3, Z_2)$, **B1 - B3;**
$(G_e, G_\nu) = (Z_2 \times Z_2, Z_2)$, **B1 - B3;**
$(G_e, G_\nu) = (Z_2, Z_2)$, **C1 - C9.**

For A_4, S_4 and A_5 the total number of models to be analysed is extremely large. However, a total of only 14 models survive the 3σ constraints on $\sin^2\theta_{ij}$ from the current data and the requirement $|\cos\delta| \leq 1$.

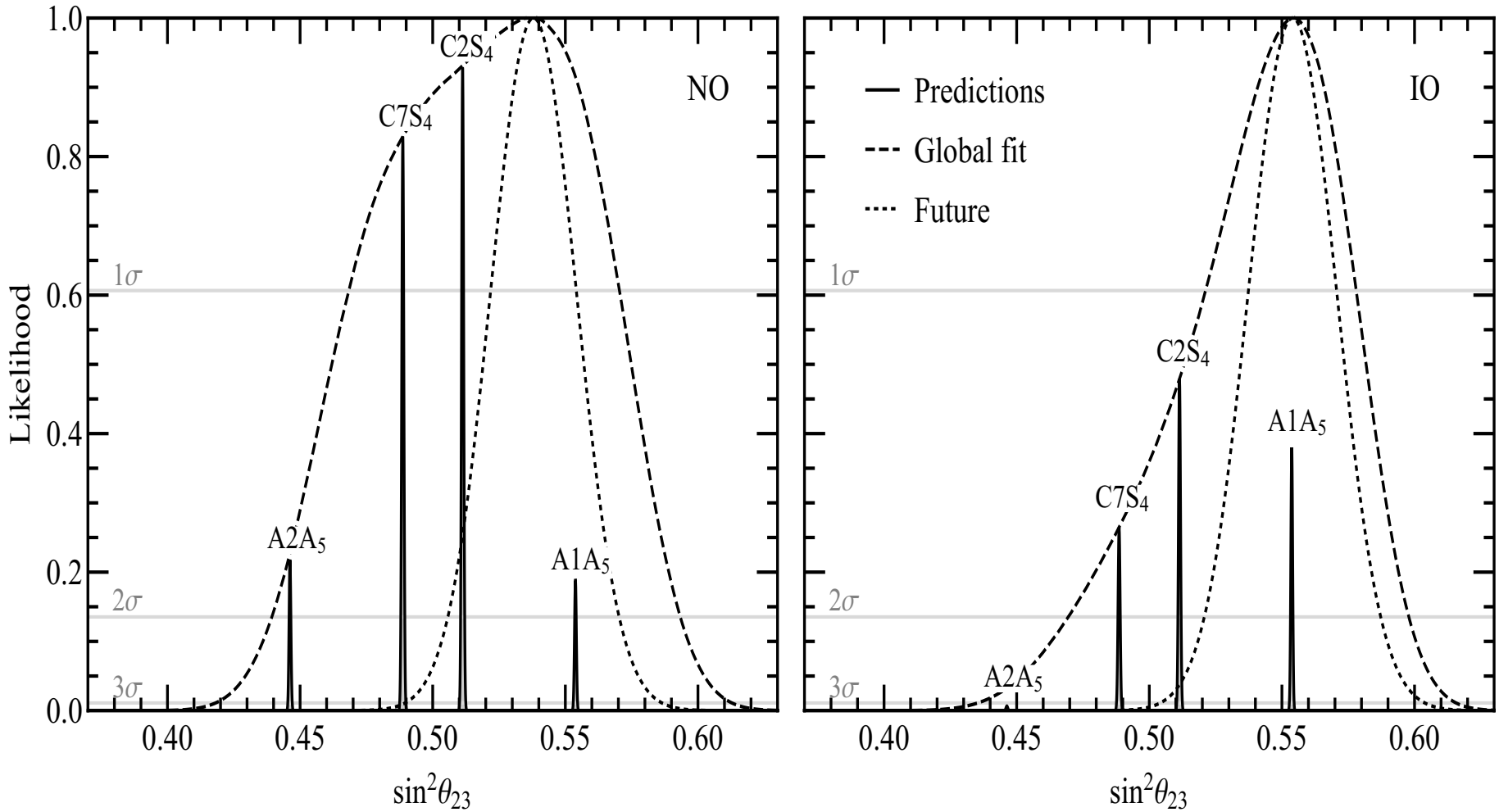
Phenomenologically Viable Predictions

A1 (A2), A_5 **(**$G_e =$**Z**$_2$**,** $G_\nu = Z_3$ **(Dirac** ν_j**)) :**
$\sin^2\theta_{23} \cong 0.553$ (0.447)**;** $\cos\delta \cong 0.716$ (-0.716)**.**

A1, S_4**:** $\sin^2\theta_{23} \cong 0.5(1-\sin^2\theta_{13}) \cong 0.489$**;**
$\cos\delta \cong -1$ **requires** $\sin^2\theta_{12} \cong 0.348$ **(!)**

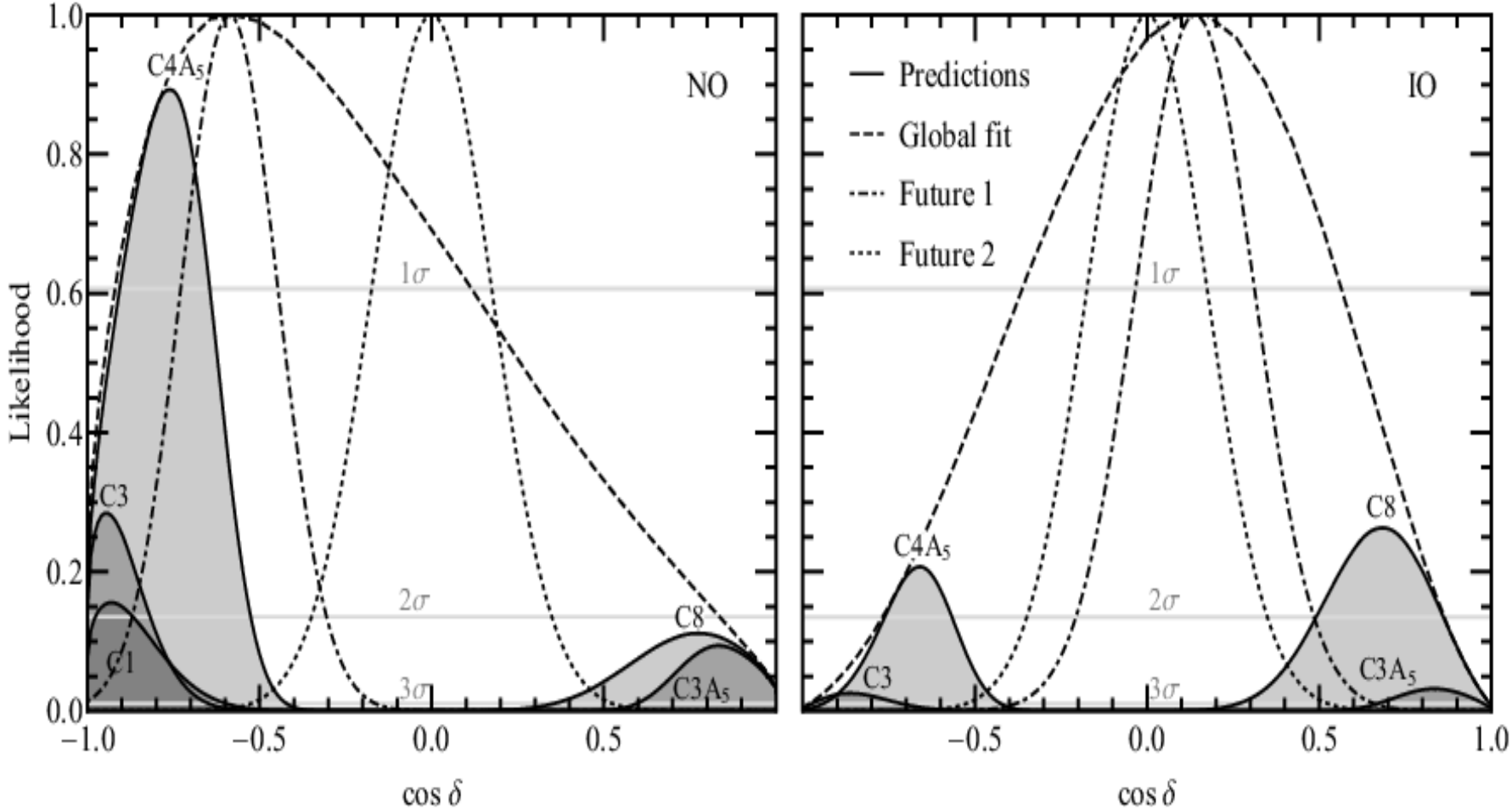
B1, A_4 **(**T'**,** S_4**,** $\mathbf{A}_5$**)** **(**$G_e = Z_3^T$**,** $G_\nu = Z_2^S$**):**
$U_{\rm PMNS} = U_{\rm TBM} U_{13}(\theta_{13}^\nu, \delta_{13}) Q_0$**;**
$\sin^2\theta_{12} = 1/(3\cos^2\theta_{13}) \cong 0.340$**;** $\cos\delta \cong 0.570$**.**

B2, S_4 **(**$G_e = Z_3^T$**,** $G_\nu = Z_2^{SU}$**):**
$\sin^2\theta_{12} \cong (1-2\sin^2\theta_{13})/3 = 0.319$**;** $\cos\delta \cong -0.269$**.**



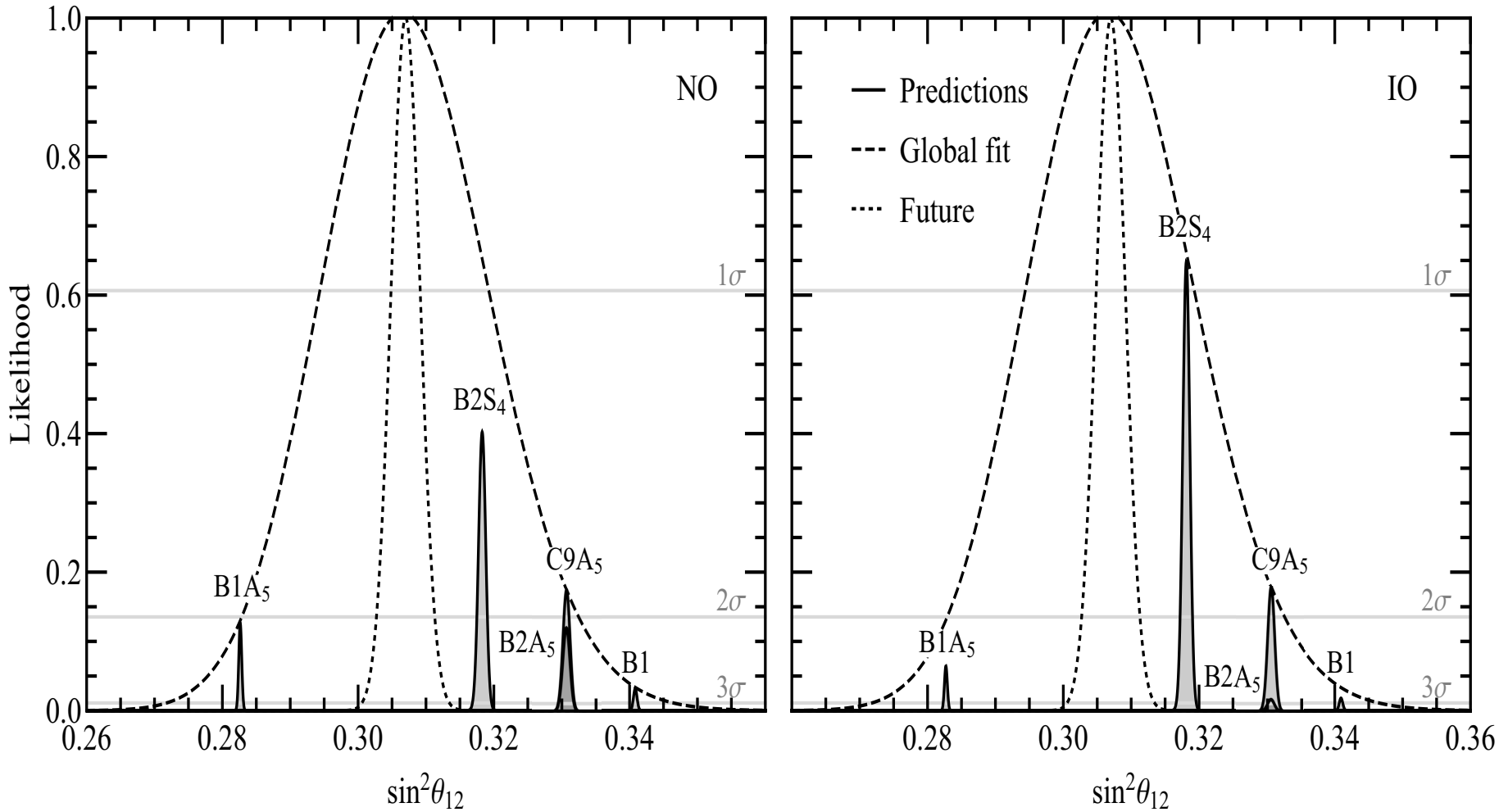
S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE).**



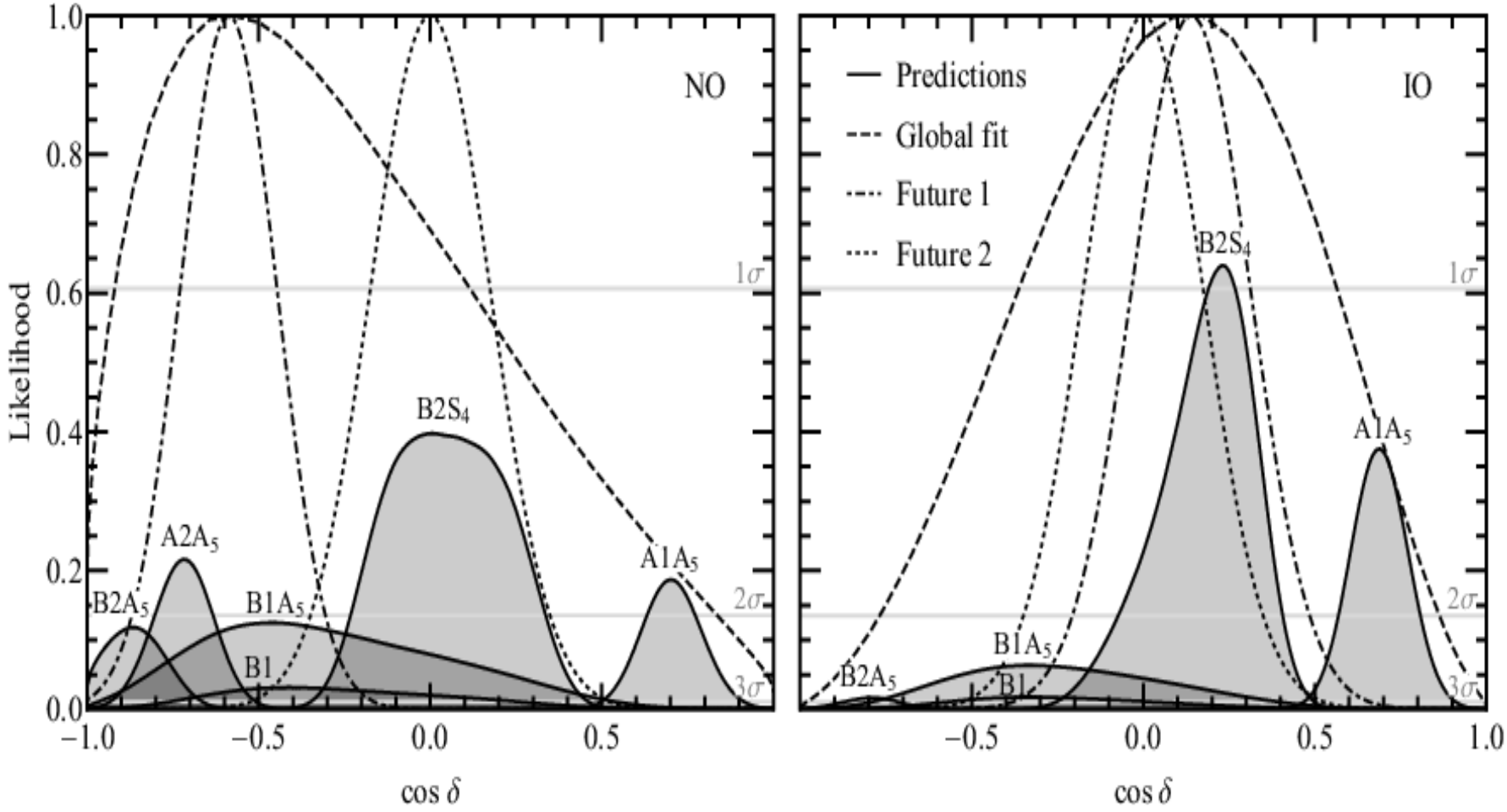
S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\delta) = 10^\circ$.



S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO).**

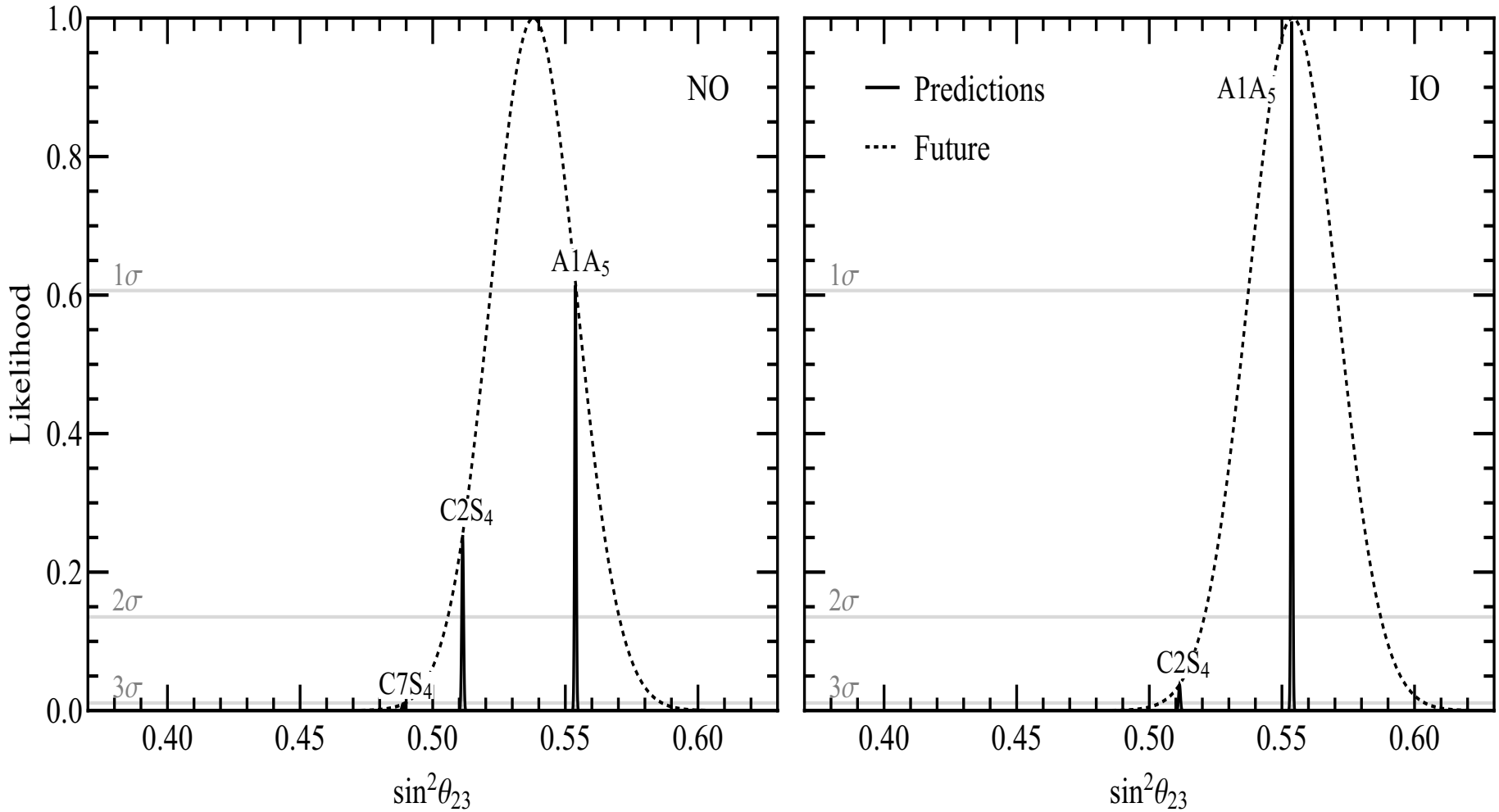


S.T.P., A. Titov, arXiv:1804.00182

A total of 6 models would survive out of the currently viable 14 (of the extremely large number) considered if $\delta(\sin^2\theta_{23}) = 3\%$, $\delta(\sin^2\theta_{12}) = 0.7\%$ and the current b.f.v. would not change:

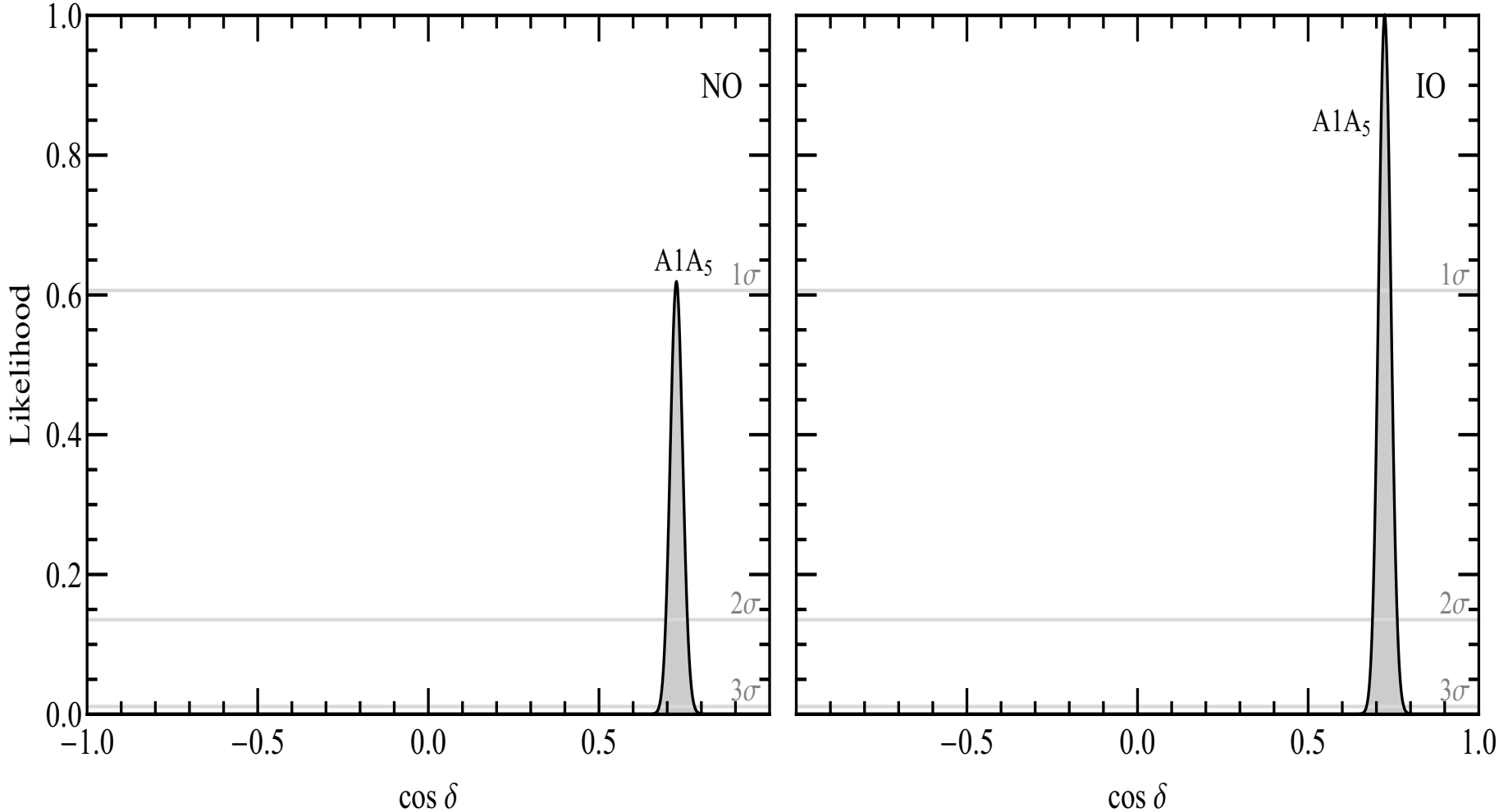
A1A$_5$, C2S$_4$, C3, C3A$_5$, C4A$_5$, C8.

Will be constrained further by the data on δ.



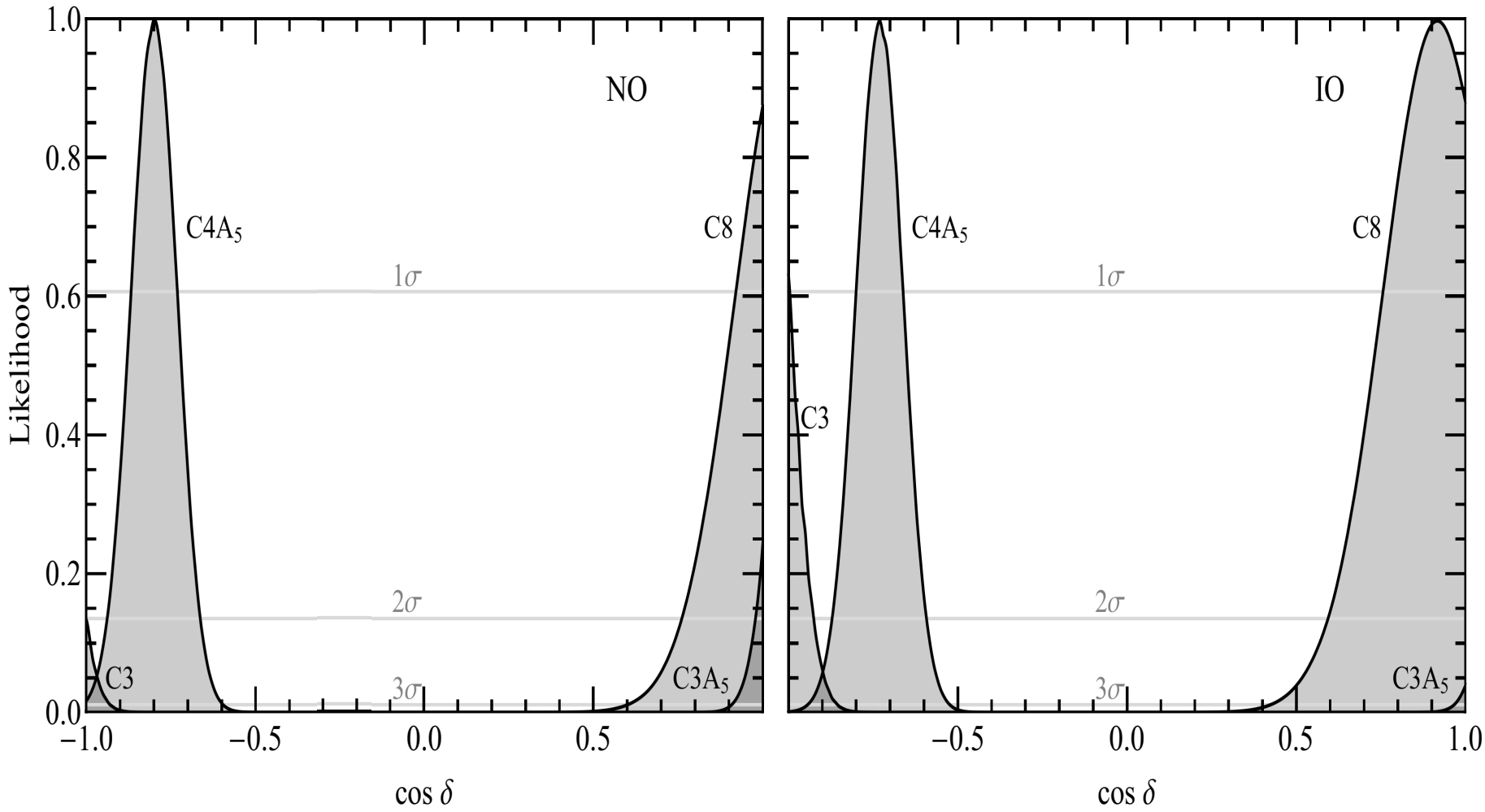
S.T.P., A. Titov, February 2018

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE); current b.f.v.**



S.T.P., A. Titov, arXiv:1804.00182

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE);**
$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO);**
current b.f.v. used.



S.T.P., A. Titov, arXiv:1804.00182

The predictions obtained for $\cos\delta$ are valid in a large class of theoretical models of (lepton) flavour based on discrete symmetries.

J. Gehrlein *et al.*, **"An $SU(5) \times A_5$ Golden Ratio Flavour Model", arXiv:1410.2095;**

I. Girardi *at al.*, **"Generalised Geometrical CP Violation in a T' Lepton Flavour Model", arXiv:1312.1966, JHEP 1402 (2014) 050.**

T' model of lepton flavour: $U_{\rm TBM}$, $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

- Light neutrino masses: type I seesaw mechanism.
- ν_j - Majorana particles.
- Diagonalisation of M_ν: $U_{\rm TBM}\Phi$, $\Phi = {\rm diag}(1, 1, 1(i))$
- $U_{\rm TBM}$ "corrected" by

$U^\dagger_{\rm lep}\, Q = R_{12}(\theta^\ell_{12})\, R_{23}(\theta^\ell_{23}) Q\,, \quad Q = {\rm diag}(1, e^{i\phi}, 1)$

T' model of lepton flavour: $U_{\rm TBM}$, $\delta \cong 3\pi/2$ or $\pi/2$.

- T': double covering of A_4 (tetrahedral symmetry group).

- T': $\mathbf{1}$, $1'$, $1''$; $\mathbf{2}$, $2'$, $2''$; $\mathbf{3}$.

- T' model: $\psi_{eL}(x)$,$\psi_{\mu L}(x)$,$\psi_{\tau L}(x)$ - triplet of T';
$e_R(x)$,$\mu_R(x)$ - a doublet, $\tau_R(x)$ - a singlet, of T';
$\nu_{eR}(x)$,$\nu_{\mu R}(x)$,$\nu_{\tau R}(x)$ - a triplet of T';
the Higgs doublets $H_u(x)$, $H_d(x)$ - singlets of T'.

- The discrete symmetries of the model are $T' \times H_{\rm CP} \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2$, the Z_n factors being the shaping symmetries of the superpotential required to forbid unwanted operators.

Predictions of the T' Model

- $m_{1,2,3}$ determined by 2 real parameters + Φ^2:

$$\frac{1}{m_1} - \frac{2}{m_2} = \frac{1}{m_3}, \quad \text{NO}$$

$$\text{NO, A}: \ (m_1, m_2, m_3) = (4.43, 9.75, 48.73) \cdot 10^{-3} \text{ eV},$$

$$\text{NO, B}: \ (m_1, m_2, m_3) = (5.87, 10.48, 48.88) \cdot 10^{-3} \text{ eV},$$

$$\text{IO}: \ (m_1, m_2, m_3) = (51.53, 52.26, 17.34) \cdot 10^{-3} \text{ eV},$$

$$\text{NO A}: \quad \sum_{j=1}^{3} m_j = 6.29 \times 10^{-2} \text{ eV},$$

$$\text{NO B}: \quad \sum_{j=1}^{3} m_j = 6.52 \times 10^{-2} \text{ eV},$$

$$\text{IO}: \quad \sum_{j=1}^{3} m_j = 12.11 \times 10^{-2} \text{ eV},$$

- $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$ determined by 3 real parameters.

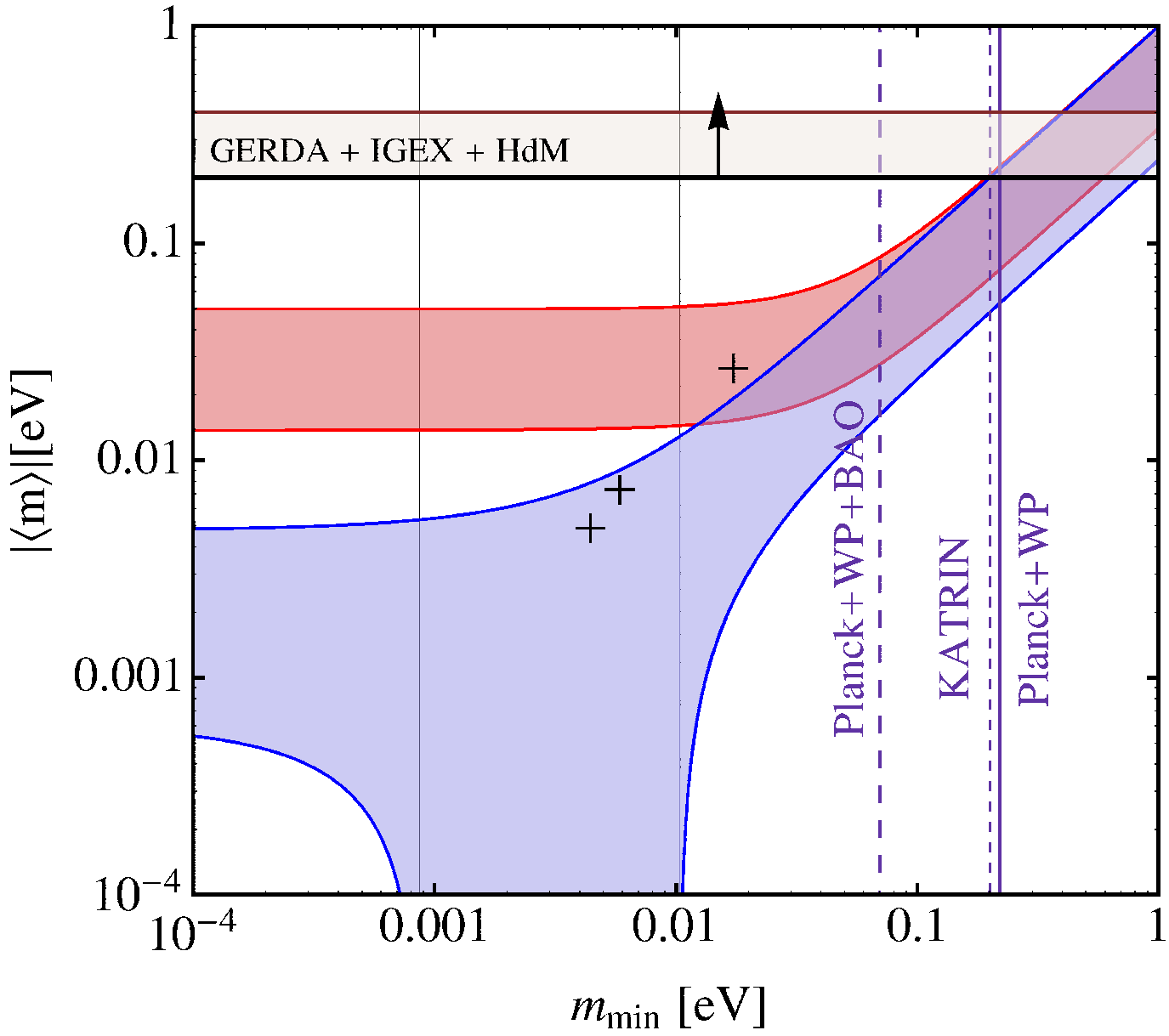
Given the values of $\theta_{12}, \theta_{23}, \theta_{13}$, $\delta, \alpha_{21}, \alpha_{31}$ are predicted:

$\delta \cong 3\pi/2$ (266°) (or $\pi/2$ (94°));

NO A: $\alpha_{21} \cong +47.0^\circ$ (or -47.0°) $(+2\pi)$,

$\alpha_{31} \cong -23.8^\circ$ (or $+23.8^\circ$) $(+2\pi)$.

The model is falsyfiable.


GERDA + IGEX + HdM
Planck+WP+BAO
KATRIN
Planck+WP
|⟨m⟩|[eV]
$m_{\min}$ [eV]
1
0.1
0.01
0.001
10^{-4}
10^{-4}
0.001
0.01
0.1
1

Conclusions.

- **The observed pattern of neutrino mixing can be due to a new fundamental (approximate) symmetry of particle interactions leading to an approximate symmetry form of the PMNS matrix. We have considered the following symmetry forms: TBM, BM (LC), GRA, GRB and HG. Each of these forms can be obtained from a specific discrete flavour symmetry.**

- **For all the forms considered $\theta^{\nu}_{13} = 0$ and $\theta^{\nu}_{23} = -\pi/4$. The forms differ by the value of θ^{ν}_{12}. Values of the neutrino mixing angles θ_{ij} compatible with the observations are obtained with the help of subleading (perturbative) corrections generated by $U_{\rm lep}$ coming from the diagonalisation of the charged lepton mass matrix.**

- **The most important testable consequence of this approach to understanding the pattern of of neutrino mixing is the correlation between the value of $\cos\delta$ and the values of the neutrino mixing angles: $\delta = \delta(\theta_{12}, \theta_{13}, \theta_{23}; \theta^{\nu}_{12})$. The correlation depends on the underlying approximate symmetry form of the $U_{\rm PMNS}$.**

- **The precise knowledge of the value of $\sin^2\theta_{23}$, in particular, is crucial for testing the predictions obtained following the approach discussed by us and for discriminating between various cases possible within this approach.**

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.