Stringent tests of QED using highly charged ions

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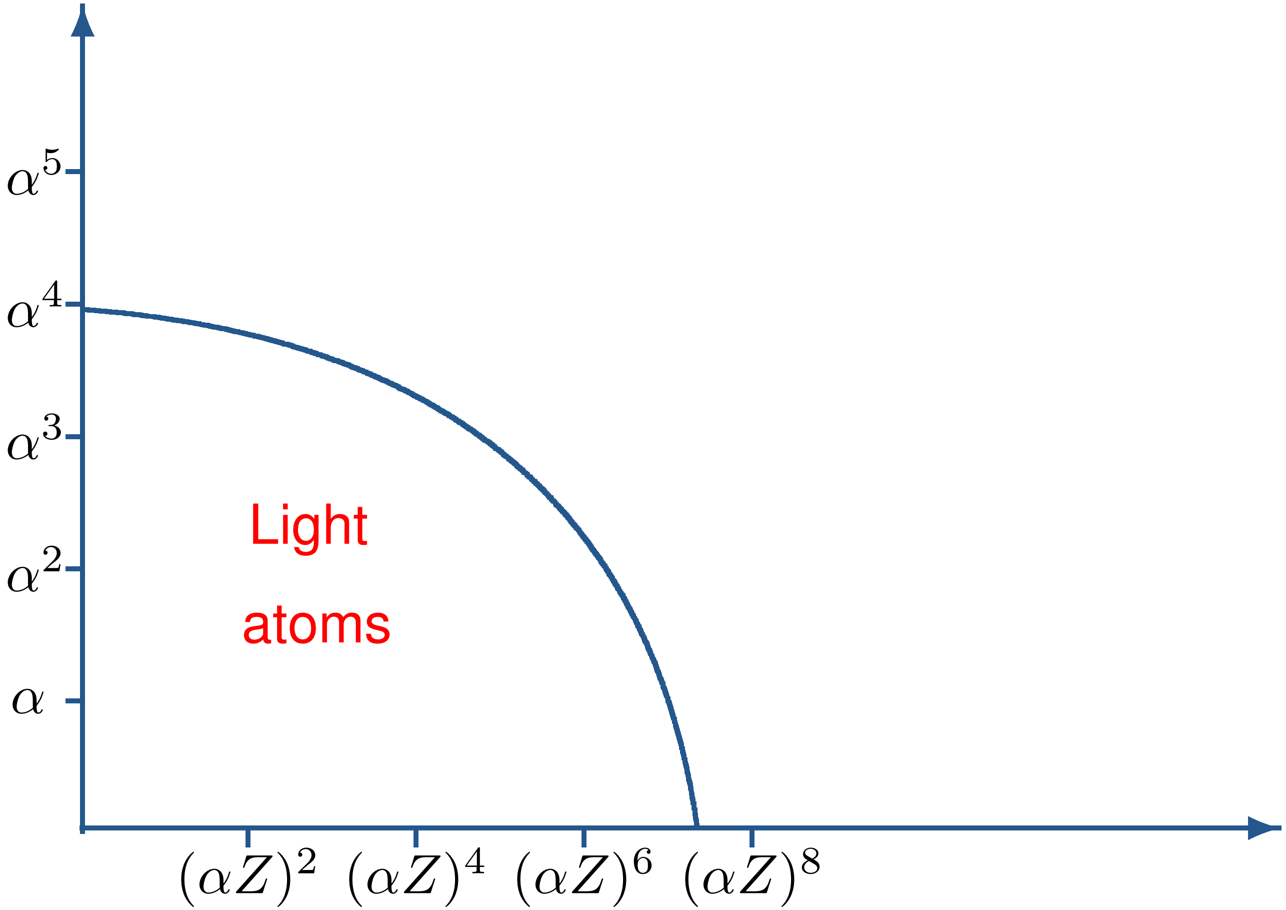
Outline of the talk

- Introduction
- Bound-state QED with highly charged ions
  - Lamb shift
  - Hyperfine splitting
  - Bound-electron g factor
- Low-energy heavy-ion collisions and supercritical fields
  - Access to supercritical fields
  - Charge transfer in the U^{91+}–U^{92+} collision
  - Pair creation in the U^{92+}–U^{92+} collision
- Conclusion

Introduction: tests of QED with light atoms

Light atoms

Tests of QED to lowest orders in $\alpha \approx \frac{1}{137}$ and in $\alpha Z \ll 1$
(Z is the nuclear charge number)



Introduction: QED theory of heavy ions

Heavy few-electron ions

$$N \ll Z\,,$$

where Z is the nuclear charge number and N is the number of electrons.

To zeroth-order approximation:

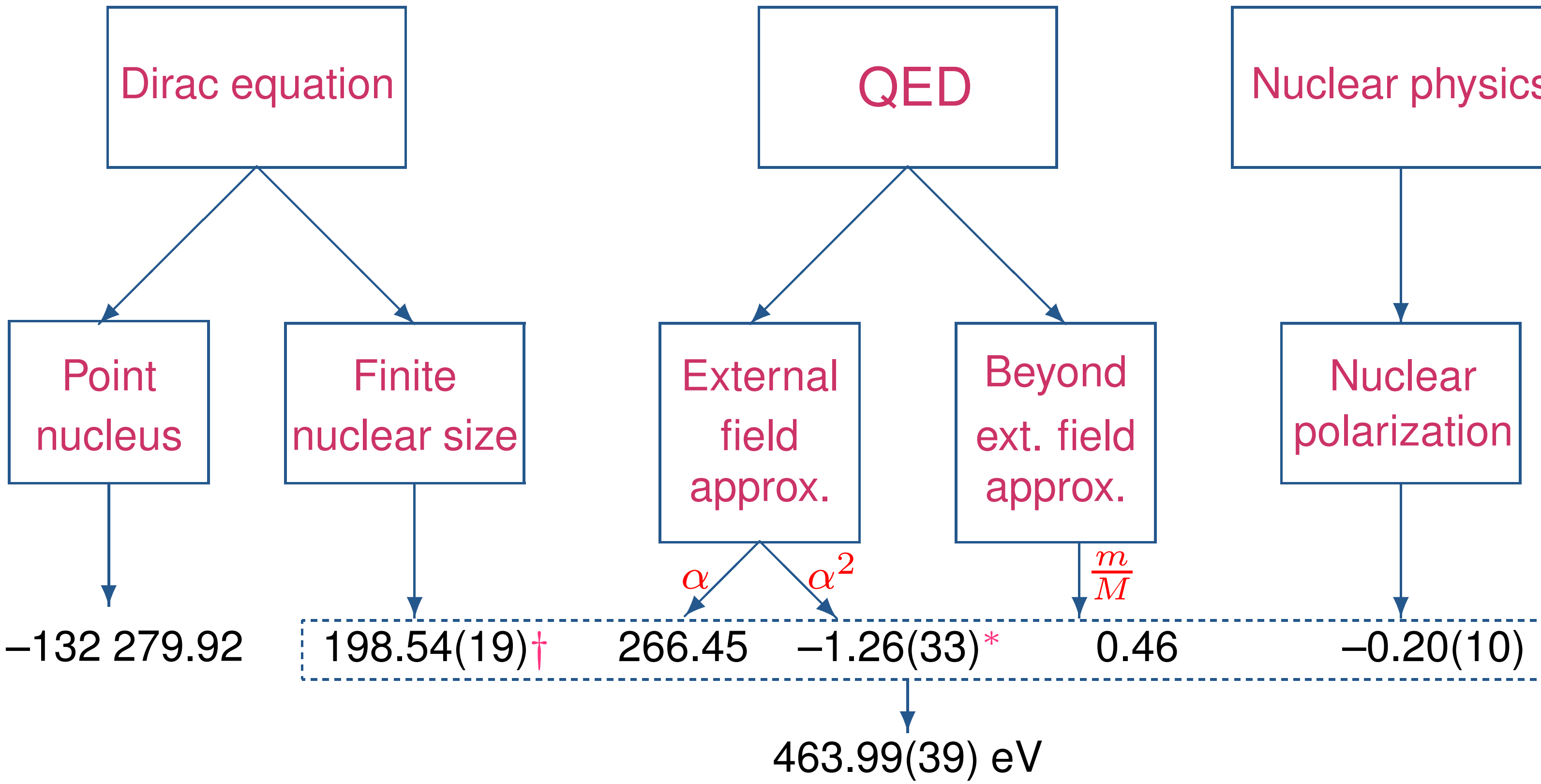
$$(-i\,\vec{\alpha}\cdot\vec{\nabla} + m\beta + V_{\mathrm{C}}(r)\,)\,\psi\,(\vec{r}) = \,E\,\psi\,(\vec{r})$$

Interelectronic-interaction and QED effects:

$$\frac{\text{Interelectronic interaction}}{\text{Binding energy}} \sim \frac{1}{Z}\,, \qquad \frac{\text{QED}}{\text{Binding energy}} \sim \alpha\,.$$

In contrast to light atoms, the parameter αZ is not small.
In uranium: $Z = 92$, $\alpha Z \approx 0.7$.

1s Lamb shift in H-like uranium, in eV



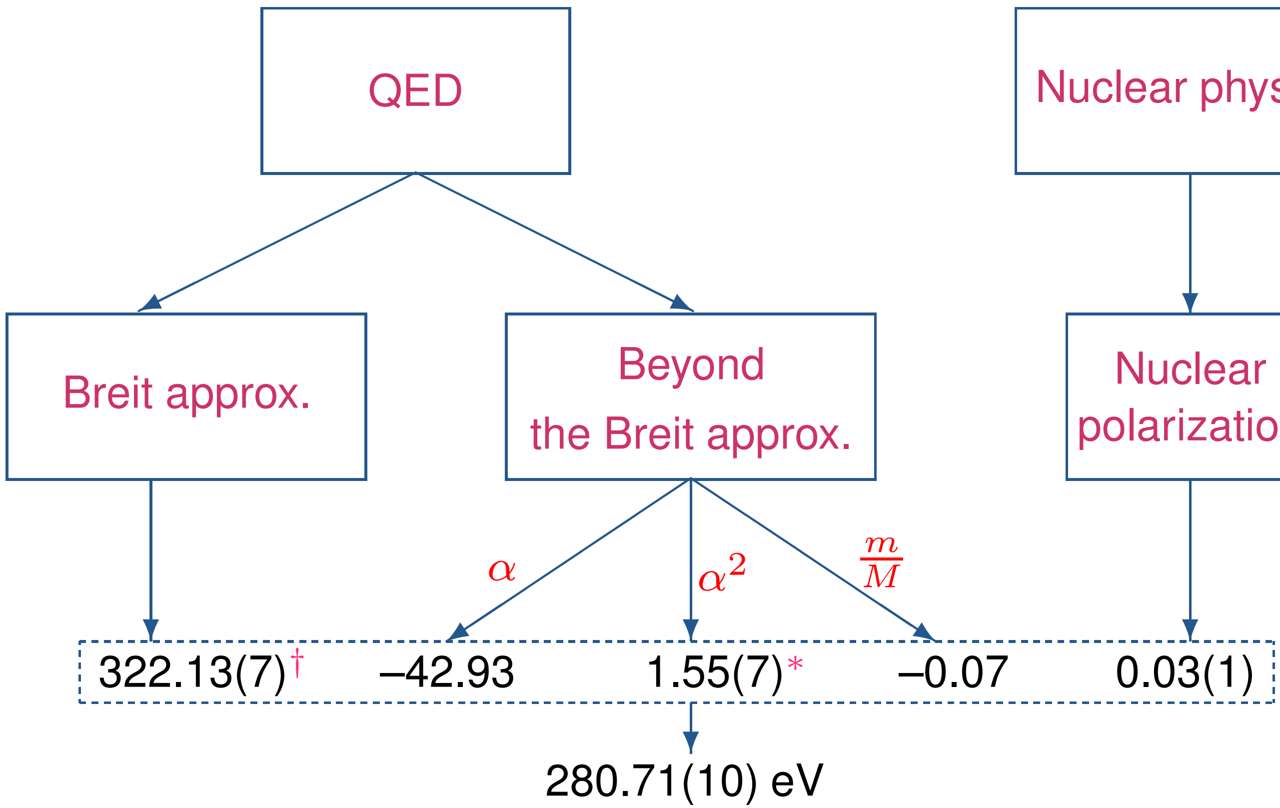
Experiment: 460.2(4.6) eV
(A. Gumberidze, T. Stöhlker, D. Banas et al., PRL, 2005)

Test of QED: $\sim$ 2%

* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

† Y.S. Kozhedub, O.V. Andreev, V.M. Shabaev et al., PRA, 2008

$2p_{1/2}$-$2s$ *transition energy in Li-like uranium, in eV*



Experiment: 280.59(10) eV (J. Schweppe et al., PRL, 1991)
280.52(10) eV (C. Brandau et al., PRL, 2003)
280.645(15) eV (P. Beiersdorfer et al., PRL, 2005)

Test of QED: $\sim$ 0.2%

* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

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Hyperfine splitting in H-like ions

I. Klaft et al., PRL, 1994:

$^{209}\mathrm{Bi}^{82+}$ ΔE^{exp}=5.0840(8) eV

J. Crespo Lopez-Urritia et al., PRL, 1996; PRA, 1998:

$^{165}\mathrm{Ho}^{66+}$ ΔE^{exp}=2.1645(6) eV

$^{185}\mathrm{Re}^{74+}$ ΔE^{exp}=2.7190(18) eV

$^{187}\mathrm{Re}^{74+}$ ΔE^{exp}=2.7450(18) eV

P. Seelig et al., PRL, 1998:

$^{207}\mathrm{Pb}^{81+}$ ΔE^{exp}=1.2159(2) eV

P. Beiersdorfer et al., PRA, 2001:

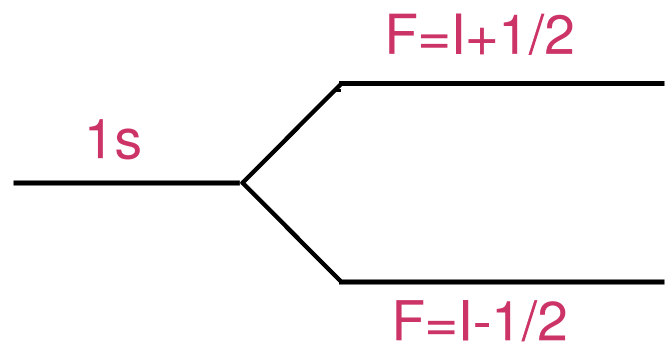
$^{203}\mathrm{Tl}^{80+}$ ΔE^{exp}=3.21351(25) eV

$^{205}\mathrm{Tl}^{80+}$ ΔE^{exp}=3.24410(29) eV

J. Ullmann et al., JPB, 2015:

$^{209}\mathrm{Bi}^{82+}$ ΔE^{exp}=5.08505(8) eV

Hyperfine splitting in H-like ions



$$\Delta E = \Delta E_{\rm Dirac}(1-\varepsilon) + \Delta E_{\rm QED}\ ,$$

where ε is the nuclear magnetization distribution correction
(the Bohr-Weisskopf effect)

Tests of QED in HFS study

We consider *(V.M. Shabaev, A.N. Artemyev, V.A. Yerokhin, O.M. Zherebtsov, and G. Soff, PRL, 2001)*

$$\Delta' E = \Delta E^{(2s)} - \xi \Delta E^{(1s)},$$

where $\Delta E^{(1s)}$ is the HFS in H-like ion, $\Delta E^{(2s)}$ is the HFS in Li-like ion, and ξ is chosen to cancel the Bohr-Weisskopf effect. In the case of Bi, ξ = 0.16886.

This method has a potential to test QED on level of a few percent, provided the HFS is measured to accuracy $\sim 10^{-6}$.

Calculations of the screened QED and higher-order interelectronic-interaction corrections to the HFS in Li-like ions have been peformed in: *[A.V. Volotka et al., PRL, 2009; D.A. Glazov et al., PRA, 2010; O.Y. Andreev et al., PRA, 2012; A.V. Volotka et al., PRL, 2012]* .

Current value for the specific HFS difference in Bi

Theoretical contributions to $\Delta' E = \Delta E^{(2s)} - \xi \Delta E^{(1s)}$ (in meV) for $\mu/\mu_N = 4.1106(2)$ *(A.V. Volotka et al., PRL, 2012; O.V. Andreev et al., PRA, 2012)*

Dirac value	-31.809
Interel. inter., $\sim 1/Z$	-29.995
Interel. inter., $\sim 1/Z^2$ and h.o.	0.255(3)
One-electron QED	0.036
Screened QED	0.193(2)
Total	-61.320(6)
Experiment *[1]*	-61.012 (5)(21)

[1] J. Ullmann et al., Nature Communications, 2017.

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We can use these theory and experiment for an independent determination of the nuclear magnetic moment. This gives $\mu/\mu_N = 4.0900(15)$. New calculations of the shielding constant and new NMR measurements in Bi(NO)$_3$ and BiF$_6^-$ yielded $\mu/\mu_N = 4.092(2)$ *(L. Skripnikov et al., PRL, 2018)*.

g factor of H-like ions

Definition: $g = -\langle \mu_z \rangle / \mu_{\rm B} \langle J_z \rangle$.

High-precision measurement of the g factor of ${}^{12}{\rm C}^{5+}$ using a single ion confined in a Penning ion trap *(H. Häffner et al., PRL, 2000)*:

$g_{\rm exp} = 2(\omega_L/\omega_c)(m_e/M)(q/|e|) = 2.001\,041\,596\,3\,(10)(44)$.

Here $\omega_c = (q/M)B$ is the cyclotron frequency, $\omega_L = \Delta E/\hbar$, M is the ion mass, and q is the ion charge. The second uncertainty (44) was due to the uncertainty of the (m_e/M) ratio. Combined with the related theory, which included evaluations of the higher-order relativistic recoil and QED corrections *(V.M. Shabaev and V.A. Yerokhin, PRL, 2002; V.A. Yerokhin et al., PRL, 2002)*, this resulted in four-times improvement of the accuracy of the electron mass.

By new experiments *(S. Sturm et al., Nature, 2014)* the precision of the atomic mass of the electron was additionally improved by a factor of 13.

Recent progress in the theory *(A. Czarnecki et al., PRL, 2018)*: the two-loop QED corrections of order $\alpha^2(\alpha Z)^5$ have been calculated.

Isotope shift of the g factor of Li-like ions

Isotope shift of the g-factor of Li-like calcium: $^{40}\mathrm{Ca}^{17+} - {}^{48}\mathrm{Ca}^{17+}$

(F. Köhler et al., Nature Communications, 2016.)

Nuclear recoil: one-electron non-QED	$0.000000012240(1)$
Nuclear recoil: interelectronic int.	$-0.000000002051(22)$
Nuclear recoil: QED $\sim m/M$	$0.000000000123(12)$
Nuclear recoil: QED $\sim \alpha(m/M)$	$-0.000000000009(1)$
Finite nuclear size	$0.000000000004(9)$
Total theory	$0.000000010305(27)$
Experiment	$0.0000000117(14)$

The calculation of the interelectronic-interaction recoil effect is based on the interpolation of the results from *[Z.-C. Yan, PRL, 2001; JPB, 2002]* which were obtained within the two-component approach *[R.A. Hegstrom, PRA, 1975]*.

Recoil effect on the g factor: QED approach

Formula for the nuclear recoil effect on the g-factor of an ion with one electron over closed shells to first order in m/M and to all orders in αZ *(V.M. Shabaev, PRA, 2001)*:

$$\Delta g = \frac{1}{\mu_0 m_a} \frac{i}{2\pi M} \int_{-\infty}^{\infty} d\omega \left[\frac{\partial}{\partial \mathcal{H}} \langle a | [\vec{p} - \vec{D}(\omega) + e\vec{A}_{\rm cl}] \right.$$
$$\left. \times G(\omega + E_a) [\vec{p} - \vec{D}(\omega) + e\vec{A}_{\rm cl}] | a \rangle \right]_{\mathcal{H}=0} .$$

Here μ_0 is the Bohr magneton, m_a is the angular momentum projection, $\vec{A}_{\rm cl} = [\vec{\mathcal{H}} \times \vec{r}]/2$ is the vector potential of the homogeneous magnetic field $\vec{\mathcal{H}}$ directed along the z axis. It is implied that all quantities are calculated in the presence of the magnetic field. The Fermi energy is chosen to be slightly above the closed schell levels.

Recoil effect on the g factor: Breit approximation

Recoil effect within the Breit approximation: four-component approach

The recoil effect on the g factor is given by a sum of two contributions. The first one is determined by the combined interaction due to $\delta V(\mathbf{x}) = -e\boldsymbol{\alpha}\cdot[\mathcal{H}\times\mathbf{r}]/2$ and the two-electron part of the effective recoil Hamiltonian:

$$H_M = \frac{1}{2M}\sum_{i,k}\Big[\mathbf{p}_i\cdot\mathbf{p}_k - \frac{\alpha Z}{r_i}\big(\boldsymbol{\alpha}_i + \frac{(\boldsymbol{\alpha}_i\cdot\mathbf{r}_i)\mathbf{r}_i}{r_i^2}\big)\cdot\mathbf{p}_k\Big]\,.$$

The second contribution is defined by the magnetic recoil operator:

$$H_M^{\rm magn} = -\mu_0\mathcal{H}\frac{m}{M}\sum_{i,k}\Big\{[\mathbf{r}_i\times\mathbf{p}_k] - \frac{\alpha Z}{2r_k}\Big[\mathbf{r}_i\times\Big(\boldsymbol{\alpha}_k + \frac{(\boldsymbol{\alpha}_k\cdot\mathbf{r}_k)\mathbf{r}_k}{r_k^2}\Big)\Big]\Big\}\,.$$

Isotope shift of the g factor of Li-like ions

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Nuclear recoil: one-electron non-QED	$0.000000012240(1)$
Nuclear recoil: interelectronic int.	-0.000000001302
Nuclear recoil: QED $\sim m/M$	$0.000000000123(12)$
Nuclear recoil: QED $\sim \alpha(m/M)$	$-0.000000000009(1)$
Finite nuclear size	$0.000000000004(9)$
Total theory *[1]*	$0.000000011056(16)$ [1]
Experiment *[2]*	$0.0000000117(14)$

[1] V.M. Shabaev, D.A. Glazov, A.V. Malyshev, and I.I. Tupitsyn, PRL, 2017.
[2] F. Köhler et al., Nature Communications, 2016.

The calculation of the interelectronic-interaction recoil effect is based on the four-component approach *[1]*.

Future prospects for the g-factor investigations

1) Tests of bound-state QED at strong fields

For stringent tests of QED in the g-factor experiments, one should study specific differences of the g factors of H-, Li- and B-like ions.

2) Tests of QED beyond the Furry picture *(A.V. Malyshev, V.M. Shabaev, D.A. Glazov, and I.I. Tupitsyn, JETP Letters, 2017).*

3) Determination of the nuclear magnetic moments

$$g_{\rm atom} = g^{(e)} \frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)} - \frac{m_e}{m_p} g^{(N)} \frac{F(F+1)+I(I+1)-J(J+1)}{2F(F+1)} .$$

4) Determination of the fine structure constant by studying the g factors of H-, Li-, and B-like ions *(V.M. Shabaev et al., PRL, 2006; V.A. Yerokhin et al., PRL, 2016).*

QED at supercritical fields

Schwinger mechanism

The rate of pair production for a constant uniform electric field E:

$$\frac{d^4 n_{e^+e^-}}{d^3x dt} \sim \frac{c}{4\pi^3 {\lambda_{\rm C}}^4} \exp\Big(-\pi \frac{E_c}{E}\Big)$$

where $\lambda_{\rm C} = \hbar/(mc)$ and

$$E_c = m^2 c^3/(e\hbar) \approx 1.3 \times 10^{16} {\rm V/cm}$$

is the critical field strength.

Unfortunately, it is not possible to produce the macroscopic static fields with the electric field strength close to E_c.

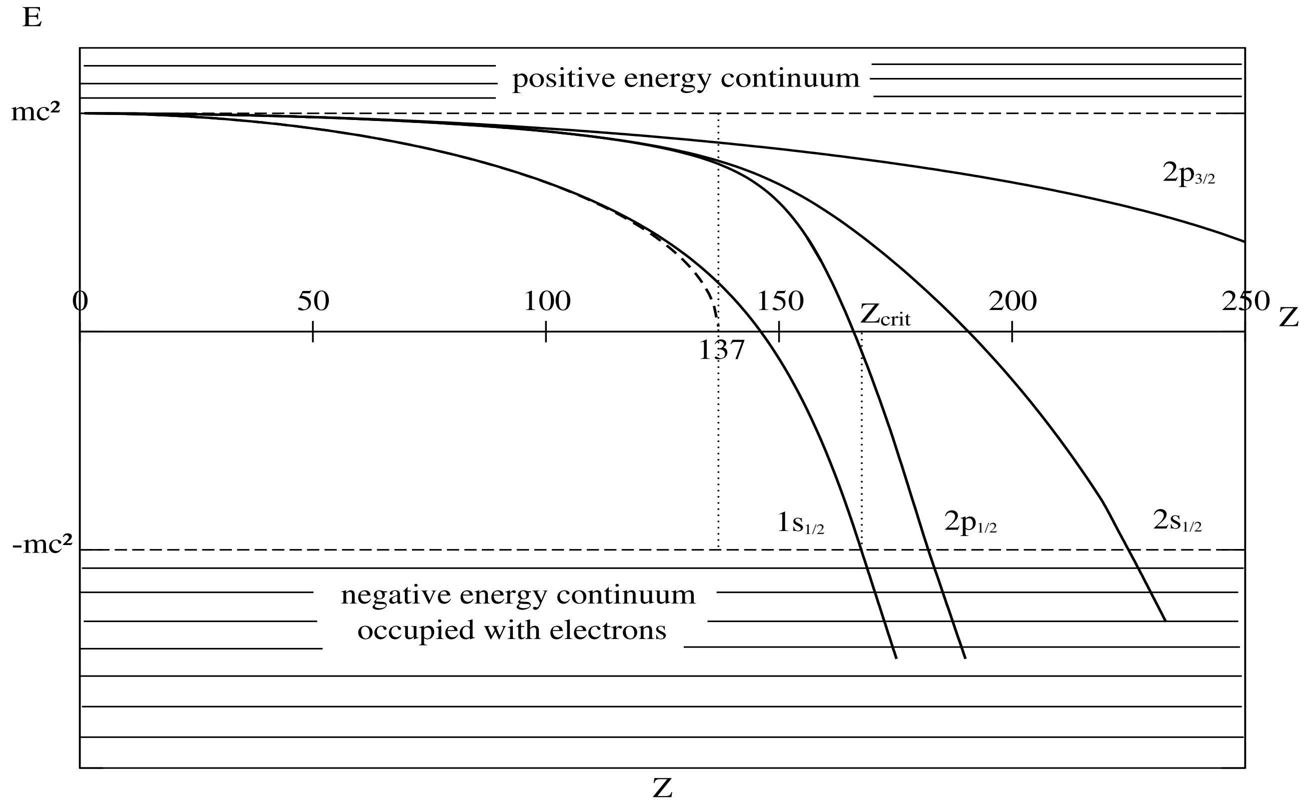
Recent progress on pair production in two counter-propagating laser pulses:

I.A. Aleksandrov, G. Plunien, and V.M. Shabaev, PRD, 2017.

Low-energy heavy-ion collisions

Access to supercritical fields

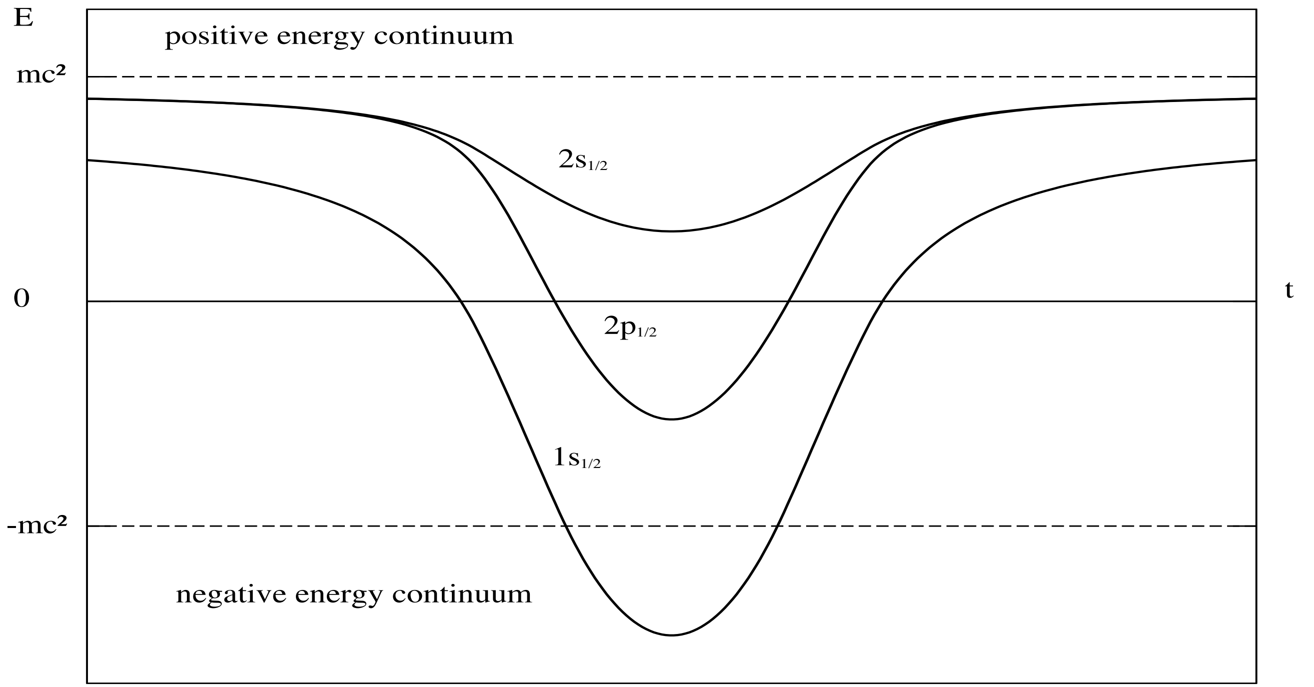
S.S. Gershtein, Ya.B. Zel'dovich, 1969; W. Pieper, W. Greiner, 1969



The $1s$ level dives into the negative-energy continuum at $Z_{\rm crit} \approx 173$.

Low-energy heavy-ion collisions

Creation of the supercritical field in heavy-ion collisions, with $Z_1 + Z_2 > 173$



The ground state dives into the negative-energy continuum for about 10^{-21} sec.

Low-energy heavy-ion collisions

New method for solving the time-dependent two-center Dirac equation (*I.I. Tupitsyn, Y.S. Kozhedub, V.M. Shabaev et al., PRA, 2010*):

$$i\frac{\partial\Psi(\vec{r},t)}{\partial t} = c(\vec{\alpha}\cdot\vec{p}) + \beta\, mc^2 + V^{(A)}_{\rm nucl}(\vec{r}_A) + V^{(B)}_{\rm nucl}(\vec{r}_B)\,,$$

where $\vec{r}_A = \vec{r} - \vec{R}_A$, $\vec{r}_B = \vec{r} - \vec{R}_B$.

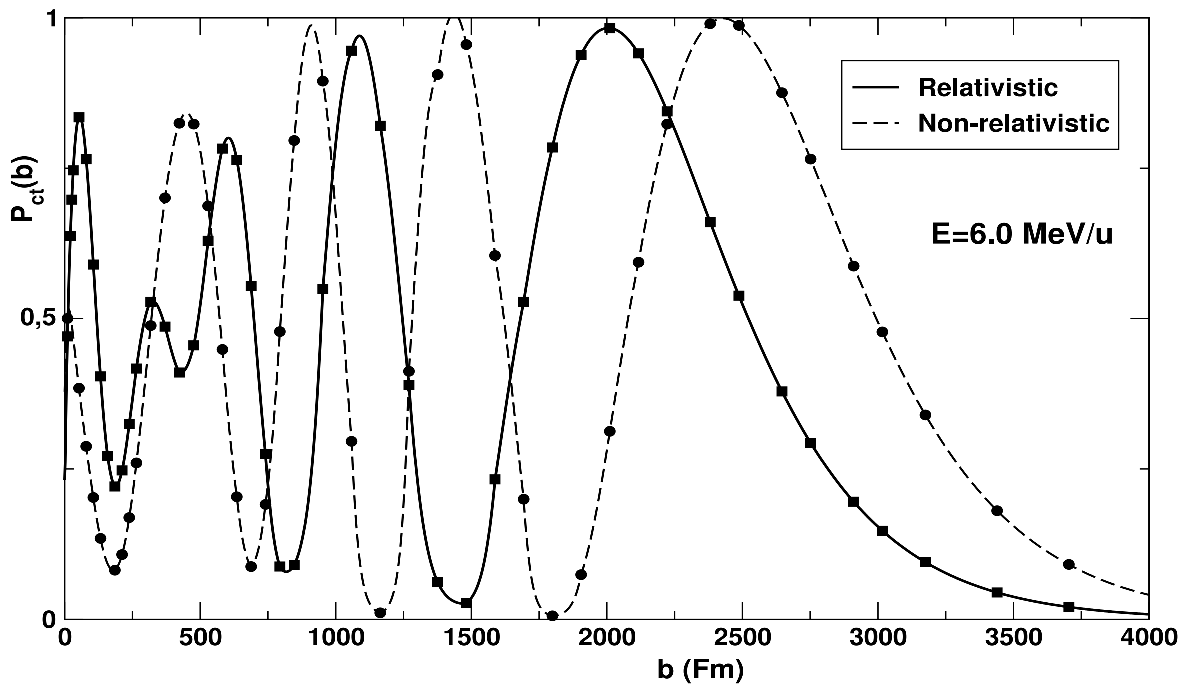
The time-dependent Dirac wave function is presented as a sum of atomic-like Dirac-Sturm orbitals localized at the ions.

The method has been tested by calculation of the charge-transfer and ionization probabilities for low-Z systems and comparison with the related nonrelativistic results.

Extention of the method to collisions of neutral atoms with H-like ions and comparison with related experiments: *I.I. Tupitsyn et al., PRA, 2012*.

Low-energy heavy-ion collisions

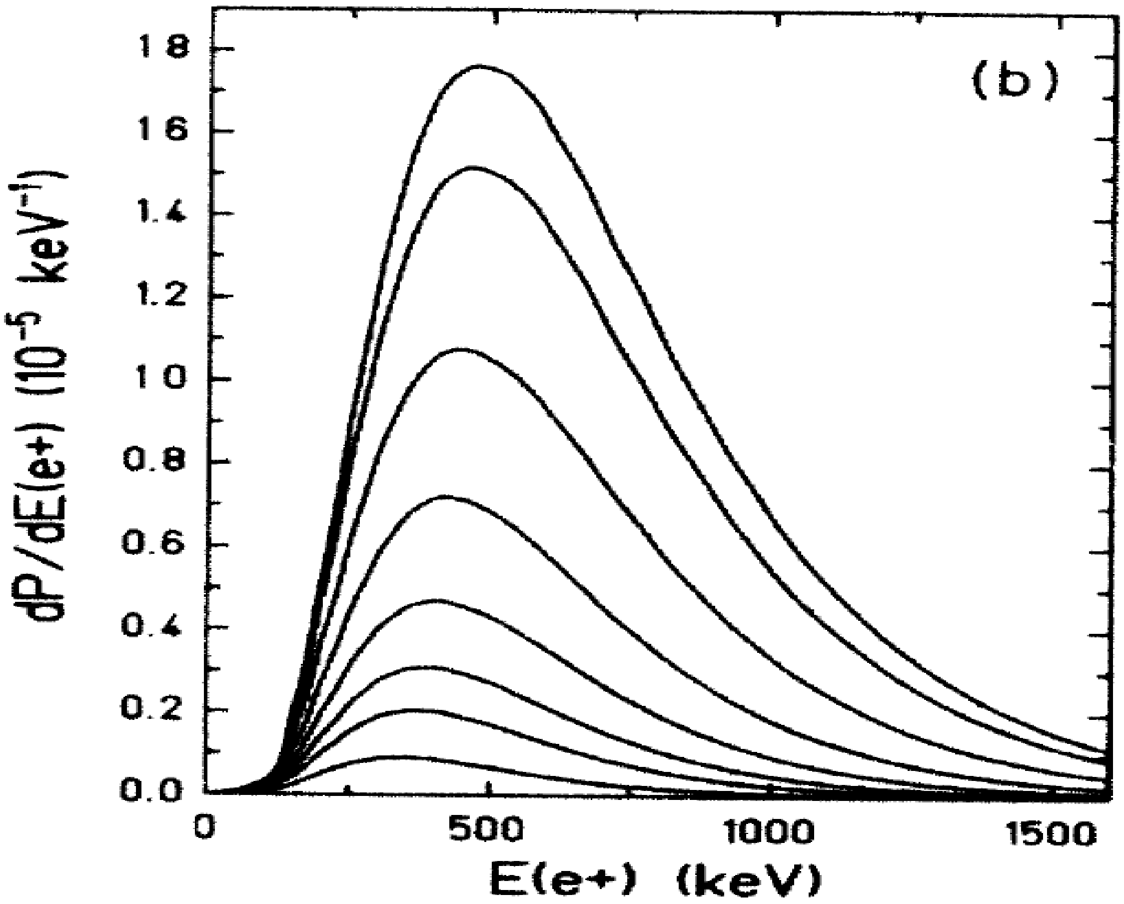
Charge-transfer probability for the $U^{91+}(1s){-}U^{92+}$ collision



Charge-transfer probability as a function of the impact parameter b for the projectile energy of 6 MeV/u *(I.I. Tupitsyn et al., PRA, 2012)*. The same results are obtained by a different method *(I.A. Maltsev et al., Phys. Scr., 2013)*.

Low-energy heavy-ion collisions

Electron-positron pair production in low-energy U-U collisions



Energy distribution of positrons emitted in U-U collisions at energy E=6.2 MeV/u for the impact parameter in the range: $b = 0 - 40$ fm *(U. Müller, T. de Reus, J. Reinhardt et al., Phys. Rev. A, 1988)*.

Low-energy heavy-ion collisions

Pair creation beyond the monopole approximation

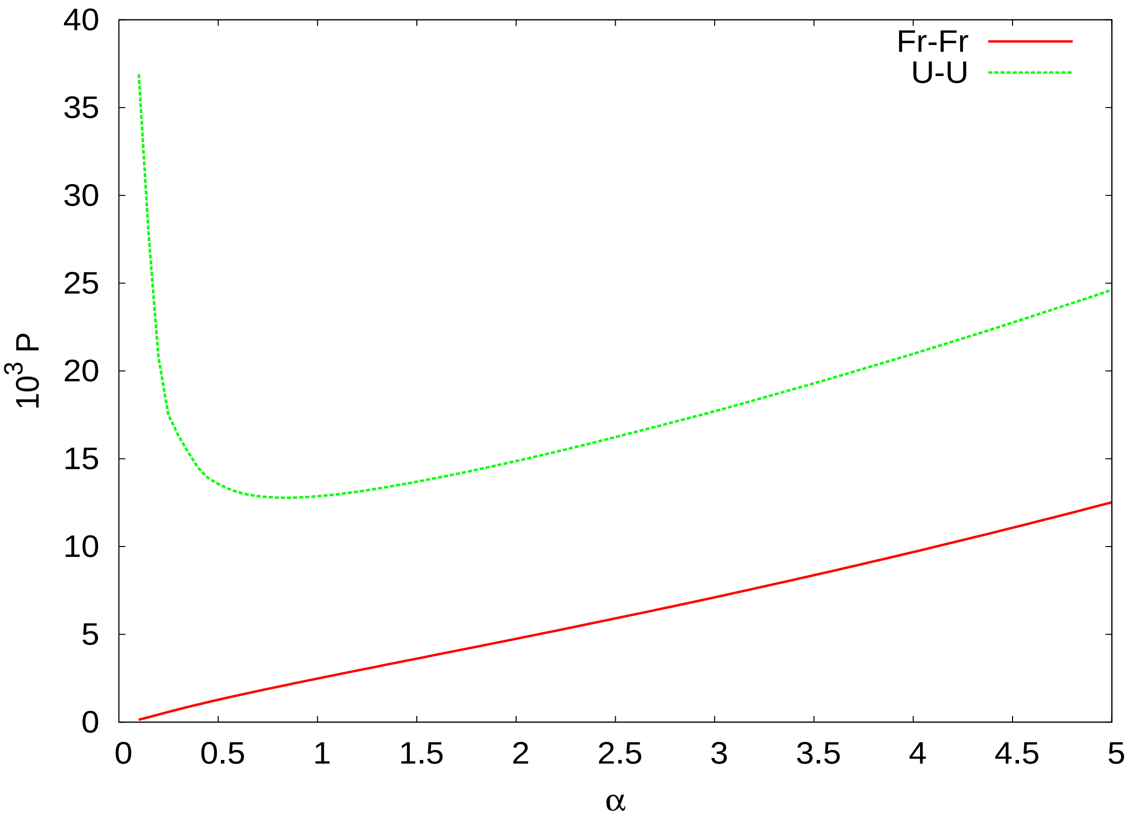
U-U, $E_{\rm cm}$ = 740 MeV

Expected number of created pairs with an electron captured into a bound state as a function of the impact parameter b *(I.A. Maltsev et al., NIMB, 2017)* .

b (fm)	Two-center approach	Monopole approximation
0	1.33×10^{-2}	1.25×10^{-2}
10	7.71×10^{-3}	7.03×10^{-3}
20	3.12×10^{-3}	2.70×10^{-3}
30	1.28×10^{-3}	1.03×10^{-3}
40	5.66×10^{-4}	4.09×10^{-4}

The two-center result for $b = 0$ has been confirmed by a different method *(R.V. Popov et al., arXiv:1802.02799)* .

Low-energy heavy-ion collisions



Pair creation with artificial trajectories for the supercritical U$-$U and subcritical Fr$-$Fr collisions at $E_{\rm cm} = 674.5$ and $E_{\rm cm} = 740$ MeV, respectively. The trajectory $R_\alpha(t)$ is defined by $\dot{R}_\alpha(t) = \alpha \dot{R}(t)$, where $R(t)$ is the classical Rutherford trajectory *(I.A. Maltsev et al., PRA, 2015)*.

Conclusion

Investigations of heavy ions at low-energy regime can provide:

- Tests of QED at strong coupling regime within and beyond the Furry picture
- Determination of the fundamental constants
- Tests of QED in supercritical fields