Proton radius and Rydberg constant from electronic and muonic atoms

Status of the "Proton Radius Puzzle"

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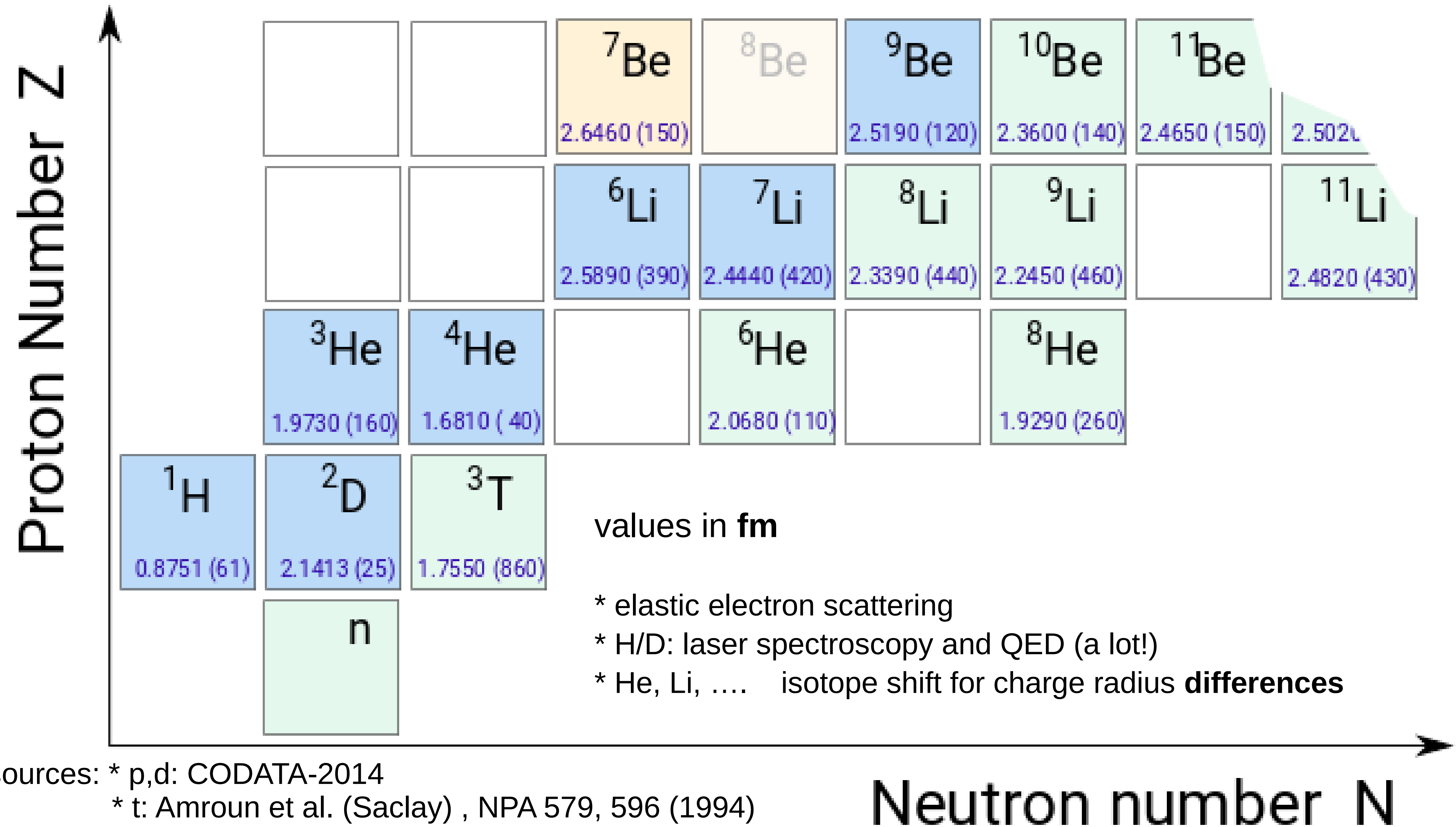
before: Max-Planck Institute of Quantum Optics

SSP2018 Aachen
June 14, 2018

Nuclear rms charge radii

from measurements with **electrons**



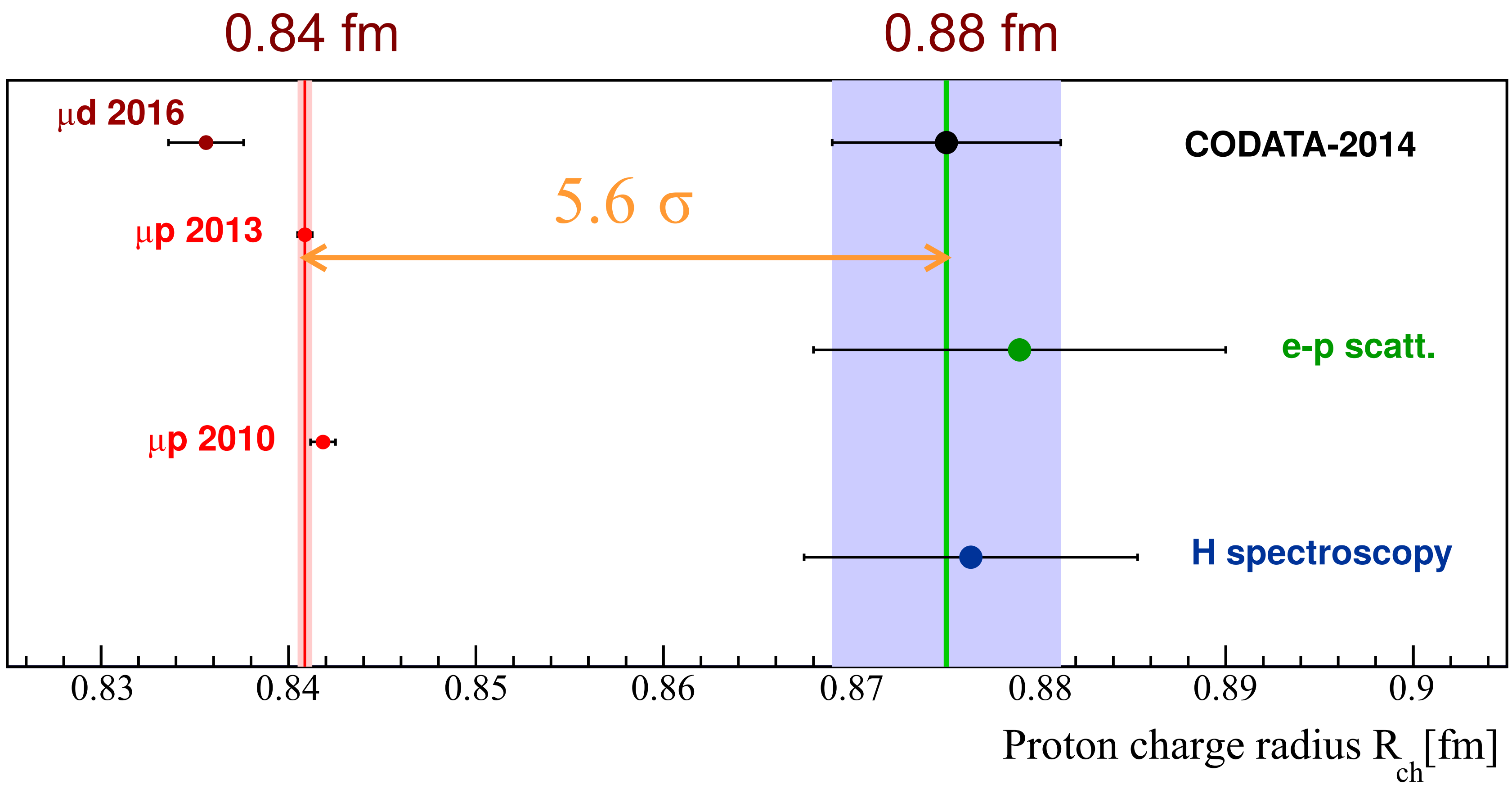
values in **fm**

* elastic electron scattering
* H/D: laser spectroscopy and QED (a lot!)
* He, Li, …. isotope shift for charge radius **differences**

sources: * p,d: CODATA-2014
* t: Amroun et al. (Saclay) , NPA 579, 596 (1994)
* ^{3,4}He: Sick, J.Phys.Chem.Ref Data 44, 031213 (2015)
* Angeli, At. Data Nucl. Data Tab. 99, 69 (2013)

The “Proton Radius Puzzle”

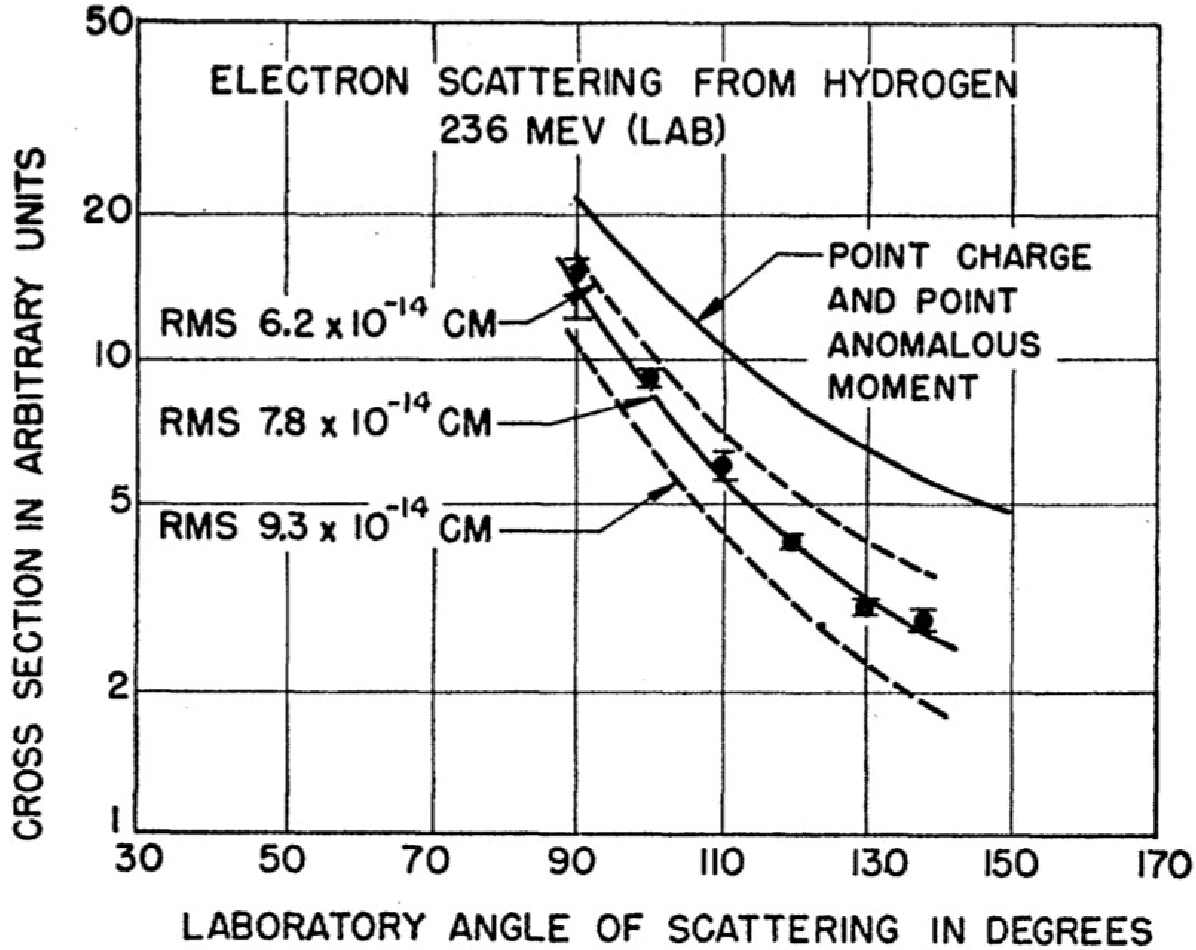
Measuring R_p using electrons: 0.88 fm (+- 0.7%)
using muons: 0.84 fm (+- 0.05%)



μd 2016: RP et al (CREMA Coll.) Science 353, 669 (2016)
μp 2013: A. Antognini, RP et al (CREMA Coll.) Science 339, 417 (2013)

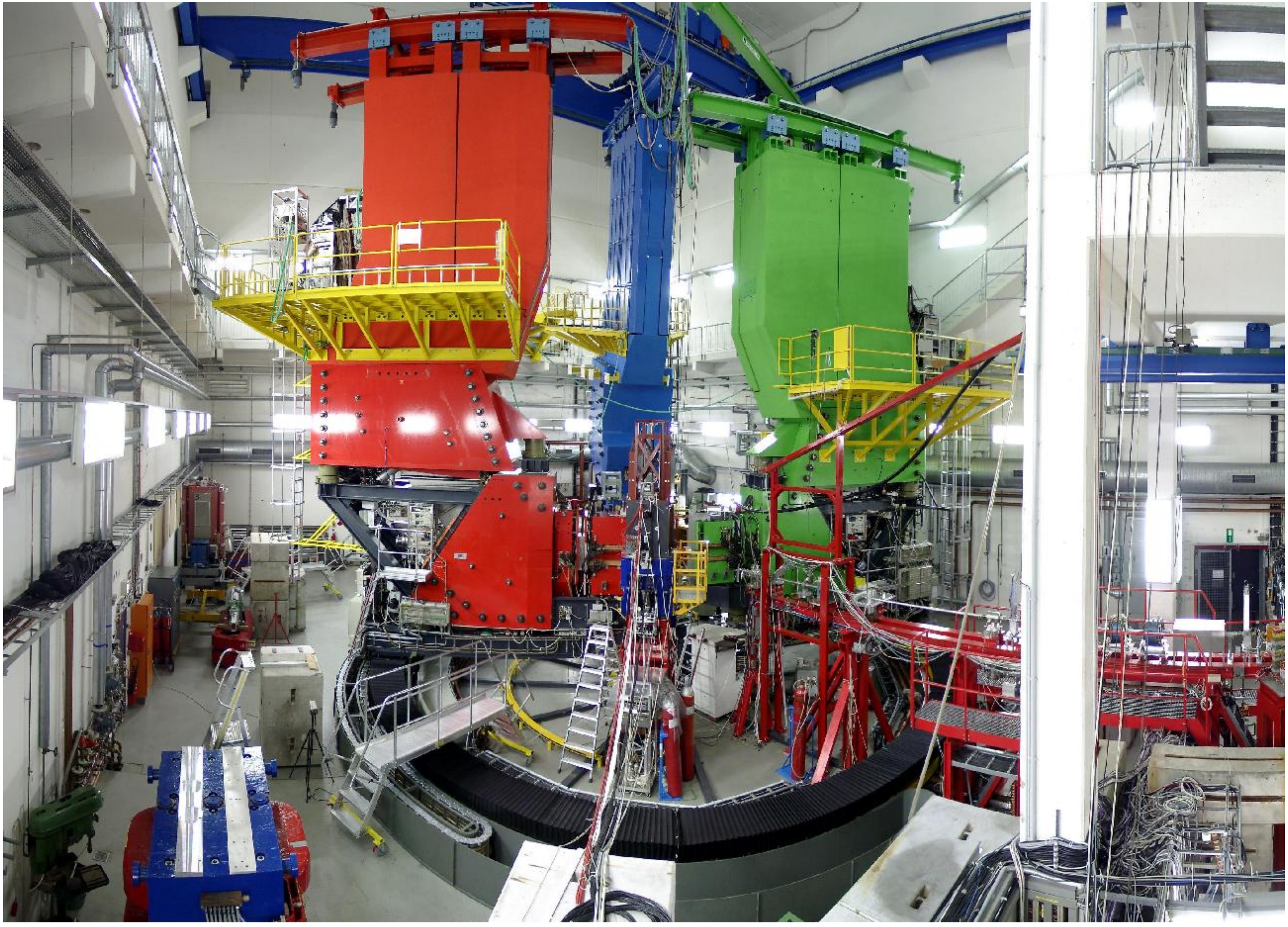
Robert Hofstadter – 1955

1915 – 1990

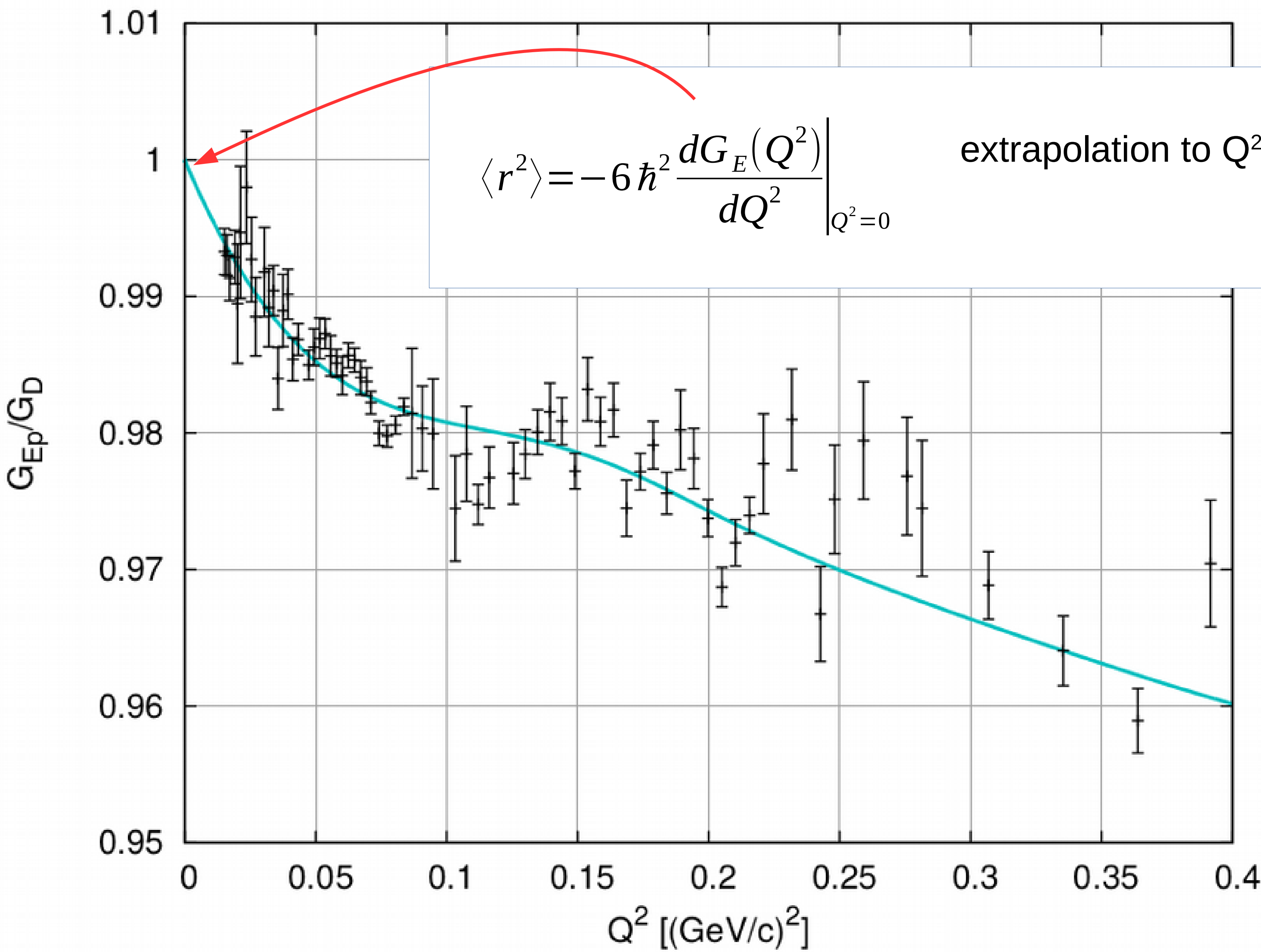


Phys. Rev. 102, 851 (1956)

Mainzer Microtron MAMI



Electron scattering - today



$$\langle r^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

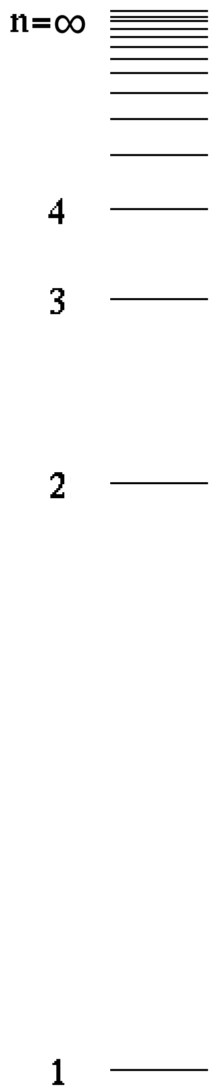
extrapolation to $Q^2 = 0$ required

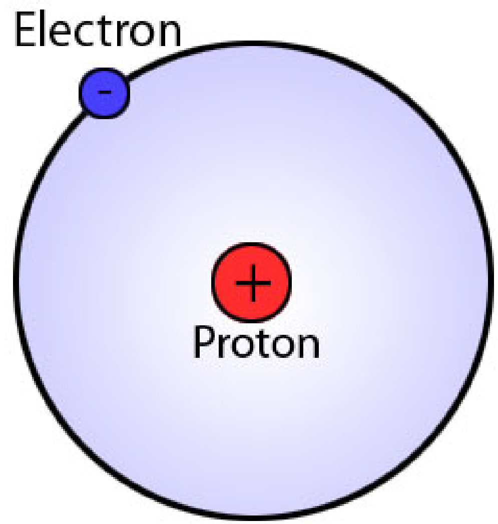
Mainz MAMI data 2010

Vanderhaeghen, Walcher: 1008.4225

Hydrogen

Energy levels of hydrogen

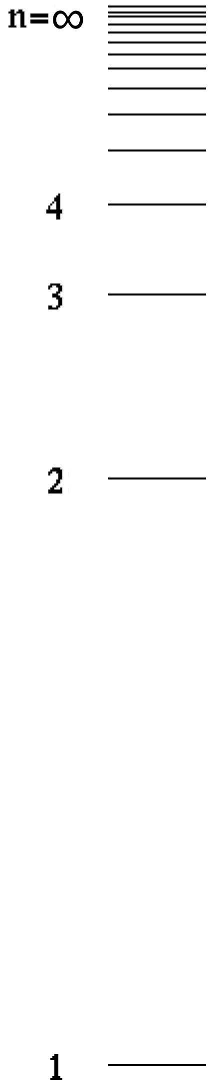




$$E_n \approx -\frac{R_\infty}{n^2}$$

Bohr formula

Energy levels of hydrogen

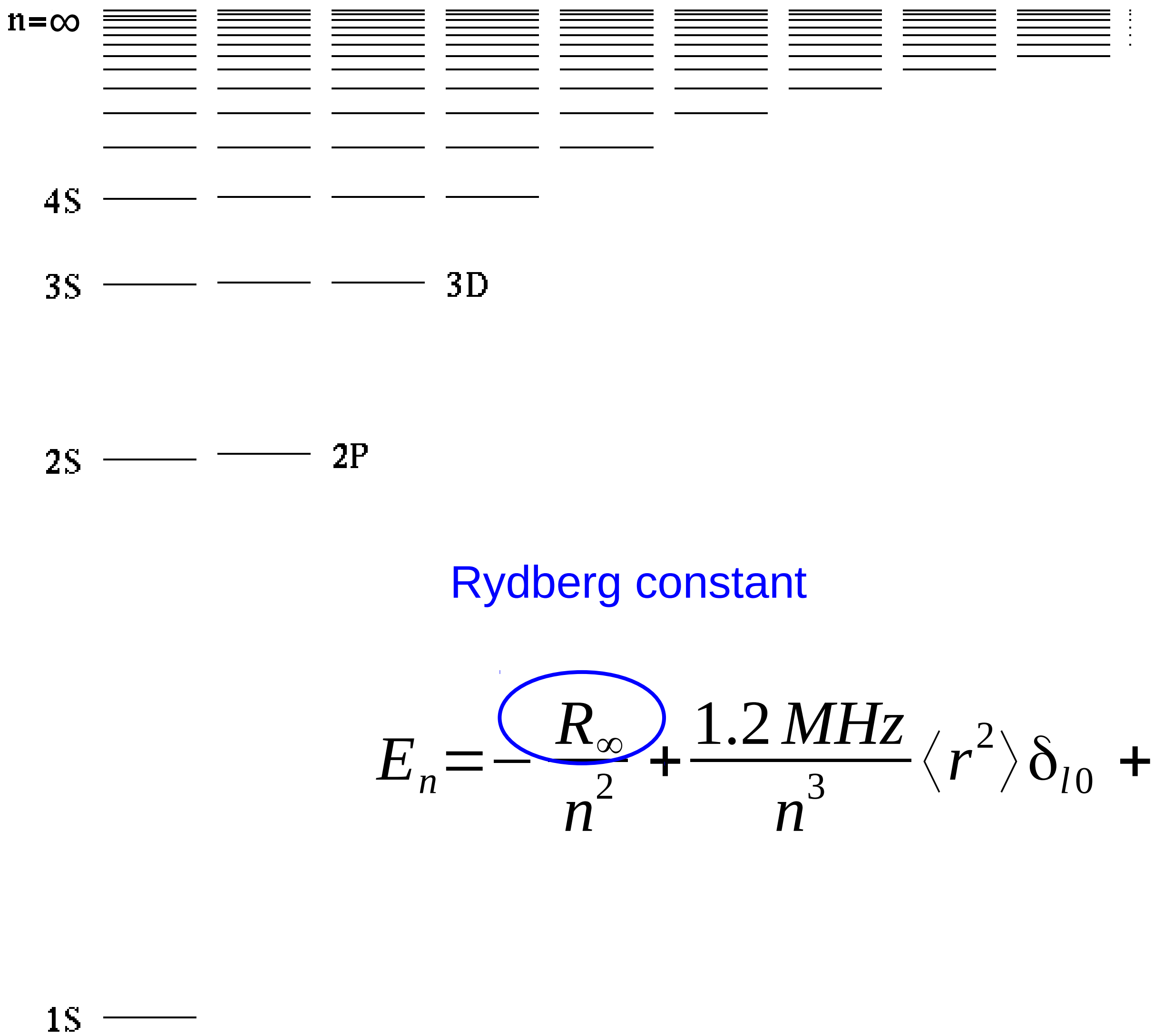


Rydberg constant

$$E_n \approx -\frac{R_\infty}{n^2}$$

Bohr formula

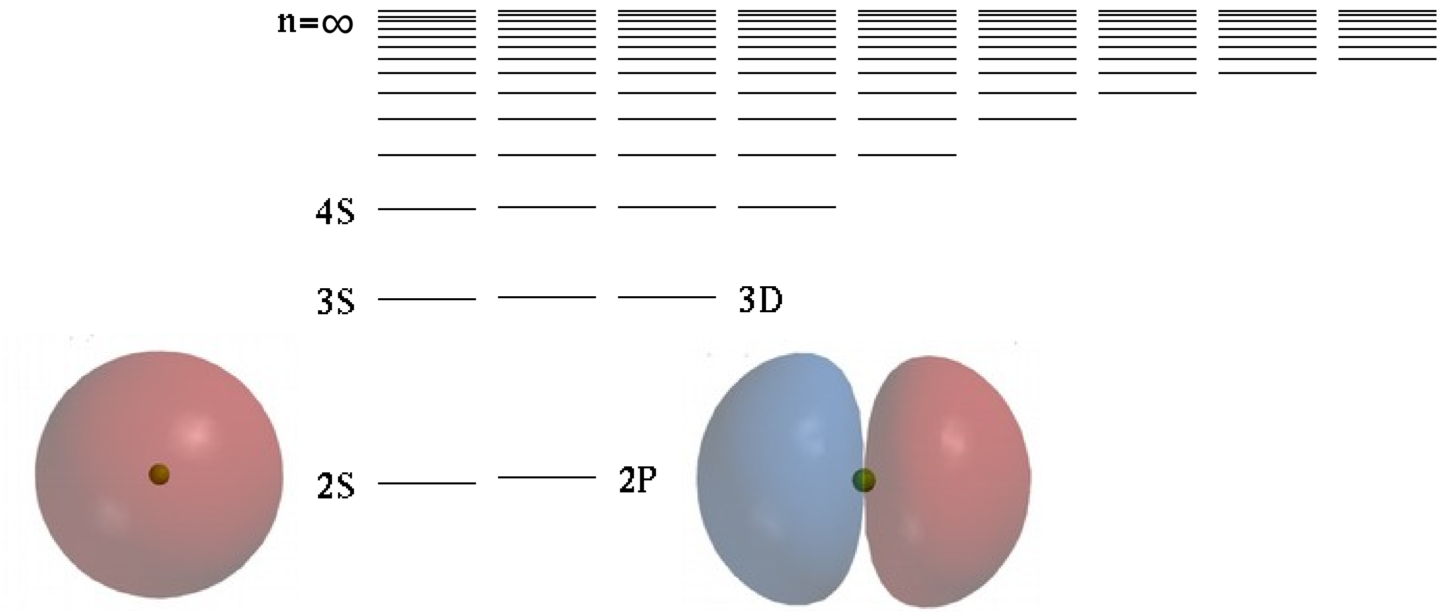
Energy levels of hydrogen



Rydberg constant

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n, l, j)$$

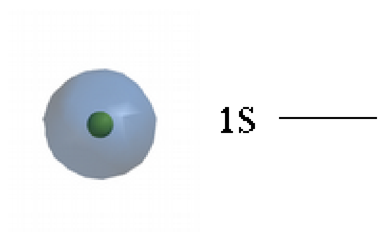
Energy levels of hydrogen



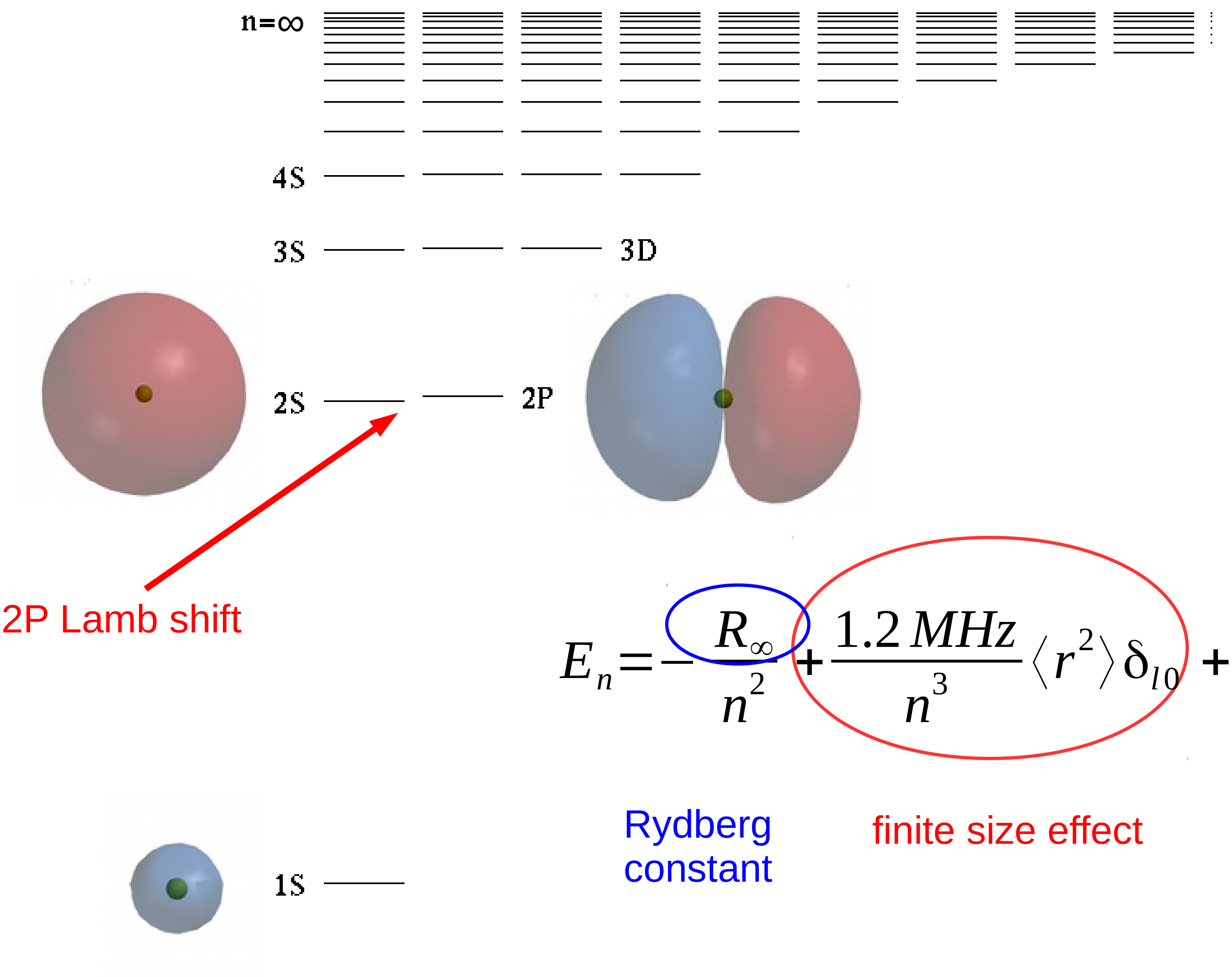
$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2 \rangle \delta_{l0} + \Delta(n, l, j)$$

Rydberg constant

finite size effect



Energy levels of hydrogen

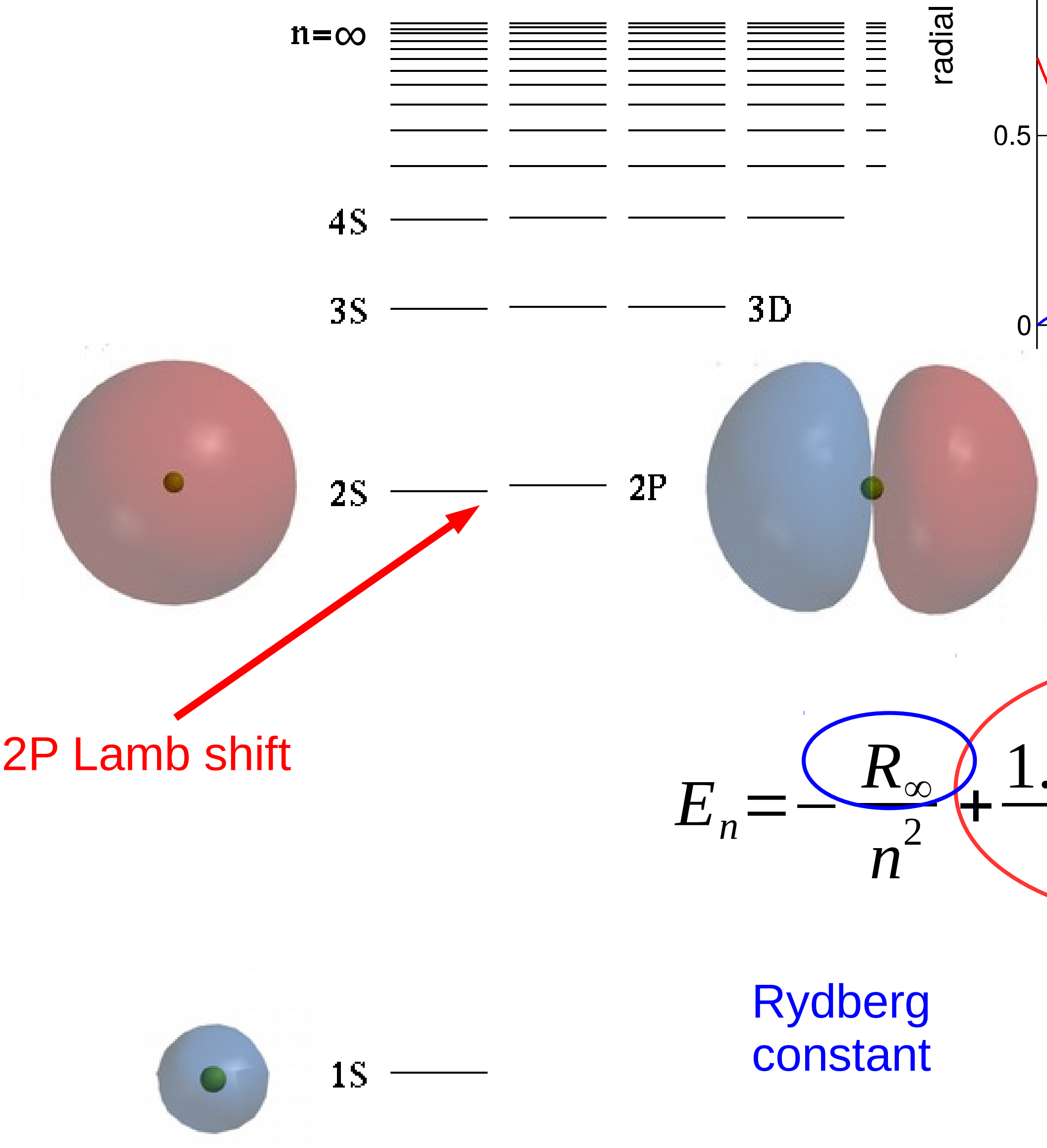


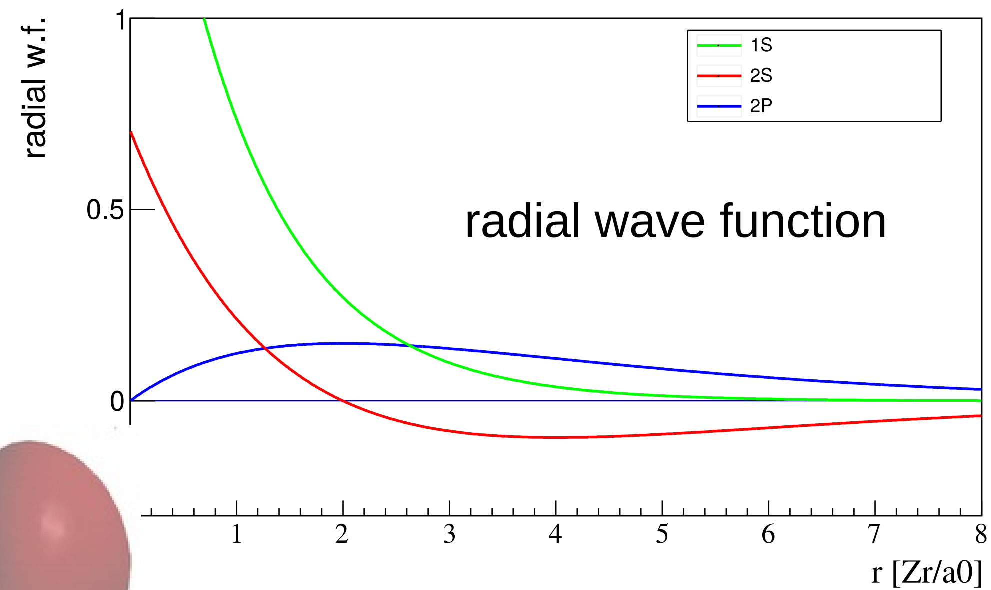
$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n,l,j)$$

Rydberg constant

finite size effect

Energy levels of hydrogen





2S-2P Lamb shift

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n,l,j)$$

Rydberg constant

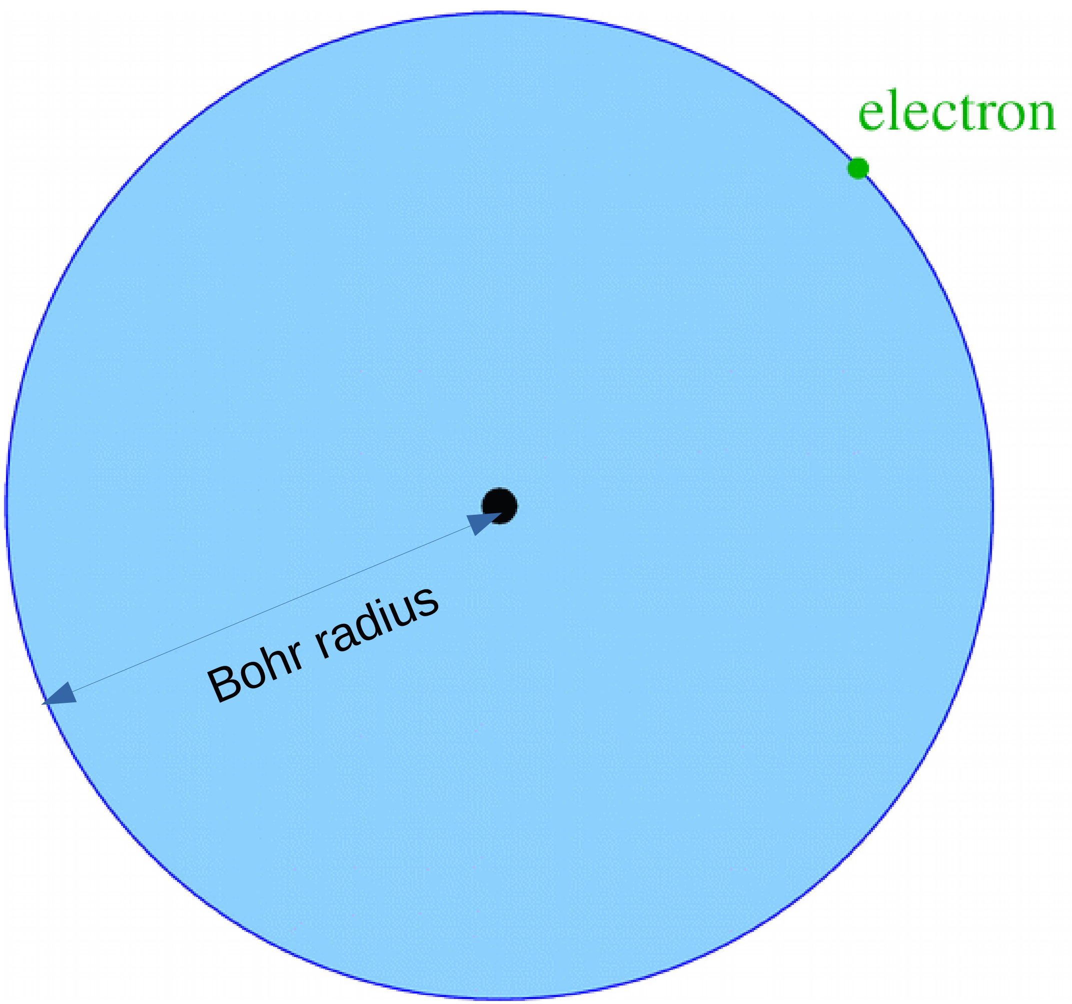
finite size effect

Muonic Hydrogen

Electronic and muonic atoms

Regular hydrogen:

Proton + Electron



Muonic hydrogen:

Proton + Muon

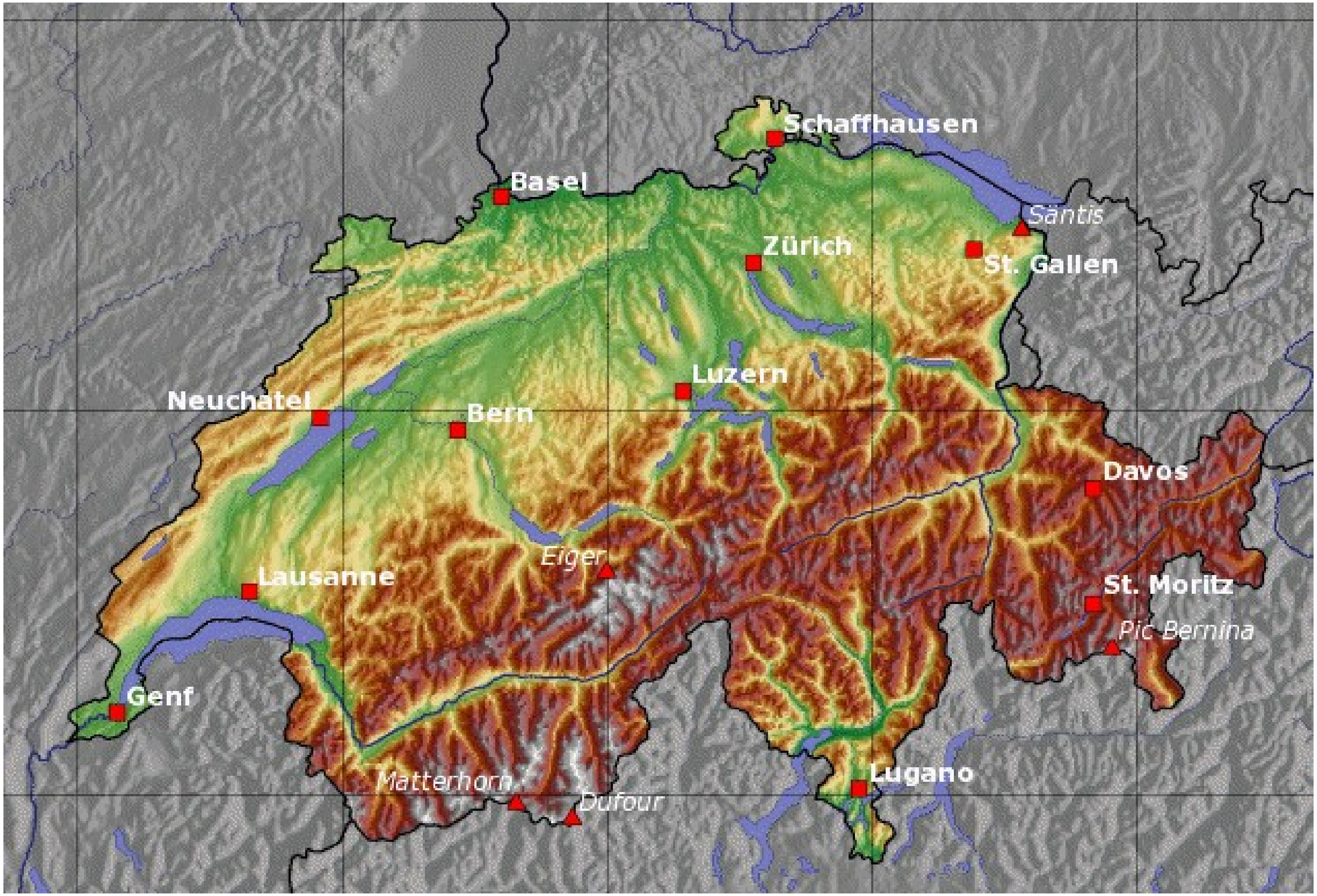
Muon **mass** = **200** * electron mass

Bohr **radius** = **1/200** of H

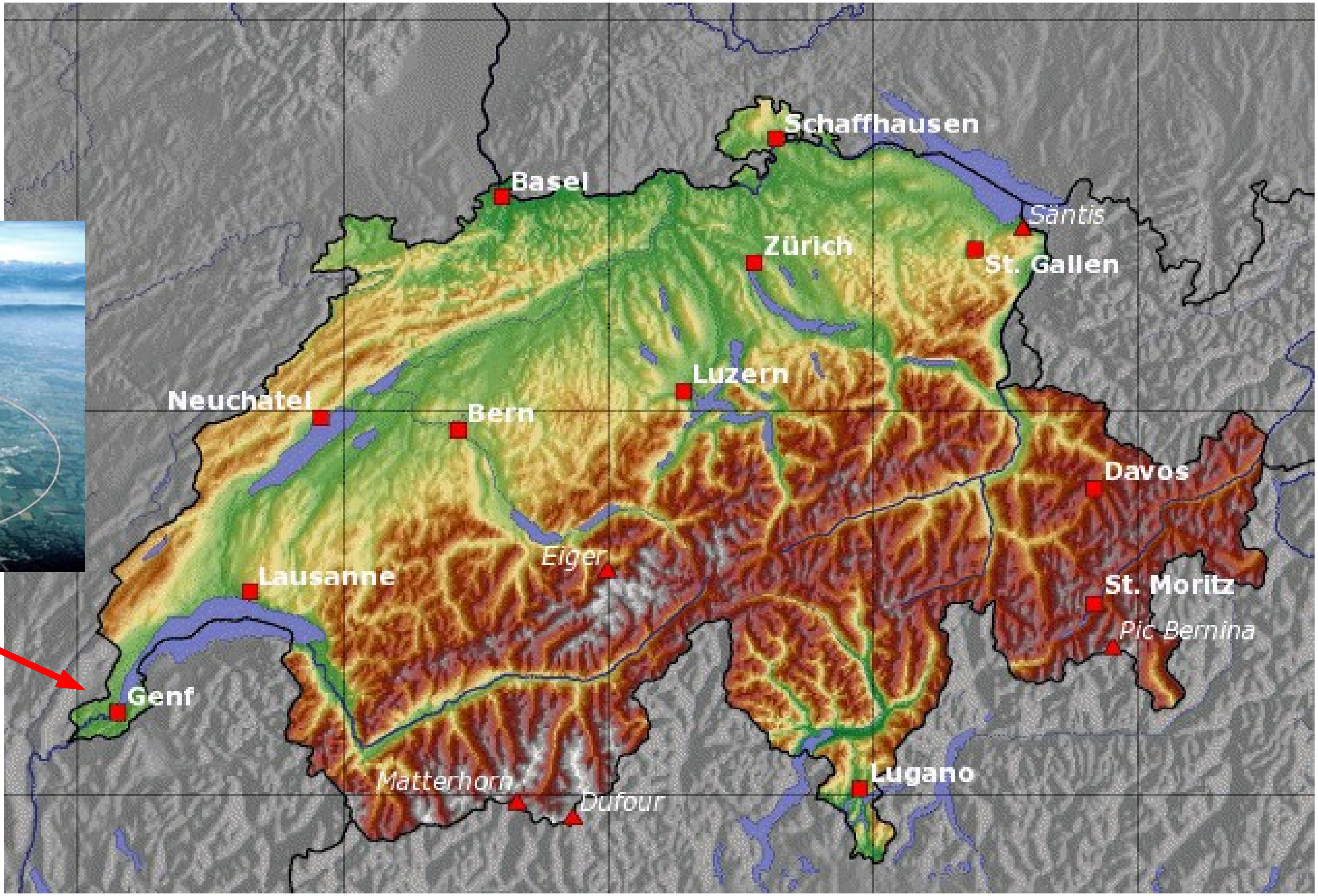
200^3 = a **few million times** more sensitive to proton size

Vastly not to scale!!

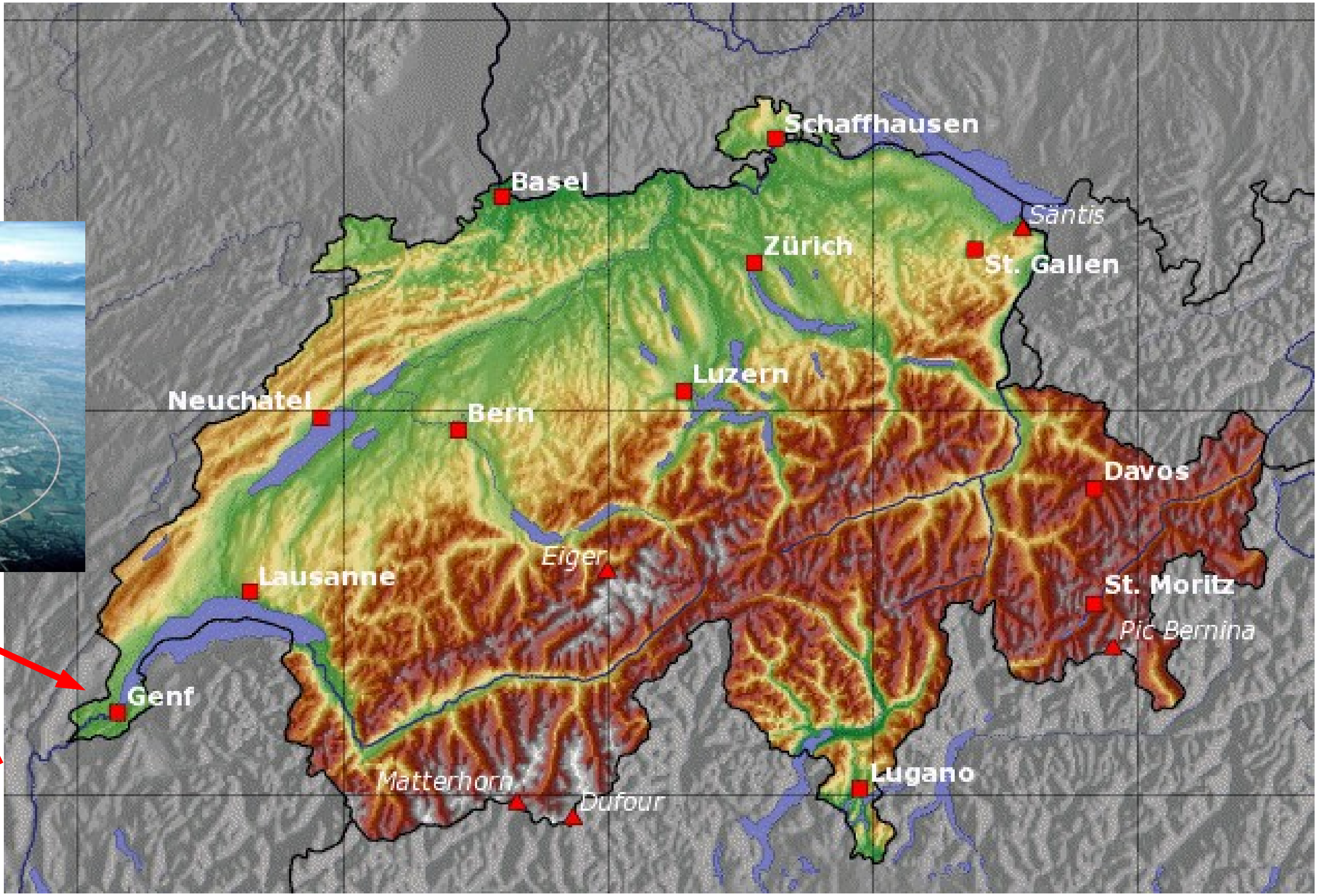
The accelerator at PSI



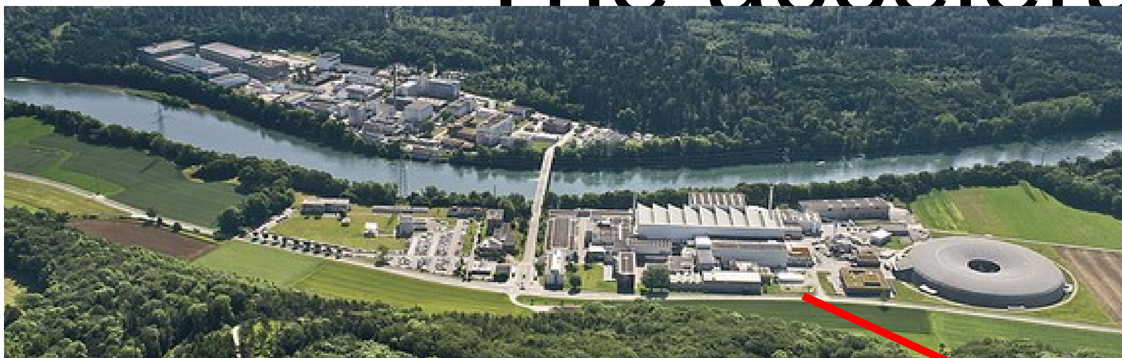
The accelerator at PSI



The accelerator at PSI

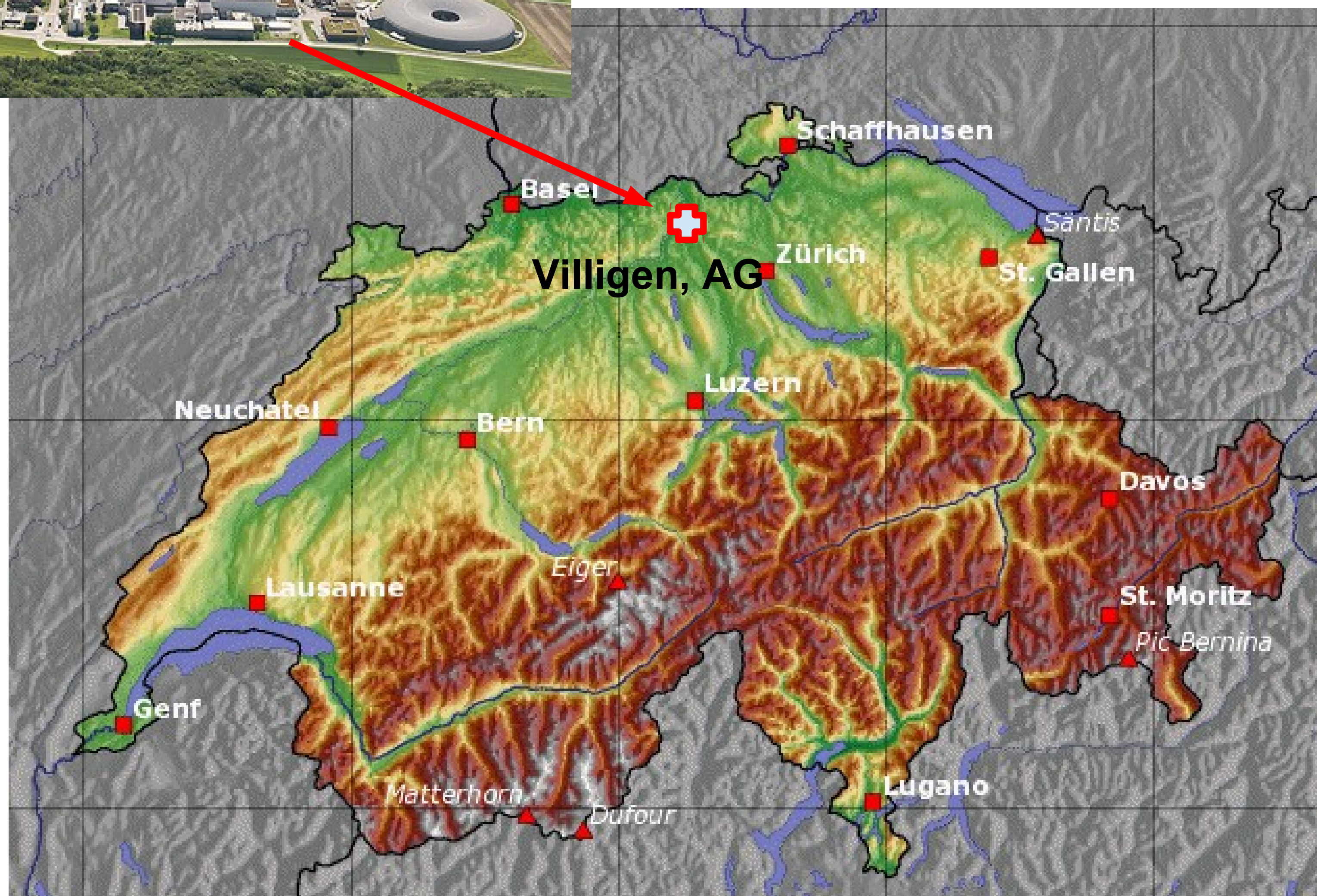


The accelerator at PSI

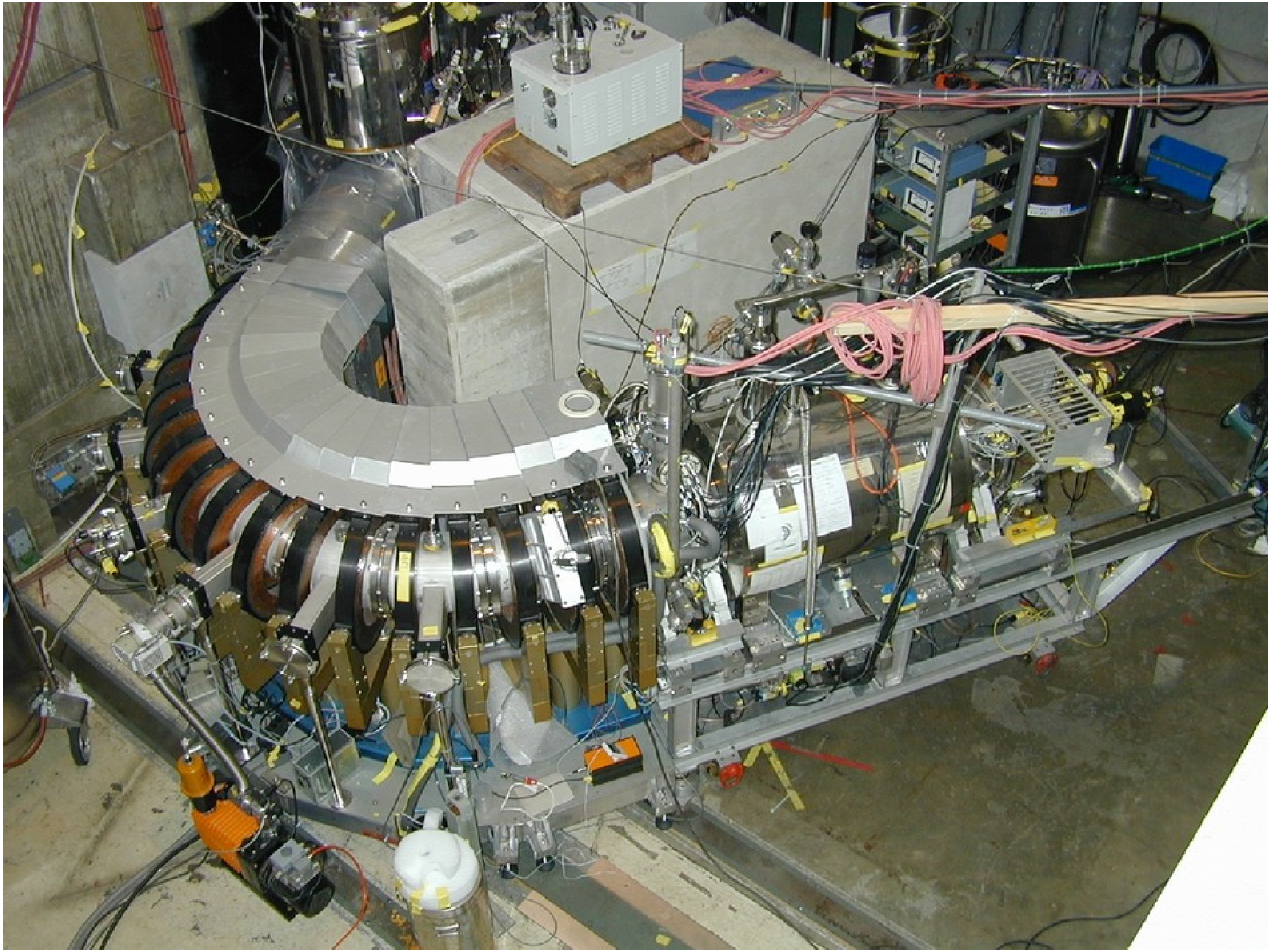


PAUL SCHERRER INSTITUT

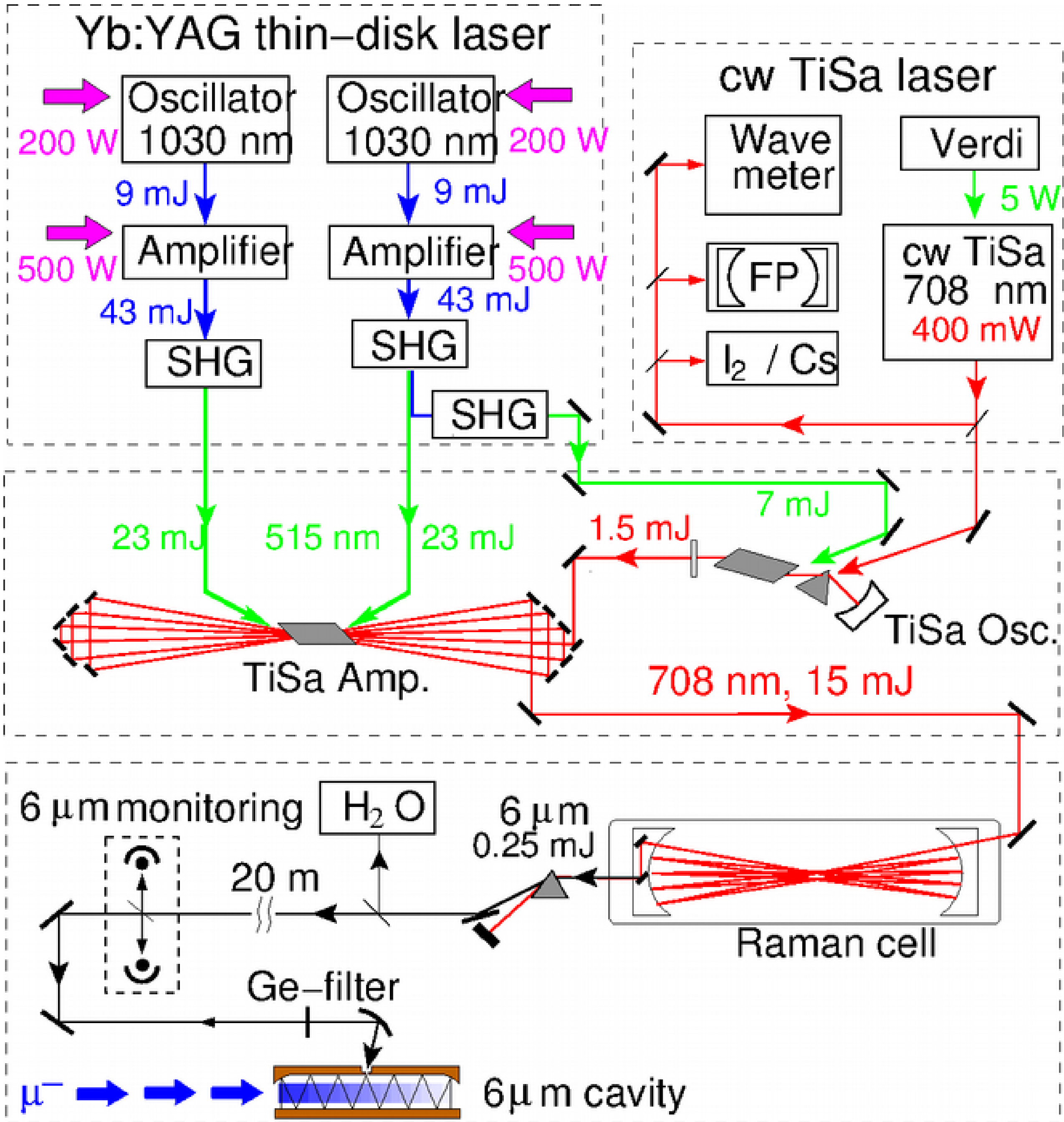
PSI



The muon beam line in πE5



The laser system



Yb:YAG Disk laser
→ fast response on µ

Frequency doubling (SHG)
→ green light to pump
Ti:sapphire laser

Ti:sapphire cw laser
→ determines laser frequency

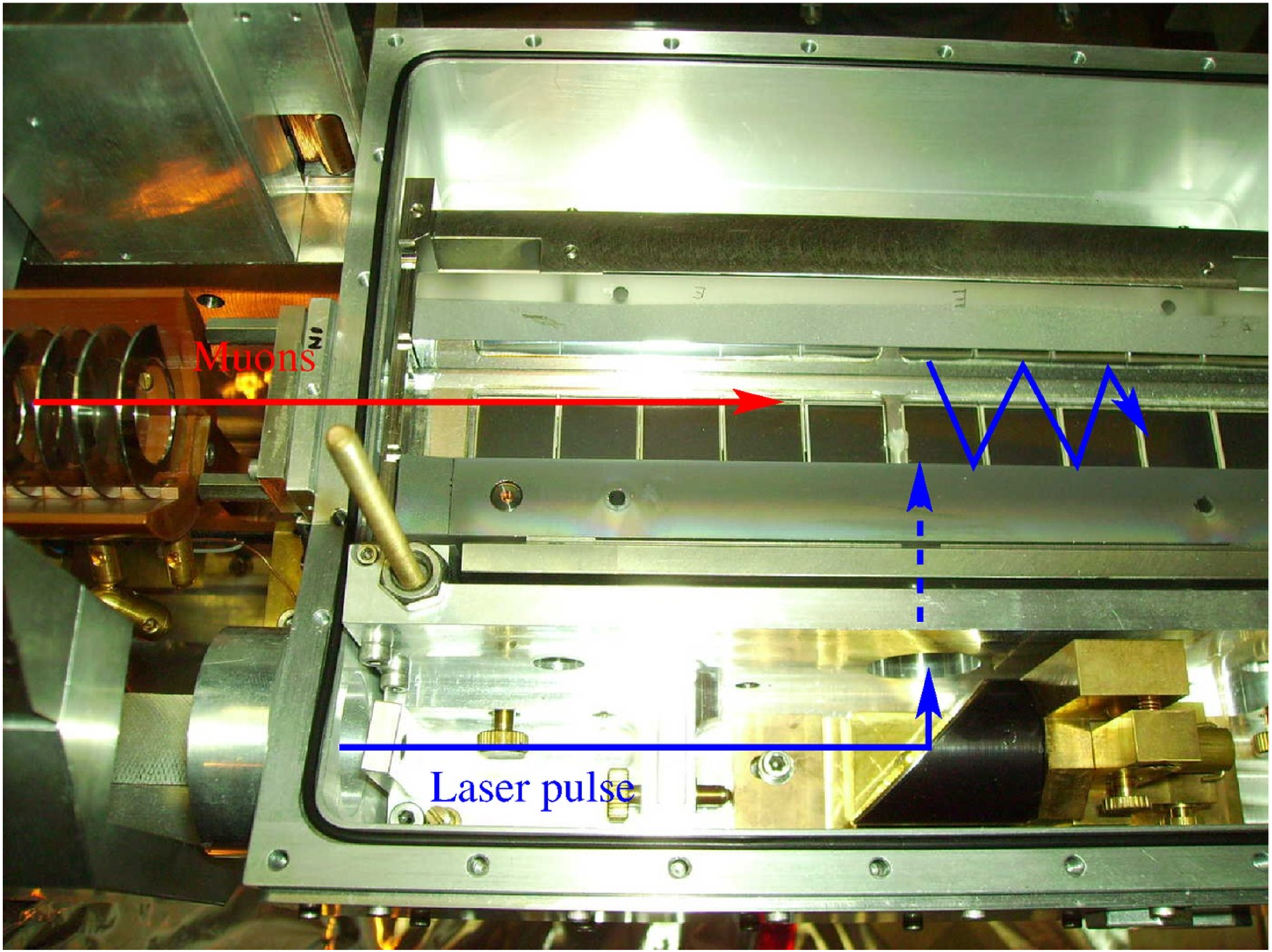
Ti:sapphire MOPA
→ high pulse energy (15 mJ)

Raman cell
→ 3 sequential stimulated
Raman Stokes shifts
Laser wave length → 6 µm

Target Cavity
→ Mirror system to fill the
muon stop volume (H_2)

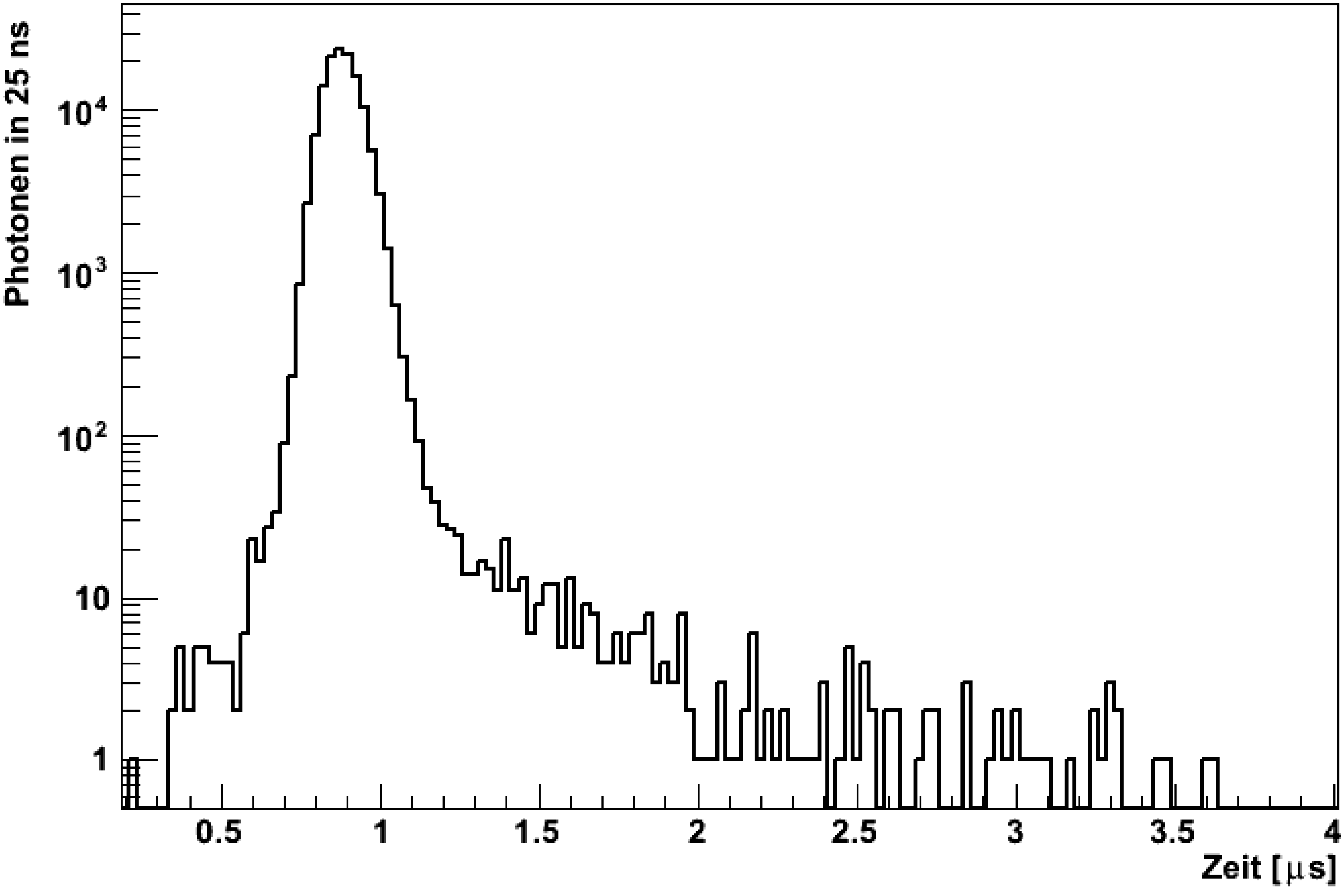
The laser hut at PSI

The hydrogen target



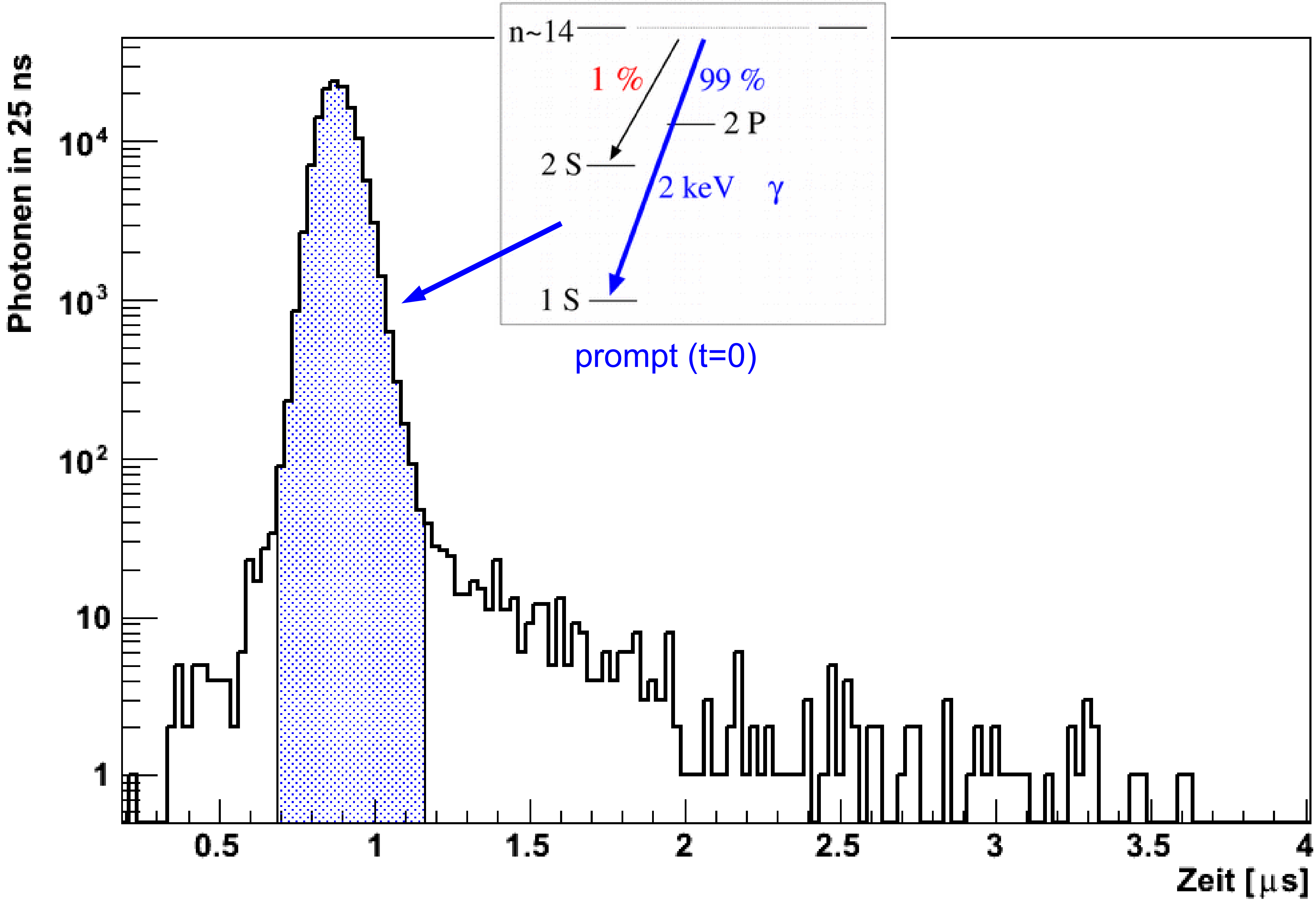
Time Spectra

13 hours of data

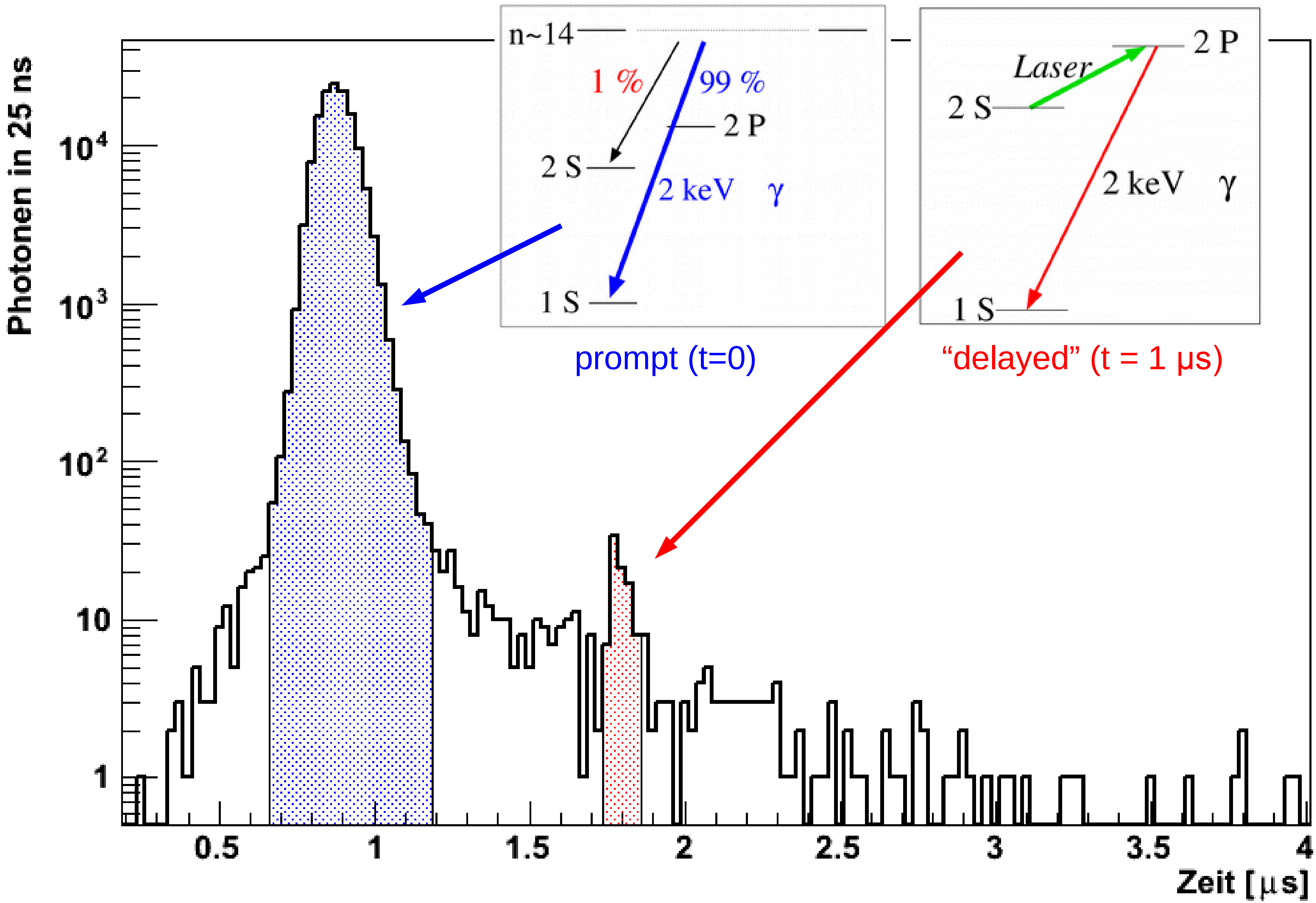


Time Spectra

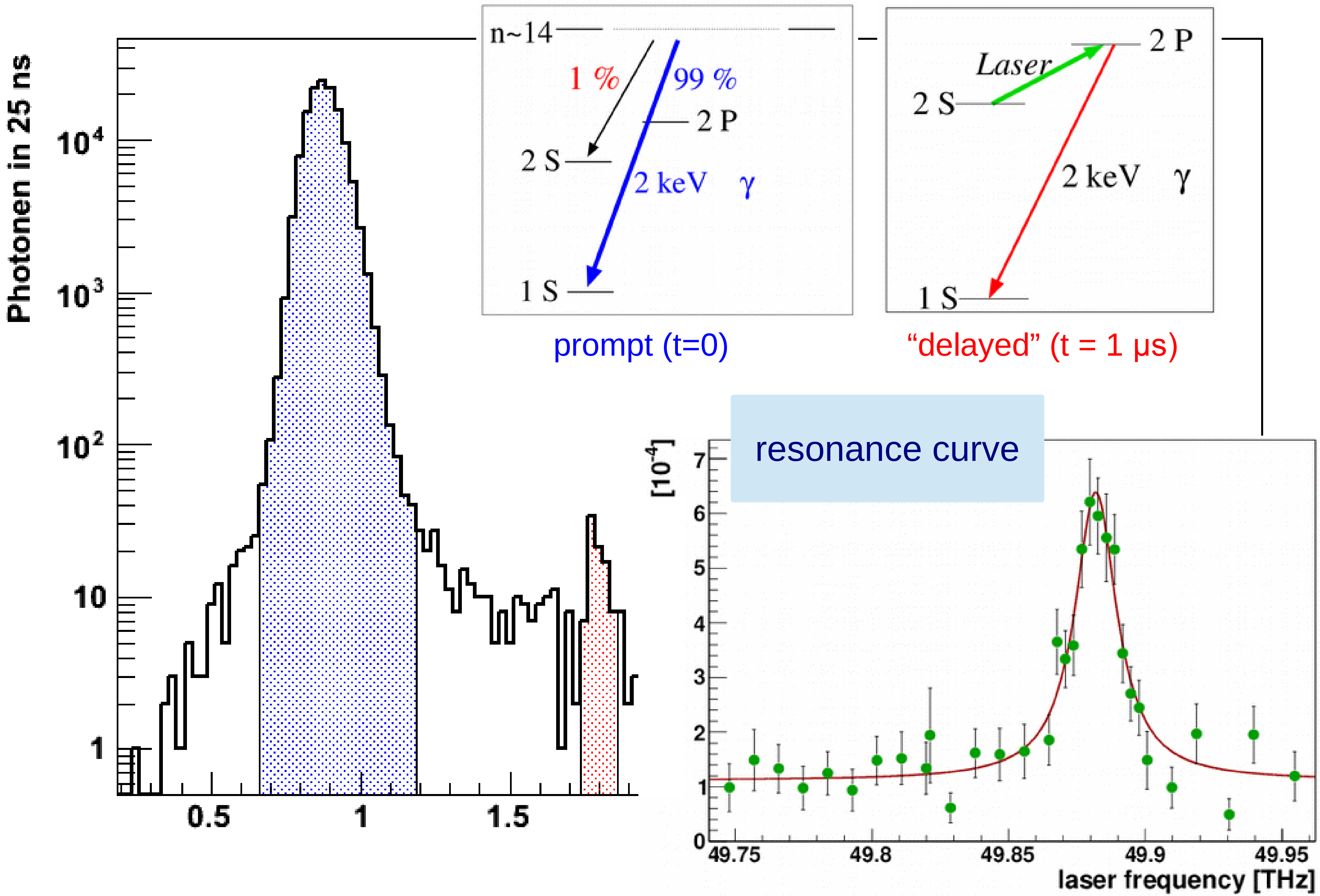
13 hours of data



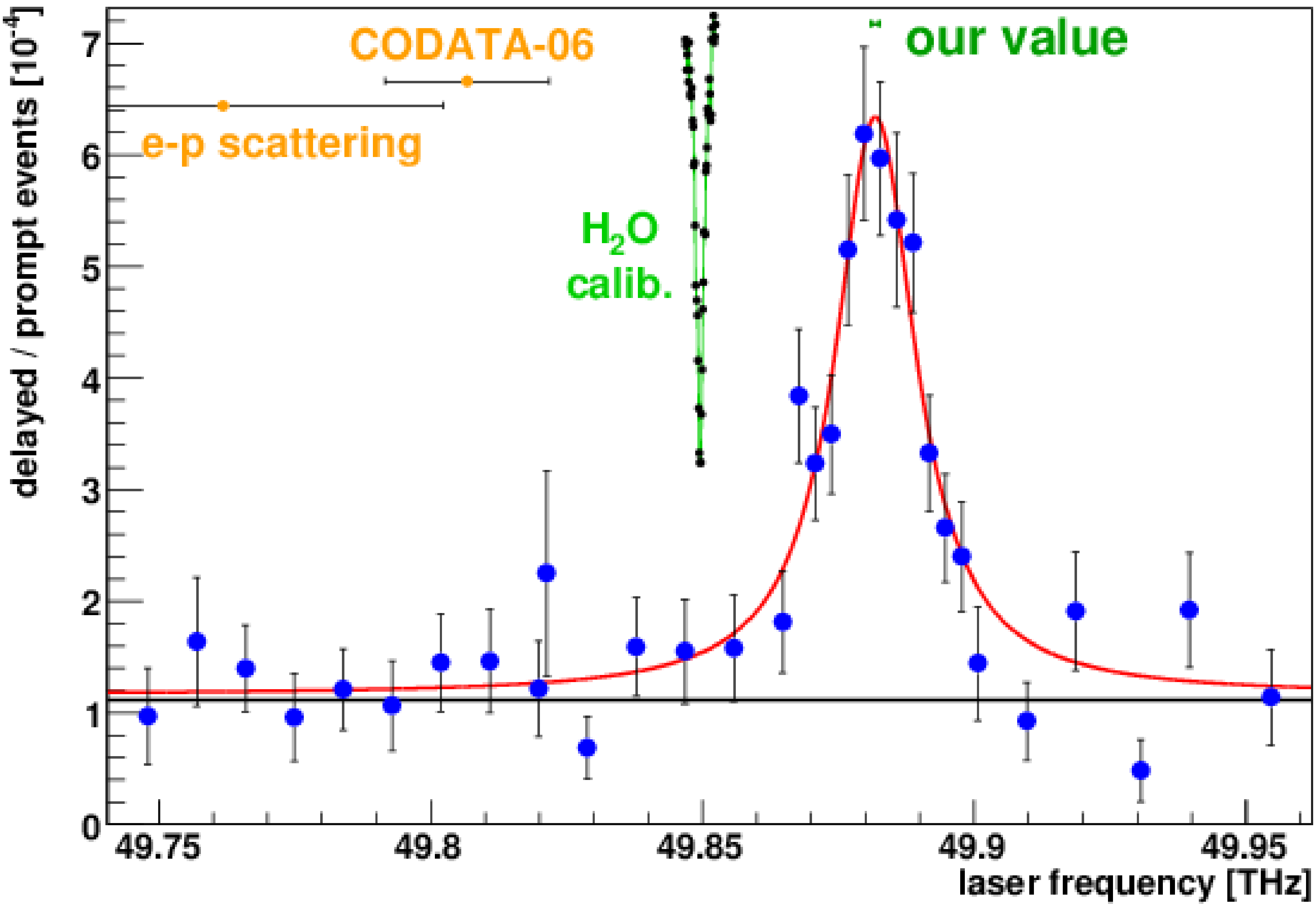
Time Spectra



Time Spectra

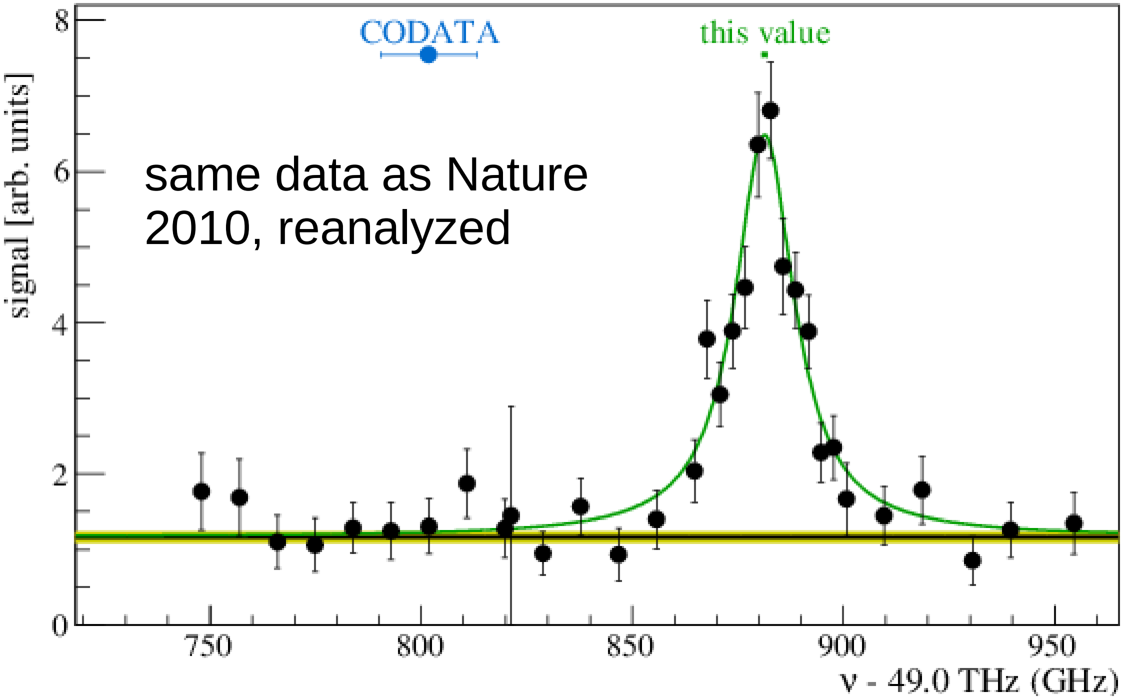


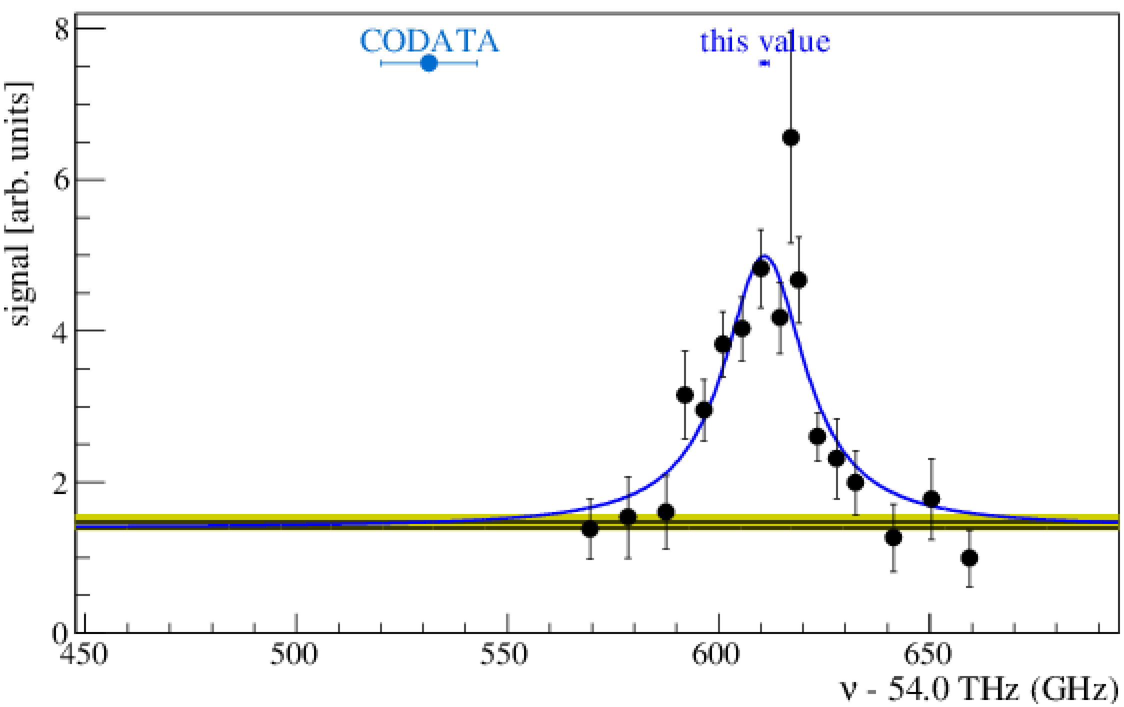
Resonance in muonic hydrogen



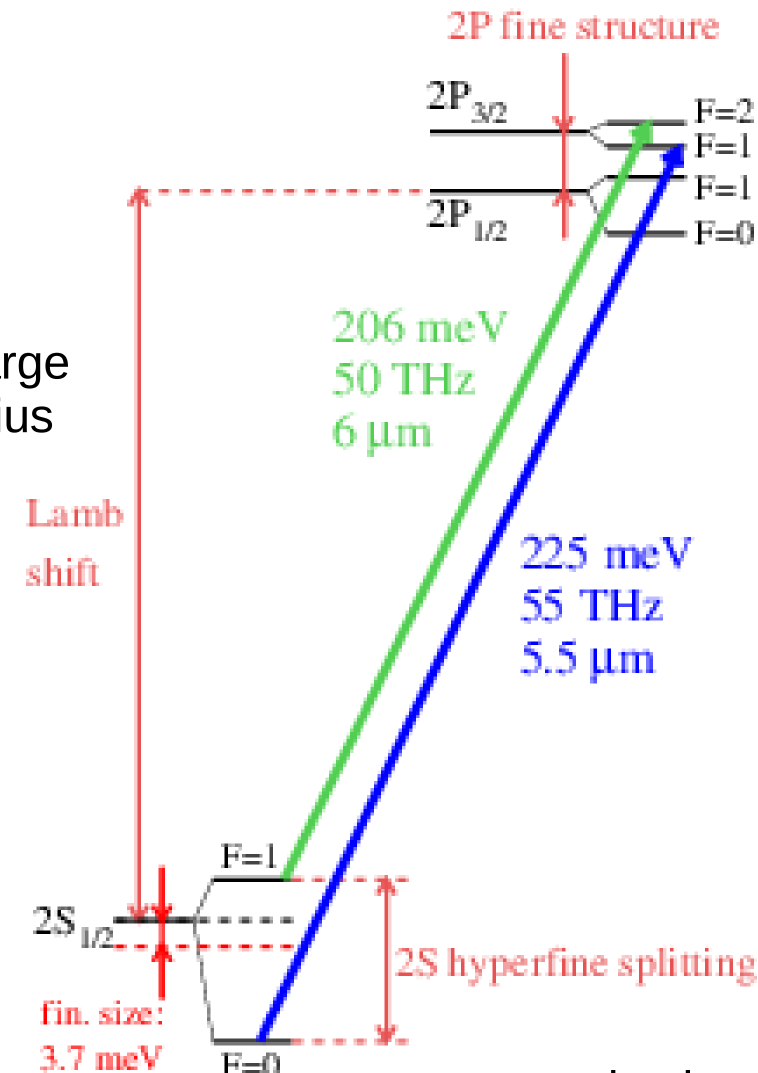
Pohl et al. (CREMA Coll.), Nature 466, 213 (2010)

2 transitions in muonic H





charge radius



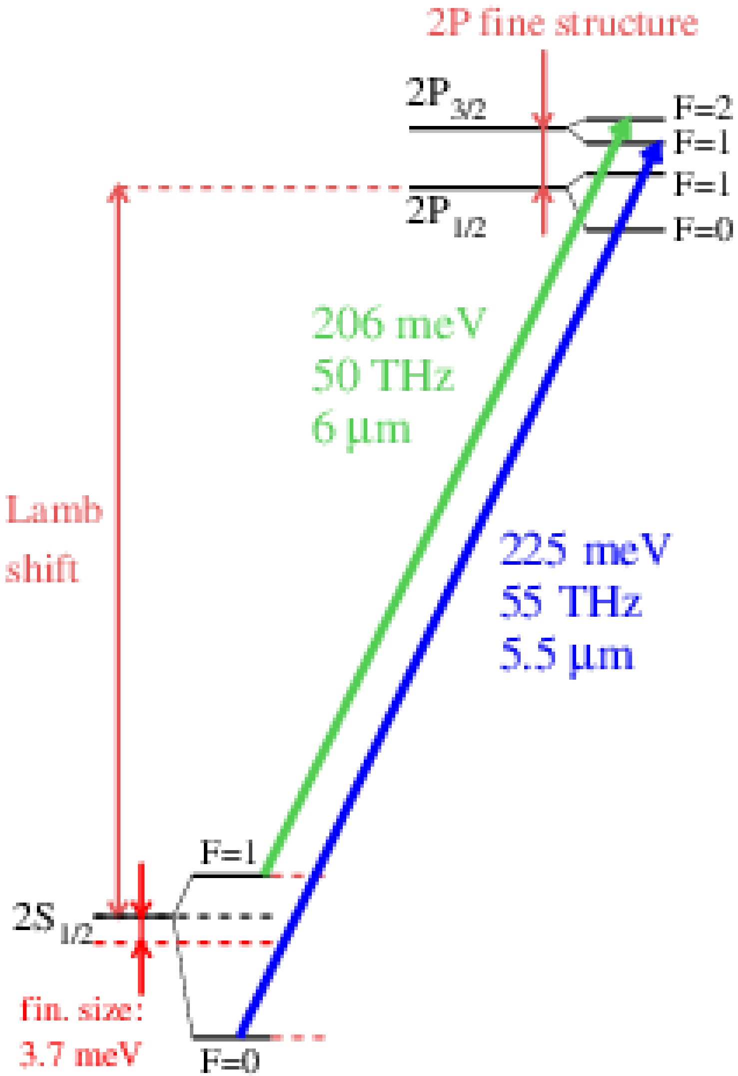
magnetization distribution (Zemach radius)

Antognini et al. (CREMA Coll.), Science 339, 417 (2013)

Theory in muonic H

$$\Delta E_{Lamb} = 206.0336\,(15)\ \text{meV}_{QED} + 0.0332\,(20)\ \text{meV}_{TPE} - 5.2275\,(10)\ \text{meV/fm}^2 * \mathbf{R_p}^2$$

Simple-looking formula
based on decades of work by
E. Borie, M.C. Birse, P. Blunden, C.E. Carlson,
M.I. Eides, R. Faustov, J.L. Friar, G. Paz,
A. Pineda, J. McGovern, K. Griffioen, H. Grotch,
H.-W. Hammer, R.J Hill, P. Indelicato,
U.D. Jentschura, S.G. Karshenboim, E.Y. Korzinin,
V.G. Ivanov, I.T. Lorenz, A.P. Martynenko,
G.A. Miller, U.-G. Meissner, J.P. Mohr,
K. Pachucki, V. Pascalutsa, J. Rafelski,
V.A. Shelyuto, I. Sick, A.W. Thomas,
M. Vanderhaeghen, V. Yerokhin,
…..
(shout if I missed your name!)



Antognini, RP at al., Ann. Phys. (N.Y.) 331, 127 (2013)

Theory in muonic H

$$\Delta E_{Lamb} = 206.0336\ (15)\ \mathrm{meV}_{QED} + 0.0332\ (20)\ \mathrm{meV}_{TPE} - 5.2275\ (10)\ \mathrm{meV/fm}^2 * \mathbf{R_p}^{\mathbf{2}}$$

Annals of Physics 331 (2013) 127–145

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Theory of the 2S–2P Lamb shift and 2S hyperfine splitting in muonic hydrogen

CrossMark

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[b] *Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P. et M. Curie, 75252 Paris, CEDEX 05, France*

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Our attempt to summarize all the original work by many theorists….

Theory I: “pure” QED

Table 1

All known radius-*independent* contributions to the Lamb shift in μp from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The “finite-size to relativistic recoil correction” (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

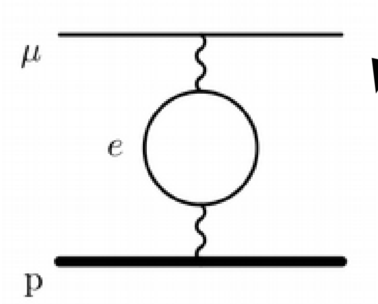
#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
1	NR one-loop electron VP (eVP)	205.0074					
2	Rel. corr. (Breit–Pauli)	0.0169[a]					
3	Rel. one-loop eVP		205.0282	205.0282	205.02821	205.02821	[80] Eq. (54)
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	(incl. in #2)[b]	−0.0041	−0.0041		−0.00208[c]	[77,78]
4	Two-loop eVP (Källén–Sabry)	1.5079	1.5081	1.5081	1.50810	1.50810	[80] Eq. (57)
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$	0.1509	0.1509	0.1507	0.15102	0.15102	[80] Eq. (60)
7	eVP corr. to Källén–Sabry	0.0023	0.00223	0.00223	0.00215	0.00215	[80] Eq. (62), [87]
6	NR three-loop eVP	0.0053	0.00529	0.00529		0.00529	[87,88]
9	Wichmann–Kroll, “1:3” LBL		−0.00103	−0.00102	−0.00102	−0.00102	[80] Eq. (64), [89]
10	Virtual Delbrück, “2:2” LBL		0.00135	0.00115		0.00115	[74,89]
New	“3:1” LBL			−0.00102		−0.00102	[89]
20	μSE and μVP	−0.6677	−0.66770	−0.66788	−0.66761	−0.66761	[80] Eqs. (72) + (76)
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	−0.005(1)	−0.00500	−0.004924[d]		−0.00254	[85] Eq. (29a)[e]
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	−0.001	−0.00150			[f]	[74,90–92]
21	Higher order corr. to μSE and μVP		−0.00169	−0.00171[g]		−0.00171	[86] Eq. (177)
13	Mixed eVP + μVP		0.00007	0.00007		0.00007	[74]
New	eVP and μVP in two Coulomb lines				0.00005	0.00005	[80] Eq. (78)
14	Hadronic VP $\alpha(Z\alpha)^4 m_r$	0.0113(3)	0.01077(38)	0.011(1)		0.01121(44)	[93–95]
15	Hadronic VP $\alpha(Z\alpha)^5 m_r$		0.000047			0.000047	[94,95]
16	Rad corr. to hadronic VP		−0.000015			−0.000015	[94,95]
17	Recoil corr.	0.0575	0.05750	0.0575	0.05747	0.05747	[80] Eq. (88)
22	Rel. RC $(Z\alpha)^5$	−0.045	−0.04497	−0.04497	−0.04497	−0.04497	[80] Eq. (88), [74]
23	Rel. RC $(Z\alpha)^6$	0.0003	0.00030		0.0002475	0.0002475	[80] Eq. (86)+Tab.II
New	Rad. (only eVP) RC $\alpha(Z\alpha)^5$					0.000136	[85] Eq. (64a)
24	Rad. RC $\alpha(Z\alpha)^n$ (proton SE)	−0.0099	−0.00960	−0.0100		−0.01080(100)	[43][h] [74]
	Sum	206.0312	206.02915	206.02862		206.03339(109)	

Antognini, RP at al., Ann. Phys. (N.Y.) 331, 127 (2013)

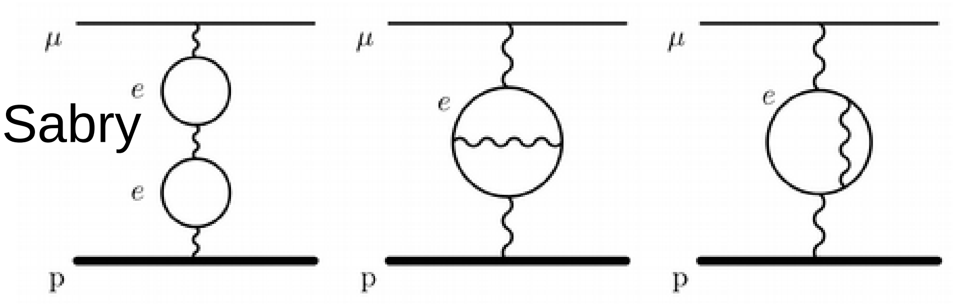
Theory in muonic H

$$\Delta E_{Lamb} = 206.0336\,(15)\ \text{meV}_{QED} + 0.0332\,(20)\ \text{meV}_{TPE} - 5.2275\,(10)\ \text{meV/fm}^2 * \mathbf{R_p}^{\mathbf{2}}$$

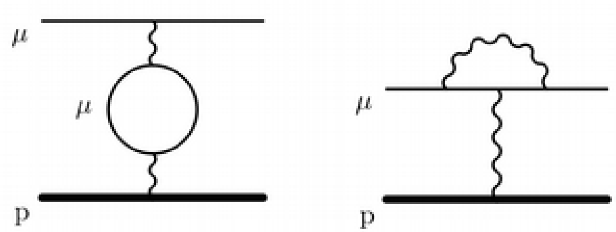
Uehling



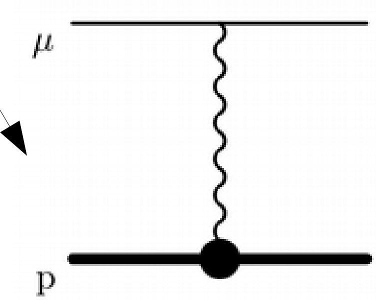
Källen-Sabry



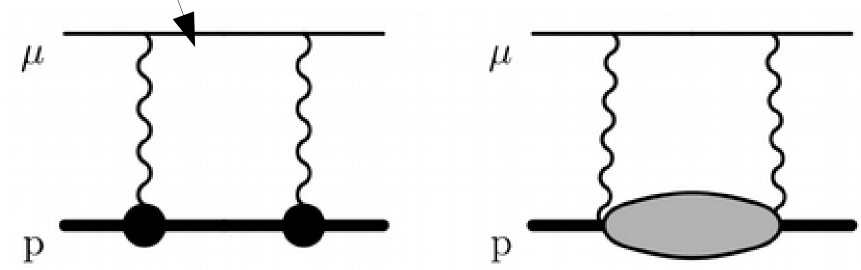
Muon SE+VP



and 20 more….



Proton form factor

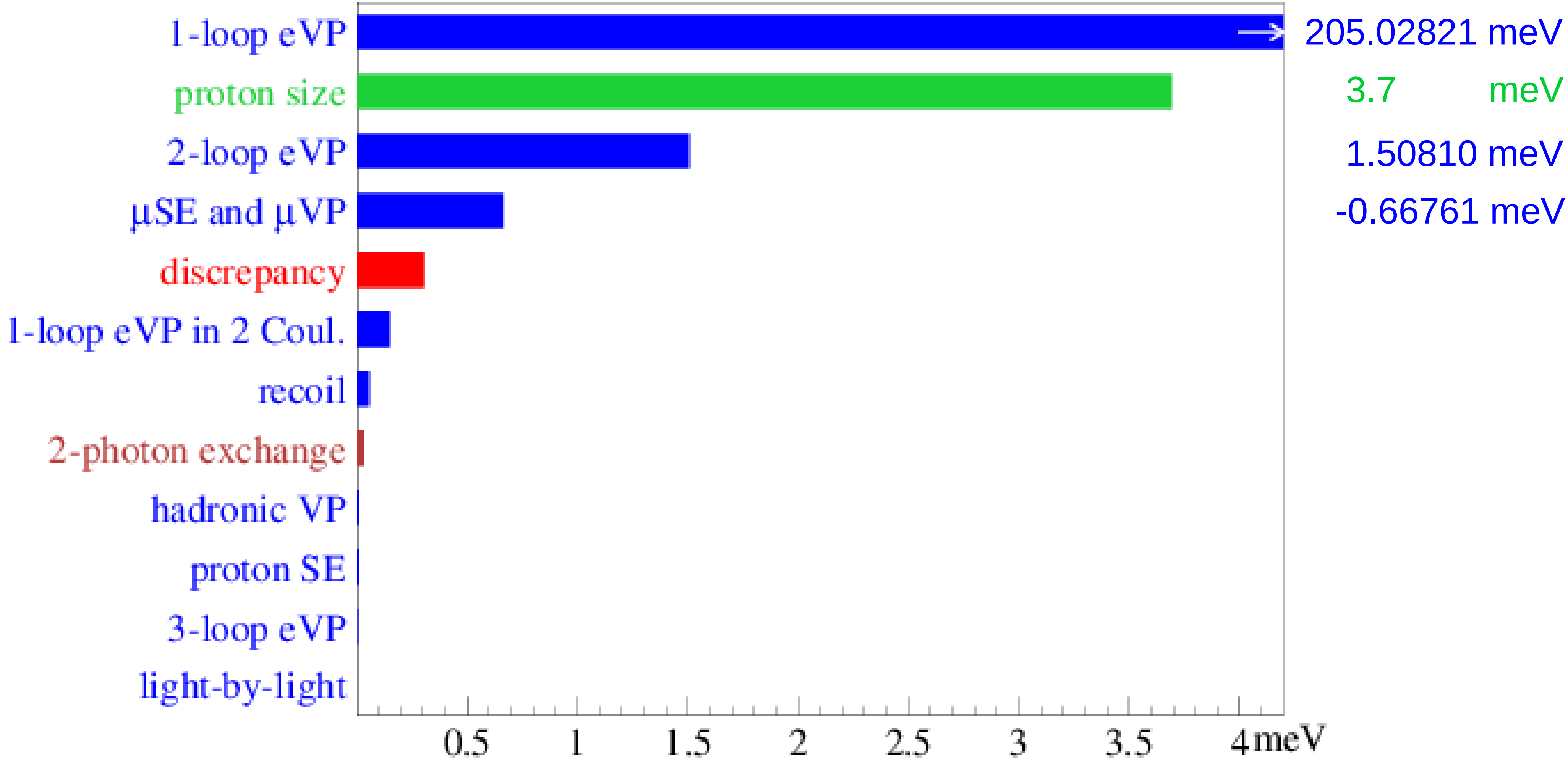


elastic and inelastic two-photon exchange
(Friar moment and polarizability)

Theory in muonic H

ΔE_{Lamb} = 206.0336 (15) meV_{QED} + 0.0332 (20) meV_{TPE} – 5.2275 (10) meV/fm^2 * $\mathbf{R_p}^{\mathbf{2}}$

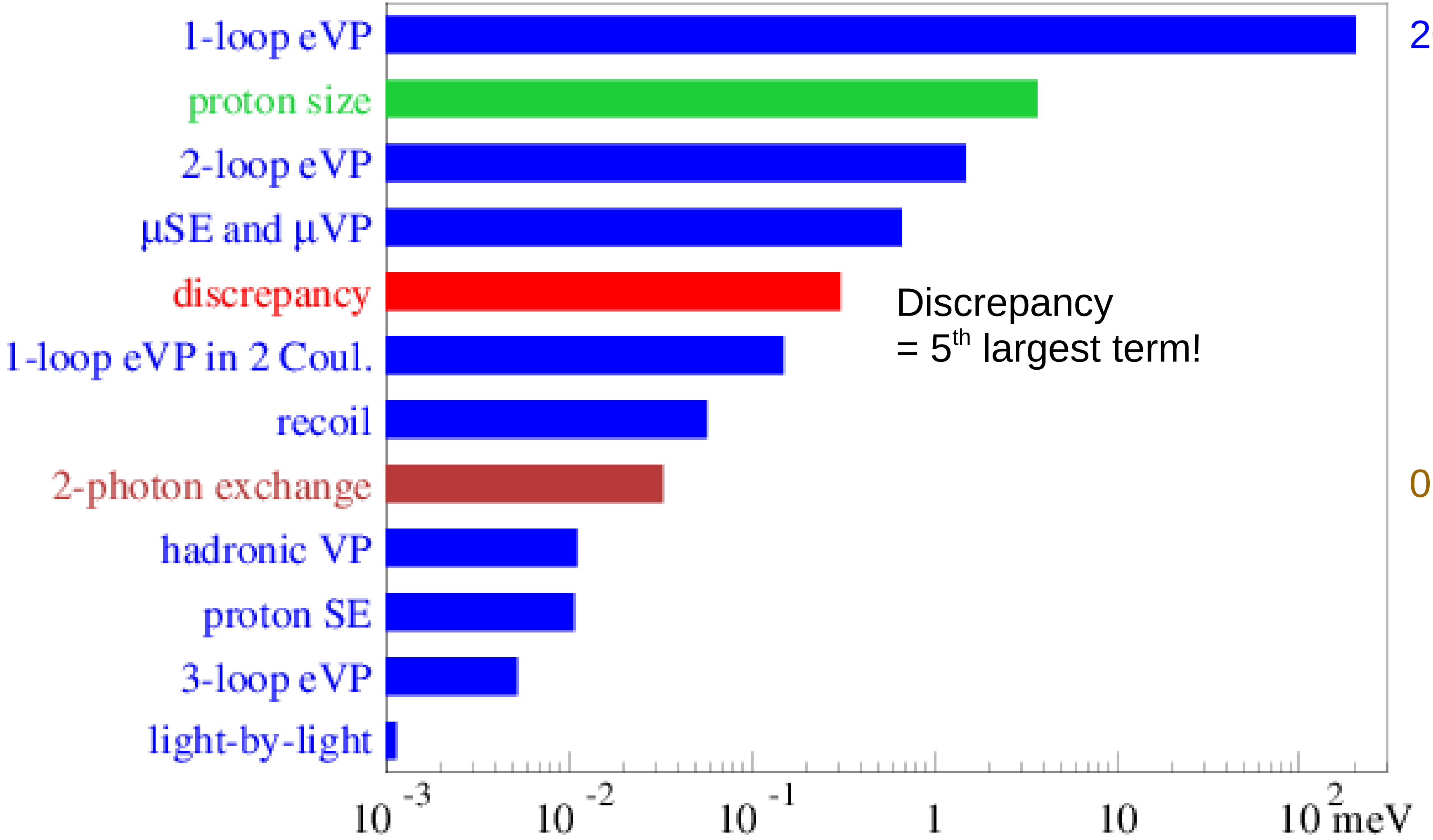
Nice hierarchy



Theory in muonic H

ΔE_{Lamb} = 206.0336 (15) meV_{QED} + 0.0332 (20) meV_{TPE} – 5.2275 (10) meV/fm^2 * R_p^2

Nice hierarchy



205.02821 meV

3.7 meV

1.50810 meV

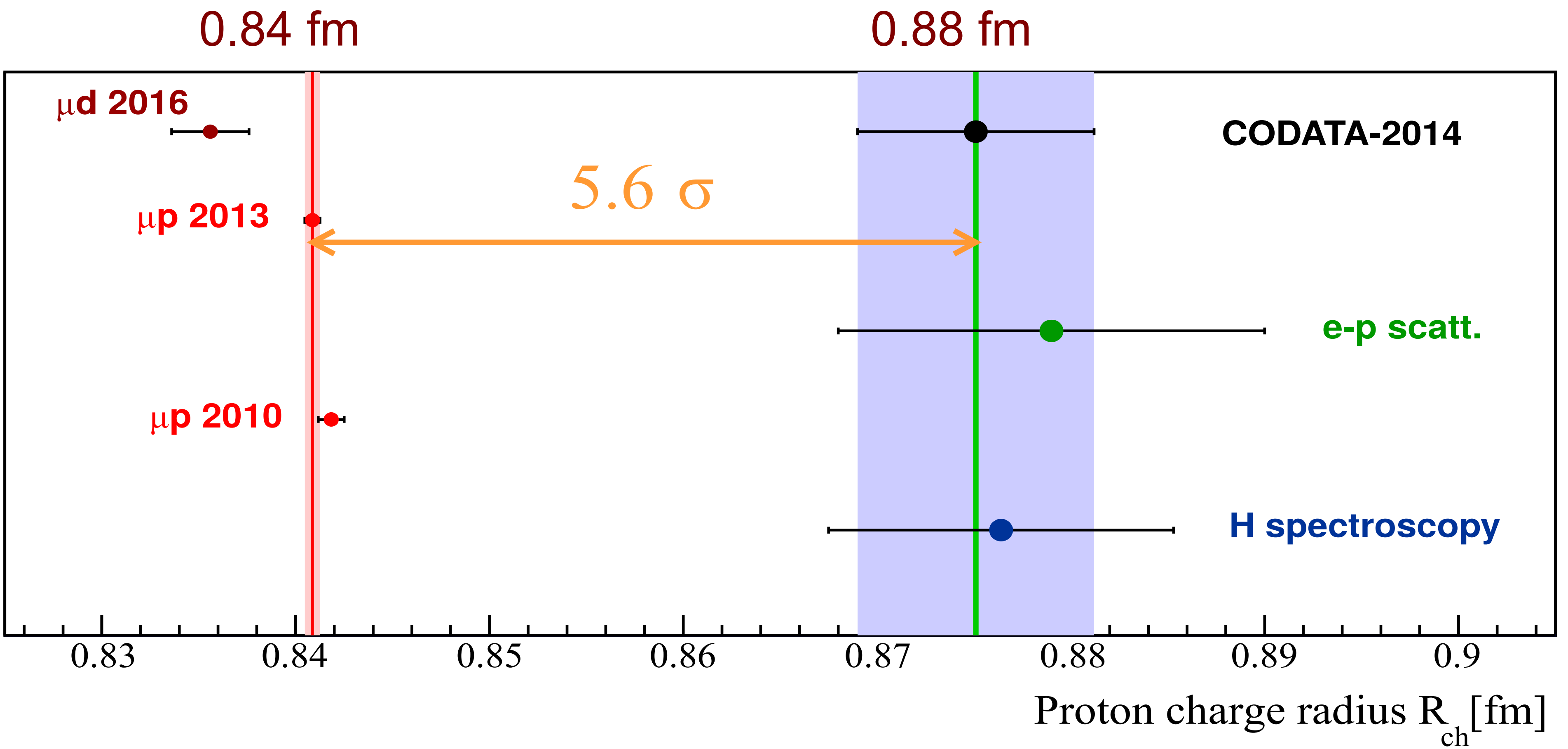
-0.66761 meV

0.33 meV

0.0332(20) meV

Discrepancy
= 5th largest term!

Muonic Hydrogen



muonic hydrogen: 0.8409 ± 0.0004 fm

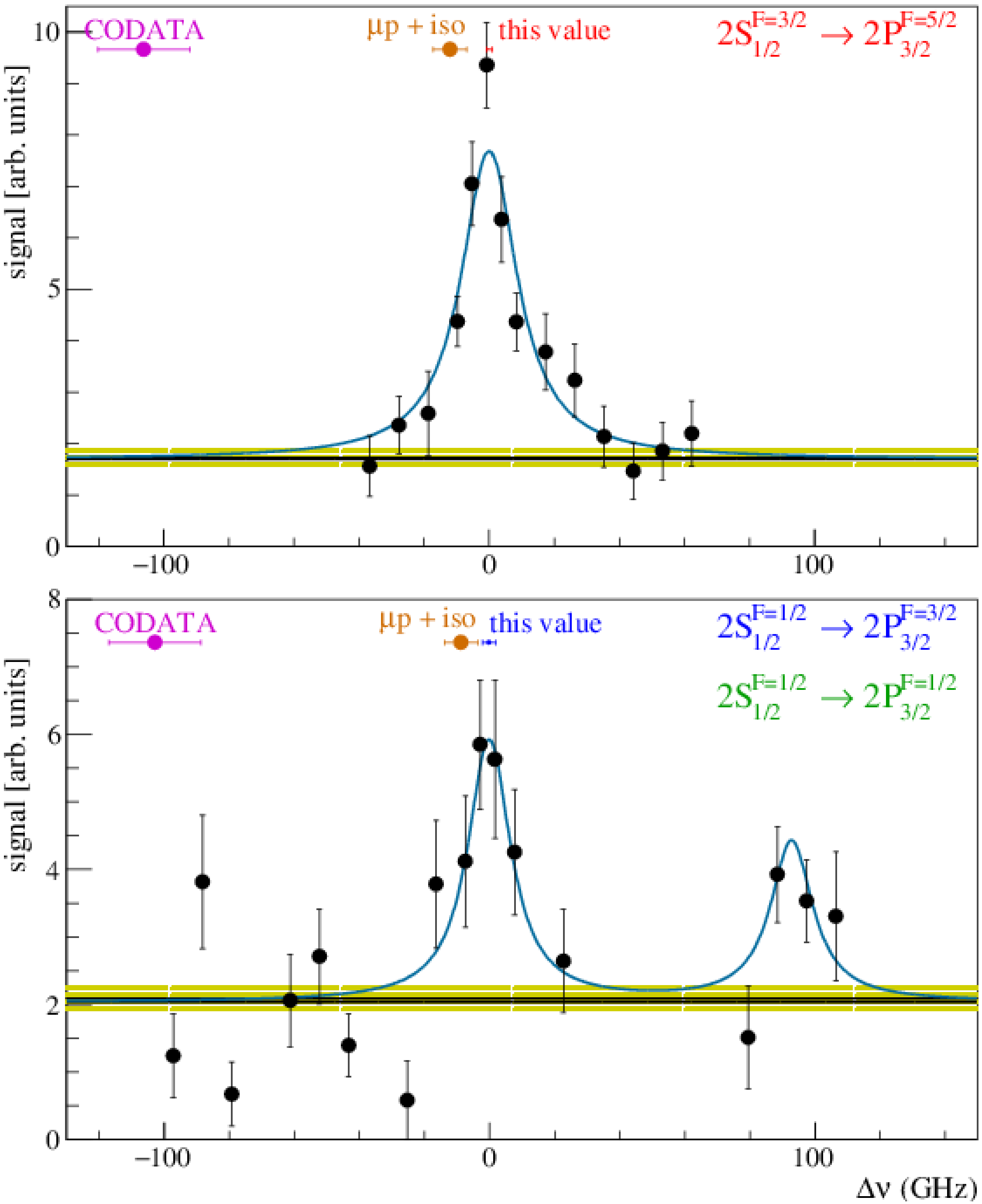
electronic hydrogen: 0.876 ± 0.008 fm

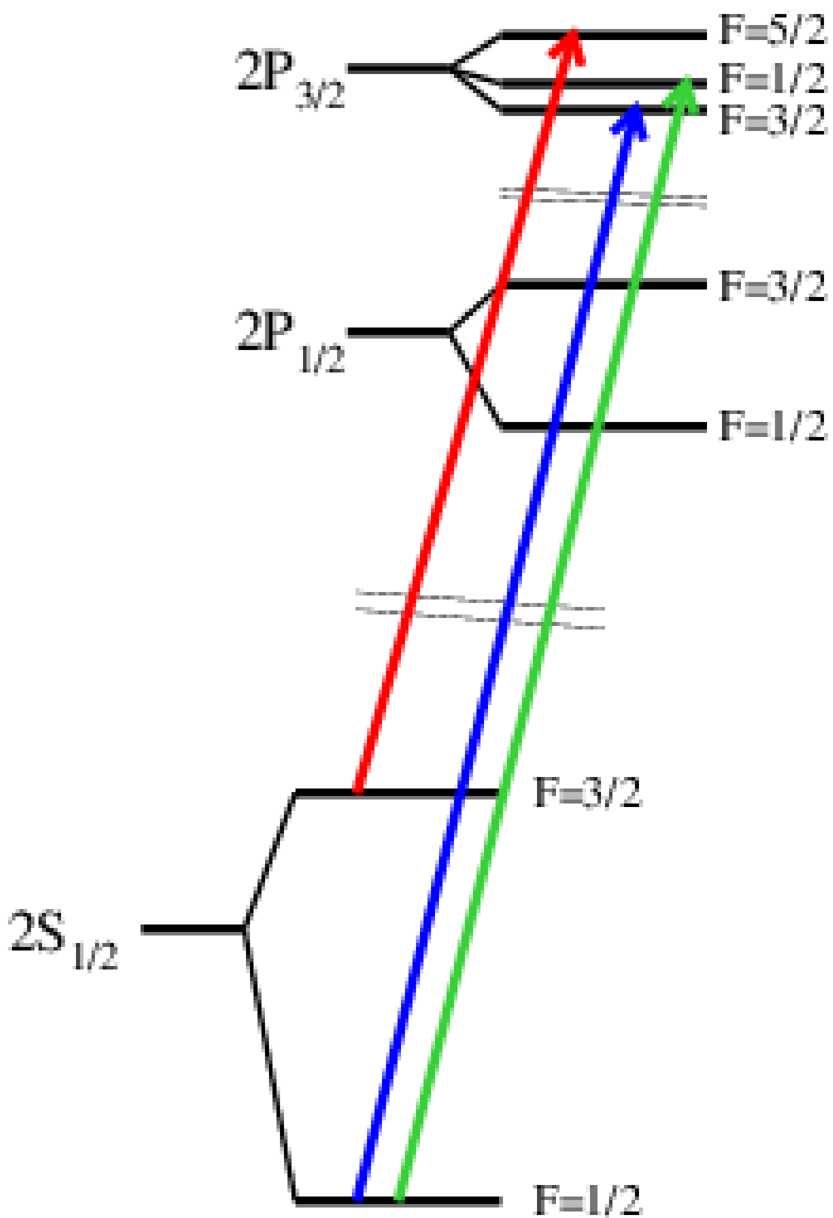
electron scattering 0.879 ± 0.011 fm

20x more precisee

Muonic Deuterium

2.5 transitions in muonic D





Pohl et al. (CREMA Coll.), Science 353, 669 (2016)

Theory in muonic D

$$\Delta E^{\mu D}_{Lamb} = 228.7766\,(10)\ \mathrm{meV}_{QED} + 1.7096\,(\mathbf{200})\ \mathrm{meV}_{TPE} - 6.1103\,(3)\ \mathrm{meV/fm^2} * \mathbf{R_d}^{\mathbf{2}}$$

$$\Delta E^{\mu H}_{Lamb} = 206.0336\,(15)\ \mathrm{meV}_{QED} + 0.0332\,(\ 20)\ \mathrm{meV}_{TPE} - 5.2275\,(10)\ \mathrm{meV/fm^2} * \mathbf{R_p}^{\mathbf{2}}$$


Annals of Physics 331 (2013) 127–145

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Theory of the 2S–2P Lamb shift and
splitting in muonic hydrogen

Aldo Antognini [a,*], Franz Kottmann [a], François
François Nez [b], Randolf Pohl [c]

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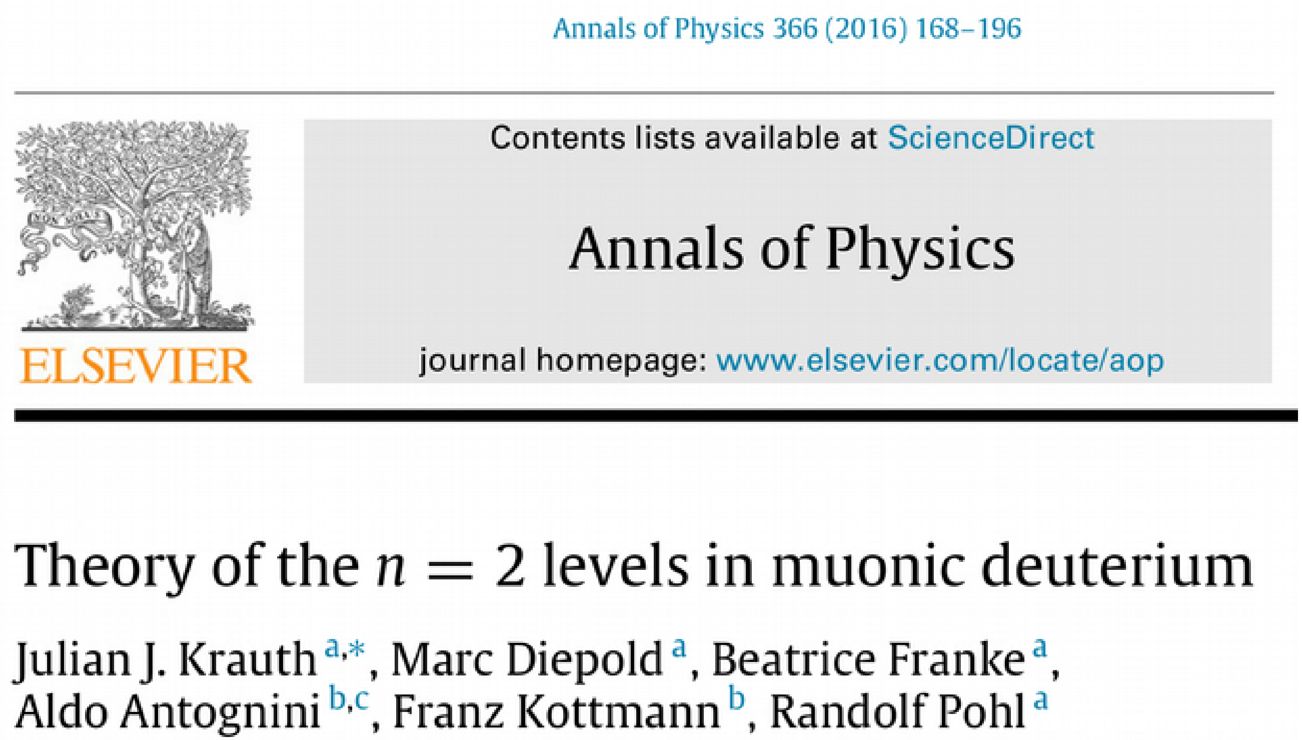
[b] Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P.
[c] Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany


Annals of Physics 366 (2016) 168–196

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Theory of the $n = 2$ levels in muonic deuterium

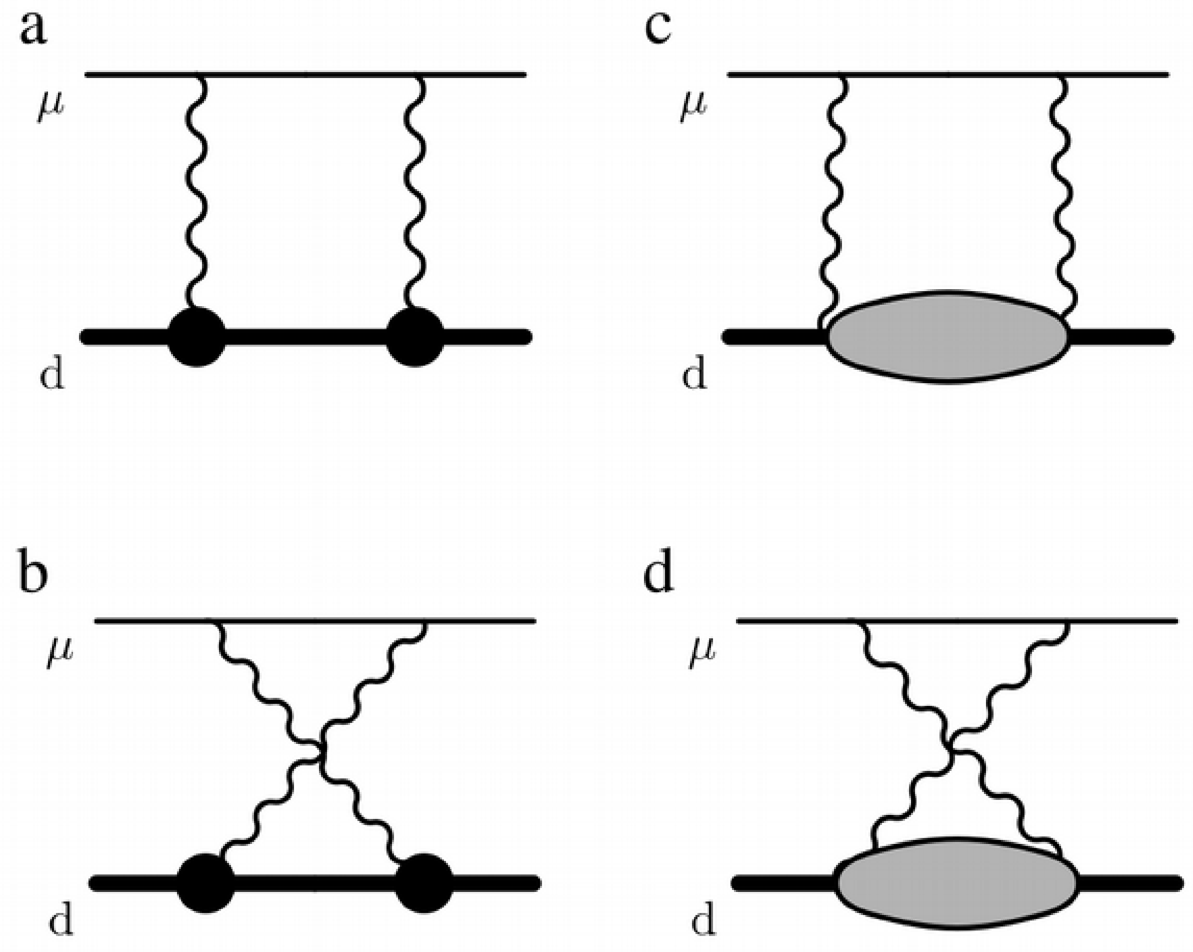
Julian J. Krauth [a,*], Marc Diepold [a], Beatrice Franke [a],
Aldo Antognini [b,c], Franz Kottmann [b], Randolf Pohl [a]

Summarizes original work by: Bacca, Barnea, Birse, Borie, Carlson, Eides, Faustov, Friar, Gorchtein, Hernandez, Ivanov, Jentschura, Ji, Karshenboim, Korzinin, Krutov, Martynenko, McGovern, Nevo~Dinur, Pachucki, Shelyuto, Sick, Vanderhaeghen, et al.

Theory in muonic D

$\Delta E^{\mu D}_{Lamb}$ = 228.7766 (10) meV$_{QED}$ + 1.7096 (**200**) meV$_{TPE}$ – 6.1103 (3) meV/fm^2 * $\mathbf{R_d^2}$

Two-photon nuclear structure contributions to the Lamb shift in muonic deuterium.



Krauth, RP at al., Ann. Phys. (N.Y.) 366, 168 (2016)

Theory in muonic D

$$\Delta E^{\mu D}_{Lamb} = 228.7766\ (10)\ \mathrm{meV}_{QED} + 1.7096\ (\mathbf{200})\ \mathrm{meV}_{TPE} - 6.1103\ (3)\ \mathrm{meV/fm}^2 * \mathbf{R_d}^{\mathbf{2}}$$

Nuclear structure contributions to the Lamb shift in muonic deuterium.

Item	Contribution	Pachucki [55]		Friar [60]		Hernandez *et al.* [58]			Pach.& Wienczek [65]		Carlson *et al.* [64]		Our choice		
		AV18		ZRA		AV18	N^3LO †		AV18		data		value		source
	Source	1		2		3	4		5		6				
p1	Dipole	1.910	$\delta_0 E$	1.925	Leading C1	1.907	1.926	$\delta^{(0)}_{D1}$	1.910	$\delta_0 E$			1.9165	± 0.0095	3-5
p2	Rel. corr. to p1, longitudinal part	−0.035	$\delta_R E$	−0.037	Subleading C1	−0.029	−0.030	$\delta^{(0)}_{L}$	−0.026	$\delta_R E$					
p3	Rel. corr. to p1, transverse part					0.012	0.013	$\delta^{(0)}_{T}$							
p4	Rel. corr. to p1, higher-order								0.004	$\delta_{HO} E$					
sum	Total rel. corr., p2+p3+p4	−0.035		−0.037		−0.017	−0.017		−0.022				−0.0195	± 0.0025	3-5
p5	Coulomb distortion, leading	−0.255	$\delta_{C1} E$						−0.255	$\delta_{C1} E$					
p6	Coul. distortion, next order	−0.006	$\delta_{C2} E$						−0.006	$\delta_{C2} E$					
sum	Total Coulomb distortion, p5+p6	−0.261				−0.262	−0.264	$\delta^{(0)}_{C}$	−0.261				−0.2625	± 0.0015	3-5
p7	El. monopole excitation	−0.045	$\delta_{Q0} E$	−0.042	C0	−0.042	−0.041	$\delta^{(2)}_{R2}$	−0.042	$\delta_{Q0} E$					
p8	El. dipole excitation	0.151	$\delta_{Q1} E$	0.137	Retarded C1	0.139	0.140	$\delta^{(2)}_{D1D3}$	0.139	$\delta_{Q1} E$					
p9	El. quadrupole excitation	−0.066	$\delta_{Q2} E$	−0.061	C2	−0.061	−0.061	$\delta^{(2)}_{Q}$	−0.061	$\delta_{Q2} E$					
sum	Tot. nuclear excitation, p7+p8+p9	0.040		0.034	C0 + ret-C1 + C2	0.036	0.038		0.036				0.0360	± 0.0020	2-5
p10	Magnetic	−0.008 ◇[a]	$\delta_M E$	−0.011	M1	−0.008	−0.007	$\delta^{(0)}_{M}$	−0.008	$\delta_M E$			−0.0090	± 0.0020	2-5
SUM_1	Total nuclear (corrected)	1.646		1.648 [b]		1.656	1.676		1.655				1.6615	± 0.0103	
p11	Finite nucleon size			0.021	Retarded C1 f.s.	0.020 ◇[c]	0.021 ◇[c]	$\delta^{(2)}_{NS}$	0.020	$\delta_{FS} E$					
p12	n p charge correlation			−0.023	pn correl. f.s.	−0.017	−0.017	$\delta^{(1)}_{np}$	−0.018	$\delta_{FZ} E$					
sum	p11+p12			−0.002		0.003	0.004		0.002				0.0010	± 0.0030	2-5
p13	Proton elastic 3rd Zemach moment	} 0.043(3)	$\delta_P E$	0.030	$\langle r^3 \rangle^{pp}_{(2)}$				} 0.043(3)	$\delta_P E$			0.0289	± 0.0015	Eq.(13)[d]
p14	Proton inelastic polarizab.					} 0.027(2)		δ^N_{pol} [64]			} 0.028(2)	ΔE^{hadr}	} 0.0280	± 0.0020	6
p15	Neutron inelastic polarizab.								0.016(8)	$\delta_N E$					
p16	Proton & neutron subtraction term												−0.0098	± 0.0098	Eq.(15)[e]
sum	Nucleon TPE, p13+p14+p15+p16	0.043(3)		0.030		0.027(2)			0.059(9)				0.0471	± 0.0101	[f]
SUM_2	Total nucleon contrib.	0.043(3)		0.028		0.030(2)			0.061(9)				0.0476	± 0.0105	
	Sum, published	1.680(16)		1.941(19)		1.690(20)			1.717(20)		2.011(740)				
	Sum, corrected			1.697(19) [g]		1.714(20) [h]			1.707(20) [i]		1.748(740) [j]		**1.7096**	**± 0.0147**	

Krauth, RP at al., Ann. Phys. (N.Y.) 366, 168 (2016)

Theory in muonic D

$$\Delta E^{\mu D}_{Lamb} = 228.7766\ (10)\ meV_{QED} + 1.7096\ (\mathbf{200})\ meV_{TPE} - 6.1103\ (3)\ meV/fm^2 * \mathbf{R_d}^{\mathbf{2}}$$

ΔE_{TPE} (theo) = 1.7096 +- 0.0200 meV

vs. +- 0.0034 meV experimental uncertainty

charge radius:

r_d (μD) = 2.12562 (13) $_{exp}$ (77) $_{theo}$ fm vs.

r_d (CODATA-14) = 2.1**4**130 (250) fm

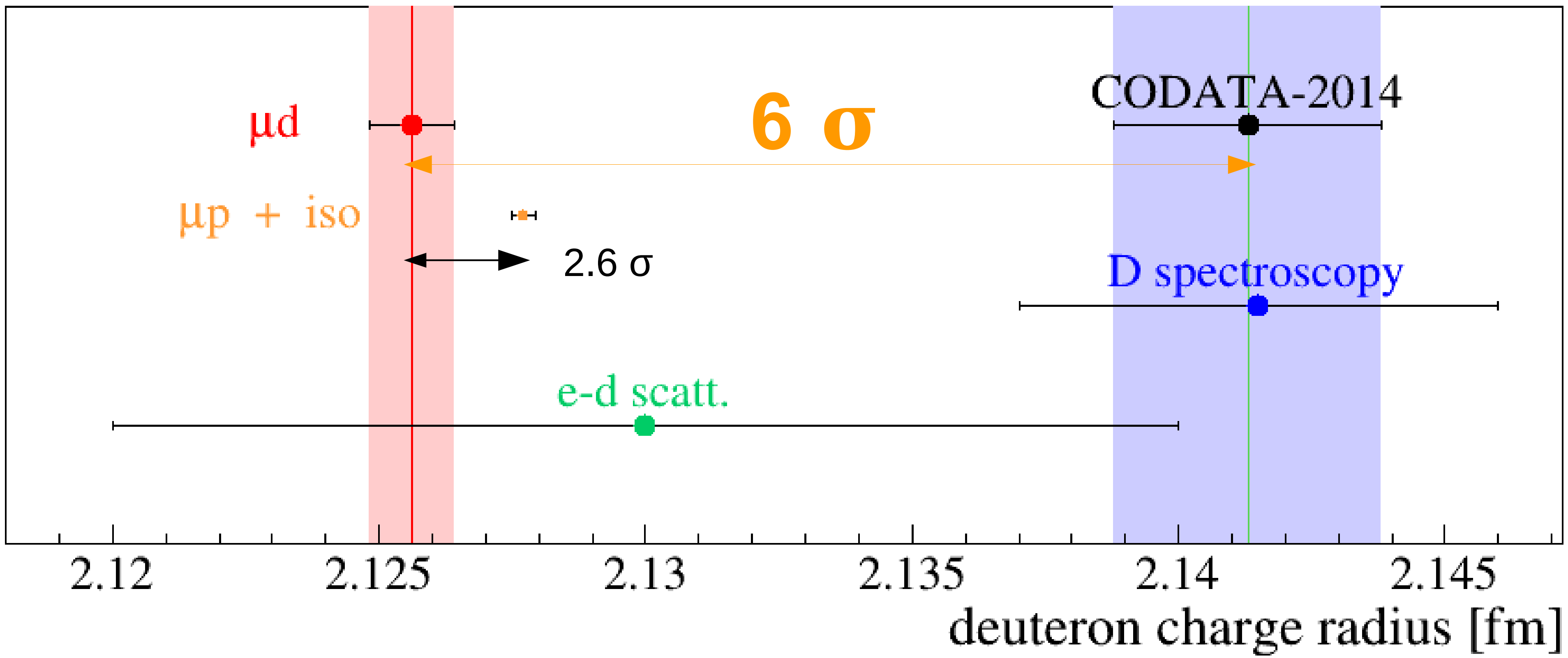
alternatively: polarizability:

ΔE_{TPE} (theo) = 1.7096 (200) meV vs.

ΔE_{TPE} (exp) = 1.7630 (68) meV 3x more accurate, 2.6σ

Krauth, RP at al., Ann. Phys. (N.Y.) 366, 168 (2016)

Muonic Deuterium



μD: 2.12562 $(13)_{exp}$ $(77)_{theo}$ fm (nucl. polarizability)

μH + H/D(1S-2S): 2.12771 (22) fm

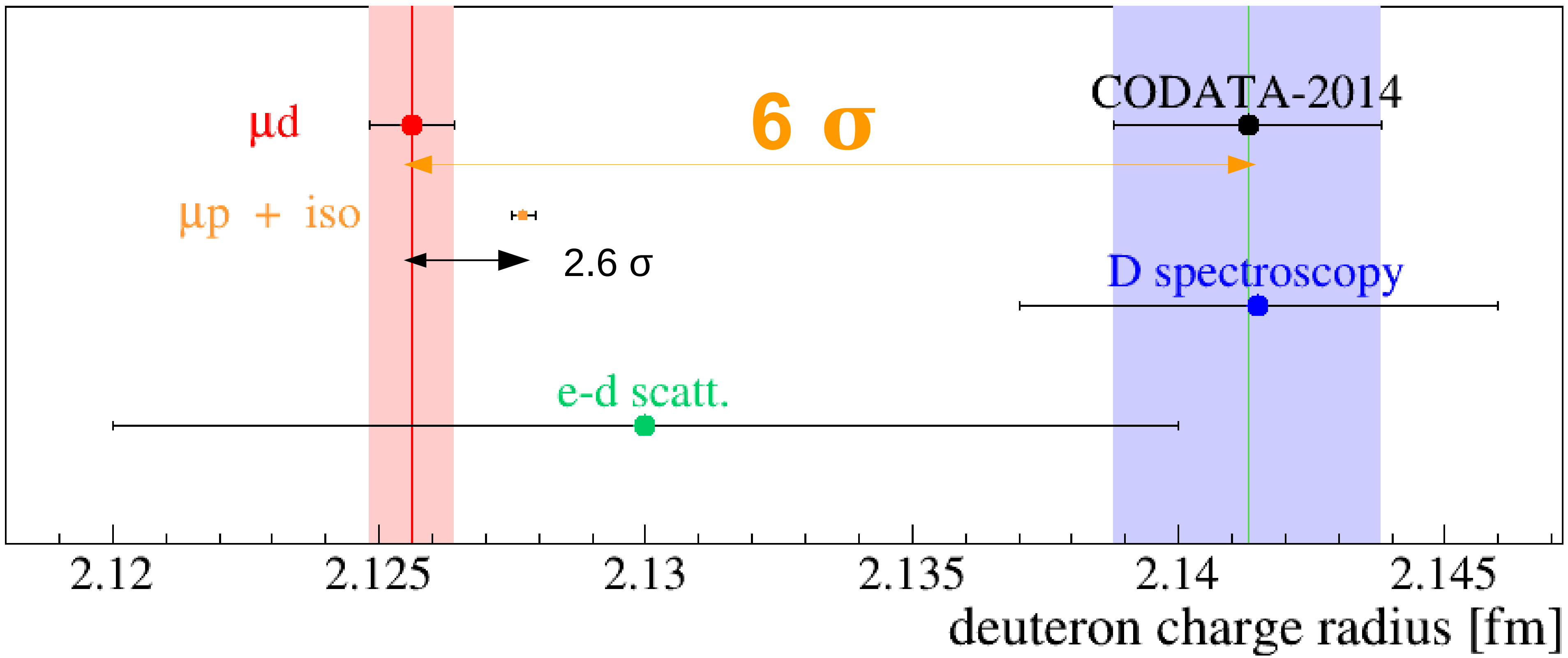
CODATA-2014: 2.1**4**130 (250) fm

H/D 1S-2S isotope shift:
$r_d^2 - r_p^2 = 3.82007(65)\ \mathrm{fm}^2$

Jentschura et al., PRA 83, 042505 (2011)

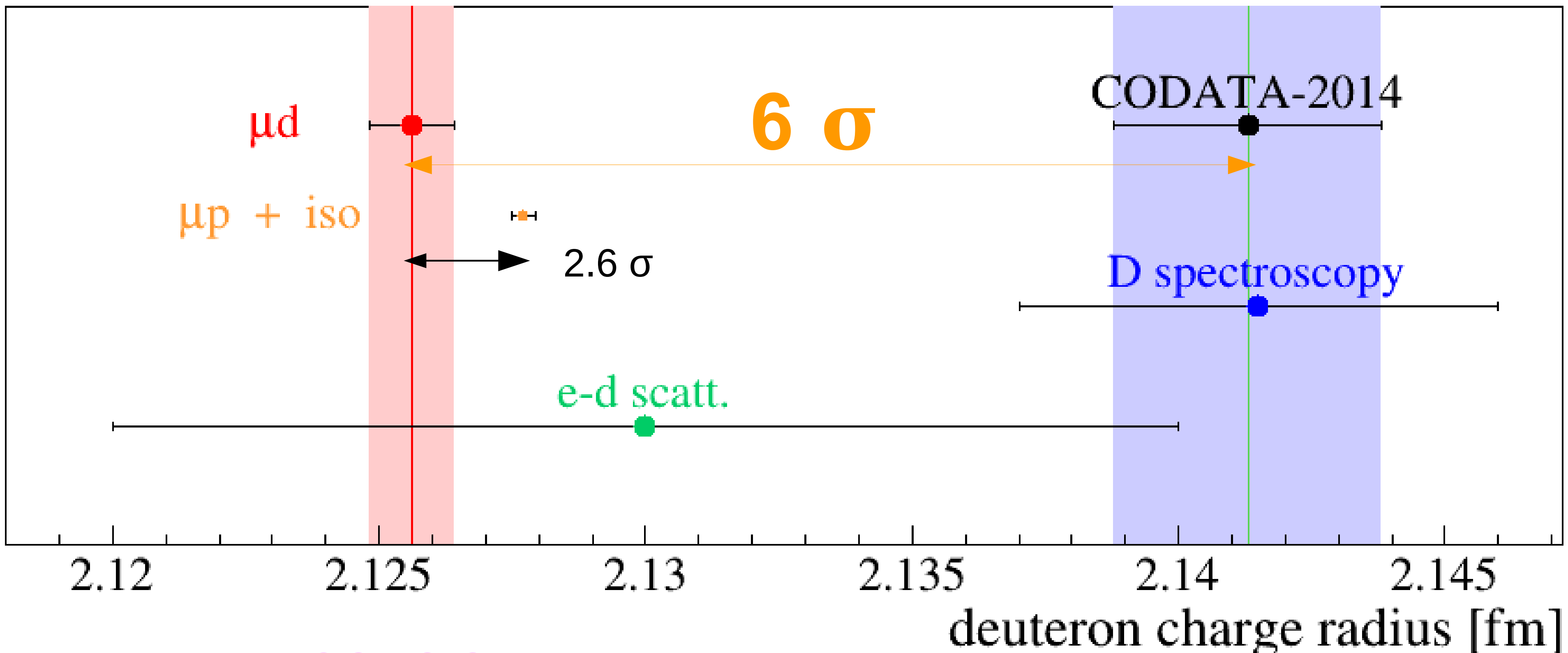
RP et al. (CREMA Coll.), Science 353, 559 (2016)

Deuteron radius



Pohl et al. (CREMA), Science 353, 669 (2016)

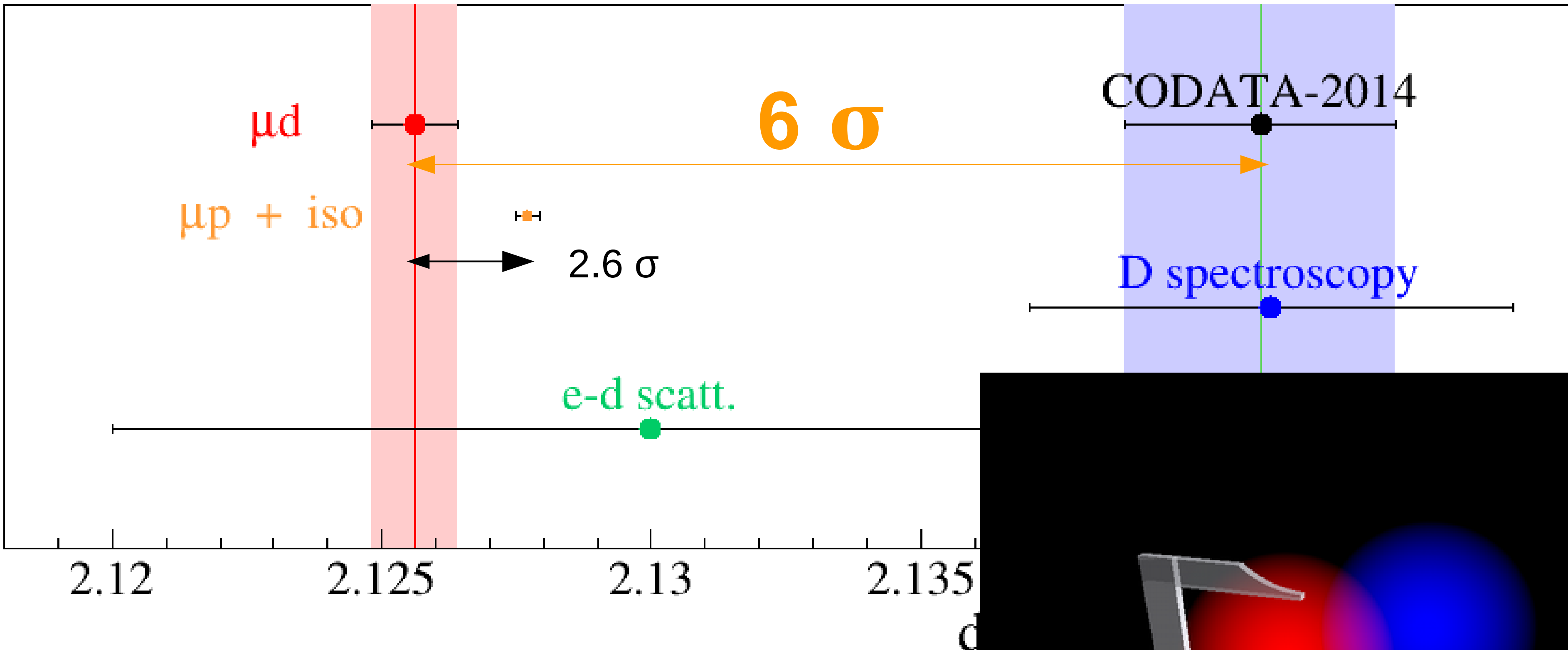
Deuteron radius

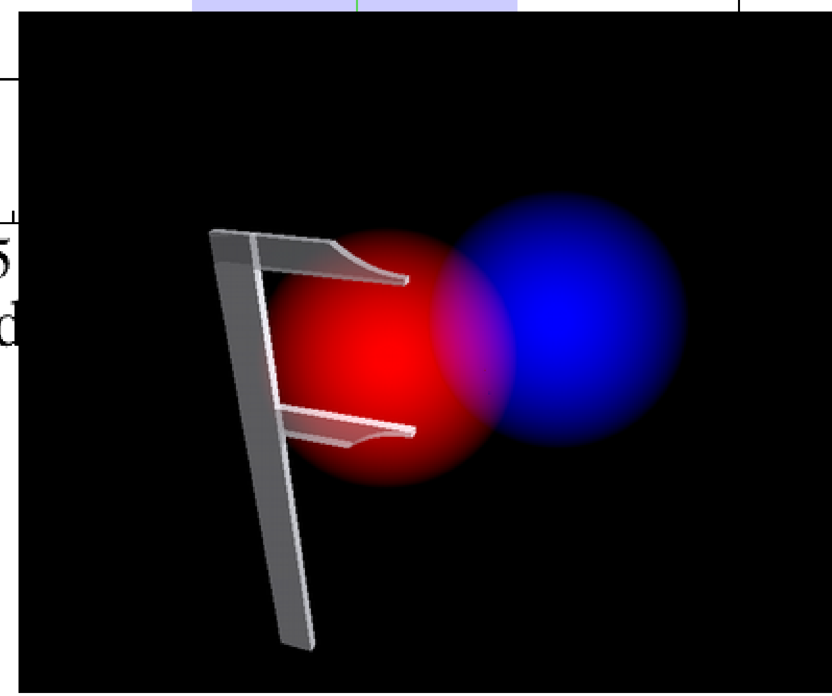


Deuteron is CONSISTENTLY smaller!

Pohl et al. (CREMA), Science 353, 669 (2016)

Deuteron radius

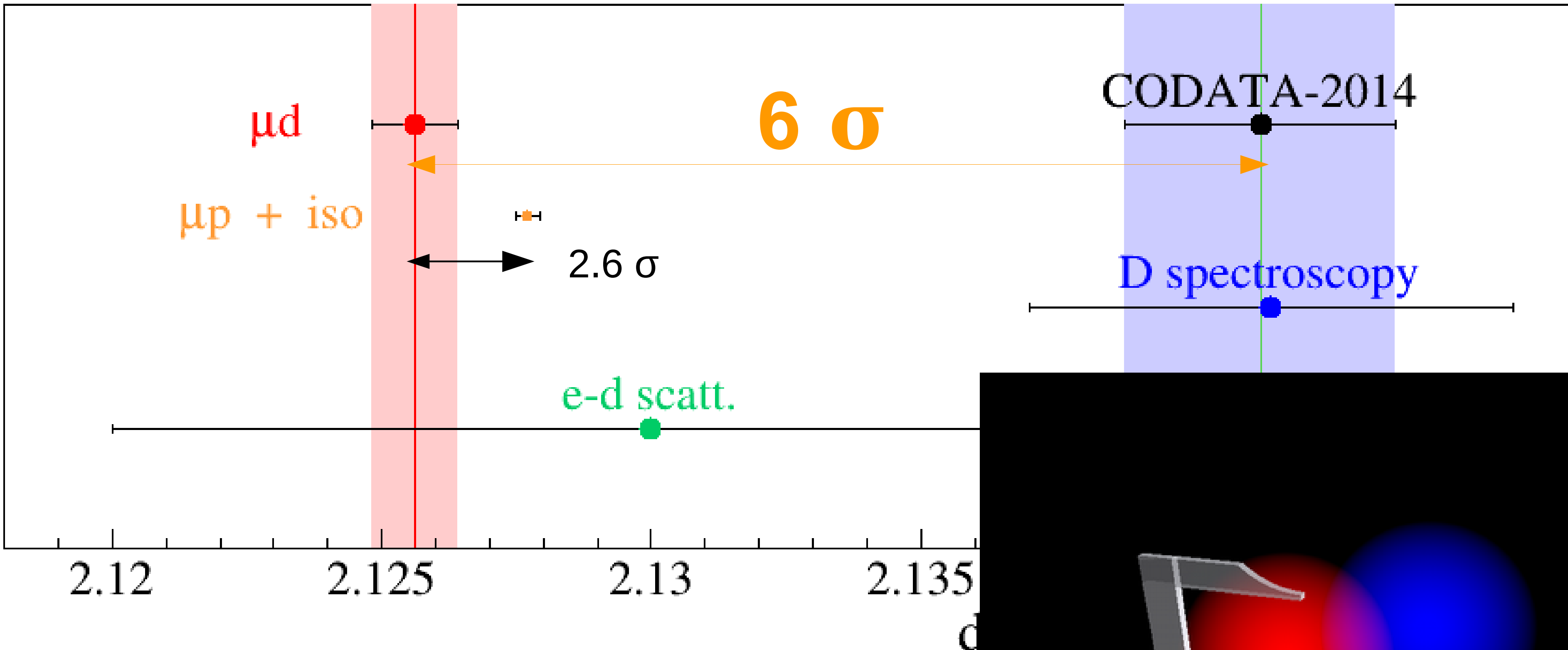


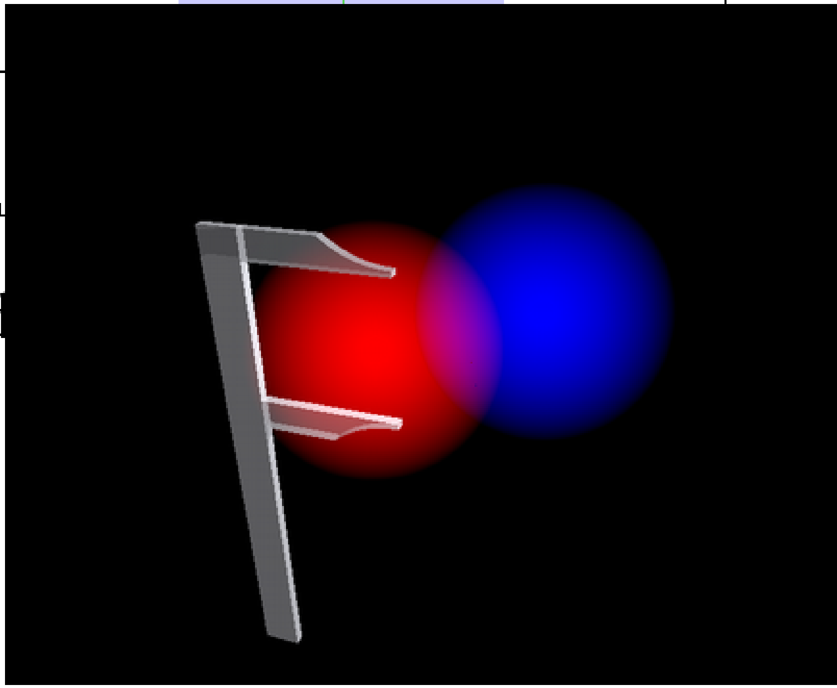


Deuteron is CONSISTENTLY smaller!

Pohl et al. (CREMA), Science 353, 669 (2016)

Deuteron radius





Deuteron is CONSISTENTLY smaller!

$$R_d^2 = R^2_{struct} + R_p^2 + R_n^2 \ (+ DF)$$

Pohl et al. (CREMA), Science 353, 669 (2016)

NEW: 3-photon contribution!

Three-photon exchange nuclear structure correction in hydrogenic systems

Krzysztof Pachucki*
Faculty of Physics, Warsaw University, Pasteura 5, 02-093 Warsaw, Poland

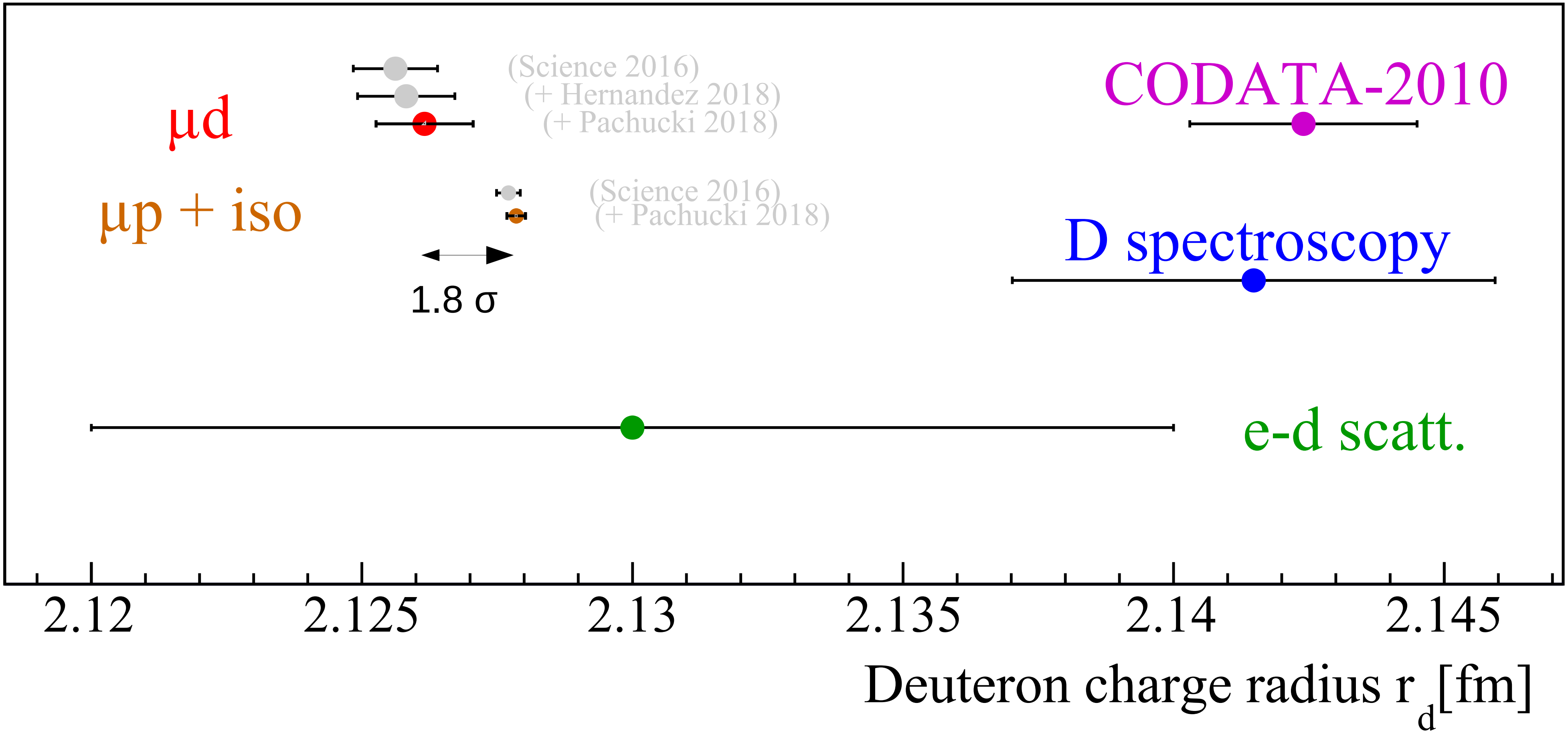
Vojtěch Patkóš
Faculty of Mathematics and Physics, Charles University, Ke Karlovu 3, 121 16 Prague 2, Czech Republic

Vladimir A. Yerokhin
Center for Advanced Studies, Peter the Great St. Petersburg Polytechnic University, Polytekhnicheskaya 29, 195251 St. Petersburg, Russia
(Dated: March 29, 2018)

The complete relativistic $O(\alpha^2)$ nuclear structure correction to the energy levels of ordinary (electronic) and muonic hydrogen-like atoms is investigated. The elastic part of the nuclear structure correction is derived analytically. The resulting formula is valid for an arbitrary hydrogenic system and is much simpler than analogous expressions previously reported in the literature. The analytical result is verified by high-precision numerical calculations. The inelastic $O(\alpha^2)$ nuclear structure correction is derived for the electronic and muonic deuterium atoms. The correction comes from a three-photon exchange between the nucleus and the bound lepton and has not been considered in the literature so far. We demonstrate that in the case of deuterium, the inelastic three-photon exchange contribution is of a similar size and of the opposite sign to the corresponding elastic part and, moreover, cancels exactly the model dependence of the elastic part. The obtained results affect the determination of nuclear charge radii from the Lamb shift in ordinary and muonic atoms.

arXiv 1803.10313

Deuteron radius



Hernandez 2018: Phys. Lett. B
Pachucki 2018: 1803.10313

Muonic Helium-3 and -4

Theory in muonic He-3

$$\Delta E^{\mu^3He}_{Lamb} = 1644.4820(149)_{QED} + 15.3000(\mathbf{5200})_{TPE} - 103.5184(10) * \mathbf{R_h}^{\mathbf{2}} / fm^2 \quad [meV]$$

$$\Delta E^{\mu D}_{Lamb} = 228.7766\,(10)_{QED} + 1.7096\,(200)_{TPE} - 6.1103\,(3) * \mathbf{R_d}^{\mathbf{2}} / fm^2 \quad [meV]$$

$$\Delta E^{\mu H}_{Lamb} = 206.0336\,(15)_{QED} + 0.0332(20)_{TPE} - 5.2275\,(10) * \mathbf{R_p}^{\mathbf{2}} / fm^2 \quad [meV]$$

Annals of Physics 331 (2013) 127–145

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Theory of the 2
splitting in mu

Aldo Antognini[a,*], F
François Nez[b], Ranc
[a] Institute for Particle Physics, ETH
[b] Laboratoire Kastler Brossel, Écol
[c] Max-Planck-Institut für Quanten

Annals of Physics 366 (2016) 168–196

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Theory of

Julian J. Krau
Aldo Antogn

Eur. Phys. J. D (2017) 71: 341
DOI: 10.1140/epjd/e2017-80296-1

THE EUROPEAN PHYSICAL JOURNAL D

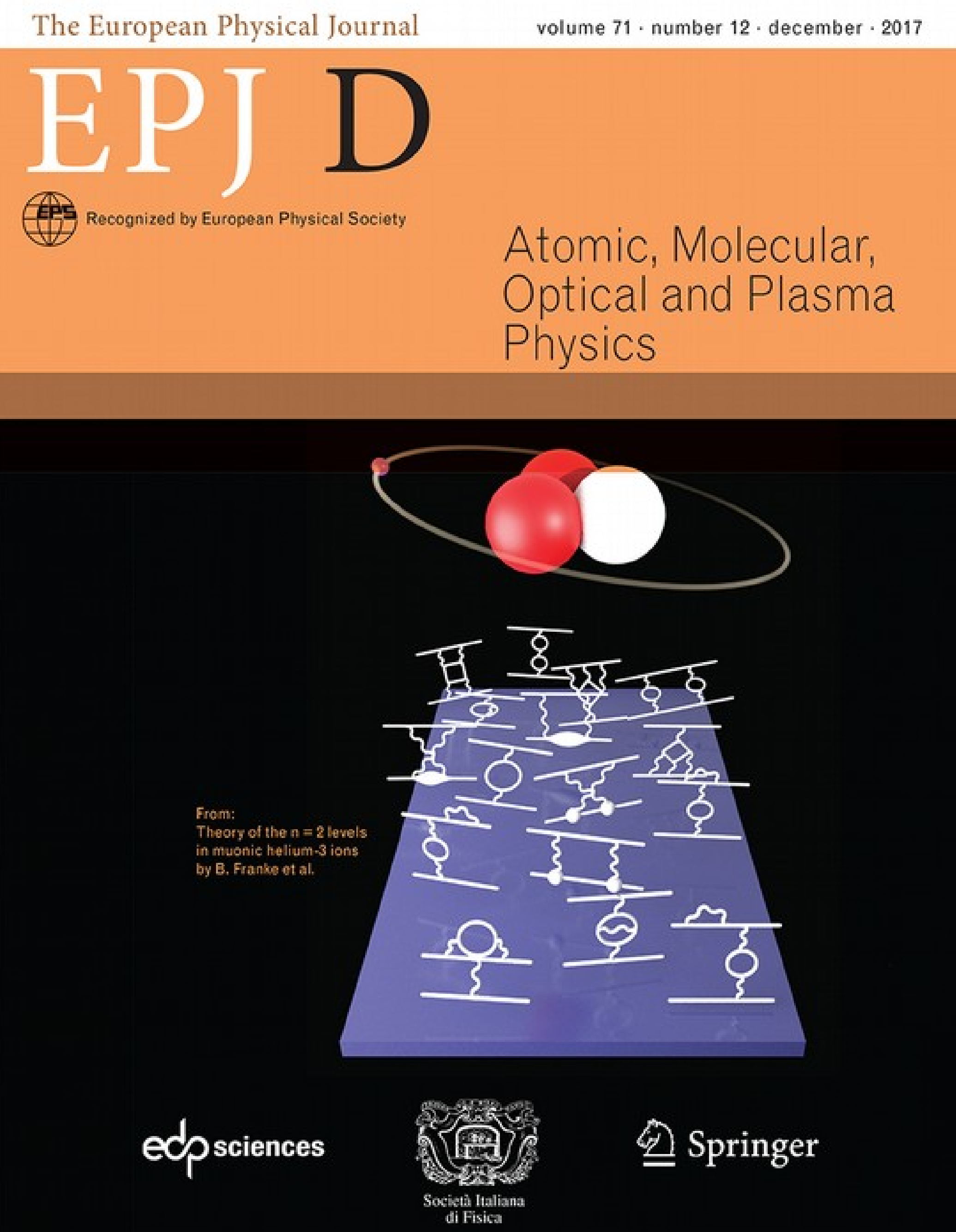
Topical Review

Theory of the $n = 2$ levels in muonic helium-3 ions

Beatrice Franke[1,2,a], Julian J. Krauth[1,3,b], Aldo Antognini[4,5], Marc Diepold[1], Franz Kottmann[4], and Randolf Pohl[3,1,c]

[1] Max–Planck–Institut für Quantenoptik, 85748 Garching, Germany
[2] TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada
[3] Johannes Gutenberg-Universität Mainz, QUANTUM, Institut für Physik & Exzellenzcluster PRISMA, 55099 Mainz, Germany
[4] Institute for Particle Physics and Astrophysics, ETH Zurich, 8093 Zurich, Switzerland
[5] Paul Scherrer Institute, 5232 Villigen, Switzerland

Theory in muonic He-3



$5200)_{TPE} - 103.5184(10) * R_h^2 /fm^2$ [meV]

$(200)_{TPE} - 6.1103(3) * R_d^2 /fm^2$ [meV]

$20)_{TPE} - 5.2275(10) * R_p^2 /fm^2$ [meV]

s 366 (2016) 168–196

THE EUROPEAN PHYSICAL JOURNAL D

levels in muonic helium-3 ions

h[1,3,b], Aldo Antognini[4,5], Marc Diepold[1], Franz Kottmann[4],

k, 85748 Garching, Germany
ouver, BC V6T 2A3, Canada
inz, QUANTUM, Institut für Physik & Exzellenzcluster PRISMA,
ophysics, ETH Zurich, 8093 Zurich, Switzerland
Switzerland

Theory in muonic He-4

$$\Delta E^{\mu^4He}_{Lamb} = 1668.5670(178)_{QED} + 9.9000(\mathbf{2800})_{TPE} - 106.3540(80) * \mathbf{R_\alpha}^2 /fm^2 \quad [meV]$$

$$\Delta E^{\mu^3He}_{Lamb} = 1644.4820(149)_{QED} + 15.3000(5200)_{TPE} - 103.5184(10) * \mathbf{R_h}^2 /fm^2 \quad [meV]$$

$$\Delta E^{\mu D}_{Lamb} = 228.7766\,(10)_{QED} + 1.7096\,(200)_{TPE} - 6.1103\,(3) * \mathbf{R_d}^2 /fm^2 \quad [meV]$$

$$\Delta E^{\mu H}_{Lamb} = 206.0336\,(15)_{QED} + 0.0332(20)_{TPE} - 5.2275\,(10) * \mathbf{R_p}^2 /fm^2 \quad [meV]$$

Annals of Physics 331 (2013) 127–145

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Theory of the 2S-2P Lamb shift and 2S hyperfine splitting in muonic hydrogen

Aldo Antognini[a,*], F...
François Nez[b], Randolf Pohl[c]
[a] Institute for Particle Physics, ETH...
[b] Laboratoire Kastler Brossel, École...
[c] Max-Planck-Institut für Quantenoptik...

Annals of Physics 366 (2016) 168–196

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Theory of the ...

Julian J. Krauth...
Aldo Antognini...

Eur. Phys. J. D (2017) 71: 341
DOI: 10.1140/epjd/e2017-80296-1

THE EUROPEAN PHYSICAL JOURNAL D

Topical Review

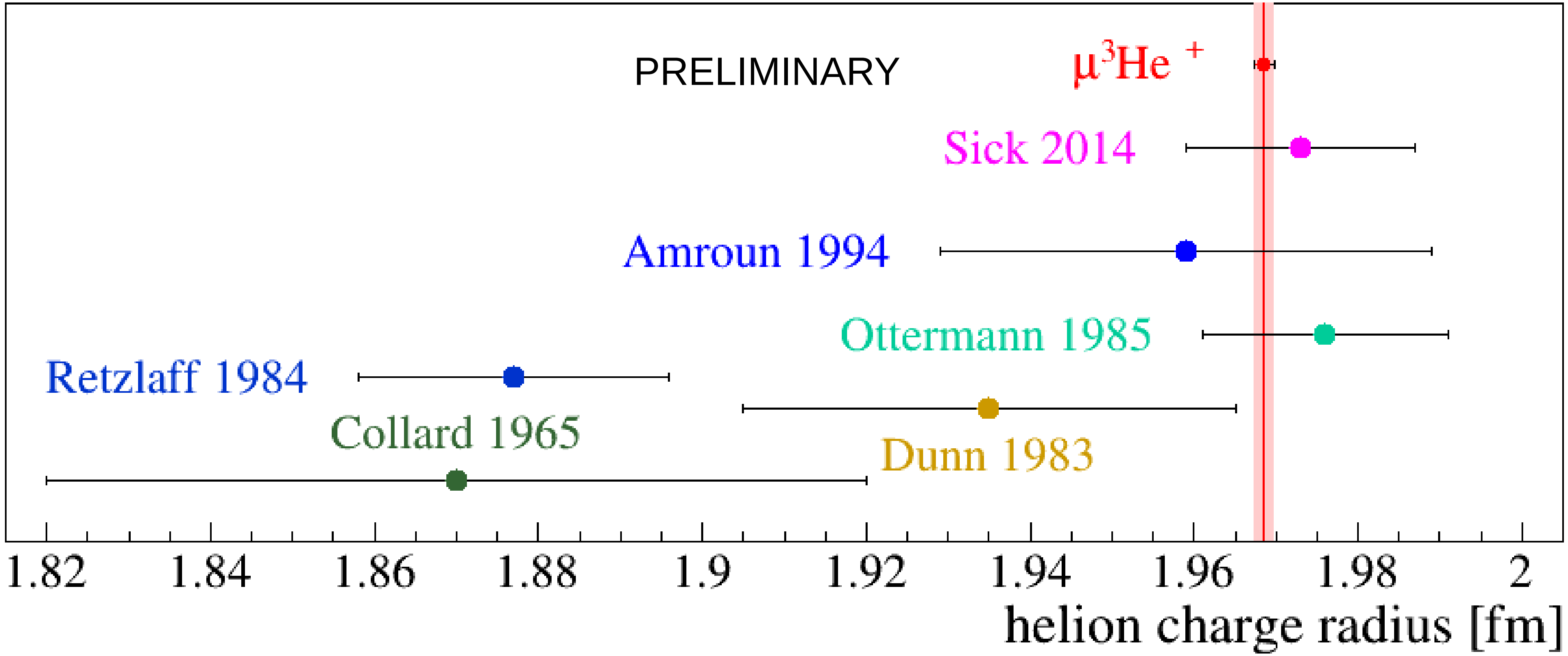
Theory of the $n = 2$ levels in muonic helium-3 ions

Beatrice ...
and Ran...
[1] Max–Pl...
[2] TRIUM...
[3] Johann...
55099 M...
[4] Institut...
[5] Paul Sc...

Theory of the Lamb Shift and Fine Structure in muonic ^{4}He ion and the muonic ^{3}He–^{4}He Isotope Shift

Marc Diepold,[1] Beatrice Franke,[1,2] Julian J. Krauth,[1,3,*] Aldo Antognini,[4,5] Franz Kottmann,[4] and
[1] Max Planck Institute of Quantum Optics, 85748 Garching, Germany
[2] TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada
[3] Johannes Gutenberg-Universität Mainz, QUANTUM, Institut für Physik & Exzellenzcluster PRISMA,
[4] Institute for Particle Physics and Astrophysics, ETH Zurich, 8093 Zurich, Switzerland
[5] Paul Scherrer Institute, 5232 Villigen-PSI, Switzerland
(Dated: April 22, 2018)

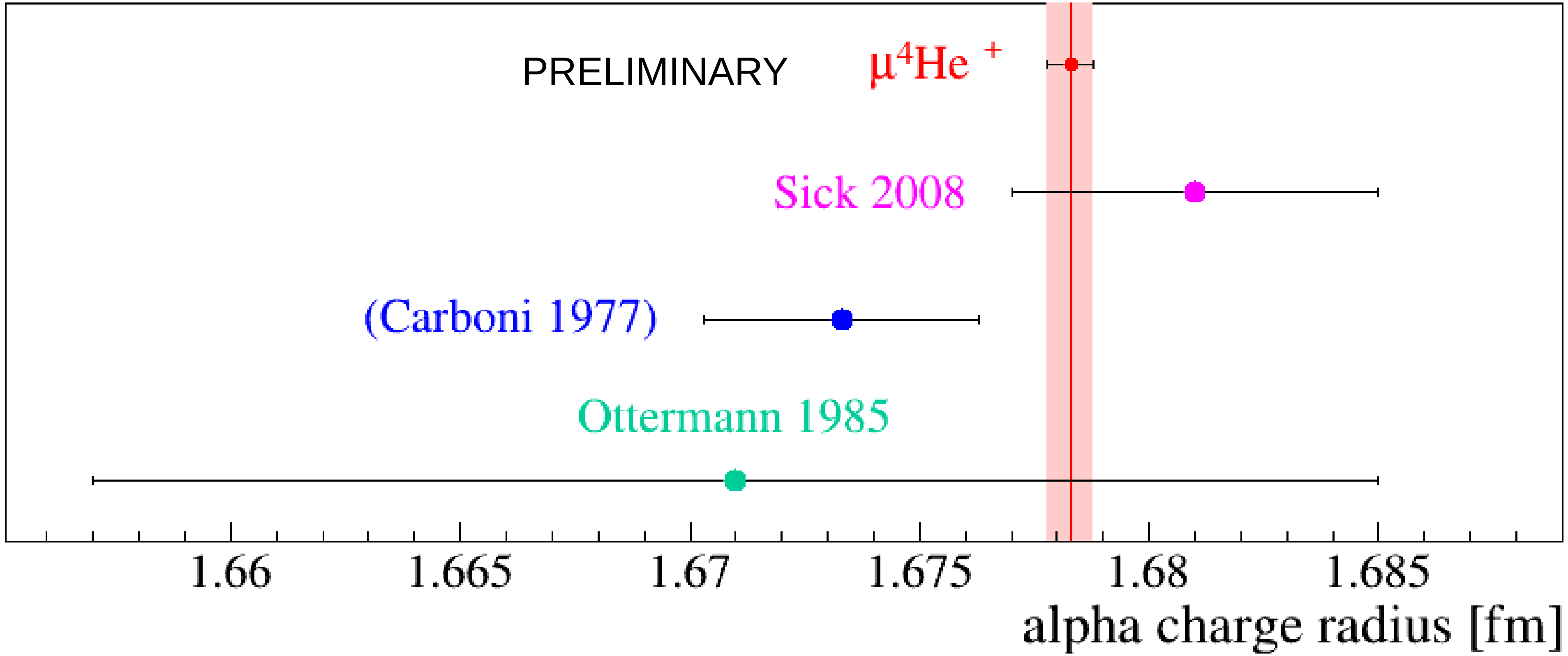
Muonic Helium-3



prel. accuracy: exp +- 0.00012 fm, theo +- 0.00128 fm (nucl. polarizability)

Theory: see Franke et al. EPJ D 71, 341 (2017) [1705.00352]

Muonic Helium-4



prel. accuracy: exp +- 0.00019 fm, theo +- 0.00058 fm (nucl. polarizability)

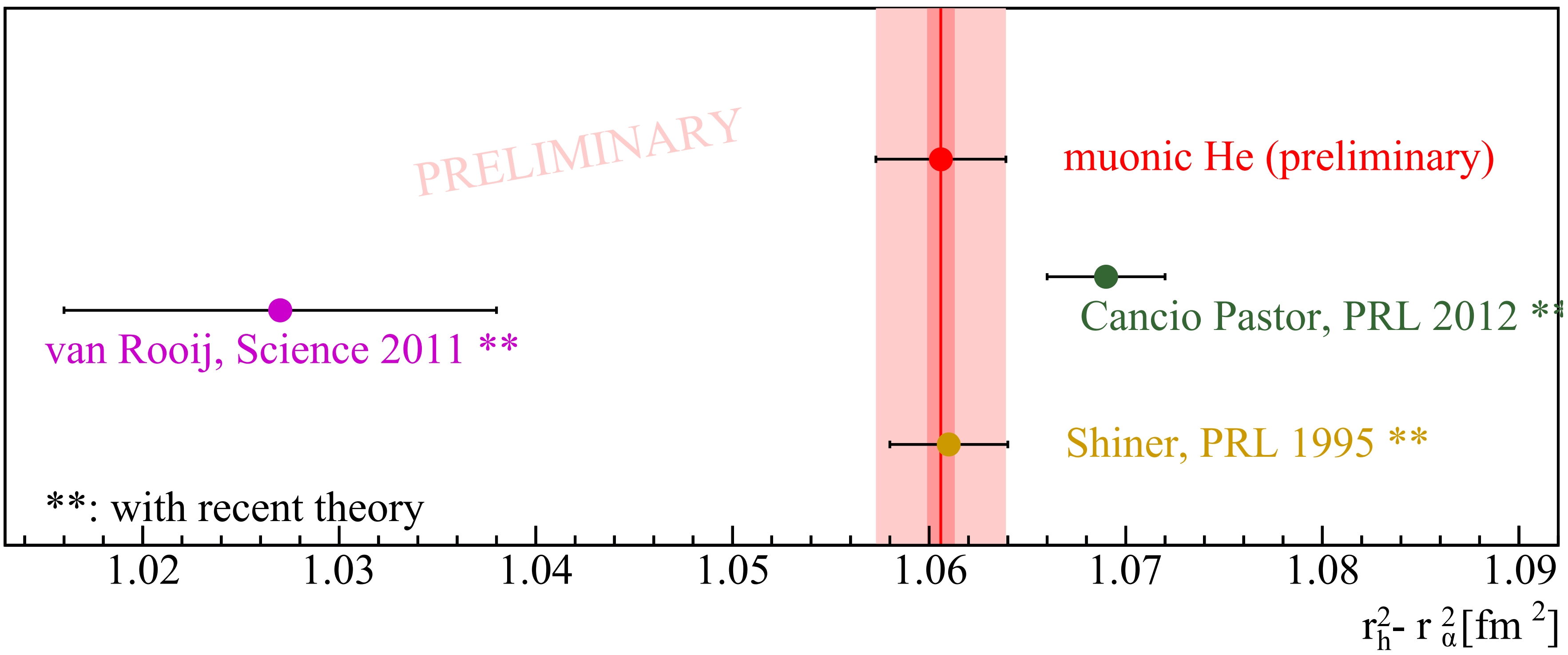
Theory: see Diepold et al. arxiv 1606.05231

Muonic conclusions

- The proton radius is 0.84087 (26)$_{exp}$ (29)$_{theo}$ fm
- The deuteron radius is 2.12771 (22) fm
- both are **>5σ smaller** than CODATA values
- No discrepancy for the **absolute radii** of the helion and alpha particle
(limited by e-scattering accuracy)
- **BUT: The helium isotope shift!!!**
- **(caveat: 3-photon, maybe more missing?)**

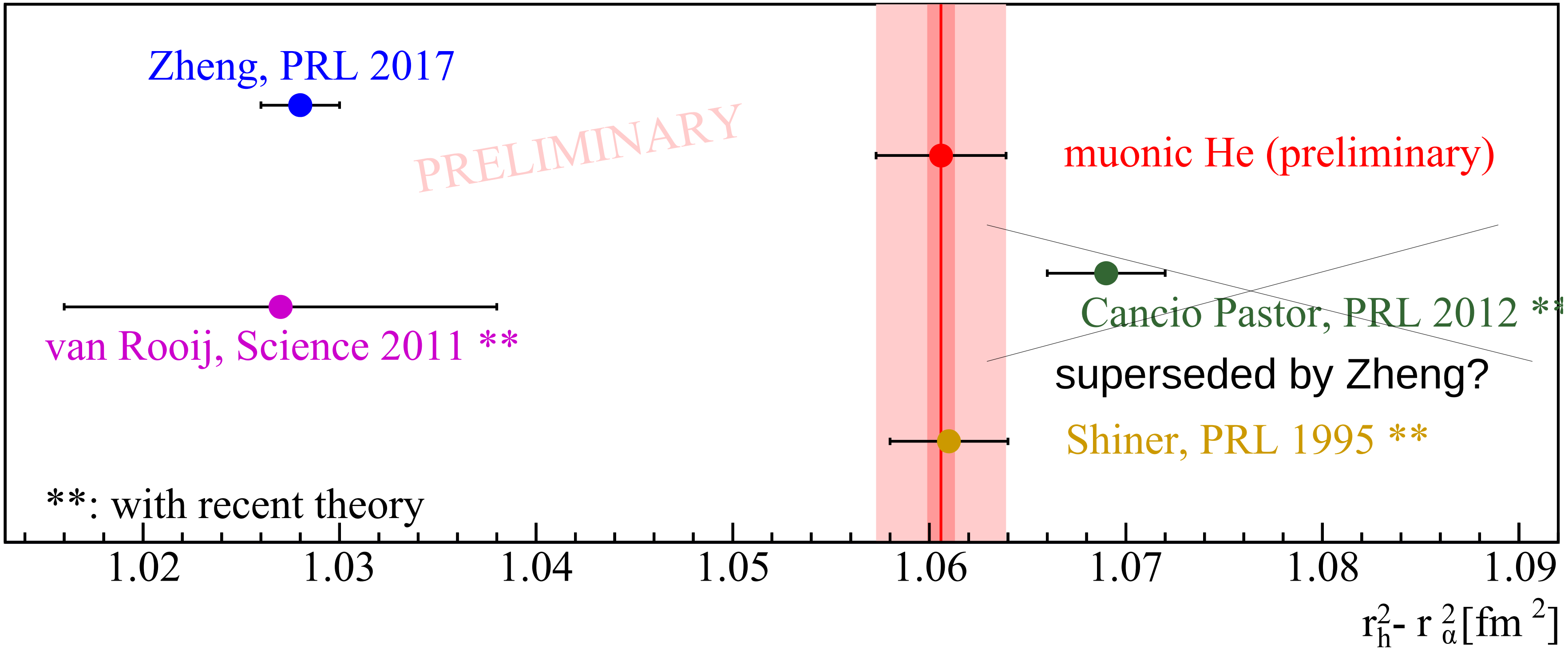
The ^{3}He – ^{4}He isotope shift

^{3}He / ^{4}He (squared) charge radius difference



The ^{3}He – ^{4}He isotope shift

^{3}He / ^{4}He (squared) charge radius difference



Another >5σ discrepancy?!

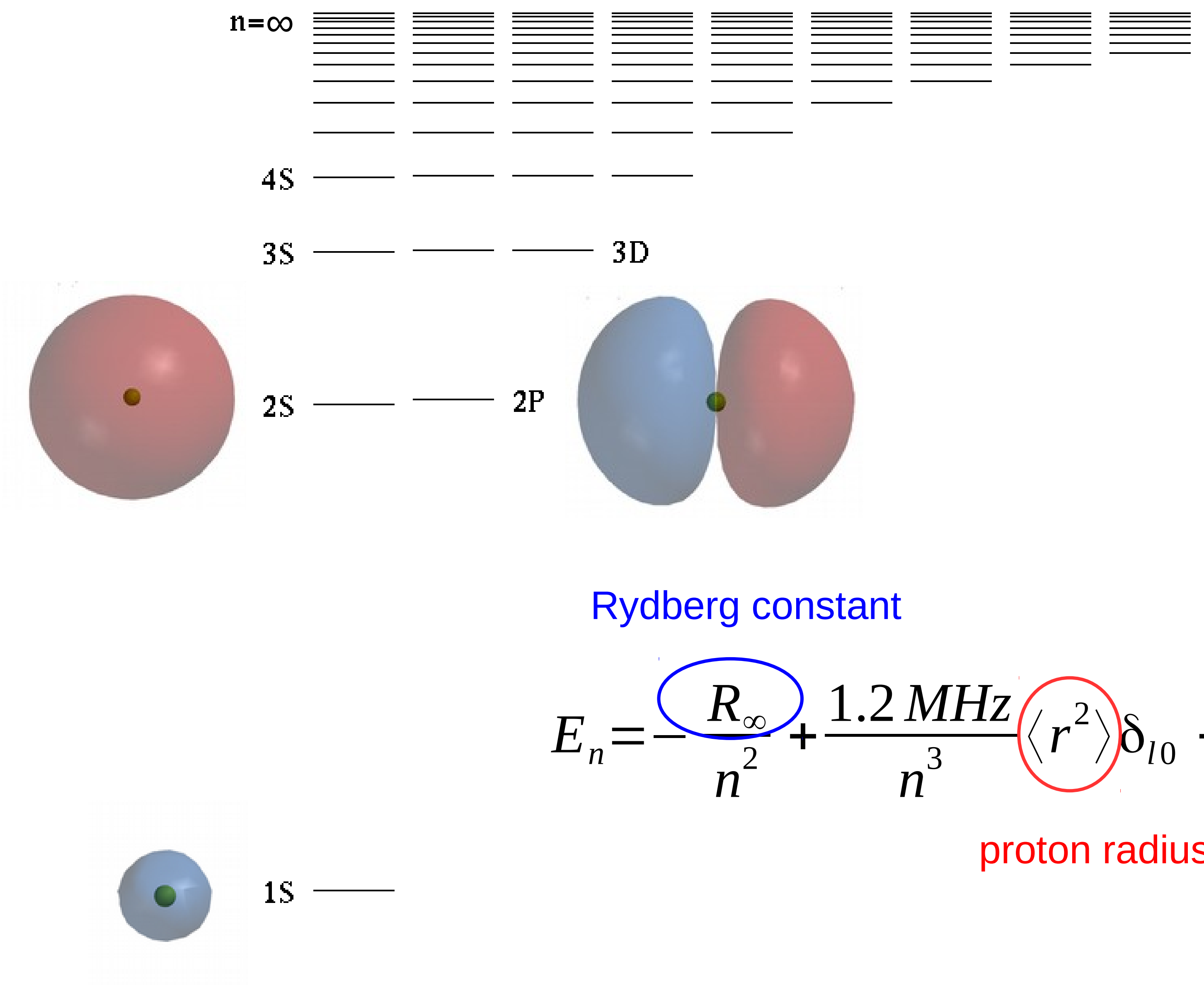
Part 2: The Rydberg constant

$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$

- most accurately determined fundamental constant u_r = 5.9 * 10^{-12}
- corner stone of the CODATA LSA of fundamental constants
  links fine structure constant α, electron mass m_e, velocity of light c and Planck's constant *h*
- correlation coefficient with proton radius: 0.9891
  → The "proton radius puzzle" could be a "Rydberg puzzle"

- R_∞ is a "unit converter": atomic units → SI (Hertz)

Energy levels of hydrogen

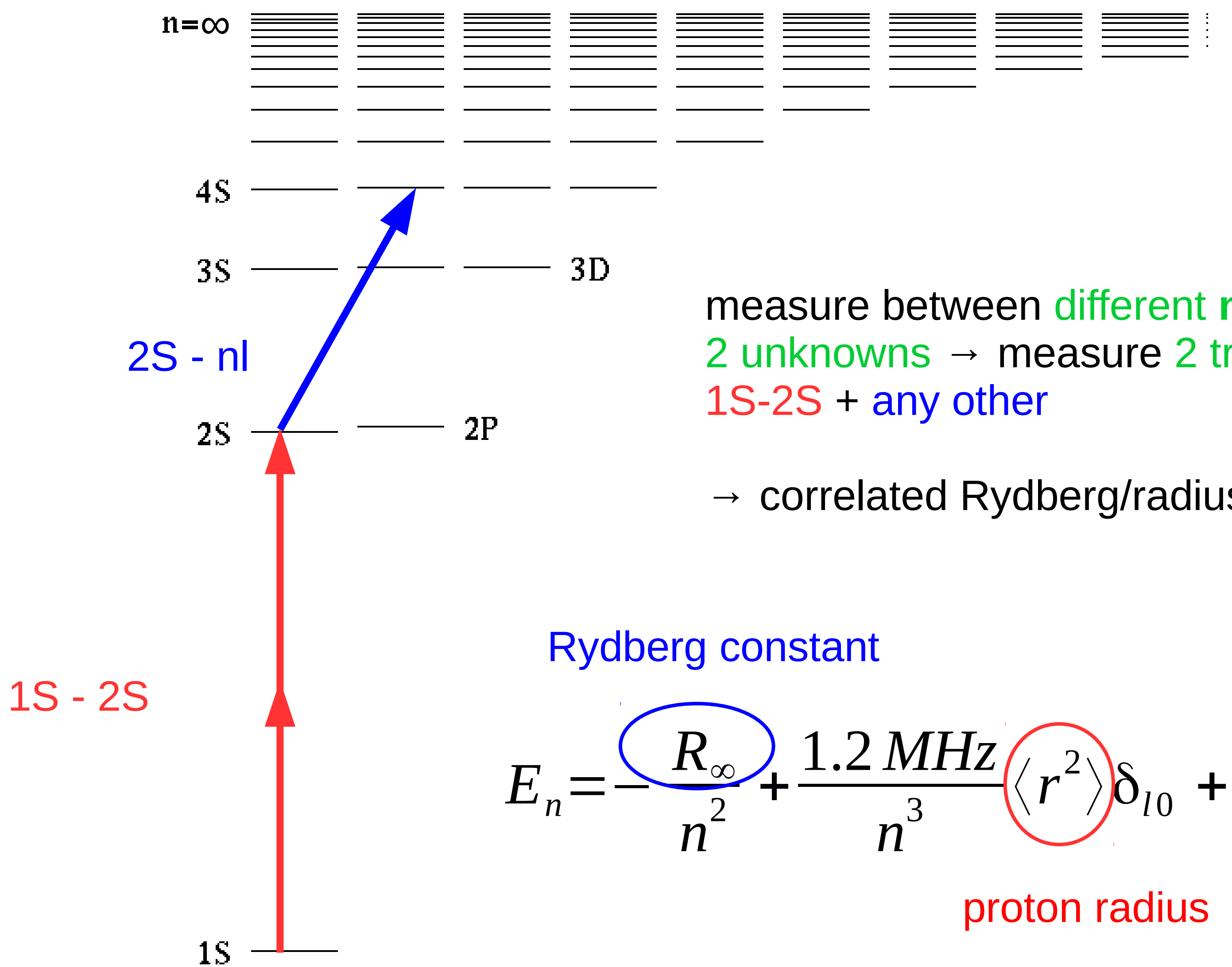


Rydberg constant

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n, l, j)$$

proton radius

Energy levels of hydrogen



measure between different **n**
2 unknowns → measure 2 transitions:
1S-2S + any other

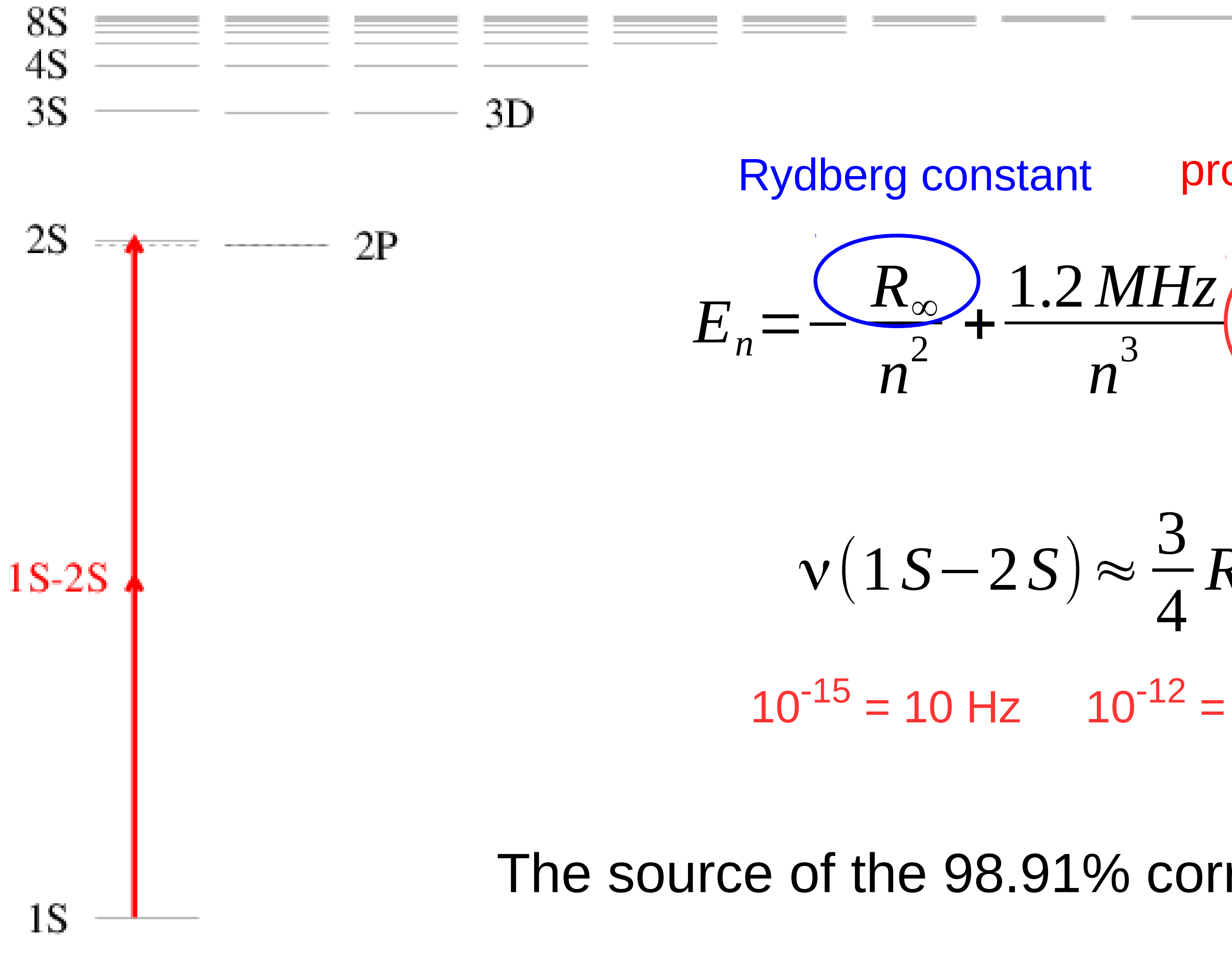
→ correlated Rydberg/radius pairs

Rydberg constant

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle\delta_{l0} + \Delta(n,l,j)$$

proton radius

Correlation between R_∞ and R_p / R_d



Rydberg constant   proton radius

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n,l,j)$$

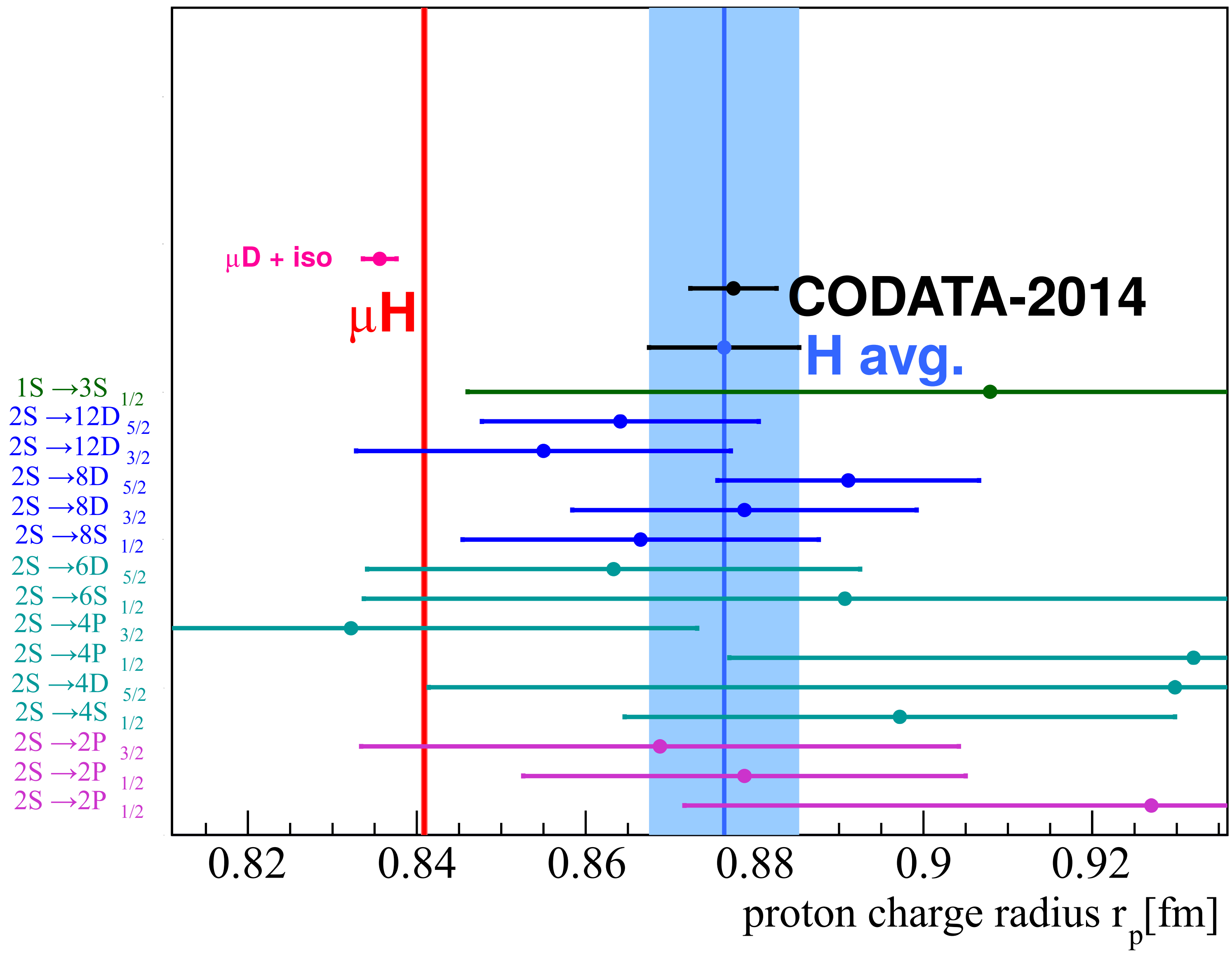
$$\nu(1S-2S) \approx \frac{3}{4}R_\infty - \frac{7}{8}E_{NS}$$

10^{-15} = 10 Hz 10^{-12} = 20 kHz

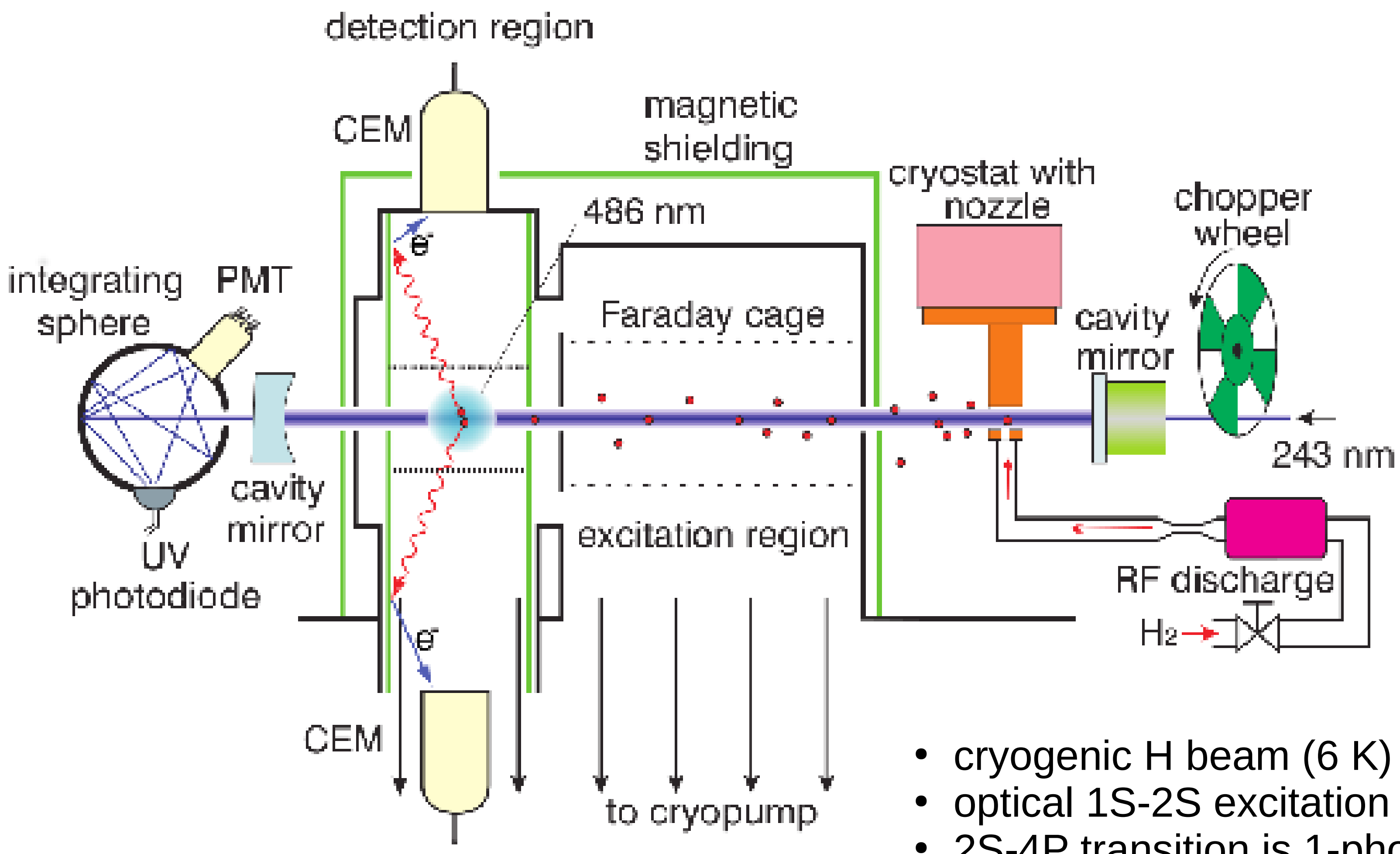
The source of the 98.91% correlation of R_∞ and R_p

1S-2S: Parthey, RP et al., PRL 107, 203001 (2011)

Rp from H spectroscopy



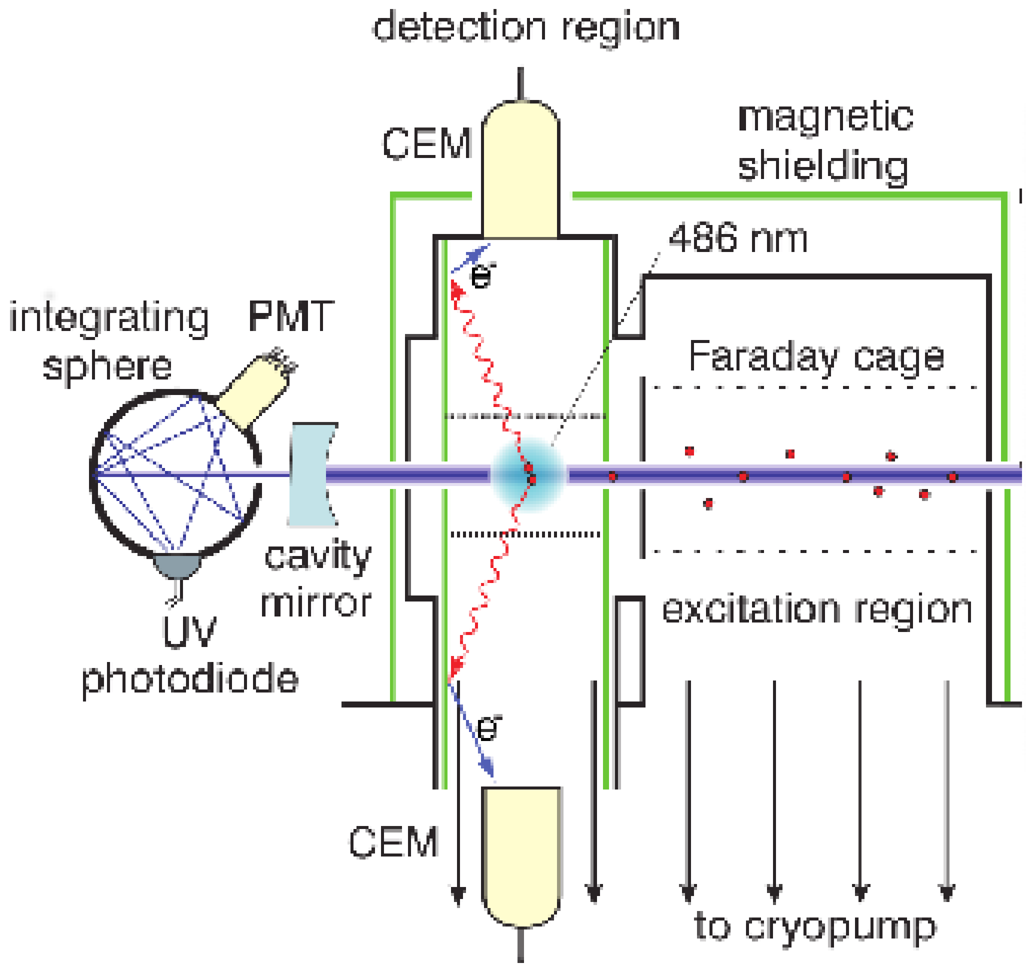
Garching H(2S-4P)



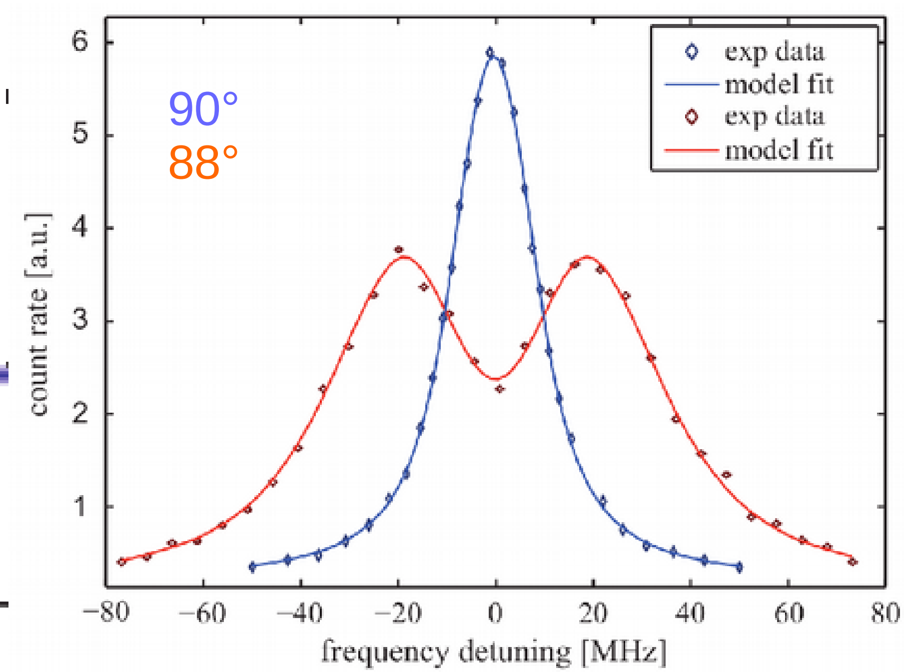
- cryogenic H beam (6 K)
- optical 1S-2S excitation (2S, F=0)
- 2S-4P transition is 1-photon: retroreflector
- split line to 10^{-4} !!!
- 2.3 kHz vs. 9 kHz PRP
- large systematics

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Garching H(2S-4P)



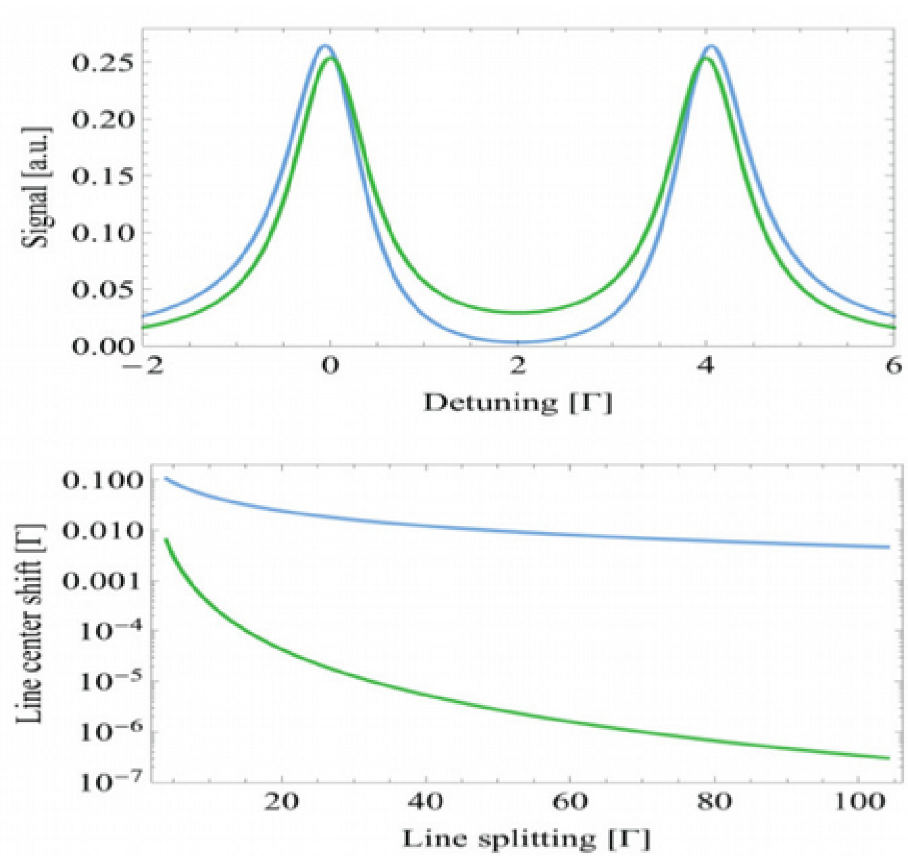
1st order Doppler cancellation

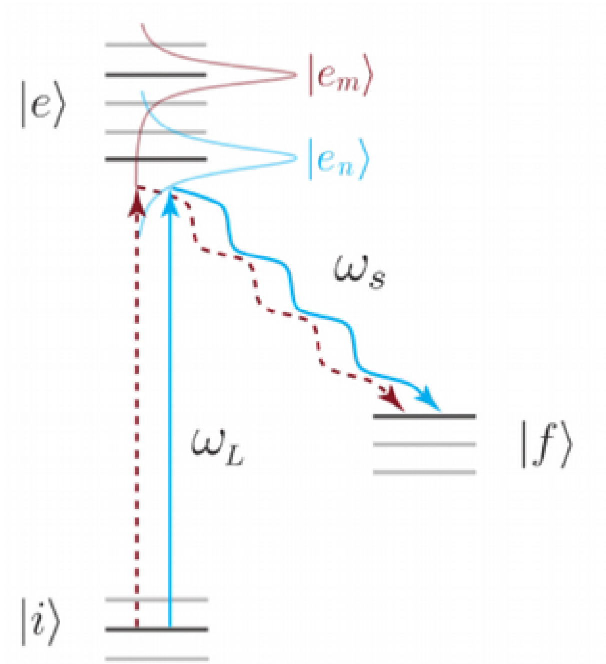


- cryogenic H beam (6 K)
- optical 1S-2S excitation (2S, F=0)
- 2S-4P transition is 1-photon: retroreflector
- split line to 10^{-4} !!!
- 2.3 kHz vs. 9 kHz PRP
- large systematics

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Quantum interference shifts





$$P(\omega) \propto \left| \frac{(\vec{d}_1 \vec{E}_0)\vec{d}_1}{\omega_1 - \omega_L + i\gamma_1/2} + \frac{(\vec{d}_2 \vec{E}_0)\vec{d}_2 e^{i\Delta\Phi}}{\omega_2 - \omega_L + i\gamma_2/2} \right|^2$$

= Lorentzian(1) + Lorentzian(2)

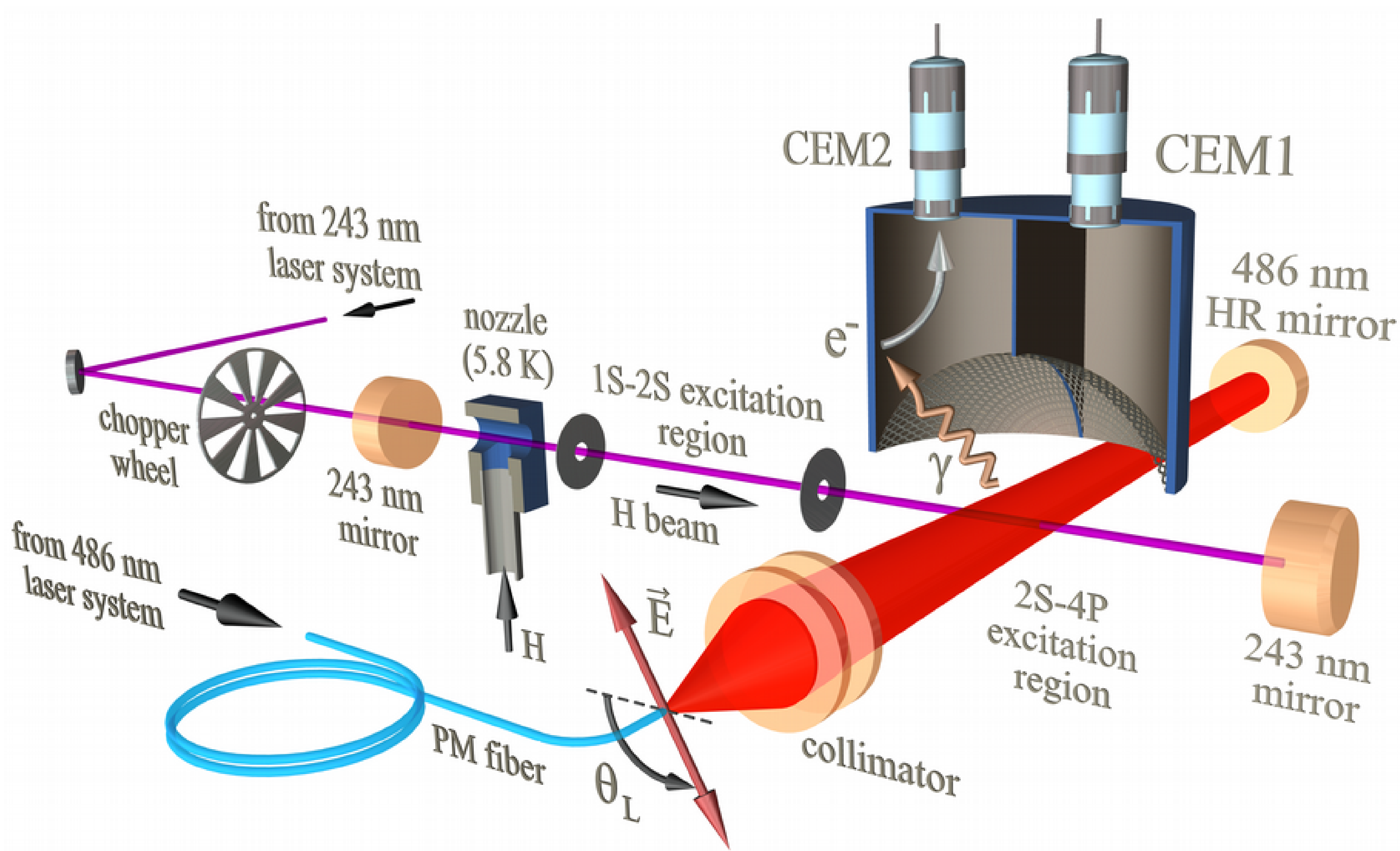
+ cross-term (QI)

Fitting this with 2 Lorentzians creates

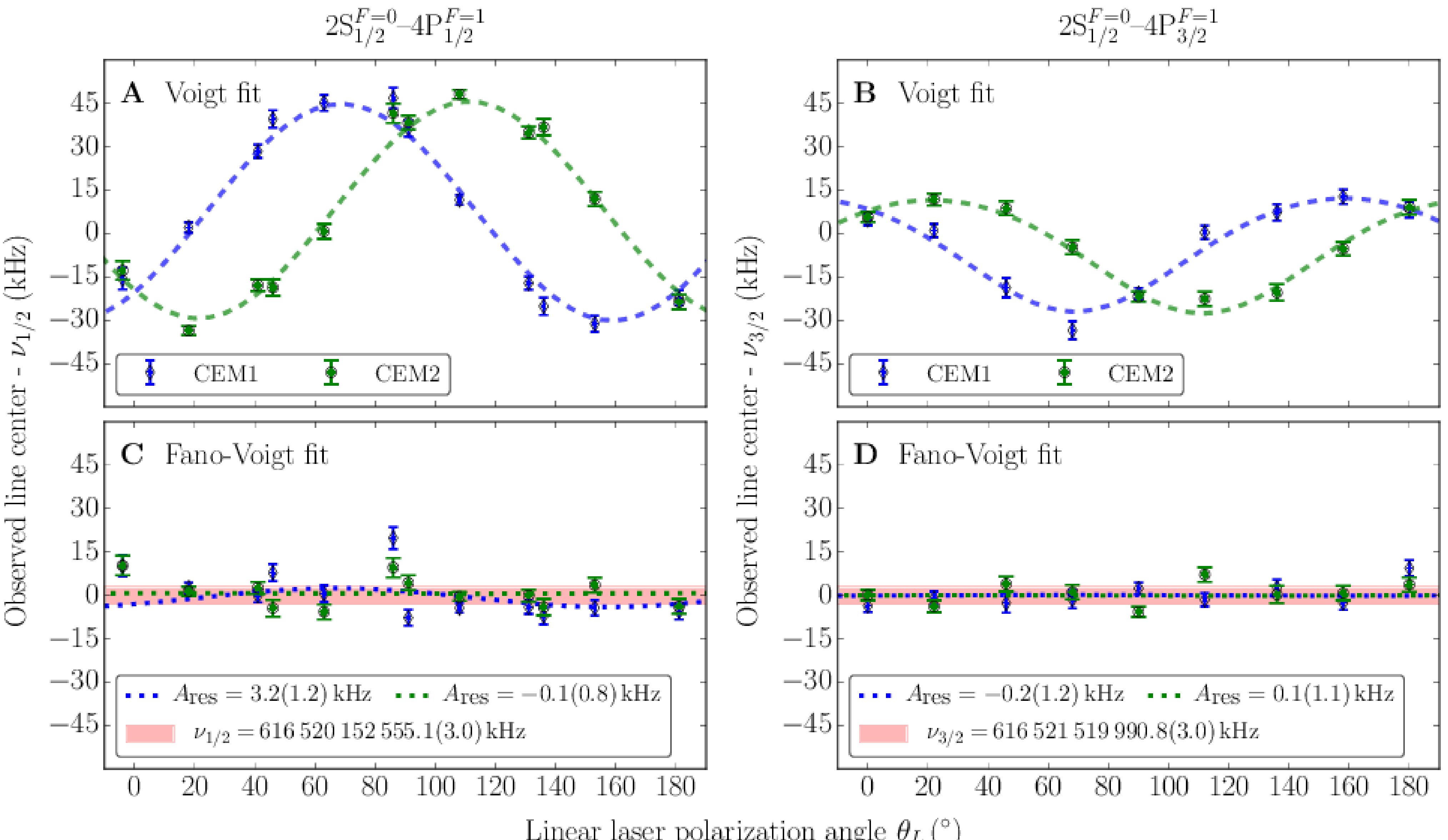
line shifts

see
Horbatsch, Hessels, PRA 82, 052519 (2010); PRA 84, 032508 (2011); PRA 86 040501 (2012)
Sansonetti et al., PRL 107, 021001 (2011)
Brown et al., PRA 87, 032504 (2013)

Studying QI in 2S-4P



QI in hydrogen   (Δ = 100 Γ)

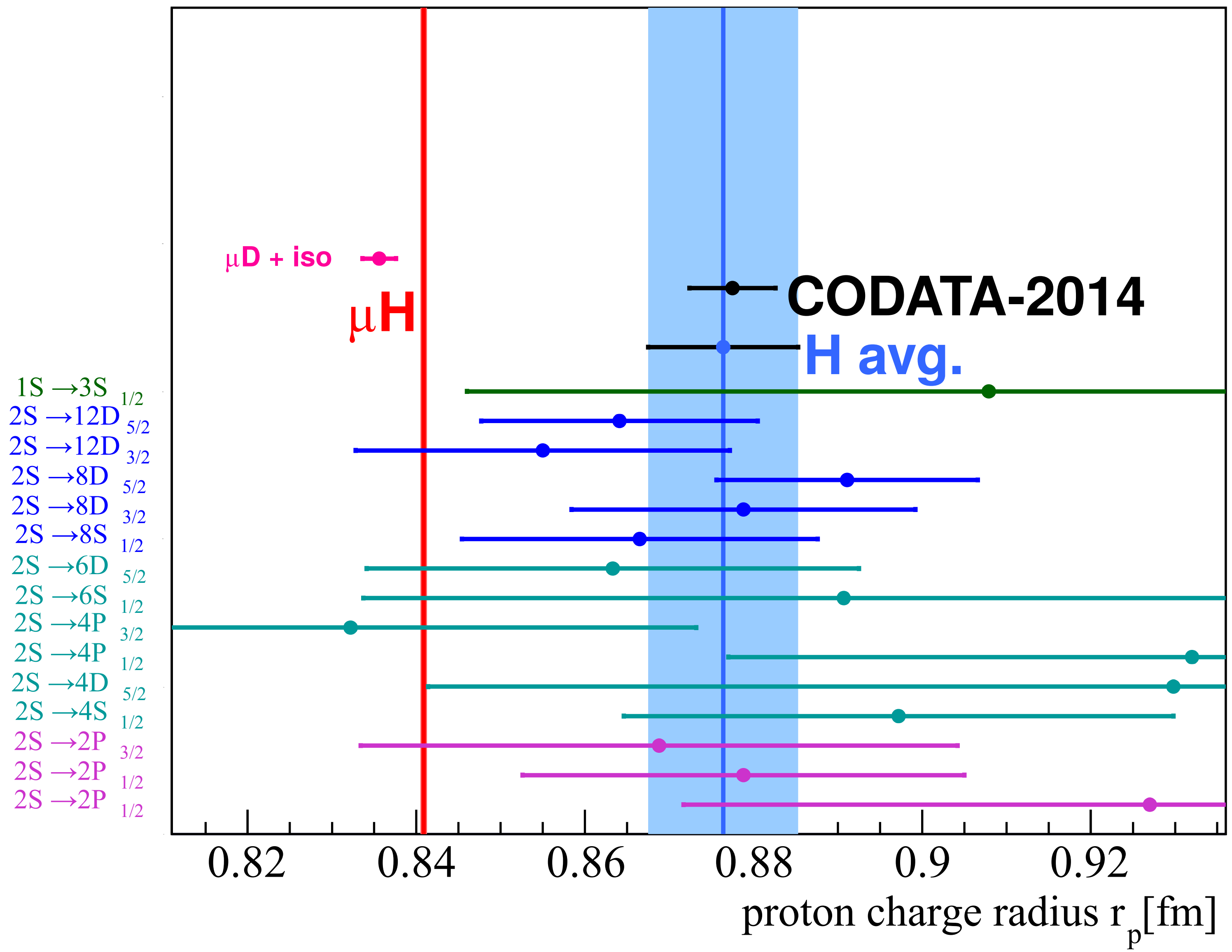


Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Systematics

Contribution	Δν (kHz)	σ (kHz)
Statistics	0.00	0.41
First-order Doppler shift	0.00	2.13
Quantum interference shift	0.00	0.21
Light force shift	–0.32	0.30
Model corrections	0.11	0.06
Sampling bias	0.44	0.49
Second-order Doppler shift	0.22	0.05
dc-Stark shift	0.00	0.20
Zeeman shift	0.00	0.22
Pressure shift	0.00	0.02
Laser spectrum	0.00	0.10
Frequency standard (hydrogen maser)	0.00	0.06
Recoil shift	–837.23	0.00
Hyperfine structure corrections	–132,552.092	0.075
Total	–133,388.9	2.3

Rp from H spectroscopy



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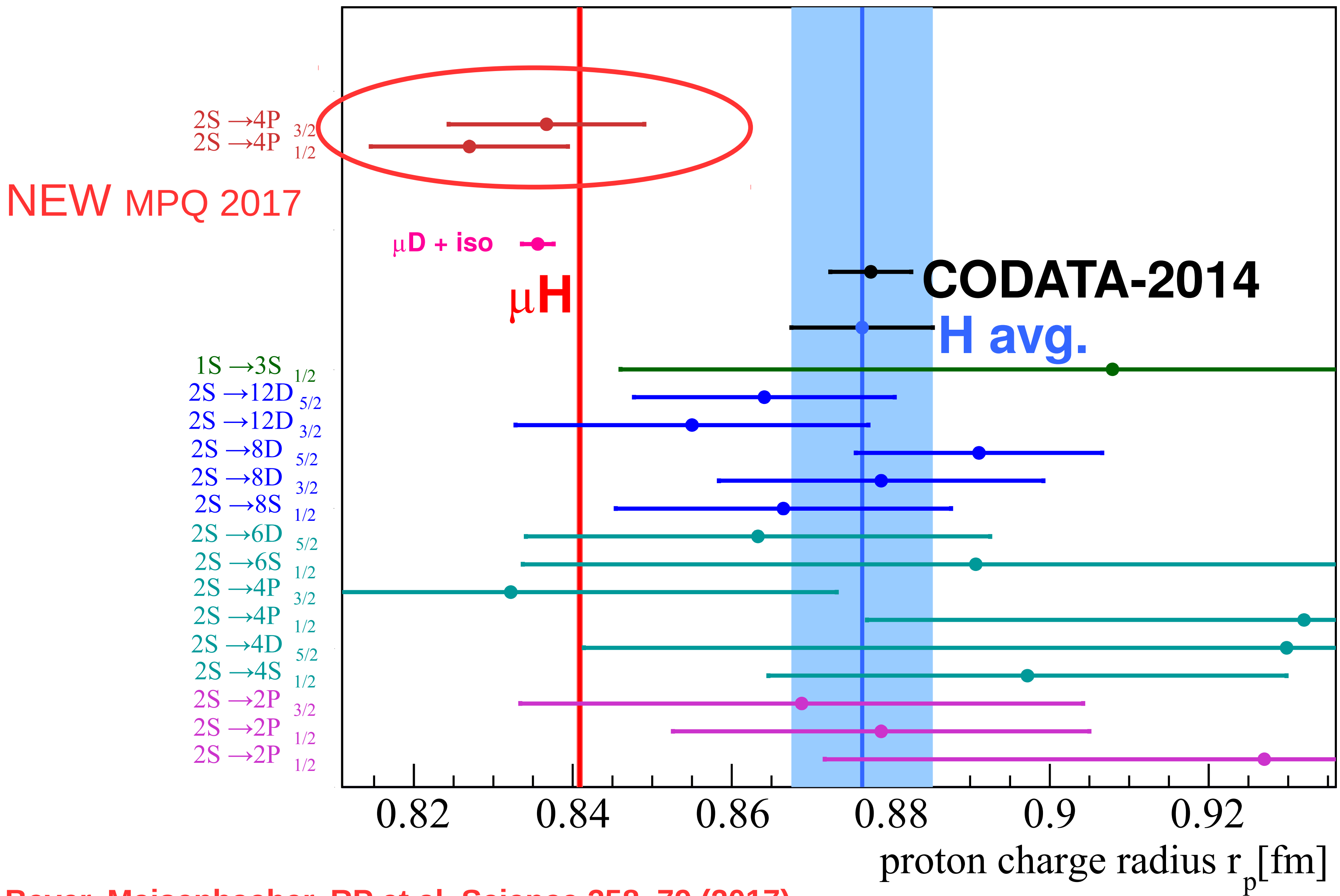

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

New Rp from Paris: 1S-3S

PHYSICAL REVIEW LETTERS **120,** 183001 (2018)

New Measurement of the $1S-3S$ Transition Frequency of Hydrogen: Contribution to the Proton Charge Radius Puzzle

Hélène Fleurbaey, Sandrine Galtier,* Simon Thomas, Marie Bonnaud, Lucile Julien, François Biraben, and François Nez
Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-Université PSL, Collège de France, 4 place Jussieu, Case 74, 75252 Paris Cedex 05, France

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LNE-SYRTE, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, 61 avenue de l'Observatoire, 75014 Paris, France

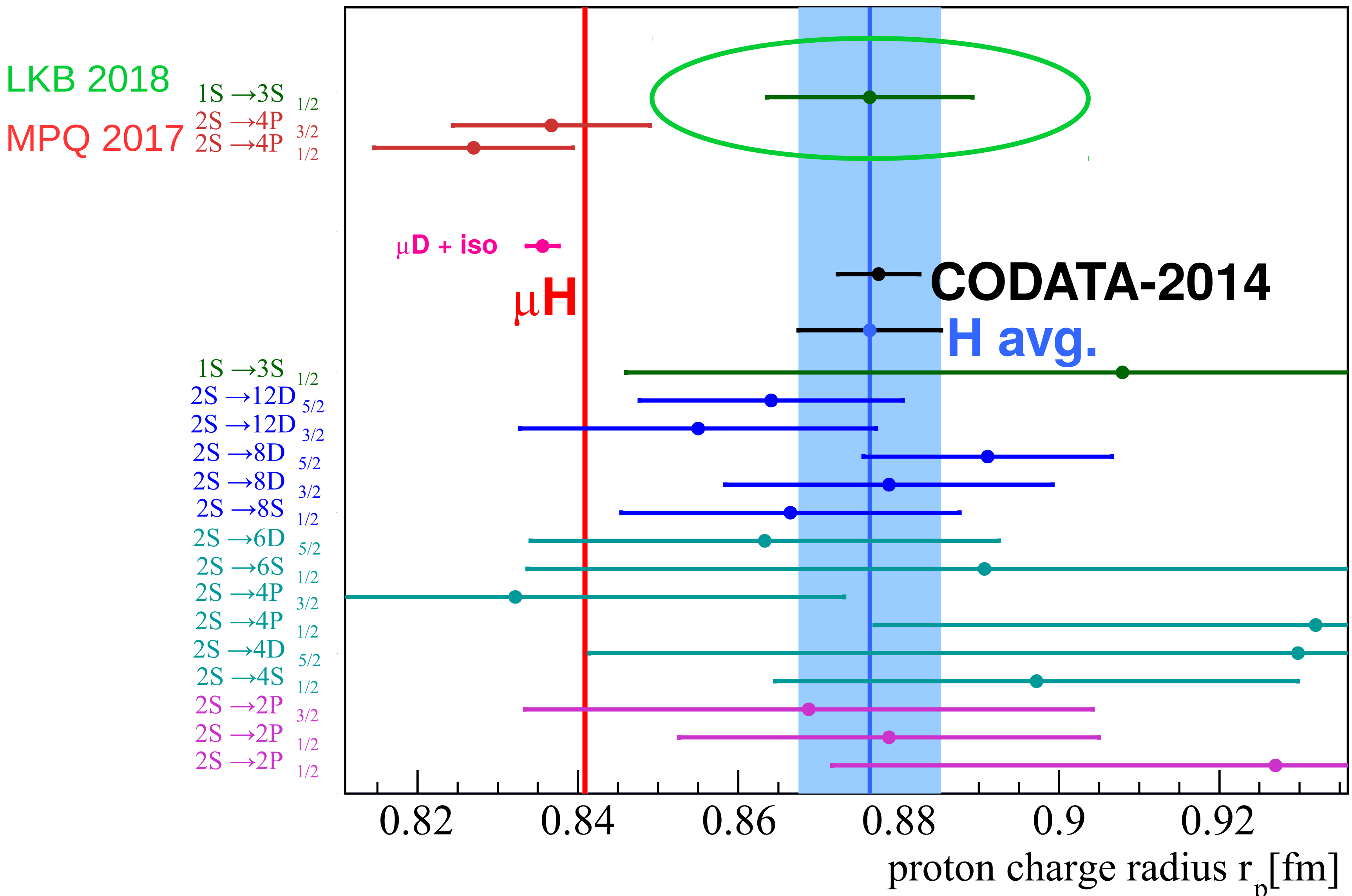
(Received 8 December 2017; revised manuscript received 9 March 2018; published 4 May 2018)

We present a new measurement of the $1S-3S$ two-photon transition frequency of hydrogen, realized with a continuous-wave excitation laser at 205 nm on a room-temperature atomic beam, with a relative uncertainty of 9×10^{-13}. The proton charge radius deduced from this measurement, $r_p = 0.877(13)$ fm, is in very good agreement with the current CODATA-recommended value. This result contributes to the ongoing search to solve the proton charge radius puzzle, which arose from a discrepancy between the CODATA value and a more precise determination of r_p from muonic hydrogen spectroscopy.

DOI: 10.1103/PhysRevLett.120.183001

arXiv: 1801.08816

Rp from H spectroscopy



Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Fleurbaey et al., 1801.08816 (submitted)

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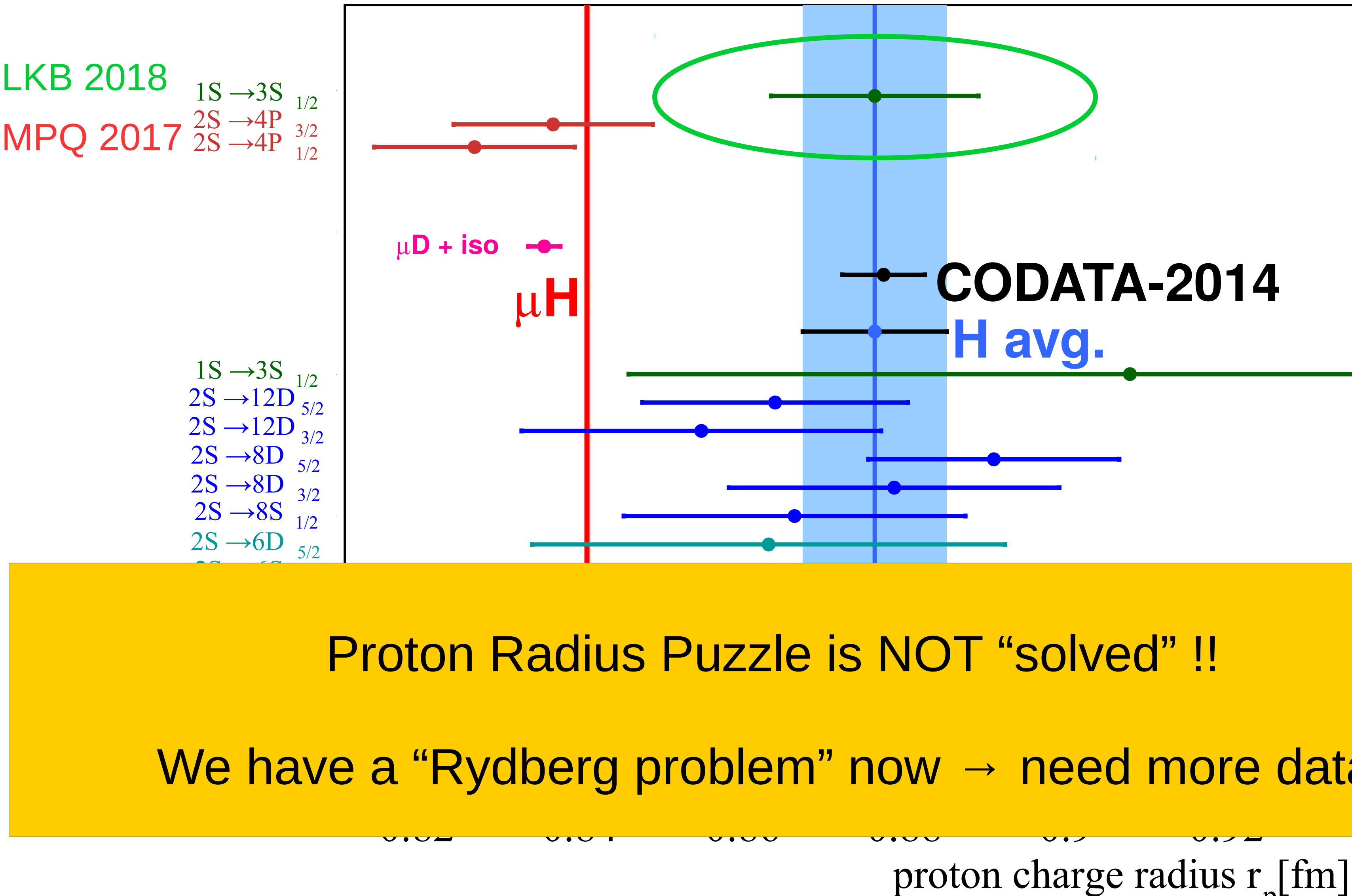

Proton Radius Puzzle is NOT “solved” !!

We have a “Rydberg problem” now → need more data!

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Fleurbaey et al., 1801.08816 (submitted)

Conclusions

- smaller radii from muonic **hydrogen** and **deuterium** imply a smaller Rydberg constant
- new H(2S-4P) gives small Rydberg constant in agreement with muonic values
- new H(2S-4P) gives thus a smaller proton radius, too
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More data needed:

- H(2S – 6P, 8P, 9P, …) and D(2S-nl) underway in Garching and Colorado
- H(1S – 3S, 4S, ..) underway in Paris and Garching
- H(2S-2P) in Toronto (Hessels)
- Muonium at PSI, J-PARC
- Positronium (Cassidy, Crivelli)
- He^+(1S-2S) underway in Garching (Udem) and Amsterdam (Eikema)
- HD^+, H_2, etc. in Amsterdam (Ubachs) and Paris (Hilico, Karr)
- He (Vassen, Amsterdam), Li^+ (Udem, Garching)
- new low-Q^2 electron scattering at MAMI, JLab, MESA
- muon scattering: MUSE @ PSI, COMPASS @ CERN

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Compare Rydberg values to test QED and SM

Up next: Hyperfine structure in μp

The 21 cm line in hydrogen (1S hyperfine splitting) has been **measured** to 12 digits (0.001 Hz) in 1971:

$$\nu_{exp} = 1\,420\,405.\,751\,766\,7 \pm 0.000\,001 \text{ kHz}$$

Essen et al., Nature 229, 110 (1971)

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$$\nu_{exp} = 1\ 420\ 405.\ 751\ 766\ 7 \ \pm 0.000\ 001 \text{ kHz}$$

Essen et al., Nature 229, 110 (1971)

QED test is limited to 6 digits (800 Hz) because of proton structure effects:

$$\nu_{theo} = 1\ 420\ 403.\ 1 \ \pm 0.6_{\text{proton size}} \ \pm 0.4_{\text{polarizability}} \text{ kHz}$$

Eides et al., Springer Tracts 222, 217 (2007)

Proton Zemach radius

HFS depends on “Zemach” radius:

$$\Delta E = -2(Z\alpha) m \langle r \rangle_{(2)} E_F$$

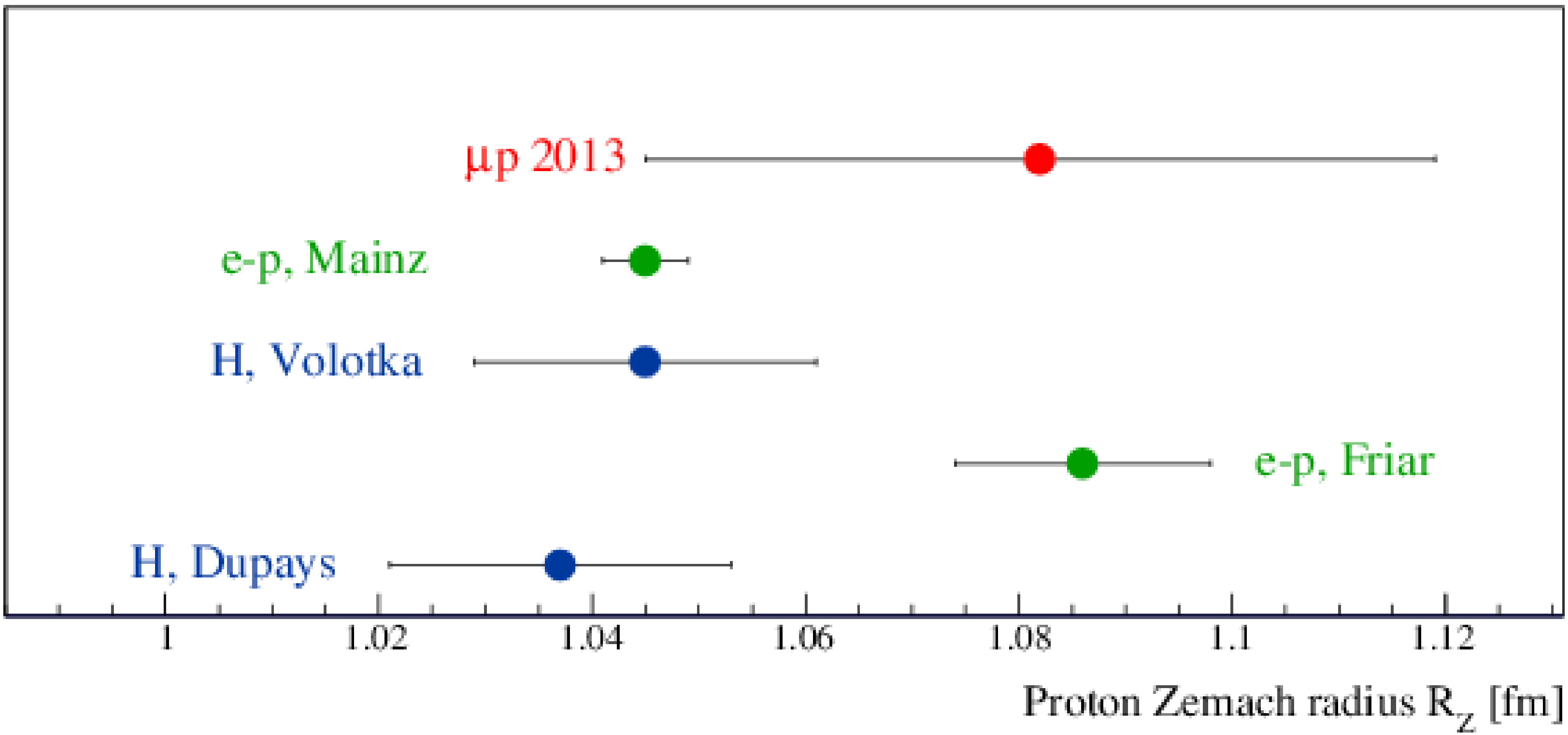
$$\langle r \rangle_{(2)} = \int d^3r\, d^3r' \rho_E(r) \rho_M(r') |r - r'|$$

Zemach, Phys. Rev. 104, 1771 (1956)

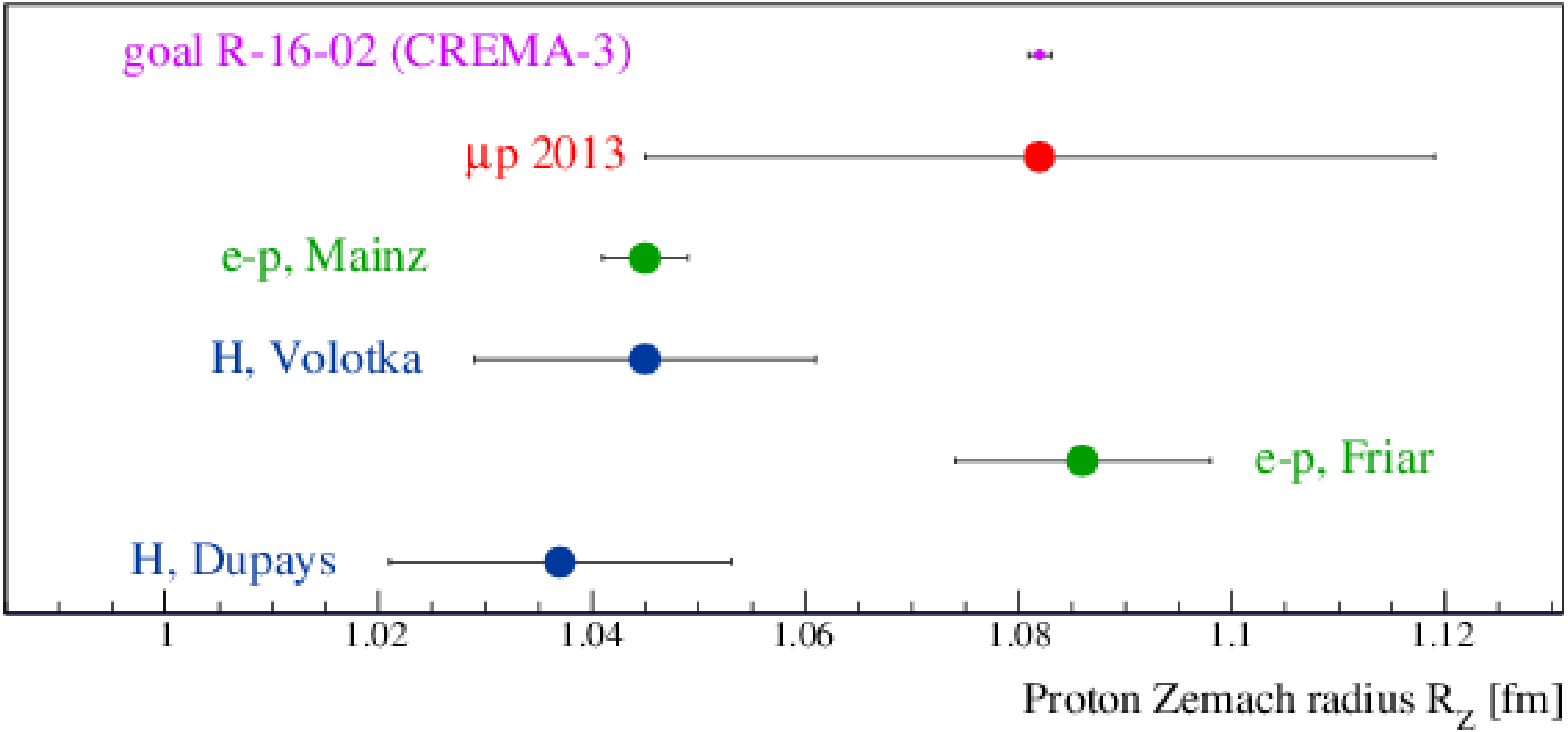
Form factors and momentum space

$$\Delta E = \frac{8(Z\alpha) m}{\pi n^3} E_F \int_0^\infty \frac{dk}{k^2} \left[\frac{G_E(-k^2) G_M(-k^2)}{1+\kappa} \right]$$

Proton Zemach radius from μp

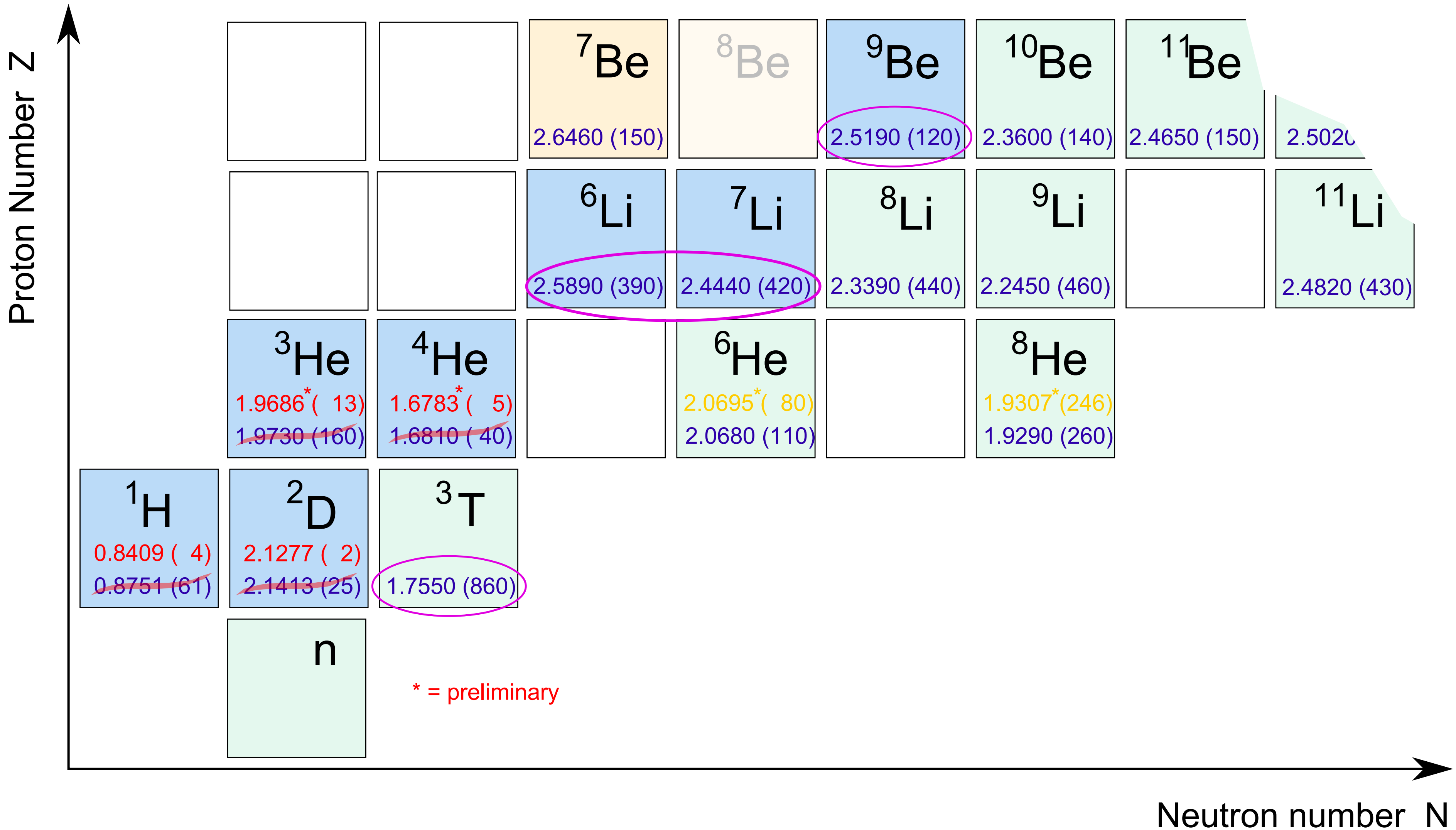


Proton Zemach radius from μp

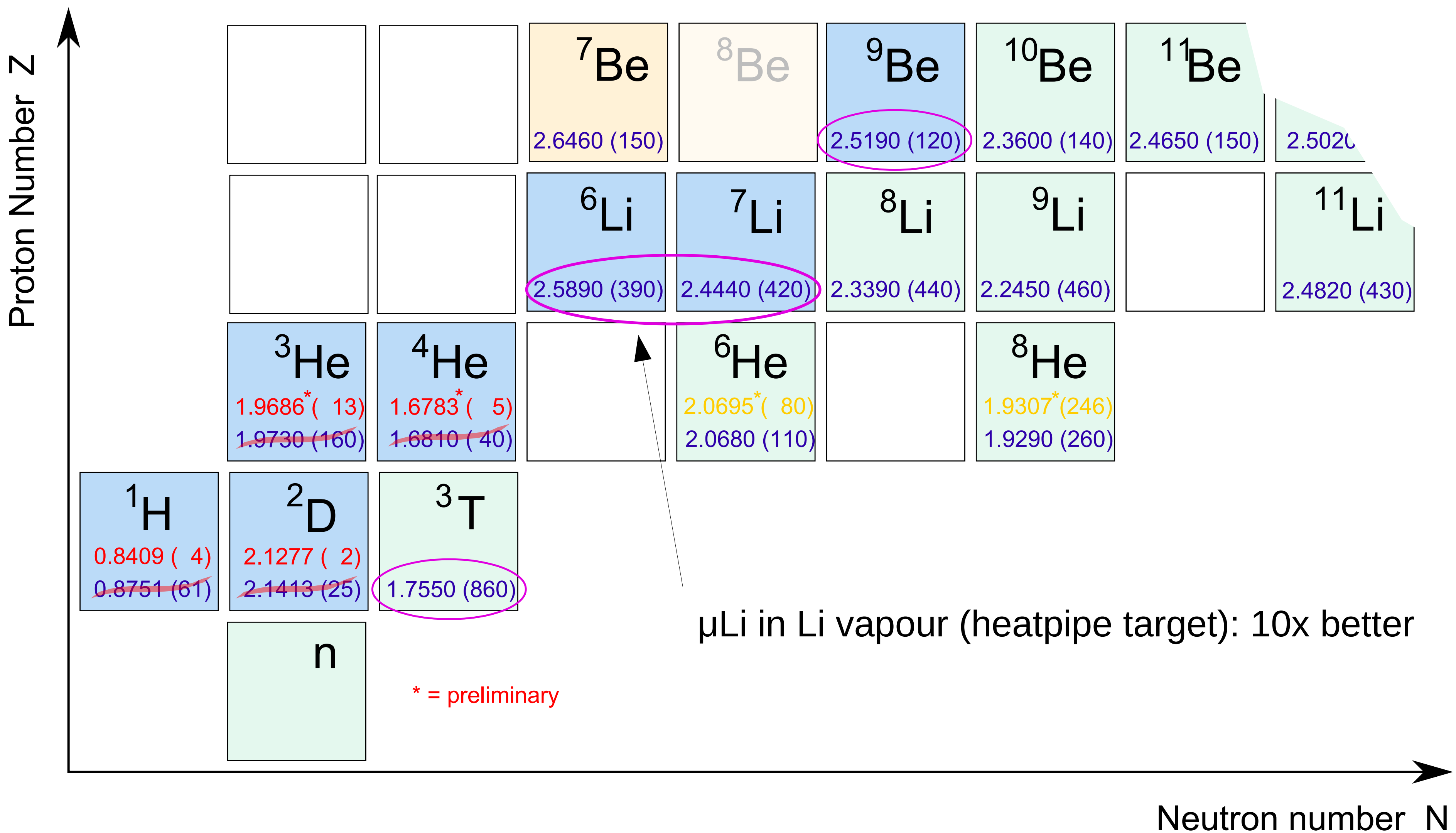


PSI Exp. R-16-02: Antognini, RP et al. (CREMA-3 / HyperMu)

Charge radii: The future



Charge radii: The future



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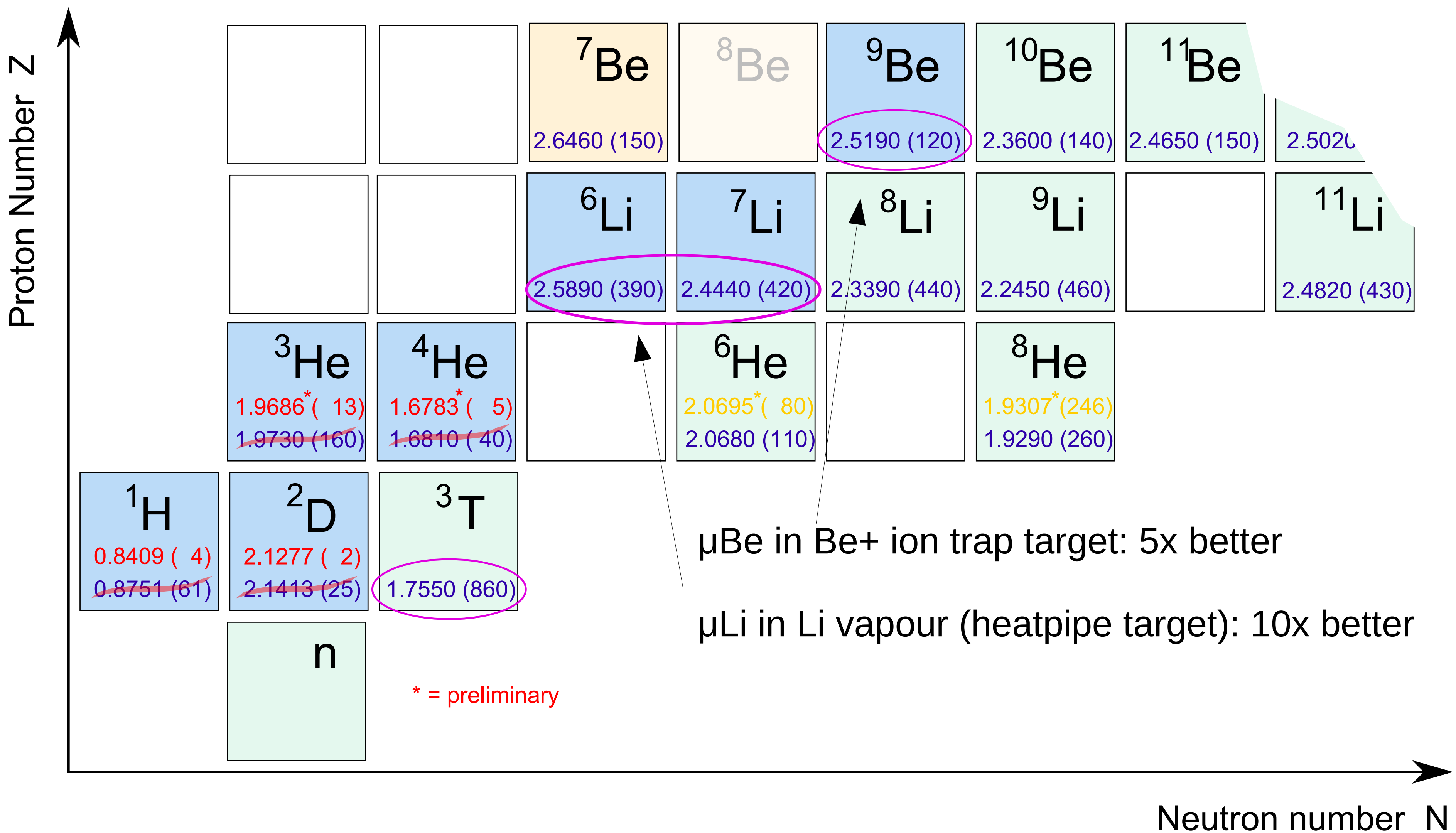
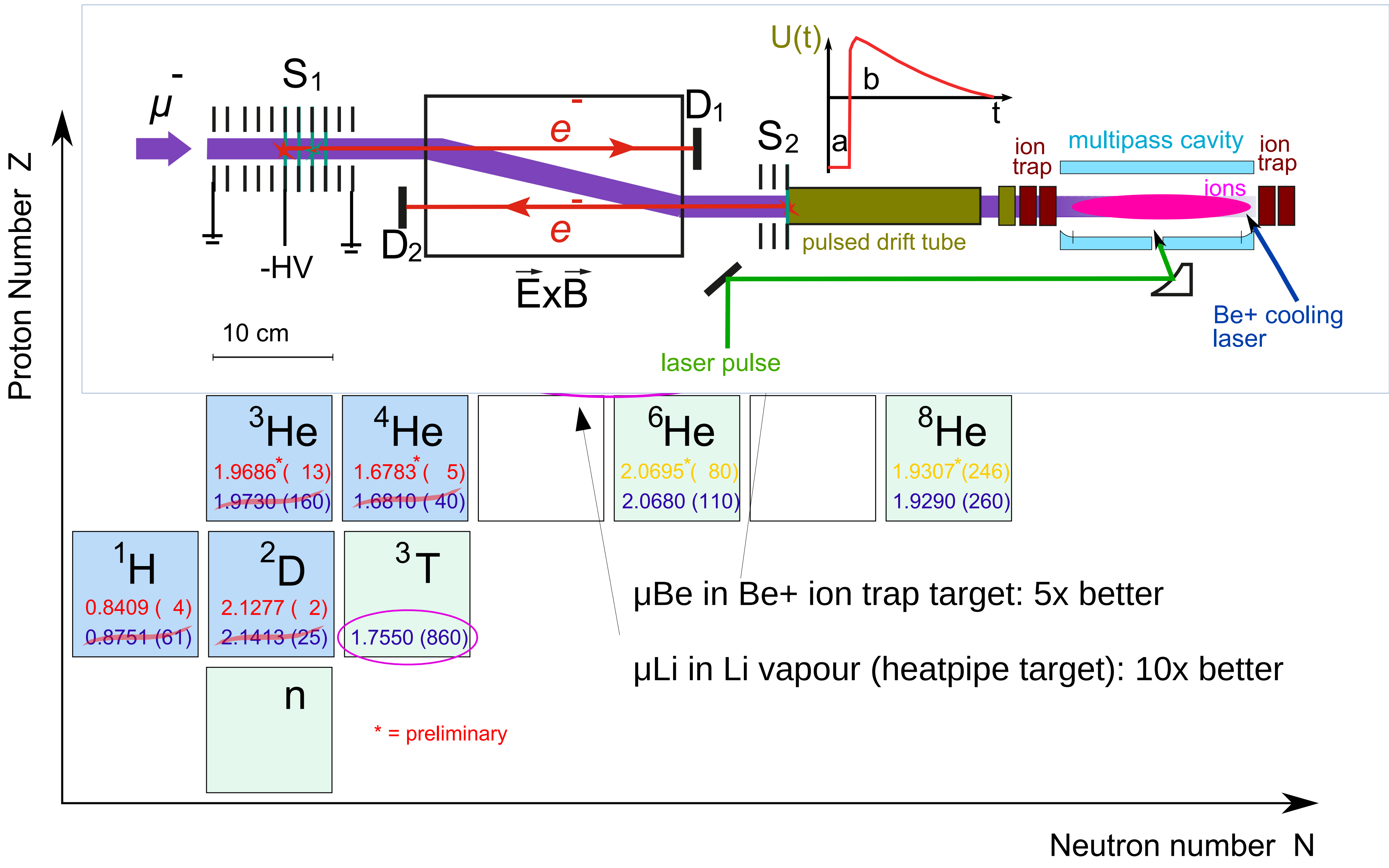
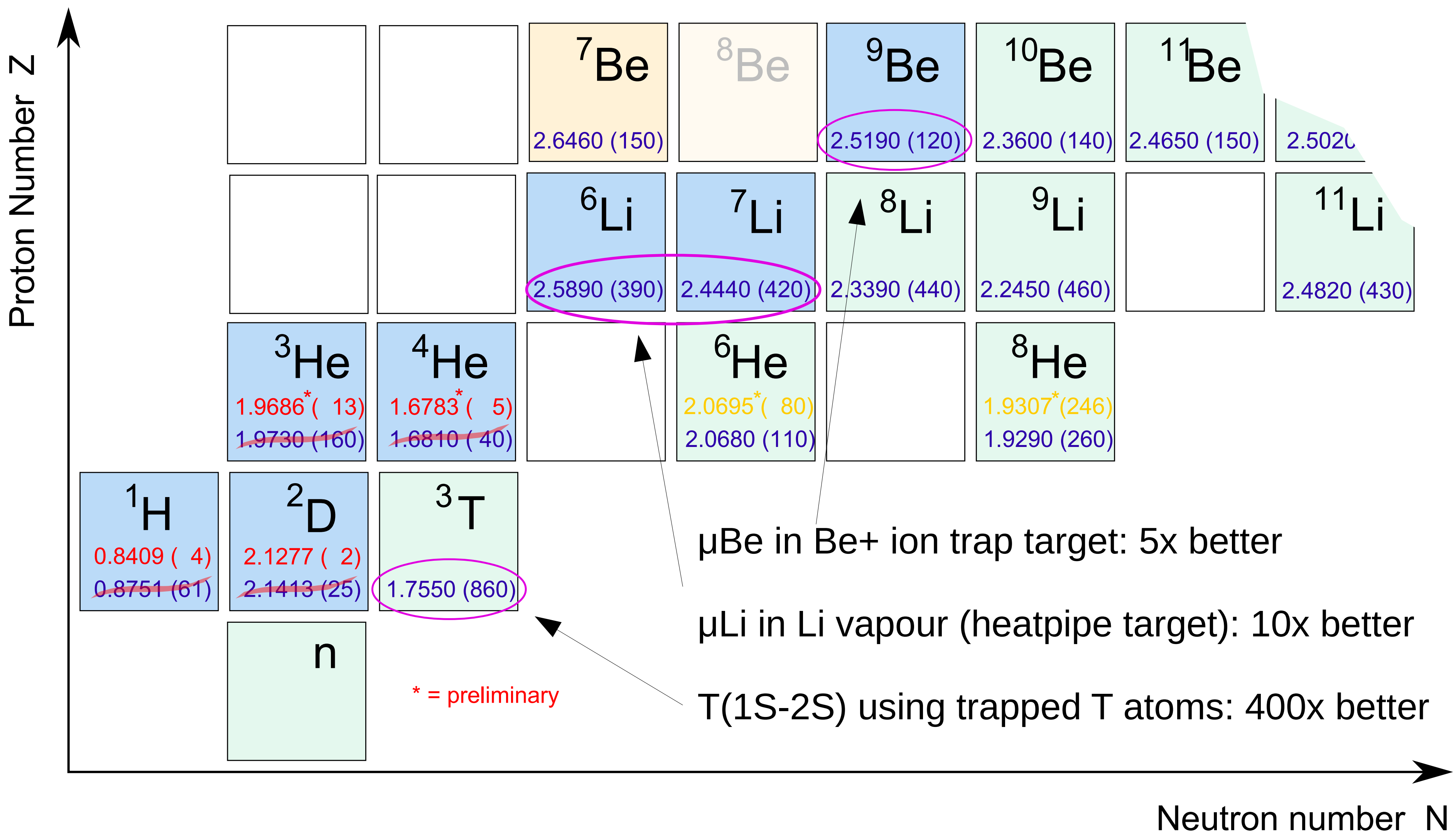


Charge radii: The future



Charge radii: The future



Thanks a lot for your attention

The Garching Hydrogen Team:

Axel Beyer, Lothar Maisenbacher, Arthur Matveev, RP,
Ksenia Khabarova, Alexey Grinin, Tobias Lamour, Dylan C. Yost,
Theodor W. Hänsch, Nikolai Kolachevsky, Thomas Udem

The CREMA Collaboration:

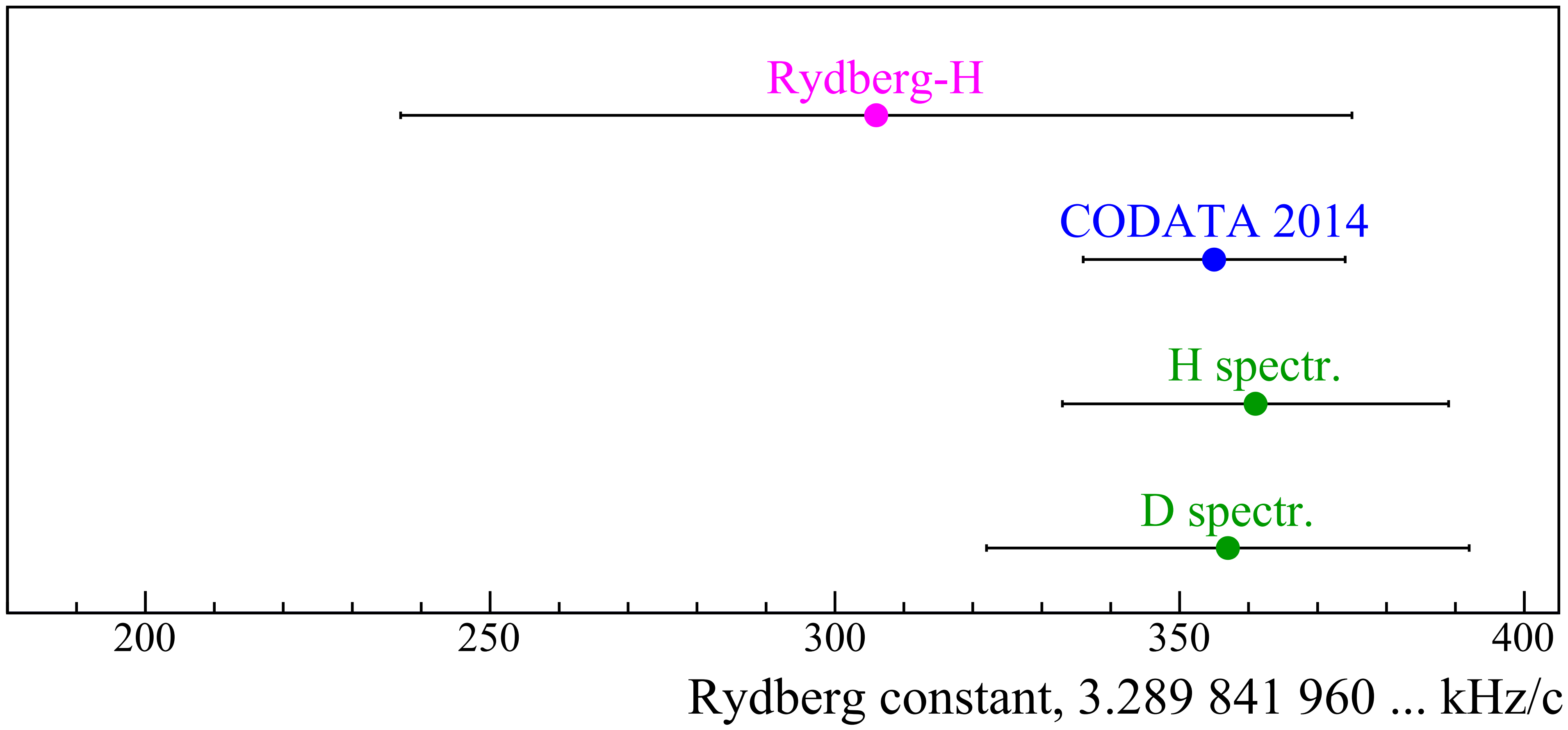
Aldo Antognini, Fernando D. Amaro, François Biraben, João M. R. Cardoso, Daniel S. Covita, Andreas Dax, Satish Dhawan, Marc Diepold, Luis M. P. Fernandes, Adolf Giesen, Andrea L. Gouvea, Thomas Graf, Theodor W. Hänsch, Paul Indelicato, Lucile Julien, Paul Knowles,Franz Kottmann, Eric-Olivier Le Bigot, Yi-Wei Liu, José A. M. Lopes, Livia Ludhova, Cristina M. B. Monteiro, Françoise Mulhauser, Tobias Nebel, François Nez, Paul Rabinowitz, Joaquim M. F. dos Santos, Lukas A. Schaller, Karsten Schuhmann, Catherine Schwob, David Taqqu, João F. C. A. Veloso, RP

My new Mainz group:

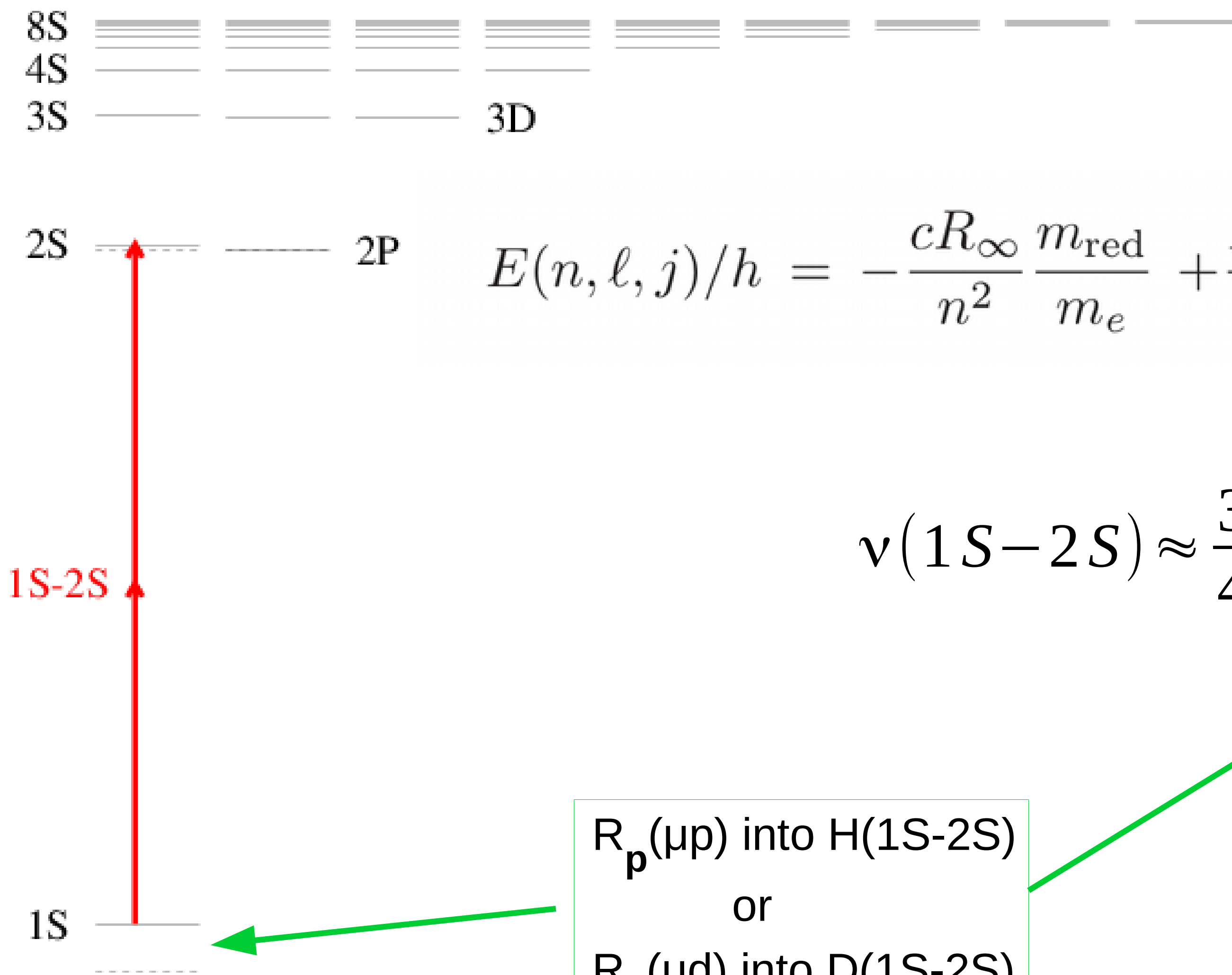
Jan Haack, Rishi Horn, Stefan Schmidt, Marcel Willig

...

Rydberg constants from H/D



And now with ***muonic*** charge radii



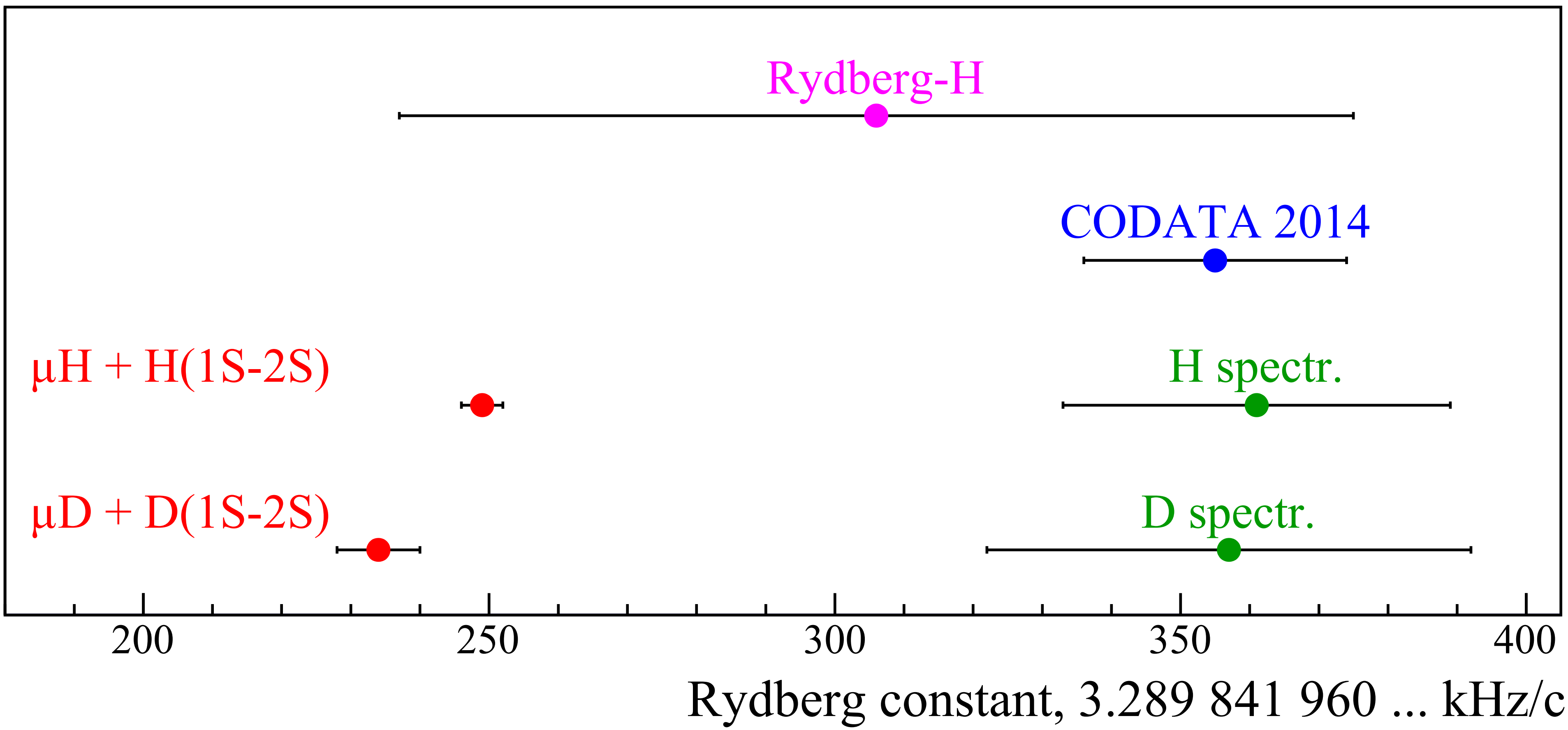
$$E(n,\ell,j)/h = -\frac{cR_\infty}{n^2}\frac{m_{\text{red}}}{m_e} + \frac{E_{NS}}{n^3}\delta_{\ell 0} + \Delta(n,\ell,j). \quad (7)$$

$$\nu(1S-2S) \approx \frac{3}{4}R_\infty - \frac{7}{8}E_{NS}$$

$R_{\mathbf{p}}$(μp) into H(1S-2S)
or
$R_{\mathbf{d}}$(μd) into D(1S-2S)

μd 2016: RP et al (CREMA Coll.) Science 353, 669 (2016)
μp 2013: A. Antognini, RP et al (CREMA Coll.) Science 339, 417 (2013)

Rydberg constants from e/μ H/D



R_∞ [μH + H(1S-2S)] = 3.289 841 960 249 **(1.0)** ^{Rp} **(2.5)** ^{QED} kHz/c

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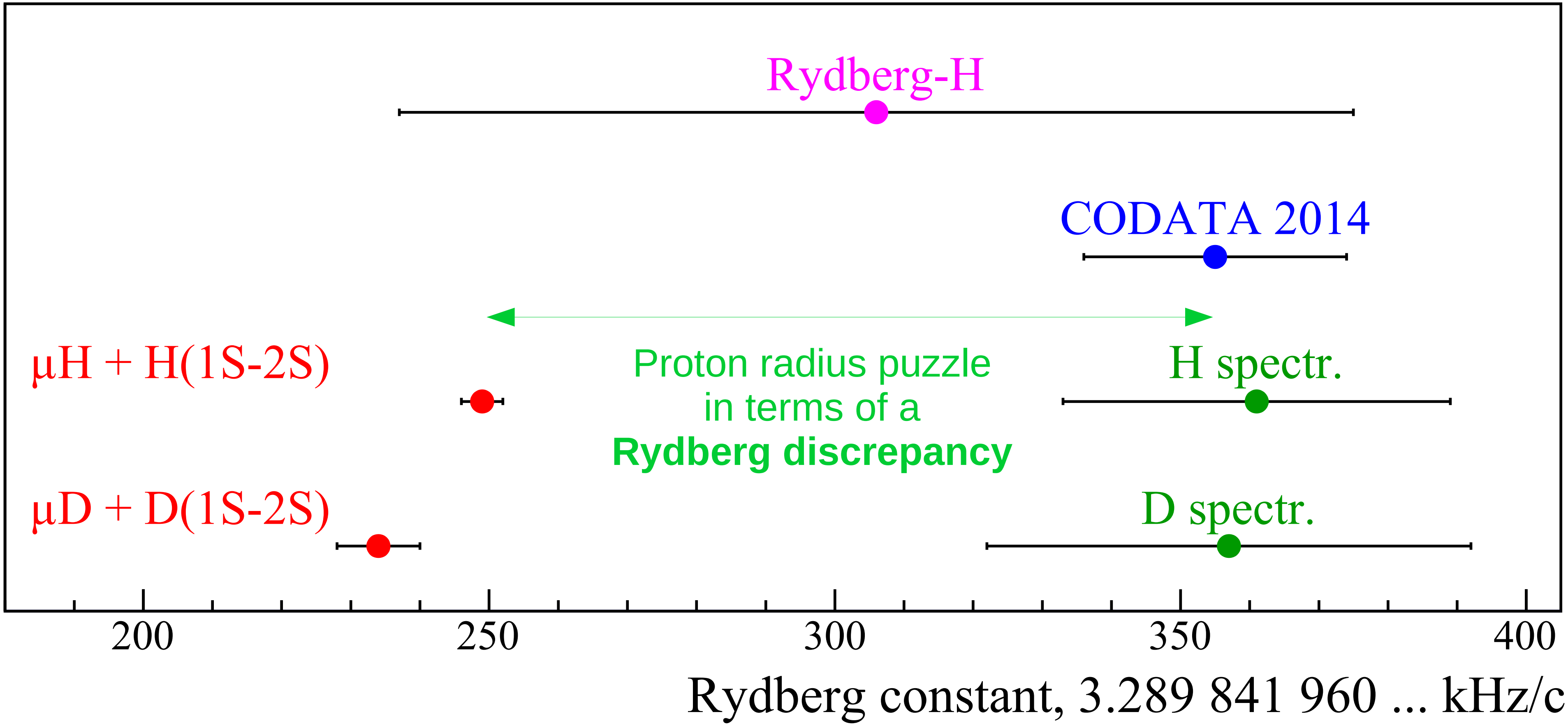

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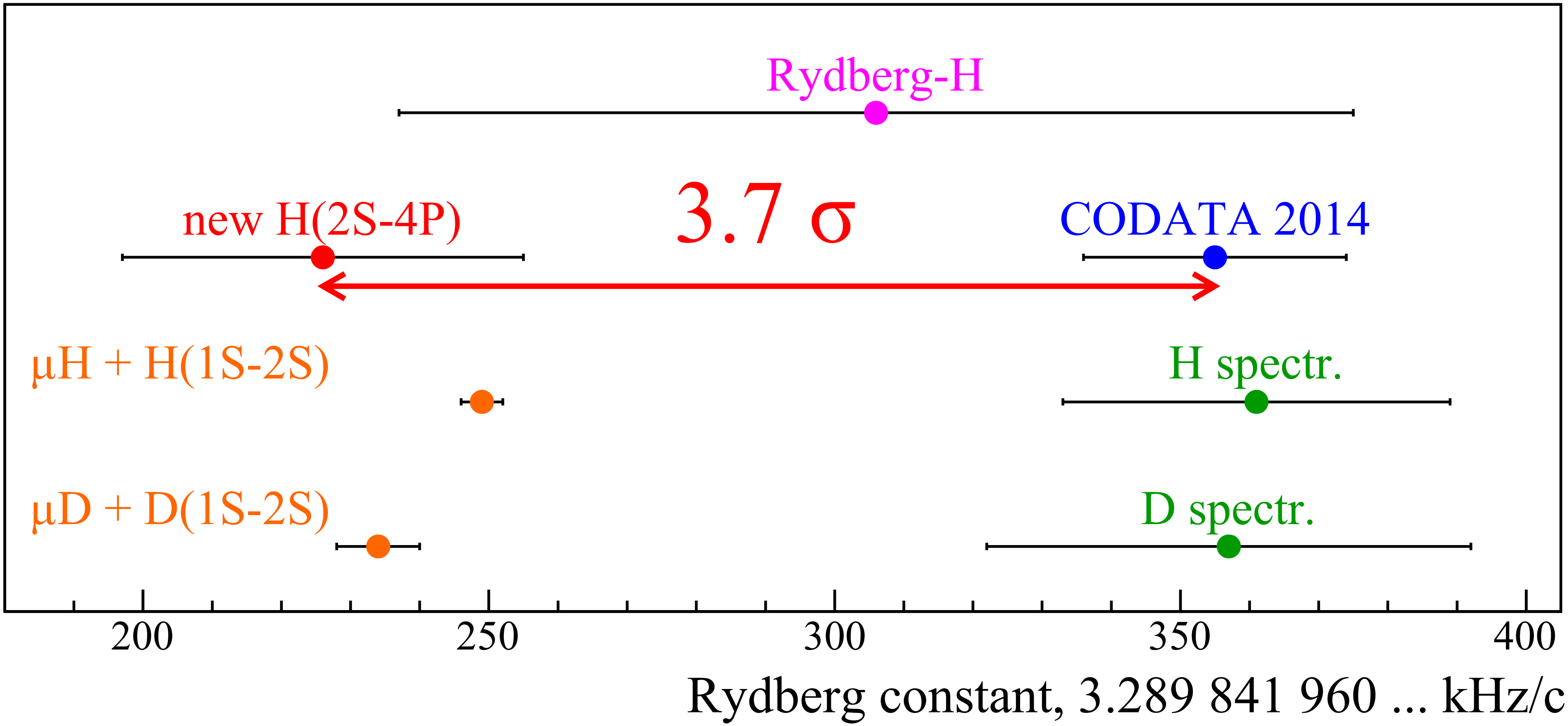
Rydberg constant from H(2S-4P)



Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

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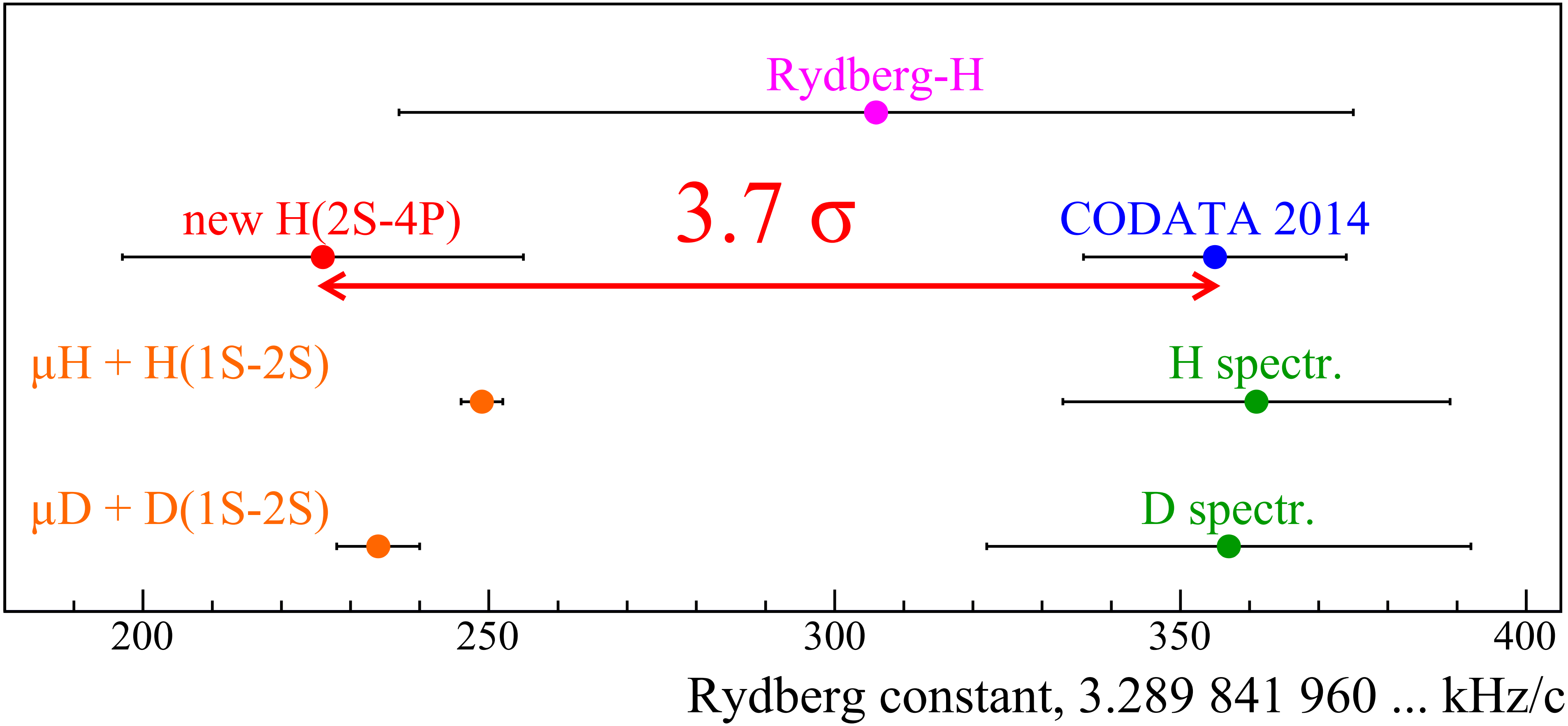

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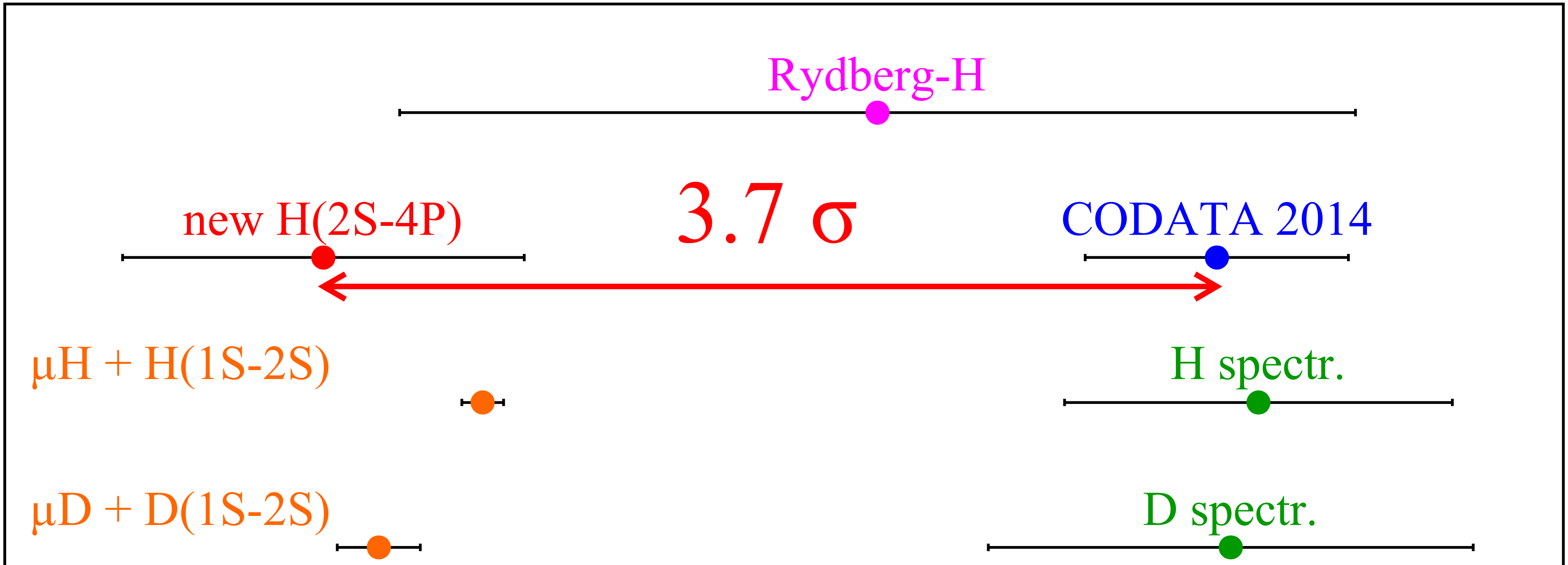

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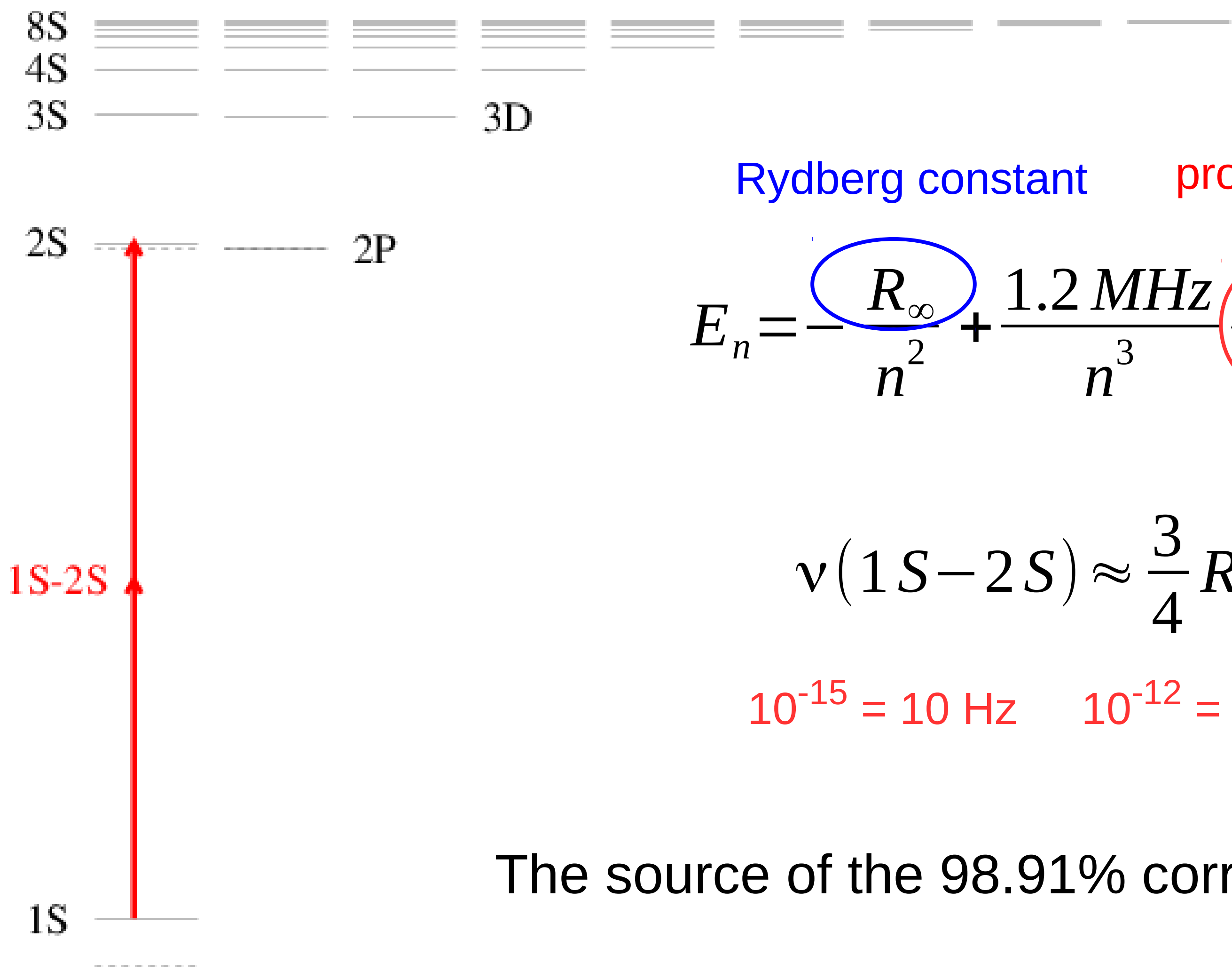
We have a “Rydberg problem” now → need more data!

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Correlation between R_∞ and R_p / R_d



Rydberg constant

proton radius

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n,l,j)$$

$$\nu(1S-2S) \approx \frac{3}{4}R_\infty - \frac{7}{8}E_{NS}$$

10^{-15} = 10 Hz 10^{-12} = 20 kHz

The source of the 98.91% correlation of R_∞ and R_p

1S-2S: Parthey, RP et al., PRL 107, 203001 (2011)