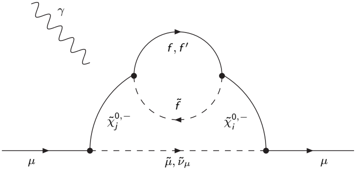
Magnetic moment $(g-2)_\mu$ theory: Standard Model and beyond

Dominik Stöckinger

TU Dresden



Symmetries in Subnuclear Physics (SSP) Aachen, June 12, 2018

Motivation: Three Fermions

Electron: $g = 2.002\,319\,304\,361\,46(56)$

Muon: $g = 2.002\,331\,841\,8(1\,3)$

Proton: $g = 5.585\,694\,713(46)$

$$g_\mu = 2(1 + a_\mu)$$

a_μ probes quantum structure of all interactions (SM and beyond!)

Overview on $g - 2$

Now: $$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.1 \pm 6.3^{\text{Exp}} \pm 3.6^{\text{Th(KNT18)}}) \times 10^{-10}$$

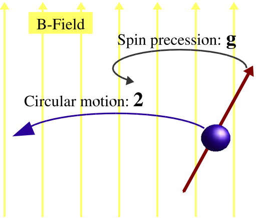
Keshavarzi,Nomura,Teubner'18; Jegerlehner'17: $\pm 4.4^{\text{Th}}$

Overview on $g-2$

Soon: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (30?? \pm 1.6^{\text{Exp}} \pm 3.4^{\text{Th}}??) \times 10^{-10}$

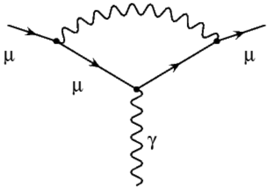
Outline

Muon magnetic moment



$$H_{\text{magnetic}} = -2(1 + a_\mu)\frac{e}{2m_\mu}\vec{B}\cdot\vec{S}$$

Quantum field theory:

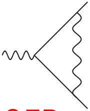


$$\tilde{=} \bar{u}(p')\left[\gamma_\mu F_1(q^2) - i\sigma_{\mu\nu}q^\nu F_M(q^2)\right]u(p)$$

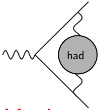
$$\rightarrow a_\mu = -2m_\mu F_M(0)$$

$\rightarrow$ Operator: $\frac{a_\mu}{m_\mu}\bar{\mu}_L\sigma_{\mu\nu}q^\nu\mu_R$

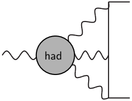
SM prediction $a_\mu^{\rm SM}$ $[10^{-10}]$ [Miller, de Rafael, Roberts, DS, Ann.Rev.Nucl.Part. (2012) 62.]



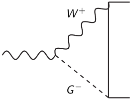
QED: 11 658 471.8 (0.0)



Had vp: 682.5 (4.2)

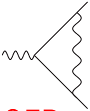


Had lbl: 10.5 (2.6)

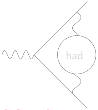


Weak: 15.36 (0.10)

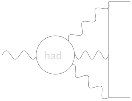
SM prediction a_μ^{SM} $[10^{-10}]$ [Miller, de Rafael, Roberts, DS, Ann.Rev.Nucl.Part. (2012) 62.]



QED: $11\,658\,471.8 \quad (0.0)$



Had vp:



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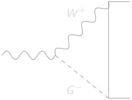


Weak:

QED:

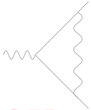
- complete 5-loop result

  [Aoyama, Hayakawa, Kinoshita, Nio '12]

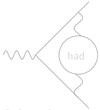
- analytic checks at 4-loop

  [Marquard,Steinhauser et al '13–'16][Laporta '17]

SM prediction a_μ^{SM} $[10^{-10}]$ [Miller, de Rafael, Roberts, DS, Ann.Rev.Nucl.Part. (2012) 62.]



QED:



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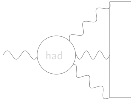

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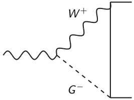
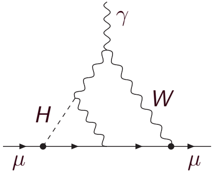


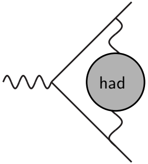
Weak: 15.36 (0.10)

Weak:

- purely EW/bosonic 2-loop result
  [Czarnecki, Krause, Marciano '95]
  [Heinemeyer, DS, Weiglein '04][Czarnecki, Gribouk '05]
- EW+quark/hadron loops
  [Peris, Perrottet, de Rafael '95] [Knecht+PPdR '02][Czarnecki, Marciano, Vainshtein '03]
- Full evaluation with M_h^{exp}
  [Gnendiger,DS,Stöckinger-Kim'13]



Progress on hadronic vacuum polarization

Basics: 

$\longleftrightarrow e^+e^- \to \gamma^* \to \text{hadrons}$

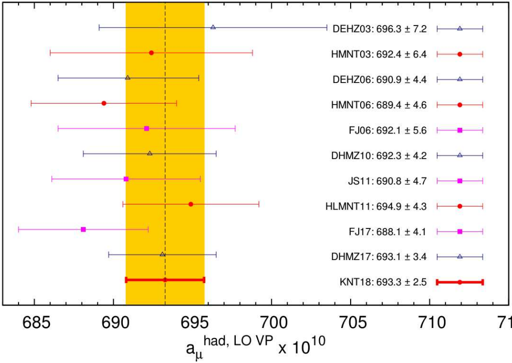
$$a_\mu^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\text{th}}}^{\infty}\frac{ds}{s}R(s)K(s)$$

- $R(s)$ from e^+e^- data
  - exclusive ($\pi\pi, 3\pi, KK$, etc) ($<\sim$ 2GeV)
    radiative return (KLOE, Babar, Belle, BES) vs E-scan (SND, CMD-II)
  - inclusive ($>\sim$ 2GeV) (Babar, KEDR, BES, ...)
- in addition/alternatively: pQCD for $>$ 2GeV
- alternative: use τ-decay data (consistency $\Rightarrow$ ρ–γ-mixing! [Jegerlehner,Szafron '11])
- or lattice

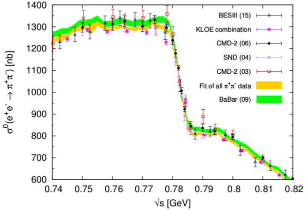
Progress on hadronic vacuum polarization

New generation of data-driven analyses: improved but consistent. E.g. KNT18 [Keshavarzi,Nomura,Teubner]:

- New $\pi\pi$ data: BES, KLOE
- use correlations (E-dependent, for data and errors)
- use improved fitting method



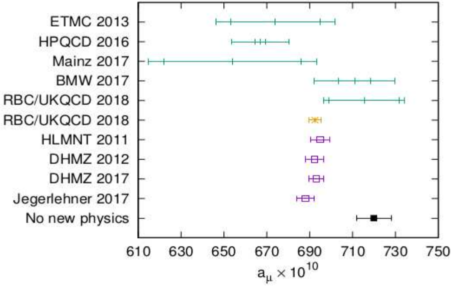
- But: imperfect agreement at ρ-peak
  - 1–2σ effect in $a_\mu^{\pi\pi}$
- rad. corr. in extraction of $R(s)$ from data
- $\Rightarrow$ important to further improve lattice



Progress on hadronic vacuum polarization

Lattice determination of $a_\mu^{\text{had, LO VP}}$

- Several pure lattice determinations: RBC/UKQCD (u,d,s,c+QED+IB), HPQCD (u,d,s,c; no QED/IB), MAINZ (u,d; no IB), BMW (u,d,s,c; no QED/IB)
  - consistent but large errors
- Combination lattice (0.4fm. . . 1fm) with data
  - Most precise $a_\mu^{\text{had,LO VP}}$!

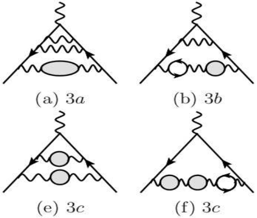


[RBC/UKQCD '18]

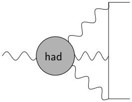
Further progress: NNLO VP

Kurz,Liu,Marquard,Steinhauser'14

$$a^{\text{had,NLO+NNLO VP}} = (-9.82(04)+1.24(01))\times 10^{-10}$$



Progress on hadronic light-by-light



- Difficult multi-scale QFT problem
- Traditionally: low-energy models
  for γ–meson/quark interactions, diagrams with π-exchange, loops...

$$a_\mu^{\text{LbL}}(\text{ Jegerlehner '18}) = \Big[[6.468 + 1.487 + 1.590](1.24)^{\pi,\eta,\eta'} \\ \underbrace{+0.76(27) - 0.60(12) - 2.0(5) + 2.23(4) + 0.11(1) + 0.3(2)}_{\text{axial, scalar mesons, } \pi,\ K\text{-loops, q loops, tensor mesons, NLO}} \Big] \times 10^{-10}$$

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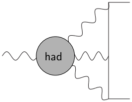

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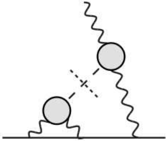
Dispersion relations for LbL:

[Colangelo et al '15] [Hoferichter,Hoid,Kubis,Leupold, Schneider '18]

- QFT definition of pion-pole contribution
- first result: $a_\mu^{\pi\text{-pole}} = 6.26^{+0.30}_{-0.25} \times 10^{-10}$
- data-driven, controlled error estimates



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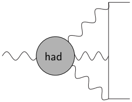

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Lattice determination:

[Blum et al '16]

$$a_\mu^{\mathrm{cHLbL}} = (11.60 \pm 0.96) \times 10^{-10}$$
$$a_\mu^{\mathrm{dHLbL}} = (-6.25 \pm 0.80) \times 10^{-10}$$
$$a_\mu^{\mathrm{HLbL}} = (5.35 \pm 1.35) \times 10^{-10},$$

- expect sizable finite-volume, finite-a corrections
- independent confirmation of model estimates
- **very unlikely that a_μ^{had} is source of exp$-$SM deviation**

Outline

Discrepancy

SM prediction too low by $\approx (30 \pm 8) \times 10^{-10}$

Note: discrepancy twice as large as $a_\mu^{\mathrm{SM,weak}}$

but we expect: $a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{SM,weak}} \times \left(\frac{M_W}{M_{\mathrm{NP}}}\right)^2 \times$ couplings

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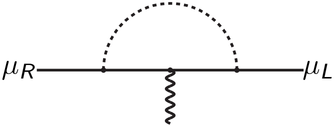
Typical scenarios with large a_μ^{NP}:

- Masses $\sim$ 1 TeV and very large couplings
- Masses $\sim$ 100 GeV and $\sim$weak-interaction couplings
- Masses $\ll$ 100 GeV and superweak couplings

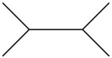
Why is a_μ special?



CP- and Flavour-conserving, chirality-flipping, loop-induced

compare: EDMs, $b \to s\gamma$, $B \to \tau\nu$, $\mu \to e\gamma$

EWPO



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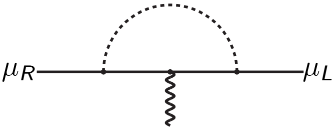

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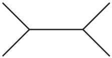
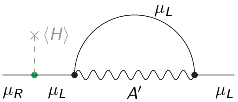
EWPO 

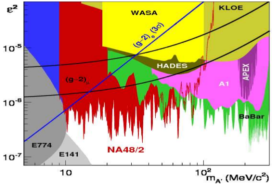
NB: large couplings could be Yukawa-like (chirality-flipping) couplings!

Light/dark sectors — compatible with large a_μ?

Very light, very weakly interacting new particles

- "dark photon" NO $\quad \mathcal{L} = -\frac{\epsilon}{2\cos\theta_W} F^{\mu\nu} B_{\mu\nu} \qquad a_\mu \sim \frac{\alpha}{2\pi}\epsilon^2$

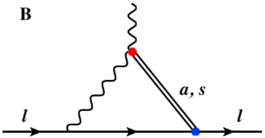




[NA48: 1504.00607]
excludes minimal dark photon for a_μ

- "ALPs" YES

$$\mathcal{L} = \frac{1}{4} g_{s\gamma\gamma} s F^{\mu\nu} F_{\mu\nu} + y_s s \bar{\mu}\mu \qquad a_\mu^{\rm BZ} \sim \frac{m_\mu}{4\pi^2} g_{s\gamma\gamma} y_s \ln(\Lambda/m_s)$$



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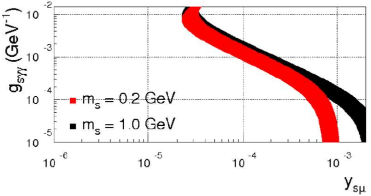


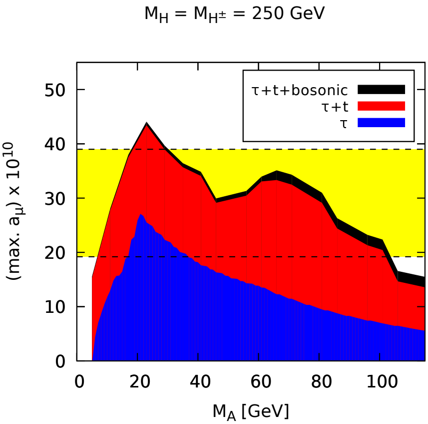
[Marciano, Masiero, Paradisi, Passera '16]

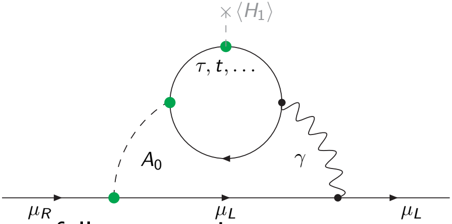
Two-Higgs doublet model — can it explain Δa_μ?

[Cherchiglia, DS, Stöckinger-Kim '17]

Answer: YES (in small par. space)!

- light A_0-boson, large couplings to leptons (and top-quarks)





- full computation

[Cherchiglia,Kneschke,DS,Stöckinger-Kim'16]

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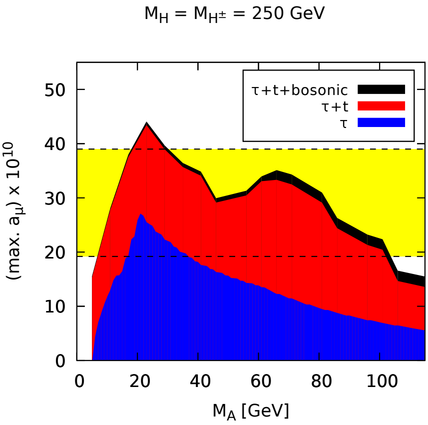


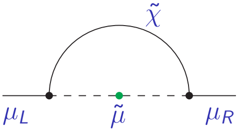
constrained/testable by

- τ-, Z-decays, LEP
- b-decays, LHC

$\Rightarrow$ maximum Yukawa couplings

- lepton Yukawa $<\sim 100$
- quark Yukawas $<\sim 0.5$
- (for $M_A = 20 \ldots 100$ GeV, else even stronger)

SUSY — still compatible with large a_μ?



- Ex. 1: "Constrained MSSM" NO:

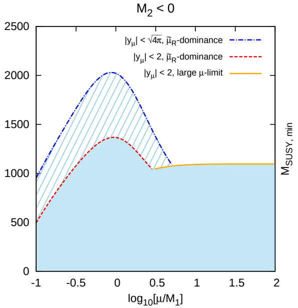
  (simple mass universality at GUT scale)

- Ex. 2: heavy SUSY, $\tan\beta \to \infty$ YES

  [Bach,JH Park,DS,Stöckinger-Kim, '15]

  large $y_\mu \rightsquigarrow m \geq 1$ TeV possible

  constraints: B-physics, Higgs-physics, vacuum stability

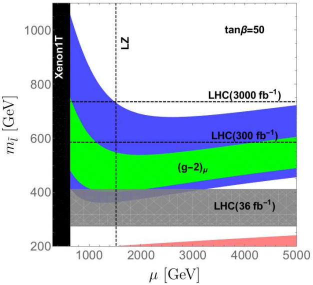


- Ex. 3: light SUSY: Dark matter+a_μ YES

  [Cox, Han, Yanagida '18]

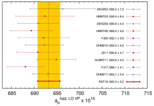
  $m_{\tilde{B}} = 185$, $m_{\tilde{W}} = 200$GeV, coannihilation

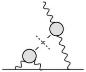
  small masses allowed since $m_{\tilde{\chi}} < m_{\tilde{l}}$



Outline

Conclusions

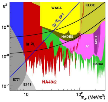


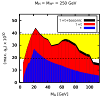


- Recent progress on all aspects of $a_\mu^{\rm SM}$
  - $a_\mu^{\rm Exp} - a_\mu^{\rm SM} \approx (30 \pm 6.3 \pm 3.6^{\rm KNT18}) \times 10^{-10}$
  - very stable and consistent
- Progress on hadronic contributions
  - VP: new data, can use correlations, τ-decays, lattice (but: lattice alone imprecise, 1–2σ variations in $a_\mu^{\pi\pi}$)
  - LbL: improved model calculations + DRs, lattice: consistent! (but: still systematic uncertainties)

    $\rightsquigarrow$ Outlook: further improvement possible
- $a_\mu^{\rm N.P.,SUSY}$ very model-dependent, typically $\mathcal{O}(\pm 1 \ldots 50) \times 10^{-10}$
  - small a_μ (Dark photon, Constrained MSSM)
  - large a_μ in special parameter regions (2HDM, heavy SUSY, light SUSY)

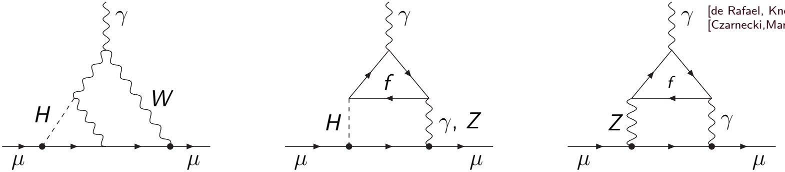
    $\rightsquigarrow$ Outlook: await new experiment!

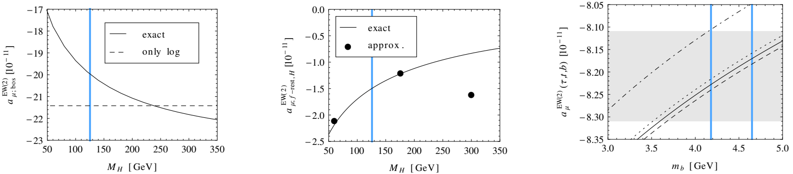




Re-evaluation of a_μ(weak) [Gnendiger, DS, Stöckinger-Kim '13]

[de Rafael, Knecht, et al '95–'02]
[Czarnecki,Marciano,Vainshtein '03]





- exact evaluation of M_H-dependent parts
- consistent parametrization of 1-, 2-, 3-loop $\propto G_F \alpha^{n-1}$
- final result: $(15.36 \pm 0.10) \times 10^{-10}$

More on bounds on dark photons, ALPs

- beam dump: dark bremsstrahlung (works for specific coupling range)
- electron fixed target (APEX,A1)
- Babar, KLOE, WASA: meson decays
- often assumed: $A' \to e^+e^-$ dominant
- if not: $K \to \pi A'$, $A' \to$invisible and $e^+e^- \to \gamma+$invisible lead to bounds

  [Davoudiasl,Lee,Marciano'14][Izaguirre et al '13]
- generalization: also mass mixing "dark Z" with more general couplings, also strongly constrained
- for ALPs: [Marciano et al '16]
  $e^+e^- \to \gamma a, e^+e^- a$ at future low-E experiments