

Validity of EFT for VBS

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Theoretical Study of the TeV scale

- From Top to Bottom: construct a full theory (renormalizable and UV complete), and describe the TeV scale in terms of the parameters of the BSM Lagrangian. For instance: MSSM has ~ 100 free parameters.
 - Advantage: a full model. Renormalizability.
 - Problems: no hints about the UV completion chosen by nature.
 - Examples: MSSM (~ 100 free parameters), non-MSSM SUSY, Technicolor, KK,...
- From Bottom to Top: construct an Effective Field Theory (EFT), based on the symmetries and available degrees of freedom at low energy.
 - Advantage: we do not rely on a specific UV completion.
 - Disadvantage: valid only at certain energy scale. Non-renormalizable in the classical QFT sense, but in the ChPT one.
 - The usual EFT approach breaks when the low energy EFT reaches the unitarity bound, becoming non-perturbative.
 - For phenomenology, EFTs with the BSM physics (resonances) as explicit degrees of freedom are used.

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Non-linear Electroweak Chiral Lagrangian

Non-linear EFT¹ for VV scattering at NLO level, minimally coupled to hh ,

$$\mathcal{L} = \frac{v^2}{4} g(h/v) \text{Tr}[(D_\mu U)^\dagger D^\mu U] + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h),$$

where

$$g(h/v) = 1 + 2a \frac{h}{v} + b \left(\frac{h}{v}\right)^2 + \dots$$

$$V(h) = V_0 + \frac{M_h^2}{2} h^2 + \sum_{n=3}^{\infty} \lambda_n h^n$$

$$D_\mu U = \partial_\mu U + i\hat{W}_\mu U - iU\hat{B}_\mu.$$

M_h and λ_n are subleading in chiral counting.

¹Yellow Report: *C.Grojean, A.Falkowski, M.Trott, B.Fuks, *G.Buchalla, T.Plehn, G.Isidori, K.Tackmann, L.Brenner,...; CERN-2017-002-M.

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$$U(x) = \mathbb{1} + i \frac{\tau^a \omega^a(x)}{v} + \mathcal{O}(\omega^2).$$

Two choices have been used:

Spherical parameterization

$$U(x) = \mathbb{1} \sqrt{1 - \frac{\omega^2(x)}{v^2}} + i \frac{\tau^a \omega^a(x)}{v}$$

Exponential parameterization (here, a cross-check for EWSBS+ $\gamma\gamma$)

$$U(x) = \exp\left(i \frac{\tau^a \pi^a(x)}{v}\right)$$

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EFT for VV scattering, minimally coupled to hh

Since we are considering scattering processes within the EWSBS, the covariant derivative reduces to

$$D_\mu U = \partial_\mu U.$$

Define

$$V_\mu \equiv (D_\mu U) U^\dagger.$$

The next counterterms are needed for the NLO computation of the VV scattering, minimally coupled to hh

$$\begin{aligned} \mathcal{L}_4 = & a_4 [\text{Tr}(V_\mu V_\nu)] [\text{Tr}(V^\mu V^\nu)] + a_5 [\text{Tr}(V_\mu V^\mu)] [\text{Tr}(V_\nu V^\nu)] \\ & + \frac{d}{v^2} (\partial_\mu h \partial^\mu h) \text{Tr}[(D_\nu U)^\dagger D^\nu U] + \frac{e}{v^2} (\partial_\mu h \partial^\nu h) \text{Tr}[(D^\mu U)^\dagger D_\nu U] \\ & + \frac{g}{v^4} (\partial_\mu h \partial^\mu h) (\partial_\nu h \partial^\nu h). \end{aligned}$$

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Validity of EFTs

- We are interested in the collider phenomenology of Vector Bosons Scattering since it is very sensitive to new physics in the EW sector in the LHC.
- The NLO-computed EFT grows with the CM energy like $A \sim s^2$. Hence, it will eventually reach the unitarity bound, becoming non-perturbative. 2 options available:
 - The common one: limit the validity range of the EFT to the perturbative region. I.e., until the unitarity bound is reached (see the talk of Matthias, *Unitarity limits for EFT parameters*). Consider the EFT as a useful parameterization of slight deviations from the SM in the range under the TeV scale.
 - Take advantage of the analytical properties of the S-Matrix (encoded inside dispersion relations and unitarization procedures) to study the non-perturbative region (TeV scale) of the theory. A decomposition in partial waves required. Going back from partial waves to scattering amplitude can be tricky, because of contributions from higher order spherical harmonics.

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Unitarity for partial waves

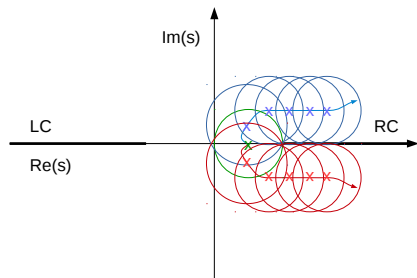
- Unit. cond. for S – *matrix*:
 $SS^\dagger = \mathbb{1}$,
- plus analytical properties of matrix elements,
- plus time reversal invariance,

Unitarity condition for partial waves

$$\text{Im } A_{IJ,p_i \rightarrow k_1}(s) = \sum_{\{a,b\}} \sqrt{1 - \frac{4m_q^2}{s}} [A_{IJ,p_i \rightarrow q_{i,ab}}(s)][A_{IJ,q_{i,ab} \rightarrow k_i}(s)]^*$$

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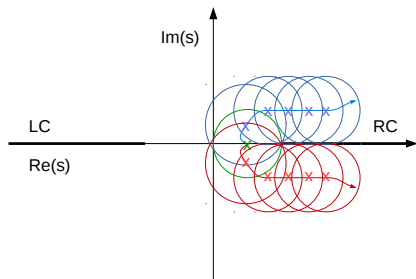


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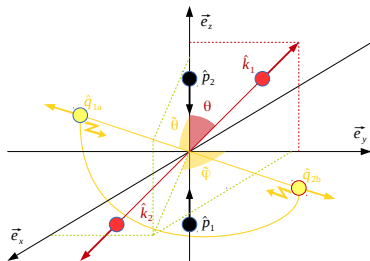


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The form of the partial wave is

$$A_{IJ}(s) = A_{IJ}^{(0)} + A_{IJ}^{(1)} + \mathcal{O}[(s/v^2)^3].$$

Which will be decomposed as

$$A_{IJ}^{(0)} = Ks$$

$$A_{IJ}^{(1)} = \left(B(\mu) + D \log \frac{s}{\mu^2} + E \log \frac{-s}{\mu^2} \right) s^2.$$

As $A_{IJ}(s)$ must be scale independent,

$$\begin{aligned} B(\mu) &= B(\mu_0) + (D + E) \log \frac{\mu^2}{\mu_0^2} \\ &= B_0 + p_4 a_4(\mu) + p_5 a_5(\mu). \end{aligned}$$

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Unitarization procedures for elastic processes

$$A^{IAM}(s) = \frac{[A^{(0)}(s)]^2}{A^{(0)}(s) - A^{(1)}(s)},$$

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PRD **91** (2015) 075017

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Usability channel of unitarization procedures

IJ	00	02	11	20	22
Method of choice	Any	N/D IK	IAM	Any	N/D IK

- The IAM method cannot be used when $A^{(0)} = 0$, because it would give a vanishing value.
- The N/D and the IK methods cannot be used if $D + E = 0$, because in this case computing $A_L(s)$ and $A_R(s)$ is not possible.
- The naive K-matrix method,

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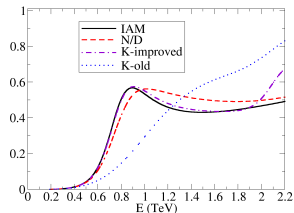
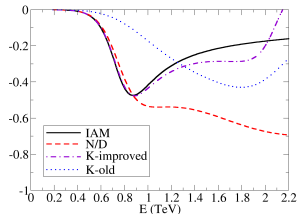
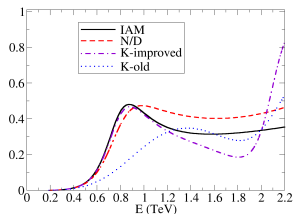
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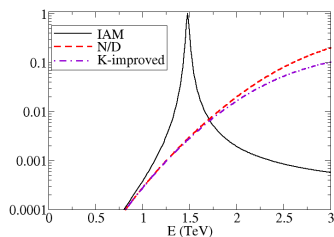
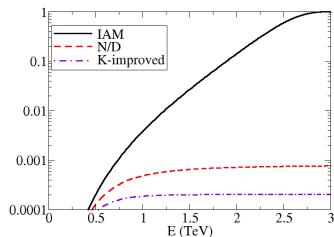
Scalar-isoscalar channels



From left to right and top to bottom, elastic $\omega\omega$, elastic hh , and cross channel $\omega\omega \rightarrow hh$, for $a = 0.88$, $b = 3$, $\mu = 3$ TeV and all NLO parameters set to 0.

PRL **114** (2015) 221803, PRD **91** (2015) 075017.

Vector-isovector channels

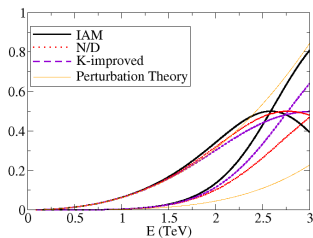
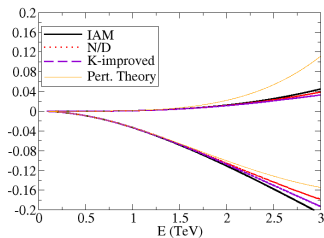


From our ref². We have taken $a = 0.88$ and $b = 1.5$, but while for the left plot all the NLO parameters vanish, for the right plot we have taken $a_4 = 0.003$, known to yield an IAM resonance according to the Barcelona group³.

²PRD **91** (2015) 075017

³PRD **90** (2014) 015035

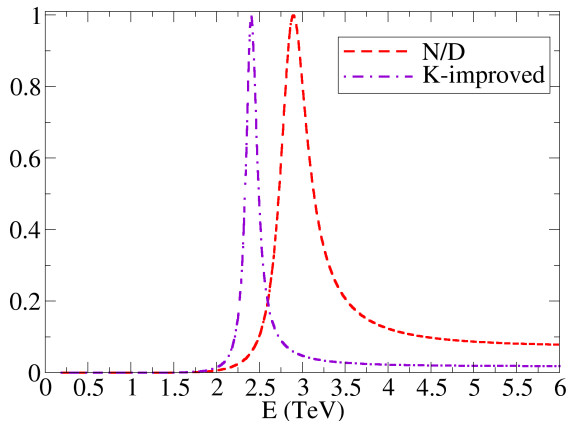
Scalar-isotensor channels ($IJ = 20$)



From our ref⁴. From left to right, $a = 0.88$, $a = 1.15$. We have taken $b = a^2$ and the NLO parameters set to zero. Both real and imaginary part shown. Real ones correspond to bottom lines at left and upper at low E at right.

⁴PRD **91** (2015) 075017

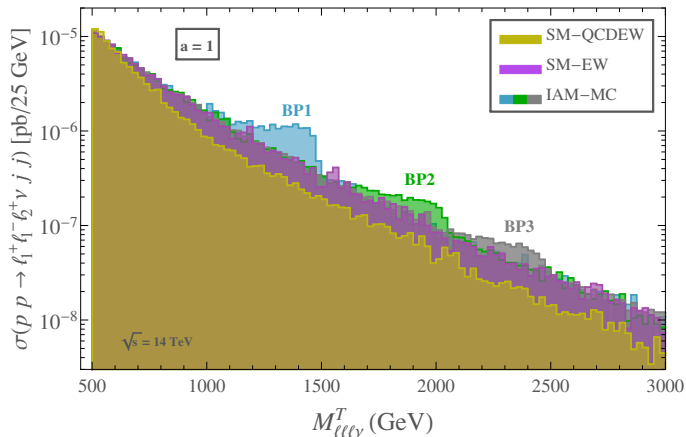
Isotensor-scalar channels ($IJ = 02$)



$a = 0.88$, $b = a^2$, $a_4 = -2a_5 = 3/(192\pi)$, all the other NLO param. set to zero.
PRD **91** (2015) 075017.

New MadGraph model, Isovector Resonance: leptonic final state

arXiv:1707.04580 [hep-ph]



$a = 1$; $a_4 \cdot 10^4 = 3.5$ (BP1), 1 (BP2), 0.5 (BP3);

Conclusions

- Studied $2 \rightarrow 2$ VBF processes within the EWSBS, including coupled channels (hh).
- An extensive analysis of the validity of unitarization methods.
- We provide a MadGraph v5 model for the unitarized ChPT, without relying on the naive K-matrix.
- Prospects for the Benchmark Points at the LHC (14 TeV, $WZ \rightarrow WZ$ process, arXiv:1707.04580 [hep-ph]):

	$\mathcal{L} = 300 \text{ fb}^{-1}$			$\mathcal{L} = 1000 \text{ fb}^{-1}$			$\mathcal{L} = 3000 \text{ fb}^{-1}$		
	N_I^{IAM}	N_I^{SM}	σ_I^{stat}	N_I^{IAM}	N_I^{SM}	σ_I^{stat}	N_I^{IAM}	N_I^{SM}	σ_I^{stat}
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BP2	0.6	0.4	-	1	1	0	4	3	0.1
BP3	0.1	0.1	-	0.4	0.3	-	1	1	0
BP1'	6	2	2.3	19	8	4.2	57	23	7.2
BP2'	2	0.9	1	6	3	1.8	19	9	3.7
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Let's start the discussion!!

Backup Slides

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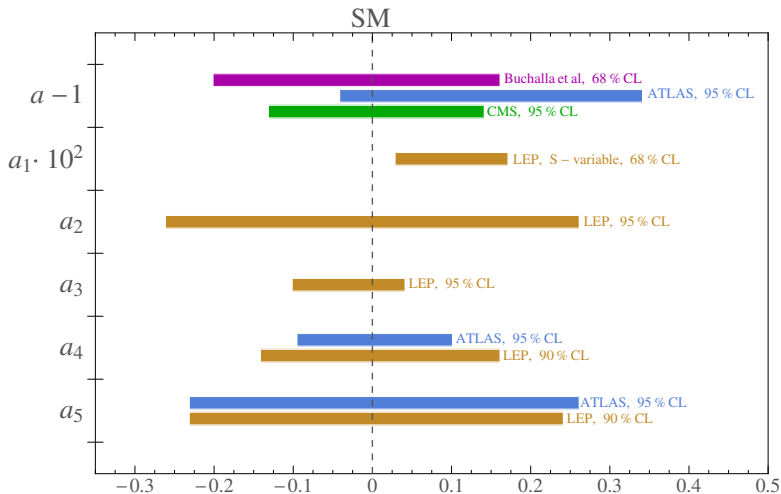
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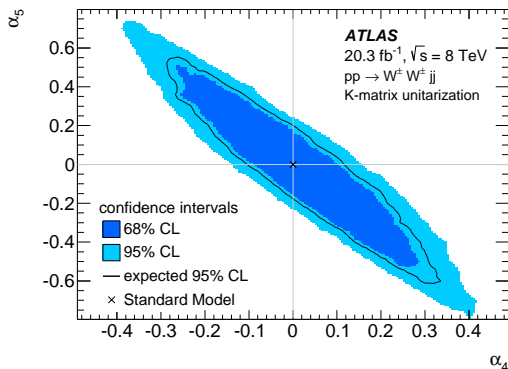
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Experimental bounds on low-energy constants



[arXiv:1707.04580 [hep-ph]]

Experimental bounds on low-energy constants, NLO a_4 - a_5

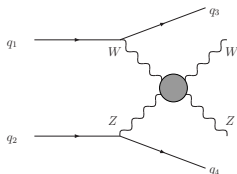


Direct constraint over a_4 - a_5 from ATLAS, [PRL**113** (2014) 141803]. Note that the naive K-matrix unitarization procedure from Kilian et al [JHEP 0811, 010] is used here.

A custom model for MadGraph v5

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by $WZ \rightarrow WZ$ scattering

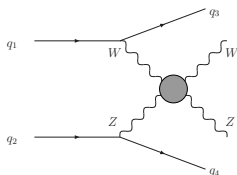


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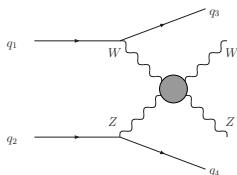


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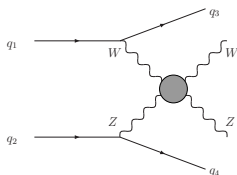


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A custom model for MadGraph v5

$$pp \rightarrow WZ j_1 j_2$$

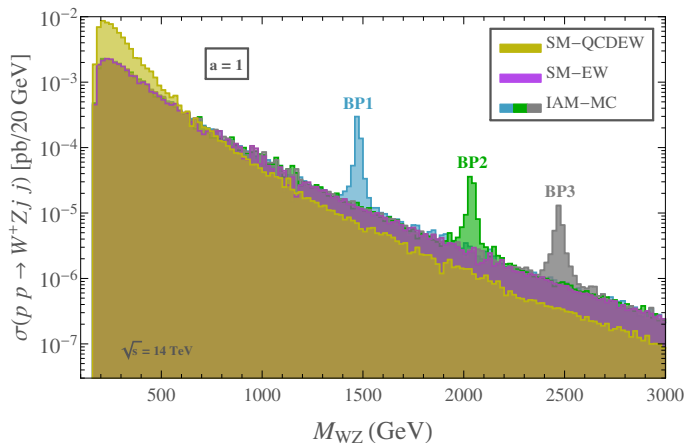
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Isvector Resonance: WZ in final state

arXiv:1707.04580 [hep-ph]

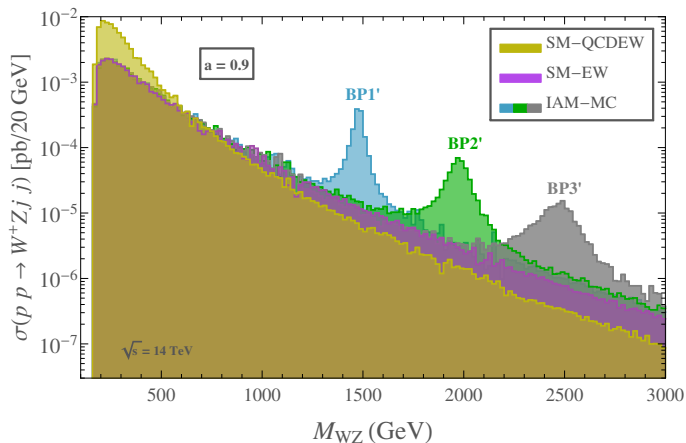


$a = 1$; $a_4 \cdot 10^4 = 3.5$ (BP1), 1 (BP2), 0.5 (BP3);

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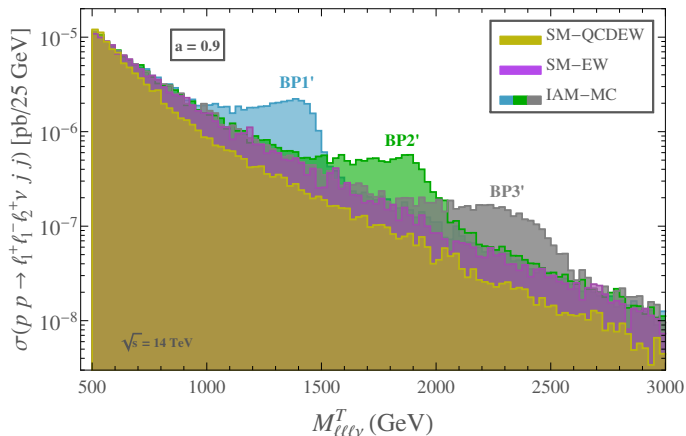


$a = 0.9$; $a_4 \cdot 10^4 = 3.5$ (BP1'), 1 (BP2'), 0.5 (BP3');

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Isvector Resonance: leptonic final state

arXiv:1707.04580 [hep-ph]



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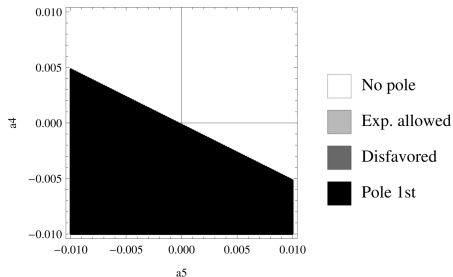
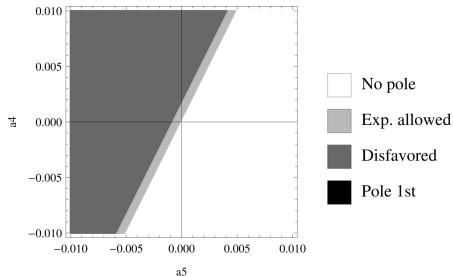
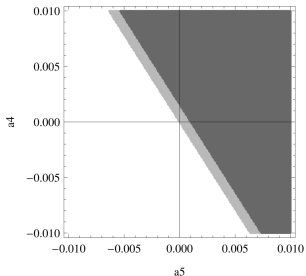
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BP	$M_V(\text{GeV})$	$\Gamma_V(\text{GeV})$	$g_V(M_V^2)$	a	$a_4 \cdot 10^4$	$a_5 \cdot 10^4$
BP1	1476	14	0.033	1	3.5	-3
BP2	2039	21	0.018	1	1	-1
BP3	2472	27	0.013	1	0.5	-0.5
BP1'	1479	42	0.058	0.9	9.5	-6.5
BP2'	1980	97	0.042	0.9	5.5	-2.5
BP3'	2480	183	0.033	0.9	4	-1

These BPs have been selected for vector resonances emerging at mass and width values that are of phenomenological interest for the LHC.

Considered backgrounds: The pure SM-EW background, of order $\mathcal{O}(\alpha_{\text{em}}^2)$.
The mixed SM-QCDEW background, of order $\mathcal{O}(\alpha_{\text{em}}\alpha_s)$.

Reson. in $W_L W_L \rightarrow W_L W_L$ due to a_4 and a_5 , ours



- $a = 0.90$, $b = a^2$
PRD **91** (2015) 075017
- From left, clockwise,
 $J = 00, 11, 20$
- Excluding resonances
 $M_S < 700 \text{ GeV}$, $M_V < 1.5 \text{ TeV}$

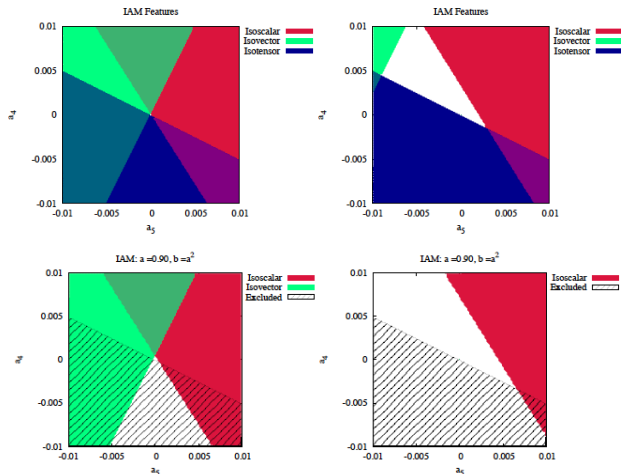
Reson. in $W_L W_L \rightarrow W_L W_L$ due to a_4 and a_5 , Barcelona

CROSS-CHECK:
Espriu, Yencho,
Mescia

PRD**88**, 055002

PRD**90**, 015035

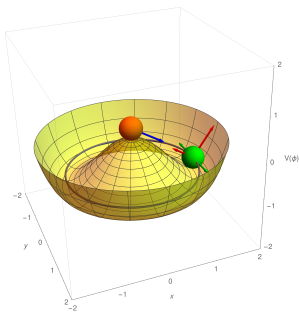
At right, exclusion
regions include reso-
nances with
 $M_{S,V} < 600$ GeV.



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- This is problematic: the massive terms are not gauge invariant. Gauge boson scattering amplitudes diverge with s at LO.
- Standard Model solution:
Higgs-mechanism, which predicts the SM Higgs boson. Global symmetry breaking pattern: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_C$.
- In 2012, ATLAS and CMS find a 125-126 GeV scalar resonance h , compatible with the Higgs of the SM.

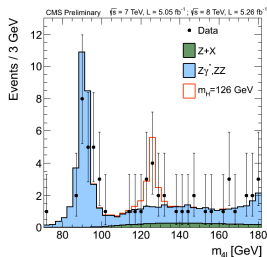
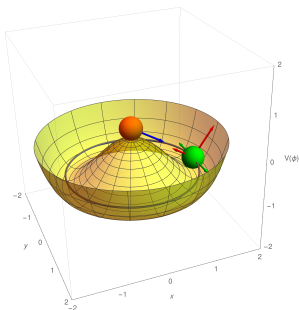
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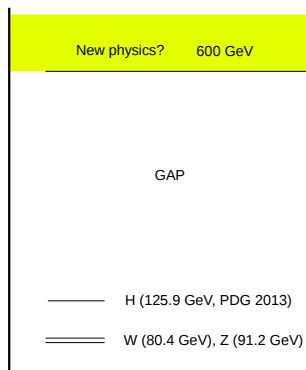
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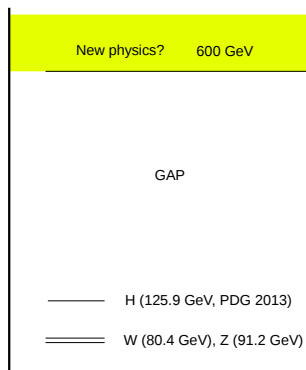


Phys. Lett. **B716**, 30-61.

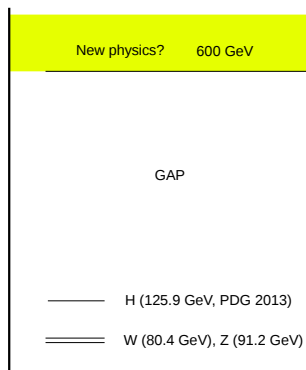
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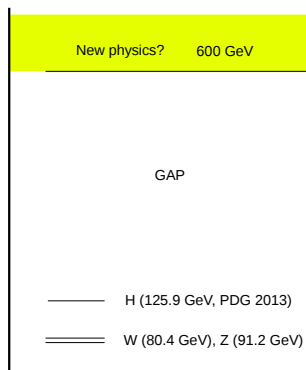
- AND: No new physics!!
If there is any...
- The SM until the Planck mass?
- Some issues: mass of neutrinos, gravity explanation (*naturalness problem*), astrophysical observation (dark matter, dark energy),...
- Four scalar light modes, a large gap.
- Natural: further spontaneous symmetry breaking at $f > v = 246 \text{ GeV}$?



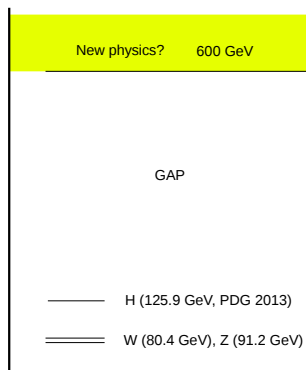
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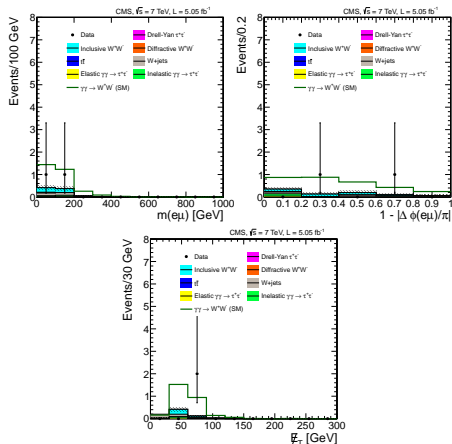
Empirical situation: $\gamma\gamma$ physics

- Current efforts to measure $\gamma\gamma \rightarrow W_L^+ W_L^-$ and $\gamma\gamma \rightarrow Z_L Z_L$ channels.
- Only 2 events measured. Graphs from CMS⁵.
- Wait for LHC Run-II, CMS-TOTEM and ATLAS-AFP.
- Efforts for measuring $\gamma\gamma$ final states: SM Higgs decay channel.

⁵JHEP **07** (2013) 116

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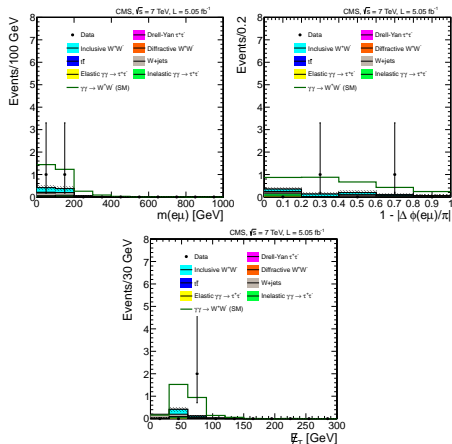
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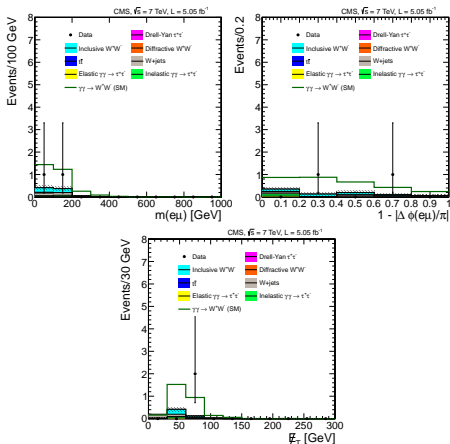
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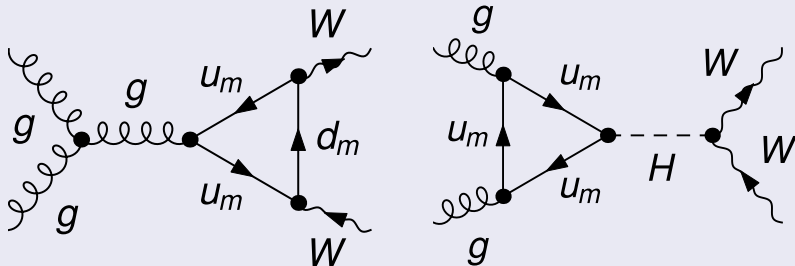


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Empirical situation: $t\bar{t}$ physics

- Initial $t\bar{t}$ states are important because of gluon fusion processes, with a large cross section at the LHC.
- The production of $t\bar{t}$ states is also a well studied experimental observable at the LHC.

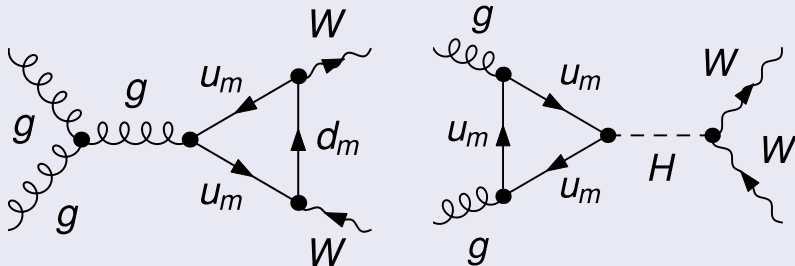
Top quark loop in the SM: VV states from gluon fusion



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Studied framework

- We consider a strongly interacting EWSBS, in contrast to the weakly interacting one of the SM.
- We study the processes $VV \rightarrow VV$, $VV \rightarrow hh$ and $hh \rightarrow hh$, and extend the result to include $\gamma\gamma$ and $t\bar{t}$ states.
- Our LO scattering amplitudes within the EWSBS diverge, but are controlled by strongly interacting dynamics which respect unitarity. This situation is similar to low-energy QCD (hadron physics).
- In order to minimize our assumptions over the (hypothetical) underlying theory, we will
 - use dispersion relations over a partial wave decomposition (the so-called unitarization procedures);
 - extend these unitarization procedures to the coupled-channels case;
 - and consider an Effective Field Theory, computed at the NLO level (within the limits of the Equivalence Theorem), with three would-be Goldstone bosons w and a Higgs-like boson h .

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Strongly Interacting Effective Field Theory + Unitarity: similarity with hadronic physics

Chiral Perturbation Theory plus Dispersion Relations.

Simultaneous description of $\pi\pi \rightarrow \pi\pi$
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Lowest order ChPT (Weinberg Theorems) and even one-loop computations are only valid at very low energies.

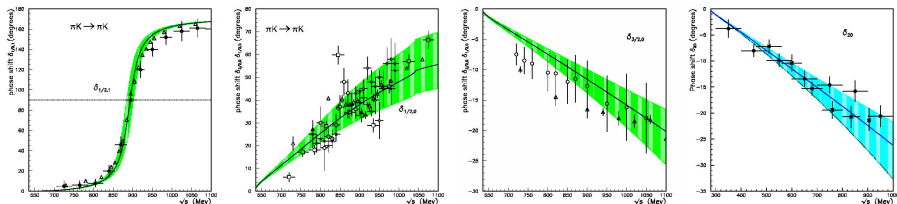
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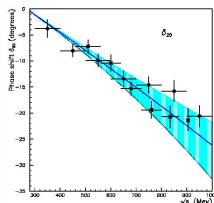
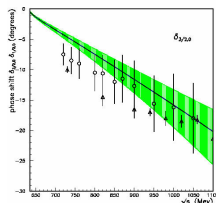
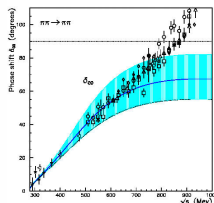
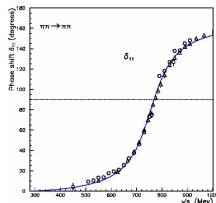
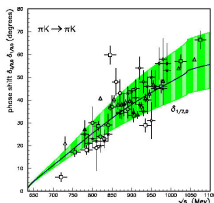
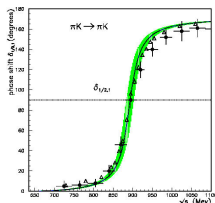
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Equivalence Theorem

- For $s \gg M_h^2, M_W^2, M_Z^2 \approx (100 \text{ GeV})^2$, longitudinal modes of gauge bosons can be identified with the would-be Goldstones. For instance,

$$T(W_L^a W_L^b \rightarrow W_L^c W_L^d) = T(\omega^a \omega^b \rightarrow \omega^c \omega^d) + \mathcal{O}(M_W/\sqrt{s})$$

- The EWSBS behaves as if the would-be Goldstone bosons were physical states. The non-gauged Lagrangian can be used directly to compute scattering amplitudes.
- During the 90's, the limits of applicability of this theorem were studied in detail, leading to the conclusion that it is valid for chiral Lagrangians, like those used in this presentation:

works from W.B.Kilgore, P.B.Pal, X.Zhang, A.Dobado, J.R.Peláez, M.T.Urdiales, H.-J.He, Y.-P.Kuang, D.Espriu, J.Matias, J.F.Donoghue, J.Tandean,...

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Equivalence Theorem

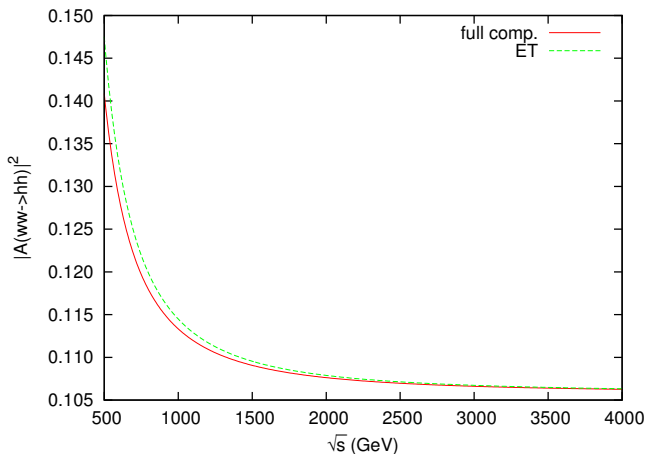
- For $s \gg M_h^2, M_W^2, M_Z^2 \approx (100 \text{ GeV})^2$, longitudinal modes of gauge bosons can be identified with the would-be Goldstones. For instance,

$$T(W_L^a W_L^b \rightarrow W_L^c W_L^d) = T(\omega^a \omega^b \rightarrow \omega^c \omega^d) + \mathcal{O}(M_W/\sqrt{s})$$

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Equivalence Theorem



Comparison between the full LO $\omega\omega \rightarrow hh$ ($\cos\theta = 3$) and that computed through the ET. The SM is used here. Work in collaboration with S. Moretti, to test a modified version of MadGraph.

Non-linear Electroweak Chiral Lagrangian

We have no clue of what, how or if new physics...

Non-linear EFT⁶ for VV scattering at NLO level, minimally coupled to hh ,

$$\mathcal{L} = \frac{v^2}{4} g(h/f) \text{Tr}[(D_\mu U)^\dagger D^\mu U] + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h),$$

where

$$g(h/v) = 1 + 2a \frac{h}{v} + b \left(\frac{h}{v}\right)^2 + \dots$$

$$V(h) = V_0 + \frac{M_h^2}{2} h^2 + \sum_{n=3}^{\infty} \lambda_n h^n$$

$$D_\mu U = \partial_\mu U + i\hat{W}_\mu U - iU\hat{B}_\mu.$$

M_h and λ_n are subleading in chiral counting.

⁶Yellow Report: *C.Grojean, A.Falkowski, M.Trott, B.Fuks, *G.Buchalla, T.Plehn, G.Isidori, K.Tackmann, L.Brenner,...; CERN-2017-002-M.

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Non-linear Electroweak Chiral Lagrangian

We need the parameterization of the $U(\omega^a) \in SU(2)_L \times SU(2)_R / SU(2)_C$ coset. In either case, whatever the non-linear term is,

$$U(x) = \mathbb{1} + i \frac{\tau^a \omega^a(x)}{v} + \mathcal{O}(\omega^2).$$

Two choices have been used:

Spherical parameterization

$$U(x) = \mathbb{1} \sqrt{1 - \frac{\omega^2(x)}{v^2}} + i \frac{\tau^a \omega^a(x)}{v}$$

Exponential parameterization (here, a cross-check for EWSBS+ $\gamma\gamma$)

$$U(x) = \exp\left(i \frac{\tau^a \pi^a(x)}{v}\right)$$

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EFT for VV scattering, minimally coupled to hh

Since we are considering scattering processes within the EWSBS, the covariant derivative reduces to

$$D_\mu U = \partial_\mu U.$$

Define

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The next counterterms are needed for the NLO computation of the VV scattering, minimally coupled to hh

$$\begin{aligned} \mathcal{L}_4 = & a_4 [\text{Tr}(V_\mu V_\nu)] [\text{Tr}(V^\mu V^\nu)] + a_5 [\text{Tr}(V_\mu V^\mu)] [\text{Tr}(V_\nu V^\nu)] \\ & + \frac{d}{v^2} (\partial_\mu h \partial^\mu h) \text{Tr}[(D_\nu U)^\dagger D^\nu U] + \frac{e}{v^2} (\partial_\mu h \partial^\nu h) \text{Tr}[(D^\mu U)^\dagger D_\nu U] \\ & + \frac{g}{v^4} (\partial_\mu h \partial^\mu h) (\partial_\nu h \partial^\nu h). \end{aligned}$$

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EFT for VV scattering, minimally coupled to hh

Using the spherical parameterization for the $SU(2)$ coset and neglecting the couplings with photons and quarks, we have the next Lagrangian describing $VV \rightarrow VV$, $VV \rightarrow hh$ and $hh \rightarrow hh$ processes:

$$\begin{aligned}\mathcal{L} = & \left[1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2 \right] \frac{\partial_\mu \omega^a \partial^\mu \omega^b}{2} \left(\delta^{ab} + \frac{\omega^a \omega^b}{v^2} \right) \\ & + \frac{4a_4}{v^4} \partial_\mu \omega^a \partial_\nu \omega^a \partial^\mu \omega^b \partial^\nu \omega^b + \frac{4a_5}{v^4} \partial_\mu \omega^a \partial^\mu \omega^a \partial_\nu \omega^b \partial^\nu \omega^b \\ & + \frac{2d}{v^4} \partial_\mu h \partial^\mu h \partial_\nu \omega^a \partial^\nu \omega^a + \frac{2e}{v^4} \partial_\mu h \partial^\mu \omega^a \partial_\nu h \partial^\nu \omega^a \\ & + \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{g}{v^4} (\partial_\mu h \partial^\mu h)^2\end{aligned}$$

Extension to $\gamma\gamma$ states

Since coupling with photons are considered⁷, the covariant derivative is defined as

$$D_\mu U = \partial_\mu U + i\hat{W}_\mu U - iU\hat{B}_\mu.$$

The photon field A arises from the couplings with $\hat{W}_{\mu\nu}$ and $\hat{B}_{\mu\nu}$ through a rotation to the physical basis; an anomalous three-particle coupling may appear

$$-c_W \frac{h}{v} \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} - c_B \frac{h}{v} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} = -\frac{c_\gamma}{2} \frac{h}{v} e^2 A_{\mu\nu} A^{\mu\nu}$$

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$$\begin{aligned} \mathcal{L}_{4'} = & a_1 \text{Tr}(U\hat{B}_{\mu\nu}U^\dagger\hat{W}^{\mu\nu}) \\ & + ia_2 \text{Tr}(U\hat{B}_{\mu\nu}U^\dagger[V^\mu, V^\nu]) \\ & - ia_3 \text{Tr}(\hat{W}_{\mu\nu}[V^\mu, V^\nu]) \end{aligned}$$

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Extension to $t\bar{t}$ states

Lagrangian additions⁸:

$$\mathcal{L}' = i\bar{Q}\partial Q - v\mathcal{G}(h) [\bar{Q}'_L U H_Q Q'_R + h.c.] .$$

This expression, for the heaviest quark generation, expands to⁹

$$\mathcal{L}_Y = -\mathcal{G}(h) \left\{ \sqrt{1 - \frac{\omega^2}{v^2}} (M_t t\bar{t} + M_b \bar{b}b) + \frac{i\omega^0}{v} (M_t \bar{t}\gamma^5 t - M_b \bar{b}\gamma^5 b) \right. \\ \left. + \frac{i\sqrt{2}\omega^+}{v} (M_b \bar{t}_L b_R - M_t \bar{t}_R b_L) + \frac{i\sqrt{2}\omega^-}{v} (M_t \bar{b}_L t_R - M_b \bar{b}_R t_L) \right\}$$

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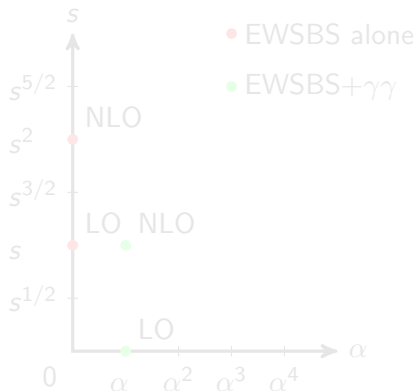
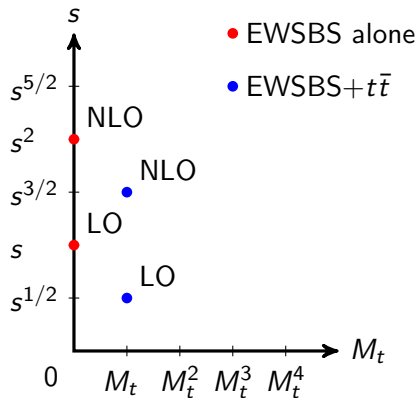
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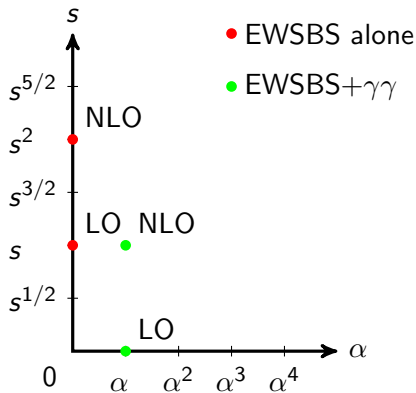
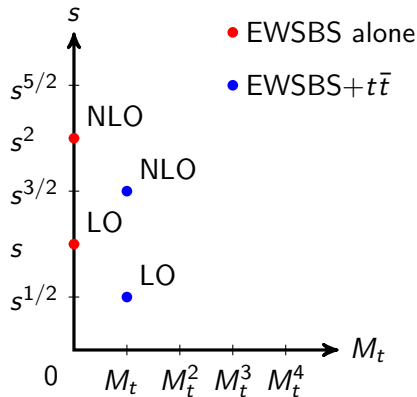
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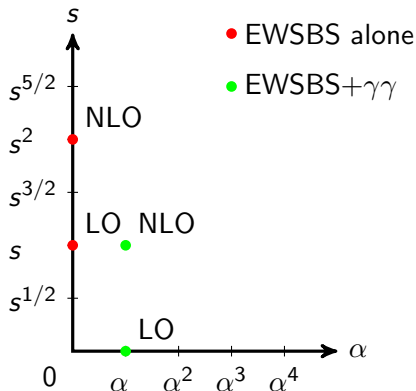
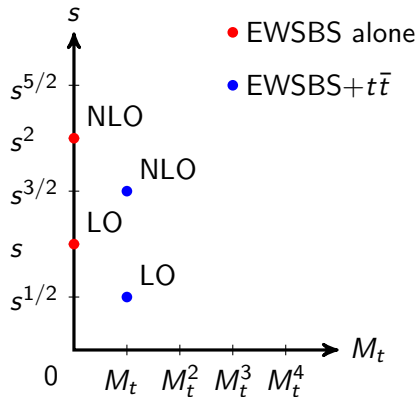
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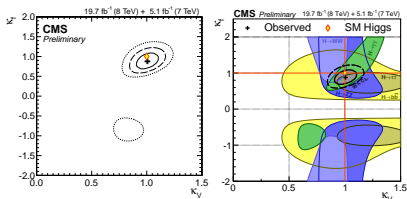
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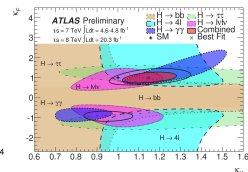
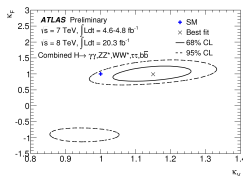
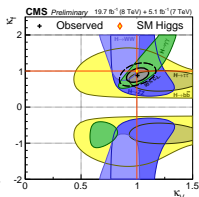
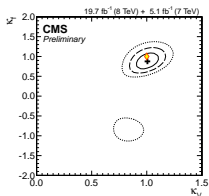
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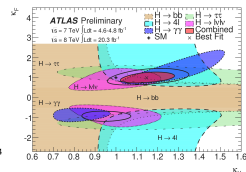
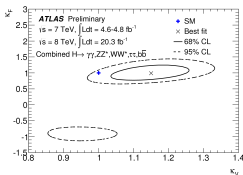
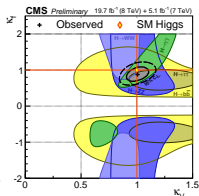
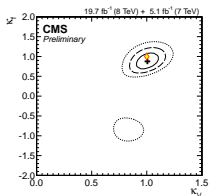
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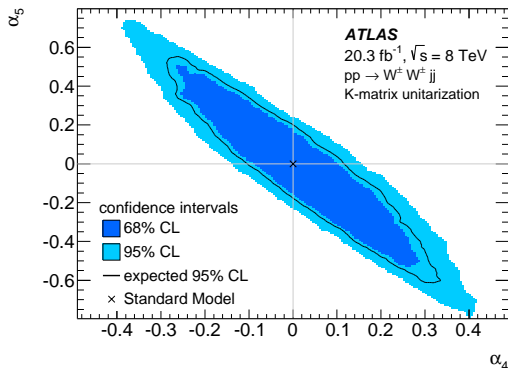
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Experimental bounds on low-energy constants, NLO a_4 - a_5



Direct constraint over a_4 - a_5 from ATLAS Collaboration¹⁵

¹⁵Taken from ref. [PRL113 (2014) 141803]. Note that CMS [PRL114 (2015) 051801] gives a constraint in terms of F_{S0}/Λ^4 and F_{S1}/Λ^4 parameters, which have no direct translation to the a_4 and a_5 ones [arXiv:1310.6708, [hep-ph]].

Partial wave decomposition

EWSBS alone (+eventually $t\bar{t}$)

$$A_{IJ}(s) = \frac{1}{32\pi K} \int_{-1}^1 dx P_J(x) A_I[s, t(s, x), u(s, x)]$$

Matrix element from partial wave decomposition

$$A_I(s, t, u) = 16\pi K \sum_{J=0}^{\infty} (2J+1) P_J[x(s, t)] A_{IJ}(s)$$

Helicity partial waves for EWSBS+ $\gamma\gamma$

$$F_{IJ}^{\lambda_1\lambda_2}(s) = \frac{1}{64\pi^2 K} \sqrt{\frac{4\pi}{2J+1}} \int d\Omega A_I^{\lambda_1\lambda_2}(s, \Omega) Y_{J, \lambda_1-\lambda_2}(\Omega)$$

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$$A_{IJ}(s) = A_{IJ}^{(0)} + A_{IJ}^{(1)} + \mathcal{O}[(s/v^2)^3].$$

Which will be decomposed as

$$A_{IJ}^{(0)} = Ks$$

$$A_{IJ}^{(1)} = \left(B(\mu) + D \log \frac{s}{\mu^2} + E \log \frac{-s}{\mu^2} \right) s^2.$$

As $A_{IJ}(s)$ must be scale independent,

$$\begin{aligned} B(\mu) &= B(\mu_0) + (D + E) \log \frac{\mu^2}{\mu_0^2} \\ &= B_0 + p_4 a_4(\mu) + p_5 a_5(\mu). \end{aligned}$$

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Note that $\gamma\gamma$ with $J = 2$, $\Lambda = \pm 2$ also couples with the EWSBS, following

$$P_{10,0}^{(0)} \propto \alpha s$$

$$P_{12,\pm 2}^{(0)} \propto \alpha$$

- Based on a collaboration with profs. M.J.Herrero and J.J.Sanz-Cillero: JHEP1407 (2014) 149.
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Unitarity for partial waves

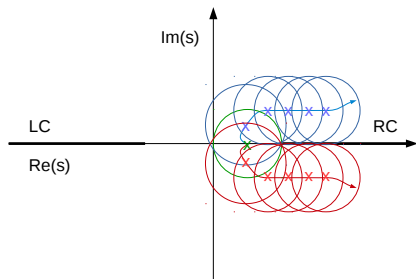
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 $SS^\dagger = \mathbb{1}$,
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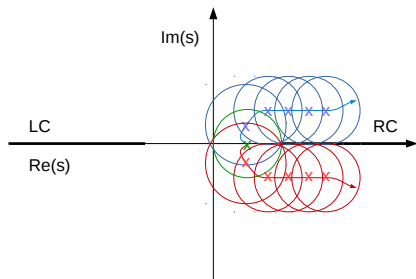


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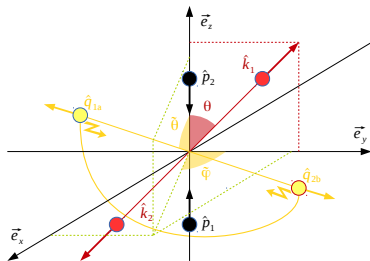


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$$A^{IAM}(s) = \frac{[A^{(0)}(s)]^2}{A^{(0)}(s) - A^{(1)}(s)},$$

$$A^{N/D}(s) = \frac{A^{(0)}(s) + A_L(s)}{1 - \frac{A_R(s)}{A^{(0)}(s)} + \frac{1}{2}g(s)A_L(-s)},$$

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$$g(s) = \frac{1}{\pi} \left(\frac{B(\mu)}{D + E} + \log \frac{-s}{\mu^2} \right)$$

$$A_L(s) = \pi g(-s) D s^2$$

$$A_R(s) = \pi g(s) E s^2$$

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PRD **91** (2015) 075017

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$$g(s) = \frac{1}{\pi} \left(\frac{B(\mu)}{D + E} + \log \frac{-s}{\mu^2} \right)$$

$$A_L(s) = \pi g(-s) D s^2$$

$$A_R(s) = \pi g(s) E s^2$$

where

PRD **91** (2015) 075017

Unitarization procedures for elastic processes

$$A^{IAM}(s) = \frac{[A^{(0)}(s)]^2}{A^{(0)}(s) - A^{(1)}(s)},$$

$$A^{N/D}(s) = \frac{A^{(0)}(s) + A_L(s)}{1 - \frac{A_R(s)}{A^{(0)}(s)} + \frac{1}{2}g(s)A_L(-s)},$$

$$A^{IK}(s) = \frac{A^{(0)}(s) + A_L(s)}{1 - \frac{A_R(s)}{A^{(0)}(s)} + g(s)A_L(s)},$$

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where

PRD **91** (2015) 075017

Matricial versions of the methods

$$F^{IAM}(s) = \left[F^{(0)}(s) \right]^{-1} \cdot \left[F^{(0)}(s) - F^{(1)}(s) \right] \cdot \left[F^{(0)}(s) \right]^{-1},$$

$$F^{N/D}(s) = \left[1 - F_R(s) \cdot \left(F^{(0)}(s) \right)^{-1} + \frac{1}{2} G(s) F_L(-s) \right]^{-1} \cdot N_0(s),$$

$$F^{IK}(s) = \left[1 + G(s) \cdot N_0(s) \right]^{-1} \cdot N_0(s),$$

where $G(s)$, $F_L(s)$, $F_R(s)$ and $N_0(s)$ are defined as

$$G(s) = \frac{1}{\pi} \left(B(\mu)(D + E)^{-1} + \log \frac{-s}{\mu^2} \right)$$

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$$N_0(s) = F^{(0)}(s) + F_L(s)$$

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Extension to $\gamma\gamma$ and $t\bar{t}$ scattering

Basic assumption

- EWSBS is strongly interacting. $\gamma\gamma$ and $t\bar{t}$ are perturbative.
- Coupling with photons, controlled by $\alpha = e^2/4\pi \ll s/v^2$.
- Coupling with top quarks, controlled by $M_t\sqrt{s}/v^2 \ll s/v^2$.

Perturbative unitarization: $\omega\omega \rightarrow \{\gamma\gamma, t\bar{t}\}$

$$\tilde{P} = \frac{\tilde{A}_{IJ}}{A_{IJ}^{(0)}} P^{(0)}$$

Perturbative unitarization: $\{\omega\omega, hh\} \rightarrow \{\gamma\gamma, t\bar{t}\}$

$$\begin{pmatrix} \tilde{P} \\ \tilde{R} \end{pmatrix} = \tilde{F} \left(F^{(0)} \right)^{-1} \begin{pmatrix} P^{(0)} \\ R^{(0)} \end{pmatrix}$$

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Usability channel of unitarization procedures

IJ	00	02	11	20	22
Method of choice	Any	N/D IK	IAM	Any	N/D IK

- The IAM method cannot be used when $A^{(0)} = 0$, because it would give a vanishing value.
- The N/D and the IK methods cannot be used if $D + E = 0$, because in this case computing $A_L(s)$ and $A_R(s)$ is not possible.
- The naive K-matrix method,

$$A_0^K(s) = \frac{A_0(s)}{1 - iA_0(s)},$$

fails because it is not analytical in the first Riemann sheet and, consequently, it is not a proper partial wave compatible with microcausality.

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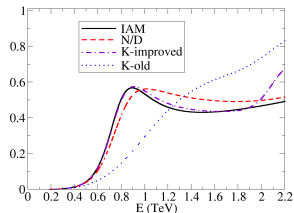
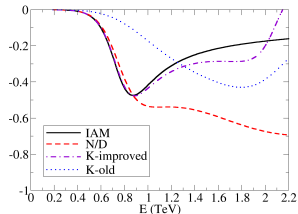
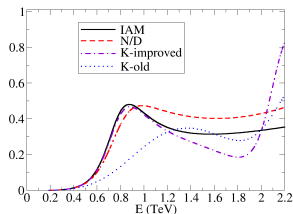
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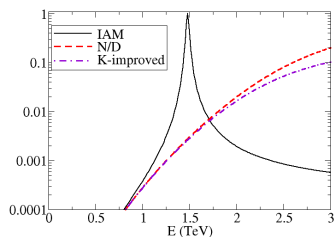
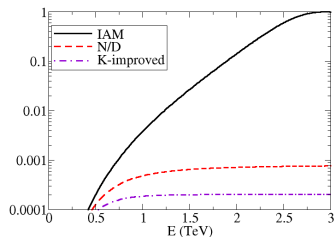
Scalar-isoscalar channels



From left to right and top to bottom, elastic $\omega\omega$, elastic hh , and cross channel $\omega\omega \rightarrow hh$, for $a = 0.88$, $b = 3$, $\mu = 3$ TeV and all NLO parameters set to 0.

PRL **114** (2015) 221803, PRD **91** (2015) 075017.

Vector-isovector channels

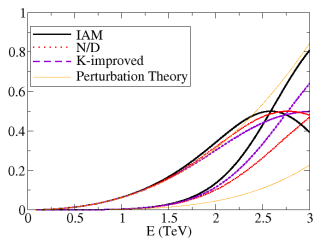
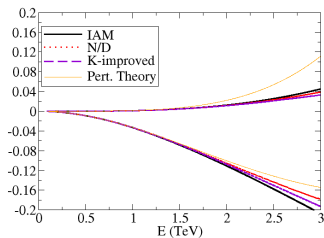


From our ref¹⁶. We have taken $a = 0.88$ and $b = 1.5$, but while for the left plot all the NLO parameters vanish, for the right plot we have taken $a_4 = 0.003$, known to yield an IAM resonance according to the Barcelona group¹⁷.

¹⁶PRD **91** (2015) 075017

¹⁷PRD **90** (2014) 015035

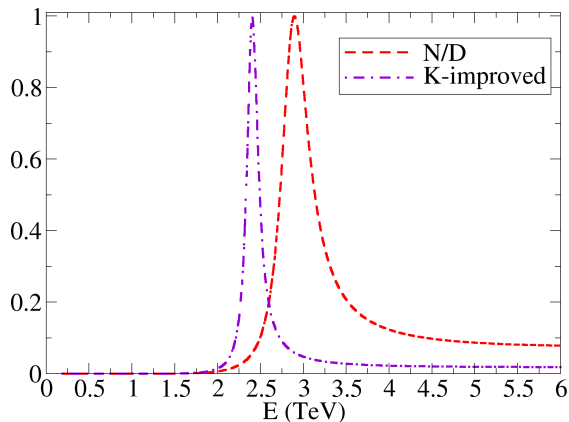
Scalar-isotensor channels ($IJ = 20$)



From our ref¹⁸. From left to right, $a = 0.88$, $a = 1.15$. We have taken $b = a^2$ and the NLO parameters set to zero. Both real and imaginary part shown. Real ones correspond to bottom lines at left and upper at low E at right.

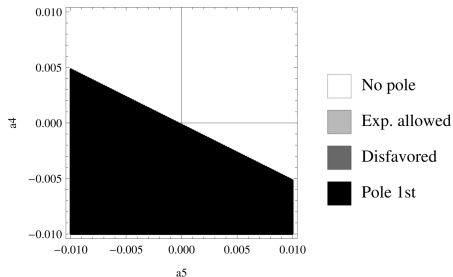
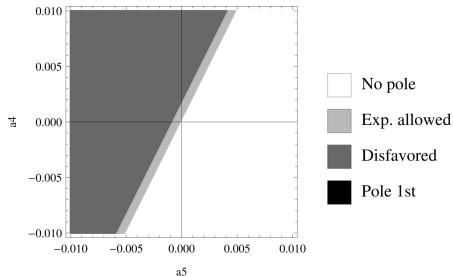
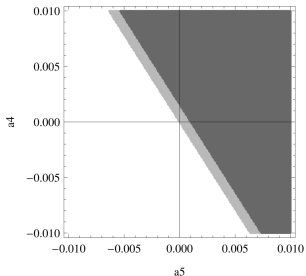
¹⁸PRD **91** (2015) 075017

Isotensor-scalar channels ($IJ = 02$)



$a = 0.88$, $b = a^2$, $a_4 = -2a_5 = 3/(192\pi)$, all the other NLO param. set to zero.
PRD **91** (2015) 075017.

Reson. in $W_L W_L \rightarrow W_L W_L$ due to a_4 and a_5 , ours



- $a = 0.90$, $b = a^2$
PRD **91** (2015) 075017
- From left, clockwise,
 $J = 00, 11, 20$
- Excluding resonances
 $M_S < 700 \text{ GeV}$, $M_V < 1.5 \text{ TeV}$

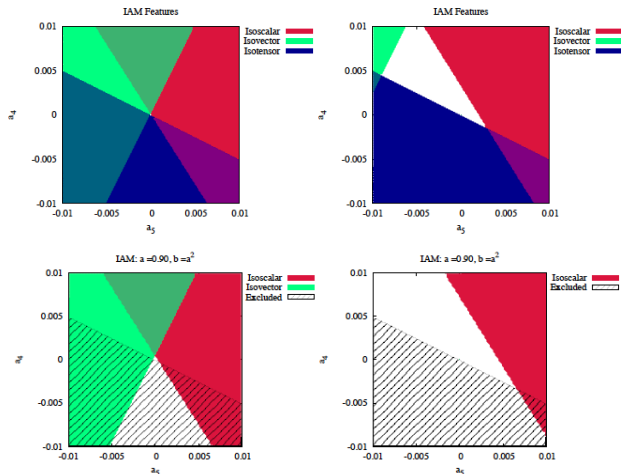
Reson. in $W_L W_L \rightarrow W_L W_L$ due to a_4 and a_5 , Barcelona

CROSS-CHECK:
Espriu, Yencho,
Mescia

PRD**88**, 055002

PRD**90**, 015035

At right, exclusion
regions include reso-
nances with
 $M_{S,V} < 600$ GeV.

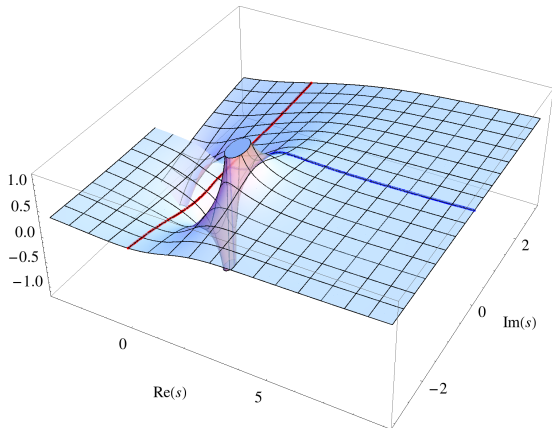


Resonance from $W_L W_L \rightarrow hh$

$a = 1$, $b = 2$, IAM,
elastic chann. $W_L W_L \rightarrow W_L W_L$,
red figure from 3D-printer

Rafael L. Delgado,
Antonio Dobado,
Felipe J. Llanes-Estrada,
*Possible New Resonance from
 $W_L W_L$ - hh Interchannel
Coupling,*

PRL **114** (2015) 221803

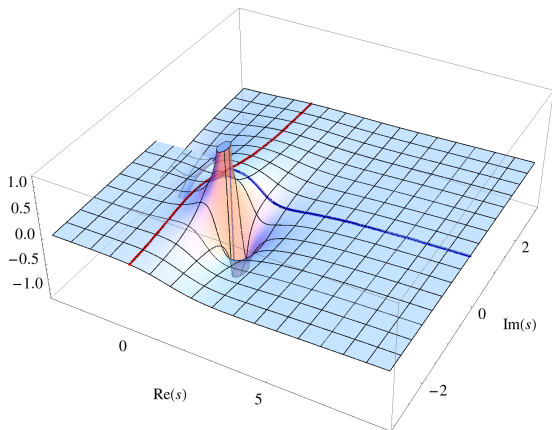


Resonance from $W_L W_L \rightarrow hh$

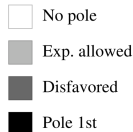
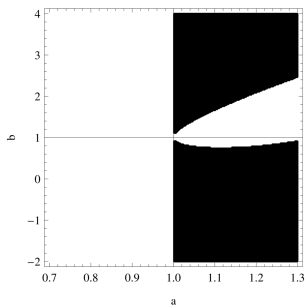
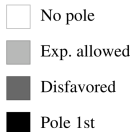
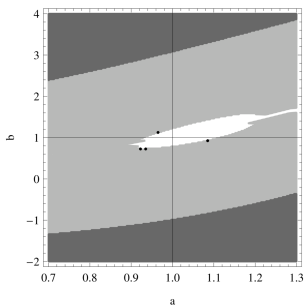
$a = 1$, $b = 2$, IAM,
inelastic chann. $W_L W_L \rightarrow hh$,
yellow figure from 3D-printer

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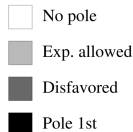
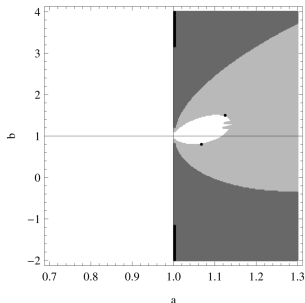
PRL **114** (2015) 221803



Resonances in $W_L W_L \rightarrow W_L W_L$ due to a and b parameters



- PRL & PRD **91** (2015) 075017
- From left, clockwise,
 $IJ = 00, 11, 20$
- Excluding resonances
 $M_S < 700 \text{ GeV}$, $M_V < 1.5 \text{ TeV}$
- Constraint over b even without data about $W_L W_L \rightarrow hh$ and $hh \rightarrow hh$ scattering processes.

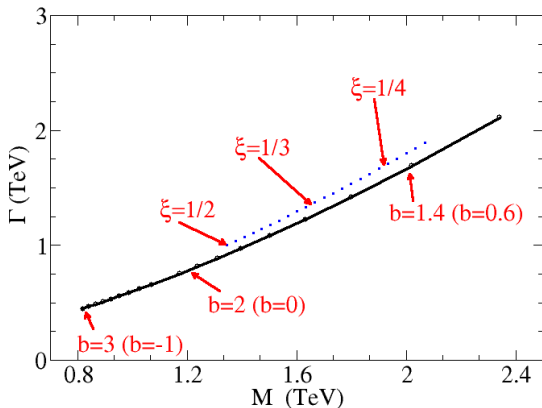


Motion of the resonance mass and width

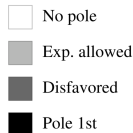
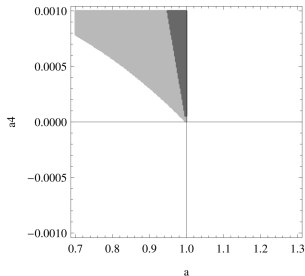
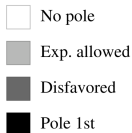
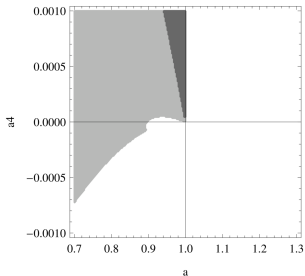
Dependence on b with $a^2 = 1$ fixed (upper curve) and for $a = 1 - \xi$ and $b = 1 - 2\xi$ with $\xi = v/f$ as in the MCHM (lower blue curve).

PRL **114** (2015) 221803

Video,
(a,b) param. space



Resonances in $W_L W_L \rightarrow W_L W_L$ due to a and a_4 parameters

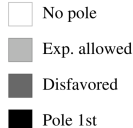
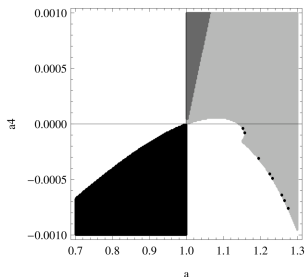


- $b = a^2$

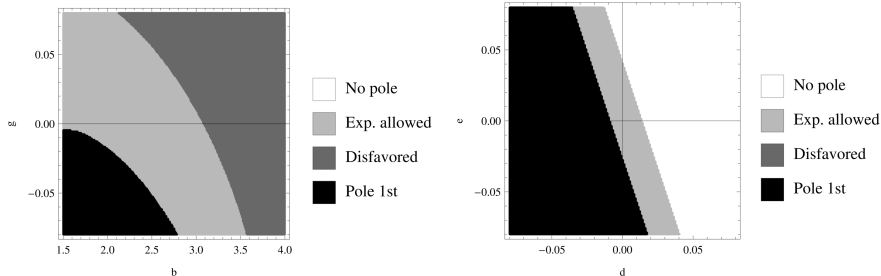
PRD **91** (2015) 075017

- From left, clockwise,
 $J = 00, 11, 20$

- Excluding resonances
 $M_S < 700 \text{ GeV}, M_V < 1.5 \text{ TeV}$



Resonances in $W_L W_L \rightarrow W_L W_L$ due to b, g, d and e parameters



Effective Theory, PRD **91** (2015) 075017, isoscalar channels ($I = J = 0$).

I) IAM method

This method needs a NLO computation,

$$\tilde{t}^\omega = \frac{t_0^\omega}{1 - \frac{t_1^\omega}{t_0^\omega}},$$

where

$$t_1^\omega = s^2 \left(D \log \left[\frac{s}{\mu^2} \right] + E \log \left[\frac{-s}{\mu^2} \right] + (D + E) \log \left[\frac{\mu^2}{\mu_0^2} \right] \right)$$

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Check at tree level

We have checked¹⁹, for the tree level case,

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}g(\varphi/f)\partial_\mu\omega^a\partial^\mu\omega^b\left(\delta_{ab} + \frac{\omega^a\omega^b}{v^2 - \omega^2}\right) \\ &\quad + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}M_\varphi^2\varphi^2 - \lambda_3\varphi^3 - \lambda_4\varphi^4 + \dots \\ g(\varphi/f) &= 1 + \sum_{n=1}^{\infty} g_n \left(\frac{\varphi}{f}\right)^n = 1 + 2\alpha\frac{\varphi}{f} + \beta\left(\frac{\varphi}{f}\right)^2 + \dots\end{aligned}$$

where $a \equiv \alpha v/f$, $b = \beta v^2/f^2$, and so on, the concordance with the methods

¹⁹See J.Phys. G41 (2014) 025002.

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$$\begin{aligned}\mathcal{L} &= \frac{1}{2}g(\varphi/f)\partial_\mu\omega^a\partial^\mu\omega^b\left(\delta_{ab} + \frac{\omega^a\omega^b}{v^2 - \omega^2}\right) \\ &\quad + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}M_\varphi^2\varphi^2 - \lambda_3\varphi^3 - \lambda_4\varphi^4 + \dots \\ g(\varphi/f) &= 1 + \sum_{n=1}^{\infty}g_n\left(\frac{\varphi}{f}\right)^n = 1 + 2\alpha\frac{\varphi}{f} + \beta\left(\frac{\varphi}{f}\right)^2 + \dots\end{aligned}$$

where $a \equiv \alpha v/f$, $b = \beta v^2/f^2$, and so on, the concordance with the methods

¹⁹See J.Phys. G41 (2014) 025002.

II) K matrix

$$\tilde{T} = T(1 - J(s)T)^{-1}, \quad , J(s) = -\frac{1}{\pi} \log \left[\frac{-s}{\Lambda^2} \right],$$

so that, for \tilde{t}_ω ,

$$\tilde{t}_\omega = \frac{t_\omega - J(t_\omega t_\varphi - t_{\omega\varphi}^2)}{1 - J(t_\omega + t_\varphi) + J^2(t_\omega t_\varphi - t_{\omega\varphi}^2)},$$

for $\beta = \alpha^2$ (elastic case),

$$\tilde{t}_\omega = \frac{t_\omega}{1 - Jt_\omega}$$

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III) Large N

$N \rightarrow \infty$, with v^2/N fixed. The amplitude A_N to order $1/N$ is a Lippmann-Schwinger series,

$$A_N = A - A \frac{N!}{2} A + A \frac{N!}{2} A \frac{N!}{2} A - \dots$$

$$I(s) = \int \frac{d^4 q}{(2\pi)^4} \frac{i}{q^2(q+p)^2} = \frac{1}{16\pi^2} \log \left[\frac{-s}{\Lambda^2} \right] = -\frac{1}{8\pi} J(s)$$

Note: actually, $N = 3$. For the (iso)scalar partial wave (chiral limit, $I = J = 0$),

$$t_N^\omega(s) = \frac{t_0^\omega}{1 - J t_0^\omega}$$

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(elastic scattering at tree level only $\beta = \alpha^2$. See ref. J.Phys. G41 (2014) 025002). Ansatz

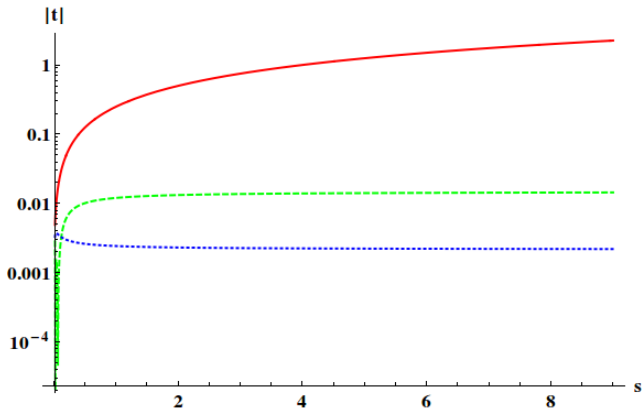
$$\tilde{t}^\omega(s) = \frac{N(s)}{D(s)},$$

where $N(s)$ has a left hand cut (and $\text{Im } N(s > 0) = 0$)
 $D(s)$ has a right hand cut (and $\Im D(s < 0) = 0$);

$$D(s) = 1 - \frac{s}{\pi} \int_0^\infty \frac{ds' N(s')}{s'(s' - s - i\epsilon)}$$

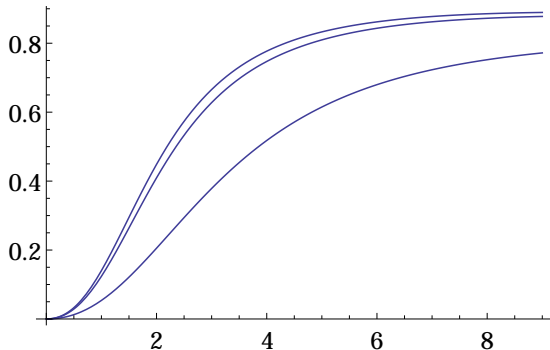
$$N(s) = \frac{s}{\pi} \int_{-\infty}^0 \frac{ds' \text{Im } N(s')}{s'(s' - s - i\epsilon)}$$

Coupled channels, tree level amplitudes



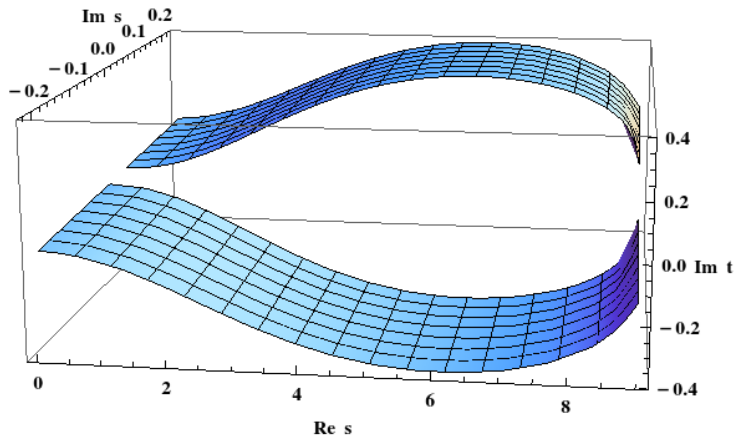
$f = 2v$, $\beta = \alpha^2 = 1$, $\lambda_3 = M_\varphi^2/f$, $\lambda_4 = M_\varphi^2/f^2$. OX axis: s in TeV^2 .

Tree level, modulus of \tilde{t}_ω , K matrix



- All units in TeV.
- From top to bottom, $f = 1.2, 0.8, 0.4$ TeV
- $\Lambda = 3$ TeV
- $\mu = 100$ GeV

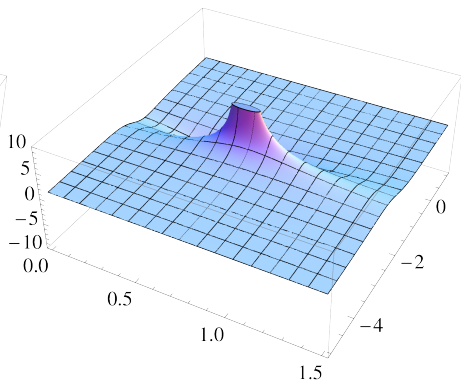
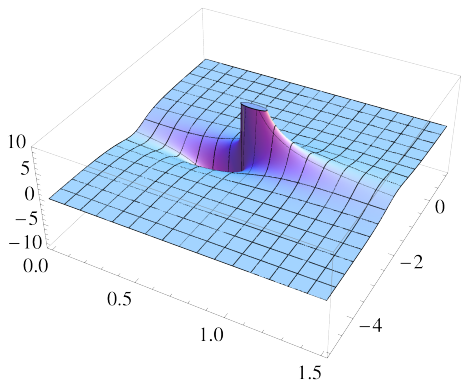
$\text{Im } t_\omega$ in the N/D method,
 $f = 1 \text{ TeV}$, $\beta = 1$, $m = 150 \text{ GeV}$



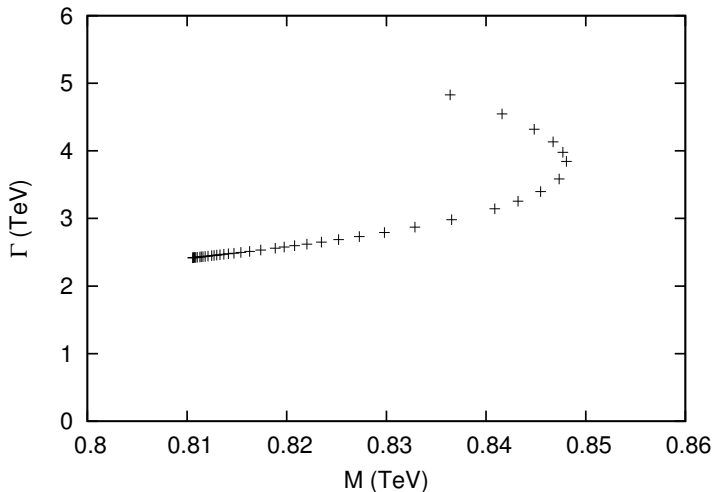
$\text{Re } t_\omega$ and $\text{Im } t_\omega$, large N , $f = 400 \text{ GeV}$



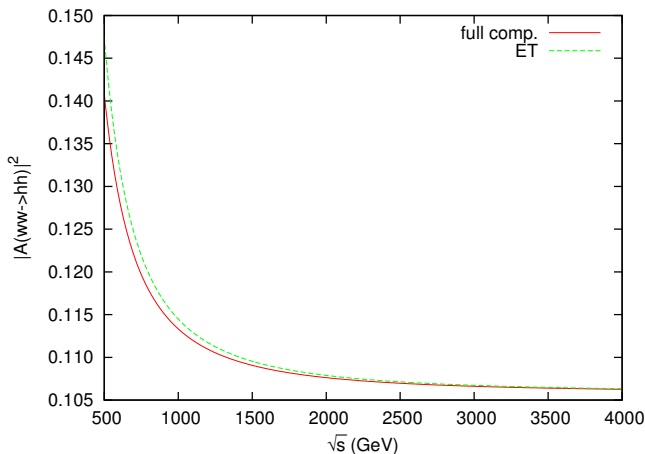
Re t_ω and Im t_ω , large N , $f = 4 \text{ TeV}$



Tree level, motion of the pole position of t_ω
K-matrix, $M_\phi = 125$ GeV, $f \in (250$ GeV, 6 TeV)

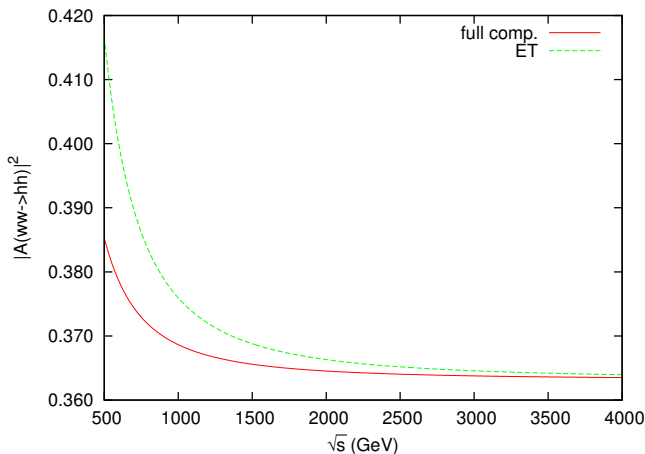


Equivalence Theorem



Comparison between the full LO $\omega\omega \rightarrow hh$ ($\cos\theta = 3$) and that computed through the ET. The SM is used here. Work in collaboration with S. Moretti, to test a modified version of MadGraph.

Equivalence Theorem



Comparison between the full LO $\omega\omega \rightarrow hh$ ($\cos\theta = 6$) and that computed through the ET. The SM is used here. Work in collaboration with S. Moretti, to test a modified version of MadGraph.