

# Dimension-8 Operators in VBS

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Vector-Boson Scattering as considered process

Look for possible deviations from SM

Assumption: new physics heavy  $\rightarrow$  SM + effective vertices

Which vertices?

- $ffV \rightarrow V$  decays
- $ffff \rightarrow$  4-fermion contact interaction
- $VVV \rightarrow$  diboson production
- ...

better sensitivity in other channels with larger cross sections

remaining vertex:

- $VVVV \leftrightarrow$  triboson production

# Motivation

How to generate  $VVVV$  from EFT?

(linear) EFT building blocks

- Higgs doublet field  $\Phi$
- covariant derivative  $D^\mu$ , which reduces to  $\partial^\mu$  for singlet fields
- field strength tensors  $G^{a,\mu\nu}$ ,  $W^{i,\mu\nu}$ ,  $B^{\mu\nu}$
- fermion fields  $\psi$

combinations to get  $V$  (all dimension-2):

- $D^\mu \Phi$
- $W^{i,\mu\nu}$  (also non-abelian part containing  $VV$ )
- $B^{\mu\nu}$

⇒ lowest-order contribution: dimension-6

$$\mathcal{O}_{WWW} = \text{Tr} \left[ \widehat{W}^\mu{}_\nu \widehat{W}^\nu{}_\rho \widehat{W}^\rho{}_\mu \right]$$

$$\mathcal{O}_W = (D_\mu \Phi)^\dagger \widehat{W}^{\mu\nu} (D_\nu \Phi)$$

$$\mathcal{O}_{\widetilde{W}WW} = \text{Tr} \left[ \widetilde{W}^\mu{}_\nu \widehat{W}^\nu{}_\rho \widehat{W}^\rho{}_\mu \right]$$

⇒ lowest-order contribution: dimension-6

	$\mathcal{O}_{WWW}$	$\mathcal{O}_W$	$\mathcal{O}_B$	$\mathcal{O}_{WW}$	$\mathcal{O}_{BB}$	$\mathcal{O}_{\phi,2}$	$\mathcal{O}_{\tilde{W}WW}$	$\mathcal{O}_{\tilde{W}}$	$\mathcal{O}_{\tilde{B}}$	$\mathcal{O}_{\tilde{W}W}$	$\mathcal{O}_{\tilde{B}B}$
$WWZ$	X	X	X				X	X	X		
$WW\gamma$	X	X	X				X	X	X		
$HWW$		X		X		X		X		X	
$HZZ$		X	X	X	X	X		X	X	X	X
$HZ\gamma$		X	X	X	X	(X)		X	X	X	X
$H\gamma\gamma$				X	X	(X)				X	X
$WWWW$	X	X					X				
$WWZZ$	X	X					X				
$WWZ\gamma$	X	X					X				
$WW\gamma\gamma$	X						X				

⇒ also give contributions to  $VVV$  vertices

⇒ test in diboson production

severely constrained (parametric uncertainty relevant? → [task](#))

⇒ look at next order of expansion

→ dimension-8

dimension-6 contribution could be loop-induced

→ additional factor  $\frac{1}{16\pi^2} \sim \frac{1}{160}$

if dimension-8 tree-level → similar size of effects possible

[Arzt, Einhorn, Wudka]

## Dimension-8

Bosonic dimension-8 operators

[Eboli, Gonzalez-Garcia]

$$\mathcal{O}_{S,0} = \left[ (D_\mu \Phi)^\dagger D_\nu \Phi \right] \times \left[ (D^\mu \Phi)^\dagger D^\nu \Phi \right]$$

$$\mathcal{O}_{S,1} = \left[ (D_\mu \Phi)^\dagger D^\mu \Phi \right] \times \left[ (D_\nu \Phi)^\dagger D^\nu \Phi \right]$$

$$\mathcal{O}_{S,2} = \left[ (D_\mu \Phi)^\dagger D_\nu \Phi \right] \times \left[ (D^\nu \Phi)^\dagger D^\mu \Phi \right]$$

$$\mathcal{O}_{M,0} = \text{Tr} \left[ \widehat{W}_{\mu\nu} \widehat{W}^{\mu\nu} \right] \times \left[ (D_\beta \Phi)^\dagger D^\beta \Phi \right]$$

$$\mathcal{O}_{M,1} = \text{Tr} \left[ \widehat{W}_{\mu\nu} \widehat{W}^{\nu\beta} \right] \times \left[ (D_\beta \Phi)^\dagger D^\mu \Phi \right]$$

$$\mathcal{O}_{M,2} = \left[ \widehat{B}_{\mu\nu} \widehat{B}^{\mu\nu} \right] \times \left[ (D_\beta \Phi)^\dagger D^\beta \Phi \right]$$

$$\mathcal{O}_{M,3} = \left[ \widehat{B}_{\mu\nu} \widehat{B}^{\nu\beta} \right] \times \left[ (D_\beta \Phi)^\dagger D^\mu \Phi \right]$$

$$\mathcal{O}_{M,4} = \left[ (D_\mu \Phi)^\dagger \widehat{W}_{\beta\nu} D^\mu \Phi \right] \times \widehat{B}^{\beta\nu}$$

$$\mathcal{O}_{M,5} = \left[ (D_\mu \Phi)^\dagger \widehat{W}_{\beta\nu} D^\nu \Phi \right] \times \widehat{B}^{\beta\mu}$$

$$\mathcal{O}_{M,7} = \left[ (D_\mu \Phi)^\dagger \widehat{W}_{\beta\nu} \widehat{W}^{\beta\mu} D^\nu \Phi \right]$$

$$\mathcal{O}_{T,0} = \text{Tr} \left[ \widehat{W}_{\mu\nu} \widehat{W}^{\mu\nu} \right] \times \text{Tr} \left[ \widehat{W}_{\alpha\beta} \widehat{W}^{\alpha\beta} \right]$$

$$\mathcal{O}_{T,1} = \text{Tr} \left[ \widehat{W}_{\alpha\nu} \widehat{W}^{\mu\beta} \right] \times \text{Tr} \left[ \widehat{W}_{\mu\beta} \widehat{W}^{\alpha\nu} \right]$$

$$\mathcal{O}_{T,2} = \text{Tr} \left[ \widehat{W}_{\alpha\mu} \widehat{W}^{\mu\beta} \right] \times \text{Tr} \left[ \widehat{W}_{\beta\nu} \widehat{W}^{\nu\alpha} \right]$$

$$\mathcal{O}_{T,5} = \text{Tr} \left[ \widehat{W}_{\mu\nu} \widehat{W}^{\mu\nu} \right] \times \widehat{B}_{\alpha\beta} \widehat{B}^{\alpha\beta}$$

$$\mathcal{O}_{T,6} = \text{Tr} \left[ \widehat{W}_{\alpha\nu} \widehat{W}^{\mu\beta} \right] \times \widehat{B}_{\mu\beta} \widehat{B}^{\alpha\nu}$$

$$\mathcal{O}_{T,7} = \text{Tr} \left[ \widehat{W}_{\alpha\mu} \widehat{W}^{\mu\beta} \right] \times \widehat{B}_{\beta\nu} \widehat{B}^{\nu\alpha}$$

$$\mathcal{O}_{T,8} = \widehat{B}_{\mu\nu} \widehat{B}^{\mu\nu} \widehat{B}_{\alpha\beta} \widehat{B}^{\alpha\beta}$$

$$\mathcal{O}_{T,9} = \widehat{B}_{\alpha\mu} \widehat{B}^{\mu\beta} \widehat{B}_{\beta\nu} \widehat{B}^{\nu\alpha}$$

Hiccups of the original version:

vanish identically:

$$\mathcal{O}_{T,3} = \text{Tr} \left[ \widehat{W}_{\alpha\mu} \widehat{W}^{\mu\beta} \widehat{W}^{\nu\alpha} \right] \times \widehat{B}_{\beta\nu}$$

$$\mathcal{O}_{T,4} = \text{Tr} \left[ \widehat{W}_{\alpha\mu} \widehat{W}^{\alpha\mu} \widehat{W}^{\beta\nu} \right] \times \widehat{B}_{\beta\nu}$$

redundant:

$$\begin{aligned} \mathcal{O}_{M,6} &= \left[ (D_\mu \Phi)^\dagger \widehat{W}_{\beta\nu} \widehat{W}^{\beta\nu} D^\mu \Phi \right] \\ &= \frac{1}{2} \mathcal{O}_{M,0} \end{aligned}$$

missing:

$$\mathcal{O}_{S,2} = \left[ (D_\mu \Phi)^\dagger D_\nu \Phi \right] \times \left[ (D^\nu \Phi)^\dagger D^\mu \Phi \right]$$

Contribution to the different vertices:

	$\mathcal{O}_{S,0}$	$\mathcal{O}_{M,0}$	$\mathcal{O}_{M,2}$	$\mathcal{O}_{T,0}$	$\mathcal{O}_{T,5}$	
	$\mathcal{O}_{S,1}$	$\mathcal{O}_{M,1}$	$\mathcal{O}_{M,3}$	$\mathcal{O}_{T,1}$	$\mathcal{O}_{T,6}$	$\mathcal{O}_{T,8}$
	$\mathcal{O}_{S,2}$	$\mathcal{O}_{M,7}$	$\mathcal{O}_{M,4}$	$\mathcal{O}_{T,2}$	$\mathcal{O}_{T,7}$	$\mathcal{O}_{T,9}$
			$\mathcal{O}_{M,5}$			
WWWW	X	X		X		
WWZZ	X	X	X	X	X	
ZZZZ	X	X	X	X	X	X
WWZ $\gamma$		X	X	X	X	
WW $\gamma\gamma$		X	X	X	X	
ZZZ $\gamma$		X	X	X	X	X
ZZ $\gamma\gamma$		X	X	X	X	X
Z $\gamma\gamma\gamma$				X	X	X
$\gamma\gamma\gamma\gamma$				X	X	X

# Defining the field-strength tensors

For dimension-6 operators, typically field-strength tensors slightly modified

$$\widehat{W}^{\mu\nu} = ig \frac{\sigma^j}{2} W^{j,\mu\nu} = ig \frac{\sigma^j}{2} \left( \partial^\mu W^{j,\nu} - \partial^\nu W^{j,\mu} - g \epsilon^{jkl} W^{k,\mu} W^{l,\nu} \right)$$

$$\widehat{B}^{\mu\nu} = ig' \frac{1}{2} B^{\mu\nu} = ig' \frac{1}{2} (\partial^\mu B^\nu - \partial^\nu B^\mu)$$

such that

$$[D^\mu, D^\nu] = \widehat{W}^{\mu\nu} + \widehat{B}^{\mu\nu}$$

→ treated on equal footing with covariant derivative  
(used in VBFNLO for both dim-6 and dim-8)

[Eboli, Gonzalez-Garcia] (also MadGraph/UFO implementation):

$$\widehat{W}^{\text{EGM},\mu\nu} = \frac{\sigma^j}{2} W^{j,\mu\nu} = \frac{\sigma^j}{2} \left( \partial^\mu W^{j,\nu} - \partial^\nu W^{j,\mu} - g \epsilon^{jkl} W^{k,\mu} W^{l,\nu} \right)$$

$$\widehat{B}^{\text{EGM},\mu\nu} = B^{\mu\nu} = (\partial^\mu B^\nu - \partial^\nu B^\mu)$$

resulting in

$$\widehat{W}^{\text{EGM},\mu\nu} = \frac{1}{ig} \widehat{W}^{\mu\nu}$$

$$\widehat{B}^{\text{EGM},\mu\nu} = \frac{2}{ig'} \widehat{B}^{\mu\nu}$$



Also possible to use non-linear EFT (electroweak chiral Lagrangian)

$$\mathcal{L}_4 = \alpha_4 (\text{Tr} [V_\mu V_\nu])^2 ,$$

$$\mathcal{L}_5 = \alpha_5 (\text{Tr} [V_\mu V^\mu])^2 ,$$

$$\mathcal{L}_{S,0} = F_{S,0} \text{Tr} [(D_\mu \hat{H})^\dagger D_\nu \hat{H}] \times \text{Tr} [(D^\mu \hat{H})^\dagger D^\nu \hat{H}] ,$$

$$\mathcal{L}_{S,1} = F_{S,1} \text{Tr} [(D_\mu \hat{H})^\dagger D^\mu \hat{H}] \times \text{Tr} [(D_\nu \hat{H})^\dagger D^\nu \hat{H}] .$$

with

$$V_\mu = \Sigma (D_\mu \Sigma)^\dagger = - (D_\mu \Sigma) \Sigma^\dagger ,$$

$$D_\mu \Sigma = \partial_\mu \Sigma + ig \frac{\sigma^a}{2} W_\mu^a \Sigma - ig' \Sigma B_\mu \frac{\sigma^3}{2} ,$$

$$\Sigma = \exp \left( -\frac{i}{v} \sigma^a w^a \right) \underset{\text{unitary gauge}}{=} 1$$

$$\hat{H} = \frac{1}{2} \begin{pmatrix} v + H - iw^3 & -i(w^1 - iw^2) \\ -i(w^1 + iw^2) & v + H + iw^3 \end{pmatrix} \underset{\text{unitary gauge}}{=} \frac{v + H}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

→ Whizard

Relations between linear and non-linear EFT

$$\alpha_4 = \frac{v^4}{16} F_{S,0} = \frac{v^4}{16} \frac{f_{S,0} + f_{S,2}}{\Lambda^4}, \quad f_{S,0} = f_{S,2}$$
$$\alpha_5 = \frac{v^4}{16} F_{S,1} = \frac{v^4}{16} \frac{f_{S,1}}{\Lambda^4}$$

Linear-EFT scenarios with  $f_{S,0} \neq f_{S,2}$  need additional operator

$$\mathcal{L}_6 = \alpha_6 \text{Tr}[V_\mu V_\nu] \text{Tr}[TV^\mu] \text{Tr}[TV^\nu]$$

with  $T = \Sigma \sigma_3 \Sigma^\dagger$

isospin-breaking

- parametrization of effects contributing to  $VVVV$  vertex
- effects not constrained by other, "easier" measurements
  - dimension-8  
(possibly leading tree-level contribution of new physics)
  - ↔ impact of dimension-6 operators taken at limits?
- subtle convention differences in implementations
  - unified definitions?
  - (↔ subtle Snowmass 2013 hint [[arXiv:1309.7890](#)] not picked up by experiments)

more detailed write-up: [[MR](#), [arXiv:1610.08420](#)]

# Backup

$$\begin{aligned}\mathcal{O}_{WWW} &= \text{Tr} \left[ \widehat{W}^\mu{}_\nu \widehat{W}^\nu{}_\rho \widehat{W}^\rho{}_\mu \right], \\ \mathcal{O}_W &= (D_\mu \Phi)^\dagger \widehat{W}^{\mu\nu} (D_\nu \Phi), \\ \mathcal{O}_B &= (D_\mu \Phi)^\dagger \widehat{B}^{\mu\nu} (D_\nu \Phi), \\ \mathcal{O}_{WW} &= \Phi^\dagger \widehat{W}_{\mu\nu} \widehat{W}^{\mu\nu} \Phi, \\ \mathcal{O}_{BB} &= \Phi^\dagger \widehat{B}_{\mu\nu} \widehat{B}^{\mu\nu} \Phi, \\ \mathcal{O}_{\phi,2} &= \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi),\end{aligned}$$

$$\begin{aligned}\mathcal{O}_{\widetilde{W}WW} &= \text{Tr} \left[ \widetilde{W}^\mu{}_\nu \widehat{W}^\nu{}_\rho \widehat{W}^\rho{}_\mu \right], \\ \mathcal{O}_{\widetilde{W}} &= (D_\mu \Phi)^\dagger \widetilde{W}^{\mu\nu} (D_\nu \Phi), \\ \mathcal{O}_{\widetilde{B}} &= (D_\mu \Phi)^\dagger \widetilde{B}^{\mu\nu} (D_\nu \Phi), \\ \mathcal{O}_{\widetilde{W}W} &= \Phi^\dagger \widetilde{W}_{\mu\nu} \widehat{W}^{\mu\nu} \Phi, \\ \mathcal{O}_{\widetilde{B}B} &= \Phi^\dagger \widetilde{B}_{\mu\nu} \widehat{B}^{\mu\nu} \Phi.\end{aligned}$$

Can we say something about size of EFT coefficients? [Arzt, Einhorn, Wudka hep-ph/9405214]

- Assumption: high-energy theory (HET) is gauge theory
- keep track of *all* possible operators  $\rightarrow$  a priori no EOM
- statement can be HET-dependent  $\rightarrow$  take maximum contribution

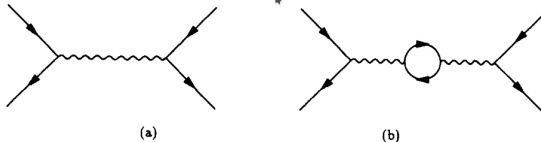


Fig. 1. The contributions of two types of underlying physics to the four-fermion operator. (a) A heavy gauge boson leads to an effective coupling of order one. (b) A heavy fermion comes in only at the one-loop level and leads to an effective coupling of order  $1/16\pi^2$ .

- $\rightarrow$  list of all possible  $\text{dim} \leq 6$  graphs from power-counting
- gauge theory restricts structure of vertices  
e.g. vector interactions: from  $F_a^{\mu\nu} F_{\mu\nu}^a$  terms in  $\mathcal{L}$   
group structure  $\rightarrow$  only AAAA, XXXX, AAXX, AXXX vertices (A SM-V, X NP-V)
- $\rightarrow$  basis splitting into tree-level and loop-induced
- dim-8 operators of VVV can be as large as dim-6

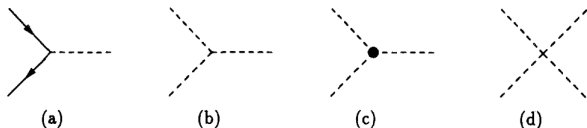


Fig. 2. These are all possible relevant vertices in an unconstrained field theory. Each dashed line may be a scalar or a vector. The dot represents a coupling constant proportional to  $\Lambda$ , the heavy scale.

In summary, the forbidden vertices are

$$\begin{array}{cccc}
 \phi_{\text{li}}\phi_{\text{li}}\phi_{\text{li}}, & \phi_{\text{li}}\phi_{\text{h}}AA, & \phi_{\text{li}}\phi_{\text{h}}A, & \phi_{\text{li}}AX, \\
 \phi_{\text{li}}AA, & \phi_{\text{h}}AA, & \phi_{\text{h}}AX, & \phi_{\text{li}}XX, \\
 AAX, & AAAX, & \psi_{\text{li}}\psi_{\text{h}}A. & 
 \end{array}$$

# Deviation Patterns (3)

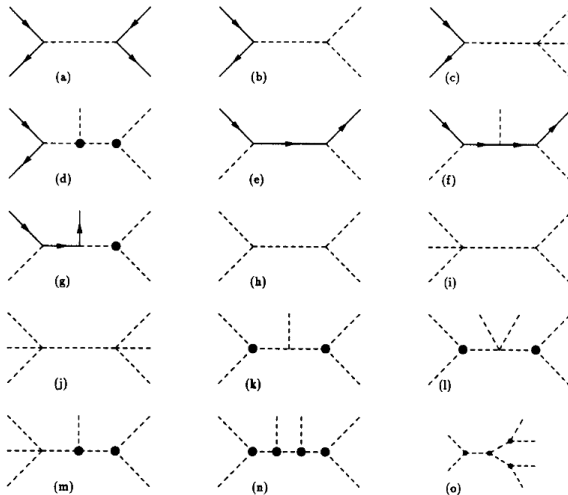


Fig. 3. These are all possible tree graphs which are suppressed by at most  $1/\Lambda^2$ , in a general field theory, with only heavy internal lines and light external lines. The dotted three-boson vertices must have a coupling constant proportional to  $\Lambda$ , the heavy mass scale. Each dashed line may be a scalar or a vector.