SWIX 1?

FIRST CONTACT WITH ANDRÉ
1967 APS MEETING TALK BY ANDRÉ:
I WAS TRANSFIXED BY HIS
(1) PURITY OF THOUGHT
(2) GRANDEUR OF AMBITION
(3) MONUMENTAL ACHIEVEMENTS
OF 1966

FIRST PESONAL INTERACTION 1970 →
ONE OF THE TOUCHING EVENTS:
Letter from André on the stationary
of LAKE UDAIPUR PALACE HOTEL
while holidaying with SCHU:
LOWER BOUNDS ON THE BOSON
STAR HAMILTON

HIGH ENERGY THEOREMS:

MARTIN'S ANALYTICITY UNITARITY PROGRAMME

S.M.R: MARTIN-FEST, CERN
27 AUG 2009

1. Genesis : 1961 FROISSART PROVES:

$$\sigma_{tot} \underset{s\to\infty}{\leq} \text{const.}\,(\ln(s/s_0))^2$$

(FROM MANDELSTAM REPRESENTATION)

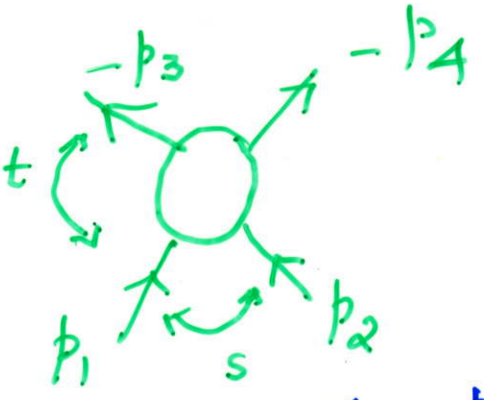
M. FROISSART: 'In the summer of 1960 the last word in elementary particle physics was the Mandelstam Representation'

Mandelstam Representation, an elegant but unproven hypothesis:

$$A(s,t,u) = \frac{s^N t^N}{\pi^2} \iint \frac{\rho(s',t')\,ds'\,dt'}{s'^N t'^N (s'-s)(t'-t)} + P_{s,t,u}$$

$$+ \sum_{p=0}^{M} \frac{t^p s^M}{\pi} \int \frac{\rho_p(s')\,ds'}{s'^M (s'-s)} + P_{s,t,u}$$

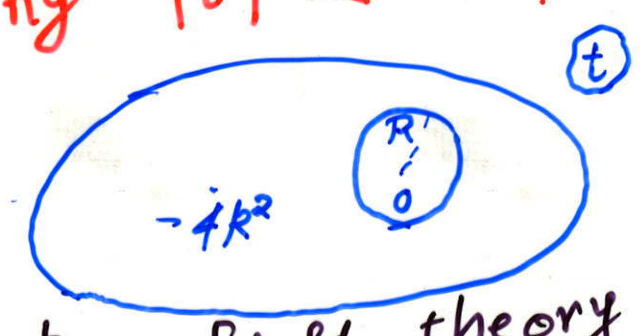
$$+ \sum_{p,q}^{L} t^p s^q \rho_{p,q} \, ,$$



$$s = (p_1 + p_2)^2, \; t = (p_1 + p_3)^2, \; u = (p_1 + p_4)^2$$

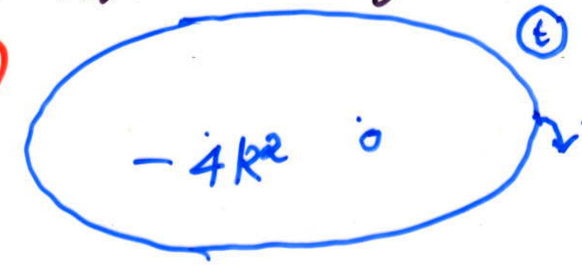
In particular, for elastic scattering it implies analyticity of $A_s(s,t)$ in an ellipse containing $|t| < R$, R indep. of energy

(i)



whereas axiomatic field theory only gives analyticity in the large LEHMANN ELLIPSE (1958)

(ii)



$T_2(s) \sim \frac{C}{s}$ for $s \to \infty$.

2. 1961-62 : H. LEHMANN and A. MARTIN ; O.W. GREENBERG & F.E. LOW wanted to prove high energy bounds from first principles (Axiomatic Field Theory, LSZ, WIGHTMAN, JAFFE, ARAKI, ...)

Greenberg-Low (1961): $\sigma_{tot} < s(\ln s/s_0)^2$ const

MARTIN (1962): Not full Mandelstam analyticity, but just analyticity of $A_s(s,t)$ in $|t| < R$ $\Rightarrow$ $\sigma_{tot} <$ Const$(\ln(\frac{s}{s_0}))^2$

FROISSART: 'A very elegant derivation'

3. 1966 : A. MARTIN PROVES

(i) LEHMANN-MARTIN ELLIPSE OF ANALYTICITY OF $A_s(s,t)$ CONTAINING CIRCLE $|t| < R$, R indep. of s.

$R = 4m_\pi^2$: $\pi\pi$, KK, $K\bar{K}$, πK, πN, $\pi\Lambda$ Scatt. (Sommer 1967; Bessis & Glaser 1967)

AND HENCE :

(ii) FROISSART - MARTIN BOUND:

$$\sigma_{tot} \underset{s\to\infty}{\leq} C\,(\ln(s/s_0))^2,$$ 'from first principles'

H. LEHMANN SENT A POSTCARD TO CONGRATULATE ANDRÉ.

A JIN-MARTIN THM. (1964) THEN GIVES VALIDITY OF DISP. RELNS. WITH AT MOST TWO SUBTRACTIONS FOR $|t| < R$

THIS FIXES THE UNKNOWN CONST. MULTIPLYING $(\ln s/s_0)^2$, [ŁUKASZUK - MARTIN 1967],

$$\sigma_{tot}(s) \underset{s\to\infty}{<} \frac{\pi}{m_\pi^2}\left(\ln(s/s_0)\right)^2$$

ACTUALLY, STRICTLY SPEAKING, MARTIN PROVIDES NOT A LOCAL BOUND ON σ_{tot} AT EACH ENERGY, BUT A BOUND ON ENERGY AVERAGES:

$$\overline{\sigma_{tot}}(s) = \frac{1}{s}\int_s^{2s} \sigma_{tot}(s')\,ds'$$

$$< \frac{\pi}{m_\pi^2}\ln^2(s/s_0) + A\ln(s/s_0) + B$$

RIGOROUS FORM OF FROISSART - MARTIN BOUND (Yndurain, 1970, Common, 1971, Martin 1985, 2009)

4. COMPARISON WITH 1960'S PHENOMENOLOGY.

Corner-stone of G. F. CHEW's (1961) resurrection of S-Matrix approach of Heisenberg (1943): CHEW: 'the S-matrix is a Lorentz-invariant analytic function of all momentum variables with only those singularities required by unitarity'

In Practice: Mandelstam Representation:

First problems: (Pre-Regge (1960) deadlock)

(i) Spin j stable particle, $A \sim \frac{P_j(\cos\theta)}{s-m^2}$ requires $M \geq j$; for high j, $\sigma_{tot}(s) \sim$ increasing polynomially with s?

Mandelstam Repres. $\Rightarrow$ FROISSART: $\sigma_{tot} < C.(\ln s)^2$

(ii) ~~2/~~ Gribov's paradox (1960)

Expts consistent with Pomeranchuk's model for high energy elastic scattering (purely absorptive diffraction scatt.):

$$A(s,t) \sim \cancel{i}\, is\, f(t), \quad \text{fixed-}t, \; s \to \infty$$

$$\therefore \rho(s,t) = s\, \mathrm{Im}\, f(t)$$

Mandelstam Continued elastic Unitarity, $4 < t < 16$

$$\Rightarrow \rho(s,t) \sim s \ln s\, |f(t)|^2 \,, \quad \text{PARADOX}$$

Gribov's Paradox also occurs if

$$A(s,t) \sim i\, s^{\alpha} (\ln s)^{\beta} f(t), \quad \alpha \text{ real}, \ \mathrm{Re}\,\beta > -1$$

$$a_J(t) \sim (J-\alpha)^{-(\beta+1)}$$

VIOLATES CONTD. UNITARITY IF α REAL, $\mathrm{Re}\,\beta > -1$

GRIBOV'S SUGGESTION: $\alpha = 1$, $\beta < -1$ (DECREASING CROSS SECTIONS)

CHEW'S SUGGESTION: $\alpha(t)$ COMPLEX, $\beta = 0$ (REGGE POLES)

Problems with Mandelstam Representation disappear if $\mathrm{Re}\,\alpha(t)$ Bounded in the cut plane

(iii) VENEZIANO FORMULA (1968)

$$F(s,t,u) = \beta \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))} + \{\text{Cyclic Perms. of } s,t,u\},$$

$$\alpha(s) = a + b s$$

Of deep significance for dual models and later for string theory.

INFINITELY RISING TRAJECTORIES $\alpha(s)$ WOULD NOT ALLOW MANDELSTAM REPRESN. WITH FINITE NO. OF SUBTRACTIONS

MARTIN'S PROOF OF $\sigma_{tot} < C(\ln s)^2$, THE ONLY ONE THAT SURVIVES.

5. RIGOROUS ANALYTICITY PROPERTIES PRE-MARTIN(1966) AND POST-MARTIN(1966):

1954: Gell-Mann - Goldberger - Thirring: Dispersion Relations Compton Scattering

Dispersion Relns. Massive Particles (e.g. πN):

[Goldberger (1955), Bogoliubov et al (1958), Symanzik (1957), Bremermann et al (1958), Lehmann (1959), ...]

proved from LSZ local field theory for $\pi K \to \pi K$, $\pi N \to \pi N$, $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$, $KK \to KK$, $\pi\Lambda \to \pi\Lambda$, $\pi\Sigma \to \pi\Sigma$

For $-t_1 \leq t \leq 0$ ($t_1 = 28\, m_\pi^2$ for $\pi\pi$, $12.4\, m_\pi^2$ for πN):

$$F(s,t) = \sum_{n=0}^{N-1} c_n(t)\, s^n + \frac{s^N}{\pi}\int_{s_{thr.}}^{\infty} \frac{A_s(s',t)\, ds'}{s'^N (s'-s)}$$

$$+ \frac{u^N}{\pi}\int_{u_{thr}}^{\infty} \frac{A_u(u',t)\, du'}{u'^N (u'-u)}$$

where the absorptive parts are continued outside the physical region $-1 \leq \cos\theta \leq 1$ using analyticity of $A_s(s; \cos\theta)$ in the Lehmann-ellipse.

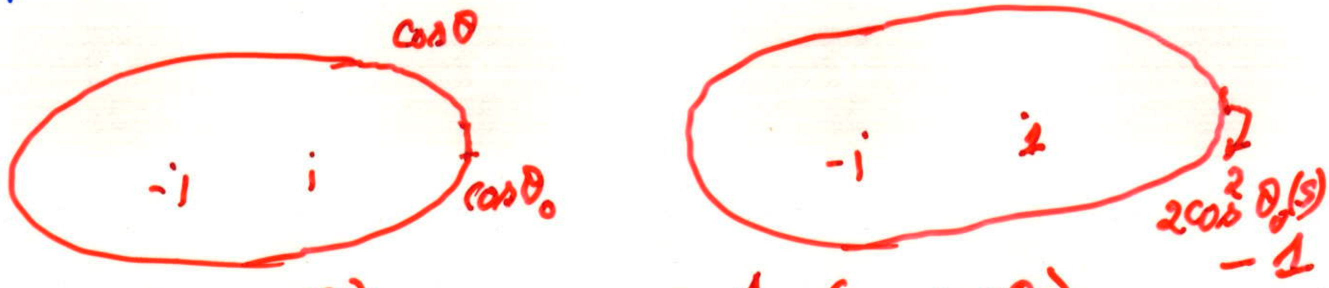
LEHMANN ELLIPSES (1958, 1959)

$F(s;\cos\theta)$ ANALYTIC INSIDE AN ELLIPSE IN COMPLEX $\cos\theta$- PLANE WITH FOCI $-1, +1$ AND SEMI- MAJOR AXIS

$$\cos\theta_0(s) = \left[1 + \frac{(M_A'^2 - M_A^2)(M_B'^2 - M_B^2)}{k^2\left(s - (M_A' - M_B')^2\right)}\right]^{1/2}$$

FOR $A + B \to A + B$, M_A, M_B PARTICLE MASSES, M_A', M_B' LOWEST MASS STATES SUCH THAT $\langle B' | j_B(0) | 0\rangle \neq 0$. $\langle A' | j_A(0) | 0\rangle \neq 0$,

For $A_s(s;\cos\theta)$ THE SEMI-MAJOR AXIS OF THE ELLIPSE OF ANALYTICITY IS $2\cos^2\theta_0(s) - 1$.



$F(s;\cos\theta)$ $\qquad$ $A_s(s;\cos\theta)$

CROSSING FOR $A + B \to C + D$:

Bros, Epstein, Glaser (1964, 1965):

$$F^{AB\to CD}(s,t) = F^{A\bar{D}\to C\bar{B}}(u,t)$$

FOR $t \leq 0$, AND $|s|$ LARGE ENOUGH, $F^{AB\to CD}$ IS ANALYTIC IN THE CUT-s PLANE EXCEPT FOR POSSIBLE SINGULARITIES IN A FINITE REGION AND CAN BE CONTINUED TO THE UPPER LIP OF THE LEFT-HAND CUT WHERE IT EQUALS THE COMPLEX CONJUGATE OF THE $A\bar{D} \to C\bar{B}$ PHYSICAL-AMPLITUDE

MARTIN'S ARSENAL : POSITIVITY OF ELASTIC ABSORPTIVE PARTS

$$F(s;\cos\theta) = \frac{\sqrt{s}}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)\,a_\ell(s)\,P_\ell(\cos\theta),$$

$$0 \le |a_\ell|^2 \le \mathrm{Im}\, a_\ell \le 1 \,,$$ IMPLY THAT,

$$\left|\frac{d^n}{d(\cos\theta)^n}\,\mathrm{Im}\,F(s,\cos\theta)\right|_{-1\le\cos\theta\le 1} \le \left|\frac{d^n}{d(\cos\theta)^n}\,\mathrm{Im}\,F(s,\cos\theta)\right|_{\cos\theta=1}$$

for $n = 0, 1, 2, 3, \ldots$

[For Particles with spin, G. MAHOUX (1976):

$$\left(\frac{d\sigma_A}{d\Omega}\right)_{\text{unpolarised, el}} = \sum_{\ell=0}^{\infty}\sigma_\ell(s)\,P_\ell(\cos\theta),$$

(absorptive part contribution)

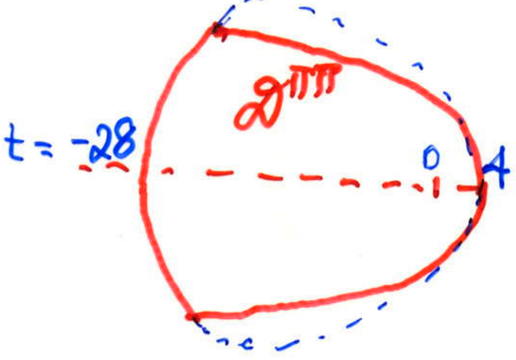
$$\sigma_\ell \geqslant 0$$]

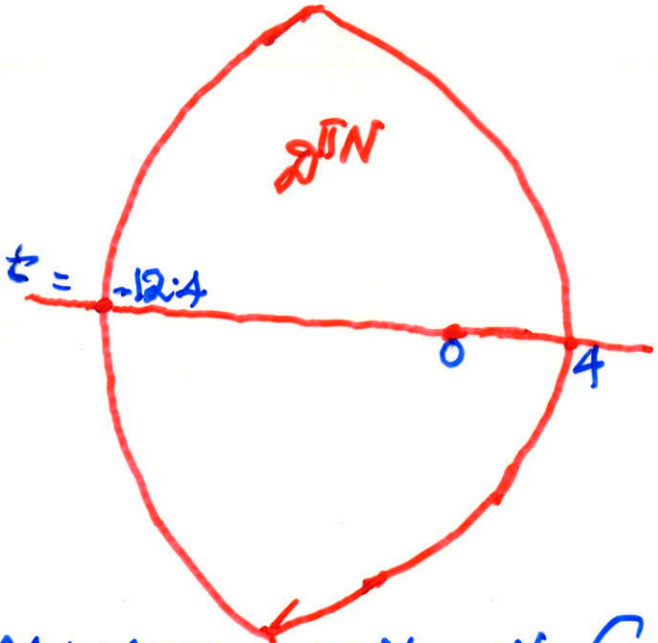
MARTIN'S RESULTS: $F(s,t)$ HAS COMBINED ANALYTICITY IN s and t in the Domain

$$C = \left\{(s,t)\,\middle|\, |t| < R,\ R > 0,\ s \in \text{CUT PLANE WITH CUTS } s \geqslant (m_A+m_B)^2,\ u \geqslant (m_A+m_B)^2\right\}$$

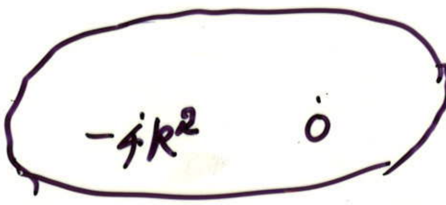
$R = 4m_\pi^2$, $\pi\pi$, KK, $K\bar{K}$, πK, πN, $\pi\Lambda$

MARTIN THEN PROVED FIXED-t
DISPERSION RELNS. IN A COMPLEX DOMAIN





BY COMBINING ANALYTICITY IN C
WITH LARGE LEHMANN ELLIPSE. IN THIS WAY
HE ALSO OBTAINED ANALYTICITY IN THE
LEHMANN-MARTIN ELLIPSE



$$t_0(s)\begin{cases} 16+\dfrac{64}{s-4}, & 4<s<16 \\ 256/s, & 16<s<32 \\ 4+\dfrac{64}{s-16}, & s>32 \end{cases}$$

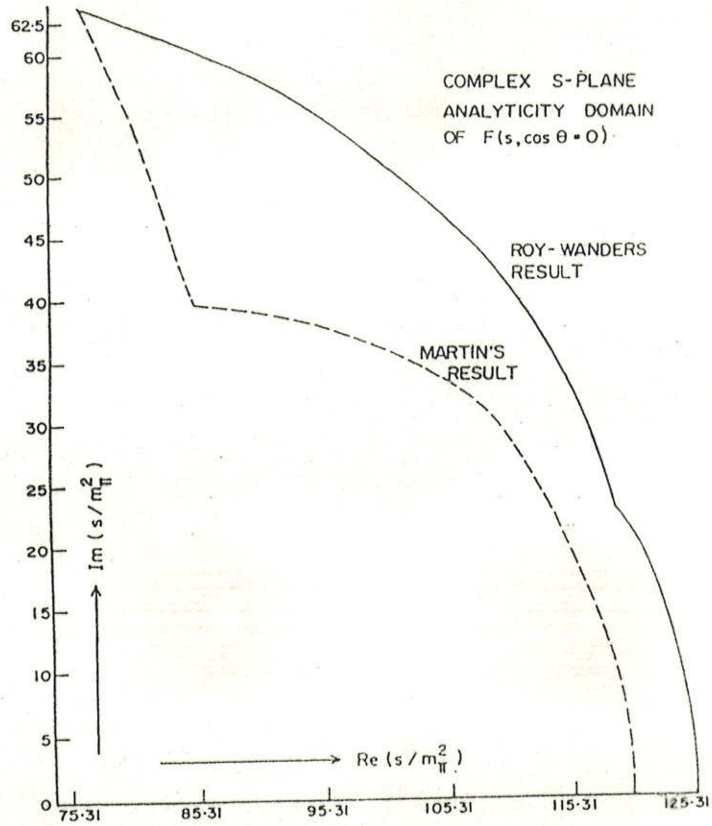
$\pi\pi$ CASE

FROISSART-MARTIN BOUND:
IS THEN A SIMPLE VARIATIONAL UPPER
BOUND ON

$$\int_{s_1}^{s_2} ds'\,\sigma_{tot}(s'),$$

given $0 \leq \operatorname{Im} a_\ell(s') \leq 1$, and

$$\int_4^\infty ds' \frac{A_s(s',t)}{s'^3} < \text{Const}, \quad 0<t<4m_\pi^2$$



APPLICATION: INTEGRAL EQNS FOR PION-PION SCATTERING INVOLVING ONLY PHYSICAL REGION PARTIAL WAVES:

S. M. Roy (1971), Mahoux-Roy-Wanders (1974)

upto $s = 60$ upto $s = 125.31$

6. COUNTEREXAMPLE : MARTIN 1967:

BY ~~UA~~ USING KNOWN AXIOMATIC FIELD THEORY RESULTS, CROSSING SYMMETRY AND ANALYTIC COMPLETION, MANDELSTAM REPRSN CANNOT BE PROVED

$$F_\nu = \int_0^1 dx \int_{p_0}^{\infty} dp \int_{q_0}^{\infty} dq \; \frac{\omega(x,p,q)}{x\,(p-s)^\nu + (1-x)(q-t)^\nu}$$

+ circular Permutations of (s,t,u)

$\nu = 1/2$: Mandelstam Reprsn

$\nu = 1$: Nakanishi-Wu Represn.

$\nu = 2/3$: natural domain of analyticity bigger than all one can get from axiomatic field theory and positivity but smaller than Mandelstam domain. ($p_0 = q_0 = 16$) (complex Singularities in s,t planes)

7. EXPTS AND PHENOMENOLOGY

13

$\sigma_{tot} = B \log^2(s/s_0) + \cdots , B = 0{\cdot}308$ mb.

Table 40.2: Total hadronic cross section. Analytic S-matrix and Regge theory suggest a variety of parameterizations of total cross sections at high energies with different areas of applicability and fits quality.

A ranking procedure, based on measures of different aspects of the quality of the fits to the current evaluated experimental database, allows one to single out the following parameterization of highest rank[1]

$$\sigma^{ab} = Z^{ab} + B\log^2(s/s_0) + Y_1^{ab}(s_1/s)^{\eta_1} - Y_2^{ab}(s_1/s)^{\eta_2} , \qquad \sigma^{\bar{a}b} = Z^{ab} + B\log^2(s/s_0) + Y_1^{ab}(s_1/s)^{\eta_1} + Y_2^{ab}(s_1/s)^{\eta_2} .$$

where Z^{ab}, B, Y_i^{ab} are in mb, and s, s_1, and s_0 are in GeV^2. The scales s_0, s_1, the rate of universal rise of the cross sections B, and exponents η_1 and η_2 are independent of the colliding particles. The scale s_1 is fixed at 1 GeV^2. Terms $Z^{ab} + B\log^2(s/s_0)$ represent the pomerons. The exponents η_1 and η_2 represent lower-lying C-even and C-odd exchanges, respectively. Requiring $\eta_1 = \eta_2$ results in somewhat poorer fits. In addition to total cross sections σ, the measured ratios of the real-to-imaginary parts of the forward scattering amplitudes $\rho = \mathrm{Re}(T)/\mathrm{Im}(T)$ were included in the fits by using s to u crossing symmetry. Global fits were made to the 2005-updated data for $\bar{p}(p)p$, $\Sigma^- p$, $\pi^\pm p$, $K^\pm p$, γp, and $\gamma\gamma$ collisions.

Exact factorization hypothesis in the form $(Z^{\gamma p}, B^{\gamma p}) = \delta \cdot (Z^{pp}, B)$, $(Z^{\gamma\gamma}, B^{\gamma\gamma}) = \delta^2 \cdot (Z^{pp}, B)$ was used to extend the universal rise of the total hadronic cross sections to the $\gamma p \to hadrons$ and $\gamma\gamma \to hadrons$ collisions. This resulted in reducing the number of adjusted parameters from 21 used for the 2002 edition to 19, and in the higher quality rank of the parameterization. The asymptotic parameters thus obtained were then fixed and used as inputs to a fit to a larger data sample that included cross sections on deuterons (d) and neutrons (n). All fits included data above $\sqrt{s_{\min}} = 5$ GeV.

Fits to $\bar{p}(p)p$, Σ^-p, $\pi^\pm p$, $K^\pm p$, γp, $\gamma\gamma$			Beam/	Fits to groups				χ^2/dof
Z	Y_1	Y_2	Target	Z	Y_1	Y_2	B	by groups
35.45(48)	42.53(1.35)	33.34(1.04)	$\bar{p}(p)/p$	35.45(48)	42.53(23)	33.34(33)	0.308(10)	
			$\bar{p}(p)n$	35.80(16)	40.15(1.59)	30.00(96)	0.308(10)	1.029
35.20(1.46)	−199(102)	−264(126)	Σ^-/p	35.20(1.41)	−199(86)	−264(112)	0.308(10)	0.565
20.86(40)	19.24(1.22)	6.03(19)	$\pi^\pm/p$	20.86(3)	19.24(18)	6.03(9)	0.308(10)	0.955
17.91(36)	7.1(1.5)	13.45(40)	$K^\pm/p$	17.91(3)	7.14(25)	13.45(13)	0.308(10)	
			$K^\pm/n$	17.87(6)	5.17(50)	7.23(28)	0.308(10)	0.669
	0.0317(6)		γ/p		0.0320(40)		0.308(10)	
	−0.61(62)E−3		γ/γ		−0.58(61)E−3		0.308(10)	0.766
$\chi^2/dof = 0.971$,	$B = 0.308(10)$ mb,		$\bar{p}(p)/d$	64.35(38)	130(3)	85.5(1.3)	0.537(31)	1.432
$\eta_1 = 0.458(17)$,	$\eta_2 = 0.545(7)$		$\pi^\pm/d$	38.62(21)	59.62(1.53)	1.60(41)	0.461(14)	0.735
$\delta = 0.00308(2)$.	$\sqrt{s_0} = 5.38(50)$ GeV		$K^\pm/d$	33.41(20)	23.66(1.45)	28.70(37)	0.449(14)	0.814

The fitted functions are shown in the following figures, along with one-standard-deviation error bands. When the reduced χ^2 is greater than one, a scale factor has been included to evaluate the parameter values, and to draw the error bands. Where appropriate, statistical and systematic errors were combined quadratically in constructing weights for all fits. On the plots, only statistical error bars are shown. Vertical arrows indicate lower limits on the $p_{\rm lab}$ or $E_{\rm cm}$ range used in the fits.

One can find the details of the global fits and ranking procedure, in the paper [1]. Database is practically the same as for the 2004 edition (it was slightly changed in the low energy regions not used in the fits).

Recently, the statement in [1] that the models with $\log^2(s/s_0)$ asymptotic terms work much better than the models with $\log(s/s_0)$ or $(s/s_0)^\epsilon$ terms was confirmed in [2] and [3], based on matching traditional asymptotic parameterizations with low energy data in different ways. Both these references, however, questioned the statement in [1] on the universality of the coefficient of the $\log^2(s/s_0)$ term for all processes with nucleon and gamma targets. The two references give different predictions at superhigh energies: $\sigma^{as}_{\pi N} > \sigma^{as}_{NN}$ [2] and $\sigma^{as}_{\pi N} \sim 2/3\sigma^{as}_{NN}$ [3]. A broader universality of σ^{as}_{tot} has been recently advocated in [4] for hadron-nucleus collisions. It should be noted that asymptotic rate universality in hadron-deuteron collisions has not been established at available energies (see Table).

Computer-readable data files are available at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS group, IHEP, Protvino, August 2005)

On-line "Predictor" to calculate σ and ρ for any energy from five high rank models is also available at `http://nuclth02.phys.ulg.ac.be/compete/predictor.html`.

References:

1. J.R. Cudell *et al.* (COMPETE Collab.), Phys. Rev. **D65**, 074024 (2002).
2. K. Igi and M. Ishida, Phys. Rev. **D66**, 034023 (2002), Phys. Lett. **B622**, 286 (2005).
3. M. M. Block and F. Halzen, Phys. Rev. **D70**, 091901 (2004), Phys. Rev. **D72**, 036006 (2005).
4. L. Frankfurt, M. Strikman, and M. Zhalov, Phys. Lett. **B616**, 59 (2005).

FIRST DISCOVERY, 1972 ISR (CERN)

σ_{tot}^{pp} RISES BY ~ 3 mb FROM ~40 mb AT $\sqrt{s} \simeq 30$ GeV to ~43 mb AT $\sqrt{s} \simeq 60$ GeV

[14]

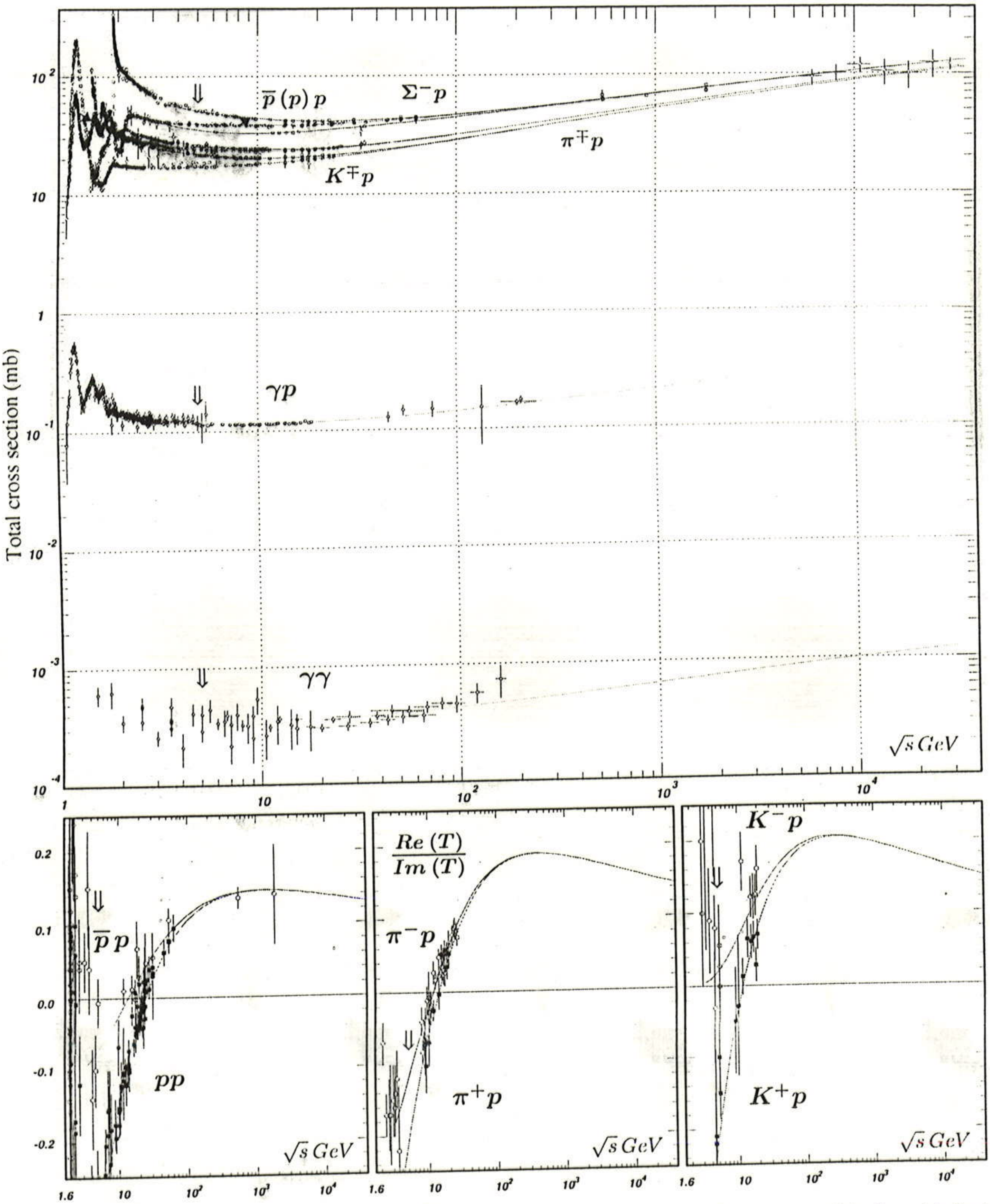


Figure 40.10: Summary of hadronic, γp, and $\gamma\gamma$ total cross sections, and ratio of the real to imaginary parts of the forward hadronic amplitudes. Corresponding computer-readable data files may be found at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS group, IHEP, Protvino, August 2005)

TYPICAL $\sqrt{s}$ (GeV): 23.5-62.5 (ISR), 540 (S$\bar{p}p$S)
1800 (Tevatron), 14000 (LHC)

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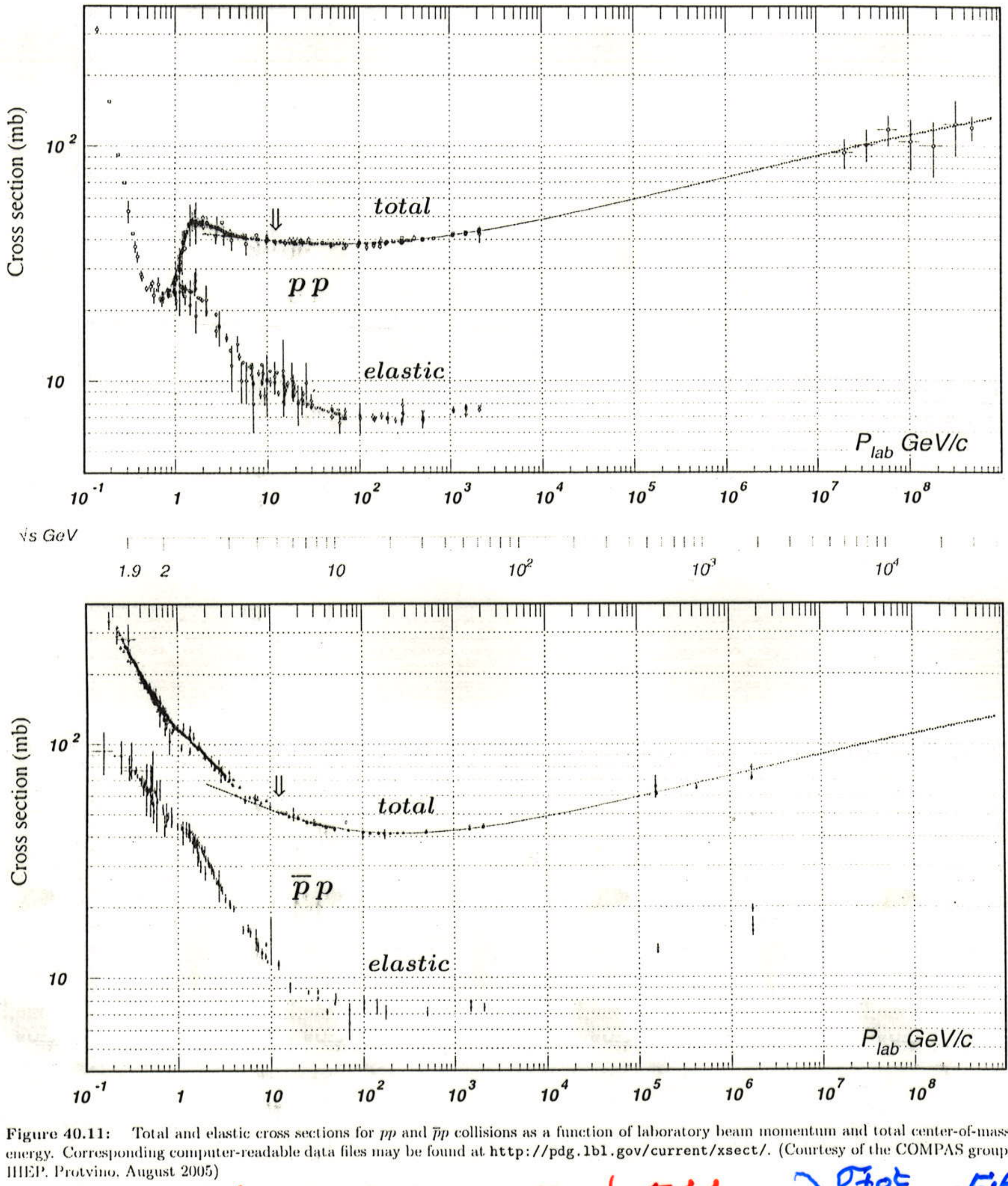

Figure 40.11: Total and elastic cross sections for pp and $\bar{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS group, IHEP, Protvino, August 2005)

$\sqrt{s}$ (GeV) →	23.5	63.5	541
σ_{tot}^{pp} (mb)	39	43	
$\sigma_{tot}^{\bar{p}p}$ (mb)	41	43	62
B (GeV^{-2})	10 – 12		15.5

$\frac{\sigma_{tot}}{\sigma_{el}} \sim 5.5 - 5.9$

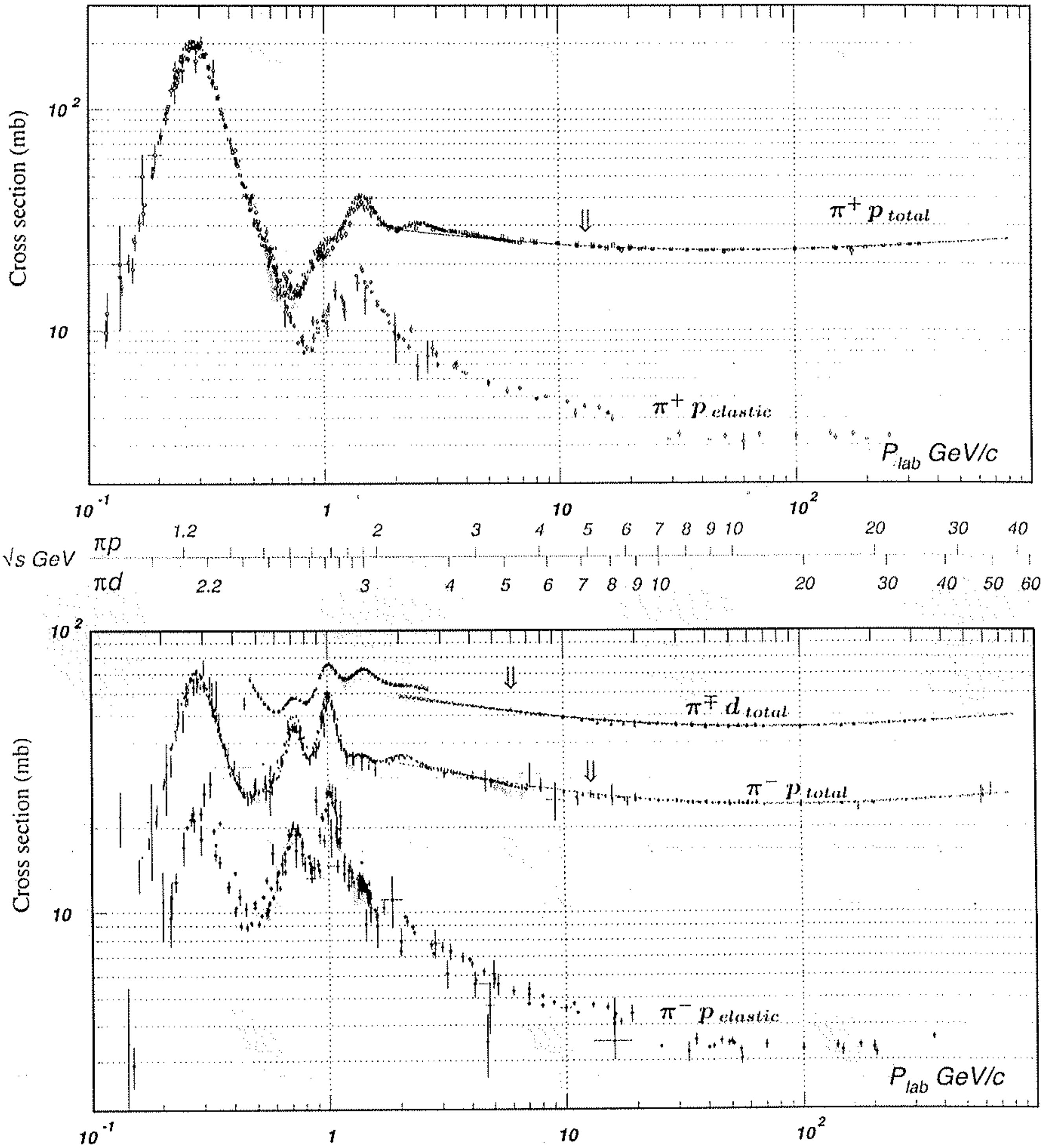

Figure 40.13: Total and elastic cross sections for $\pi^\pm p$ and $\pi^\pm d$ (total only) collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS Group, IHEP, Protvino, August 2005)

L16

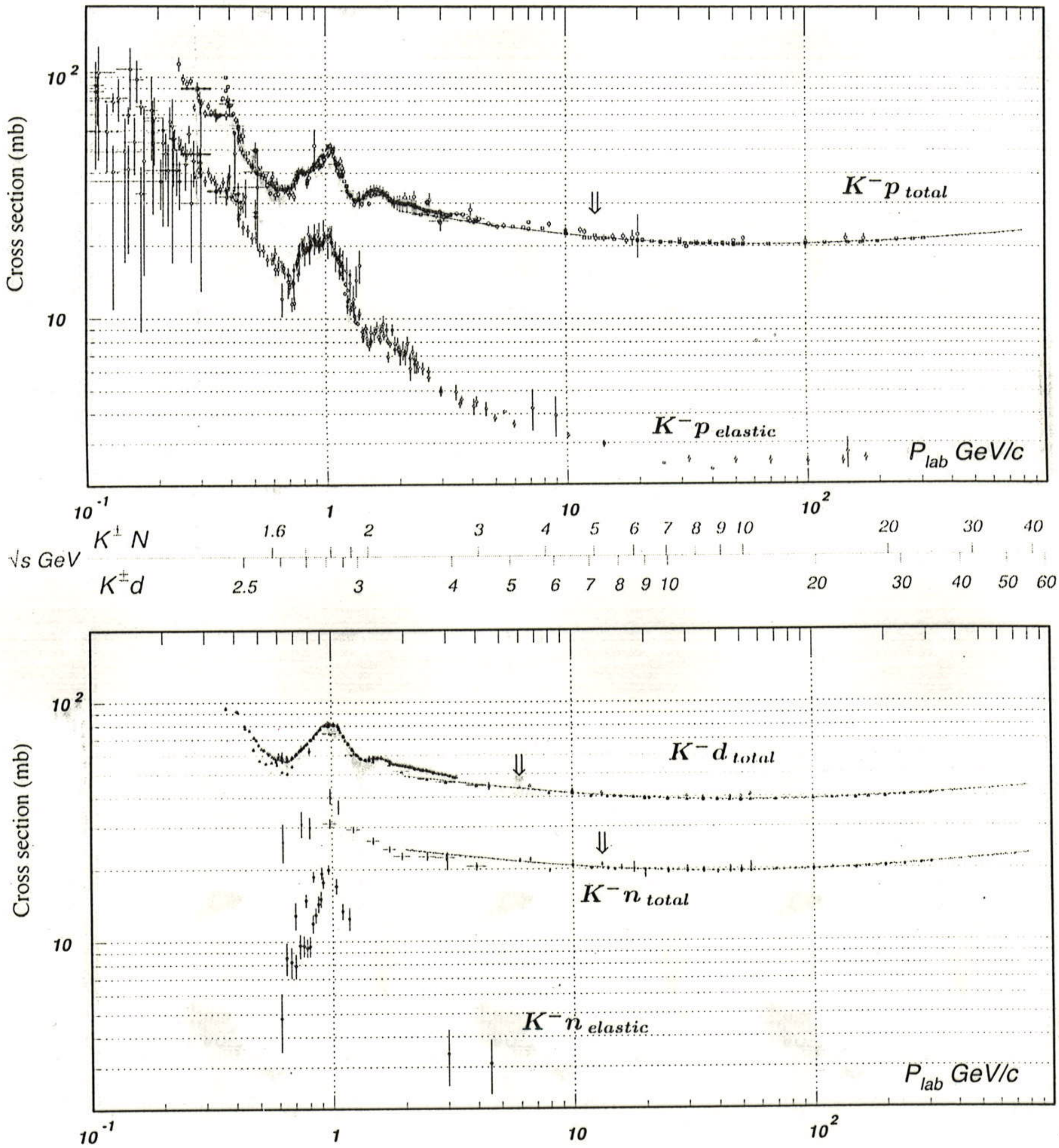

Figure 40.14: Total and elastic cross sections for K^-p and K^-d (total only), and K^-n collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS Group, IHEP, Protvino, August 2005)

L17.

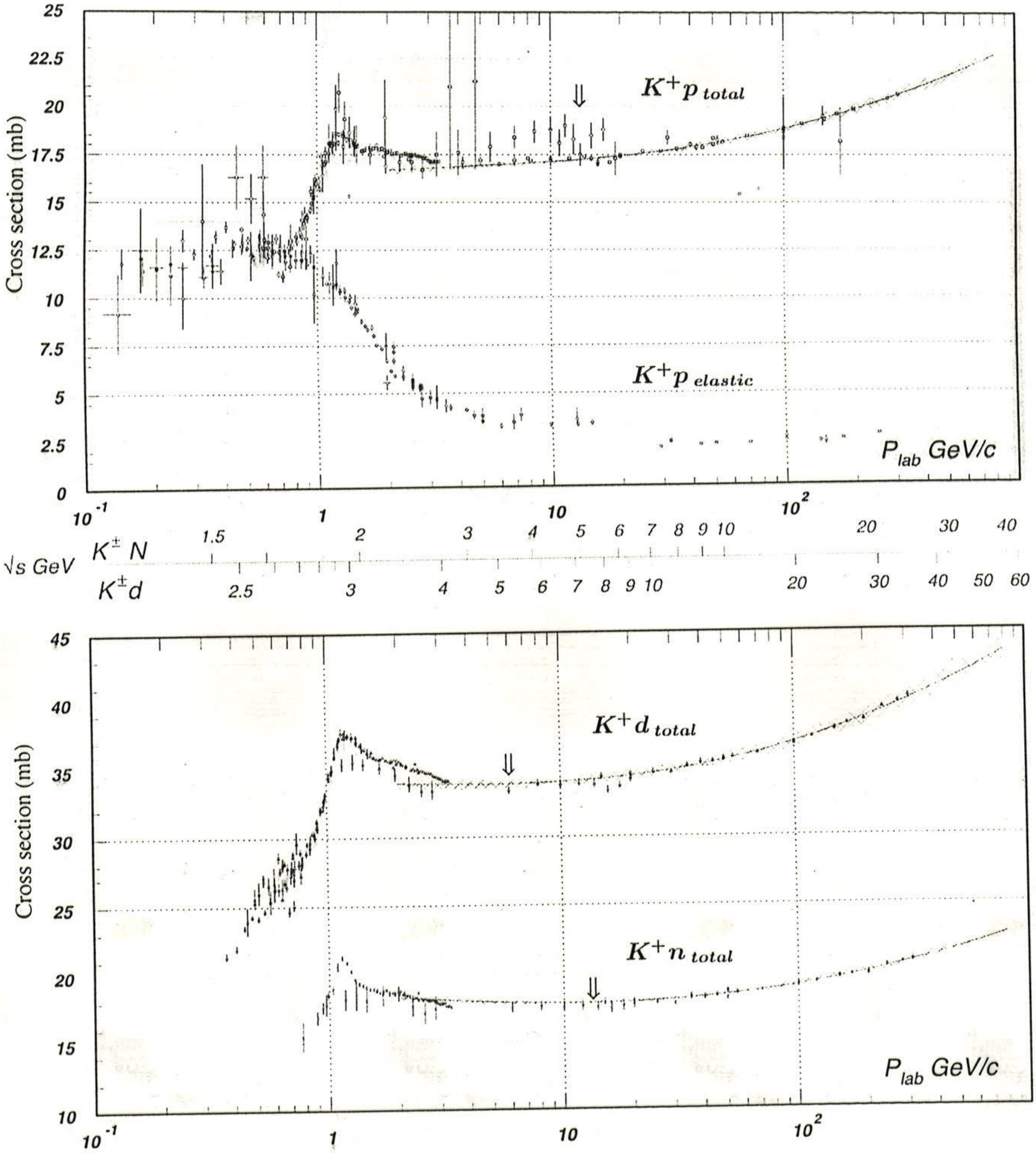

Figure 40.15: Total and elastic cross sections for K^+p and total cross sections for K^+d and K^+n collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS Group, IHEP, Protvino, August 2005)

Summary of Analyticity - Unitarity Bounds on Cross Sections for $s \to \infty$:

$$\sigma_{tot} < \frac{\pi}{m_\pi^2}\left[\ln(s/s_0)\right]^2$$

Martin (1966)
Lukaszuk-Martin (1967)

$$\left(\frac{d\sigma^{ab\to cd}}{d\Omega}\right)_{\cos\theta=1} \leq \frac{s}{64\pi m_\pi^2}\,\sigma^{ab\to cd}\left[\ln\left(\frac{s}{\sigma^{ab\to cd}}\right)\right]^2$$

$$\left(\frac{d\sigma^{ab\to cd}}{d\Omega}\right)_{\cos\theta\neq\pm1} \leq \frac{\sqrt{s}}{8\pi^2 m_\pi}\,\frac{\sigma^{ab\to cd}}{\sin\theta}\,\ln\left(\frac{s}{\sigma^{ab\to cd}}\right)$$

$$\underset{\text{fixed-}t}{\approx} \frac{s}{16\pi^2 m_\pi}\,\frac{\sigma^{ab\to cd}}{\sqrt{-t}}\,\ln\left(\frac{s}{\sigma^{ab\to cd}}\right)$$

Singh-Roy (1970), Roy-Singh (1970), and With unknown consts. on R.H.S. Bessis (1966), Eden (1966), Kinoshita (1966), Martin (1963), Logunov & Van Hieu (1968),

$$\left|\sigma_{tot}^{\pi^-p} - \sigma_{tot}^{\pi^+p}(s)\right| \leq \frac{\sqrt{2\pi}}{m_\pi}\sqrt{\sigma^{\pi^-p\to\pi^0n}}\,\ln\left(\frac{s}{\sigma^{\pi^-p\to\pi^0n}}\right)$$

IF ISO-SPIN INVARIANCE HOLDS.

IN ADDITION:

$$\left|\lim_{s\to\infty}\left(\sigma_{tot}^{\pi^-p} - \sigma_{tot}^{\pi^+p}\right)\right| \leq \frac{\pi^{3/2}}{m_\pi\sqrt{2}}\lim_{s\to\infty}\sqrt{\sigma^{\pi^-p\to\pi^0n}}$$

IF THE LIMITS EXIST

Roy-Singh (1970)

EXPTL. CONTRADICTION ?

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$\sigma_{ex} \sim s^{-1.23 \pm .02}$ Apel et al Serpukhov 1979 15–40 GeV/c + Barnes et al Fermilab 23-100 GeV/c

$\sim s^{-1.25 \pm .01}$ (Barnes et al 23- 200 GeV/c)

MY OWN FIT

$(\Delta\sigma_{tot})^2 \sim s^{-.9 \pm .06}$

Carroll et al Fermilab 23-240 GeV/c

$(\alpha_\rho \sim .55)$

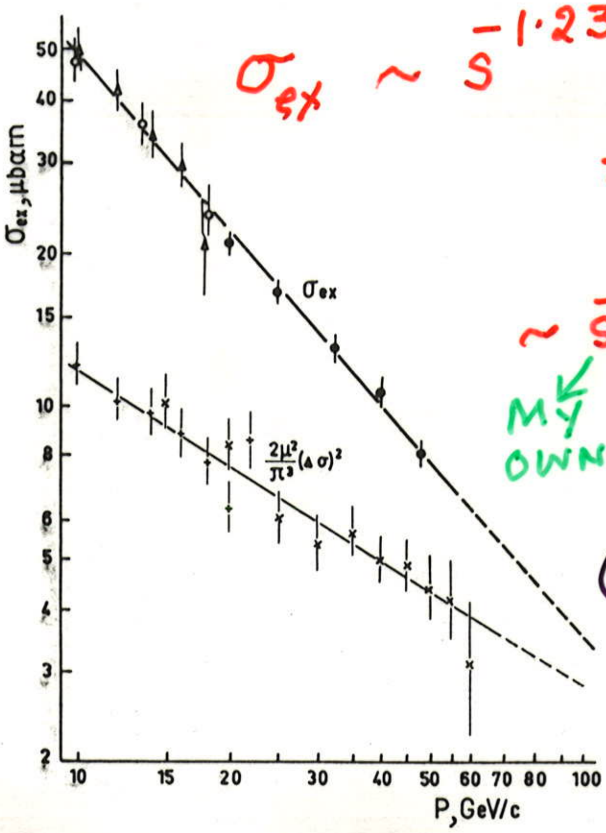

Fig. 4. The Serpukhov data of Bolotov et al. [B13, G1] on the $\pi^-p \to \pi^0 n$ integrated cross section, and the experimental values [A1] of the quantity $(2m_\pi^2/\pi^3)(\sigma_{tot}^{\pi^-p} - \sigma_{tot}^{\pi^+p})^2$, occurring in the bound (6.12) on the charge exchange cross section. The interesting speculations raised by these data are discussed in section 6.3.

$$|\sigma_{tot}^{\pi^-p}(s) - \sigma_{tot}^{\pi^+p}(s)| \underset{s\to\infty}{\leqslant} \frac{\sqrt{2\pi}}{m_\pi}\sqrt{\sigma^{\pi^-p\to\pi^0n}(s)}\,\ln\left(\frac{s}{s_0^2\sigma^{\pi^-p\to\pi^0n}}\right), \tag{6.10}$$

and

$$|\sigma_{tot}^{K^+n}(s) - \sigma_{tot}^{K^-n}(s)| \underset{s\to\infty}{\leqslant} \frac{2\sqrt{\pi}}{m_\pi}\sqrt{\sigma^{K_Lp\to K_Sp}(s)}\,\ln\left(\frac{s}{s_0^2\sigma^{K_Lp\to K_Sp}(s)}\right) \tag{6.11}$$

where $\sigma^{\pi^-p\to\pi^0n}$ and $\sigma^{K_Lp\to K_Sp}$ denote the integrated $\pi^-p \to \pi^0 n$ and $K_L p \to K_S p$ cross-sections respectively, and s_0 is an unknown but constant scale factor.

The elementary bound (6.10) is violated by the present asymptotic parametrizations (6.8) and (6.9) of the Serpukhov data [D2, D3, G1, B13]; this indicates either that isotopic spin invariance is violated at high energies, or that the present experimental parametrizations do not hold at asymptotic energies.

ii) Roy and Singh [R3] have shown that, *if isotopic spin invariance holds at high energies*, then,

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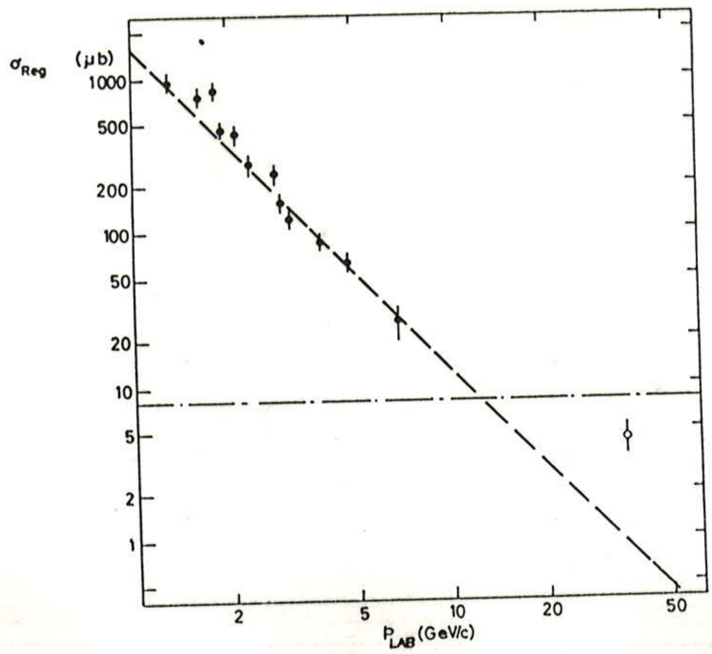

Fig. 5. The experimental values of the integrated $K_Lp \to K_Sp$ regeneration cross section [B10], σ_{Reg} are compared with the asymptotic lower bound on it obtained from eq. (6.13) under the assumption that $\sigma_{tot}^{K^-n} - \sigma_{tot}^{K^+n} \to 2.25$ mb, for $s \to \infty$; this assumption about the total cross section difference would be disproved if the present trend of σ_{Reg} continues to higher energies. The solid black circles are the data points, and the open point is explained in ref. [F3].

$$\left|\lim_{s\to\infty} [\sigma_{tot}^{\pi^-p}(s) - \sigma_{tot}^{\pi^+p}(s)]\right| \leqslant \frac{\pi^{3/2}}{\sqrt{2}m_\pi} \lim_{s\to\infty} \sqrt{\sigma^{\pi^-p\to\pi^0n}(s)}, \tag{6.12}$$

provided the above limits exist.

Similarly, Finkelstein and Roy [F3] have obtained, *if isotopic spin invariance holds at high energies,* then,

$$\left|\lim_{s\to\infty} [\sigma_{tot}^{K^+n}(s) - \sigma_{tot}^{K^-n}(s)]\right| \leqslant (\pi^{3/2}/m_\pi) \lim_{s\to\infty} \sqrt{\sigma^{K_Lp\to K_Sp}} \tag{6.13}$$

provided the limits in (6.13) exist.

The results (6.12) and (6.13) also hold if the $\lim_{s\to\infty}$ on the right hand sides are replaced by $\lim_{s\to\infty}$ sup, in the cases where $\sigma^{\pi^-p\to\pi^0n}(s)$ and $\sigma^{K_Lp\to K_Sp}(s)$ have oscillatory behaviour for $s \to \infty$.

From the practical point of view, the bounds (6.12) and (6.13) constitute a rather satisfactory substitute for the Pomeranchuk theorem. For example eq. (6.12) implies that, *if the integrated πN charge exchange cross-section vanishes for $s \to \infty$, $|\sigma_{tot}^{\pi^-p}(s) - \sigma_{tot}^{\pi^+p}(s)|$ must also vanish for $s \to \infty$.* An analogous result for KN scattering follows from eq. (6.13). Experimentally, both $\sigma^{\pi^-p\to\pi^0n}(s)$ and $\sigma^{K_Lp\to K_Sp}(s)$ seem to be decreasing fast [G1, B10] with increasing s (see figs. 4 and 5), suggesting that $[\sigma_{tot}^{\pi^-p}(s) - \sigma_{tot}^{\pi^+p}(s)]$ and $[\sigma_{tot}^{K^+n}(s) - \sigma_{tot}^{K^-n}(s)]$ should also vanish for $s \to \infty$. For comparisons at high but finite energies we note that the inequalities corresponding to (6.12) and (6.13) are expected to hold at such energies, only if $\sigma^{\pi^-p\to\pi^0n}$ and $\sigma^{K_Lp\to K_Sp}$ do not vanish for

BOUNDS ON THE DIFFRACTION PEAK

$$b(s) \equiv \frac{d}{dt} \ln \left(\frac{d\sigma_A}{dt} \right) \Bigg|_{t=0} \geqslant \frac{2}{9} \left[\frac{\sigma_{tot}^2}{4\pi\sigma_{el}} - \frac{1}{R^2} \right]$$

MacDowell-Martin (1964), spinless case
Auberson-Martin-Mennessier (1977) general spins, with an extra factor 0.9965 on R.H.S.

$$b(s) \underset{s\to\infty}{\leq} \frac{1}{16\, m_\pi^2} \left[\ln \left(\frac{s^2}{\frac{d\sigma^A}{dt}(s,0)} \right) \right]^2 \equiv b_{Max}(s)$$

Singh (1971) spinless case; Auberson-Roy (1976): general spins

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ELASTIC SPINLESS PARTICLES SCATT.

$$\left| \frac{\operatorname{Im} F(s,t)}{\operatorname{Im} F(s,0)} \right| \underset{s\to\infty}{\leq} \left[1 - \frac{r}{9} + \frac{3}{8}\left(\frac{r}{9}\right)^2 - \frac{21}{320}\left(\frac{r}{9}\right)^3 + \cdots \right]$$

$$\text{if } 0 \leq r \equiv (-t) \frac{\sigma_{tot}^2}{4\pi\sigma_{el}} \leq 2.5$$

SINGH - ROY (1970)

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$$\Sigma_{el} = \frac{\sigma_{tot}^2}{16\pi B}$$

MacDowell-Martin Bound:

$$\frac{\sigma_{el}}{(\Sigma_{el})^{Im}} \geq \frac{8}{9}$$

$\sqrt{B} \to (B)$ Im

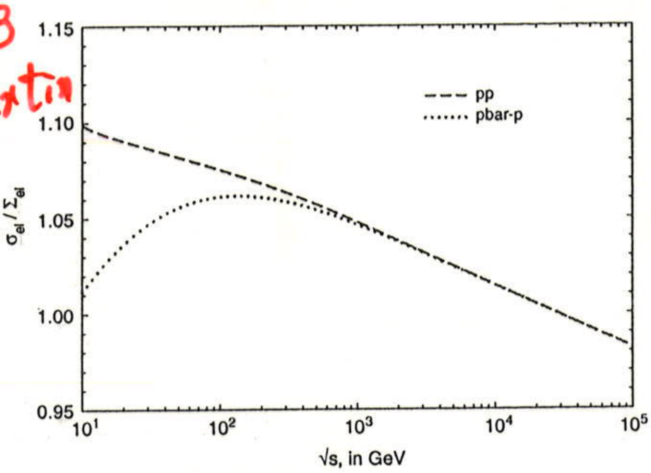

Fig. 15. The ratio σ_{el}/Σ_{el} vs. the c.m. energy $\sqrt{s}$, in GeV, using a constrained Aspen model fit (QCD-inspired theory). The elastic cross section is σ_{el} and $\Sigma_{el} \equiv \sigma_{tot}^2/16\pi B$. The dashed curve is for pp and the dotted curve is for $\bar{p}p$.

As expected from diffractive shrinkage, the first minimum of the 14 TeV curve moves to lower $|t|$ than the first minimum of the 1.8 TeV plot. Our new prediction at 14 TeV for the first sharp minimum at $|t| \sim 0.4$ $(\mathrm{GeV}/c)^2$ and a second shallow minimum at $|t| \sim 2$ $(\mathrm{GeV}/c)^2$ should be readily verified when the LHC becomes operative.

In Fig. 15, the ratio σ_{el}/Σ_{el} is plotted against the c.m. energy $\sqrt{s}$, in GeV, where $\sigma_{el} = \int_{-\infty}^{0} (\mathrm{d}\sigma_{el}/\mathrm{d}t)\,\mathrm{d}t$ is the true total elastic scattering cross section, while $\Sigma_{el} = \sigma_{tot}^2/(16\pi B)$, which was defined by Eq. (56) in Section 5. We recall to your attention that what is typically measured by experimenters is Σ_{el} and *not* the real σ_{el}. From Fig. 15 we see that the error made is $\sim$ 5–10% for energies less than 100 GeV, being $\sim$ 5% at the $S\bar{p}pS$, $\sim$ 4% at the Tevatron and less than 1% at the LHC, and hence, the MacDowell–Martin bound [21], which states that $\sigma_{el}/\Sigma_{el} \geqslant 8/9$, is clearly satisfied.

13.1.3. Rapidity gap survival probabilities

We now turn to some interesting properties of the Aspen eikonal, concerning the validity of the factorization theorem for nucleon–nucleon, γp and $\gamma\gamma$ collisions. It was shown that the survival probabilities of large rapidity gaps in high-energy $\bar{p}p$ and pp collisions are identical (at the *same* energy) for γp and $\gamma\gamma$ collisions, as well as for nucleon–nucleon collisions [52]. We will show that neither the factorization theorem nor the reaction-independence of the survival probabilities depends on the assumption of an additive quark model, but, more generally, depends on the *opacity* of the eikonal being *independent* of whether the reaction is n–n, γp or $\gamma\gamma$.

Rapidity gaps are an important tool in new-signature physics for ultra-high-energy $\bar{p}p$ collisions. Block and Halzen [51] used the Aspen model (QCD-inspired eikonal model) to make a reliable calculation of the survival probability of rapidity gaps in nucleon–nucleon collisions. We sketch below their arguments.

From Section 13.1, using Eq. (307), we write the inelastic cross section, $\sigma_{inel}(s)$, as

$$\sigma_{inel}(s) = \int [1 - \mathrm{e}^{-2\chi_I(b,s)}]\,\mathrm{d}^2\vec{b}. \tag{313}$$

It is readily shown, from unitarity and Eq. (313), that the differential probability in impact parameter space b, for *not* having an inelastic interaction, is given by

$$\frac{\mathrm{d}^2 P_{\text{no inelastic}}}{\mathrm{d}^2\vec{b}} = \mathrm{e}^{-2\chi_I(b,s)}. \tag{314}$$

Because the parameterization is both unitary and analytic, its high-energy predictions are effectively model-independent, if you require that the proton is asymptotically a black disk.

As an example of a large rapidity gap process, consider the production cross section for Higgs-boson production through W fusion. The inclusive differential cross section in impact parameter space b is given by

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V. SINGH & S. M. ROY (1970)

Table I. Upper bounds on the imaginary part $A(s,t)$ in the diffraction-peak region as a function of $\rho = (-t)(\sigma_{tot}^2/4\pi\sigma_{el})$.

ρ	Upper bound on $A(s,t)/A(s,0)$	ρ	Upper bound on $A(s,t)/A(s,0)$
0	1.000	4.50	0.630
0.50	0.945	5.00	0.610
1.00	0.893	5.50	0.593
1.50	0.846	6.00	0.578
2.00	0.795	6.50	0.566
2.50	0.749	7.00	0.554
3.00	0.710	7.50	0.543
3.50	0.677	3.00	0.533
4.00	0.652	8.42	0.523

BOUNDS ON ABSORPTIVE CONTRIBUTION TO DIFFERENTIAL CROSS SECTIONS CAN BE TESTED EXPTLLY

L24

SINGH-ROY BOUND (1979)
(NEARLY SATURATED
BY EXPTL DATA)

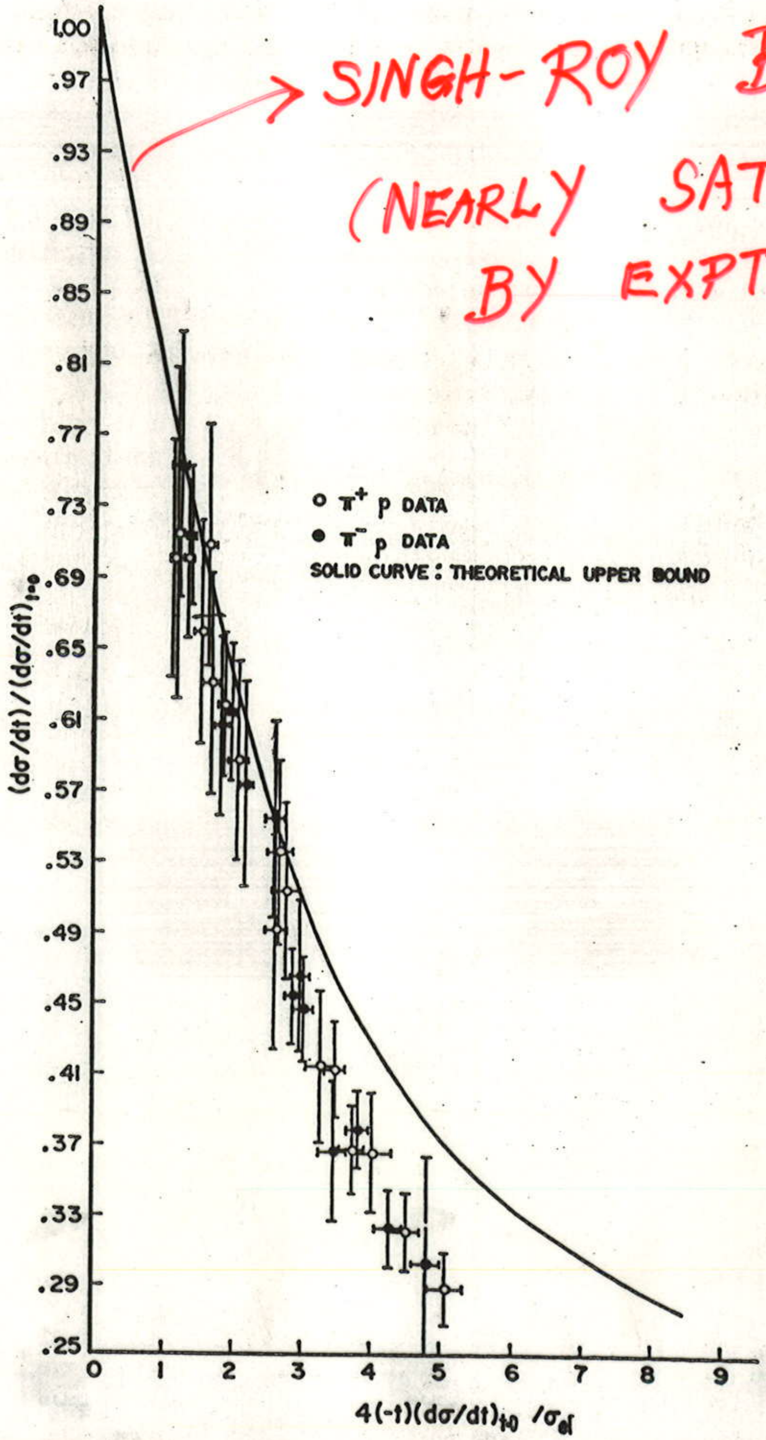

Fig. 11. Theoretical unitarity upper bound [S3] on the curve of $[A(s,t)/A(s,0)]^2$ versus $\rho \equiv (-t)\sigma_{tot}^2/(4\pi\,\sigma_{el})$ (solid line) is compared with the experimental curve of $(d\sigma/dt)/(d\sigma/dt)_{t=0}$ versus $4(-t)(d\sigma/dt)_{t=0}/\sigma_{el}$ in the diffraction peak region for π^+p and π^-p elastic scattering. The quantities plotted in the theoretical and experimental curves are equal for purely absorptive and spin-independent scattering. The data are from ref. [F6] in the lab-momentum range 6.8 to 13.0 GeV/c (see section 8).

POMERANCHUK-LIKE THEOREMS

If $\lim_{s\to\infty} (\sigma_{tot}^{AB} - \sigma_{tot}^{\bar{A}B})$ exists, the limit being finite or infinite, and if

$$\lim_{s\to\infty} \frac{F(s, t=0)}{s \ln(s/s_0)} = 0 , \quad F = F^{AB} - F^{\bar{A}B},$$

$\downarrow_{K\, AB\to AB}$

then $\lim_{s\to\infty} (\sigma_{tot}^{AB} - \sigma_{tot}^{\bar{A}B}) = 0$.

MARTIN (1965)

If σ_{tot}^{AB} or $\sigma_{tot}^{\bar{A}B} \to \infty$ for $s \to \infty$

then $\sigma_{tot}^{AB} / \sigma_{tot}^{\bar{A}B} \xrightarrow[s\to\infty]{} 1$

if the ratio has a limit

Eden (1966), Kinoshita (1966), Grunberg and Truong (1973, 1974)

Inside the diffraction peak

$$\frac{(d\sigma/dt)^{AB\to AB}}{(d\sigma/dt)^{\bar{A}B\to\bar{A}B}}(s, t(s)) \xrightarrow[s\to\infty]{} 1$$

if the limit exists,

and provided that $\dfrac{(d\sigma/dt)(s,0)}{(d\sigma/dt)(s,t(s))}$ stays finite for $s \to \infty$

Cornille and Martin (1972, 1974)

REAL PARTS :

For $F(s,t) = F^{AB \to AB} + F^{A\bar{B} \to A\bar{B}}$

If

$$\mathrm{Im}\, F(s,0) \sim c\, s (\ln s)^{\gamma} (\ln \ln s)^{\beta} \ldots, \quad s \to \infty$$

with $0 < \gamma \leq 2$, $c \neq 0$, then

$$\rho = \frac{\mathrm{Re}\, F(s,0)}{\mathrm{Im}\, F(s,0)} \sim \frac{1}{2} \frac{\pi \gamma}{\ln s}, \quad s \to \infty$$

KHURI-KINOSHITA (1965).
Martin suggested high energy measurements of ρ, which have [Bartenev et al 1973, Amaldi et al 1977, Augier et al 1993] qualitatively verified the prediction.
At LHC ($\sqrt{s} = 14$ TeV) SUCH MEASUREMENTS WOULD NEED GOING TO $\theta_0 = .0037$ mrad

SCALING THEOREMS

If $\sigma_{tot}^{AB} \sim const (\ln(s/s_0))^2$, for $s \to \infty$, then every sequence of $s \to \infty$ must contain a subsequence such that

$$\lim_{s\to\infty,\ \tau\text{-fixed}} \frac{F^{AB\to CD}(s, t = -\tau[\ln(s/s_0)]^{-2})}{F^{AB\to CD}(s,0)} = f(\tau),$$

where $f(\tau)$ is an entire function of τ of order half

Auberson - Kinoshita - Martin (1971). (Spinless case)

For arbitrary spin elastic scattering if $\frac{b(s)}{b_{max}(s)} \geq b_0 \neq 0$, $s \to \infty$, then every sequence of $s \to \infty$ must contain a subsequence such that

$$\lim_{s\to\infty,\ \tau\text{-fixed}} \frac{\frac{d\sigma_A}{dt}(s, t = -\frac{\tau}{b(s)})}{\frac{d\sigma_A}{dt}(s, 0)} = f(\tau)$$

where $f(\tau)$ is an entire function of order half obeying $f(0) = 1$, $f'(0) = -1$, and

$$f(\tau) = \int_{\lambda=0}^{2/\sqrt{b_0}} d\mu(\lambda)\, J_0(\lambda\sqrt{\tau})$$

$\int d\mu(\lambda) = 1$, $\int d\mu(\lambda) \cdot \lambda^2 = 4$

Auberson - Roy (1976)

CAN THE FROISSART-MARTIN BOUND BE IMPROVED?

UNITARITY GIVES

$\mathrm{Im}\, a_\ell \geqslant |a_\ell|^2$ All Energies (i)

$\mathrm{Im}\, a_\ell = |a_\ell|^2$ elastic region (ii)

ONLY (i) NEEDED TO PROVE THE BOUND.

THE GRIBOV PARADOX SHOWS THAT $s \to \infty$ BEHAVIOUR CONSTRAINED BY ELASTIC UNITARITY IN t-CHANNEL. MOREOVER MARTIN-RICHARD (2000) EXAMPLE:

$$F = \mathrm{Const}\left(4 - \sqrt{(4-t)(4-u)}\right) \exp\left[-2(4-s)^{1/4}\right]$$

+ Circular permuts. in (s, t, u)

WHICH BEHAVES LIKE $s f(t)$ AT HIGH ENERGY AND OBEYS (i) SHOWS THAT ELASTIC UNITARITY IS ESSENTIAL FOR THE GRIBOV THM.

ATKINSON (1970) HAS AN AMPLITUDE OBEYING (i) and (ii) BUT WITH $\sigma_{tot} \sim (\ln s)^{-3}$.

KUPSCH (1982) HAS AN AMPLITUDE "QUALITATIVELY" SATURATING THE FROISS-MARTIN BOUND, $\sigma_{tot} \sim \mathrm{Const.}(\ln s)^2$ OBEYING (i) BUT NOT (ii)

NO EXAMPLE SATURATING $\sigma_{tot} \sim \mathrm{Const.}(\ln s)^2$ AND OBEYING (i) AND (ii) KNOWN.