From Quanta to Noncommutative Field Theory

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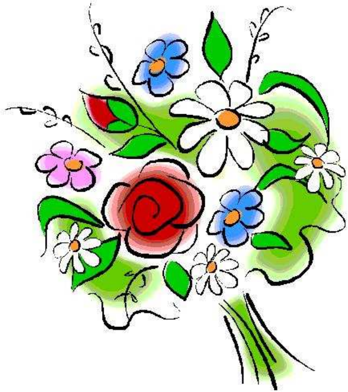
Congratulation

to **André**

Congratulation to your 80th birthday



Introduction

- Quarkonia
- THEOREM

QFT

Deformed Minkowski Space-Time

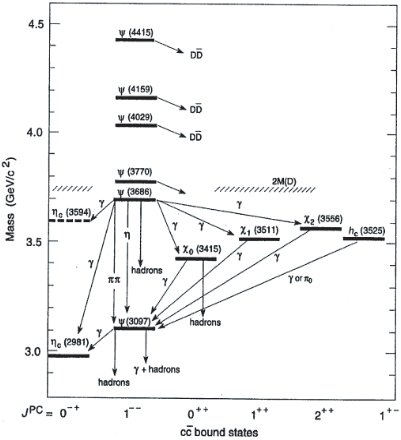
- Wedge Locality H G + Lechner

Deformed Euclidean Space-Time

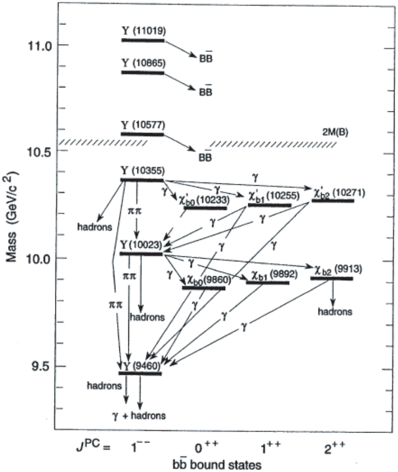
- Cure IR/UV mixing Matter fields
  modified actions H G + Wulkenhaar,...
- Taming the Landau ghost H G + Wulkenhaar

Summary

Quarkonia



Mass (GeV/c²)
4.5
4.0
3.5
3.0
ψ (4415)
DD̄
ψ (4159)
ψ (4029)
ψ (3770)
2M(D)
ψ (3686)
η_c (3594)
χ_2 (3556)
h_c (3525)
χ_1 (3511)
χ_0 (3415)
hadrons
γ or π_0
ψ (3097)
η_c (2981)
γ + hadrons
J^PC = 0^-+ 1^-- 0^++ 1^++ 2^++ 1^+-
cc̄ bound states



Mass (GeV/c²)
11.0
10.5
10.0
9.5
Υ (11019)
BB̄
Υ (10865)
Υ (10577)
2M(B)
Υ (10355)
χ'_b0 (10233)
χ'_b1 (10255)
χ'_b2 (10271)
hadrons
Υ (10023)
χ_b0 (9860)
χ_b1 (9892)
χ_b2 (9913)
Υ (9460)
γ + hadrons
J^PC = 1^-- 0^++ 1^++ 2^++
bb̄ bound states

History

- 1974 Observation of J/Ψ, $c\bar{c}$ quark-antiquark system
- 1977 Order of Levels perturbation results
- Further Bounds: Level differences, wave functions,...
- treated inverse problem,...
- 1984 Level Ordering nonperturbative results

treated Stability of matter problem: Quasiclassical estimates,... nonlinear PDE,...

- 2-dim QFT: solvable S-matrices
- Quantize inverse scattering method, ...
  Met many people

Order of Levels

Formulation

Let $E_{n,l}$ be an energy level in a central potential, n being the number of nodes of the radial wave function, l the orbital angular momentum. Then

$$r^2 \Delta V(r) > 0 \Rightarrow E_{n+1,l} > E_{n,l+1}$$

- Condition is a strong version of asymptotic freedom
- Holds also for opposite signs applicable to atomic physics
- The $c\bar{c}$ potential is concave and increasing:

Implies:

$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}V(r) < 0 \Rightarrow E_{n+1,l} < E_{n,l+2}$$

Sketch of proof: Use SUSY QM

If $u_{n,l}$ solves the radial Schrödinger equation,

$\bar{u} = (\frac{d}{dr} - \frac{u'_{0,l}}{u_{0,l}})u_{n,l}$ solves (Darboux)

$$(-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} - 2\frac{d}{dr}(\frac{u'_{0,l}}{u_{0,l}}) + V(r) - E)\bar{u} = 0$$

...use André 's tricks....Wronkian, nodal theorem,....

Lemma: If $\Delta V(r) > 0$ than

$$-\frac{u'_{0,l}}{u_{0,l}} - \frac{l+1}{r} > 0$$

Compare with Coulomb,...

- Apply to magnetic field Schrödinger operators
- spacing of levels
- wave function at origin, decays, kinetic energy
- Dirac equation, inverse problem,...

QFT

- Define Wightman or Schwinger functions:
  $S_N(z_1, ..., z_N) = \int \Phi(z_1)...\Phi(z_N) d\nu(\Phi)$ $d\nu = \frac{1}{Z} e^{-\int L_{int}(\Phi)} d\mu(\Phi)$,
  $d\mu$ is reg. Gauss measure
- QFT needs regularization and renormalization
- UV, IR, convergence problem use RG FLOW
- Landau ghost, trivial Higgs model?
- add ”Gravity” or deform Space-Time

Project

merge general relativity with quantum physics through noncommutative geometry

Space-Time Concepts

in Newton gr., QM, ED, GR, QFT and QG ?
Limited localisation of events in space-time

$$D \geq R_{ss} = \frac{G}{c^4}\frac{hc}{\lambda} \geq \frac{G}{c^4}\frac{hc}{D}$$

- gives Planck lenght as a lower bound
- Connes: Noncommutative Geometry

- Replace manifold by algebra deform it
- keep differential calculus derivations....
- Replace fields by projective modules
- Replace integrals by traces
- use renormalized perturbation expansion

Deform Minkowski

H G Gandalf Lechner, AQFT: Buchholz and Summers
Quantum fields over deformed Minkowski space time
NC coordinates: $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$, $[\hat{x}^\mu, \theta^{\sigma\tau}] = 0$
Field operators are defined as tensor product:

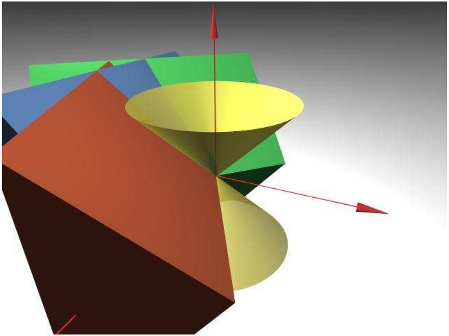
$$\Phi^{\otimes}(x) = \int d\mu_p (e^{ipx} e^{ip\hat{x}} \otimes \Phi_p)$$

Vacuum states: take: $\omega_\theta = \nu \otimes < \Omega, .\Omega >$
ω_θ is independent on ν
GNS rep. of pol algebra build from $\Phi^{\otimes}(f)$ wrt ω_θ
Deformation like Balachandran et.al., Chaichian et.al, Soloviev,..

- ϕ^θ acts on Hilbert space H of the undeformed theory
  $\Rightarrow$ cov. prop. of ϕ^θ w.r.t. undeformed rep. U
- Consider family of fields

Wedges

We relate the antisymmetric matrices to Wedges:
$W_1 = \left\{ x \in \mathbb{R}^D | x_1 > |x_0| \right\}$ act on standard wedge by proper Lorentz transformations $i_\Lambda(W) = \Lambda W$. Stabilizer group is $SO(1,1)xSO(2)$, which corresponds to boosts and rotations.



Wedges and Wedge local QF

$\mathcal{A} = \{\gamma_\Lambda(\theta_1) | \Lambda \in \mathcal{L}_+\}$, $\theta(\Lambda W_1) := \gamma_\Lambda(\theta_1) = \Lambda\theta_1\Lambda^\dagger$
We define wedge local fields through: $\phi = \{\phi_W | W \subset \mathcal{W}_0\}$ get family of fields, covariance and localization in wedges.

$$U_{y,\Lambda}\Phi_W(x)U^\dagger_{y,\Lambda} = \Phi_{\Lambda W}(\Lambda x + y)$$

Theorem

Let $\kappa_e \geq 0$ the family $\Phi_W(x)$ is a wedge local quantum field:

$$[\phi_{W_1}(f), \phi_{-W_1}(g)](\psi) = 0,$$

for $supp(f) \subset W_1$, $supp(g) \subset -W_1$.

Proof relies on spectrum condition and support properties

DeformEuclideanΦ^4_θ

Formulation

ϕ^4 on nc $\mathbb{R}^4$, $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$ antisymmetric,
or equivalently star product

$$(a * b)(x) = \int dy \int dk a(x + \frac{\theta k}{2}) b(x + y) e^{iky}$$

differential calculus,....

ϕ^4 action

$$S = \int dp (p^2 + m^2)\phi_p \phi_{-p} + \lambda \int \prod_{j=1}^{4} (dp_j \phi_{p_j})\, \delta(\sum_{j=1}^{4} p_j) e^{-i \sum_{i<j} p_i \theta p_j}$$

Feynman rules

cyclic order of momenta leads to ribbon graphs
Model is not renormalizable, implied by IR/UV mixing
One possible solution: modify action

Theorem

H. G. and R. Wulkenhaar ϕ^4 model modified,
IR/UV mixing: short and long distances related
Theorem: Action

$$S = \int d^4x \Big(\frac{1}{2} \partial_\mu \phi \star \partial^\mu \phi + \frac{\Omega^2}{2} (\tilde{x}_\mu \phi) \star (\tilde{x}^\mu \phi) + \frac{\mu_0^2}{2} \phi \star \phi + \lambda \phi \star \phi \star \phi \star \phi \Big)(x)$$

for $\tilde{x}_\mu := (\theta^{-1})_{\mu\nu} x^\nu$
is perturbativly renormalizable to all orders in λ
3 proofs: Polchinski-Wilson approach in matrix base
Rivasseau et al: Multiscale analysis: matrix base and position space

Action has Langmann-Szabo position-momentum duality
$S[\phi; \mu_0, \lambda, \Omega] \mapsto \Omega^2 S[\phi; \frac{\mu_0}{\Omega}, \frac{\lambda}{\Omega^2}, \frac{1}{\Omega}]$

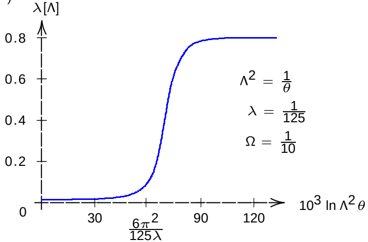
Taming Landau Ghost

evaluate β function, H. G. and R. Wulkenhaar,

$$\beta_\lambda = c\lambda^2 \frac{(1-\Omega^2)}{(1+\Omega^2)^3} + \mathcal{O}(\lambda^3)$$

flow bounded, L. ghost killed!
Due to wave fct. renormalization
$\Omega = 1$ betafunction vanishes
to all orders!

$\lambda[\Lambda]$ diverges in comm. case



- perturbation theory remains valid at all scales!
- non-perturbative construction of the model possible!

Gurau, Magnen, Rivasseau, Tanasa new ren.m: add $\int \Phi^2 \frac{\alpha}{p^2}$

Symmary

- Found new type of Wedge local quantum fields
- Found renormalizable ncQFT for matter fields
- β - function vanishes identically (to all orders of pertubation theory) in a special model,
  constructive procedure possible?
- Gauge fields formulated,
  BRST, BV formulation ok, ghost for ghost needed,
  renormalizable? deformed Borel summable Standard model?
- nc gravity?

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