BCFT: Anomalies, Entanglement, Holography

Workshop "Boundary and Defect Conformal Field Theories: Open Problems and Applications"
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topics of the talk:

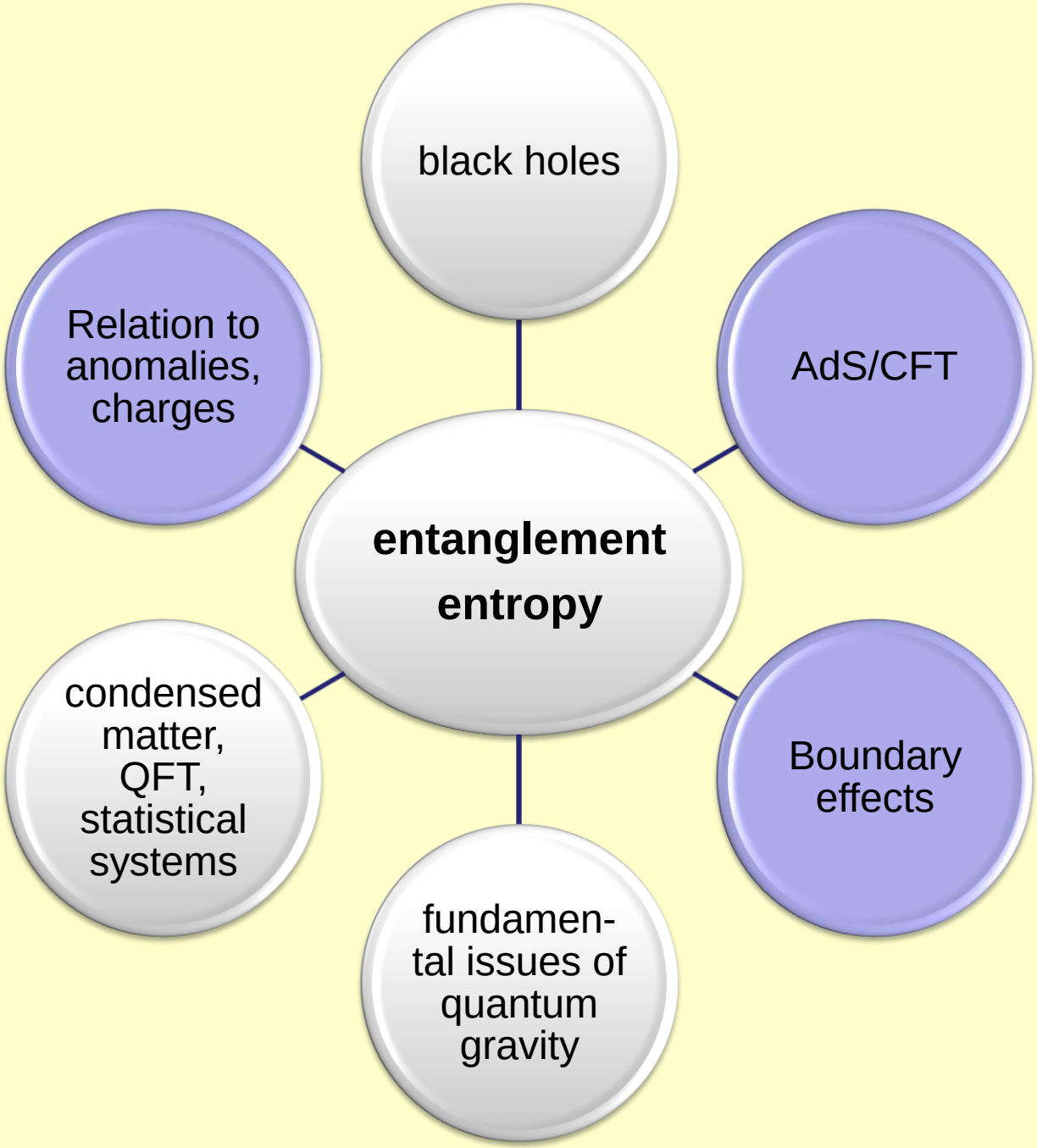
anomalies in BCFT and the entropy of entanglement (EE), when the entangling surface crosses the boundary;

the focus is on the following:

- boundary charges in the BCFT anomaly in d=4, relation between these charges and other properties of BCFT (bulk charges, correlation fns);

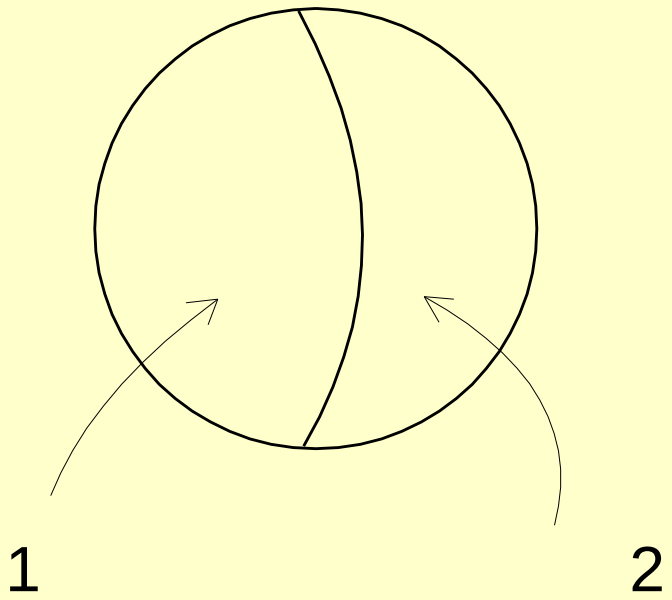
- logarithmic terms in EE for BCFT;

- recent progress in AdS/CFT description of boundary terms in the anomaly and EE.


black holes
Relation to anomalies, charges
AdS/CFT
entanglement entropy
condensed matter, QFT, statistical systems
Boundary effects
fundamen-tal issues of quantum gravity

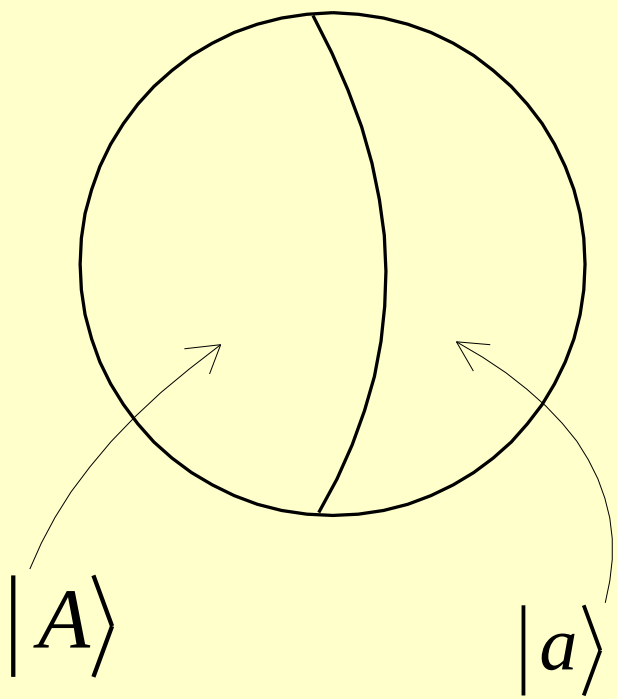
Quantum entanglement

<u>quantum mechanics</u>:
states of subsystems may not be described independently
= states are entangled



<u>importance:</u>
studying correlations of different systems (especially at strong coupling), critical phenomena and etc

reduced density matrix- a general definition



$$\rho(A,a\,|\,B,b)$$

$$\rho_1(A\,|\,B)=\sum_a \rho(A,a\,|\,B,a),$$

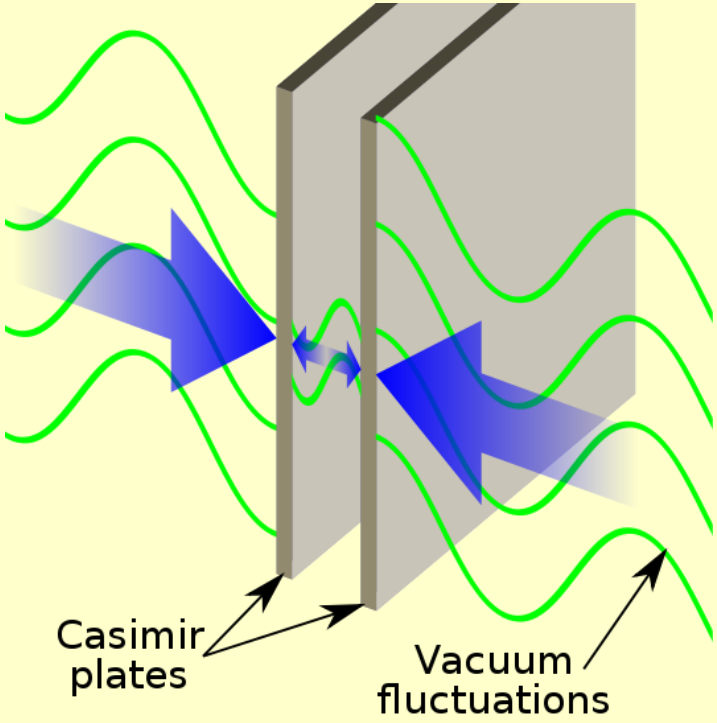
$$\rho_2(a\,|\,b)=\sum_A \rho(A,a\,|\,A,b),$$

$$\rho_1 = Tr_2\,\rho, \quad \rho_2 = Tr_1\,\rho,$$

$$S_1 = -Tr_1\,\rho_1 \ln \rho_1, \quad S_2 = -Tr_2\,\rho_2 \ln \rho_2$$

$S_1 = S_2$ for pure states

Why boundaries and EE?



- Boundaries result in observable effects in QFT (the Casimir forces);

- Casimir energies in BCFT are determined by charges in anomalies;

- In odd dimensions anomalies are pure boundary effects;

- EE is sensitive to boundaries: new information about physics of boundaries in QFT (how states are entangled across the boundary): applications….

- EE and Renyi E on manifolds with conical singularities, EE in BCFT= CFT with intersecting boundaries and defects (new types of charges, F-theorems in 3d, relation to correlation functions,…)

We consider EE when an entangling surface crosses the boundary

Works on low-dimensional BCFT related to this talk

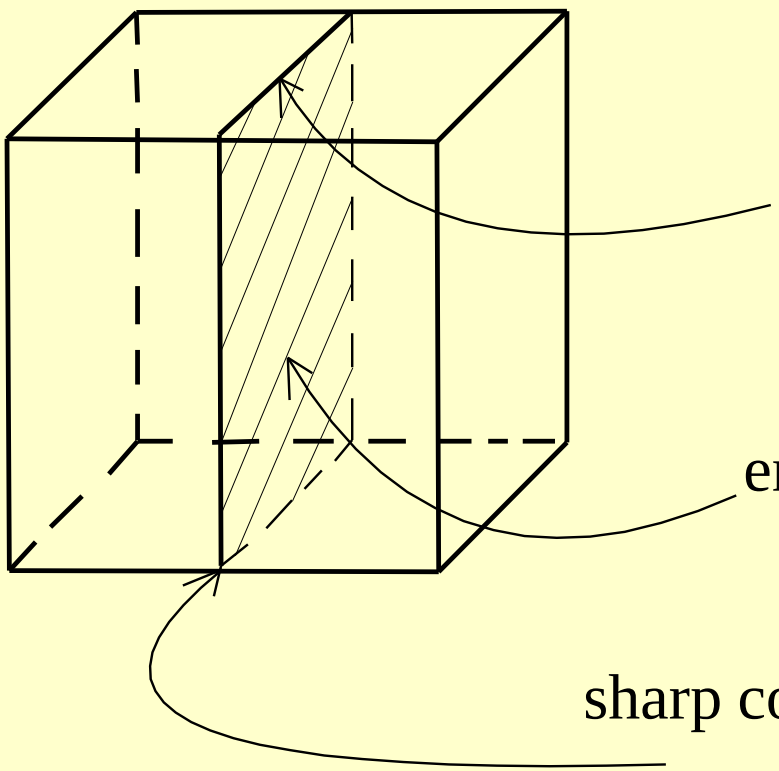
2D:

J. L. Cardy, Nucl. Phys. B 324, 581 (1989);
I. Affleck and A. W. W. Ludwig, Phys. Rev. Lett. 67, 161 (1991);
…..

3D:

R.C. Myers, A. Sinha, PRD 82 (2010) 046006,….
H.Casini and M.Huera, PRD 85 (2012) 125016
K. Jensen and A. O'Bannon, PRL 116 (2016) 091601
S. Solodukhin, PLB 752 (2016) 131
….
D. Seminara, J. Sisti, E. Tonni, 1708.05080 [hep-th]

first studies of boundary effects in 4D QFT's



$$S(B) \sim \frac{A(B)}{\varepsilon^2} + \frac{P}{\varepsilon} + \boxed{s_{\log}} \ln \varepsilon, \qquad \boxed{s_{\log} = C(\alpha_i),}$$

ε is UV cutoff

Fursaev, PRD73, 124025 (2006)

Wilczek, Hertzberg, PRL 106, 050404 (2011)

Boundary terms appear in

$s_{\log}$ - the ‘logarithmic part’ of EE

$$S(B) \sim \frac{A(B)}{\varepsilon^2} + \frac{P}{\varepsilon} + s_{\log} \ln \varepsilon,$$

one expects that the logarithmic part of EE is related to the conformal anomaly and may have a holographic description

trace anomaly in d=4:

local conformal anomaly

$$\left\langle T^{\mu}_{\mu} \right\rangle = -2aE - cI - \frac{c'}{24\pi^2}\nabla^2 R$$

$$E = \frac{1}{16\pi^2}\left(R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \right)$$ – "density" of the Euler n.

$$I = -\frac{1}{16\pi^2} C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho}, \quad C_{\mu\nu\lambda\rho}$$ – the Weyl tensor

"bulk charges" a, c

a- monotonically decreases under RG flow from UV to IR

suggested by J. Crardy, PLB 215, 749-752 (1988),

proved by Z.Komargodski and A.Schwimmer, JHEP 12 (2011)099

bulk charges in d=4:

relation to correlation functions

$$\left\langle T^{\mu}_{\mu} \right\rangle = -2aE - cI - \frac{c'}{24\pi^2}\nabla^2 R$$

$$\left\langle T_{\mu\nu}(x) T_{\lambda\rho}(0) \right\rangle \simeq \frac{C_T}{x^8} I_{\mu\nu,\lambda\rho}(x)$$

$$c = \frac{\pi^2}{40} C_T$$

H.Osborn, A.C.Petkou (hep-th/9307010)

Logarithmic term in EE in d=4
(arguments for general CFT, checked for free CFT)

$$S(B) \sim \frac{A(B)}{\varepsilon^2} + \frac{P}{\varepsilon} + s_{\log} \ln \varepsilon,$$

$$s_{\log} = aF_a + cF_c + bF_b \quad \text{(no boundaries)}$$

- Ryu,Takayanagi, JHEP 0608, 045 (2006),
- Solodukhin, PLB 665, 305 (2008)
- Fursaev, Patrushev, Solodukhin, PRD 88, 044054 (2013)

$$c = b \qquad \text{for CFT's}$$

conformal charges in the trace anomaly of a CFT uniquely fix the logarithmic term in EE (no boundaries) !

3 invariants on a smooth entangling surface *B* in d=4 (no boundaries)

$$F_a = -\frac{1}{2\pi}\int_B \sqrt{\sigma}\, d^2x\, R(B) \quad , \qquad R(B) - \text{scalar curvature of } B$$

$$F_c = \frac{1}{2\pi}\int_B \sqrt{\sigma}\, d^2x\, C_{\mu\nu\lambda\rho} n_i^\mu n_j^\nu n_i^\lambda n_j^\rho \quad , \quad C_{\mu\nu\lambda\rho} - \text{Weyl tensor of } M \text{ at } B,$$

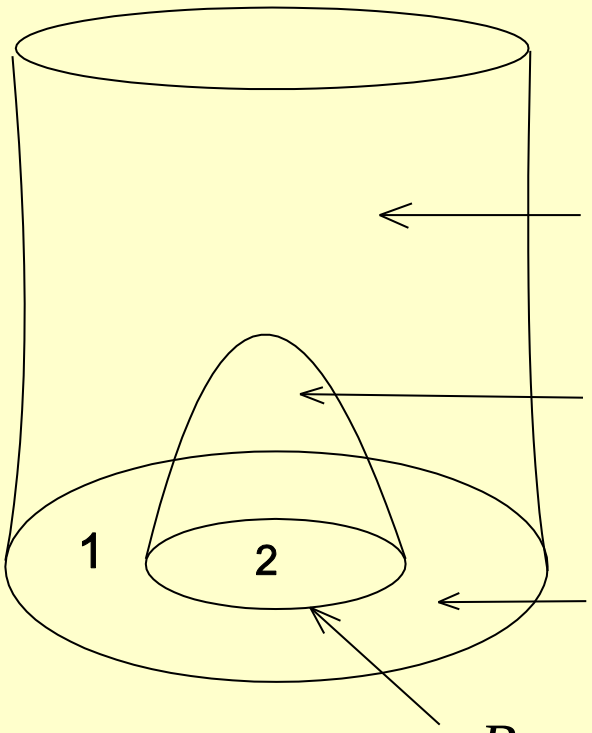
$$F_b = \frac{1}{2\pi}\int_B \sqrt{\sigma}\, d^2x \left(\frac{1}{2}\mathrm{Tr}(k_i)\mathrm{Tr}(k_i) - \mathrm{Tr}(k_i k_i) \right),$$

$(k_i)_{\mu\nu}$ – extrinsic curvatures of B, n_i – normal vectors

F_a, F_b, F_c – are invariant with respect to the Weyl transformations $g_{\mu\nu}'(x) = e^{2\omega(x)} g_{\mu\nu}(x)$

Holographic Formula for Entanglement Entropy

Ryu and Takayanagi,
hep-th/0603001, 0605073



AdS_{d+1} (bulk space)

$\tilde{B}$ minimal (least area) surface in the bulk

4d space-time manifold (asymptotic boundary of AdS)

B separating surface

entropy of entanglement

$$S = \frac{\tilde{A}}{4G^{(d+1)}}$$

is measured in terms of the area of $\tilde{B}$

$G^{(d+1)}$ is the gravity coupling in AdS

Holographic EE, strong couplings (Ryu-Takayanagi formula)

volume of a holographic surface $\tilde{B}$ in AdS

$$A(\tilde{B}) = \frac{1}{2\varepsilon^2} A(B) + \frac{\pi}{2}(F_a + F_c + F_b)\ln\frac{\mu}{\varepsilon} + \ldots$$

$z = \varepsilon -$ position of the boundary (a UV cutoff in CFT)

(expansion for $A(\tilde{B})$ first found by A.Schwimmer and S.Theisen, arXiv:0802.1017)

$$S(B) = \frac{A(\tilde{B})}{4G_5} \sim \frac{N^2\Lambda^2}{4\pi} A(B) + \frac{1}{4} N^2 (F_a + F_c + F_b)\ln \mu\Lambda + \ldots$$

use AdS/CFT dictionary: $\frac{1}{G_5} = \frac{2N^2}{\pi}, \quad \varepsilon = 1/\Lambda$

one reproduces correctly the structutre of the leading divergences and exact value of the logarithmic part of the entropy

The integrated conformal anomaly (ICA)

If a classical theory is scale invariant :

$$g'_{\mu\nu}(x) = e^{2\sigma(x)} g_{\mu\nu}(x),$$

the trace of the stress - energy tensor is zero, $T^{\mu}_{\mu} = 0$; classical property is

broken for quantum everages of the corresponding (renormalized) operators

$$\boxed{\left\langle \hat{T}^{\mu}_{\mu} \right\rangle \neq 0 -}$$ local (trace) anomaly

the property is known as the conformal or scale anomaly;

we also use the integrated conformal anomaly (ICA)

$$\boxed{\mathrm{A} = \partial_{\sigma} W[e^{2\sigma} g_{\mu\nu}]_{\sigma=0}} = \int_{M} \left\langle \hat{T}^{\mu}_{\mu} \right\rangle \sqrt{g}\, d^{n}x + \text{b.t.+c.s+...}$$

of the effective action W

ICA includes contributions of boundary terms, conical singularities, defects

ICA and Renyi entropies:

$$S_q = \frac{1}{1-q}(\ln \mathrm{Tr}\,\bar{\rho}^q - q \ln \mathrm{Tr}\,\bar{\rho})$$ - Renyi entropy,

where $\rho = \bar{\rho} / \mathrm{Tr}\bar{\rho}$ is normalized density matrix, calculate S_q

for $q = 2, 3, \ldots$ and continue, $S_{EE} = \lim_{q \to 1} S_q$ - entanglement entropy;

effective action on manifolds with conical singularities (q-replicas)

$$W_q \equiv -\ln \mathrm{Tr}\,\bar{\rho}^q,$$

$$S_q = \frac{1}{1-q}(qW_1 - W_q), \quad S_{EE} = W_1' - W_1, \quad W_q' \equiv \partial_q W_q$$

ICA for renormalized W_q: $\mathrm{A}_q \equiv \partial_\sigma W_q[e^{2\sigma} g]$

$$s_{\log} = s_{\log,1} = \mathrm{A}_1' - \mathrm{A}_1$$

Some new results for ICA:

$$S_1' = \frac{1}{2} W_1'' \sim c \sim \mathrm{C}_T$$ E.Perelmutter (1308.1083)

for a spherical entangling surface, low spin CFT, any dimension;

a recent suggestion by A.Tseytlin and M.Beccaria (1707.0245)

$$\boxed{\mathrm{C}_T \sim c = -\frac{1}{4}\mathrm{A}_1'' - \frac{1}{2}\mathrm{A}_1', \qquad a = -\frac{1}{4}\mathrm{A}_1}$$

works for the anomaly for free conformal higher spin (CHS) fields

on squashed 4-spheres with conical singularities with angle def. $2\pi(1-\mathrm{q})$

$$c - 3a = -\frac{1}{4} s_{\log,1}' - \frac{3}{4} s_{\log,1}$$

$s_{\log,\, q}$ – logarithmic part of Renyi entropy

the rest of the talk:

BCFTs in d=4, EE, intersecting boundaries and conical defects

For free BCFTs computations are based on conformal invariance of the heat coefficient

- Let the classical action be invariant

$$I[\phi, g] = \int d^d x \sqrt{g}\, \phi(x) L \phi(x)$$

under conformal transformations:

$$g_{\mu\nu}'(x) = e^{2\omega(x)} g_{\mu\nu}(x), \quad \phi'(x) = e^{k\omega(x)} \phi(x),$$

$$I[\phi, g] = I[\phi', g']$$

- Let boundary conditions respect the conformal invariance,

for an example: $\phi\big|_{\partial\Sigma} = 0$ (the Dirichlet condition)

heat trace asymptotics $\mathrm{Tr}\, e^{-tL} \sim \sum_{p=0} A_p t^{p-d/2}$

Then the heat coefficient $A_{p=d}$ is a conformal invariant:

$$\boxed{\mathrm{A} = A_{p=d}[g] = A_{p=d}[g']}$$

Boundary terms in ICA in d=4:

a general structure of the integrated anomaly in the presence of boundaries

$$\mathrm{A} = -2a\chi_4 - ci_4 + q_1 j_1 + q_2 j_2 \quad , \; i_4 = \int_M I$$

$$\chi_4 = \int_M E + \frac{1}{32\pi^2}\int_{\partial M} Q$$ – Euler characteristic of M;

$$Q = -8\left[\det K_{ab} + (\hat{R}_{ab} - \frac{1}{2} g_{ab}\hat{R})K^{ab}\right]$$

$$j_1 = \frac{1}{16\pi^2}\int_{\partial M} C_{\mu\nu\lambda\rho} n^\nu n^\rho \hat{K}^{\mu\lambda} \quad , \quad j_2 = \frac{1}{16\pi^2}\int_{\partial M} \mathrm{Tr}(\hat{K}^3)$$

$\hat{K}^{\mu\lambda}$ – traceless part of the extrinsic curvature of the boundary ∂M,

conformal structure of A has been studied first for a scalar field

with the Dirichlet boundary condition (Dowker & Schofield, 1990),

also C.Herzog, K.W.Huang, K.Jensen, JHEP 1601 (2016) 162.

Results for boundary charges in d=4

(DF, JHEP 1512, 112 (2015))

- boundary "charges" q_k are calculated for CFT's, spins 0, 1/2, 1

Results for d=4

CFT	a	c	q1	q2	b.cond.
Scalar	1 / 360	1 / 120	1 / 15	2 / 35	Dirichlet
Scalar	1 / 360	1 / 120	1 / 15	2 / 45	Robin
Spinor	11 / 360	1 / 20	2 / 5	2 / 7	Mixed
Maxwell	31 / 180	1 / 10	12 / 15	16 / 35	Absolute
Maxwell	31 / 180	1 / 10	12 / 15	16 / 35	Relative

- For an Abelian gauge field "charges" do not depend on the boundary conditions:

$$\vec{E}_{\parallel} = \vec{B}_{\perp} = 0 \quad \text{or} \quad \vec{E}_{\perp} = \vec{B}_{\parallel} = 0$$

Properties of boundary chargers in d=4

- $q_1 = 8c,$
- as consequence, integrated anomaly has a correct Gibbons-Hawking type

boundary term: the functional

$$c\int_M C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho} + q_1\int_{\partial M} C_{\mu\nu\lambda\rho}n^\nu n^\rho \hat{K}^{\mu\lambda},$$

under variations has no normal derivatives of the bulk metric on the boundary

(Solodukhin, PLB 752, 131 (2016))

- Boundaries yield a single independent boundary charge q_2 (at $\int \mathrm{Tr}\,\hat{K}^3$)
- q_2 is sensitive to boundary conditions
- q_2 appears in RG equation for 3-point correlation function of the stress-energy

tensor near the boundary (Kuo-Wei Huang (2016), 1604.02138[hep-th])

relation to 2-point correlation functions

C.Herzog and Kuo-Wei Huang (1707.06224[hep-th])

- relation $q_1 = 8c$ is required by derivation of the bulk 2-point correlation fn in free BCFT:

$$\left\langle T_{\mu\nu}(x) T_{\lambda\rho}(0) \right\rangle \simeq \frac{C_T}{x^8} I_{\mu\nu,\lambda\rho}(x) \quad , \quad C_T \sim c$$

- q_1 determines the boundary correlator (including interacting BCFTs)

$$\left\langle T_{nn}(x) T_{nn}(0) \right\rangle \simeq \frac{3}{4} \frac{q_1}{x^8}$$

$$T_{nn}(x) = n^\mu n^\nu T_{\mu\nu}(x), \quad x \in \partial M$$

boundary charges determine the Casimir energies

R.-X. Miao and C.-S. Chu (1706.09652[hep-th]) by using results

by D.Deutch and P.Candelas (PRD 20 (1979) 3063)

$$\boxed{\left\langle T_{\mu\nu}(x,y)\right\rangle \simeq \frac{1}{16\pi^2}\left[\frac{q_1}{y^3}\hat{K}_{\mu\nu}(x)+\frac{t_{\mu\nu}(x)}{y^2}+\ldots\right]}$$

$x-$ coordinate on the boundary, $y-$ proper distance from a point to ∂M

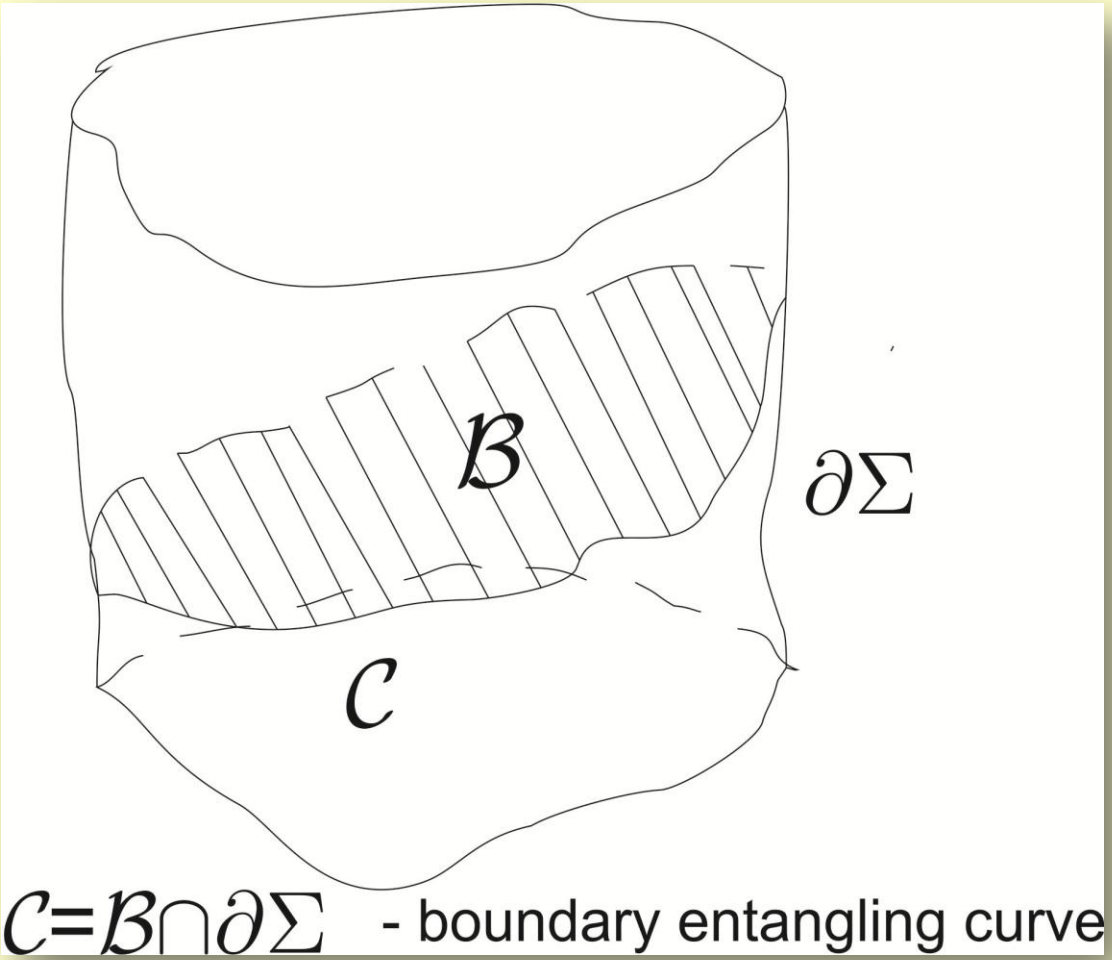
$$t_{\mu\nu}(x)=\frac{1}{3}q_1\left(-2n_{(\mu}h^{\lambda}_{\nu)}(\partial_\lambda K+R_{\lambda\alpha}n^\alpha)+(n_\mu n_\nu-h_{\mu\nu}/3)\hat{K}_{\alpha\beta}\hat{K}^{\alpha\beta}\right)$$

$$+\frac{1}{6}q_1K\hat{K}_{\mu\nu}-\left(2q_1+3q_2\right)\left(\hat{K}_{\mu\beta}\hat{K}^{\beta}_{\nu}-\frac{1}{3}h_{\mu\nu}\hat{K}_{\alpha\beta}\hat{K}^{\alpha\beta}\right)$$

holds for free spin 0 (Dirichlet and Robin) and gauge BCFT;

$$g^{\mu\nu}\left\langle T_{\mu\nu}(x,y)\right\rangle = O(1)$$

EE for entangling surface crossing the boundary



Logarithmic terms in EE in CFT's (d=4)

$$s_{\log}(B) = aF_a + cF_c + bF_b + dF_d + eF_e$$

terms on $C = B \bigcap \partial M$

$$F_a = -\frac{1}{2\pi}\left(\int_B \sqrt{\sigma} d^2x\, R(B) + \int_C ds\, k\right) = -2\chi_2(B) \quad ,$$

$\chi_2(B) -$ Euler characteristics of B

$F_c, F_b -$ are not modified in the presence of boundaries

$F_d = F_d(C), \quad F_e = F_e(C) -$ terms of a new type (pure boundary effects)

$F_d, F_e -$ are dimensionless Weyl invariant (for CFT's) integrals on C

$d, e -$ are boundary coefficients in the entropy

Do d, e are related to charges in the integrated conformal anomaly?

Invariants and coefficients

$$F_d = \frac{3}{2\pi}\int_C ds\, \psi_1\, \hat{K}_{\mu\nu} u^\mu u^\nu \,, \quad u^\nu - \text{tangent vector to } C$$

$$F_e = \frac{1}{\pi}\int_C ds\, \psi_2\, (N\cdot p_i)(\hat{k}_i)_{\mu\nu} u^\mu u^\nu \quad ,$$

$(\hat{k}_i)_{\mu\nu} -$ traceless part of extrinsic curvature of B,

$\psi_1(\alpha), \psi_2(\alpha) -$ are unknown functions of α - a tilt angle of B and ∂M

(between normal vector to ∂M and a normal vector to ∂M in B)

coefficient d at F_d can be calculated when B is orthogonal to ∂M ($\psi_1(0) \equiv 1$)

Fursaev, JHEP 1307, 119 (2013), Fursaev, Solodukhin, Berthiere,

Astaneh, PRD (2017)

Results for d=4 (orthogonal configuration)

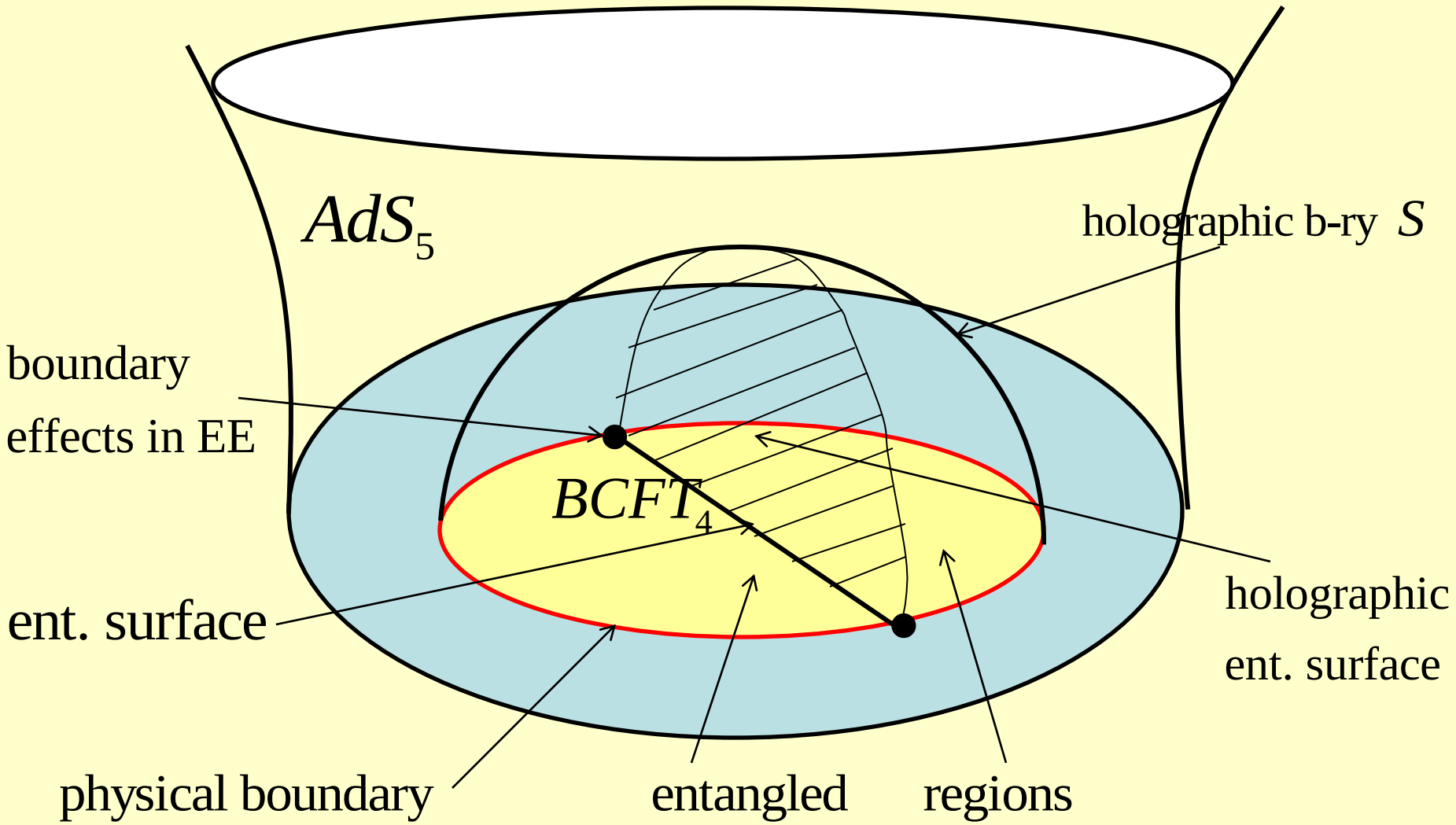
CFT	a	c	q2	d	b.cond.
Scalar	1 / 360	1 / 120	2 / 35	1/60	Dirichlet
Scalar	1 / 360	1 / 120	2 / 45	-1/90	Robin
Spinor	11 / 360	1 / 20	2 / 7	1/60	Mixed
Maxwell	31 / 180	1 / 10	16 / 35	7/60	Absolute
Maxwell	31 / 180	1 / 10	16 / 35	7/60	Relative

- For gauge fields extra arguments are needed
- A new 'magic' relation !

$$d = 3\,a - 14\,c + 35/12\;\; q_2$$

- d depends on boundary conditions

Holographic BCFT



BCFT in D=4:

$\mathrm{N} = 4, \quad SU(N)$ super YM at weak coupling with b.c. which break 1/2 of supersymmetries

boundary effects can be calculated at a weak coupling:

- boundary terms in the integarted conformal anomaly
- boundary terms in EE

Integrated anomaly in 4D BCFT

$\mathrm{N}=4, \quad SU(N)$ super YM at weak coupling, 1/2 of susy's are broken

$$\boxed{\mathrm{A} = -2a\chi_4 - ci_4 + 8cj_1 + q_2 j_2}$$

$$a = c = \frac{N^2-1}{4}, \quad q_2 = \frac{4}{3}(N^2-1)$$

see Astaneh, Solodukhin PLB 769 (2017) 25

Log-term in EE in 4D BCFT

$\mathrm{N} = 4, \quad SU(N)$ super YM

$$s_{\log} = \frac{N^2 - 1}{8\pi}\left[\left(\int_B R_B + 2\int_C k_B\right) + \int_B \mathrm{Tr}\, k_i^2 - 2\int_C \hat{K}_{\mu\nu} u^\mu u^\nu\right]$$

M is flat, B is orthogonal to ∂M $(\psi_1(0) \equiv 1)$, $C = \partial M \cap B$,

see Astaneh, Berthiere, Fursaev, Solodukhin, PRD (2017)

Definition of the 'holographic boundary' (HB)?:

- Takayanagi, PRL107 (2011) 101602, (restricted version – Miao, Chu, Guo): HB is determined by properties of boundary terms in gravity action

$$I_{AdS} = I_{\text{bulk}} + I_{\text{bound}}$$

$$I_{\text{bound}} = -\frac{1}{8\pi G}\int_S \left(K_S + T\right) \quad , \; T \text{ - a free parameter}$$

HB equation $\quad K_S = -\dfrac{d}{d-1}T \quad ,$ consistent with varaitional principle

- Astaneh and Solodukhin, PLB 769 (2017) 25: HB is a kind of brane governed by Nambu-Goto eqs

$$I_{\text{bound}} = -\frac{\lambda}{8\pi G}\int_S \quad , \; \lambda \text{ - is a constant}$$

HB equation $\quad K_S = 0 \quad ,$ minimal syrface equation

Prescription for the holographic EE:

$$S = \frac{A(\tilde{B})}{4G_5} \quad - \text{Ryu-Takayanagi formula}$$

$\tilde{B}$ – holographic surface in the bulk,

$\tilde{B}$ – is extended in AdS till the holographic boundary S

Results:

- minimal HB surface (Astaheh-Solodukhin prescription, Takayanagi, Miao et al prescription) reproduce exactly weak coupling results for the integrated anomaly and EE in 4D BCFT with ½ susy's,
  - if correct, it implies that new boundary charges in the anomaly and EE do not receive quantum corrections (same as for the bulk charges) ;

- for non-minimal HB surface (in restricted Takayanagi's prescription) boundary charges differ from charges at weak couplings:
  - the charges are not protected from ?
  - BCFT has different b.c.

GOOD NEWS: Holography seems to be able to deal with boundary effects

MORE WORK is to be done to fix prescriptions and draw conclusions

Geometric configuration

bulk metric

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}(-dt^2 + dr^2 + (\gamma_{ij} - k_{ij} r)^2 dx^i dx^j)$$

M is flat, $\partial M: \quad r = 0,$ holographic boundary: $r = f(\rho)$

entangling surface $B: \quad x^1 = 0$

holographic entangling surface $\tilde{B}: \quad x^1 = f(r, \rho)$

Comments:

- similar computations in d=3 (no local anomaly), similar conclusions;

- boundary charges and Casimir energies, boundary charges and two-point functions (including interacting theories) are related, an interesting topic for future research;

- analogs of C-theorems for boundary charges in 4d BCFT: unclear;

- entanglement in 4d: curvature effects are important to learn the full structure of boundary terms (have EE not been calculated so far by other methods);

- holography: there can be other versions of Ryu-Takayanagi formula for holographic EE with boundaries

Thank you for attention