QFT in AdS

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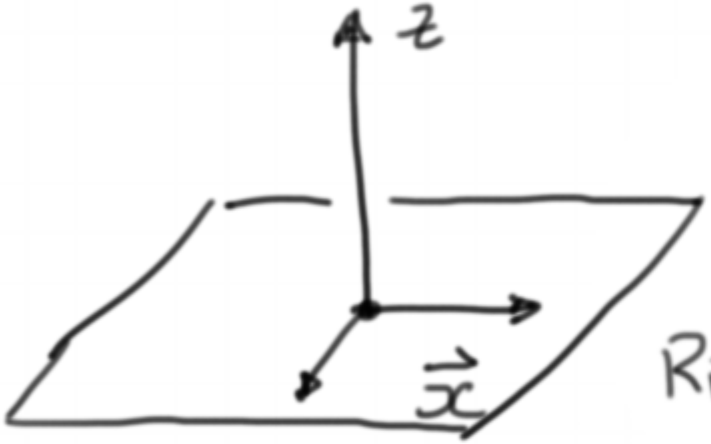
based on work with
M. Paulos, J. Penedones, J. Toledo, P. Vieira

Main idea: put a D-dim QFT on a fixed hyperbolic background space (AdS_D or H^D).

Geometry:

$$ds^2 = \frac{R^2}{z^2}\left(dz^2 + d\vec{x}^2\right)$$

$$z > 0, \quad \vec{x} \in \mathbb{R}^{D-1}$$



$$ds^2 = \left(\frac{r^2}{R^2}+1\right)d\tau^2 + \frac{dr^2}{\left(\frac{r^2}{R^2}+1\right)} + r^2 d\Omega^2_{D-2}$$

$$r > 0$$

$$Ric = -D(D-1)R^{-2}$$

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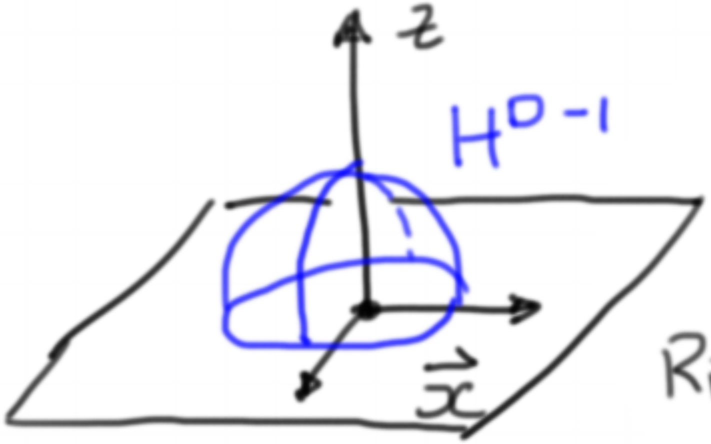

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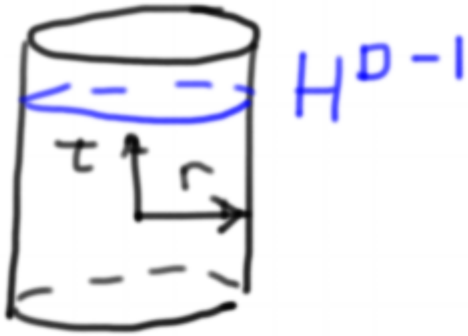

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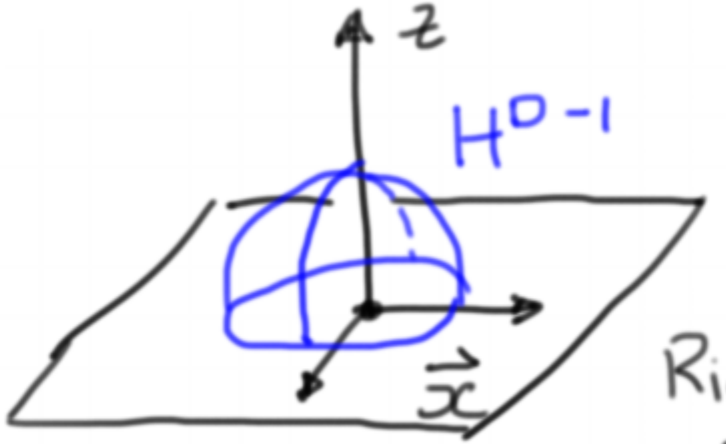

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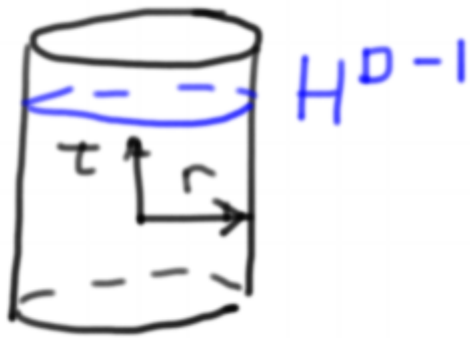


- isometries: $SO(D,1) \sim$ conf. algebra in $D-1$ dims.
- note $z\partial_z + \vec{x}\cdot\partial_{\vec{x}} \sim \partial_\tau$
- in "radial quantization": $\mathcal{H}$ on H^{D-1}

Before we start...

Q: Boundary conditions?

A: agnostic, but assume that $SO(D,1)$ is preserved.

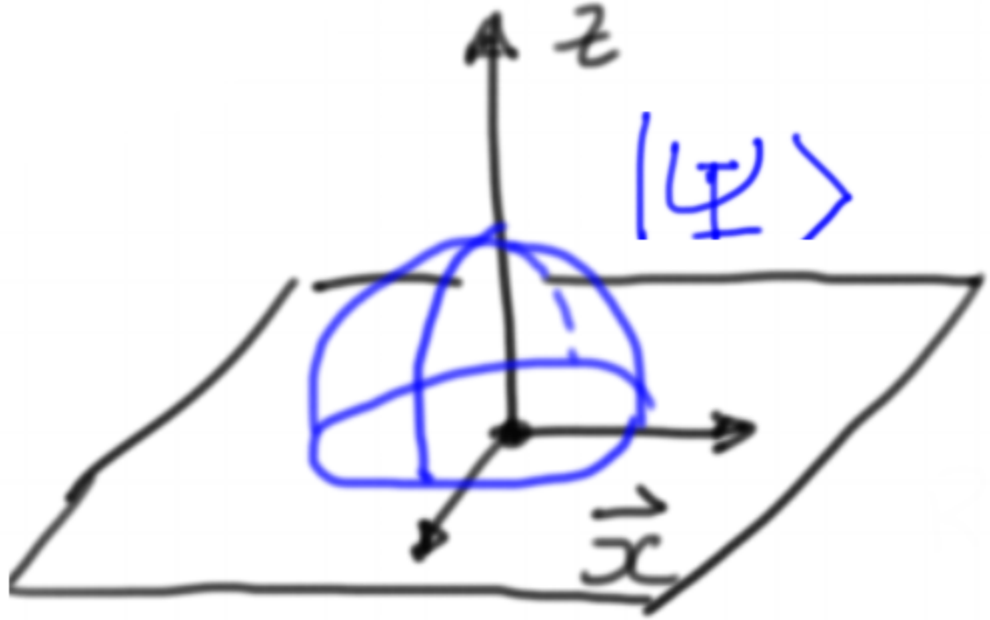
Q: Curvature couplings?

A: idem

Q: Older work?

A: Yes, [Callan-Wilczek 1990; Aharony, Berkooz, Tong, Yankielowicz 2012; ...]

Q: What is $\mathcal{H}$ on H^{D-1}?



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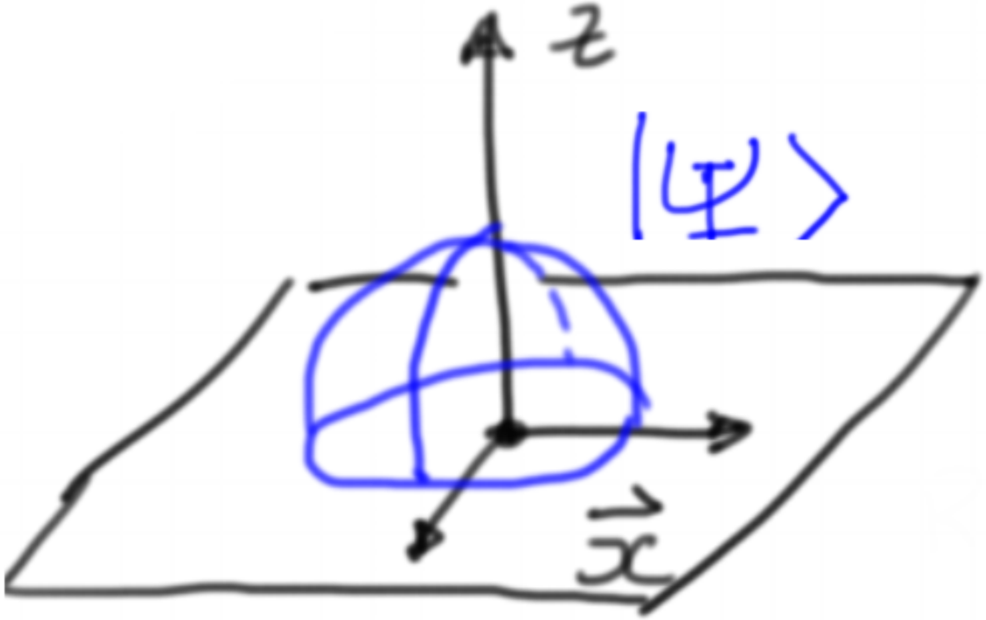


- Almost by construction, there exists a <u>bulk state – bdy operator correspondence</u>.

Q: What is $\mathcal{H}$ on H^{D-1}?

- We can use intuition from $D-1$-dim. CFT:

$\mathcal{O}_{\mu_1 \ldots \mu_N} |0\rangle$ primary

"Energy" Δ $\sim$ ∂_τ

Spin N $\sim$ S^{D-2}

$D-1$-dim. Lor. index

$[P_{\nu_1}, \ldots P_{\nu_M}, \mathcal{O}_{\mu_1 \ldots \mu_N}] \ldots] |0\rangle$

descendants

[Note: no stress tensor $T_{\mu\nu}$.]

Q: What is $\mathcal{H}$ on H^{D-1}?

- For a free massive QFT, clear bulk interpretation:

$\phi(0)$
single particle
$\Delta_\phi(\Delta_\phi - D + 1) = m^2 R^2$

$\partial .. \partial \phi(0)$
moving particle
$\Delta_\phi + L$

$\phi .. \partial .. \phi$
2-particles, moving
$2\Delta_\phi + L$

Q: What is $\mathcal{H}$ on $\mathbb{H}^{D-1}$?

- For an interacting gapped QFT we expect a similar pattern:

$\phi(0)$ single particle $\Delta_\phi(\Delta_\phi - D + 1) = m^2 R^2$ ✓

$\partial .. \partial \phi(0)$ moving particle $\Delta_\phi + L$ ✓

$\phi .. \partial .. \phi$ 2-particles, moving $2\Delta_\phi + L + \gamma_2$

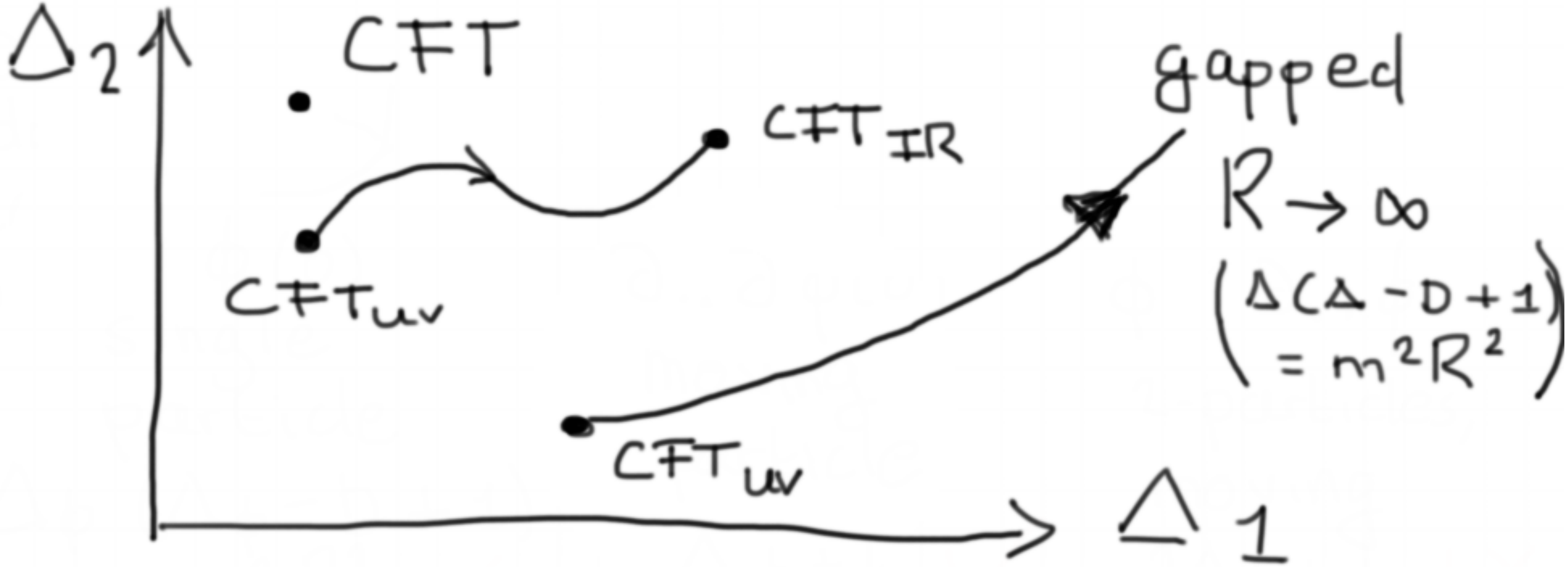
Q: What is $\mathcal{H}$ on H^{D-1}?

- For a CFT in AdS_D the curvature is irrelevant $\rightsquigarrow$ this is just BCFT.

Q: What is $\mathcal{H}$ on H^{D-1}?

- Note: Δ's depend on R

(for a QFT with a fixed scale μ:
$$\Delta(\mu R)$$
)



One way to construct boundary operators is as limits of bulk operators, i.e.

$$\mathcal{O}^B_I(\vec{x}) = \lim_{z \to 0} \hat{\mathcal{O}}(z, \vec{x})\, z^{-\Delta_I}$$

with $\Delta_I(\Delta_I - D + 1) = m_I^2 R^2$

if $\langle 0 | \hat{\mathcal{O}}(z, \vec{x}) | \mathcal{O}^B_I \rangle \neq 0$

We can think of this as an LSZ prescription for QFT on AdS.

The correlation functions of the boundary operators are familiar from (boundary) CFT:

$$\langle \mathcal{O}(x) \rangle = 0$$

$$\langle \mathcal{O}_I(x)\, \mathcal{O}_J(y) \rangle = \frac{\delta_{IJ}}{|x-y|^{2\Delta_I}}$$

etc.

What's more, one expects the conf. block decomposition:

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle = \sum_k \lambda_{12k} \lambda_{34k}$$ 

to converge as usual.

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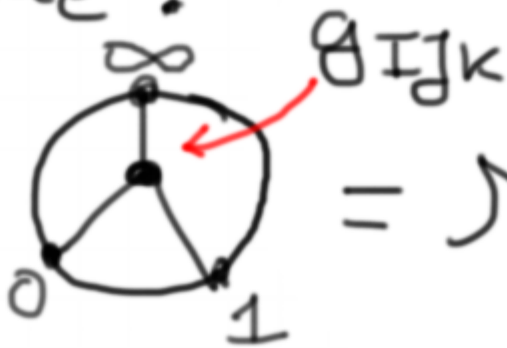
to converge as usual. $\Rightarrow$ Can we bootstrap QFT in AdS?

Let's take a closer look at 3-pt functions:

$$\langle \mathcal{O}_I(0)\, \mathcal{O}_J(1)\, \mathcal{O}_k(\infty) \rangle = \lambda_{IJk}$$

Suppose $\mathcal{O}_I$ correspond to 1-particle states. Then λ_{IJk} corresponds to 3-pt. bulk coupling and we can compute:

$\lambda_{IJk} =$ 

$= \mathcal{N}[\Delta_I, \Delta_J, \Delta_k]\, g_{IJk}$

$\lambda_{IJK} =$ 

$= \mathcal{N}[\Delta_I, \Delta_J, \Delta_K]\, g_{IJK}$

For later:

$$\mathcal{N}[\Delta_1, \Delta_1, \Delta_2] \underset{\substack{\Delta_i \to \infty \\ \mu = \Delta_2/\Delta_1 \text{ fixed}}}{=} P(\mu, D)\, \Delta_1^{(3-D)/4}\, Q(\mu)^{-\Delta_1} + \dots$$

(with $Q(\mu) > 1$)

so if g remains finite then $f \to 0$.

Suppose we have a boundary op. $\mathcal{O}_1(\vec{x})$ with self-OPE:

$$\mathcal{O}_1(\vec{x})\mathcal{O}_1(0) \sim \frac{\mathbb{1}}{|x|^{2\Delta_1}} + \lambda_{112}\frac{\mathcal{O}_2(0) + \text{desc.}}{|x|^{2\Delta_1 - \Delta_2}} + \left(\ldots \underset{\Delta \geqslant 2\Delta_1}{\text{op's with}} \ldots\right)$$

and assume $\mathcal{O}_1$ and $\mathcal{O}_2$ create stable 1-particle states (so $m_i^2 = \Delta_i(\Delta_i - D + 1)$, $i = 1, 2$, and $\Delta_2 \leq 2\Delta_1$).

Q: Can we bound $g_{112} = \lambda_{112}/\mathcal{N}$?

Q: Upper bound on λ_{112}^2 for

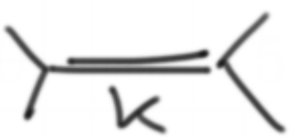
$$\mathcal{O}_1 \times \mathcal{O}_1 \to \mathbb{1} + \lambda_{112}\mathcal{O}_2 + (\ldots \Delta \geq 2\Delta_1 \ldots)$$

in <u>one-dimensional</u> conformal theory?

Methodology:

Conformal bootstrap methods of [Rattazzi, Rychkov, Tonni, Vichi (2008)] applied to

$$\langle \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_1 \rangle =$$

$\sum_k \lambda_{11k}^2$  $= \sum_k \lambda_{11k}^2$ 

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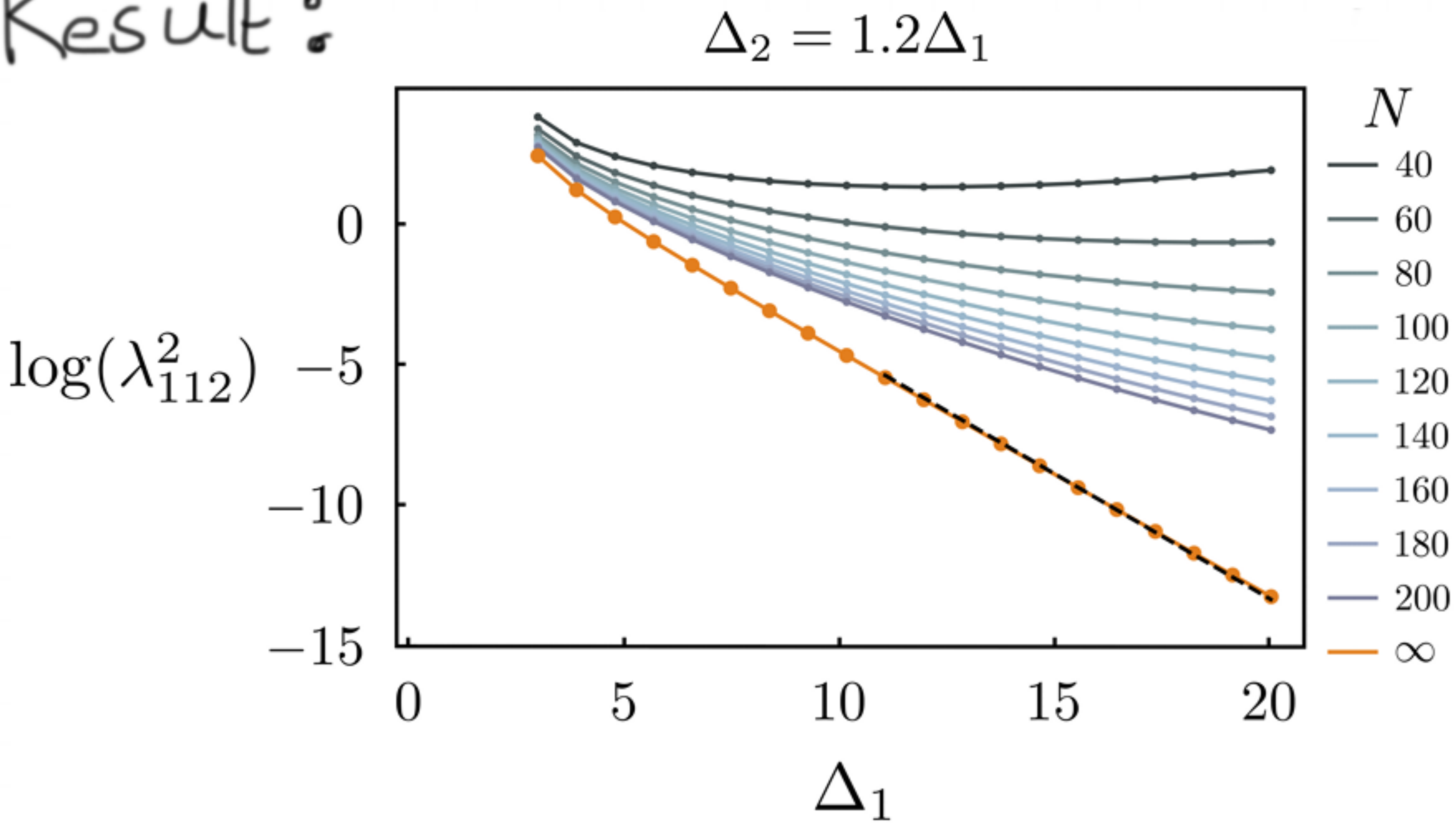
Result:



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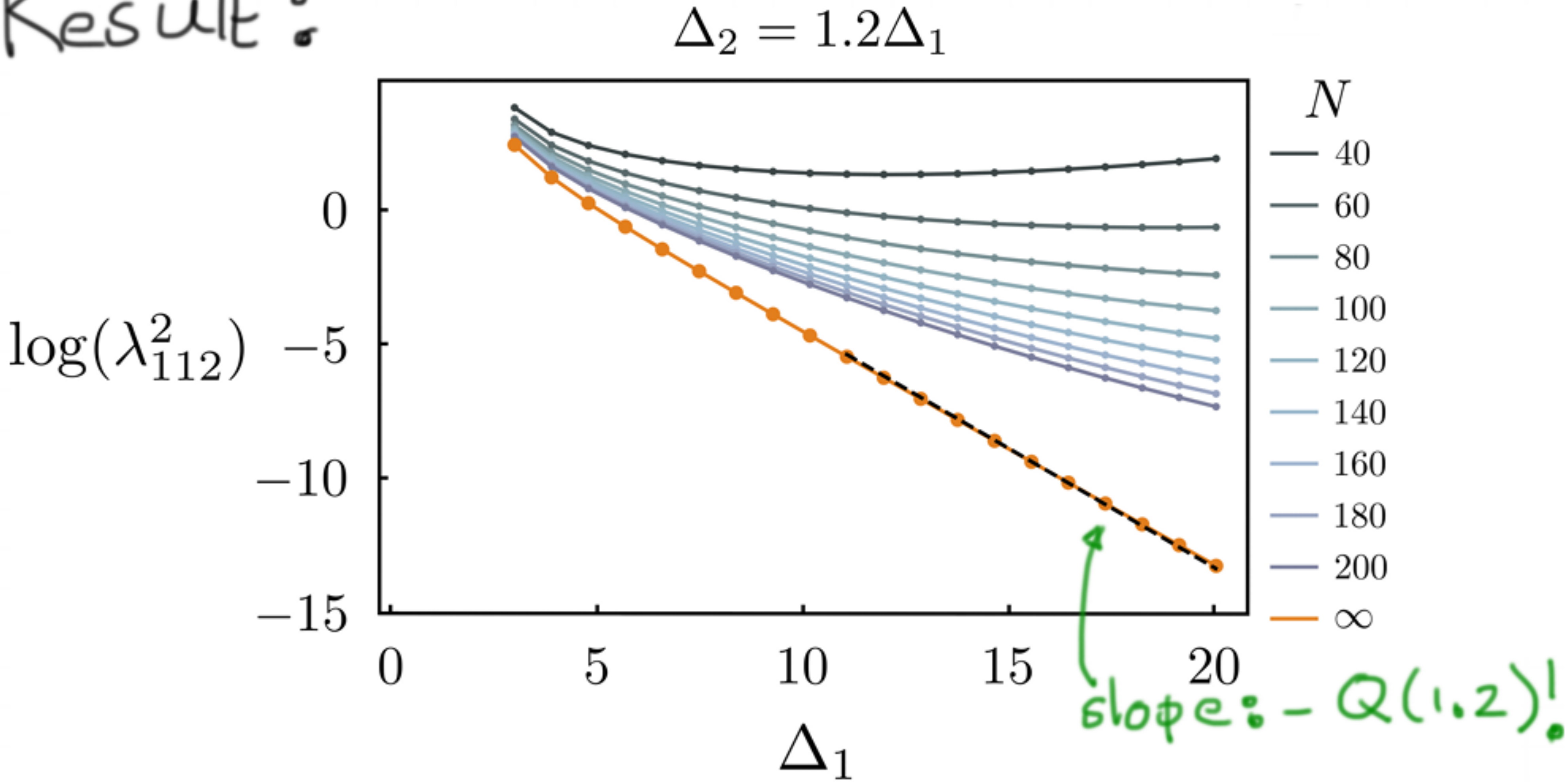
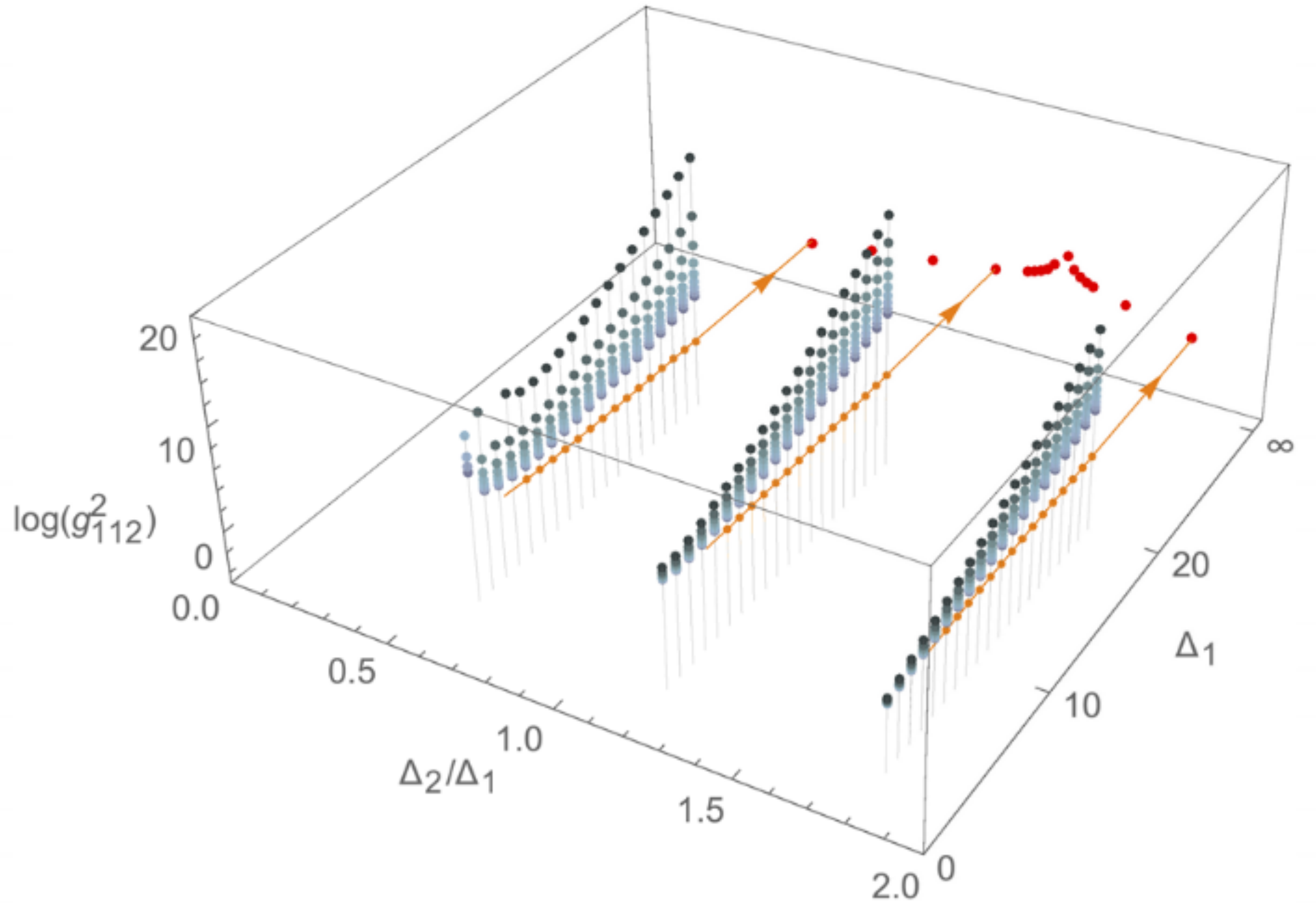
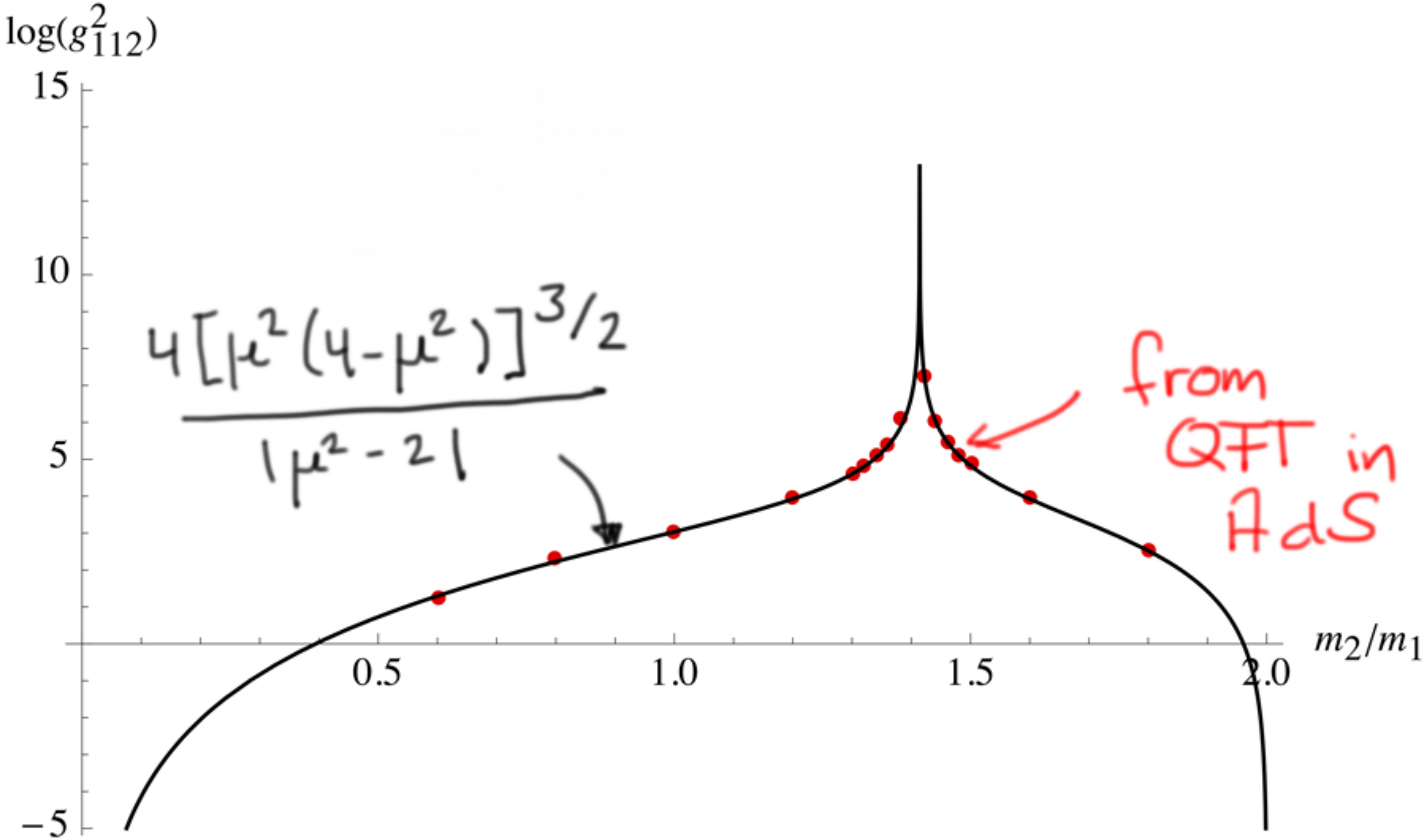


Multiple extrapolations to flat-space limit:



Final result:



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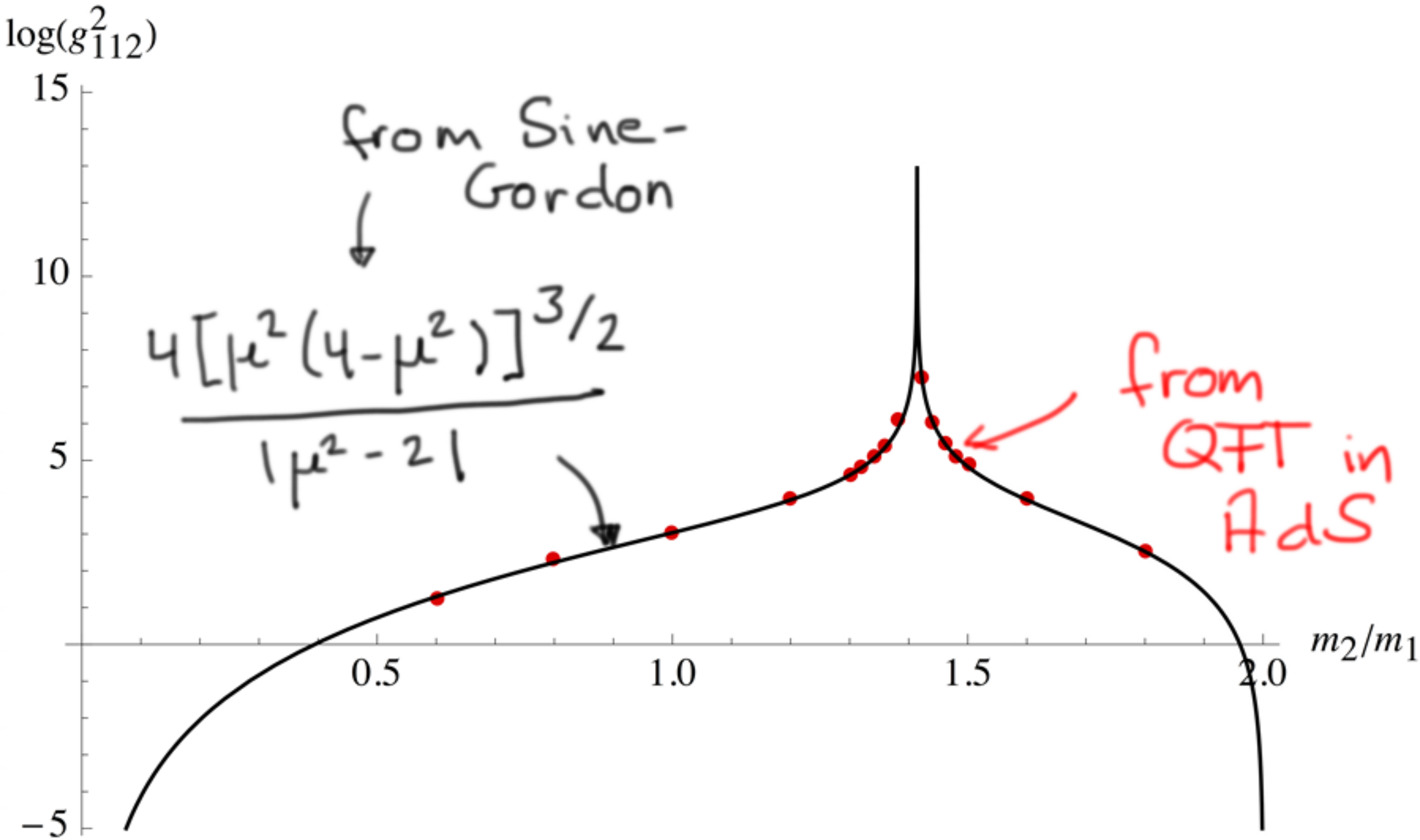

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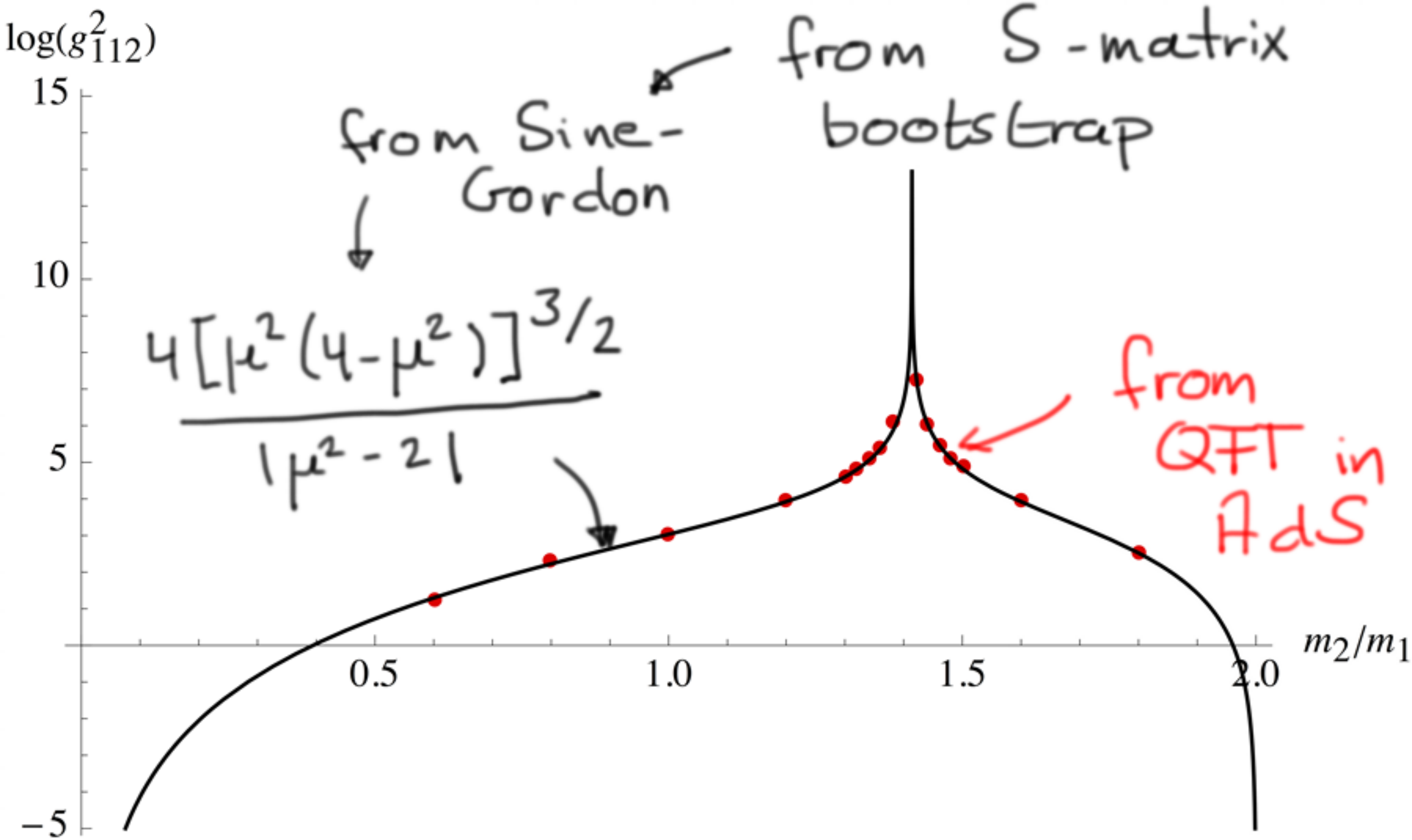

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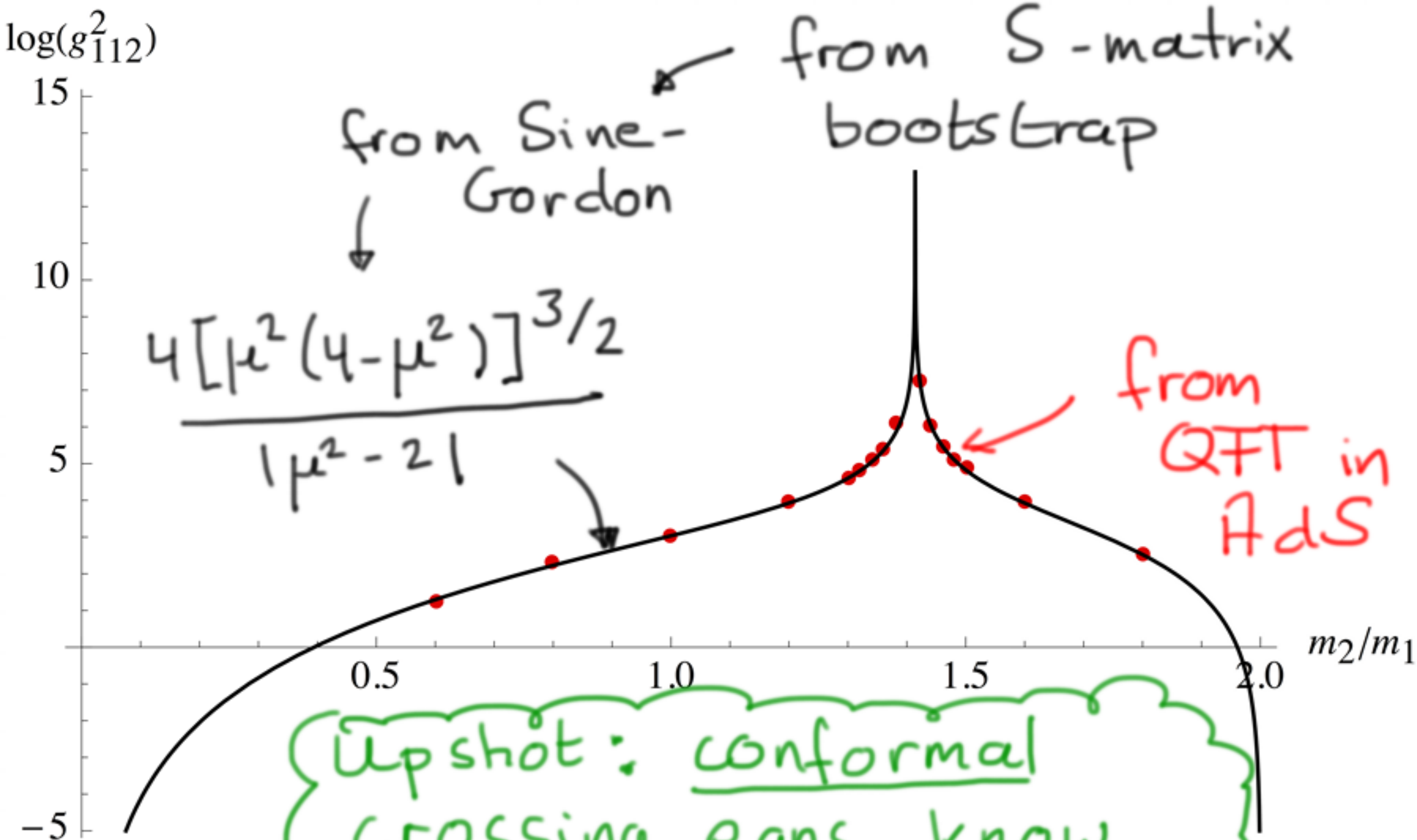
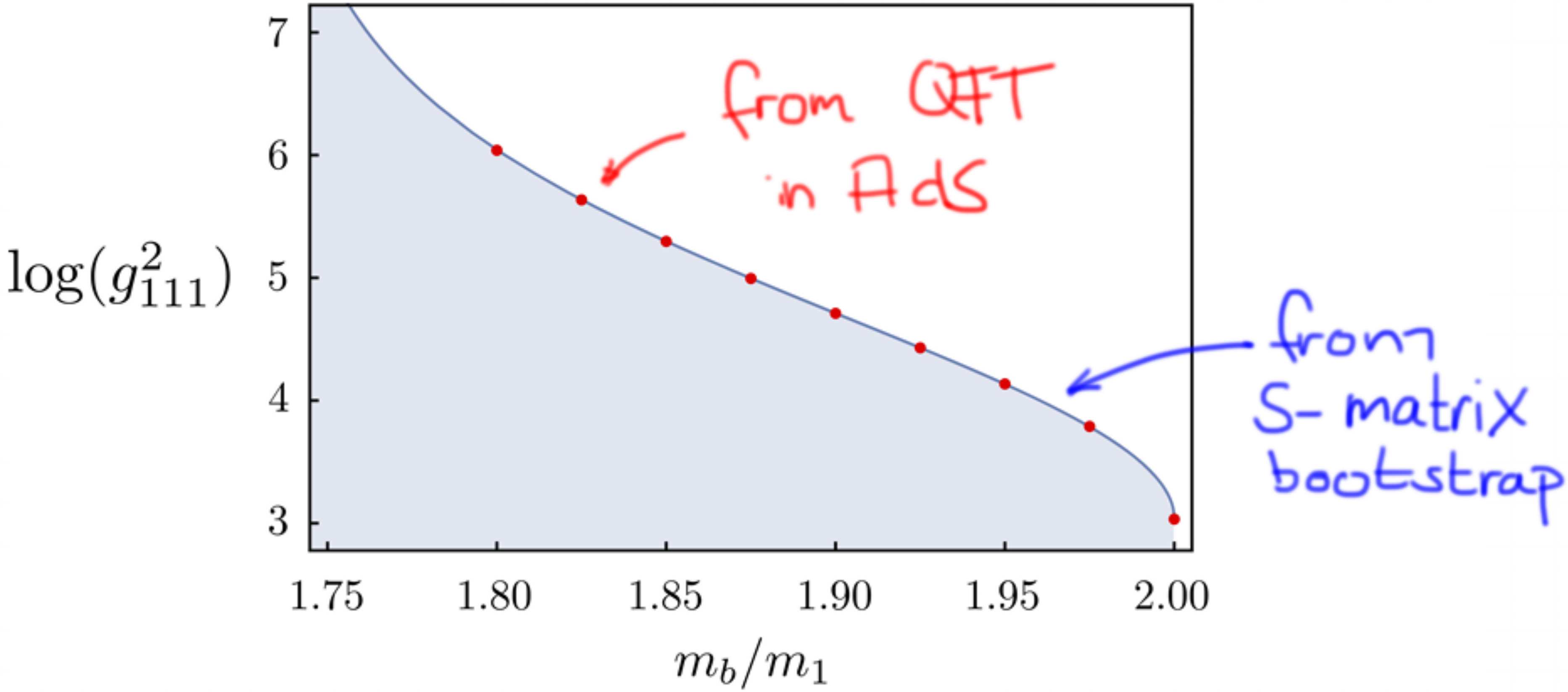


Upshot: conformal crossing eqns. know about flat-space QFT!

Second result:

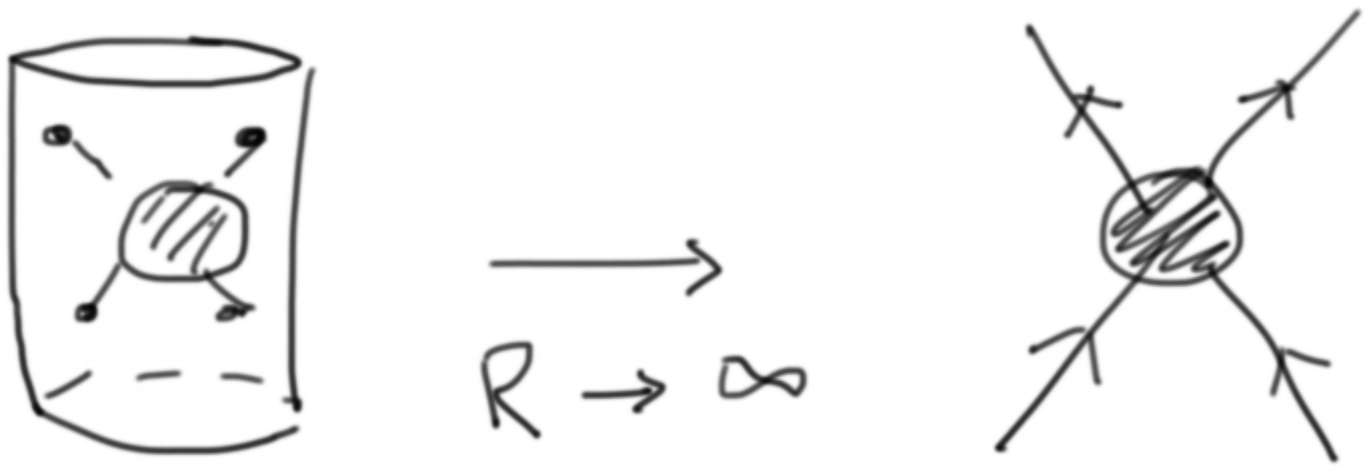
$$\mathcal{O}_1 \times \mathcal{O}_1 = \mathbb{1} + \lambda_{111} \, \underline{1} + (\dots \Delta \geq \Delta_b \dots)$$



Intuitively, we expect that

$$\langle O_1 O_1 O_1 O_1 \rangle \Rightarrow {}_{\text{out}}\langle 1 \underline{1} \,|\, 1 \underline{1} \rangle_{\text{in}}$$

in the flat-space limit.



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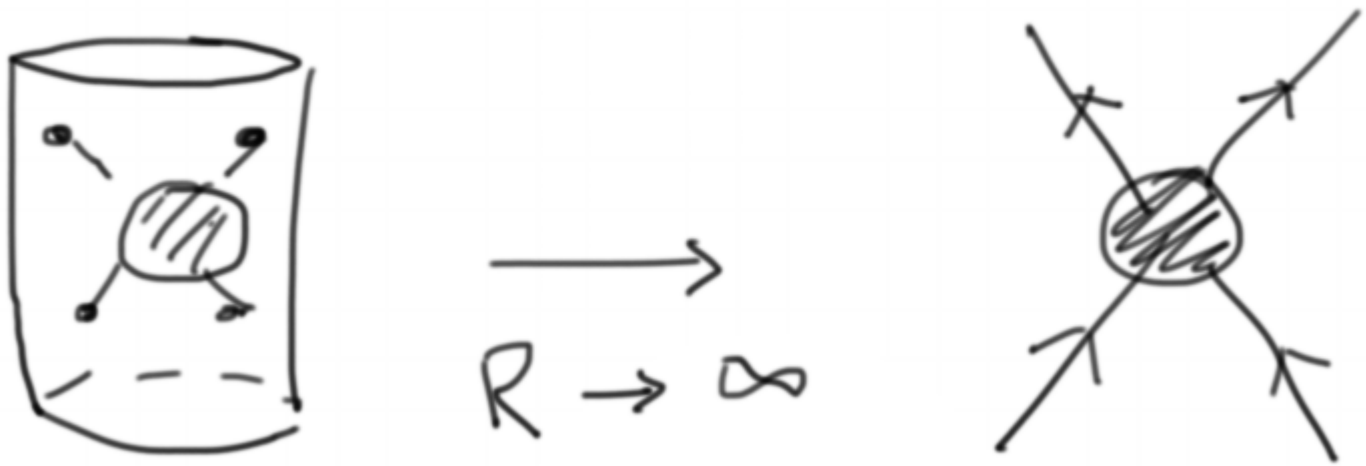
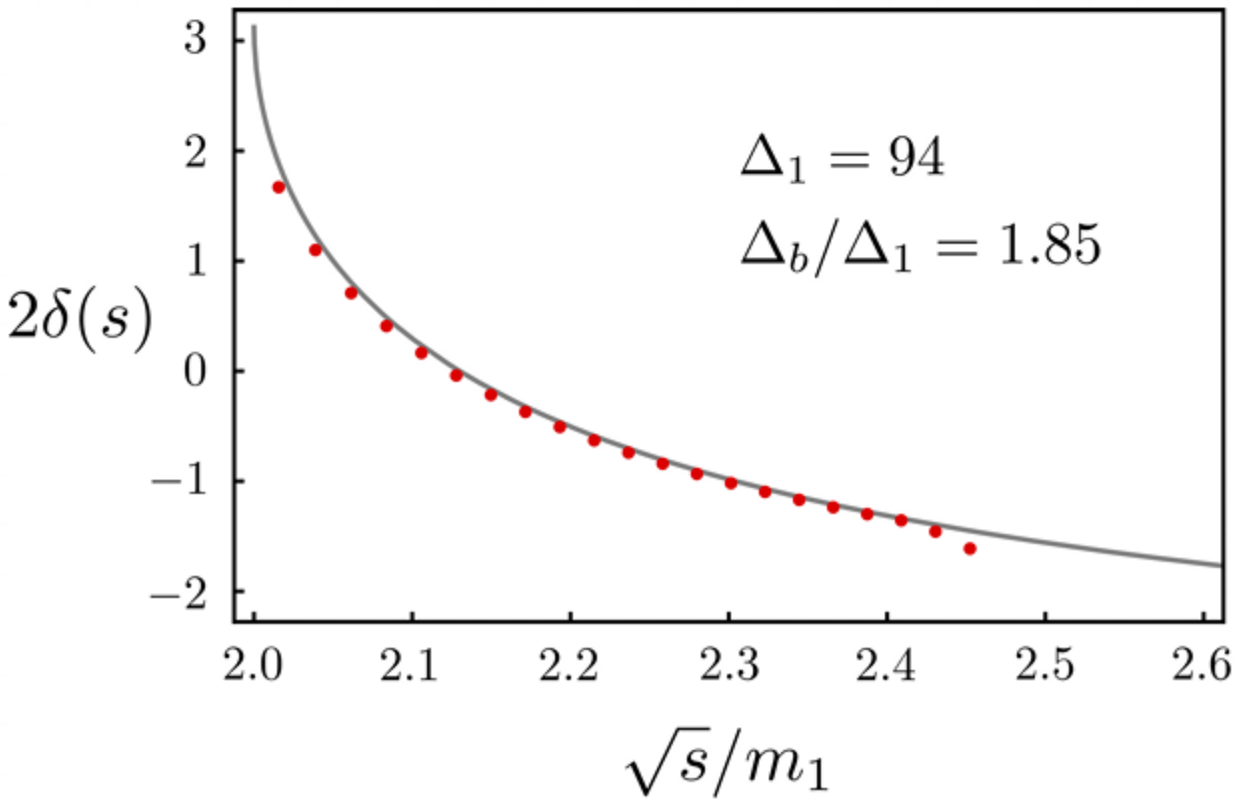


Q: Can we make this precise?

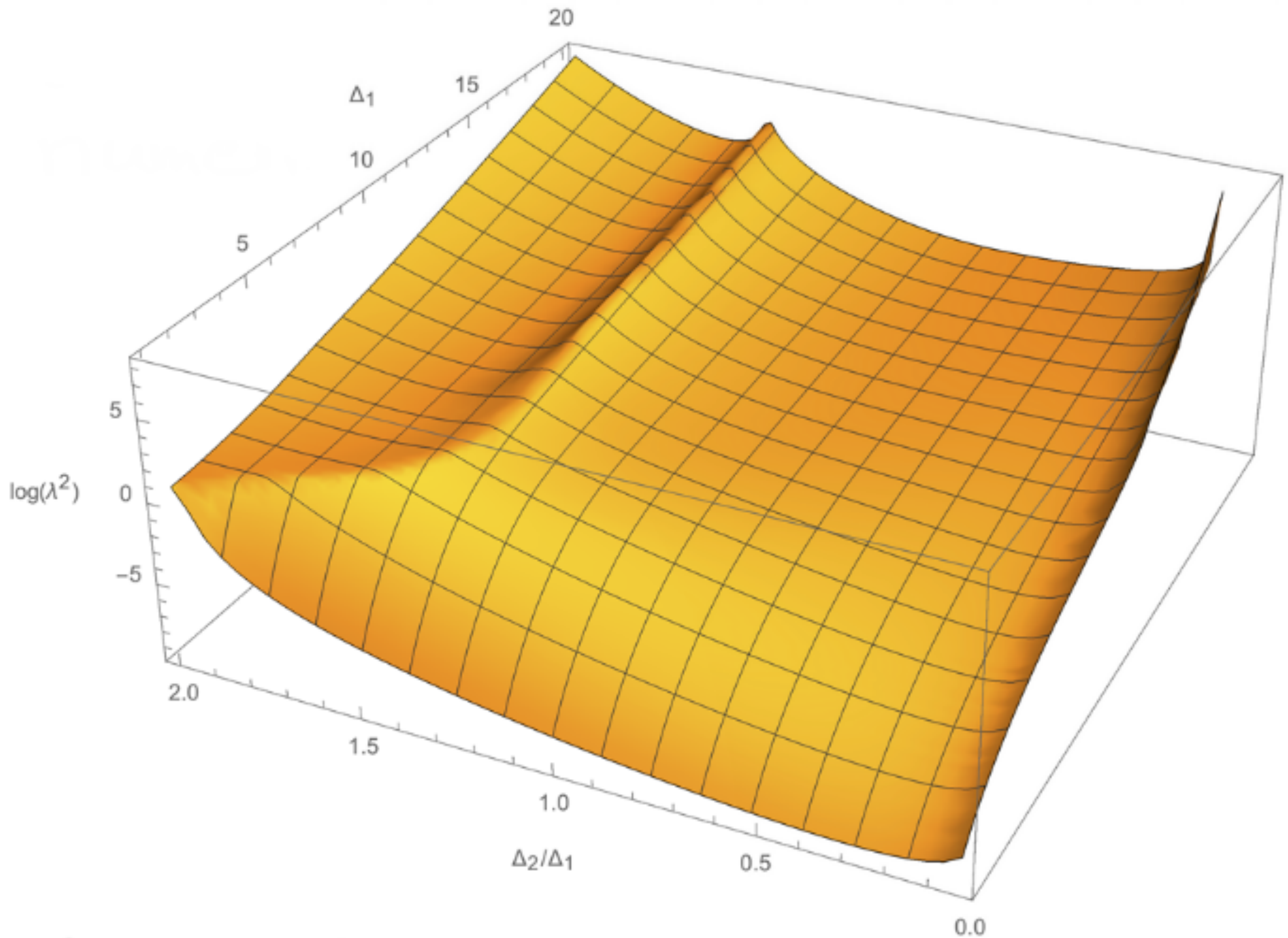
A: Yes, e.g. in Mellin space...

This allows us to extract the flat-space phase shift

$$\left({}_{out}\langle 1\,1 | 1\,1 \rangle_{in} \sim e^{2i\delta(s)} \right)$$



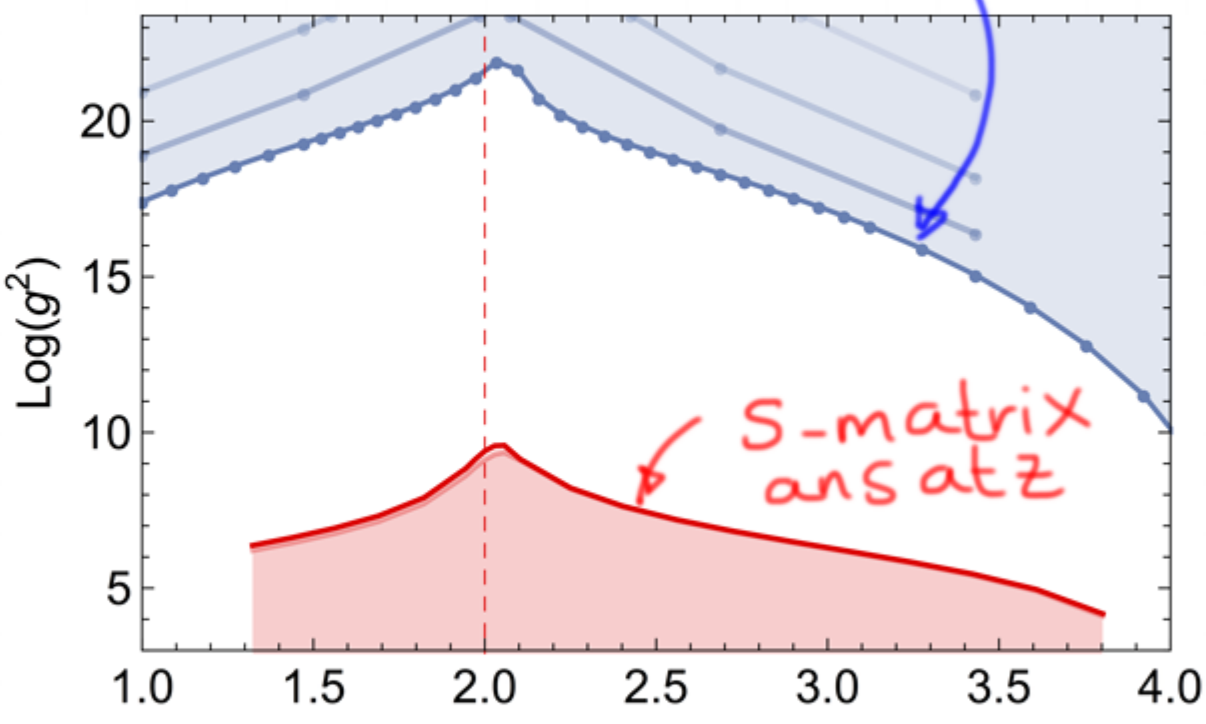
We can repeat the analysis in 2+1 dimensions:



but extrapolations do not work as well.

For finite Δ (i.e. finite R) we get a qualitative (but not quantitative) match:

QFT in AdS_3, $\Delta_1 = 17$



$$m_b^2 \simeq \frac{\Delta_b(\Delta_b - 2)}{\Delta_1(\Delta_1 - 2)}$$

[Q: extrapolation?]

Conclusions

- conformal crossing equations know about QFT in AdS
- extrapolations as $\Delta \to \infty$ give general flat-space QFT constraints.
- Can we "bootstrap" more complicated QFTs?
  - in $D = 2$?
  - in $D > 2$?

Open questions:

- Confining theories in AdS?
- SUSY theories in AdS?
- IR divergences?
- Scattering amplitudes from AdS?

Thank you!