Double Trace Interfaces

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Work in collaboration with

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Boundary and Defect Conformal Field Theory
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Outline

Introduction
- Double trace renormalization group interfaces
- Setup

Propagators and correlation functions
- Bulk Green's function
- Bulk-boundary propagator
- Two-point correlation functions

Spectra, OPE coefficients, and interface free energy

CFT comparison
- Large-N CFTs
- Perturbative verifications
- Comparison with predictions from Gaiotto's interfaces

Outlook

Renormalization group interfaces

RG Interfaces:

- Interfaces between CFTs corresponding to RG flow (non-perturbative) Brunner-Roggenkamp 07
- "Minimal" interfaces? Douglas 10, Bachas *etal* 13, Brunner-SC 15
- Realisation for basis of space of RG flows Gukov 15, 16
- Class of particularly tractable conformal (non-topological) interfaces

Double trace renormalization group interfaces

Examples for RG interfaces:

- see talks by John Cardy, Anatoly Konechny, Shinsei Ryu
- 2d $\mathcal{N} = 2$ (topol. twisted) SCFTs — Brunner-Roggenkamp 07
- Some analytic results — Gaiotto 12
- Some numerical results — Gliozzi-Liendo-Meineri-Rago 15
- Some perturbative results — Konechny-SC 14, Brunner-SC 15
- Results from holography — Bobev-Pilch-Warner 14, Karndumri-Upathambhakul 17

Double trace renormalization group interfaces

Double trace interfaces in a nutshell...

- Correspond to double-trace CFT deformations (perturbation by φ^2 for scalar φ)
- Some standard RG flows in this class (deformed free theories, $O(N)$ theories, coset models ...)
- Particular simplicity if dual classical AdS theory

Setup

- Scalar on AdS: $\phi(\rho,\chi) = J(\chi)\rho^{d-\Delta_\varphi} + \psi(\chi)\rho^{\Delta_\varphi} + \cdots$
- In range $-\frac{d^2}{4} \leq m^2 \leq -\frac{d^2}{4} + 1$, both parts lead to unitary CFTs: $CFT_\pm$ with scalars $\varphi_\pm$, $\Delta_\pm = \frac{d}{2} \pm \nu$ for $\nu^2 = m^2 + \frac{d^2}{4}$ Klebanov-Witten 99
- Deformation of CFT_- by φ_-^2 flows to CFT_+.

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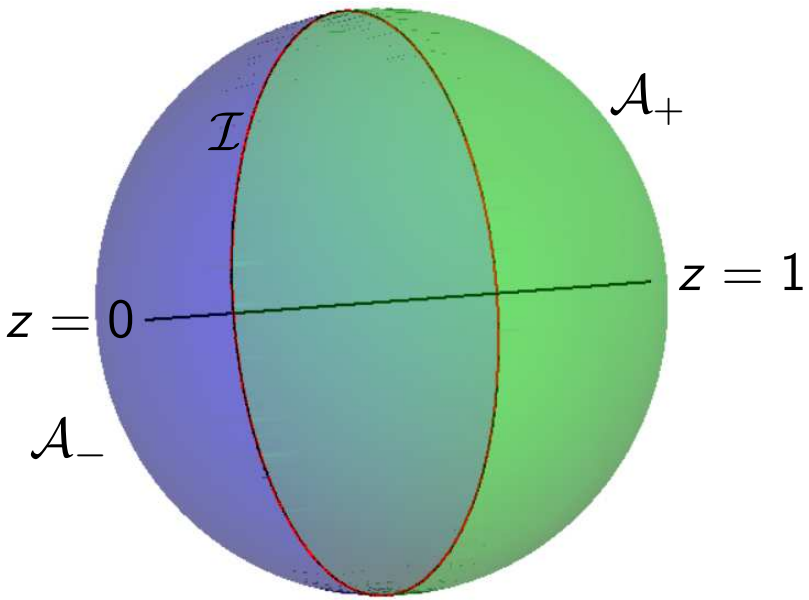
- Two regions $\mathcal{A}_\pm$ on boundary of AdS, with $\pm$ asymptotics: RG interface between $\mathcal{A}_\pm$
- Janus coordinates $ds^2_{H^{d+1}} = \frac{dz^2}{4z^2(1-z)^2} + \frac{ds^2_{H^d}}{4z(1-z)}$, $z \in (0,1)$: Bak *etal* 03



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- $(-\nabla^2_{H^{d+1}} + m^2)\Phi\Psi_s = 0$ solved by functions involving ${}_2F_1$:

$$\begin{aligned} \Phi_L^{\pm}(s;z) &: \quad \Phi_L^{\pm} \sim z^{\Delta_\pm/2} && (z \to 0) \\ \Phi_R^{\pm}(s;z) &: \quad \Phi_R^{\pm} \sim (1-z)^{\Delta_\pm/2} && (z \to 1) \end{aligned}$$

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- Basis transformation $\Phi_L^{\pm} \mapsto \Phi_R^{\pm}$ by Kummer's connection coefficients
- $G^{ab}(X,X') = \int d\mu(s)\, \Psi_s(x)\overline{\Psi}_s(x') \begin{cases} \Phi_L^a(s;z)A_s(z'), & z > z' \\ \Phi_R^b(s;z)B_s(z'), & z < z' \end{cases}$
- Can solve for A_s,B_s, obtain
  $G^{ab}(X,X') = \int_0^\infty d\sigma\, \mathcal{A}_\sigma^{ab} J_\sigma(x,x') \begin{cases} \Phi_L^a(\sigma;z')\Phi_R^b(\sigma;z), & z > z' \\ \Phi_L^a(\sigma;z)\Phi_R^b(\sigma;z'), & z < z' \end{cases}$

Bulk-boundary propagator

Methods for obtaining the bulk-boundary propagator $K(X, x')$

- $K(\rho, x; x') = \frac{1}{d-2\Delta} \lim_{\rho' \to 0} \rho'^{-\Delta} G(\rho, x; \rho, x')$
- Mixed boundary value problem: Find K from defining equations (rather than G). In order to match the prescribed asymptotics, method uses integral representations of Heaviside theta function.

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For spherical interface, obtain K^{+-} from G^{+-} as limit:

$$K^{+-}(z, x; x') =$$

$$\frac{\sin(\pi\nu)}{\pi} \frac{\Gamma(\frac{d}{2})}{\pi^{d/2}} \left(\frac{\sqrt{z(1-z)}}{2(\xi + z)} \right)^{\Delta_+} \left(\frac{\xi + z}{1 - z} \right)^{\nu} {}_2F_1\left(\begin{matrix} d/2, -\nu \\ 1 - \nu \end{matrix} \middle| -\frac{1-z}{\xi + z} \right),$$

with x' in $\mathcal{A}^+$, and $\xi(x, x')$ the conformal cross ratio.

Two-point correlation functions

In K^{+-}, push insertion X to boundary to obtain correlation functions:

$$\langle \varphi_+(x)\varphi_+(x')\rangle = \frac{1}{|x-x'|^{2\Delta_+}}\left(1 + B\,\xi^{\Delta_+}{}_2F_1\left(\begin{matrix} d/2, \Delta_+ \\ \Delta_+ + 1 \end{matrix}\middle| -\xi\right)\right)$$

with $B = \frac{\Gamma(d/2)\Gamma(\nu+1)}{\Gamma(\Delta_+ + 1)}\frac{\sin(\pi\nu)}{\pi}$,

$$\langle \varphi_-(x)\varphi_+(x')\rangle = \sqrt{\frac{\sin(\pi\nu)}{\pi\nu}}\,\frac{\Gamma(d/2)}{\sqrt{\Gamma(\Delta_+)\Gamma(\Delta_-)}}\,\frac{(-\xi)^{-\frac{d}{2}}}{(2y')^{\Delta_+}(2y)^{\Delta_-}}\,.$$

Interface spectrum

Janus setup preserves $SO(d,1)$: Use AdS/CFT on boundary to infer the spectrum. Aharony *etal* 03

- Write EOM as $(-\nabla^2_{H^d} + \mathcal{D}(z))\phi = 0$
- Separate coordinates $\phi = \sum_\alpha \psi_\alpha(x)\phi_\alpha(z)$: $\psi_\alpha(x)$ will have dual operators on interface
- Masses from $\mathcal{D}(z)\phi_\alpha(z) = m^2_\alpha \phi_\alpha(z) \Rightarrow$ interface fields have $\Delta_\alpha = \frac{d-1}{2} + \nu_\alpha$
- Solutions $\Phi^{a,b}_{L,R}(\sigma; z)$ with σ continued to imaginary values: $\sigma_\alpha = -i\nu_\alpha$
- bc $\Delta_\pm$ for ϕ_α in addition enforce $\cos(\pi\nu_\alpha) = 0$
- $\Rightarrow$ defect spectrum: single trace operators have

$$\Delta_\alpha = \tfrac{d}{2} + \alpha\,, \qquad \alpha \in \mathbb{N}_0$$

- Classical limit: That's all there is
- Displacement operator: Double trace of $\alpha = 0$

OPE coefficients

From standard expressions for two-point correlation functions, can read off bulk and bulk-interface OPE spectrum and coefficients McAvity-Osborn 95

Decomposition by combination of identities for hypergeometric functions (Gaussian Summation formulae, Saalschütz' theorem, quadratic identitities, Euler- and Pfaffian formulae...) Erdélyi 53

Results:

- $\varphi_+ \times \varphi_+$ contains primary operators with

  $\Delta_n = 2\Delta_+ + 2n$: Double trace operators from φ_+ descendents
- ${C_{++}}^n {B_n}^0 = \frac{\sin(\pi\nu)}{\pi} \frac{\Gamma(\frac{d}{2})\Gamma(1+n+\nu)}{\Gamma(n+\nu+1)} \frac{(\nu)_n(\Delta_+)_n}{(\Delta_++\nu+n)_n}$
- $\varphi_\pm$ create the primary interface operators with

  $\Delta_\alpha = \frac{d}{2} + \alpha$, $\alpha \in \mathbb{N}_0$: Single-trace spectrum
- $B_+^\alpha = \left(\frac{\sin(\pi\nu)}{\pi} \frac{\Gamma(\frac{d}{2})\Gamma(\nu)}{\Gamma(\Delta_+)} \frac{\alpha!(\frac{d}{2})_\alpha(1+\nu)_\alpha}{(2\alpha)!(1-\nu)_\alpha} \right)^{\frac{1}{2}}$

Interface free energy

- Calculation of (1-loop) free energy leads to UV, IR divergences.
- Consider $CFT_+ \otimes CFT_-$ with and without interface:
  $F_{+-} + F_{-+}, \quad F_{++} + F_{--}$
- Contribution of interface to free energy:
  $2F_{\text{interface}} = F_{+-} + F_{-+} - F_{++} - F_{--} = \frac{1}{2}\log\left(\frac{\det \mathcal{D}^{+-} \det \mathcal{D}^{-+}}{\det \mathcal{D}^{++} \det \mathcal{D}^{--}}\right)$
  with $\mathcal{D} = -\Box + m^2$
- IR regularisation and $\frac{d}{dm^2}[\mathrm{tr}\log \mathcal{D}]^{ab} = \int d^{d+1}X \sqrt{g_{H^{d+1}}}\, G^{ab}(X,X)$ yields Diaz-Dorn 07

$$\frac{d}{d\nu} 2F_{\text{interface}} \left(= \frac{d}{d\nu}\log g^2\right) = -\nu \frac{\cos \pi\nu}{\cos \frac{\pi d}{2}} \frac{\Gamma(\Delta_+)\Gamma(\Delta_-)}{\Gamma(1+d)}$$

- Compare: Diaz-Dorn 07

$$\frac{d}{d\nu}(S_{\mathrm{CFT}_+} - S_{\mathrm{CFT}_-}) = \nu \frac{\sin \pi\nu}{\sin \frac{\pi d}{2}} \frac{\Gamma(\Delta_+)\Gamma(\Delta_-)}{\Gamma(1+d)}$$

Large-N CFTs

Dual CFTs of AdS bulk with double trace interface are of large N type:

- Correlation functions reduce to Wick contractions
- Scalar φ with $\varphi \times \varphi = 1 + C'\varphi^2 + \dots$
- Double trace operator $\varphi^2 \times \varphi^2 = 1 + C\varphi^2 + \dots$, with $C = 2C'$

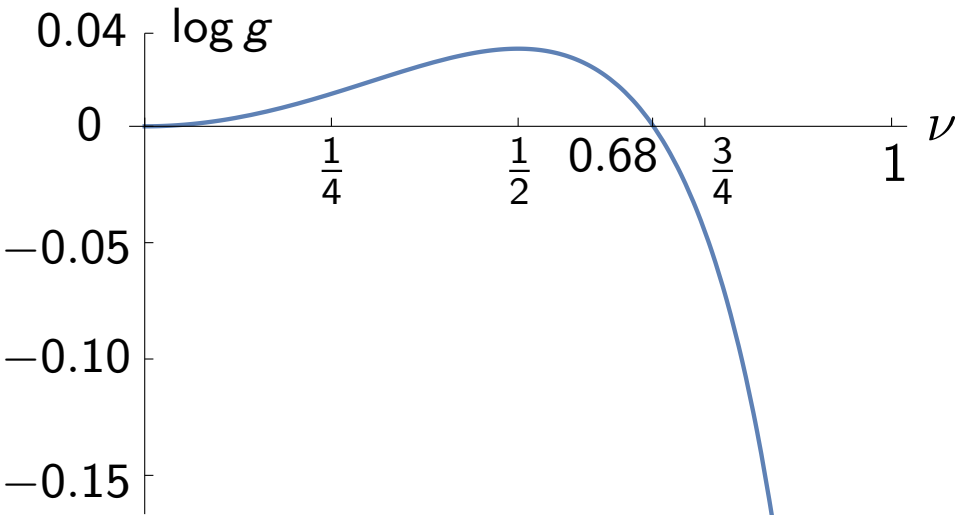
Therefore, RG flow triggered by φ^2 particularly simple.

Example $O(N)$:

$$\varphi_- \times \varphi_- = 1 + \sqrt{\frac{2}{N}}\varphi_- + \frac{\sqrt{2(N^2-N+3)}}{N}\varphi_-^2 + \dots$$
$$\varphi_-^2 \times \varphi_-^2 = 1 + \frac{4\sqrt{2N}(N+2)}{N^2-N+3}\varphi_- + \frac{\sqrt{8}(N+8)}{\sqrt{N^2-N+3}}\varphi_-^2 + \dots$$

Perturbative verifications

- For small values of ν, RG flow is short: $2\Delta_- = d - 2\nu$
- Coupling at IR $\kappa \sim \nu$ (OPE-scheme): Can perturbatively verify results at small ν
- Exact bulk results give predictions for all higher perturbative terms
  E.g. $d = 2$: RG interfaces for $0 \leq \nu < 1$ have $\log g$



$\rightarrow$ value of g away from criticality (in appropriate scheme)

Comparison with predictions from Gaiotto's interfaces

In $d = 2$, exact data in HS/CFT from (limit of) Gaiotto's interface construction. Gaiotto 12, Brunner-SC 16

RG flow of coset model CFTs Gaberdiel-Gopakumar 12

$$\frac{\mathfrak{su}(N)_k \otimes \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}} \to \frac{\mathfrak{su}(N)_{k-1} \otimes \mathfrak{su}(N)_1}{\mathfrak{su}(N)_k}$$

dual to Vasiliev bosonic Higher Spin theory. Vasiliev 96

Duality holds for finite N, k; our bulk results for 't Hooft limit

$$\nu = \frac{N}{N+k}\,, \quad N, k \to \infty$$

Reflection and transmission: Runkel *etal* 07

$$\left\langle T^{UV}\,\tilde{T}^{UV}\right\rangle = \frac{\nu^2}{2}(1+\nu)\,, \quad \left\langle T^{IR}\,\tilde{T}^{IR}\right\rangle = \frac{\nu^2}{2}(1-\nu)\,, \quad \left\langle T^{UV}\,T^{IR}\right\rangle = \frac{N}{2}(1-\nu^2)$$

Outlook

- Compute interface OPE coefficients
- Bulk-interface scalar OPE closes on single-trace operators, bulk-bulk on double trace: Unique?
- Method for mixed boundary values can be applied to more complicated configurations: Fusion of RG interfaces at large N?
- Extension to non-unitary boundary CFTs (*e.g.* classical finite N limit in $d = 2$)?