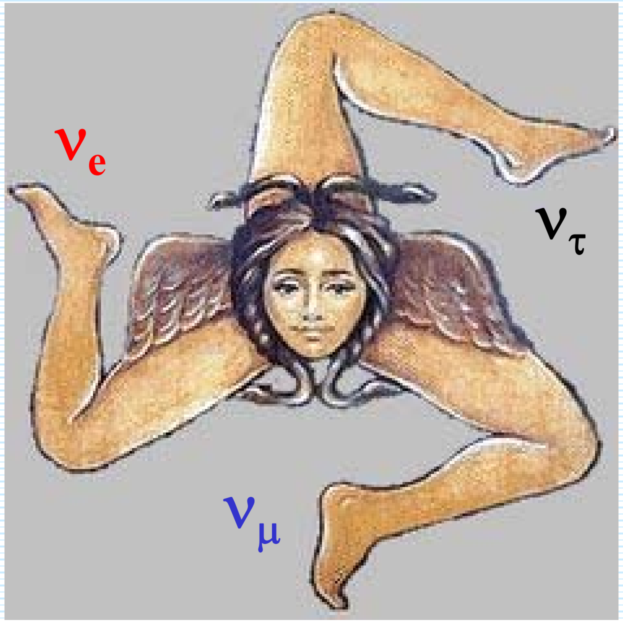
Colloquium Prague ν15
Prague, Czech Republic, November 1-3, 2017



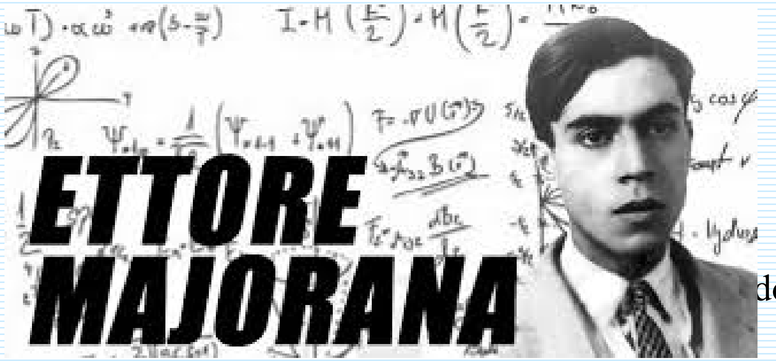
Double Beta Decay Theory

Fedor Šimkovic

c

Majorana fermion

https://en.wikipedia.org/wiki/File:Ettore_Majorana.jpg



dor Si

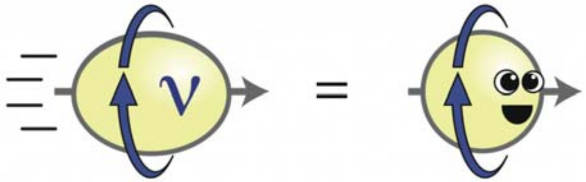
TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di Ettore Majorana

Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Sunto. - *Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di* Dirac *ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di «antiparticelle» corrispondenti ai «vuoti» di energia negativa.*

L'interpretazione dei cosidetti « stati di energia negativa » proposta da Dirac (1) conduce, come è ben noto, a una descrizione sostanzialmente simmetrica degli elettroni e dei positroni. La sostanziale simmetria del formalismo consiste precisamente in questo, che fin dove è possibile applicare la teoria girando le difficoltà di convergenza, essa fornisce realmente risultati del tutto simmetrici. Tuttavia gli artifici suggeriti per dare alla teoria una forma simmetrica che si accordi … isfacenti; sia perchè si … trica, sia perchè la sin… iante tali procedimenti … che possibilmente dov… nuova via che conduce più direttamente alla meta.



Per quanto riguarda gli elettroni e i positroni, da essa si può veramente attendere soltanto un progresso formale; ma ci sembra importante, per le possibili estensioni analogiche, che venga a cadere la nozione stessa di stato di energia negativa. Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi.

(1) P. A. M. Dirac, « Proc. Camb. Phil. Soc. », **30**, 150, 1924. V. anche W. Heisenberg, « ZS. f. Phys. », **90**, 209, 1934.

MESONIUM AND ANTIMESONIUM

B. PONTECORVO

Joint Institute for Nuclear Research

Submitted to JETP editor May 23, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 549-551 (August, 1957)

INVERSE BETA PROCESSES AND NONCONSERVATION OF LEPTON CHARGE

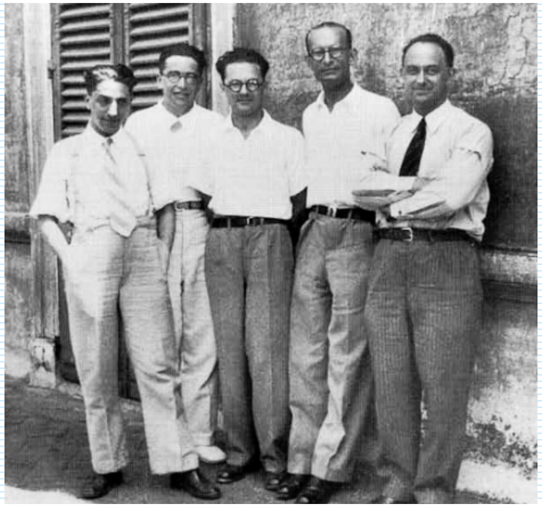
B. PONTECORVO

Joint Institute for Nuclear Research

Submitted to JETP editor October 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 247-249 (January, 1958)

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are "mixed" particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.[5]



I ragazzi di via Panisperna

1968 Gribov, Pontecorvo [PLB 28(1969) 493]

oscillations of neutrinos - a solution of deficit of solar neutrinos in Homestake exp.

OUTLINE

- ***Introduction***
  ν-oscillations and ν-masses
- ***The 0νββ-decay scenarios due neutrinos exchange***
  (simpliest, sterile ν, LR-symmetric model)
- ***DBD NMEs and Quenching of g_A***
  (nuclear structure issues)
- ***DBD with emission of a single electron***
- ***DBD NMEs within schematic models***
  (SU(4) symmetry, nonlinear QRPA)
- ***Conclusion***

Acknowledgements: **A. Faesler** **(Tuebingen)**, **P. Vogel** **(Caltech)**, **S. Kovalenko** **(Valparaiso U.)**, **M. Krivoruchenko** **(ITEP Moscow)**, **D. Štefánik, R. Dvornický** **(Comenius U.)**, **A. Babič, A. Smetana, J. Terasaki** **(IEAP CTU Prague), …**

Observation of ν-oscillations = the first prove of the BSM physics

mass-squared differences: $\Delta m^2_{SUN} \cong 7.5\ 10^{-5}\ eV^2$, $\Delta m^2_{ATM} \cong 2.4\ 10^{-3}\ eV^2$

The observed small neutrino masses (limits from tritium β-decay, cosmology) have profound implications for our understanding of the Universe and are now a major focus in astro, particle and nuclear physics and in cosmology.

PMNS unitary mixing matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

large off-diagonal values

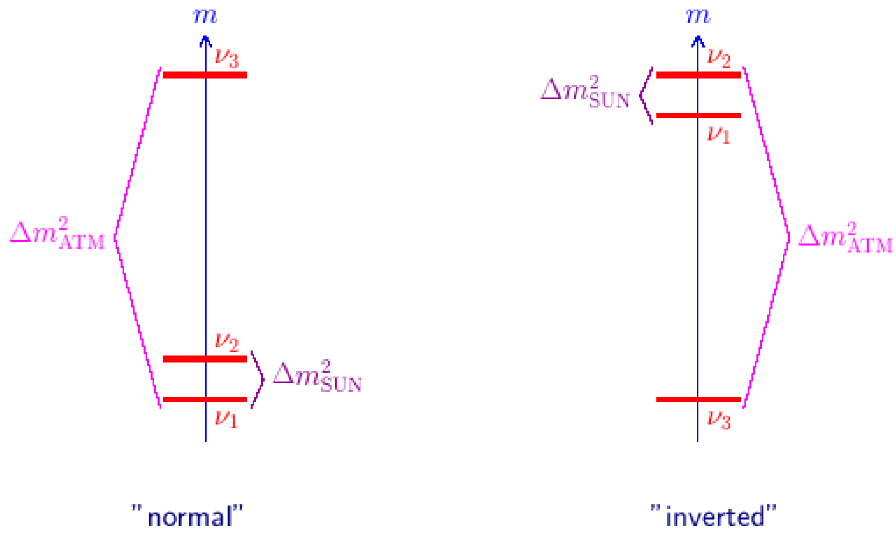
$$\begin{pmatrix} 0.82 & 0.54 & -0.15 \\ -0.35 & 0.70 & 0.62 \\ 0.44 & -0.45 & 0.77 \end{pmatrix}$$

3 angles: θ_{12}=33.36° (solar), θ_{13}=8.66° (reactor), θ_{23}=40.0° or 50.4° (atmospheric)

$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

unknown (CP violating) phases: δ, α_1, α_2

Neutrinos mass spectrum

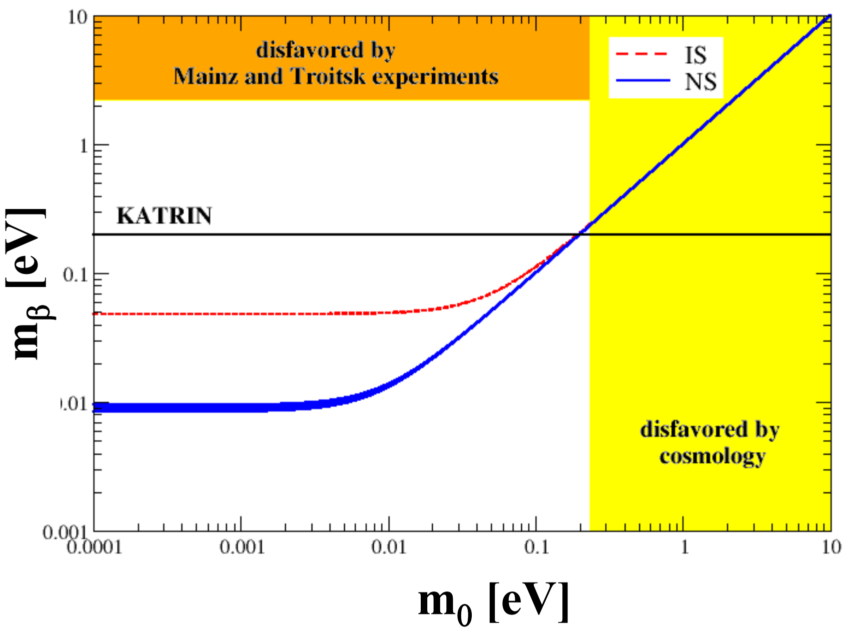


0νββ Measurements

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3 \right|$$

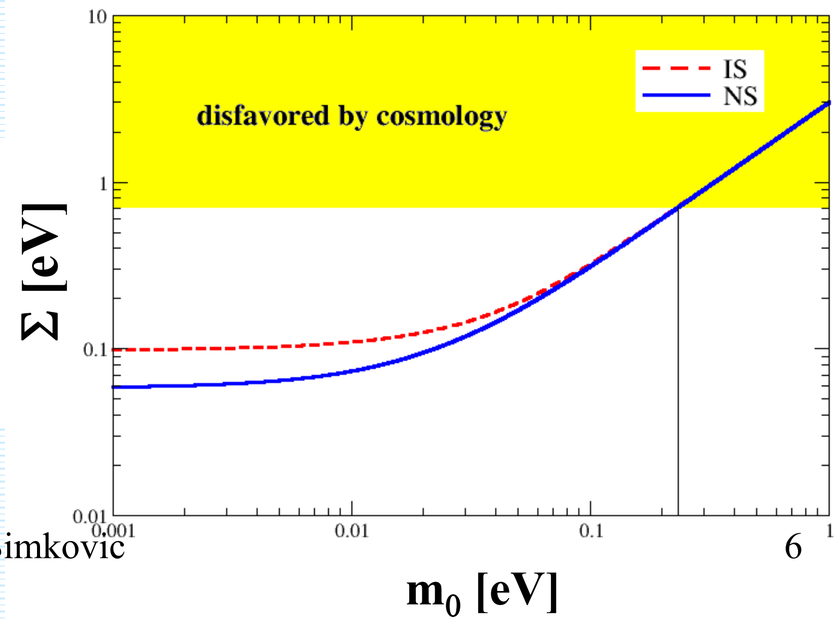
Beta Decay Measurements

$$m_\beta = \sqrt{c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2}$$



Cosmological Measurements

$$\Sigma = m_1 + m_2 + m_3$$



The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

What is the nature of neutrinos?

$\nu \Rightarrow$

GUT's

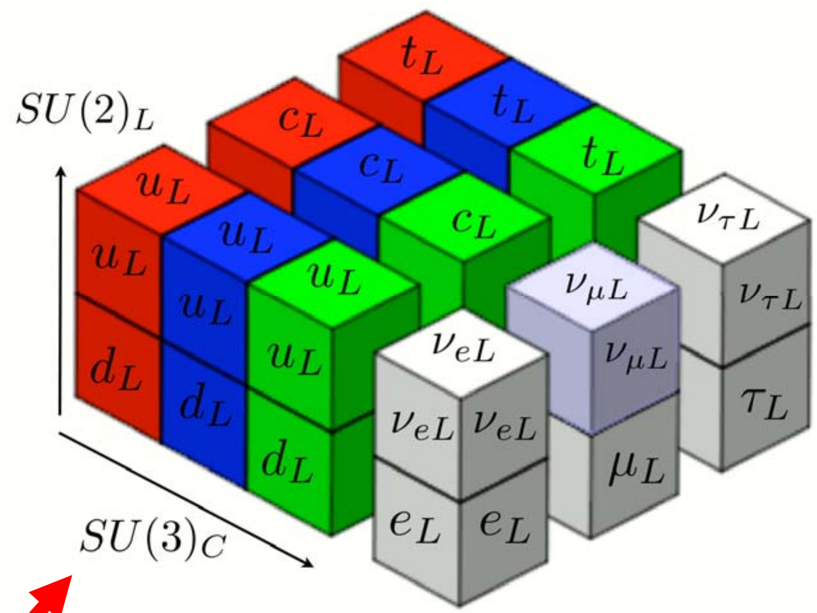
Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Only the 0νββ-decay can answer this fundamental question

Analogy with kaons: K_0 and $\bar{K}_0$

Analogy with π_0

Beyond the Standard model physics
(EFT scenario)



$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$

Minimal SM + EFT

S.M. Bilenky, Phys.Part.Nucl.Lett. 12 (2015) 453-461

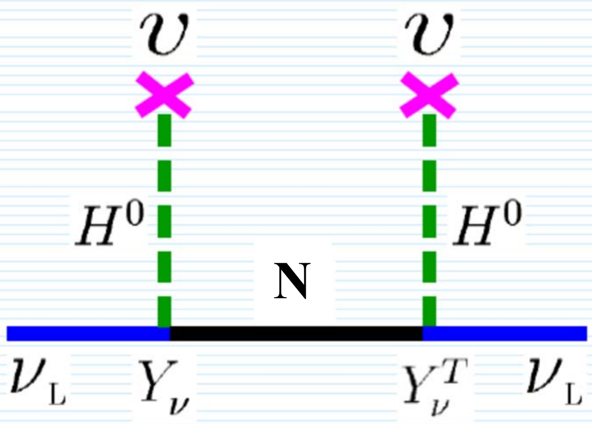
The **absence of the right-handed neutrino fields** in the Standard Model is the simplest, most economical possibility. In such a scenario **Majorana mass term** is the only possibility for neutrinos to be massive and mixed. This mass term is generated by the **lepton number violating Weinberg effective Lagrangian.**

$$\mathcal{L}_5^{eff} = -\frac{1}{\Lambda} \sum_{l_1 l_2} \left(\overline{\Psi}^{lep}_{l_1 L} \tilde{\Phi} \right) \acute{Y}_{l_1 l_2} \left(\tilde{\Phi}^T (\Psi^{lep}_{l_2 L})^c \right)$$

$$m_i = \frac{v}{\Lambda} (y_i v), \quad i = 1, 2, 3$$

$\Lambda \geq 10^{15}$ GeV

Heavy Majorana leptons N_i ($N_i = N^c_i$)
singlet of $SU(2)_L x U(1)_Y$ group
Yukawa lepton number violating int.



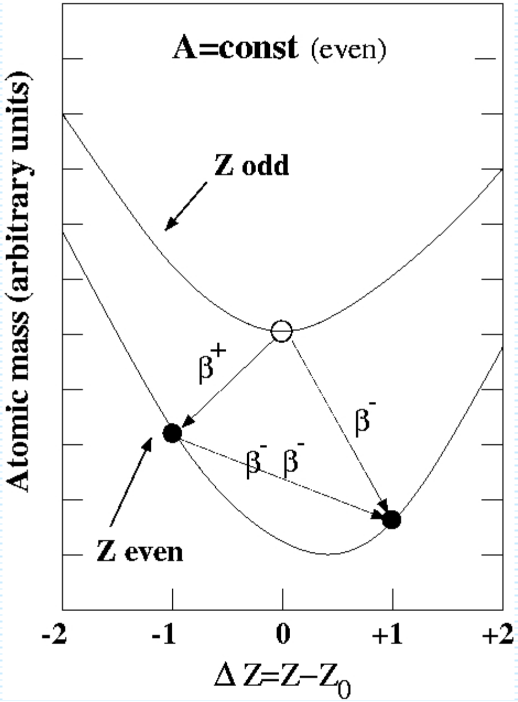
The three Majorana neutrino masses are suppressed by the ratio of the electroweak scale and a scale of a lepton-number violating physics.

The discovery of the ββ-decay and absence of transitions of flavor neutrinos into sterile states would be evidence in favor of this minimal scenario.

I. *The simplest $0\nu\beta\beta$-decay scenario*
(SM + EFT scenario)

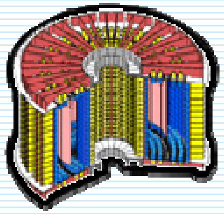
$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 \; g_A^4 \; \left|M^{0\nu}_{\nu}\right|^2 \; G^{0\nu}$$

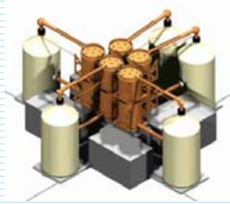
(A,Z) → (A,Z+2) + e⁻ + e⁻



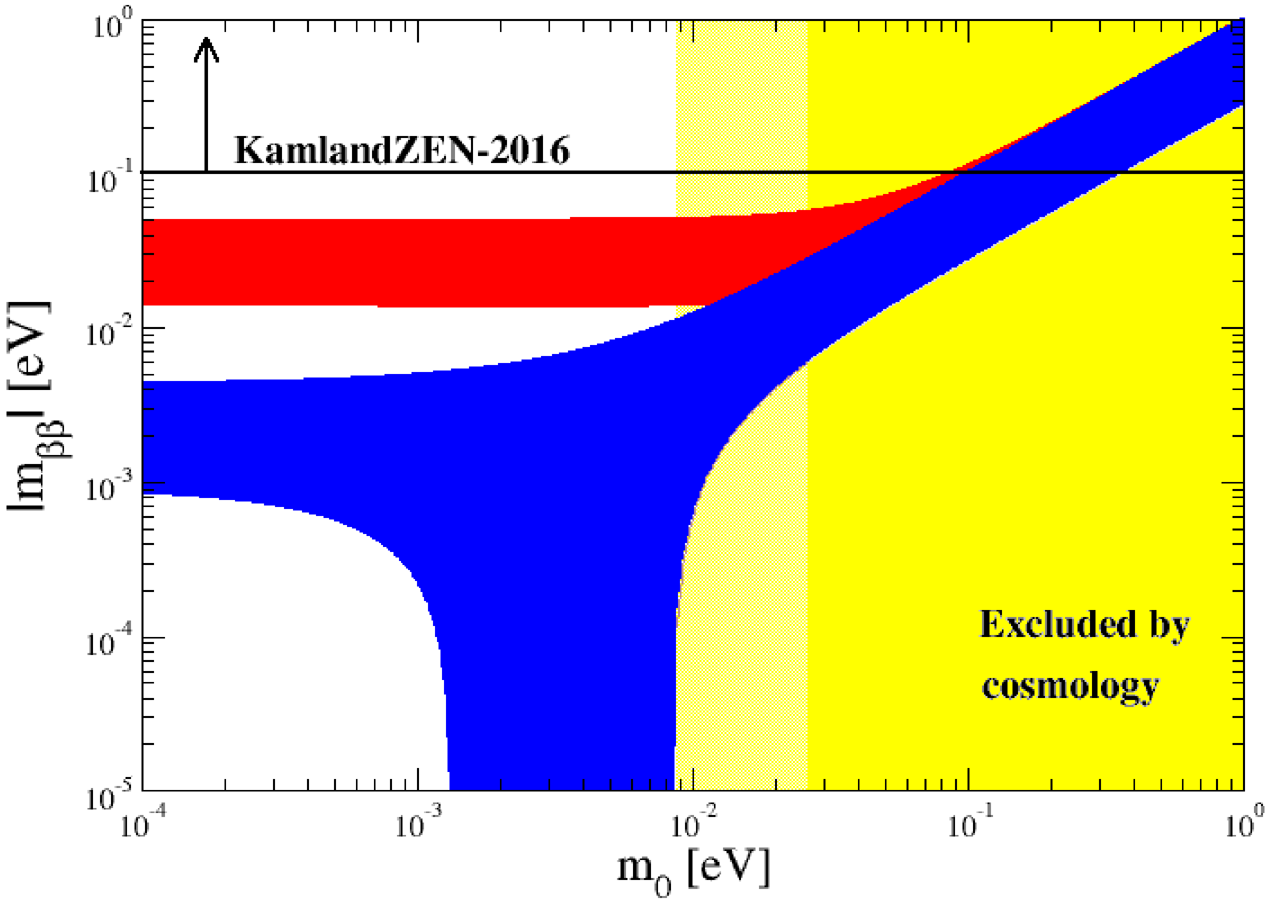
transition	$G^{01}(E_0,Z)$ $\times 10^{14} y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\lvert M^{0\nu}\rvert^2$
$^{150}Nd \rightarrow {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \rightarrow {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \rightarrow {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \rightarrow {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \rightarrow {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \rightarrow {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \rightarrow {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \rightarrow {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \rightarrow {}^{76}Se$	0.79	2.040	8	?

The NMEs for $0\nu\beta\beta$-decay must be evaluated using tools of nuclear theory





Effective mass of Majorana neutrinos



Complementarity of 0νββ-decay, β-decay and cosmology

β-decay (Mainz, Troitsk)

$$m_\beta^2 = \sum_i |U^L_{ei}|^2 m_i^2 \leq (2.2\ \mathrm{eV})^2$$

KATRIN: $(0.2\ \mathrm{eV})^2$

Cosmology (Planck)

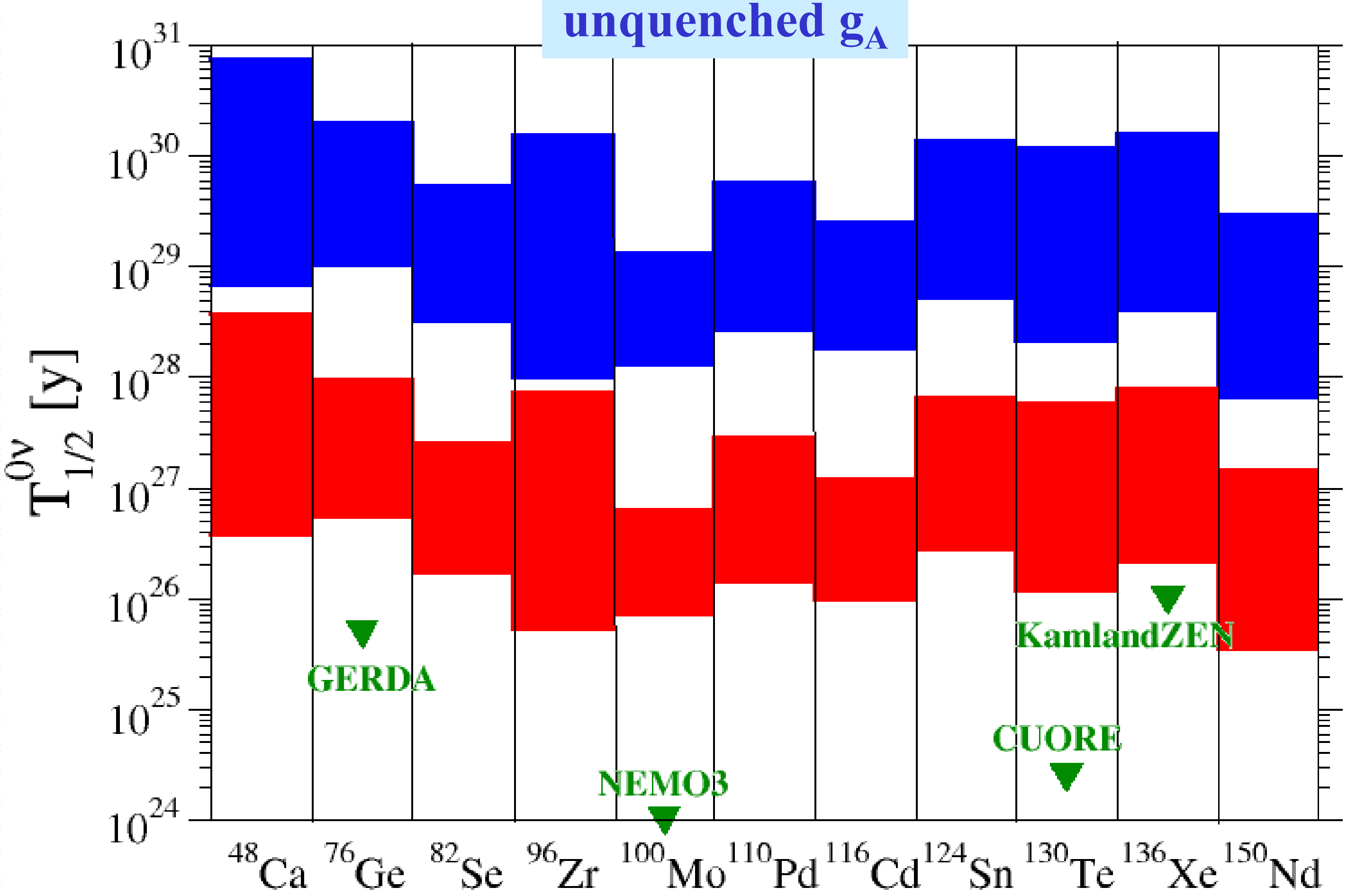
$$\Sigma < 110\ \mathrm{meV}$$

$$m_0 > 26\ \mathrm{meV}\ (\mathrm{NS})$$
$$87\ \mathrm{meV}\ (\mathrm{IS})$$

GUT's

m_1, m_2, m_3, θ_{12}, θ_{13}, α_1, α_2
(3 unknown parameters)

$0\nu\beta\beta$ –half lives for NH and IH with included undertainties in NMEe



NH: $m_1 \ll m_2 \ll m_3$ $m_3 \simeq \sqrt{\Delta m^2}$

$m_1 \ll \sqrt{\delta m^2}.$ $m_2 \simeq \sqrt{\delta m^2}$

$1.4 \text{ meV} \leq m_{\beta\beta} \leq 3.6 \text{ meV}$

IH: $m_3 \ll m_1 < m_2$ $m_1 \simeq m_2 \simeq \sqrt{\Delta m^2}$

$m_3 \ll \sqrt{\Delta m^2}$

$20 \text{ meV} \leq m_{\beta\beta} \leq 49 \text{ meV}$

Lightest ν-mass equal to zero

II. *The sterile ν mechanism of the 0νββ-decay*

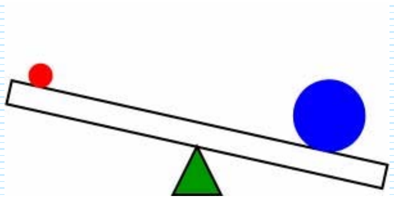
(D-M mass term, V-A SM int.)

$$N = \sum_{\alpha=s,e,\mu,\tau} U_{N\alpha}\, \nu_\alpha$$

Mixing of active-sterile neutrinos

Dirac-Majorana mass term

$$\begin{pmatrix} 0 & m_D \\ m_D & m_{LNV} \end{pmatrix}$$



Light ν mass ≈ $(m_D/m_{LNV})\, m_D$
Heavy ν mass ≈ m_{LNV}

small ν masees due to see-saw mechanism

Possible lepton number violating scale - m_{LNV}

Neutrinos masses may offer a great opportunity to jump beyond the EW framework via see-saw ...

10^{19} GeV — **Planck**

How heavy are the heavy Majorana neutrinos?

10^{16} GeV — **GUT**

to unify strong, weak & electromagnetic forces?

Conventional (Type-one) Seesaw Picture: close to the GUT scale

10^{3} GeV — **TeV**

TeV Seesaw Idea: driven by testability at LHC

to solve the unnatural gauge hierarchy problem?

GeV/5 — **Fermi**

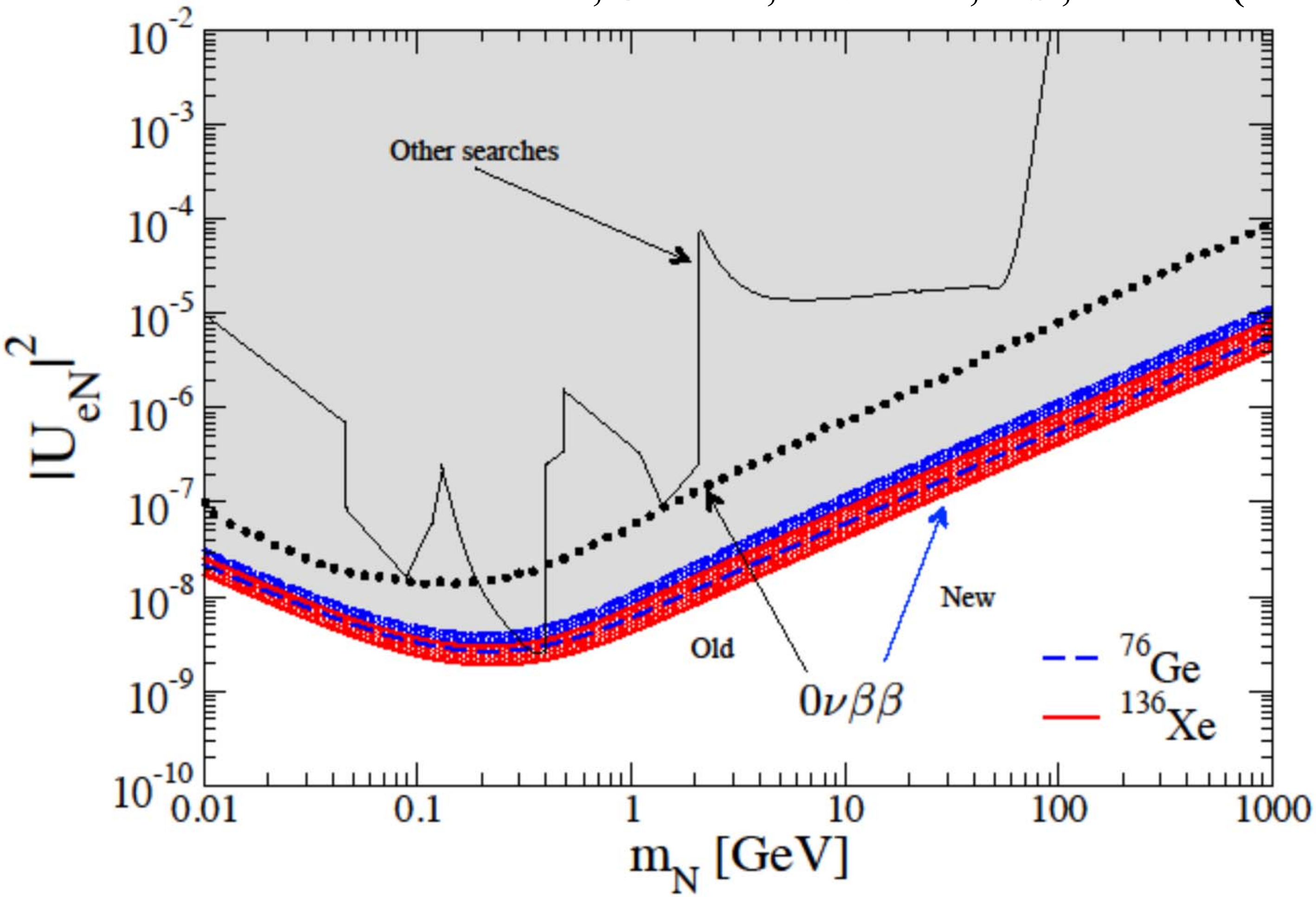
10^{-6} GeV — **kev**

Hot dark mattter

Exclusion plot in $|U_{eN}|^2 - m_N$ plane

$T^{0\nu}_{1/2}(^{76}Ge) \geq 3.0\ 10^{25}$ yr
$T^{0\nu}_{1/2}(^{136}Xe) \geq 3.4\ 10^{25}$ yr

Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]



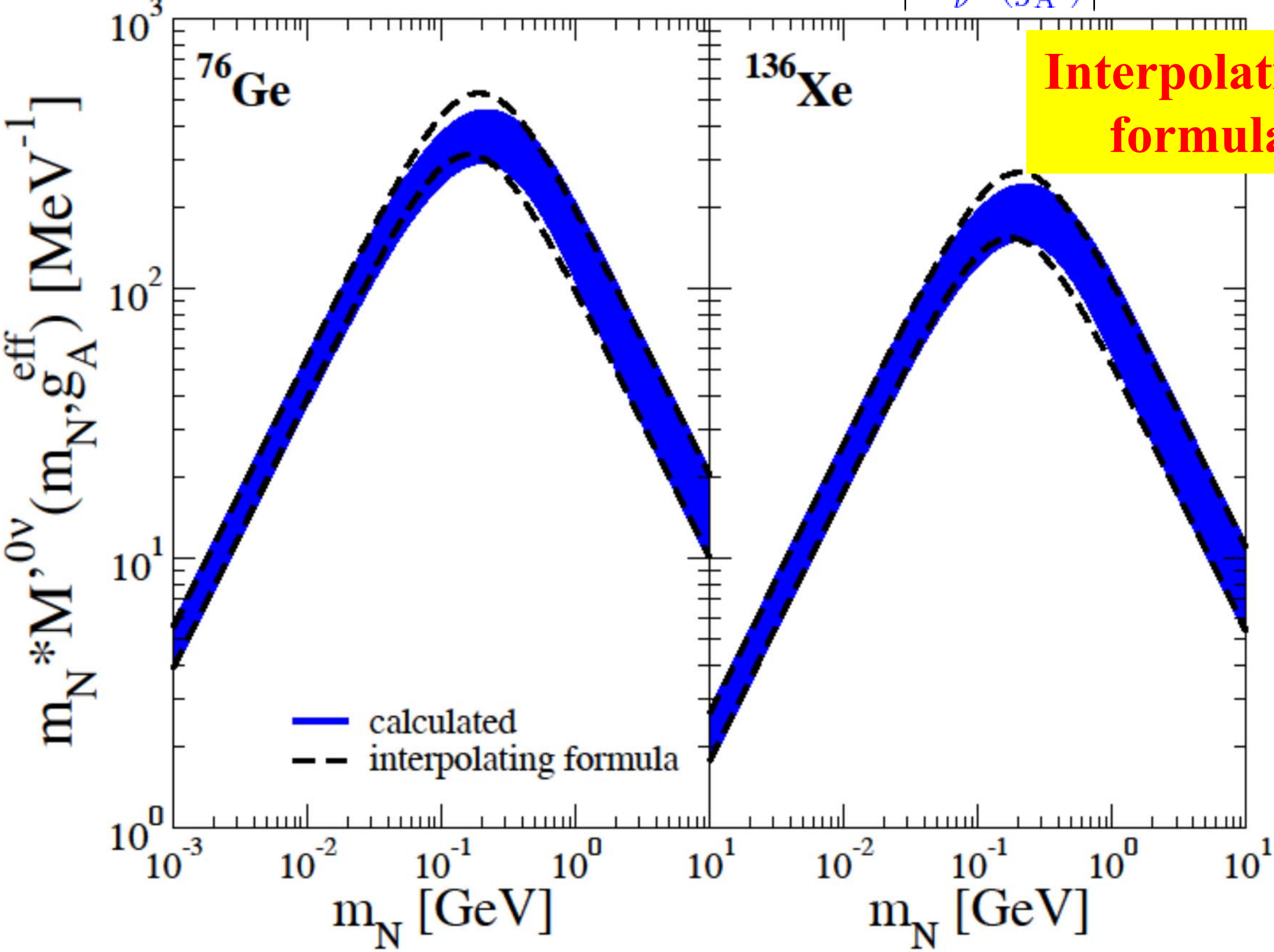
Improvements: i) QRPA (constrained Hamiltonian by 2νββ half-life, self-consistent treatment of src, restoration of isospin symmetry …), ii) More stringent limits on the 0νββ half-life

$$[T_{1/2}^{0\nu}]^{-1} = \mathcal{A} \cdot \left| m_p \sum_N U_{eN}^2 \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|^2 ,$$

$$\mathcal{A} = G^{0\nu} g_A^4 \left| M_N'^{0\nu}(g_A^{\rm eff}) \right|^2 ,$$

$$\langle p^2 \rangle = m_p m_e \left| \frac{M_N'^{0\nu}(g_A^{\rm eff})}{M_\nu'^{0\nu}(g_A^{\rm eff})} \right| \approx 200 \ MeV$$

Interpolating formula



III. *The $0\nu\beta\beta$-decay within L-R symmetric theories*

(D-M mass term, see-saw, V-A and V+A int., exchange of light neutrinos)

Effective β-decay Hamiltonian

$$H^\beta = \frac{G_\beta}{\sqrt{2}}\left[j_L{}^\rho J_{L\rho} + \chi\, j_L{}^\rho J_{R\rho} + \eta\, j_R{}^\rho J_{L\rho} + \lambda\, j_R{}^\rho J_{R\rho} + h.c.\right].$$

left- and right-handed lept. currents

$$j_L{}^\rho = \bar{e}\gamma^\rho(1-\gamma_5)\nu_{eL}$$
$$j_R{}^\rho = \bar{e}\gamma^\rho(1+\gamma_5)\nu_{eR}$$

Mixing of vector bosons W_L and W_R

$$\begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix} = \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} W_1^- \\ W_2^- \end{pmatrix}$$

$$\eta = -\tan\zeta, \quad \chi = \eta,$$
$$\lambda = (M_{W_1}/M_{W_2})^2$$

The $0\nu\beta\beta$-decay half-life

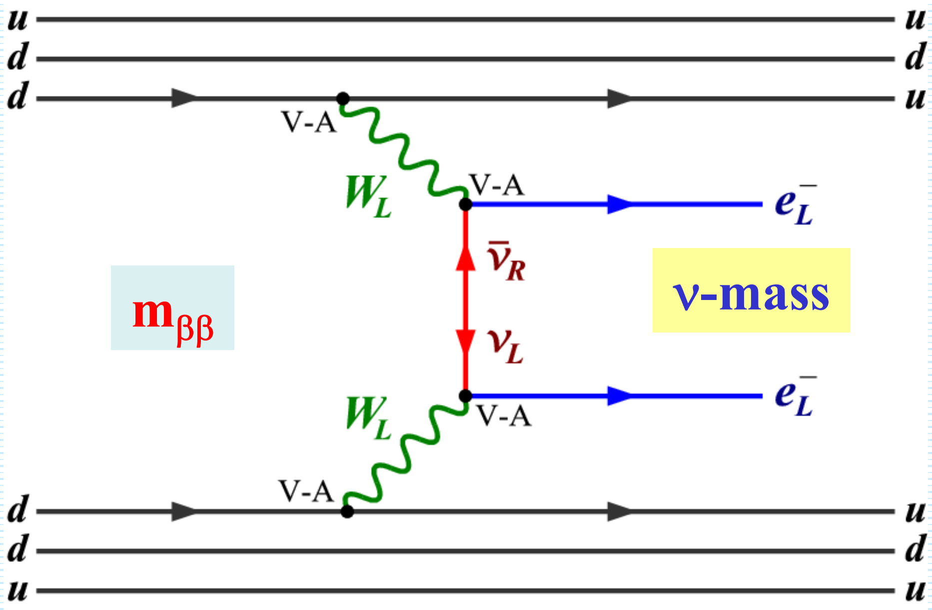
$$\left[T^{0\nu}_{1/2}\right]^{-1} = \frac{\Gamma^{0\nu}}{\ln 2} = g_A^4\, |M_{GT}|^2 \left\{ C_{mm}\frac{|m_{\beta\beta}|^2}{m_e} + C_{m\lambda}\frac{|m_{\beta\beta}|}{m_e}\langle\lambda\rangle\cos\psi_1 + C_{m\eta}\frac{|m_{\beta\beta}|}{m_e}\langle\eta\rangle\cos\psi_2 + C_{\lambda\lambda}\langle\lambda\rangle^2 + C_{\eta\eta}\langle\eta\rangle^2 + C_{\lambda\eta}\langle\lambda\rangle\langle\eta\rangle\cos(\psi_1-\psi_2) \right\}$$

$\langle\lambda\rangle$ - W_L-W_R exch.

$\langle\eta\rangle$ - W_L-W_R mixing

D. Štefánik, R. Dvornický, F.Š., P. Vogel, PRC 92, 055502 (2015)

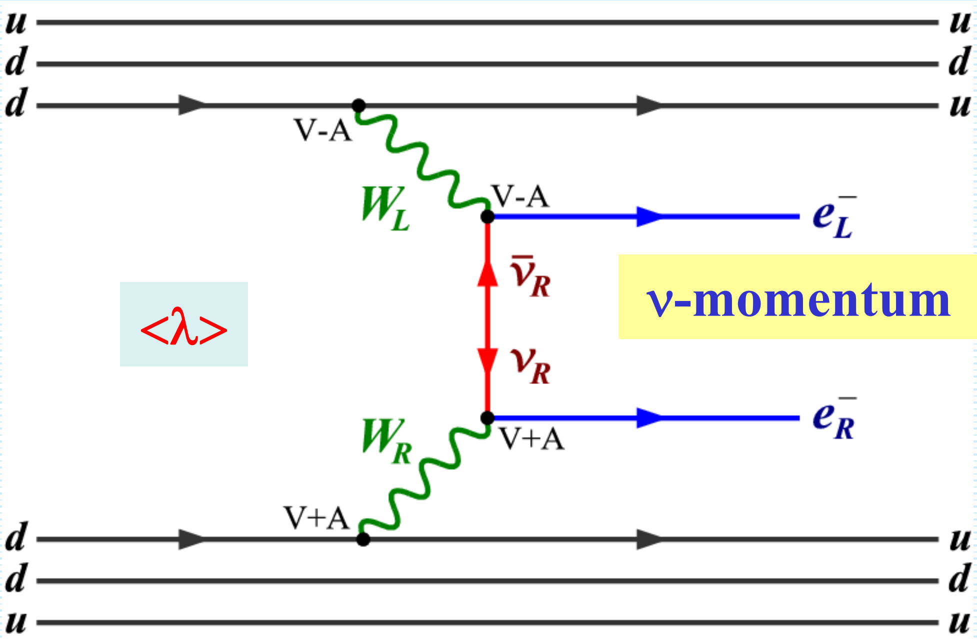
Left-right symmetric models SO(10)



Mixing of ligt and heavy neutrinos

$$\nu_{eL} = \sum_{j=1} \left(U_{ej}\nu_{jL} + S_{ej}(N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^{3} \left(T_{ej}^{*}(\nu_{jL})^C + V_{ej}^{*} N_{jR} \right)$$



Effective LNV parameters due to RHC

$$\langle\lambda\rangle = \lambda \left| \sum_{j=1}^{3} U_{ej} T_{ej}^{*} \right|$$

$$\langle\eta\rangle = \eta \left| \sum_{j=1}^{3} U_{ej} T_{ej}^{*} \right|$$

Mixing and masses of vector bosons

$$\eta = -\tan\zeta, \quad \chi = \eta,$$

$$\lambda = (M_{W_1}/M_{W_2})^2$$

3x3 block matrices U, S, T, V are generalization of PMNS matrix

Zhi-zhong Xing, Phys. Rev. D **85**, 013008 (2012)

Basis

6x6 neutrino mass matrix

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

$$(\nu_L, (N_R)^c)^T$$

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}$$

15 angles, 10+5 phases

Decomposition

$$\mathcal{U} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

The see-saw structure and neglecting mixing between different generations

Approximation

$$A \approx \mathbf{1}, \quad B \approx \mathbf{1}, \quad R \approx \frac{m_D}{m_{LNV}}\,\mathbf{1}, \quad S \approx -\frac{m_D}{m_{LNV}}\,\mathbf{1}$$

$$U_0 \simeq V_0$$

LNV parameters

$$|\langle\lambda\rangle| \approx \frac{m_D}{m_{LNV}} \left(\frac{M_{W_1}}{M_{W_2}}\right)^2 |\xi|$$

$$|\langle\eta\rangle| \approx \frac{m_D}{m_{LNV}} \tan(\zeta)\, |\xi|$$

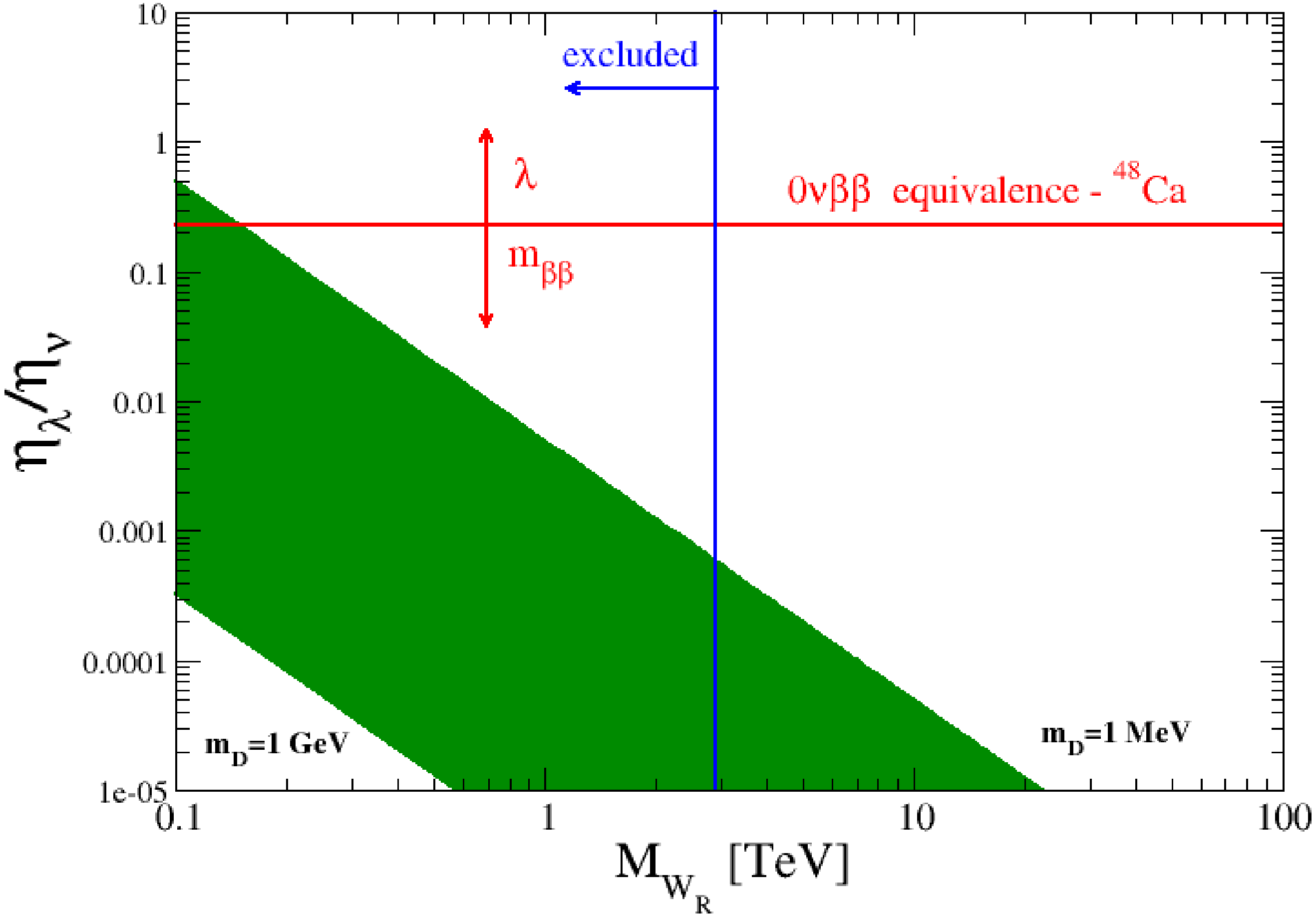
$$|\xi| \simeq 0.82$$

$$\eta_\nu = \frac{m_{\beta\beta}}{m_e} = \sum_i ((U_0)_{ei})^2 \frac{m_i}{m_e}$$

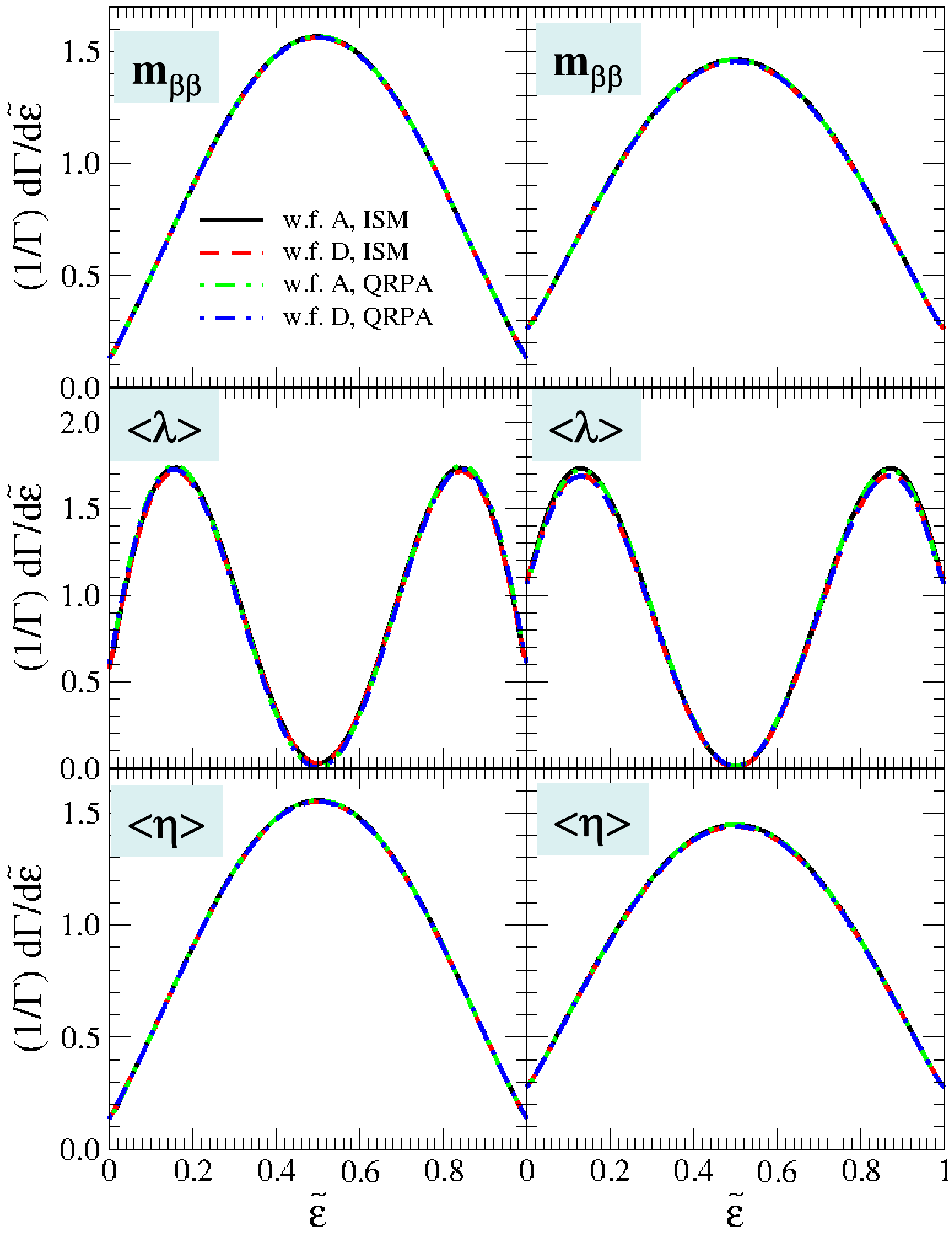
$$\approx \frac{m_D}{m_{LNV}} \frac{m_D}{m_e} \sum_i (U_0)^2_{ei} \frac{m_i\, m_{LNV}}{m_D^2}$$

if ≈ 1

$$|\langle\lambda\rangle| \approx \frac{m_D}{m_{LNV}} \left(\frac{M_{W_1}}{M_{W_2}}\right)^2 |\xi|$$

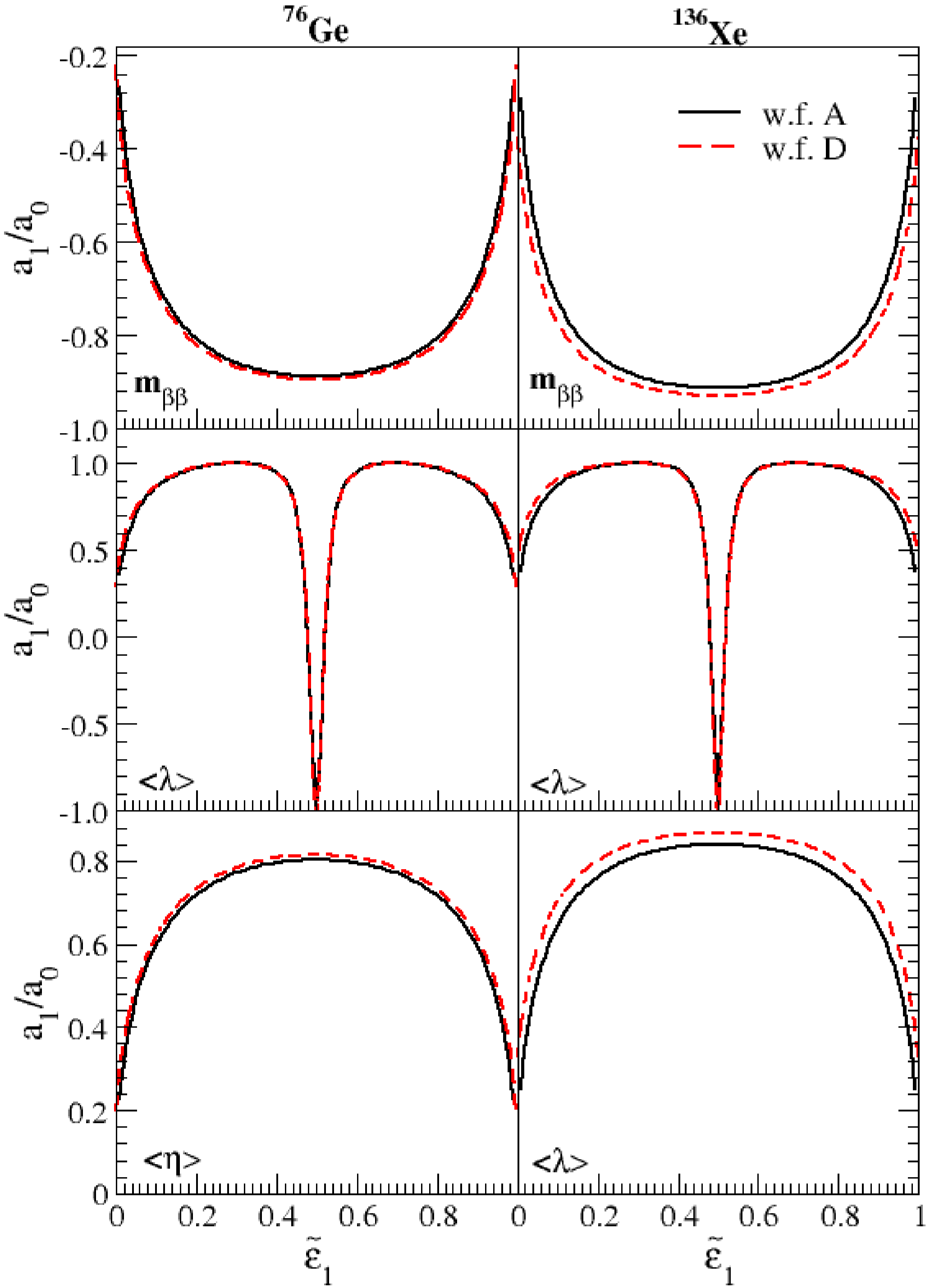


Clear dominance of $m_{\beta\beta}$ over <λ> mechanism by current constraint on mass of heavy vector boson and 1 MeV ≤ m_D ≤ 1 GeV



The single differential decay rate normalized to the total decay rate as function of electron energy for 3 limiting cases:

^{82}Se

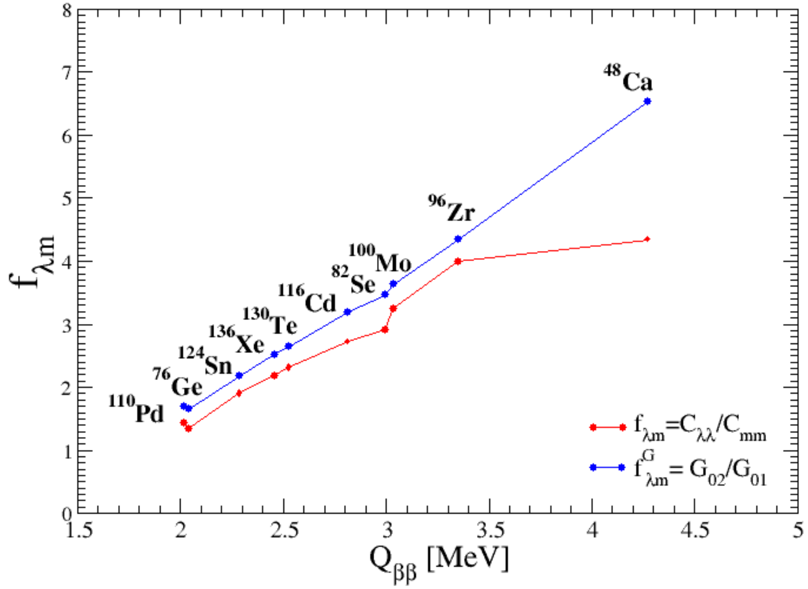


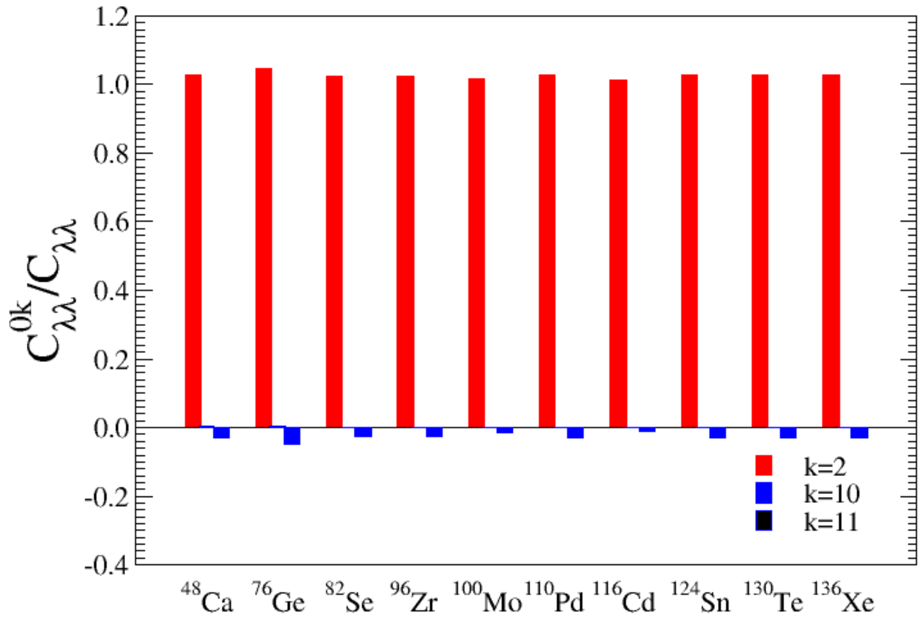
Angular correlation factor as function of electron energy

$$\frac{d\Gamma}{d\cos\theta \, d\tilde{\varepsilon}_1} = a_0 \left(1 + a_1/a_0 \cos\theta\right)$$

SuperNEMO experiment could see it

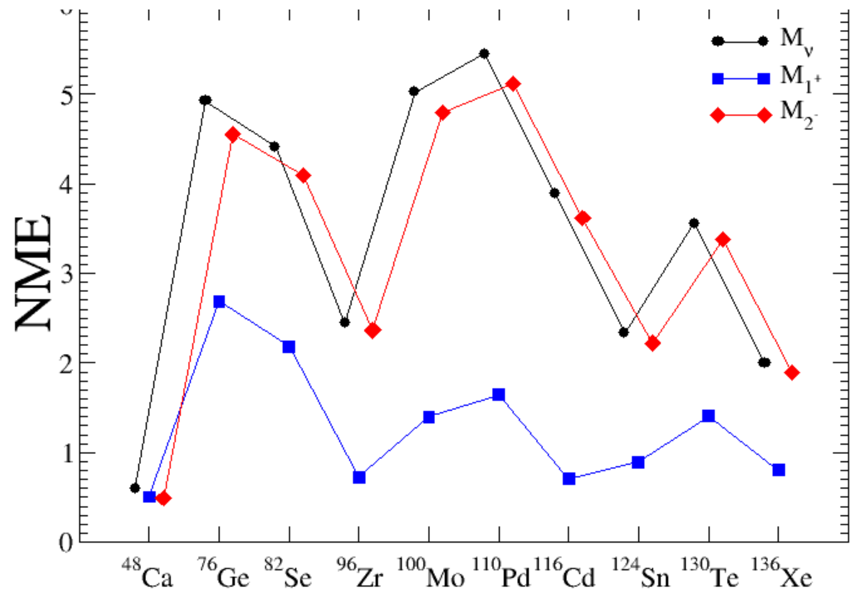
$m_{\beta\beta}$ and λ mechanisms





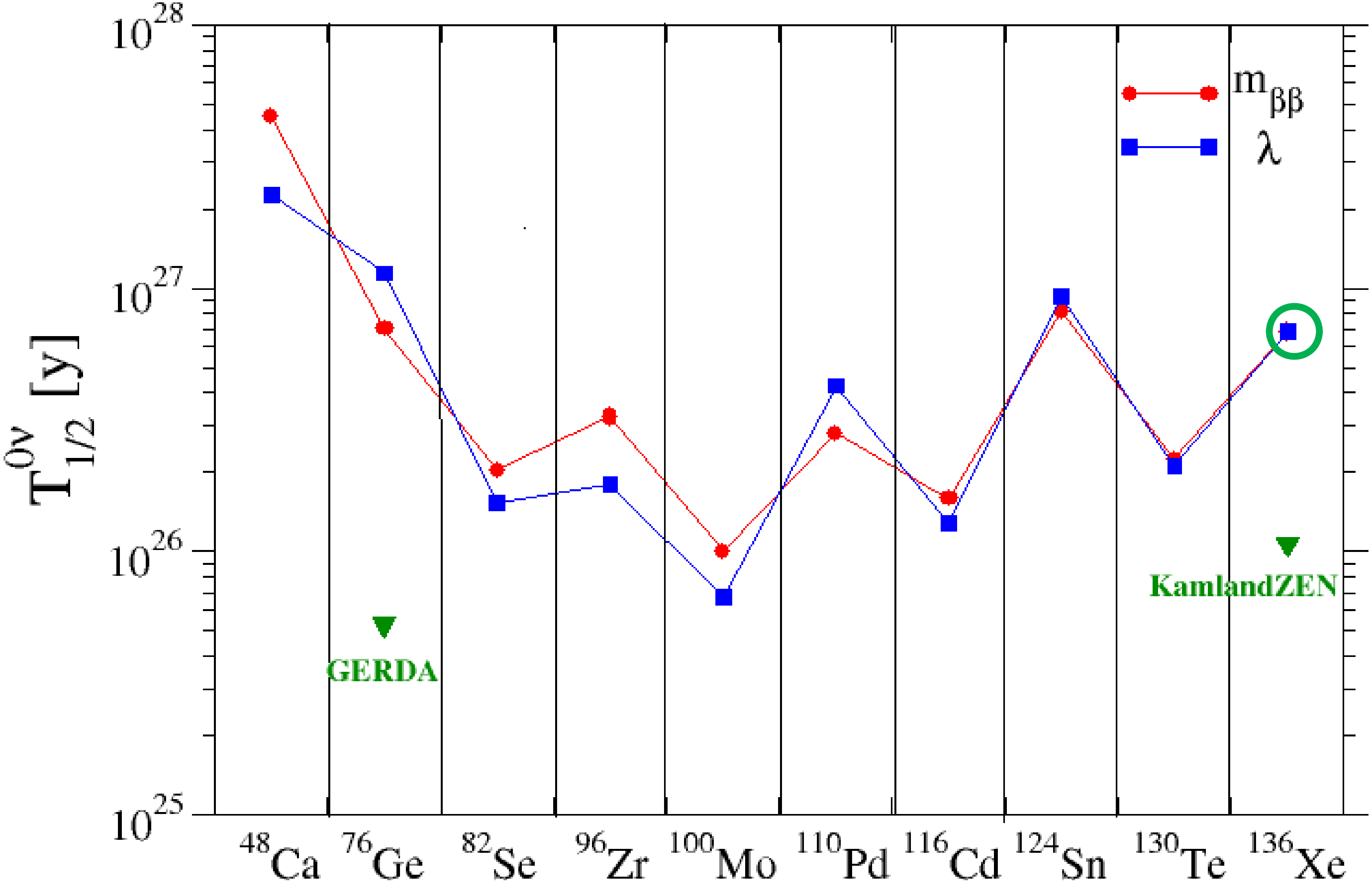
$$\left[T^{0\nu}_{1/2}\right]^{-1} = \left(\eta_\nu^2 + \eta_\lambda^2 f_{\lambda m}\right) C_{mm}$$
$$\simeq \left(\eta_\nu^2 + \eta_\lambda^2 f^G_{\lambda m}\right) g_A^4 M_\nu^2 G_{01}$$

$$f_{\lambda m} = \frac{C_{\lambda\lambda}}{C_{mm}} \simeq f^G_{\lambda m} = \frac{G_{02}}{G_{01}}$$



F.Š., R. Dvornický, R. Štefánik, submitted to Found. Phys.

$m_{\beta\beta}$= 50 meV (^{136}Xe), g_A=1.269, QRPA NMEs



IV. *The 0νββ-decay within L-R symmetric theories*

(D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)

J.D.Vergados, H. Ejiri, , F.Š., Int. J. Mod. Phys. E25, 1630007(2016)

$$\left(T^{0\nu}_{1/2} G^{0\nu} g_A^2\right)^{-1} = \left|\eta_\nu\ M^{0\nu}_\nu + \eta^L_N\ M^{0\nu}_N\right|^2 + \left|\eta^R_N\ M^{0\nu}_N\right|^2$$

$$\eta_\nu = \frac{m_{\beta\beta}}{m_e} = \sum_i ((U_0)_{ei})^2 \frac{m_i}{m_e} \approx \frac{m_p}{m_{LNV}} \frac{m_D^2}{m_e m_p} \sum_i (U_0)^2_{ei} \frac{m_i\, m_{LNV}}{m_D^2}$$

~1

$$\eta^L_N = \frac{m_p}{m_{LNV}} \sum_i (U^{(12)}_{ei})^2 \frac{m_{LNV}}{M_i} \approx \frac{m_p}{m_{LNV}} \left(\frac{m_D}{m_{LNV}}\right)^2 \sum_i \frac{m_{LNV}}{M_i}$$

~1

$\eta_\nu >> \eta^L_N$

$$\eta^R_N = \frac{m_p}{m_{LNV}} \left(\frac{M_{W_1}}{M_{W_2}}\right)^2 \sum_i (U^{22}_{ei})^2 \frac{m_{LNV}}{M_i} \approx \frac{m_p}{m_{LNV}} \left(\frac{M_{W_1}}{M_{W_2}}\right)^2 \sum_i (V_0)^2_{ei} \frac{m_{LNV}}{M_i}$$

~1

Sensitivity of 0.1 eV scale to LNV comparable
to sensitivity of TeV scale to LNV

η_ν and η^R_N might be comparable, if e.g.

$$\sum_i (U_0)^2_{ei} \frac{m_i\, m_{LNV}}{m_D^2} \simeq \sum_i (V_0)^2_{ei} \frac{m_{LNV}}{M_i}$$

$$\frac{m_D^2}{m_e m_p} M^{0\nu}_\nu \simeq \left(\frac{M_{W_1}}{M_{W_2}}\right)^2 M^{0\nu}_N$$

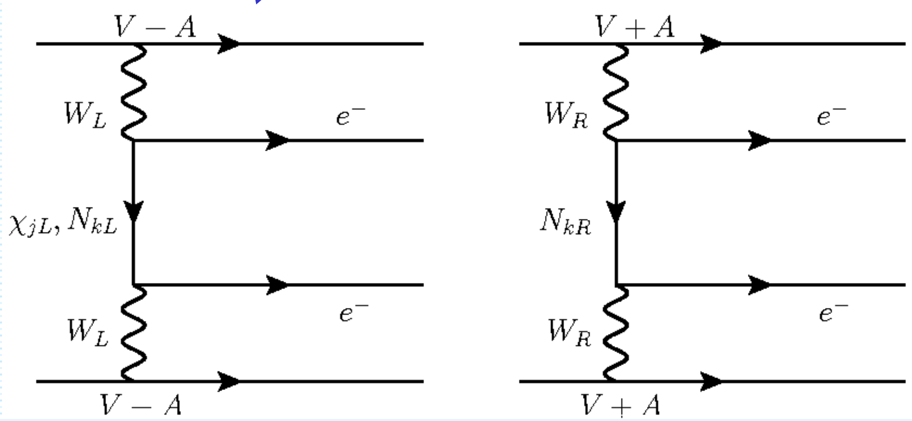
$m_D \approx m_e$ | **~5** | **$<10^{-4}$** | **~200**

Two non-interfering mechanisms of the $0\nu\beta\beta$-decay (light LH and heavy RH neutrino exchange)

Half-life:

$$\frac{1}{T^{0\nu}_{1/2,i} G^{0\nu}_i(E,Z)} \cong |\eta_\nu|^2 |M'^{0\nu}_{i,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{i,N}|^2$$

$$\eta_\nu = \frac{m_{\beta\beta}}{m_e}$$



Set of equations:

$$\frac{1}{T_1 G_1} = |\eta_\nu|^2 |M'^{0\nu}_{1,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{1,N}|^2$$

$$\frac{1}{T_2 G_2} = |\eta_\nu|^2 |M'^{0\nu}_{2,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{2,N}|^2$$

Solutions:

$$|\eta_\nu|^2 = \frac{|M'^{0\nu}_{2,N}|^2/T_1 G_1 - |M'^{0\nu}_{1,N}|^2/T_2 G_2}{|M'^{0\nu}_{1,\nu}|^2 |M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2 |M'^{0\nu}_{2,\nu}|^2}$$

$$|\eta_R|^2 = \frac{|M'^{0\nu}_{1,\nu}|^2/T_2 G_2 - |M'^{0\nu}_{2,\nu}|^2/T_1 G_1}{|M'^{0\nu}_{1,\nu}|^2 |M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2 |M'^{0\nu}_{2,\nu}|^2}$$

$$\eta^R_N = \left(\frac{M_W}{M_{WR}}\right)^4 \sum_k^{heavy} V^2_{ek} \frac{m_p}{M_k}$$

A. Faessler, A. Meroni, S.T. Petcov, F. Š., J.D. Vergados, Phys. Rev. D 83, 113003 (2011); JHEP 1302, 025 (2013)

Two non-interfering mechanisms of the $0\nu\beta\beta$-decay
(light LH and heavy RH neutrino exchange)

Pure $m_{\beta\beta}$ mech.

The positivity condition:

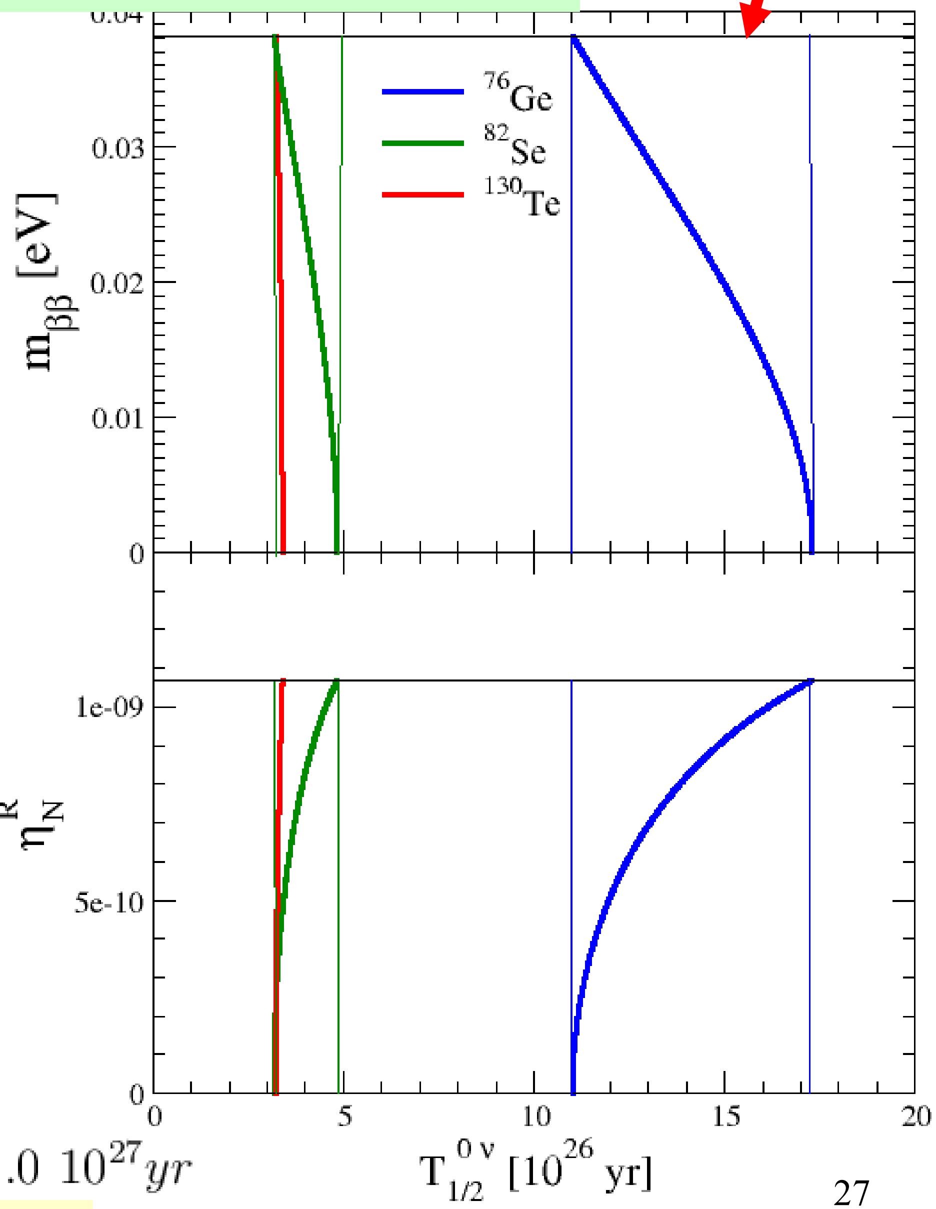
$$\frac{T_1 G_1 |M'^{0\nu}_{1,N}|^2}{G_2 |M'^{0\nu}_{2,N}|^2} \leq T_2 \leq \frac{T_1 G_1 |M'^{0\nu}_{1,\nu}|^2}{G_2 |M'^{0\nu}_{2,\nu}|^2}$$

Very narrow ranges!

$$1.10 \leq \frac{T^{0\nu}_{1/2}(^{76}Ge)}{T^{0\nu}_{1/2}(^{136}Xe)} \leq 1.73$$

$$3.17 \leq \frac{T^{0\nu}_{1/2}(^{82}Se)}{T^{0\nu}_{1/2}(^{136}Xe)} \leq 4.83$$

$$3.22 \leq \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{136}Xe)} \leq 3.40$$

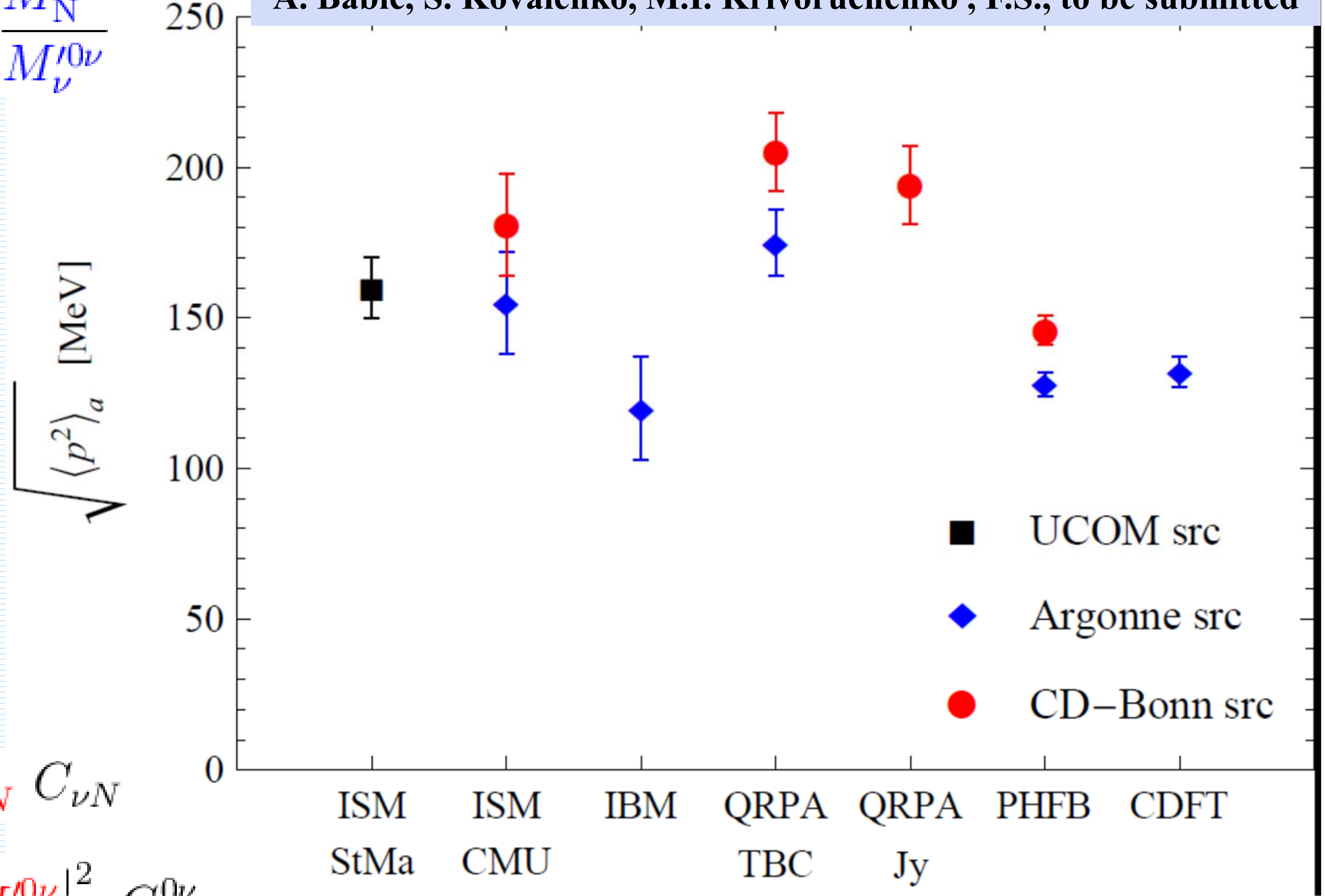


$$T^{0\nu}_{1/2}(^{136}Xe) = 1.0\ 10^{27} yr$$

Assumption

$$\langle p^2 \rangle = m_\mathrm{p} m_\mathrm{e} \frac{M'^{0\nu}_\mathrm{N}}{M'^{0\nu}_\nu}$$

A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., to be submitted



$$[T^{0\nu}_{1/2}]^{-1} = \eta^2_{\nu N}\ C_{\nu N}$$

$$C_{\nu N} = g_\mathrm{A}^4 \left| M'^{0\nu}_\nu \right|^2 G^{0\nu}$$

$$\eta^2_{\nu N} = \left| \sum_{j=1}^{3} \left(U^2_{ej} \frac{m_j}{m_e} + S^2_{ej} \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2 + \lambda^2 \left| \sum_{j=1}^{3} \left(T^2_{ej} \frac{m_j}{m_e} + V^2_{ej} \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2$$

$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta\,\mathbf{1} \\ -\zeta\,\mathbf{1} & V_0 \end{pmatrix}$$

$$m_i M_i \simeq m_D^2$$

$$M^{\rm R}_{\beta\beta} = \lambda \frac{\langle p^2\rangle_a}{m_D^2} \left| \sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\; m_j \right|$$

$$m_i \simeq \zeta^2 M_i$$

$$M^{\rm R}_{\beta\beta} = \lambda\, \zeta^2 \left| \sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\; \frac{\langle p^2\rangle_a}{m_j} \right|$$

$V_0 = U^\dagger_{PMNS}$

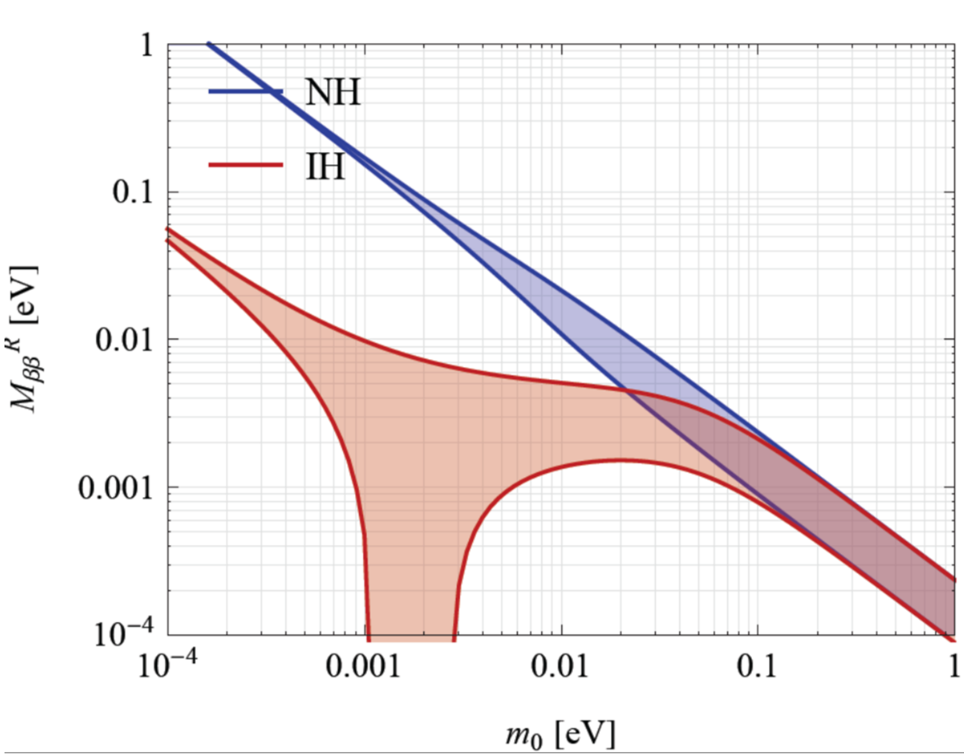
$M_i = m_D^2/m_i \quad m_D \simeq 5$ MeV

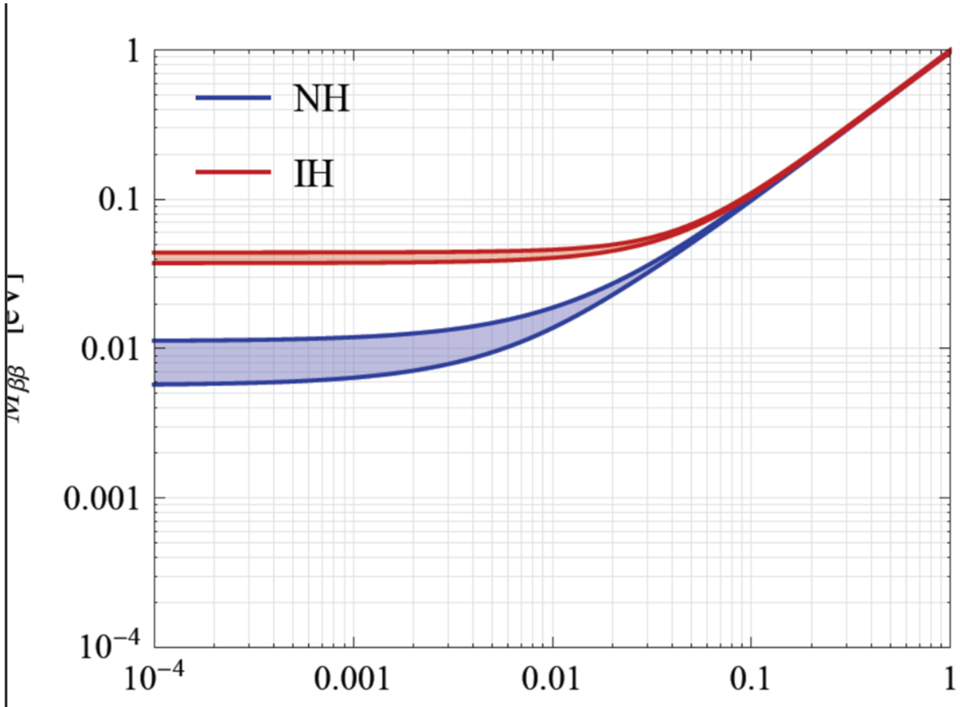
$\lambda = 7.7 \times 10^{-4}$

$V_0 = U^\dagger_{PMNS}$

$\zeta = m_i/M_i \quad \zeta^2 \simeq 5 \times\ 10^{-17}$

$\lambda = 7.7 \times 10^{-4}$





See-saw scenario

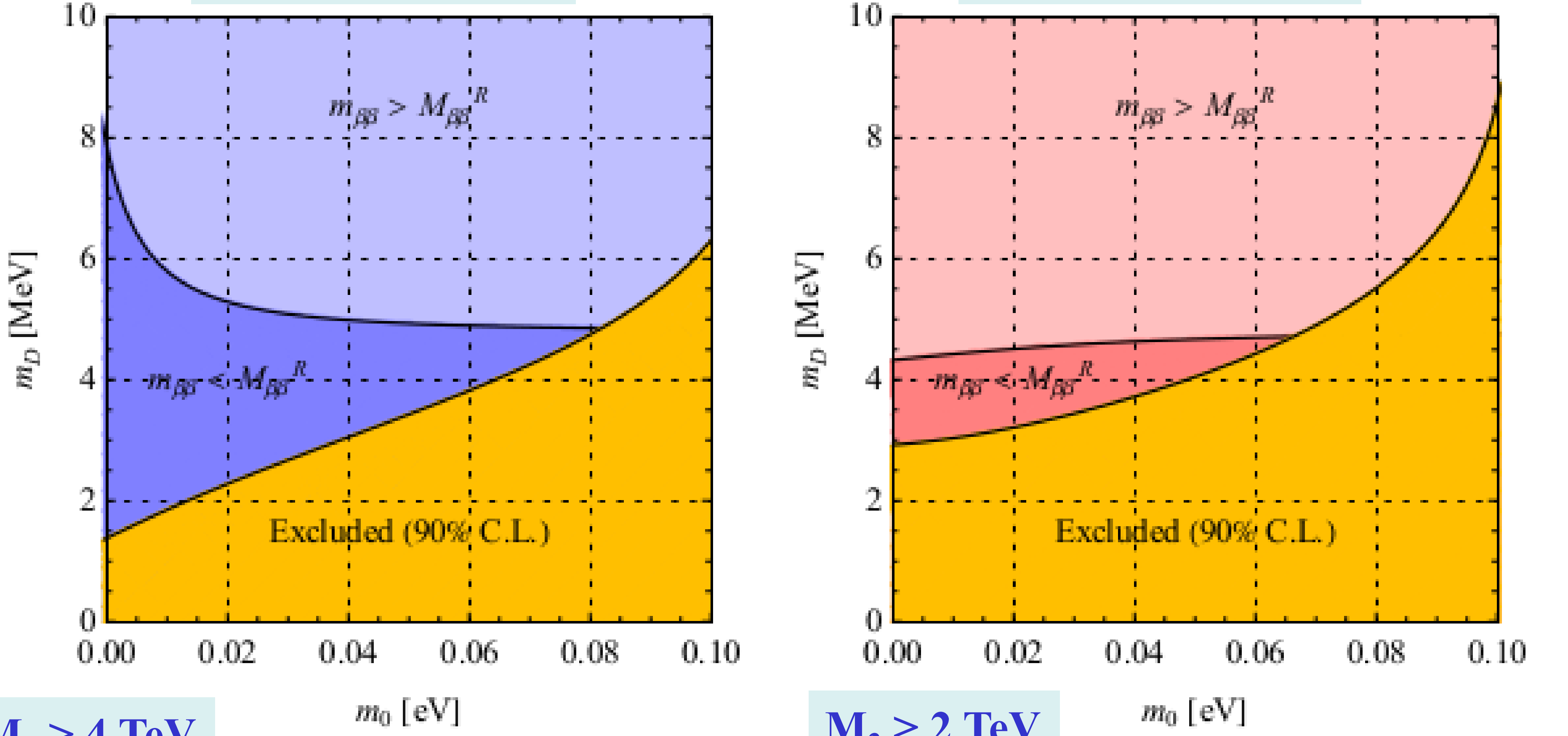
$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta\,\mathbf{1} \\ -\zeta\,\mathbf{1} & V_0 \end{pmatrix}$$

$$m_i M_i \simeq m_D^2$$

$$M^{\rm R}_{\beta\beta} = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\, m_j \right|$$

$$\eta^2_{\nu N} = \frac{1}{m_e^2}\left(m^2_{\beta\beta} + (M^R_{\beta\beta})^2\right)$$

Normal spectrum | **Inverted spectrum**



M_3 > 4 TeV | M_3 > 2 TeV

A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., to be submitted

See-saw scenario

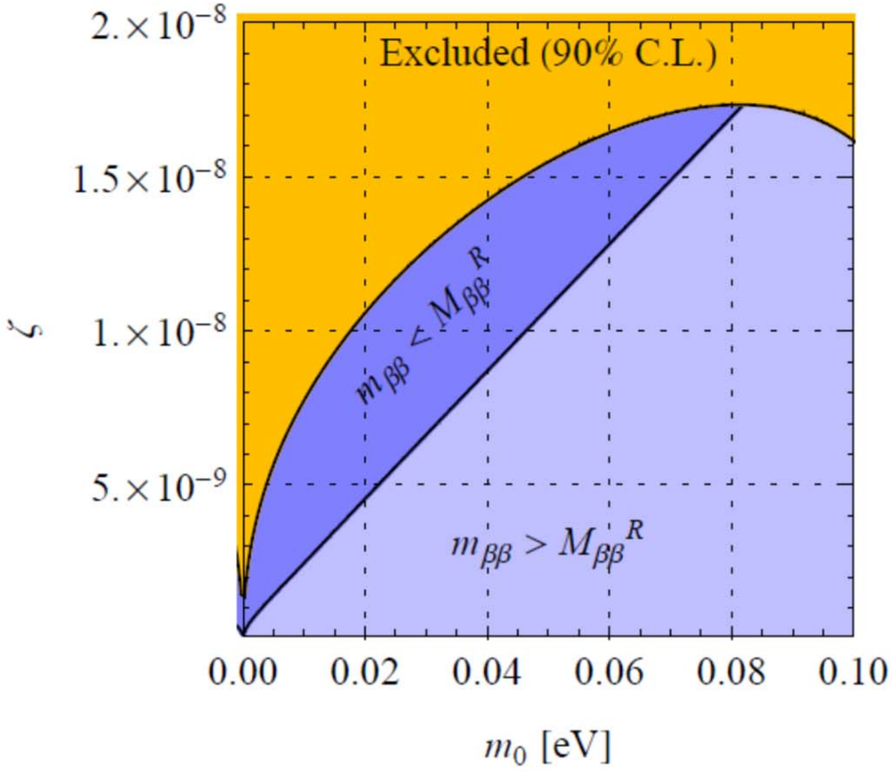
$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta\,\mathbf{1} \\ -\zeta\,\mathbf{1} & V_0 \end{pmatrix}$$

$$m_i \simeq \zeta^2 M_i$$

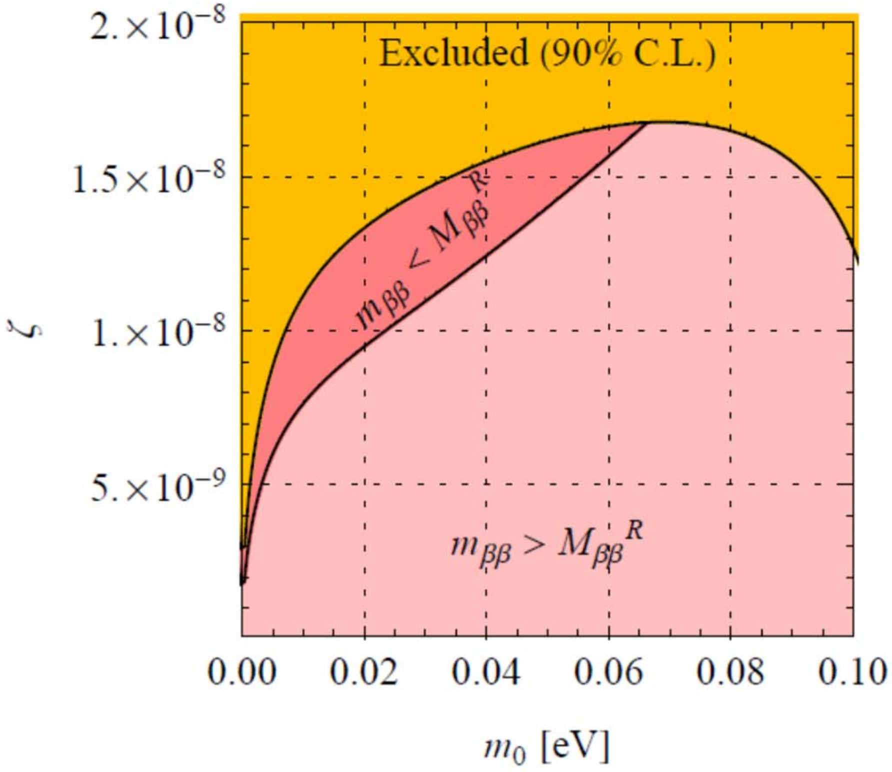
$$M^{\mathrm{R}}_{\beta\beta} = \lambda\,\zeta^2 \left| \sum_{j=1}^{3} (U_0^\dagger)^2_{ej} \frac{\langle p^2\rangle_a}{m_j} \right|$$

$$\eta^2_{\nu N} = \frac{1}{m_e^2}\left(m^2_{\beta\beta} + (M^R_{\beta\beta})^2\right)$$

Normal spectrum



Inverted spectrum



A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., to be submitted

Calculation of $0\nu\beta\beta$ decay NMEs

Method	g_A	src	$M_\nu^{0\nu}$					
			^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd
ISM-StMa	1.25	UCOM	0.85	2.81	2.64			
ISM-CMU	1.27	Argonne	0.80	3.37	3.19			
		CD-Bonn	0.88	3.57	3.39			
IBM	1.27	Argonne	1.75	4.68	3.73	2.83	4.22	4.05
QRPA-TBC	1.27	Argonne	0.54	5.16	4.64	2.72	5.40	5.76
		CD-Bonn	0.59	5.57	5.02	2.96	5.85	6.26
QRPA-Jy	1.26	CD-Bonn		5.26	3.73	3.14	3.90	6.52
dQRPA-NC	1.25	without		5.09				
PHFB	1.25	Argonne				2.84	5.82	7.12
		CD-Bonn				2.98	6.07	7.42
NREDF	1.25	UCOM	2.37	4.60	4.22	5.65	5.08	
REDF	1.25	without	2.94	6.13	5.40	6.47	6.58	
Mean value			1.34	4.55	4.02	3.78	5.57	6.12
variance			0.81	1.20	0.91	2.49	0.58	1.78

Method	g_A	src	$M_\nu^{0\nu}$					
			^{116}Cd	^{124}Sn	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
ISM-StMa	1.25	UCOM		2.62		2.65	2.19	
ISM-CMU	1.27	Argonne		2.00		1.79	1.63	
		CD-Bonn		2.15		1.93	1.76	
IBM	1.27	Argonne	3.10	3.19	4.10	3.70	3.05	2.67
QRPA-TBC	1.27	Argonne	4.04	2.56	4.56	3.89	2.18	
		CD-Bonn	4.34	2.91	5.08	4.37	2.46	3.37
QRPA-Jy	1.26	CD-Bonn	4.26	5.30	4.92	4.00	2.91	
dQRPA-NC	1.25	without				1.37	1.55	2.71
PHFB	1.27	Argonne			3.90	3.81		2.58
		CD-Bonn			4.08	3.98		2.68
NREDF	1.25	UCOM	4.72	4.81	4.11	5.13	4.20	1.71
REDF	1.25	without	5.52	4.33		4.98	4.32	5.60
Mean value			4.34	3.07	4.34	3.42	2.59	3.01
variance			0.79	1.01	0.23	1.67	1.10	1.34

NMEs for unquenched value of g_A

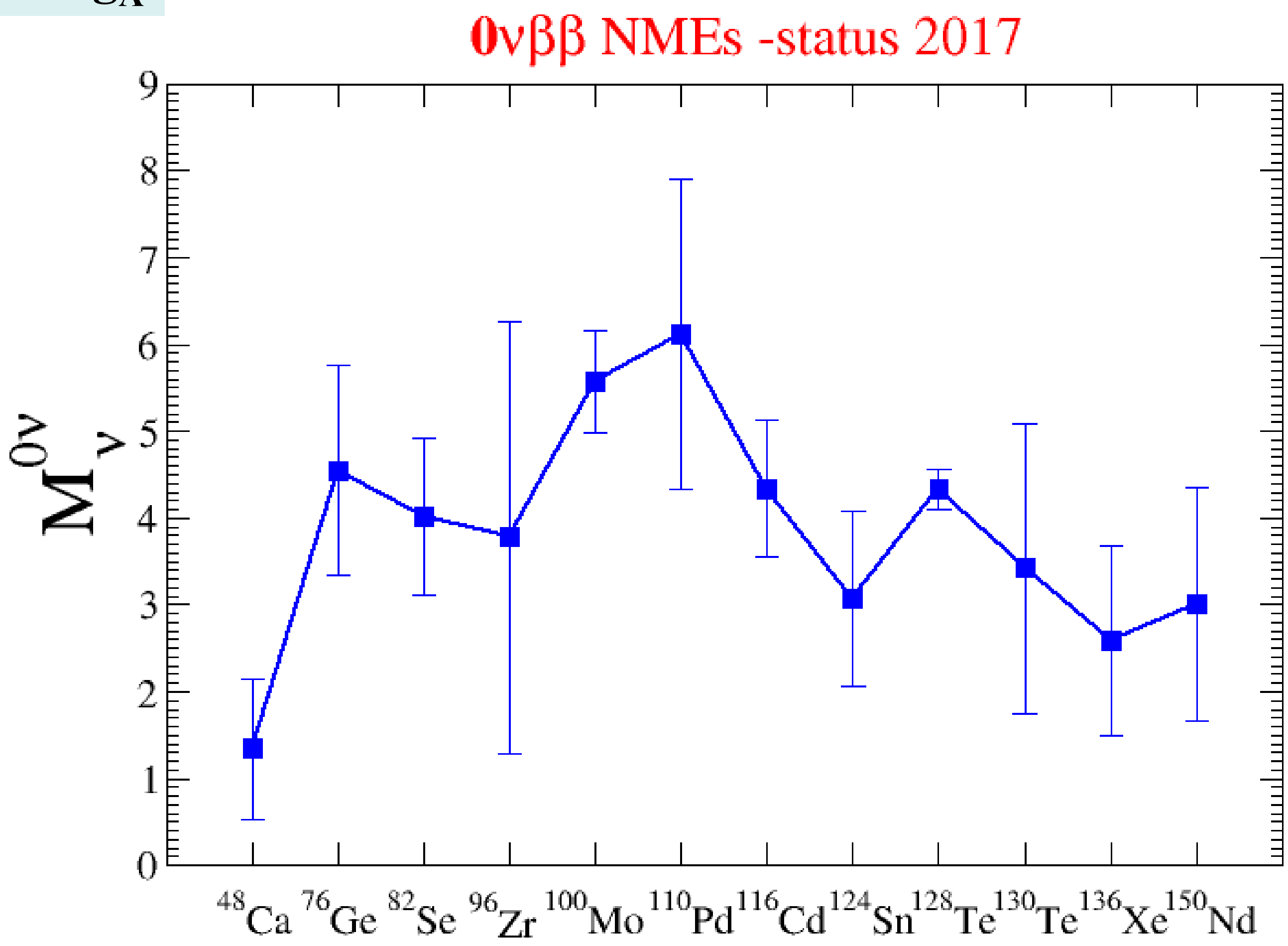
Mean field approaches
(PHFB, NREDF, REDF)
⇒ Large NMEs

Interacting Shell Model
(ISM-StMa, ISM-CMU)
⇒ small NMEs

Quasiparticle Random Phase Approximation
(QRPA-TBC, QRPA-Jy, dQRPQ-NC)
⇒ Intermediate NMEs

Interacting Boson Model
(IBM)
⇒ Close to QRPA results

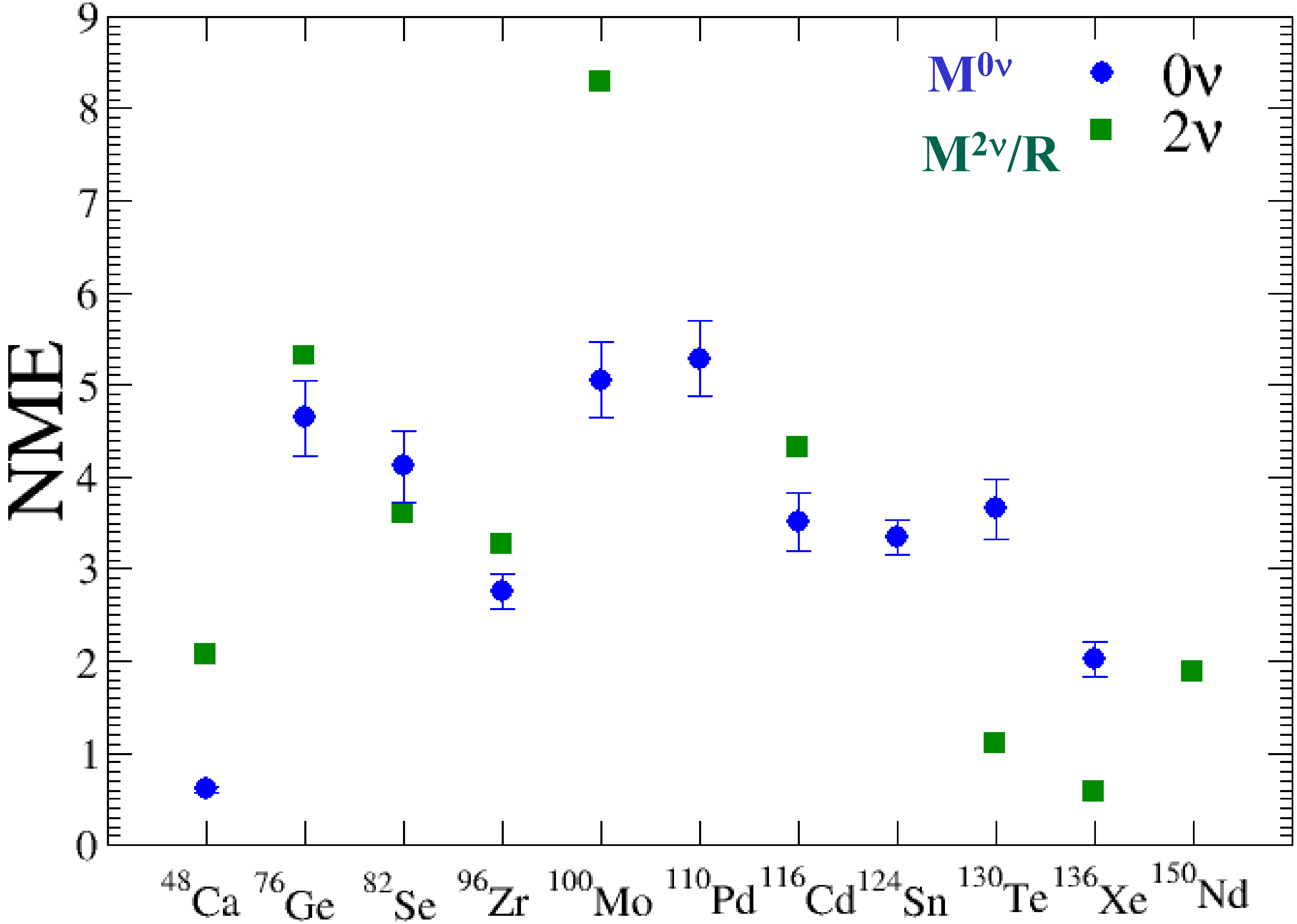
unquenched g_A



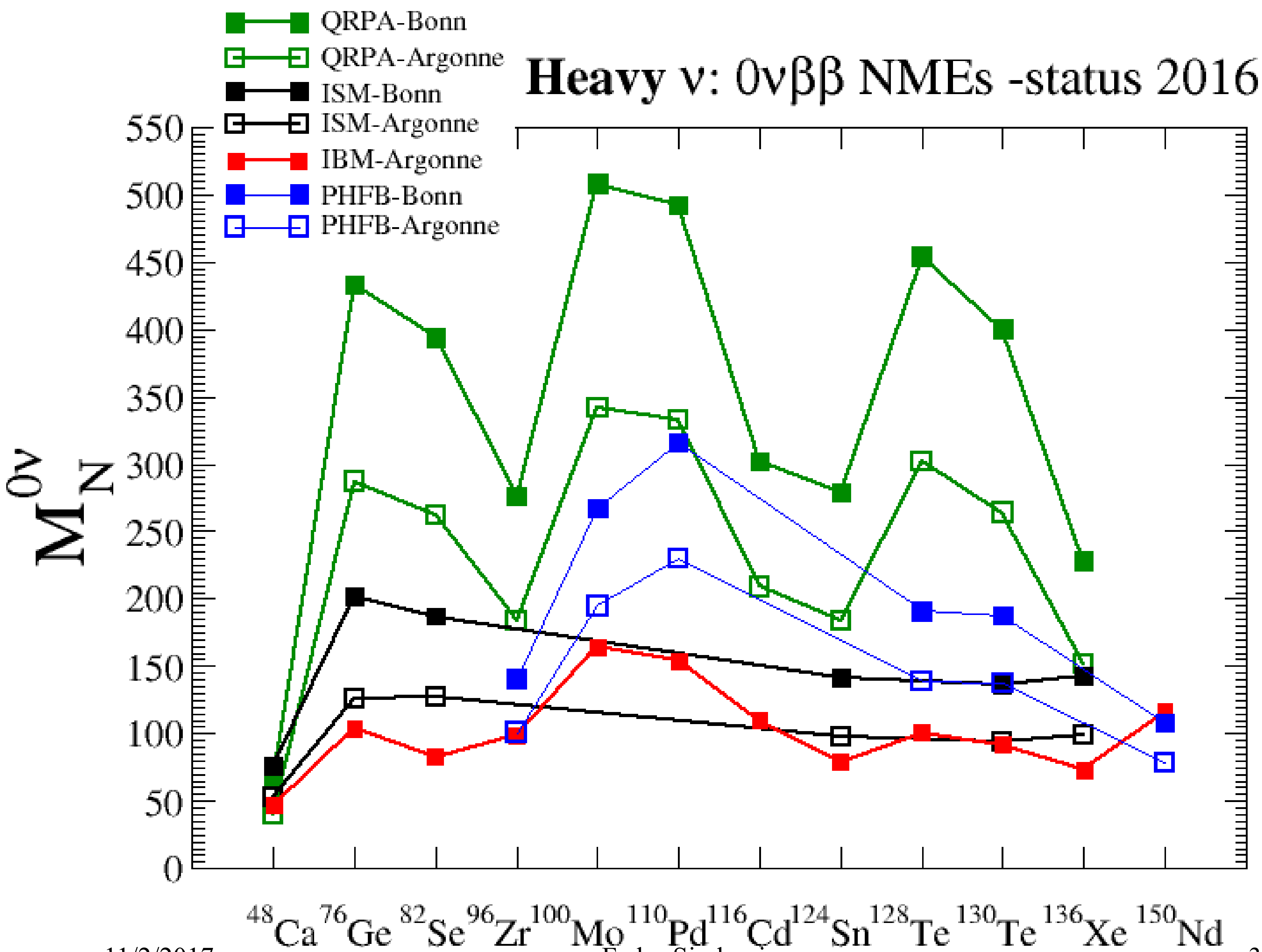
J.D.Vergados, H. Ejiri, , F.Š., Int. J. Mod. Phys. E25, 1630007(2016)

	mean field meth.	ISM	IBM	QRPA
Large model space	yes	no	yes	yes
Constr. Interm. States	no	yes	no	yes
Nucl. Correlations	limited	all	restricted	restricted

Is there a scaling factor between 0νββ- and 2νββ-decay NMEs?



F. Š., Nucl.Part.Phys.Proc. 265-266 (2015) 19-24


QRPA-Bonn
QRPA-Argonne
ISM-Bonn
ISM-Argonne
IBM-Argonne
PHFB-Bonn
PHFB-Argonne
Heavy ν: 0νββ NMEs -status 2016
$M^{0\nu}_N$
0
50
100
150
200
250
300
350
400
450
500
550
^{48}Ca ^{76}Ge ^{82}Se ^{96}Zr ^{100}Mo ^{110}Pd ^{116}Cd ^{124}Sn ^{128}Te ^{130}Te ^{136}Xe ^{150}Nd

Quenching of g_A

$g_A^4 = (1.269)^4 = 2.6$

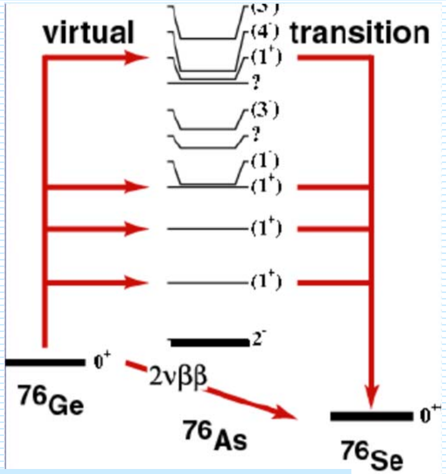
Quenching of g_A *(from exp.: $T_{1/2}^{0\nu}$ up 2.5 x larger)*

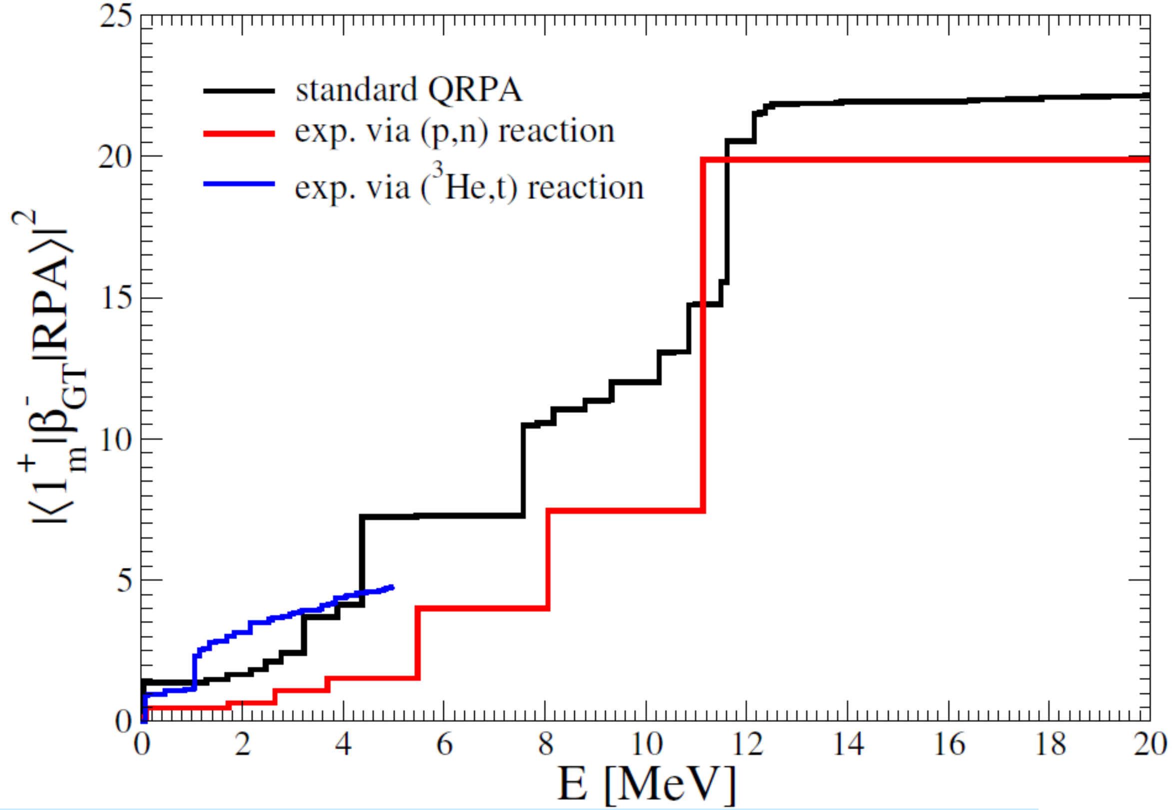
$(g^{eff}_A)^4 = 1.0$

Strength of GT trans. (approx. given by Ikeda sum rule =3(N-Z)) has to be quenched to reproduce experiment

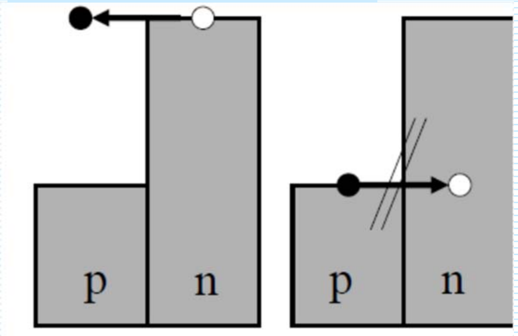
$^{76}_{32}Ge_{44} \Rightarrow$

$S_{\beta}^{-} - S_{\beta}^{+} = 3(N-Z) = 36$





Pauli blocking



Cross-section for charge exchange reaction:

$$\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{\mu}{\pi\hbar}\right]^2 \frac{k_f}{k_i} N_d \, |V_{\sigma\tau}|^2 \, |\langle f \,|\, \sigma\tau \,|\, i \rangle|^2$$

$q = 0!!$

largest at 100 - 200 MeV/A

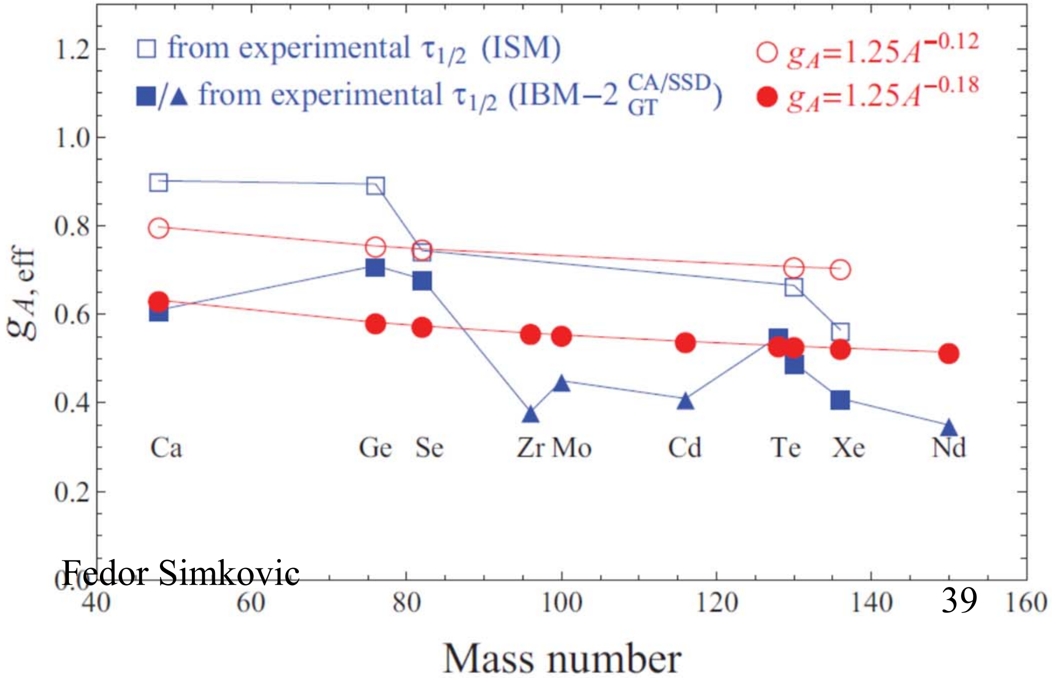
Quenching of g_A (from theory: $T_{1/2}^{0\nu}$ up 50 x larger)

$(g^{eff}_A)^4 \simeq$ 0.66 (^{48}Ca), 0.66 (^{76}Ge), 0.30 (^{76}Se), 0.20 (^{130}Te) and 0.11 (^{136}Xe)
The Interacting Shell Model (ISM), which describes qualitatively well energy spectra, does reproduce experimental values of $M^{2\nu}$ only by consideration of significant quenching of the Gamow-Teller operator, typically by 0.45 to 70%.

$(g^{eff}_A)^4 \simeq (1.269\ A^{-0.18})4 = 0.063$ (The Interacting Boson Model). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like 60%.

J. Barea, J. Kotila, F. Iachello, PRC 87, 014315 (2013).

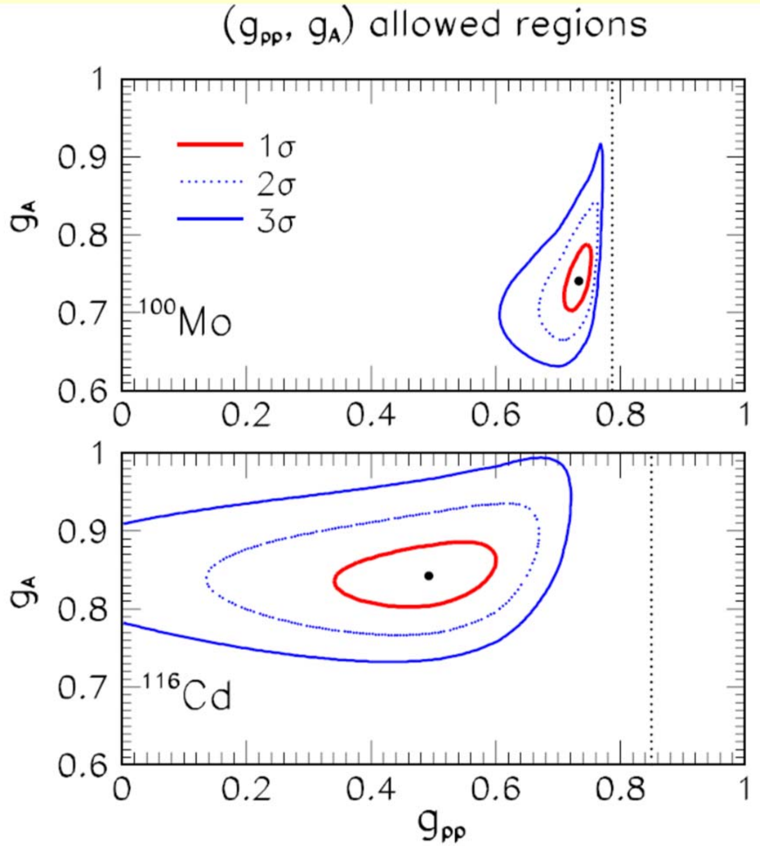
It has been determined by theoretical prediction for the 2νββ-decay half-lives, which were based on within closure approximation calculated corresponding NMEs, with the measured half-lives.

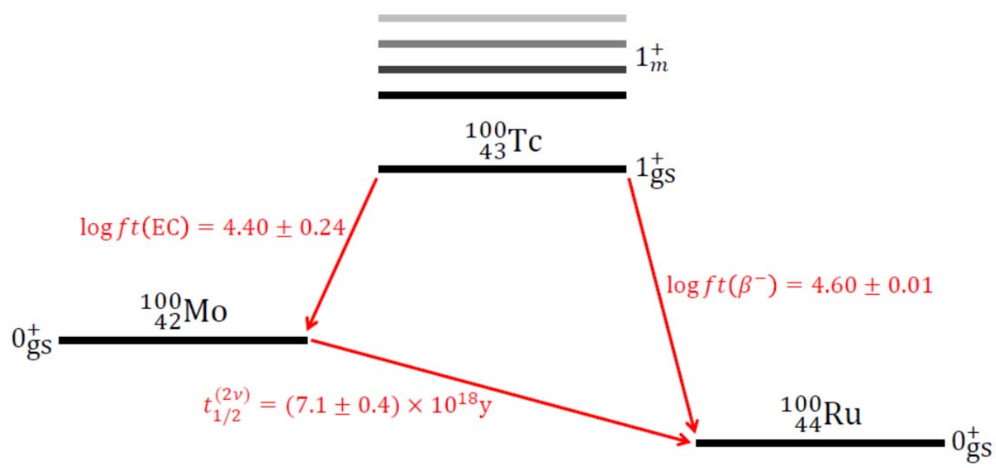


Faessler, Fogli, Lisi, Rodin, Rotunno, F. Š, J. Phys. G 35, 075104 (2008).

$(g^{eff}_A)^4$ = 0.30 and 0.50 for ^{100}Mo and ^{116}Cd, respectively (**The QRPA prediction**).

g^{eff}_A was treated as a completely free parameter alongside g_{pp} (used to renormalize particl-particle interaction) by performing calculations within the QRPA and RQRPA. It was found that a least-squares fit of g^{eff}_A and g_{pp}, where possible, to the **β-decay rate** and **β+/EC rate** of the J = 1^+ ground state in the intermediate nuclei involved in double-beta decay in addition to the **2νββ rates** of the initial nuclei, leads to an effective g^{eff}_A of about 0.7 or 0.8.





Extended calculation also for neighbor isotopes performed by

F.F. Depisch and J. Suhonen, PRC 94, 055501 (2016)

Dependence of g^{eff}_A on **A** was not established.

A novel method to determine effective g_A

F. Š., R. Dvornický, D. Štefánik, A. Faessler, to be submitted

Improved description of the 0νββ–decay rate

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

Let perform Taylor expansion

$$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2} \qquad \epsilon_{K,L} \in \left(-\frac{Q}{2}, \frac{Q}{2}\right)$$

We get

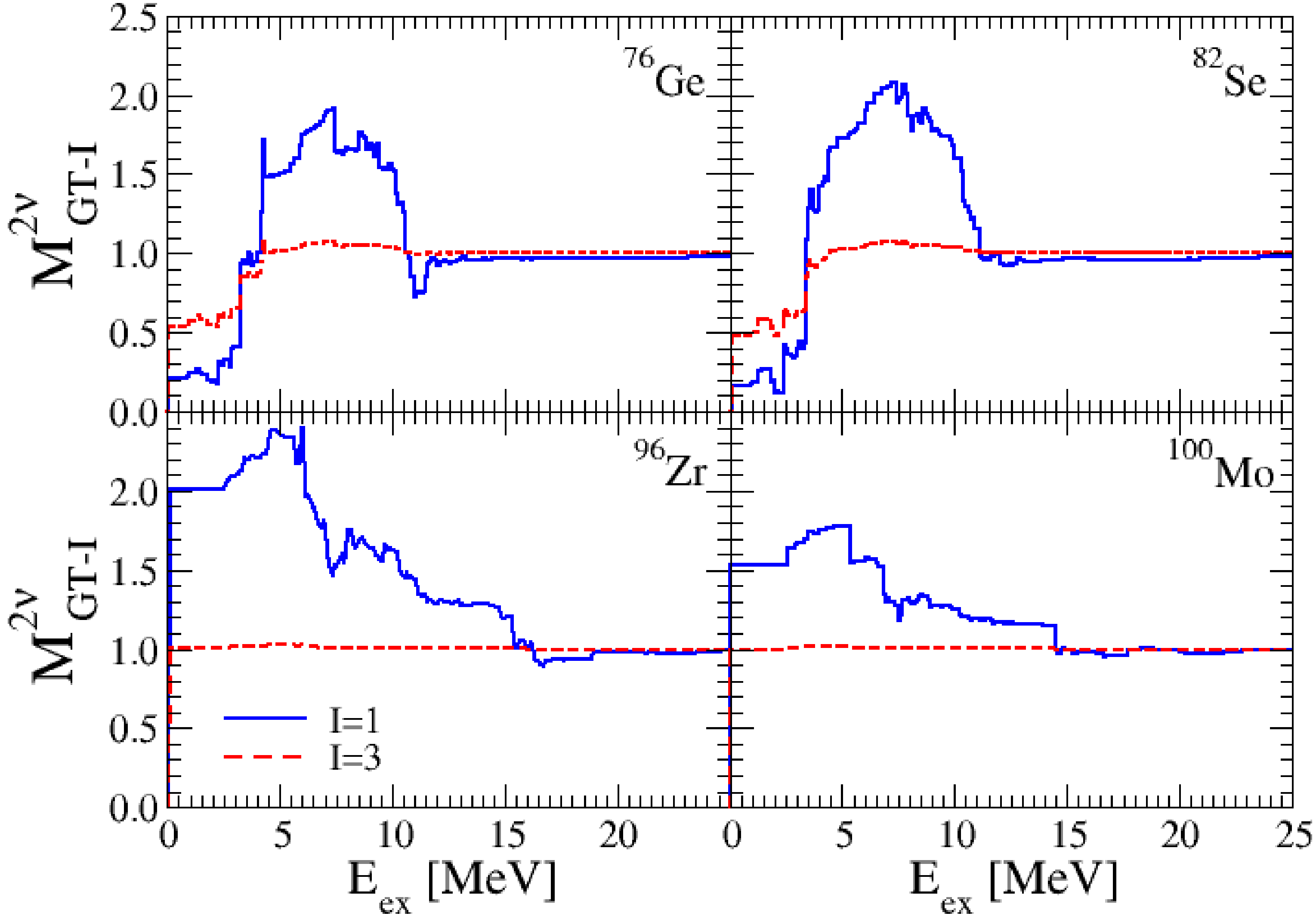
$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\rm eff}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{\left|\xi_{13}^{2\nu}\right|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

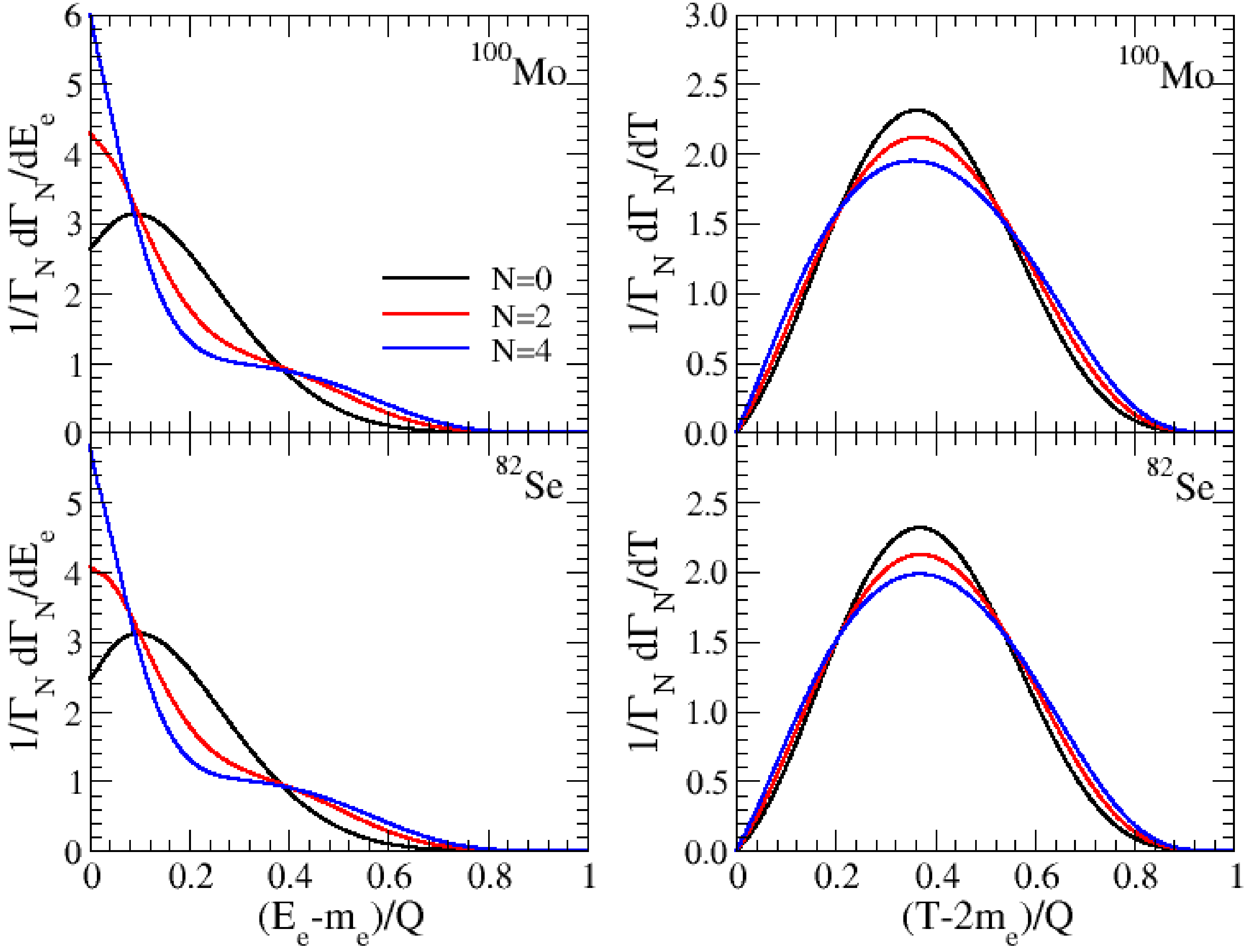
$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4\, m_e^3}{(E_n - (E_i + E_f)/2)^3} \qquad \xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

The g_A^{eff} can be deterimed with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM?)

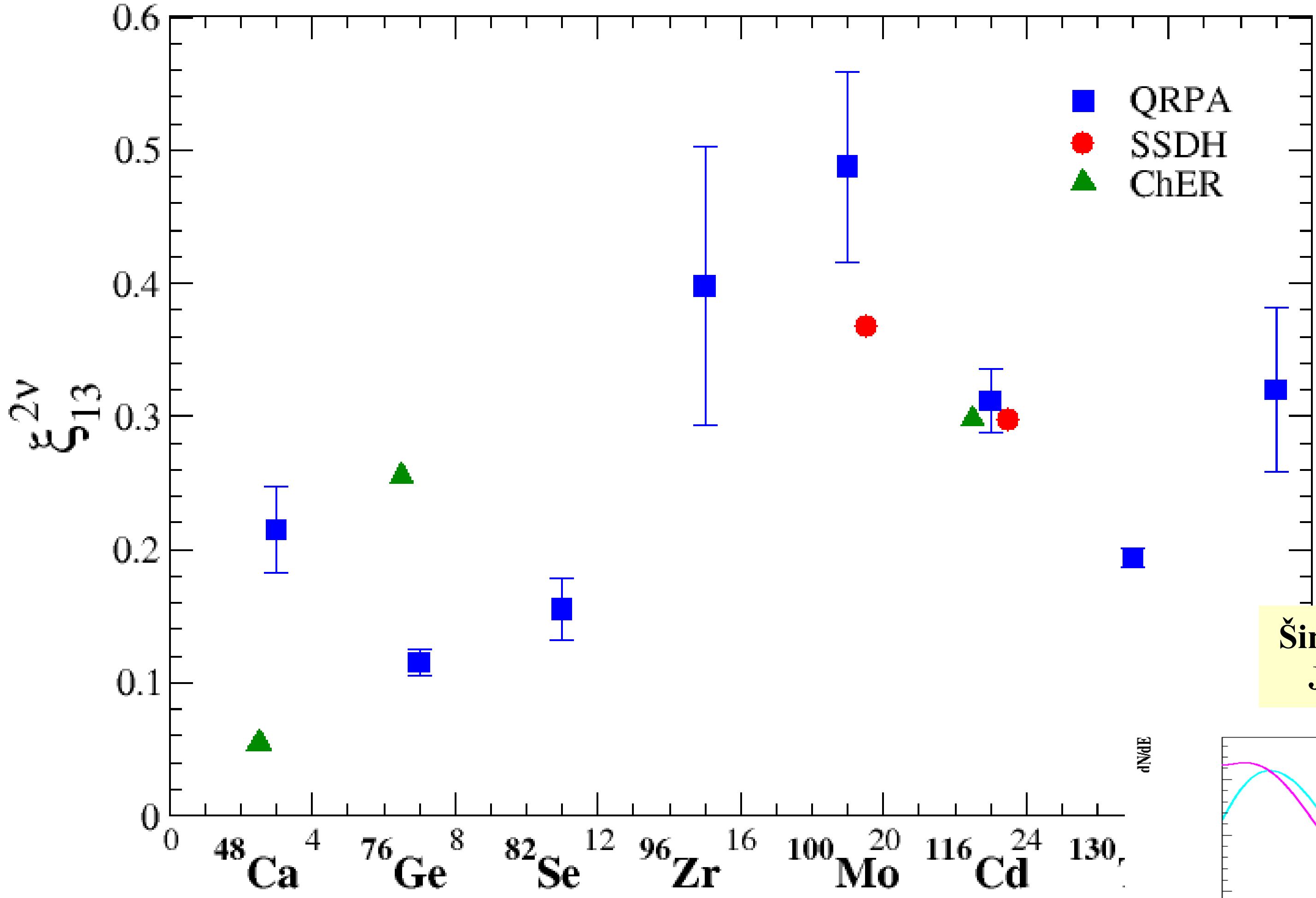
The running sum of the 2νββ–decay NMEs



Normalized to unity different partial energy distributions

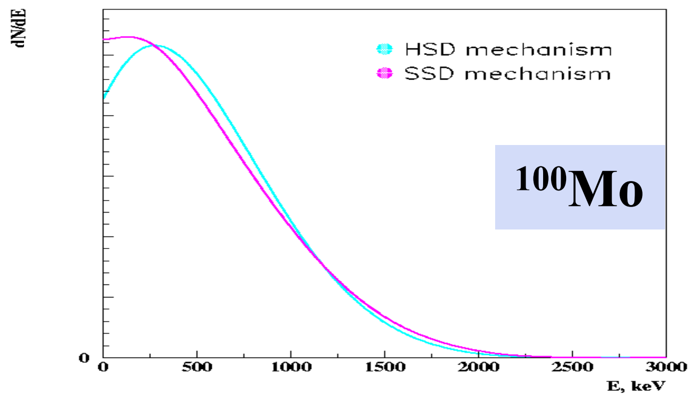


ξ_{13} tell us about importance of higher lying states of int. nucl.



HSD: ξ_{13}=0

Šimkovic, Šmotlák, Semenov
J. Phys. G, 27, 2233, 2001



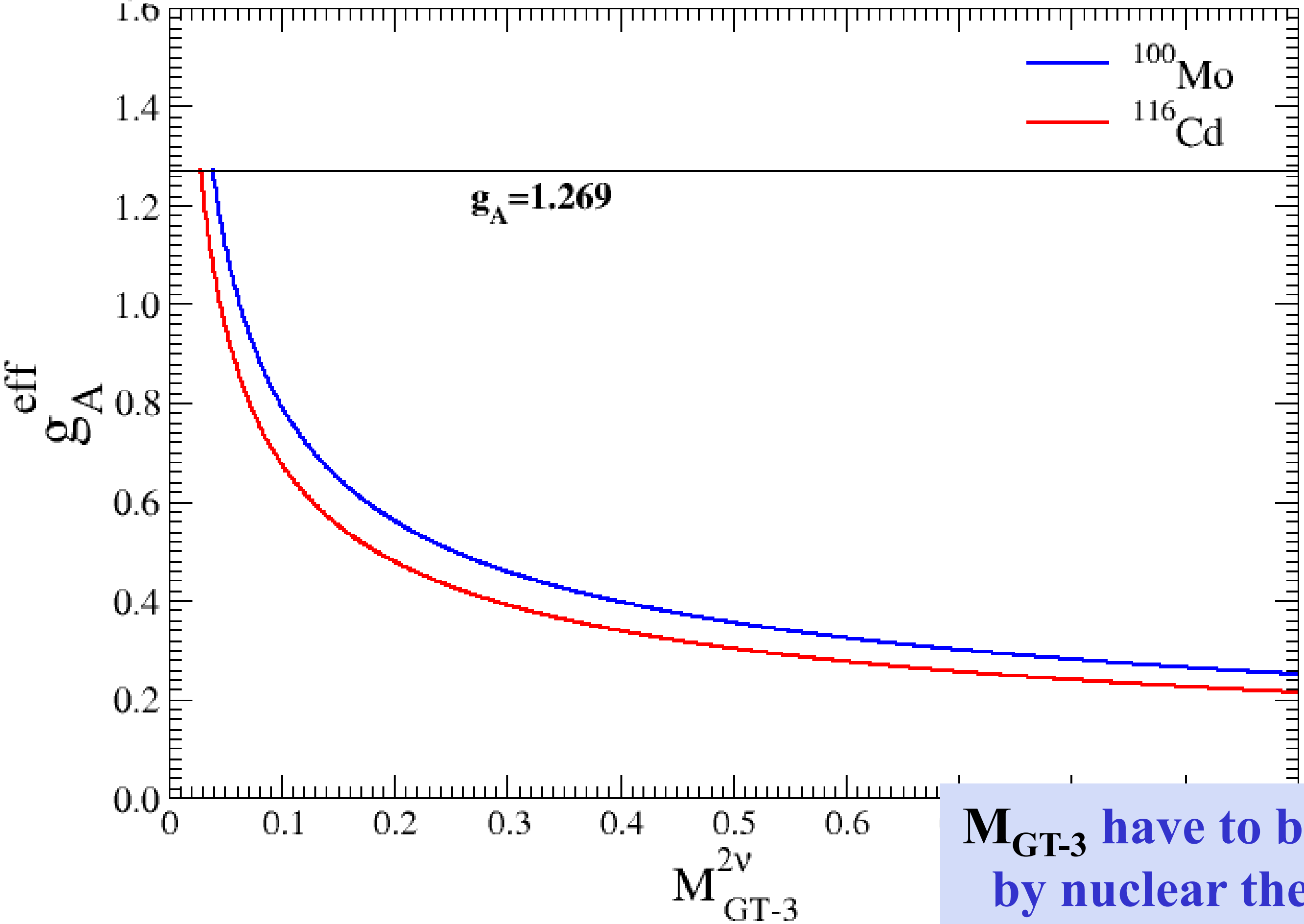
ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

Solution: NEMO3/Supernemo measurement of ξ and calculation of M_{GT-3}

$$\left(g_A^{\text{eff}}\right)^2 = \frac{1}{\left|M_{GT-3}^{2\nu}\right|} \frac{\left|\xi_{13}^{2\nu}\right|}{\sqrt{T_{1/2}^{2\nu-exp}\left(G_0^{2\nu}+\xi_{13}^{2\nu}G_2^{2\nu}\right)}}$$

$$g_A^{\text{eff}}(^{100}\text{Mo}) = \frac{0.251}{\sqrt{M_{GT-3}^{2\nu}}}$$

$$g_A^{\text{eff}}(^{100}\text{Cd}) = \frac{0.214}{\sqrt{M_{GT-3}^{2\nu}}}$$



M_{GT-3} have to by calculated by nuclear theory - ISM

Double beta decay with emission of a single electron

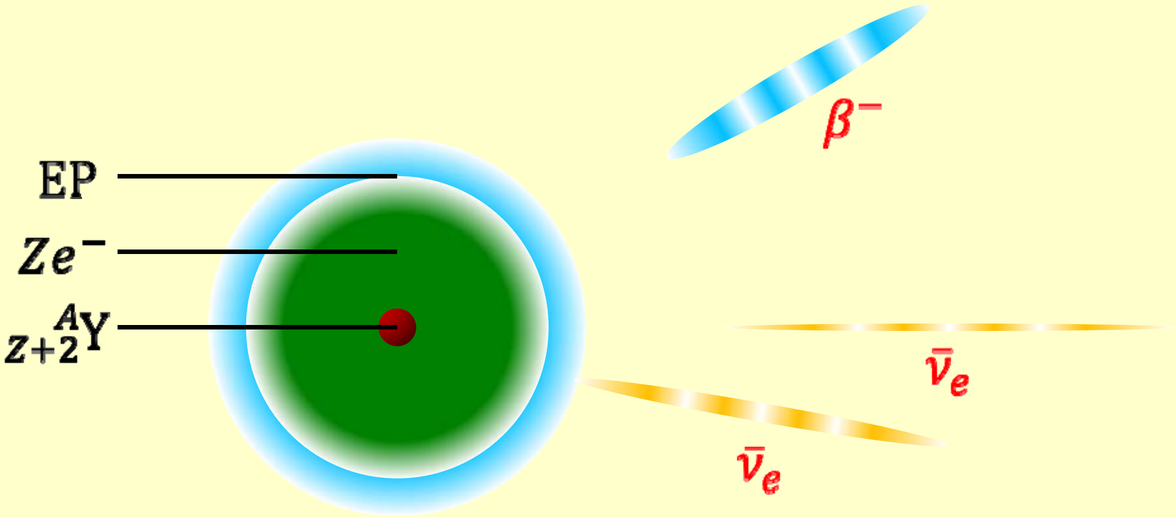
Double Beta Decay with emission of a single electron

A. Babič, M.I. Krivoruchenko, F.Š., tobe submitted to PRC

[Jung *et al.* (GSI), 1992] observed beta decay of $^{163}_{66}\mathrm{Dy}^{66+}$ ions with Electron Production (EP) in K or L shells: $T_{1/2}^{\mathrm{EP}} = 47$ d

Bound-state double-beta decay $0\nu\mathrm{EP}\beta^-$ $(2\nu\mathrm{EP}\beta^-)$ with EP in available $s_{1/2}$ or $p_{1/2}$ subshell of daughter 2+ ion:

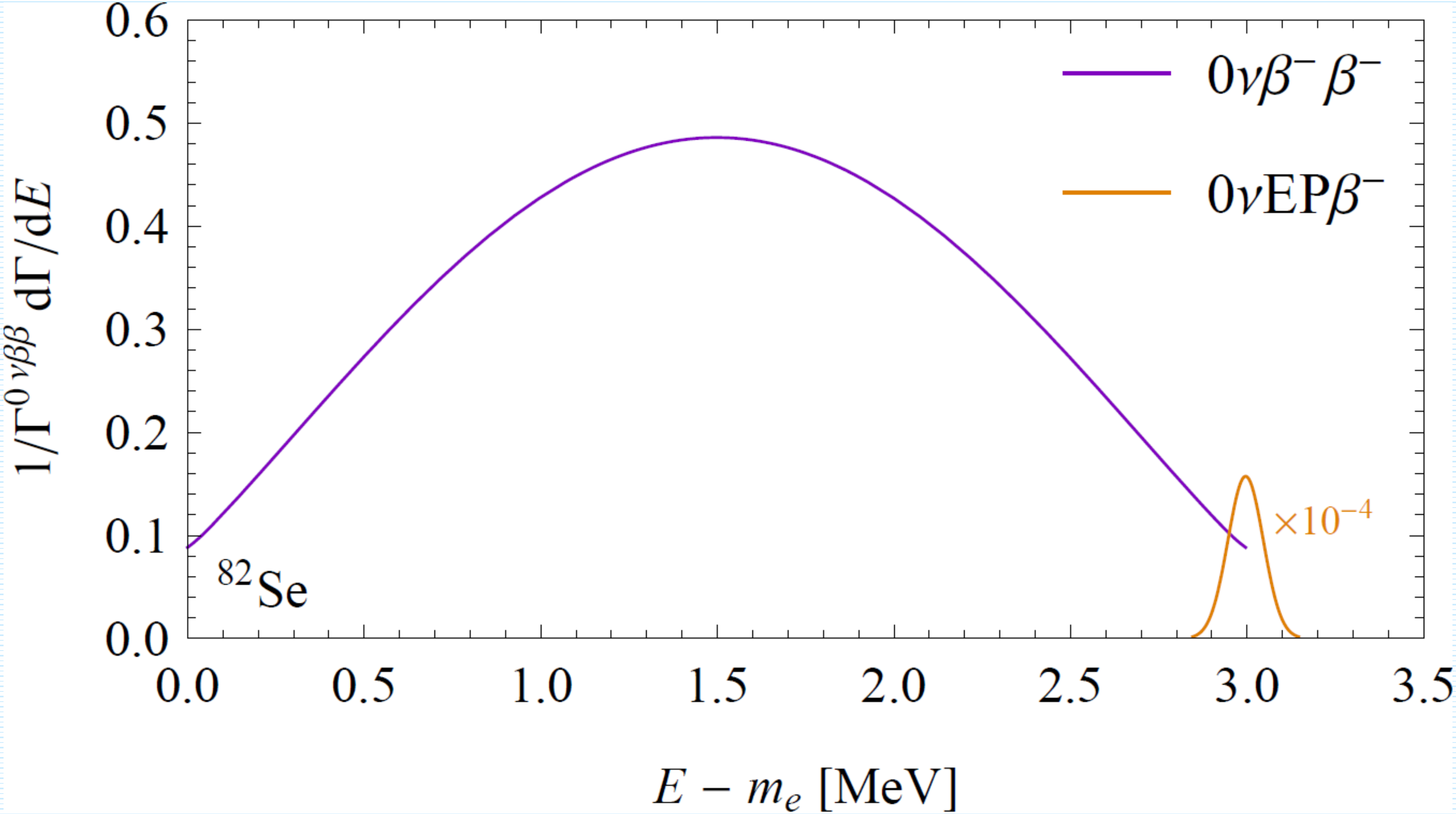
$$^{A}_{Z}\mathrm{X} \longrightarrow {}_{Z+2}^{\;\;A}\mathrm{Y} + e_{\mathrm{b}}^- + e^- + (\bar{\nu}_e + \bar{\nu}_e)$$



Search for possible manifestation in single-electron spectra…

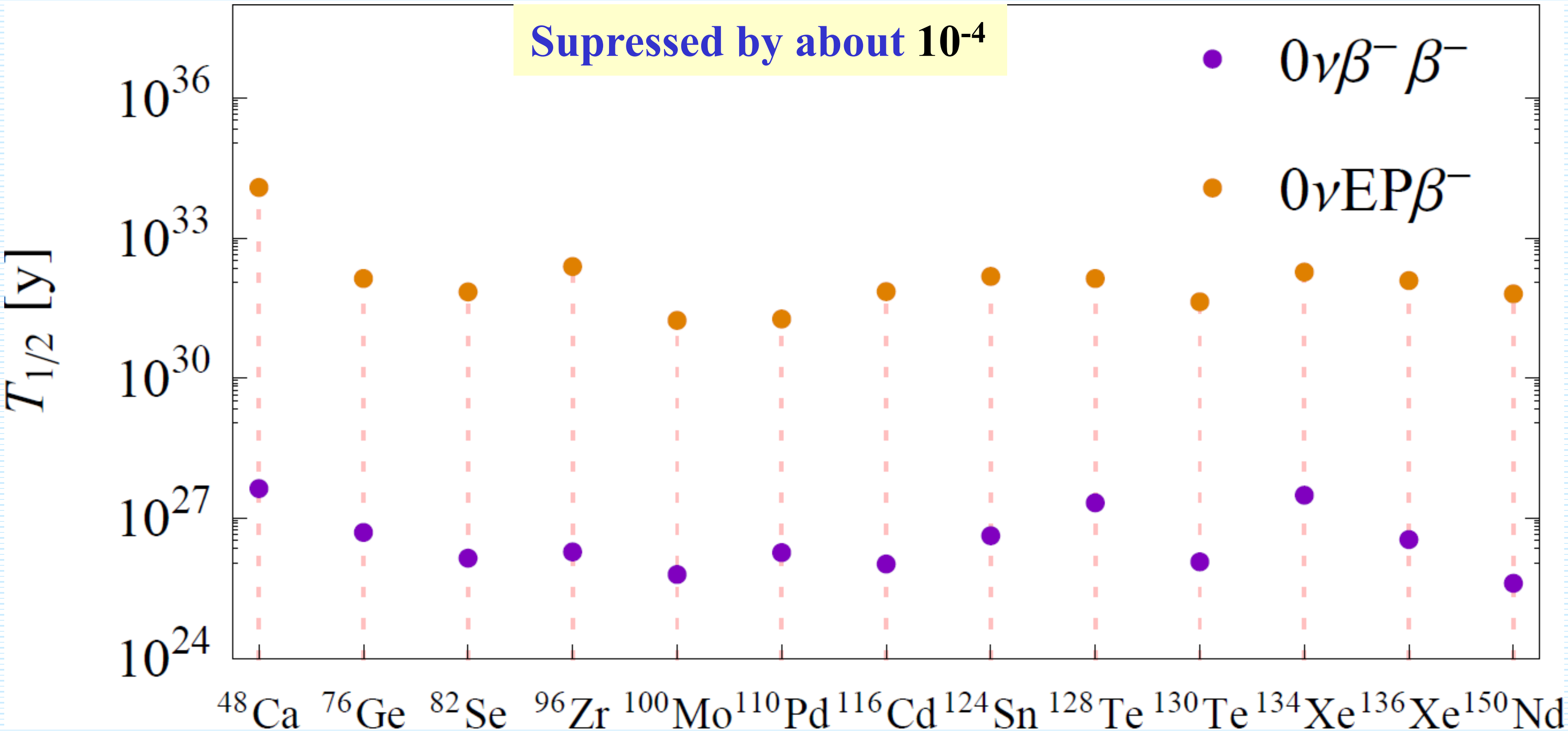
$0\nu\text{EP}\beta^-$ Single-Electron Spectrum (^{82}Se)

$0\nu\beta^-\beta^-$ **and** $0\nu\text{EP}\beta^-$ **single-electron spectra** $1/\Gamma^{0\nu\beta\beta}\,\mathrm{d}\Gamma/\mathrm{d}E$ ***vs.*** **electron kinetic energy** $E - m_e$ **for** ^{82}Se ($Q = 2.996$ MeV)



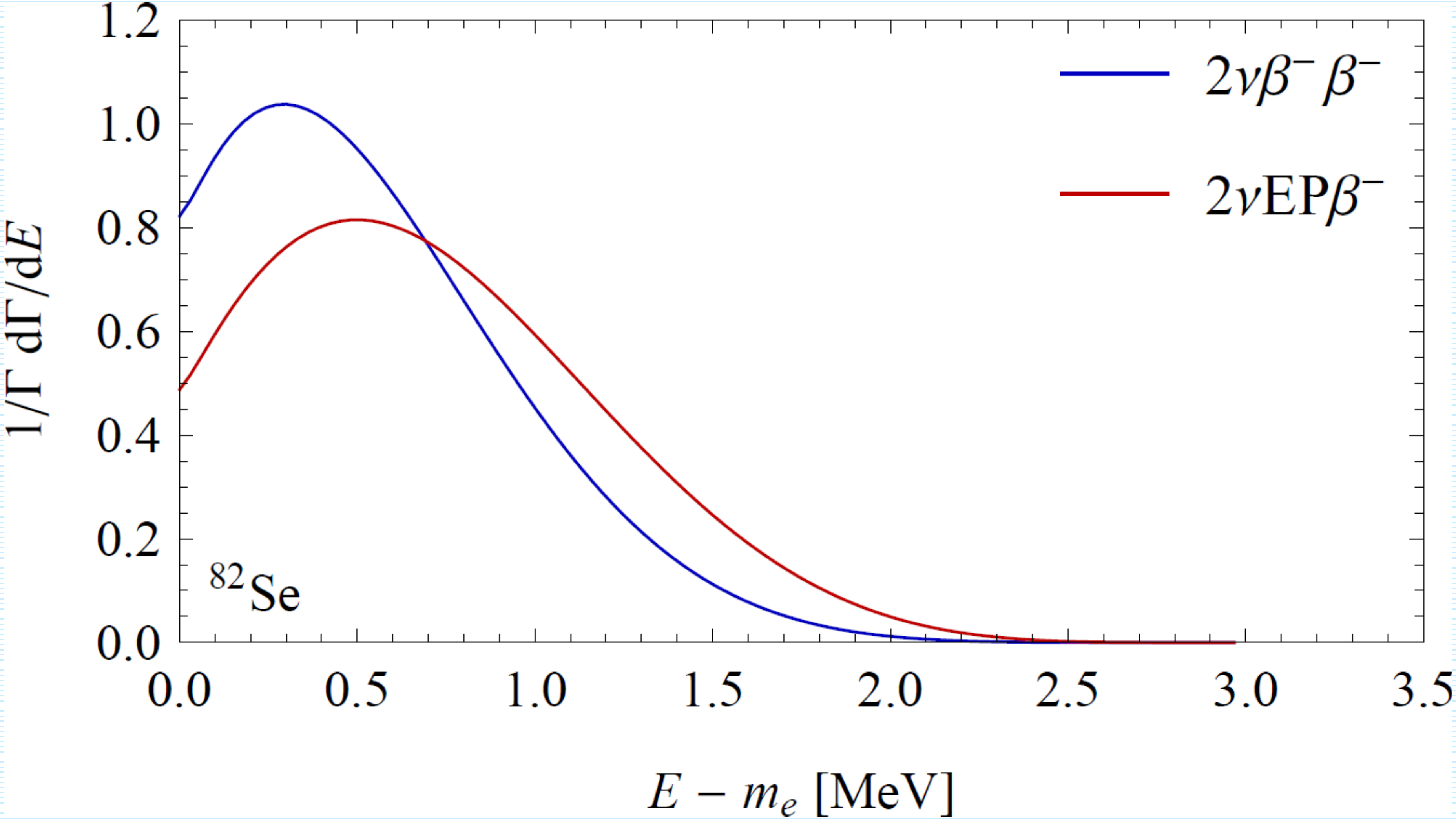
$0\nu\text{EP}\beta^-$ Half-Lives

$0\nu\beta^-\beta^-$ and $0\nu\text{EP}\beta^-$ half-lives $T_{1/2}^{0\nu\beta\beta}$ and $T_{1/2}^{0\nu\text{EP}\beta}$ estimated for $\beta^-\beta^-$ isotopes with known NME $\left|M^{0\nu\beta\beta}\right|$, assuming unquenched $g_A = 1.269$ and $\left|m_{\beta\beta}\right| = 50\ \text{meV}$



$2\nu\mathrm{EP}\beta^-$ Single-Electron Spectrum (^{82}Se)

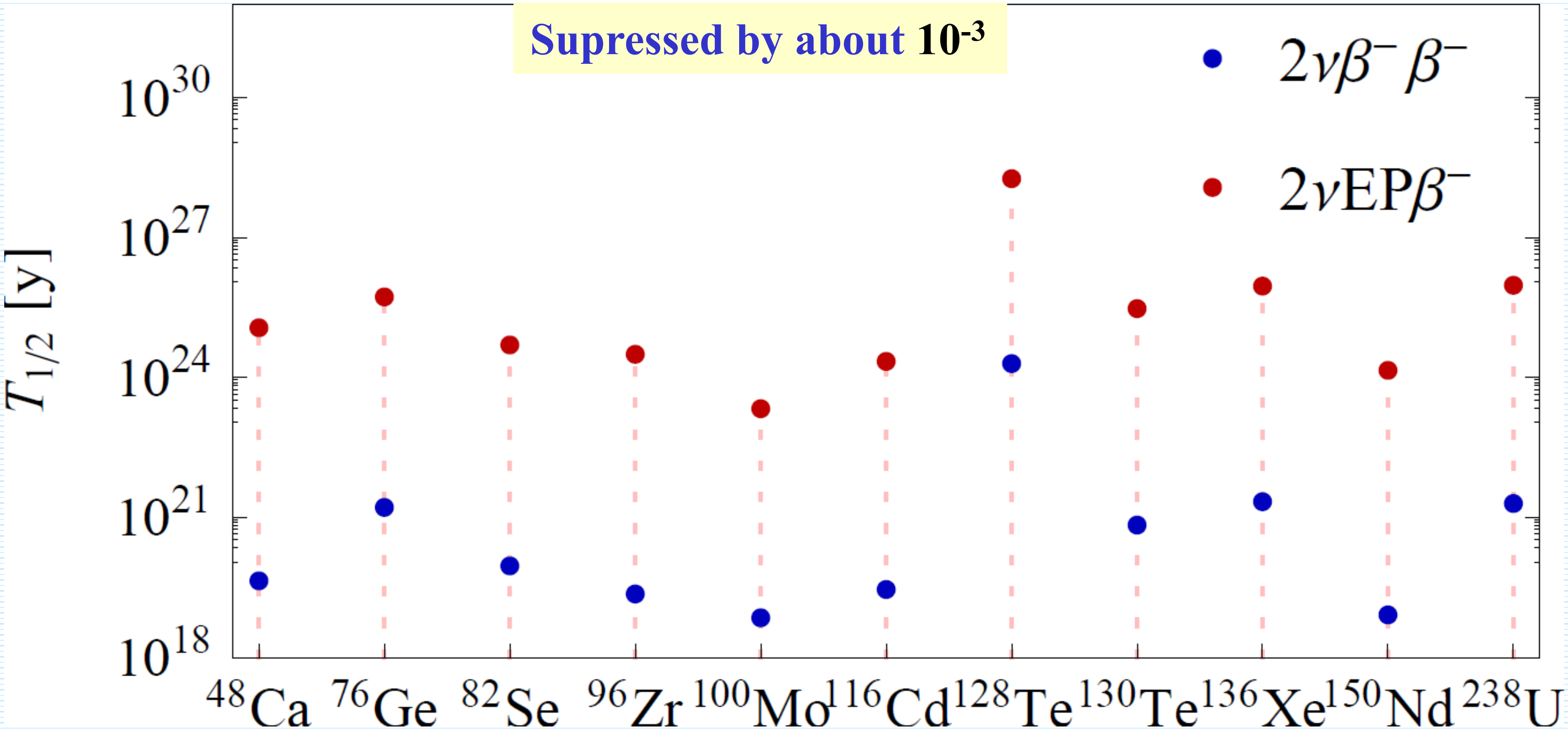
$2\nu\beta^-\beta^-$ **and** $2\nu\mathrm{EP}\beta^-$ **single-electron spectra** $1/\Gamma\,\mathrm{d}\Gamma/\mathrm{d}E$ ***vs.*** **electron kinetic energy** $E - m_e$ **for** ^{82}**Se** ($Q = 2.996$ MeV)



$2\nu EP\beta^-$ Half-Lives predictions

(independent on g_A and value of NME)

$2\nu\beta^-\beta^-$ **and** $2\nu EP\beta^-$ **half-lives** $T_{1/2}^{2\nu\beta\beta}$ **and** $T_{1/2}^{2\nu EP\beta}$ **calculated for** $\beta^-\beta^-$ **isotopes observed experimentally, assuming unquenched** $g_A = 1.269$



Nuclear structure studies within schematic models

Understanding of the 2νββ-decay NMEs is of crucial importance for correct evaluation of the 2νββ-decay NMEs

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

Both 2νββ and 0νββ operators connect the same states.
Both change two neutrons into two protons.

Explaining 2νββ-decay is necessary but not sufficient

There is no reliable calculation of the *2νββ-decay NMEs*

Calculation via intermediate nuclear states: QRPA (sensitivity to pp-int.)
ISM (quenching, truncation of model space, spin-orbit partners)

Calculation via closure NME: IBM, PHFB

No calculation: EDF

The DBD Nuclear Matrix Elements and the SU(4) symmetry

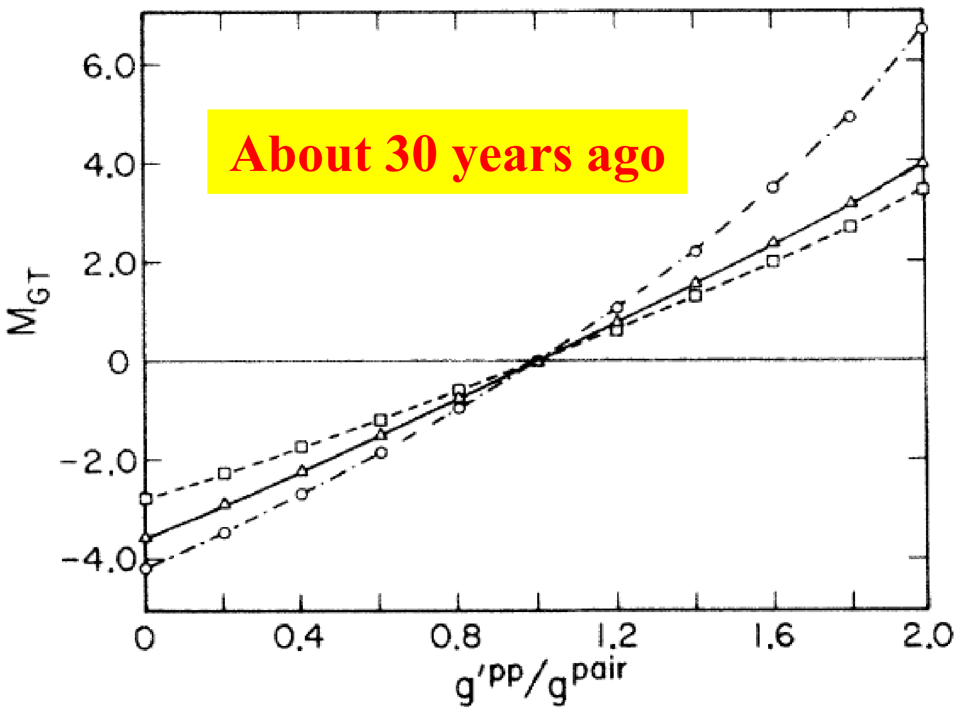
D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects

P. Vogel, M.R. Zirnbauer, PRL (1986) 3148

O. Civitarese, A. Faessler, T. Tomoda, PLB 194 (1987) 11
E. Bender, K. Muto, H.V. Klapdor, PLB 208 (1988) 53

…



The isospin is known to be a good approximation in nuclei

In heavy nuclei the SU(4) symmetry is strongly broken by the spin-orbit splitting.

What is beyond this behavior? Is it an artifact of the QRPA?

s.p. mean-field

Conserves SU(4) symmetry

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair}\left(\sum_{M_T=-1,0,1} A^{\dagger}_{0,1}(0,M_T)A_{0,1}(0,M_T) + \sum_{M_S=-1,0,1} A^{\dagger}_{1,0}(M_S,0)A_{1,0}(M_S,0)\right) + g_{ph}\sum_{a,b} E^{\dagger}_{a,b}E_{a,b}}_{H_0}$$

$$\underbrace{+ (g_{pair} - g_{pp}^{T=0}) \sum_{M_S=-1,0,1} A^{\dagger}_{1,0}(M_S,0)A_{1,0}(M_S,0) + (g_{pair} - g_{pp}^{T=1})A^{\dagger}_{0,1}(0,0)A_{0,1}(0,0)\,.}_{H_I}$$

H_I violates SU(4) symmetry

g_{pair}- strength of isovector like nucleon pairing (L=0, S=0, T=1, M_T=±1)

$g_{pp}^{T=1}$- strength of isovector spin-0 pairing (L=0, S=0, T=1, M_T=0

$g_{pp}^{T=0}$- strength of isoscalar spin-1 pairing (L=0, S=1, T=0)

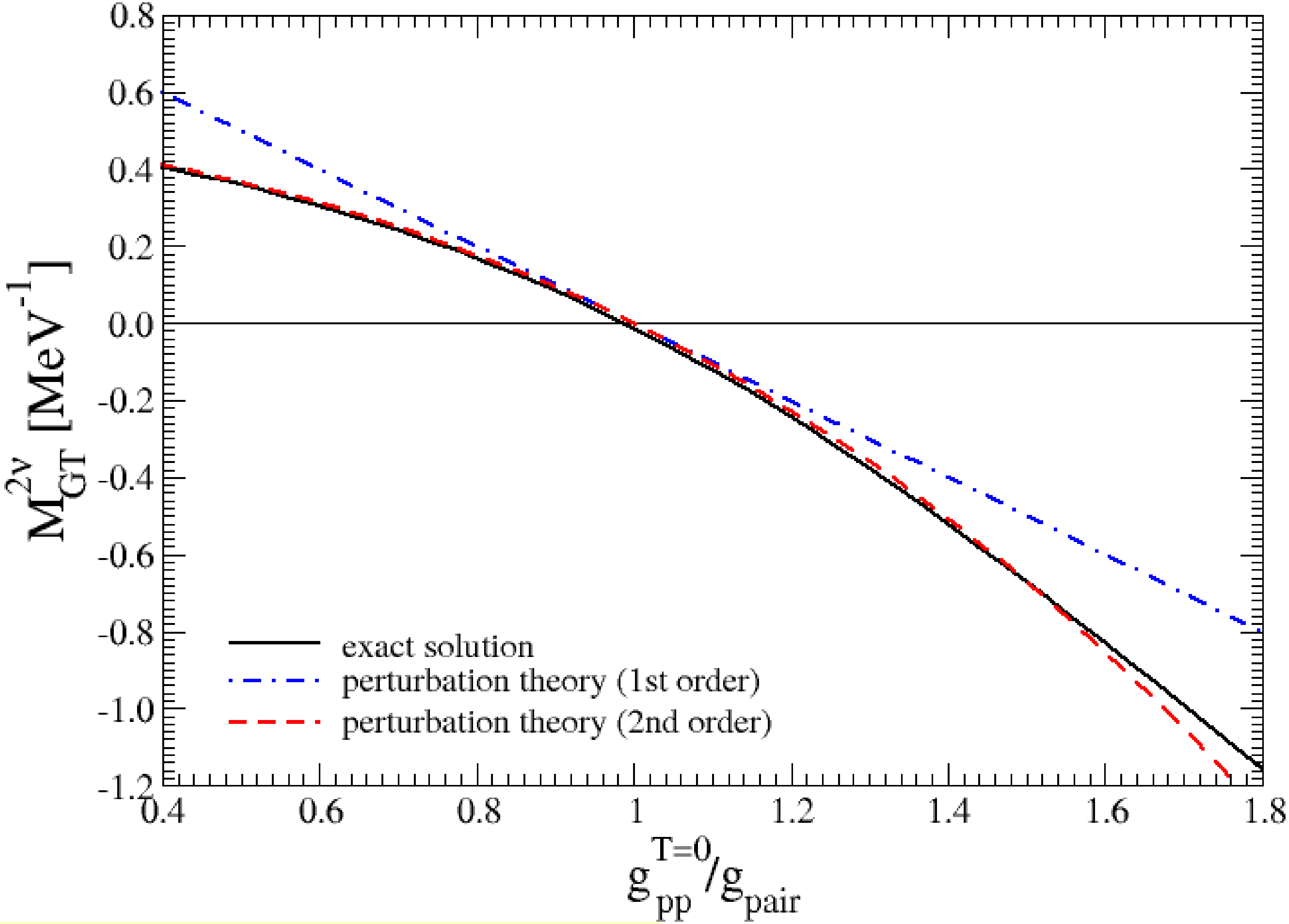
g_{ph}- strength of particle-hole force

M_F and M_{GT} do not depend on the mean-field part of **H** and are governed by a weak violation of the **SU(4)** symmetry by the particle-particle interaction of **H**

$$M_F^{2\nu} = -\frac{48\sqrt{\frac{33}{5}}\,(g_{pair} - g_{pp}^{T=1})}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}$$

$$M_{GT}^{2\nu} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair} + 9g_{ph}}\left\{\frac{(g_{pair} - g_{pp}^{T=0})}{(10g_{pair} + 20g_{ph})} + \frac{2g_{ph}(g_{pair} - g_{pp}^{T=1})}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})}\right\}$$

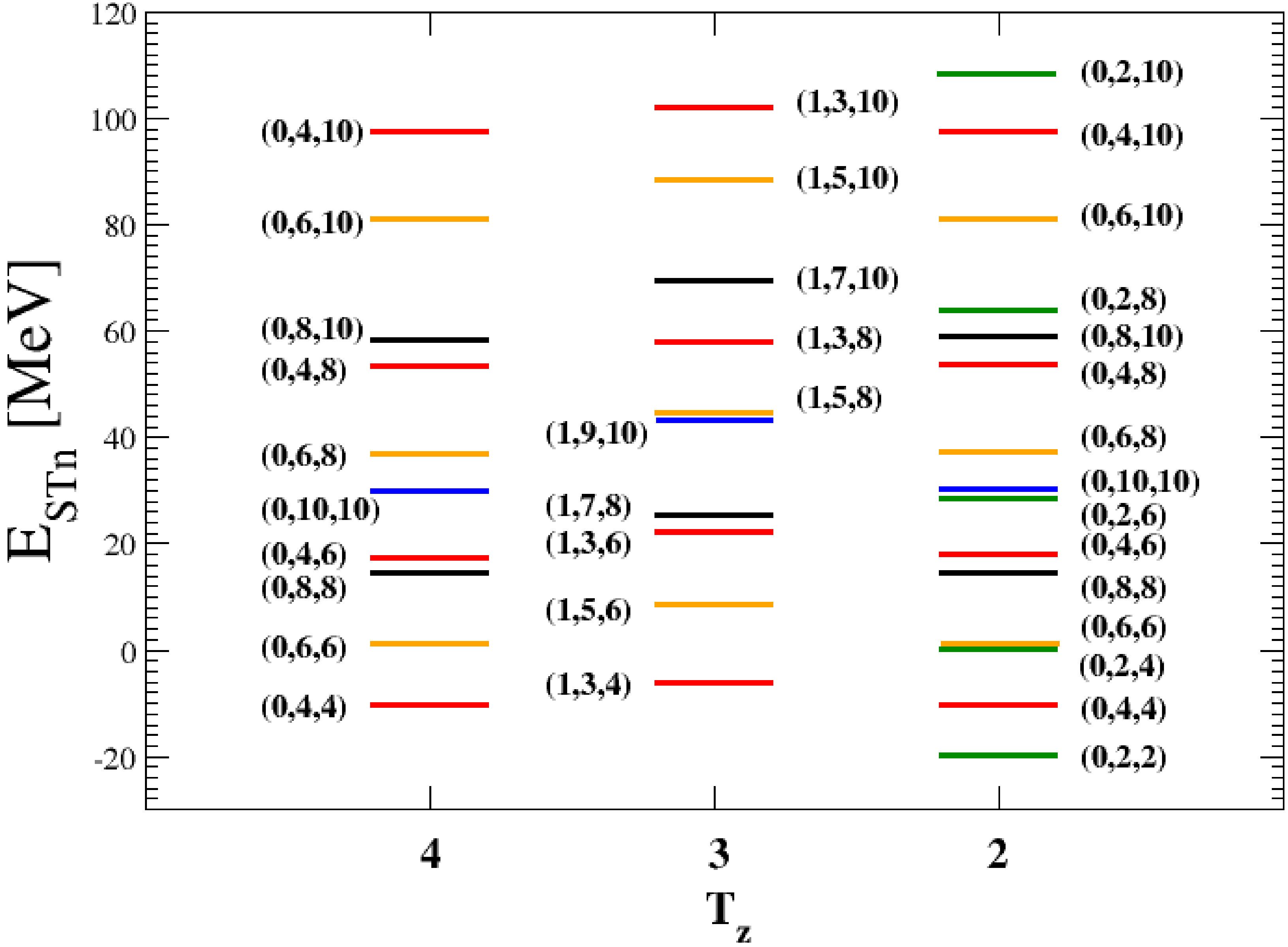
M_{GT} up to the second order of perturbation theory due to violation of the SU(4) symmetry by the particle-particle interaction of H



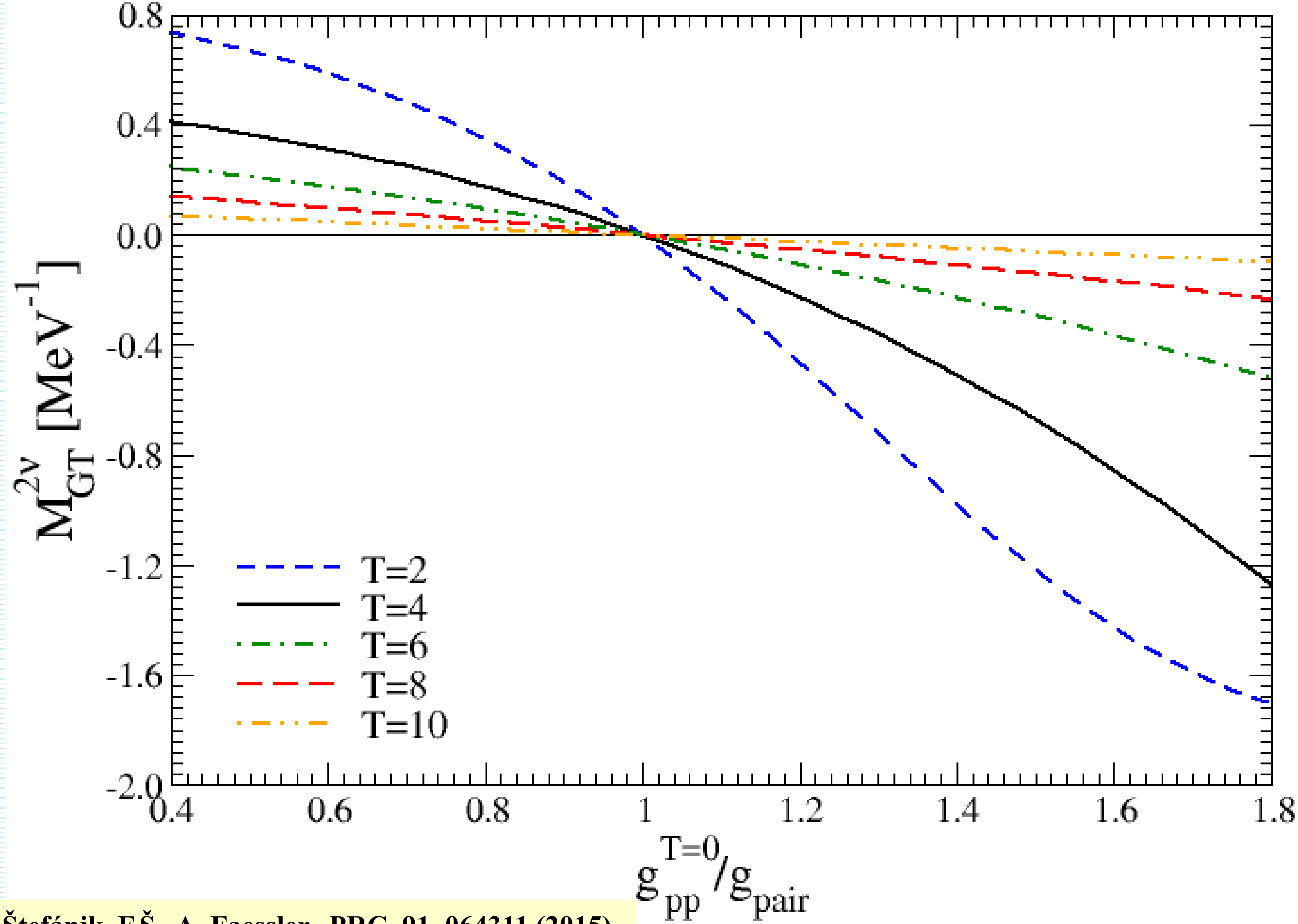
D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

Energies of excited states for the case of conserved SU(4) symmetry

$M_F=0$, $M_{GT}=0$ (see SU(4) multiplets)



By assuming a fixed violation of the SU(4) symmetry by particle-particle int.
M_{GT} decreases by increase of isospin of the ground state



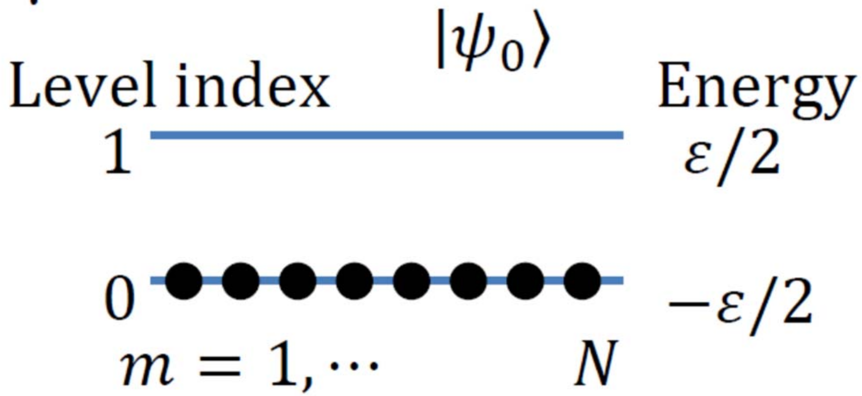
D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

Reproduction of exact solutions of Lipkin model by nonlinear higher random-phase approximation

J. Terasaki, A. Smetana, F. Š., M.I. Krivoruchenko, arXiv:1701.08368 [nucl-th]

Useful for test of theory often used.
H.J. Lipkin et al., N.P. **62**, 188 (1965)

Lipkin model



Hamiltonian

$$H = \varepsilon J_z + \frac{V}{2}\left(J_+^2 + J_-^2\right)$$

The nonlinear phonon operator

$$Q_k^{o\dagger} = \sum_{l=1}^{n} \left(X_{2l-1}^k \mathcal{J}_+^{2l-1} + Y_{2l-1}^k \mathcal{J}_-^{2l-1}\right),$$

(odd-order subspace)

$$Q_k^{e\dagger} = c_k + \sum_{l=1}^{n} \left(X_{2l}^k \mathcal{J}_+^{2l} + Y_{2l}^k \mathcal{J}_-^{2l}\right),$$

(even-order subspace)

Algebra

$$[J_z, J_+] = J_+$$
$$[J_z, J_-] = -J_-$$
$$[J_+, J_-] = 2J_z$$

RPA ground state

$$Q_k|\Psi_0\rangle = 0$$

Eigen states, wave functions, total energies, excitation energies and phonon-creation operators obtained for N=2 by the nonlinear higher RPA.

Eigenstate	Wave function	Total energy
Ground	$\lvert\Psi_0\rangle = \frac{V}{\sqrt{2E^o_{10}(E^o_{10}-\varepsilon)}}\left(1 - \frac{E^o_{10}-\varepsilon}{2V}J_+^2\right)\lvert\psi_0\rangle$	$-E^o_{10}$
Odd-order excited	$Q_1^{o\dagger}\lvert\Psi_0\rangle = \frac{1}{\sqrt{2}}J_+\lvert\psi_0\rangle$	0
Even-order excited	$Q_1^{e\dagger}\lvert\Psi_0\rangle = \frac{V}{\sqrt{2E^o_{10}(E^o_{10}+\varepsilon)}}\left(1 + \frac{E^o_{10}+\varepsilon}{2V}J_+^2\right)\lvert\psi_0\rangle$	E^o_{10}

Eigenstate	Excitation energy	Phonon-creation operator
Ground	0	
Odd-order excited	$E^o_{10} = \sqrt{\varepsilon^2 + V^2}$	$Q_1^{o\dagger} = \frac{\sqrt{E^o_{10}}}{2\varepsilon}\left(\frac{V}{\lvert V\rvert}\sqrt{E^o_{10}+\varepsilon}J_+ + \sqrt{E^o_{10}-\varepsilon}J_-\right)$
Even-order excited	$E^e_{10} = 2E^o_{10}$	$Q_1^{e\dagger} = \frac{V}{\lvert V\rvert}\left(\frac{V}{2\varepsilon} + \frac{E^o_{10}+\varepsilon}{4\varepsilon}J_+^2 + \frac{E^o_{10}-\varepsilon}{4\varepsilon}J_-^2\right)$

J. Terasaki, A. Smetana, F. Š., M.I. Krivoruchenko, arXiv:1701.08368 [nucl-th]

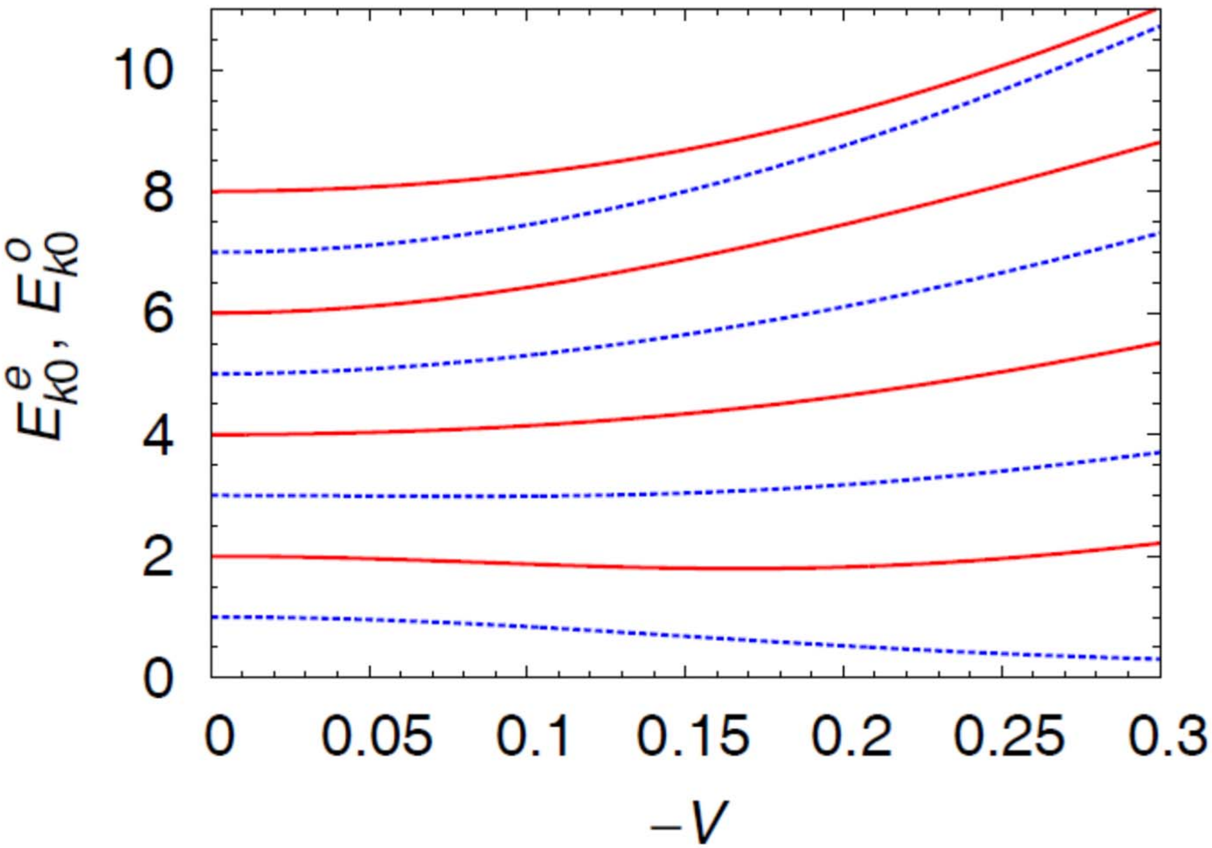
RPA equation

$$\begin{pmatrix} A^o & B^o \\ B^o & A^o \end{pmatrix} \begin{pmatrix} X_k^o \\ Y_k^o \end{pmatrix} = E_{k0}^o \begin{pmatrix} U^o & O \\ O & -U^o \end{pmatrix} \begin{pmatrix} X_k^o \\ Y_k^o \end{pmatrix}$$

$$\begin{aligned} A_{ij}^o &= \langle \Psi_0 | [\mathcal{J}_-^{2i-1}, H, \mathcal{J}_+^{2j-1}] | \Psi_0 \rangle \\ B_{ij}^o &= \langle \Psi_0 | [\mathcal{J}_-^{2i-1}, H, \mathcal{J}_-^{2j-1}] | \Psi_0 \rangle \\ U_{ij}^o &= \langle \Psi_0 | [\mathcal{J}_-^{2i-1}, \mathcal{J}_+^{2j-1}] | \Psi_0 \rangle \end{aligned}$$

$$[A, B, C] = (1/2)[[A, B], C] + (1/2)[A, [B, C]]$$

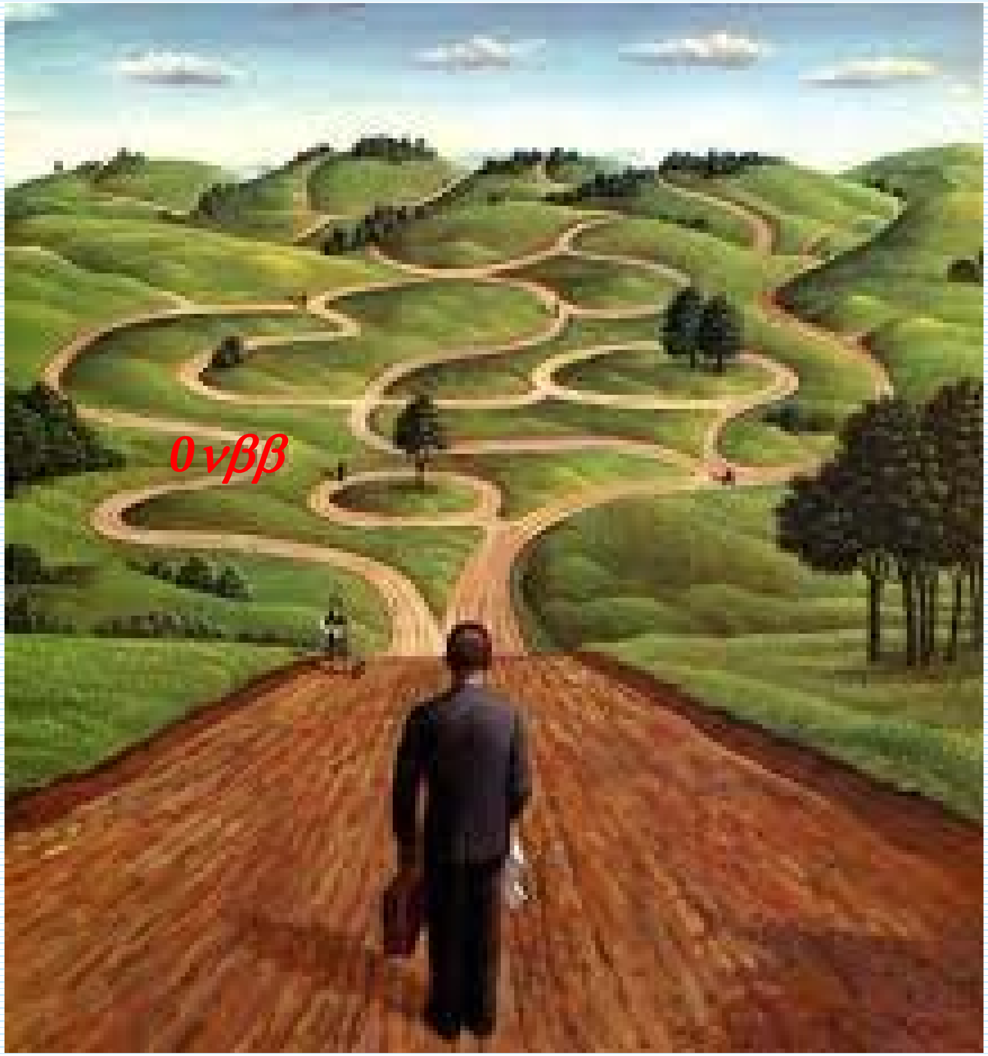
N=8, ε=1
Breaking point of RPA is V=-0.143



Exact agreement of RPA results with those obtained by diagonalization of H

Instead of Conclusions

Progress
in
nuclear
structure
calculations
is
highly
required



We are at the beginning of the BSM Road…