

Primordial Black Holes as (part of the) dark matter

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Motivation

Formation

Constraints

For further details on these topics (and also PBH binary mergers as source of GWs)
see recent review by Sasaki, Suyama, Tanaka & Yokoyama arXiv:1801.05235.

Motivation

Cosmological observations indicate that dark matter (DM) has to be cold and non-baryonic.

Primordial Black Holes (PBHs) form before nucleosynthesis and are therefore non-baryonic.

PBHs evaporate (Hawking radiation), lifetime longer than the age of the Universe for $M > 10^{15}$ g.

A DM candidate which (unlike WIMPs, axions, sterile neutrinos,...) isn't a new particle (however their formation does usually require Beyond the Standard Model physics).

LIGO has detected gravitational waves from mergers of $10+$ M_{sun} BHs. Could be formed by astrophysical processes, but a large population of massive BH binaries was possibly somewhat unexpected (stellar winds from progenitors must be weak & hence metallicity low + natal kicks must be small).

Could PBHs be the CDM?

(and potentially also the source of the BH-BH GW events?? Bird et al.; Sasaki et al.)

Formation: collapse of density perturbations^{\$}

During radiation domination an initially large (at horizon entry) density perturbation can collapse to form a PBH with mass of order the horizon mass.

Zeldovich & Novikov; Hawking; Carr & Hawking

For gravity to overcome pressure forces resisting collapse, size of region at maximum expansion must be larger than Jean's length.

Simple analysis:

Carr; see Harada, Yoo & Kohri for refinements

density contrast: $\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$

threshold for PBH formation: $\delta \geq \delta_c \sim w = \frac{p}{\rho} = \frac{1}{3}$

PBH mass: $M \sim w^{3/2} M_H$ $M_H \sim 10^{15} \text{ g} \left(\frac{t}{10^{-23} \text{ s}} \right)$

^{\$} Other formation mechanisms include collapse of cosmic string loops Hawking; Polnarev & Zemboricz, bubble collisions Hawking, fragmentation of inflaton/scalar condensate into oscillons. Cotner & Kusenko; Cotner, Kusenko & Takhistov

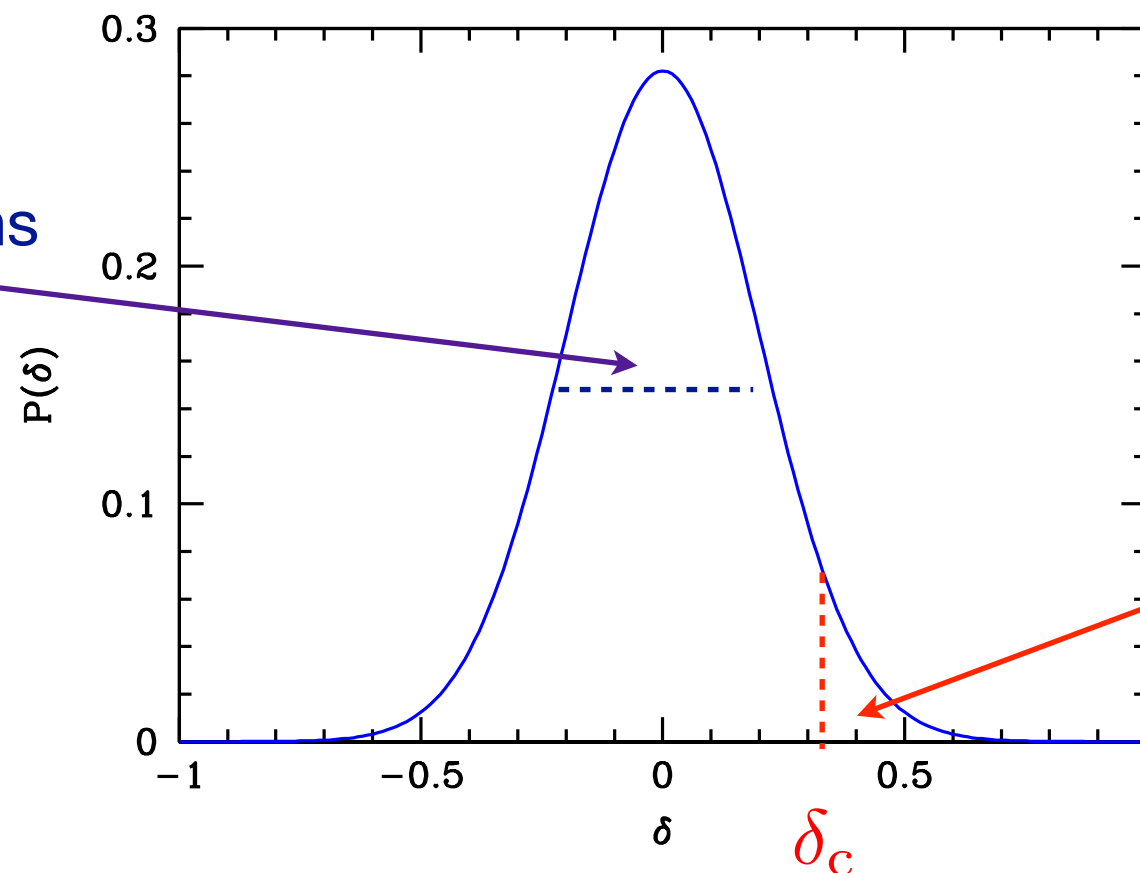
initial PBH mass fraction (fraction of universe in regions dense enough to form PBHs):

$$\beta(M) \sim \int_{\delta_c}^{\infty} P(\delta(M_H)) d\delta(M_H)$$

assuming a gaussian probability distribution:

$$\beta(M) = \text{erfc} \left(\frac{\delta_c}{\sqrt{2}\sigma(M_H)} \right)$$

$\sigma(M_H)$ (mass variance)
typical size of fluctuations



PBH forming
fluctuations

but in fact β must be small, and hence $\sigma \ll \delta_c$

PBH abundance

Since PBHs are matter, during radiation domination the fraction of energy in PBHs grows with time:

$$\frac{\rho_{\text{PBH}}}{\rho_{\text{rad}}} \propto \frac{a^{-3}}{a^{-4}} \propto a$$

Relationship between **PBH initial mass fraction, β** , and **fraction of DM in form of PBHs, f** :

$$\beta(M) \sim 10^{-9} f \left(\frac{M}{M_{\odot}} \right)^{1/2}$$

i.e. initial mass fraction must be small, but non-negligible.

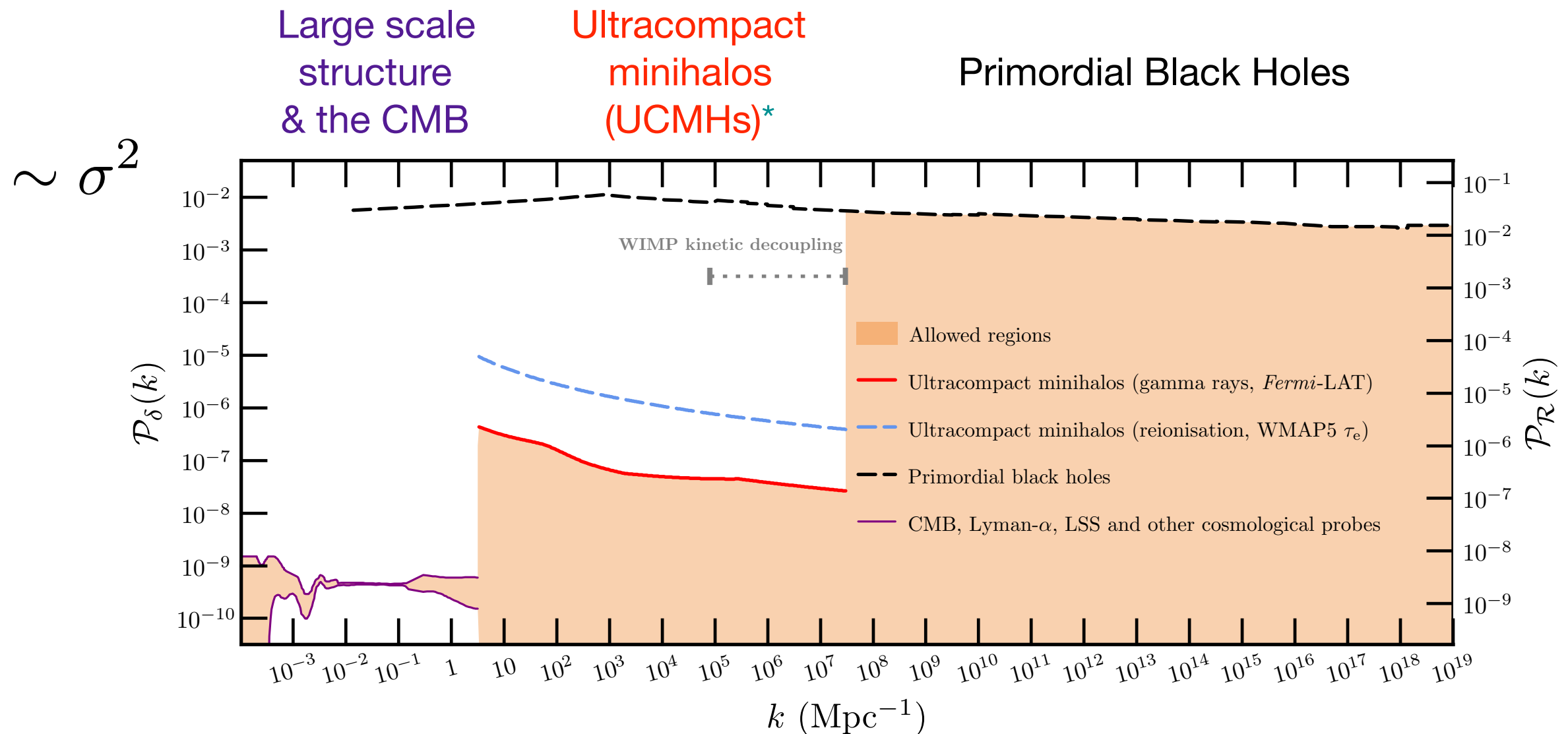
On CMB scales the primordial perturbations have amplitude $\sigma(M_{\text{H}}) \sim 10^{-5}$

If the primordial perturbations are close to scale-invariant the number of PBHs formed will be completely negligible:

$$\beta(M) \sim \text{erfc}(10^5) \sim 10^5 \exp [-(10^5)^2]$$

To form an interesting number of PBHs the primordial perturbations must be significantly larger ($\sigma(M_{\text{H}}) \sim 0.01$) on small scales than on cosmological scales.

Constraints on the primordial power spectrum



Bringmann, Scott & Akrami

* UCMH constraints only hold if most of the DM is WIMPs. Also recent studies find UCMHs have shallower density profiles than assumed in this calc [Gosenca et al.](#), [Delos et al.](#) which will affect constraints.

Deviations from simple scenario:

i) non-gaussianity

Since PBHs are formed from rare large density fluctuations, changes in the shape of the tail of the probability distribution (i.e. non-gaussianity) can significantly affect the PBH abundance. Bullock & Primack; Ivanov;... Byrnes, Copeland & Green;...

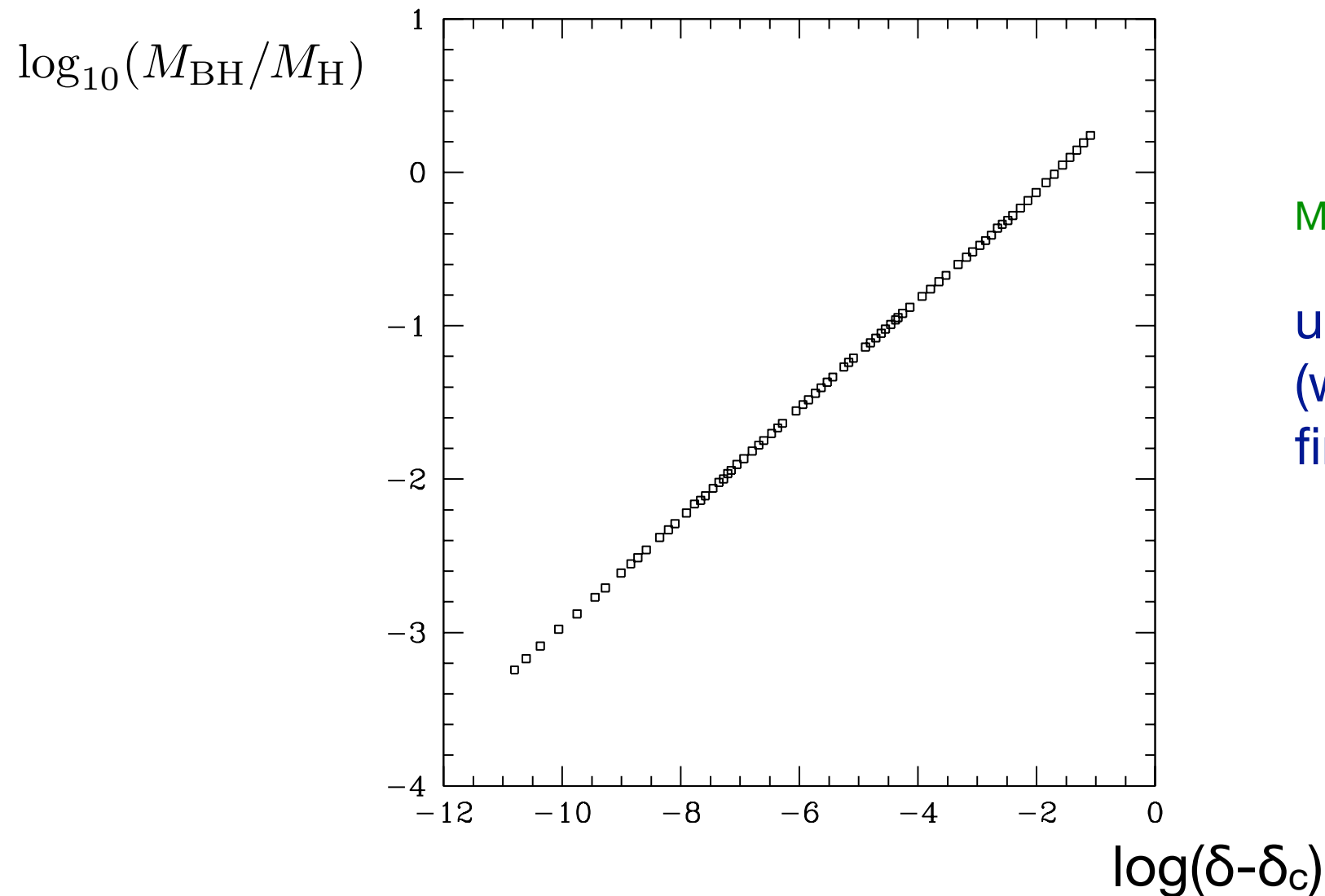
Franciolini, Kehagias, Matarrese & Riotto use a path integral formalism to derive an exact expression for the PBH abundance. However it involves all of the smoothed N-point connected correlation functions...

ii) critical collapse

Choptuik; Evans & Coleman; Niemeyer & Jedamzik

BH mass depends on size of fluctuation it forms from:

$$M = kM_{\text{H}}(\delta - \delta_{\text{c}})^{\gamma}$$



Musco, Miller & Polnarev

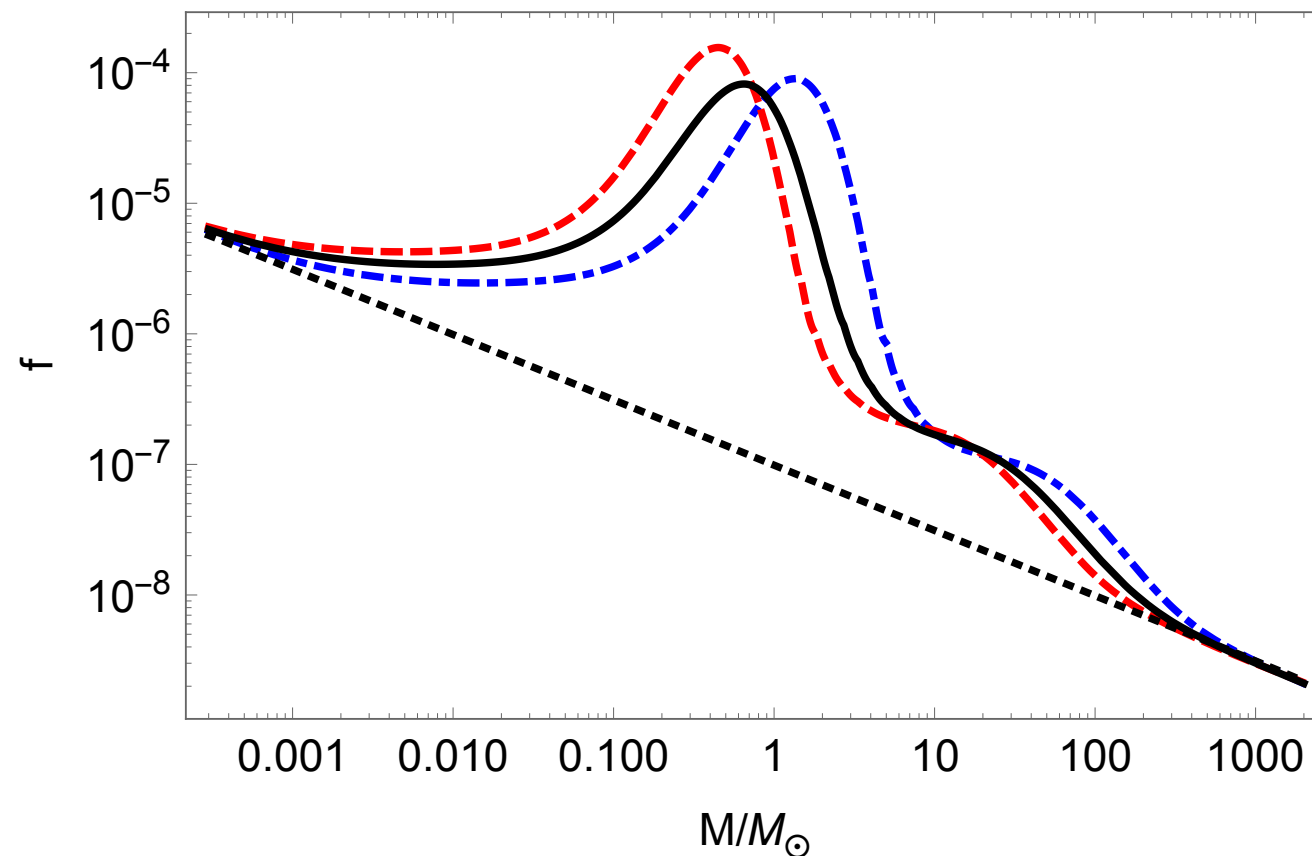
using numerical simulations
(with appropriate initial conditions)
find $k=4.02$, $\gamma=0.357$, $\delta_{\text{c}} = 0.45$

Get PBHs with range of masses produced even if they all form at the same time
i.e. we don't expect the PBH MF to be a delta-function

iii) phase transitions

Reduction in the equation of state parameter ($w=\rho/p$) at phase transitions decreases the threshold for PBH formation δ_c and enhance the abundance of PBHs formed on this scale. (Horizon mass at QCD phase transition is of order a solar mass.) [Jedamzik](#)

Using new lattice calculation of QCD phase transition [Byrnes et al.](#) transition find a 2 order of magnitude enhancement in β (but still need a mechanism for amplifying the primordial perturbations):



Inflation: a crash course

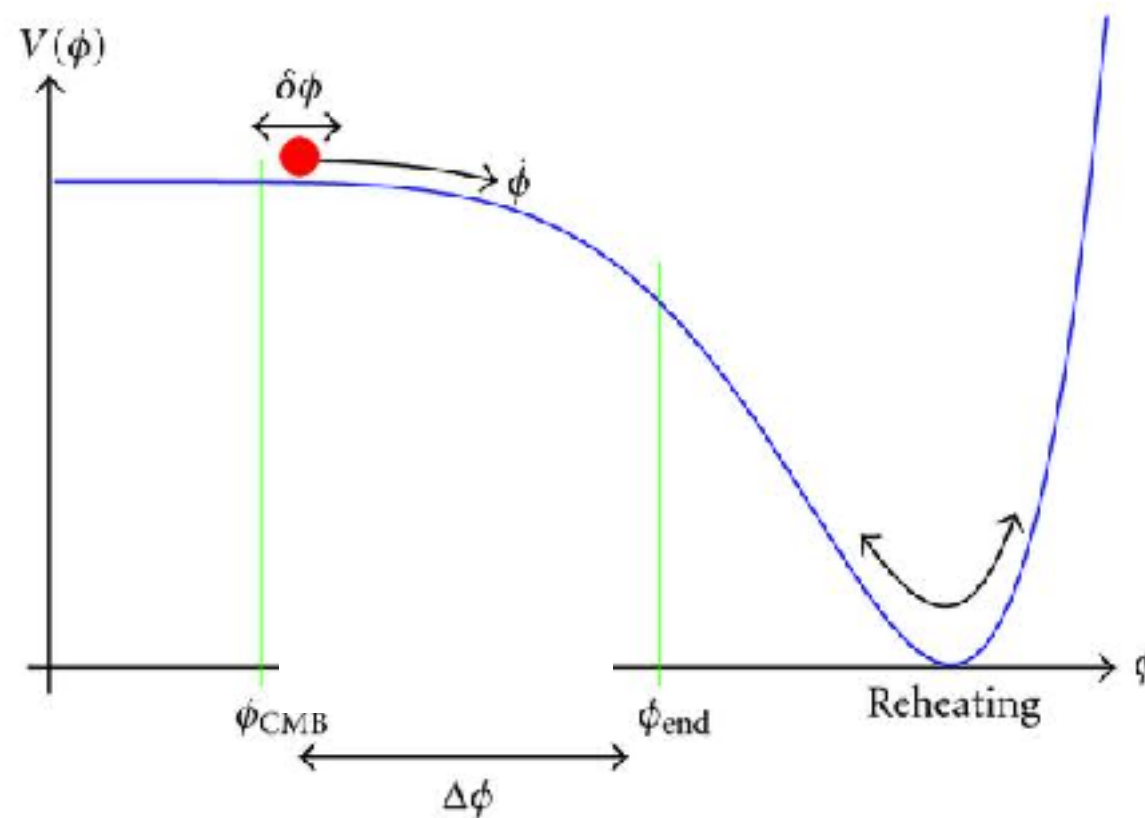
A postulated period of accelerated expansion in the early Universe, proposed to solve various problems with the Big Bang (flatness, horizon & monopole).

Driven by a 'slowly rolling' scalar field.

Quantum fluctuations in scalar field generate primordial density perturbations.

Scale dependence of primordial perturbations depends on shape of potential:

Yadav & Wandelt



in slow-roll approx

$$\sigma^2(M_H) \propto \frac{V^3}{(V')^2}$$

Scales probed by:



Large scale structure
& the CMB

Primordial Black Holes

(an incomplete selection of) Inflation models with large primordial perturbations on small scales

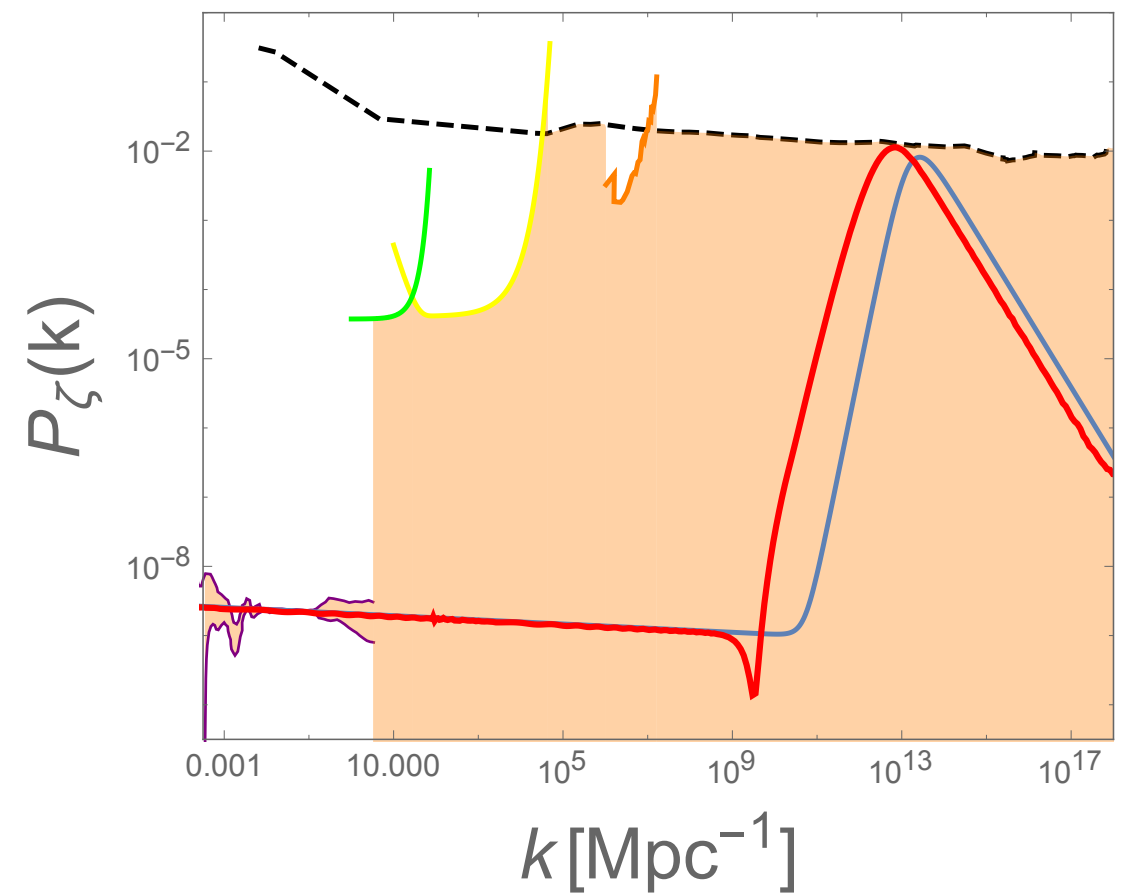
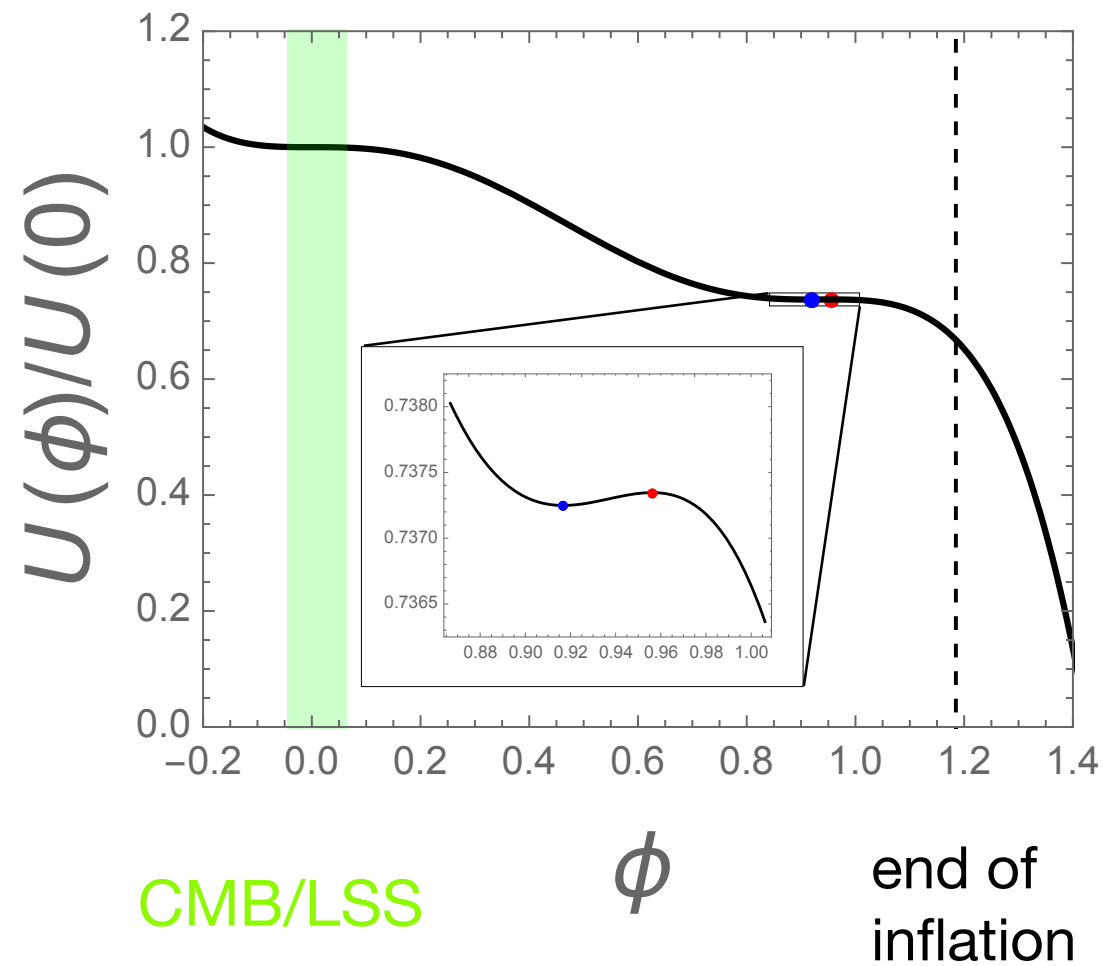
In single field models need to violate slow roll (and hence standard expressions for amplitude of fluctuations aren't valid).

Models which might naively be expected to produce large perturbations (e.g. potentials with an inflection point, $V'(\phi) \rightarrow 0$ 'ultra-slow-roll') don't. Kannike et al.; Germani & Prokopec; Motohashi & Hu; Ballesteros & Taoso

i) over-shoot a local minimum

Ballesteros & Taoso; Herzberg & Yamada

Potential fine-tuned so that field goes past local max, but with reduced speed

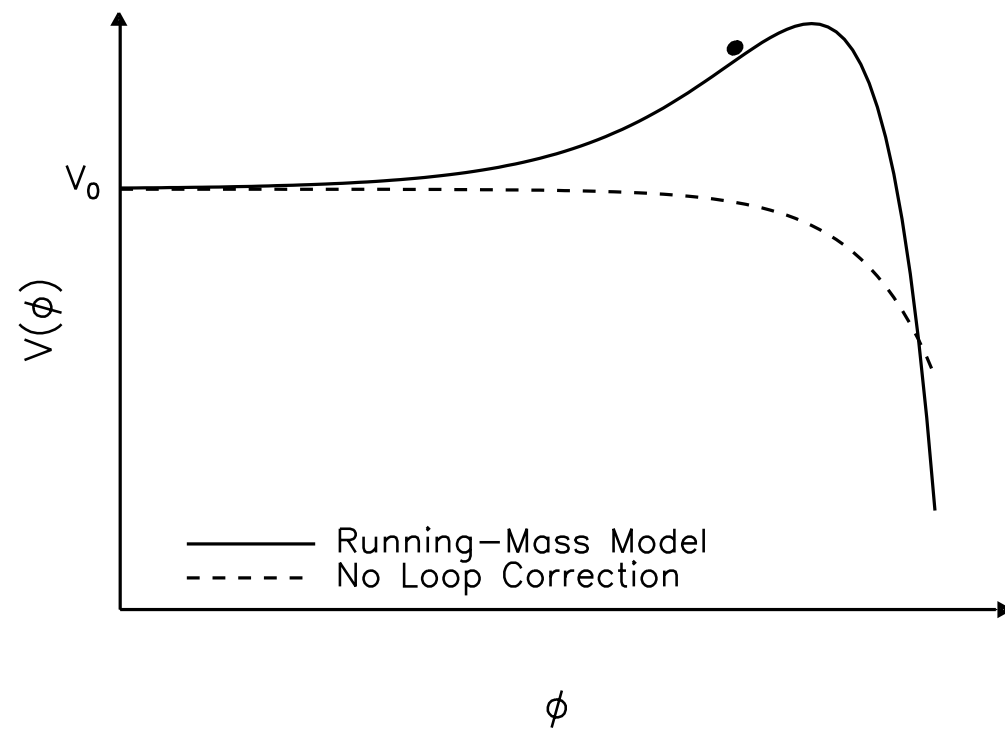


Can be done with quintic potential, with fine-tuning at $\sim 10^{-8.5}$ level...

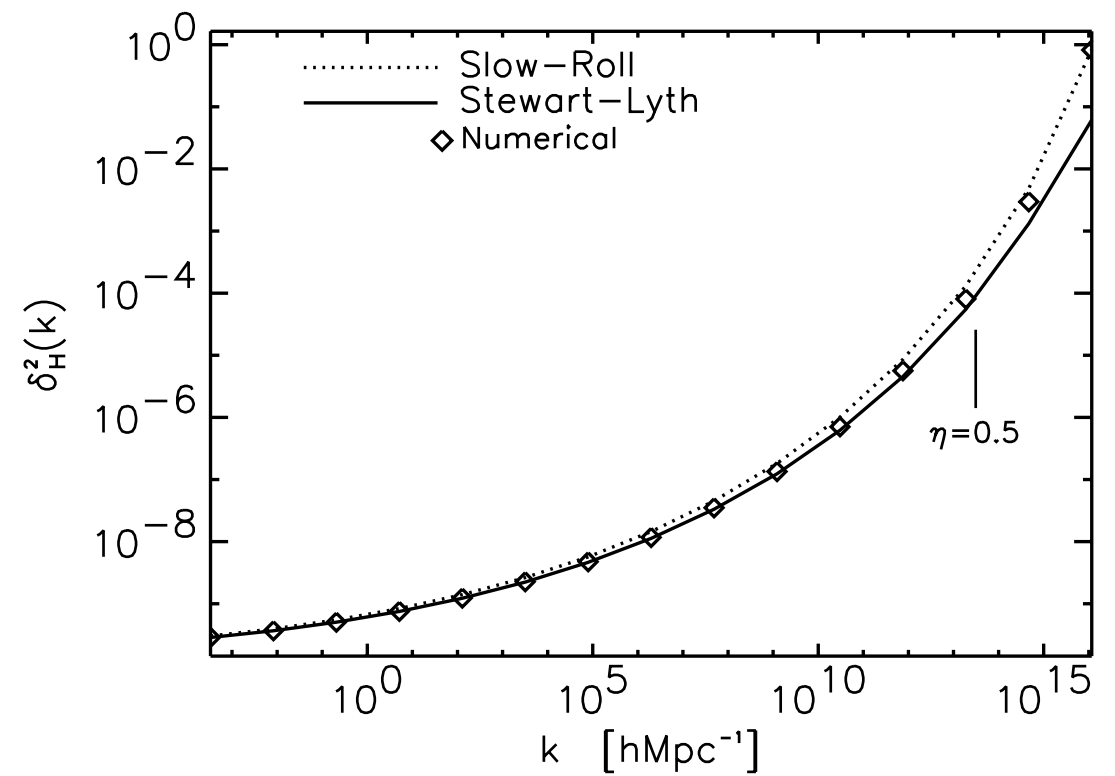
ii) running-mass inflation **Stewart**

$$V(\phi) = V_0 + \frac{1}{2}m_\phi^2(\phi)\phi^2$$

potential



primordial power spectrum

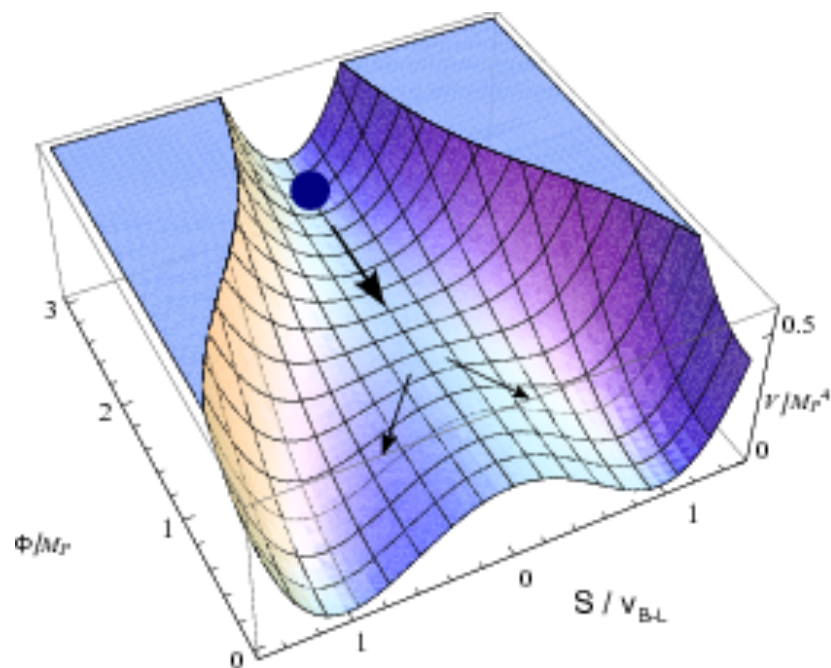


iii) double inflation

Perturbations on scales which leave the horizon close to the end of the 1st period, of inflation get amplified during the 2nd period. Saito, Yokoyama & Nagata; Kannike et al.

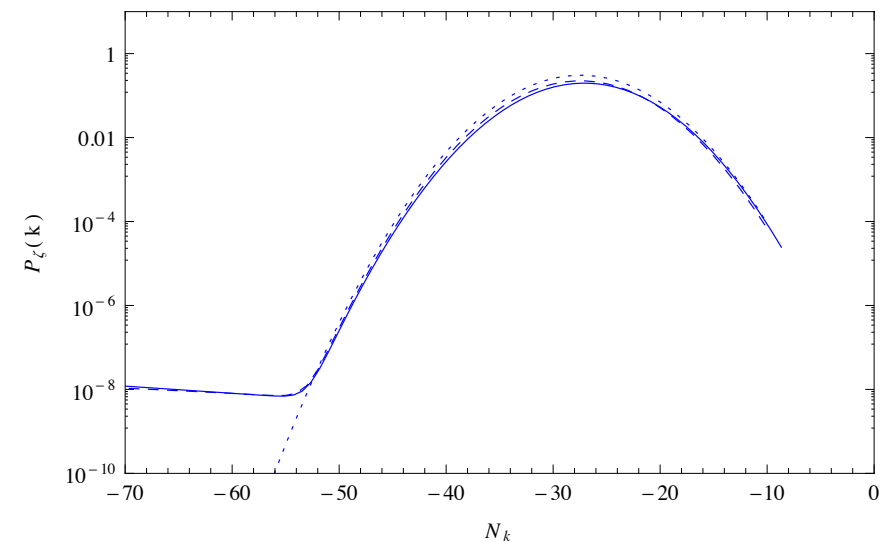
Also double inflation models where large scale perturbations are produced during 1st period, and small scale (PBH forming) perturbations during 2nd (Kawasaki et al.; Kannike et al.; Inomata et al.).

iv) hybrid inflation with a mild waterfall transition Garcia-Bellido, Linde & Wands



Buchmuller

primordial power spectrum



Clesse & Garcia-Bellido

v) axion-like curvaton

Large scale perturbations generated by inflaton, small scale (PBH forming) perturbations by curvaton (a spectator field during inflation gets fluctuations and decays afterwards producing perturbations Lyth & Wands) Kawasaki, Kitajima & Yanagida

PBH formation during an early (pre nucleosynthesis) period of matter domination

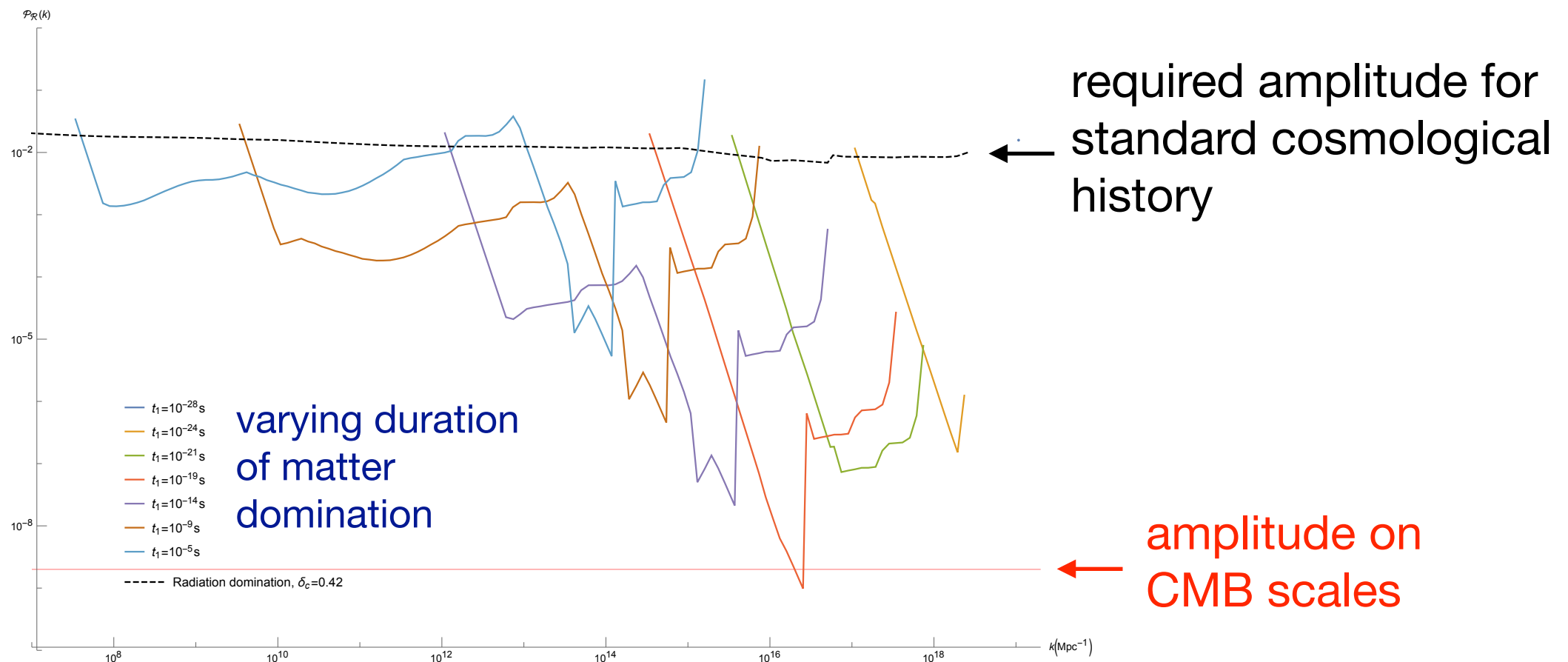
During matter domination PBHs can form from smaller fluctuations (no pressure to resist collapse) in this case fluctuations must be sufficiently spherically symmetric

Yu, Khlopov & Polnarev; Harada et al. and

$$\beta(M) \approx 0.056\sigma^{5(+1.5?)}$$

The required increase in the amplitude of the perturbations is reduced Georg, Sengör & Watson; Georg & Watson; Carr, Tenkanen & Vaskonen; Cole & Byrnes:

Primordial
curvature
perturbation
power
spectrum



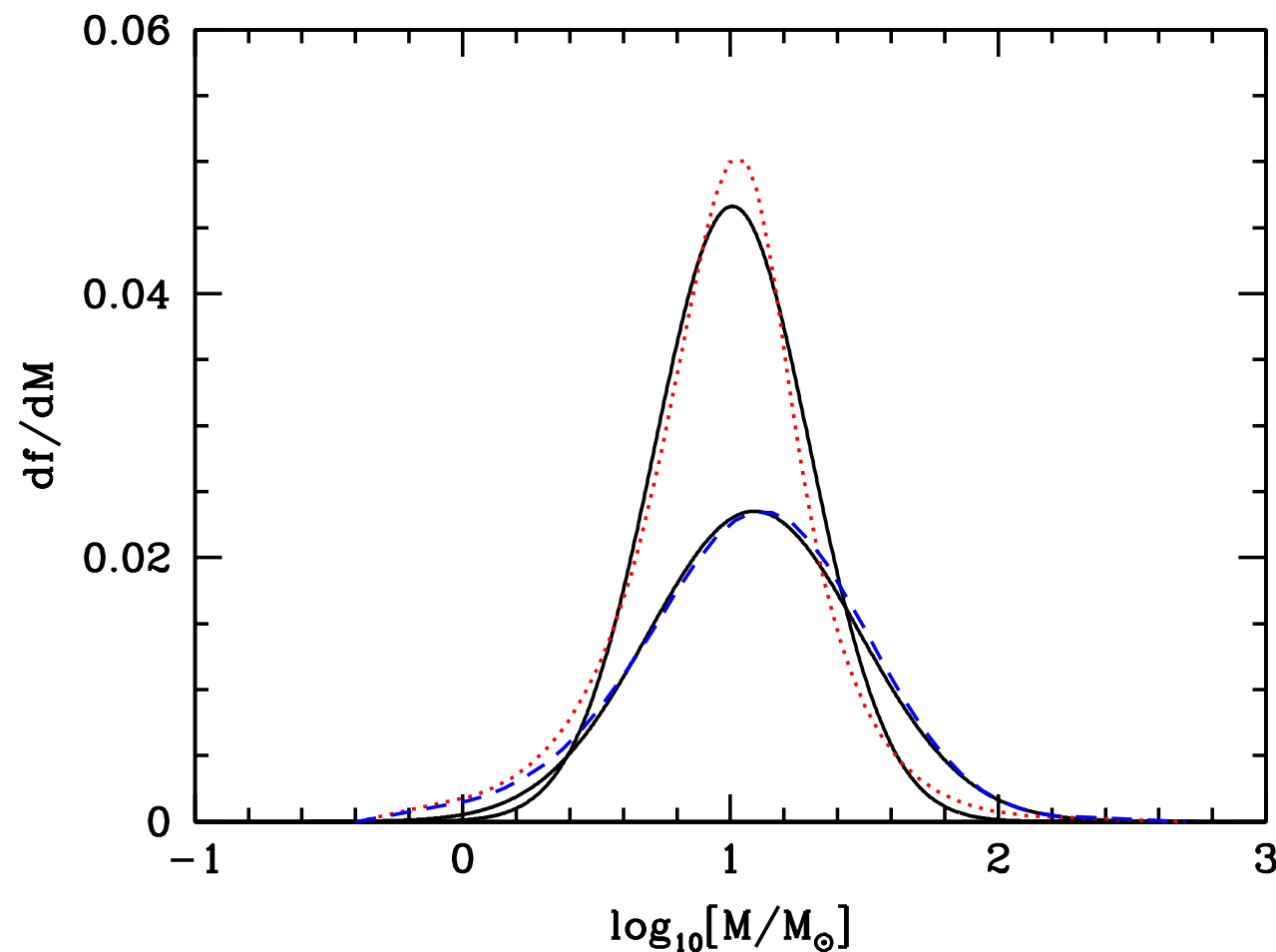
Cole & Byrnes

k

Mass function

Extended MFs produced by inflation models with finite width peak in power spectrum, taking into account critical collapse, often well approximated by a log-normal distribution: Green; Kannike et al.

$$\frac{dn}{dM} \propto \exp \left\{ -\frac{[\log(M/M_c)]^2}{2\sigma^2} \right\}$$



axion-like curvaton

running mass inflation

Constraints

Gravitational lensing

Microensing of stars in the Magellanic Clouds (EROS and MACHO), in M31 (Hyper Subprime-CAM) and nearby (Kepler).

Microensing of quasars (Mediavilla et al.) and supernovae (Zumalacarregui & Seljak).

Millilensing of radio sources (Wilkinson et al.).

Femtolensing of GRBs (Barnacka, Glickenstein & Moderski).

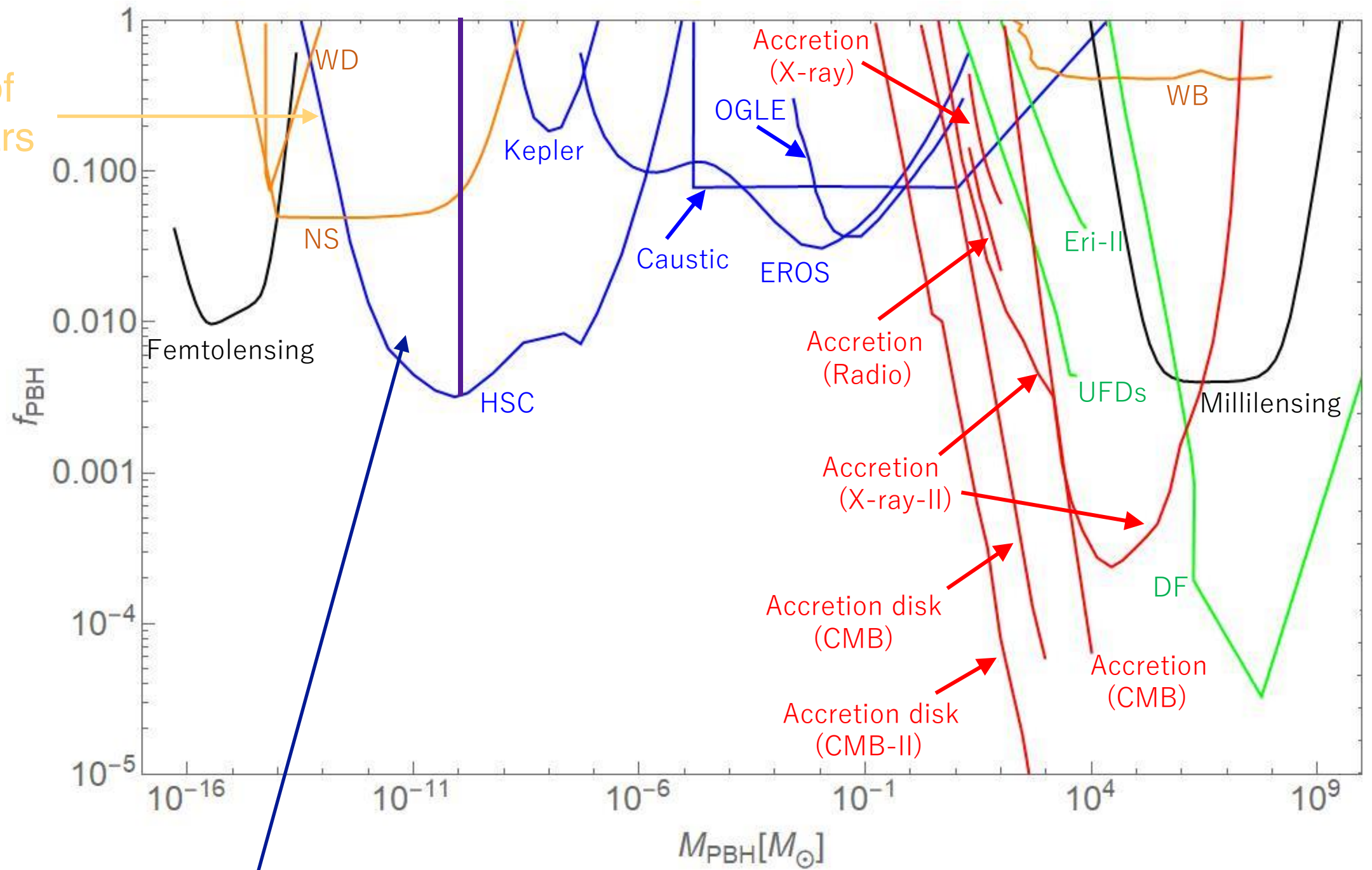
Dynamical effects: on dwarf galaxies (Brandt; Koushiappas & Loeb) and wide binaries (Yoo, Chaname & Gould; Quinn et al.; Monroy-Rodriguez & Allen).

Accretion: effect on CMB (Ricotti et al; Ali-Hamoud & Kamionkowski; Horowitz; Blum, Aloni & Flauger) and X-ray/radio emission (Gaggero et al.; Inoue & Kusenko).

Evaporation: extra-galactic gamma-rays (Carr et al.).

Destruction: of neutron stars (Capela, Pshirkov & Tinyakov; Pani & Loeb) and white dwarfs (Graham, Rajendran & Varela).

Requires existence of neutron stars in high DM density regions



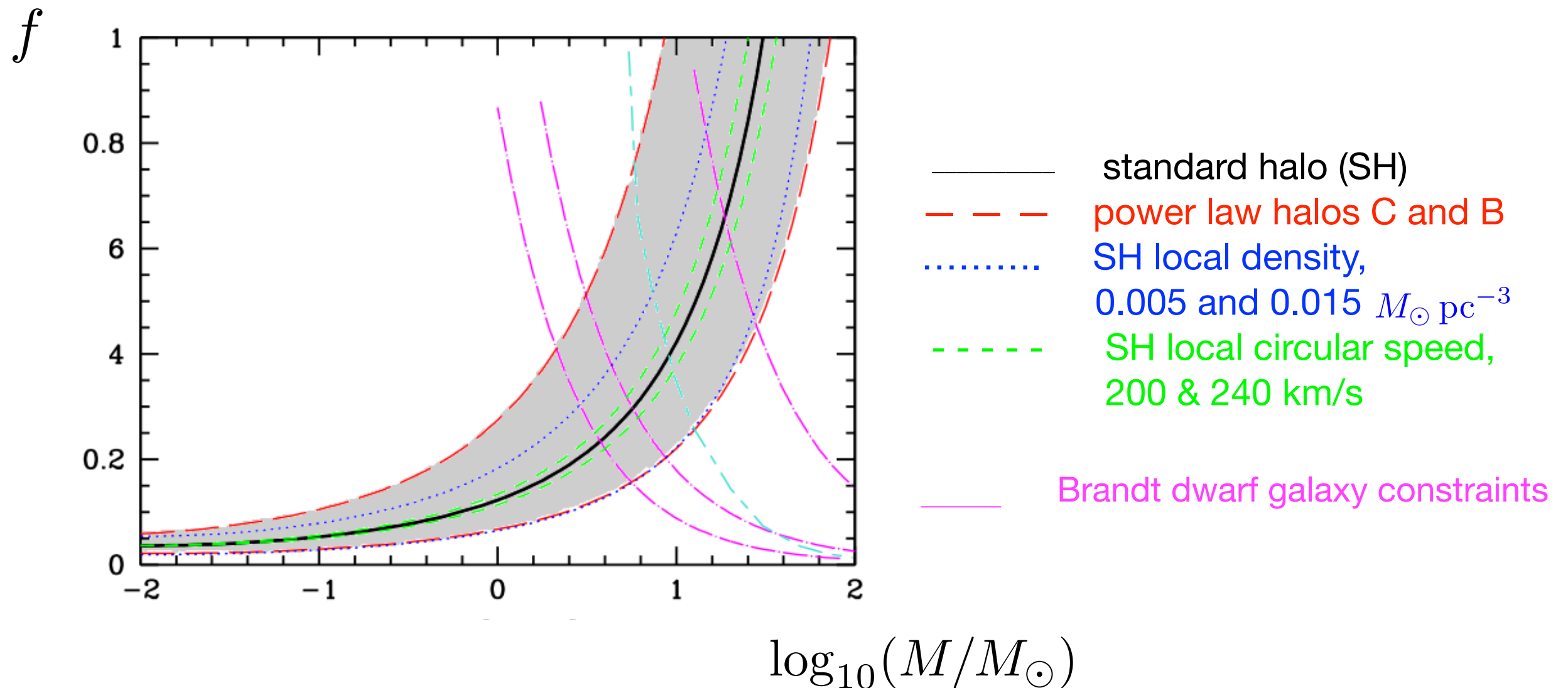
Sasaki et al.

HSC analysis assumes geometric optics, however for $M \lesssim 10^{-10} M_{\odot}$ wavelength of light is larger than Schwarzschild radius of lens diffraction occurs and lowers maximum magnification. Inomata et al.

Caveat

Constraints often depend on the dark matter distribution

For example, for the EROS microlensing constraints (assuming a delta-function MF):



Clustering of PBHs would also affect microlensing (and other) constraints.

Garcia-Bellido & Clesse

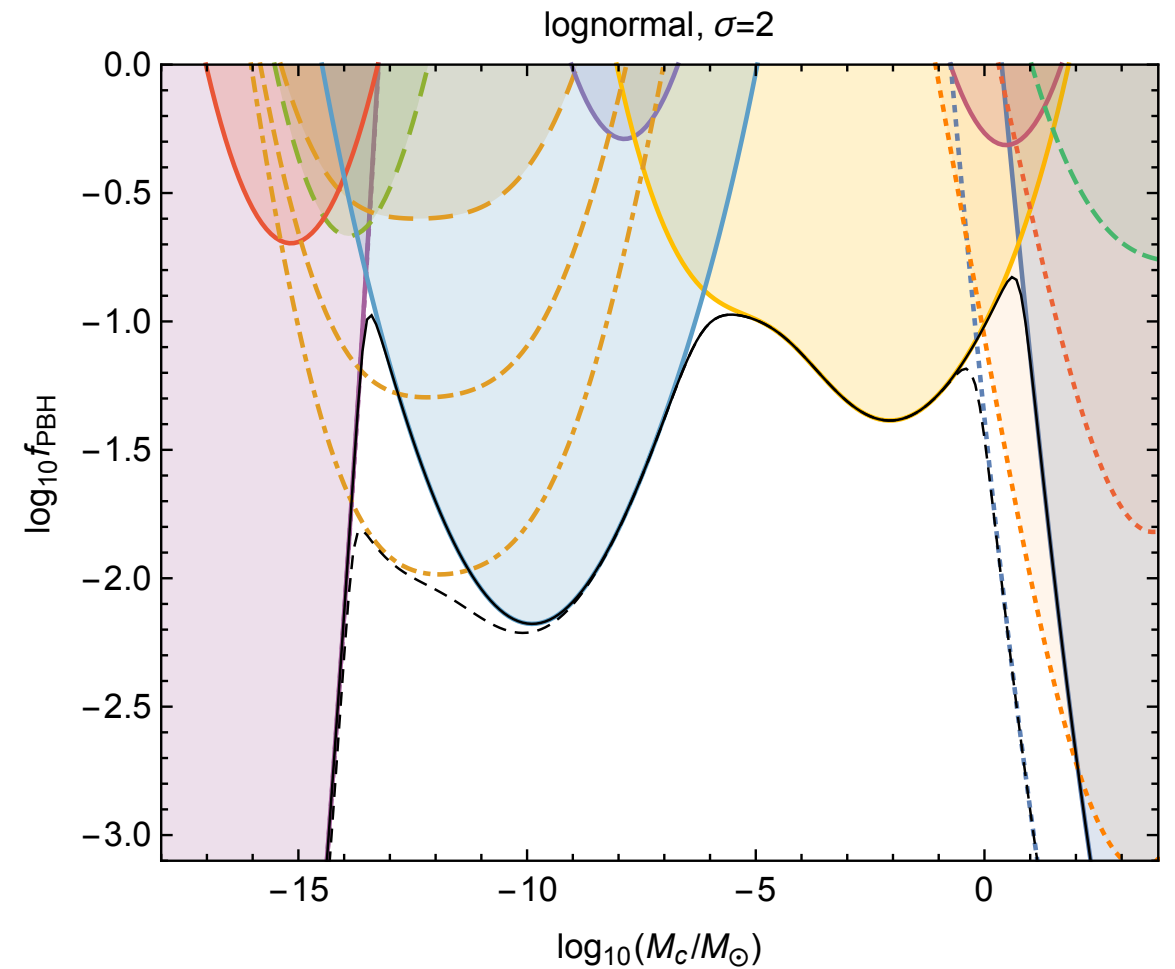
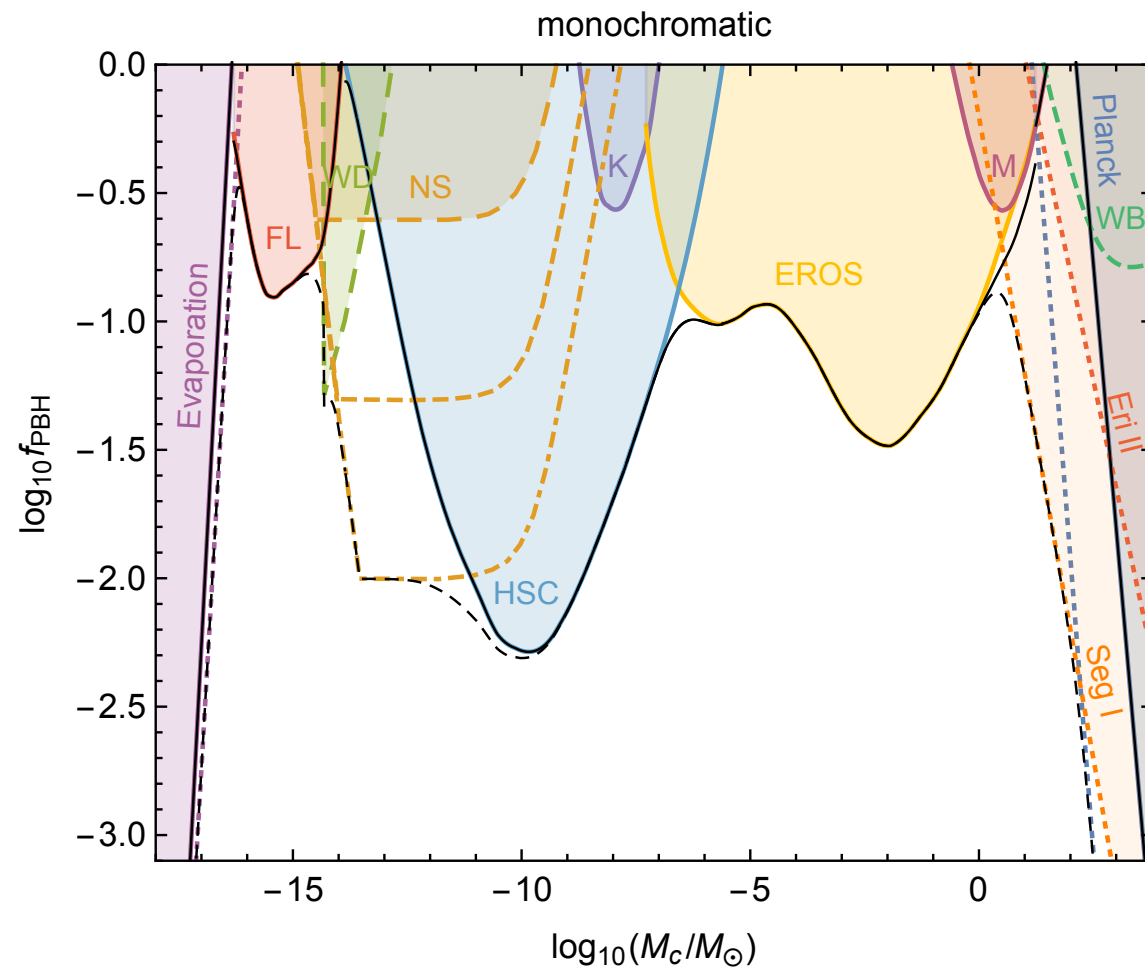
Extended mass functions

For realistic extended mass functions, individual constraints are smoothed out, but when all constraints considered maximum allowed PBH fraction is reduced Carr et al.

monochromatic

log-normal
(fixed width)

$\log_{10} f$

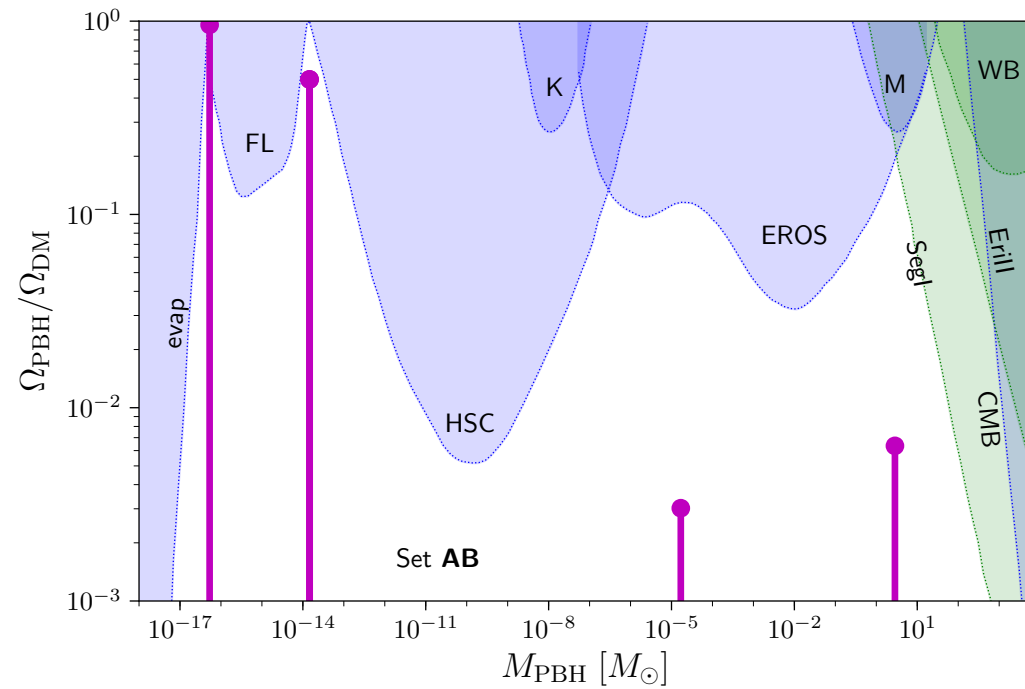


Carr et al.

$$\log_{10} \left(\frac{M}{M_\odot} \right)$$

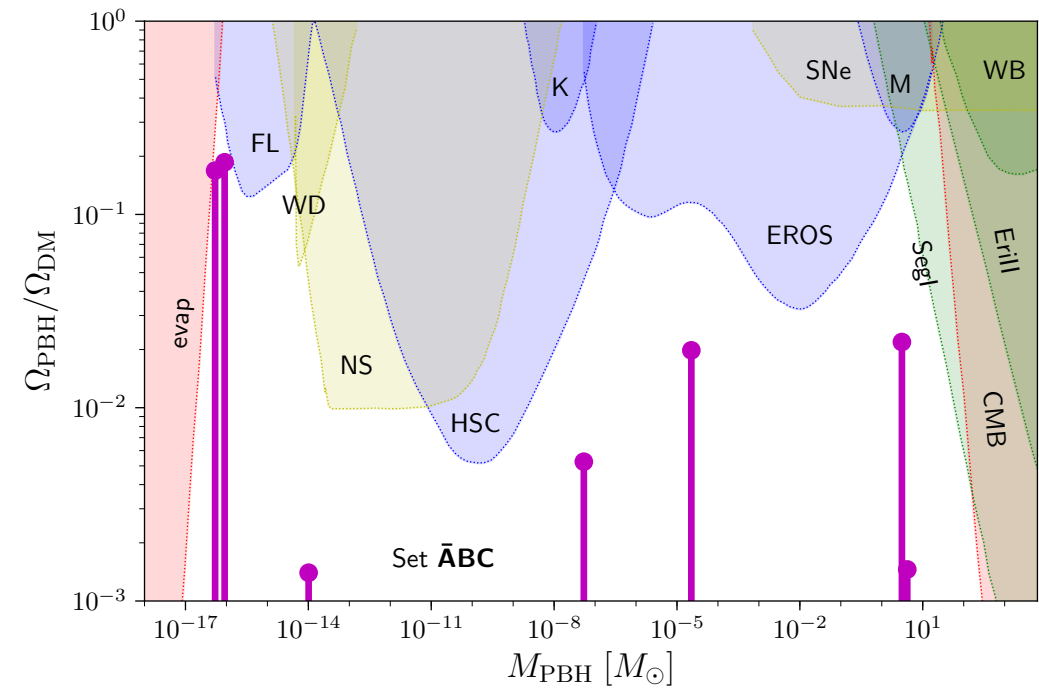
PBH fraction (considering all constraints) maximised by a MF which is a sum of delta-functions [Lehmann, Profumo & Yant](#):

'robust' constraints



$$f_{\text{max}} = 2.0$$

all constraints



$$f_{\text{max}} = 0.4$$

Summary

Primordial Black Holes can form in the early Universe, for instance from the collapse of large density perturbations during radiation domination.

A non-negligible number of PBHs will only be produced if the amplitude of the fluctuations is ~ 3 orders of magnitude larger on small scales than on cosmological scales (required amplification can be reduced with a period of early matter domination).

This can be achieved in inflation models (e.g. with a feature in the potential or multiple fields). However this is not generic/natural.

PBHs are expected to have an extended mass function (due to critical collapse and also width of primordial power spectrum).

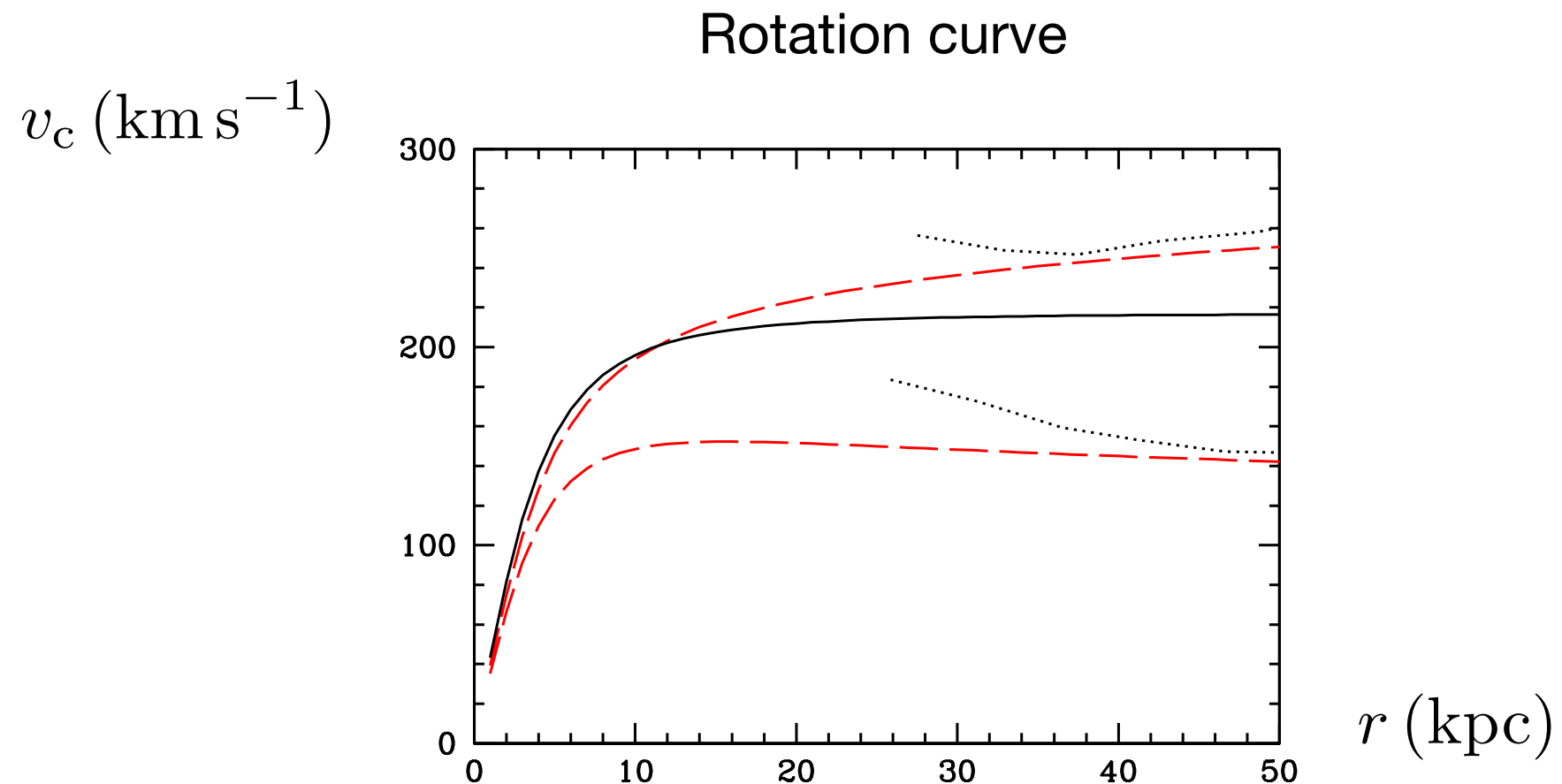
There are numerous constraints on the abundance of PBHs from gravitational lensing, their dynamical effects, accretion and other astrophysical processes.

$(1 - 100)M_{\odot}$ PBHs making up all of the DM appears to be excluded, but there may be an allowed mass window at $(10^{-14} - 10^{-10})M_{\odot}$.

(realistic) extended mass functions are more tightly constrained than delta-function MF (which is usually assumed when calculating constraints).

Back-up slides

Evans power law halo models: self-consistent halo models, which allow for non-flat rotation curves. Traditionally used in microlensing studies since there are analytic expressions for velocity distribution.



- standard halo (SH)
- — — top: power law halo B (massive halo, rising rotation curve)
bottom: power law halo C (light halo falling rotation curve)
- envelope of MW rotation curve data [Bhattacharjee et al.](#)