

Minimal Flavor Violation with Axion-like Particles

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Based on

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Theoretical Physics

Outline

- MFV structure of ALP couplings
- Implication for UV completed ALPs
 - Field theoretic ALP
 - String theoretic ALP
- Constraint from experiments
- Conclusion

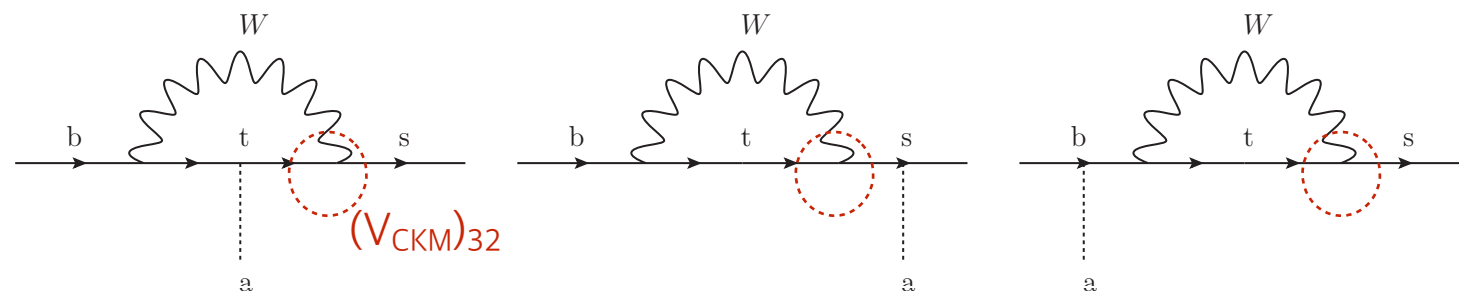
SM Flavor violating couplings with ALP

General ALP couplings to SM fermions

$$(y_u)_{ij} e^{i c_{ij}^u a/f_a} u_i^c Q_j H_u + (y_d)_{ij} e^{i c_{ij}^d a/f_a} d_i^c Q_j H_d + (y_e)_{ij} e^{i c_{ij}^e a/f_a} e_i^c L_j H_e$$

$$c_{ij} \propto \delta_{ij} \quad \text{No FCNC coupling to ALP at tree level}$$

ALP-FCNC at loop level through the SM Yukawa couplings (MFV) :



Batell, Pospelov, Ritz '09

Dolan, Kahlhoefer, McCabe, Schmidt-Hoberg '14

$B \rightarrow K a$ ($b \rightarrow s a$)

$$i c_{32}^d m_b \frac{a}{f_a} \bar{b}_R s_L$$

$$c_{32}^d \propto \ln \left(\frac{\Lambda}{m_t} \right) \rightarrow \text{UV cut-off}$$

: UV dominated

- As the radiatively induced ALP-FCNC is **logarithmically divergent**, the dominant contribution comes from high energy scale where **gauge invariance is restored**.
- Manifestly gauge invariant description** is necessary to clarify the structure and see the connection to underlying UV physics.

Georgi-Kaplan-Randall (GKR) field basis

Georgi, Kaplan, Randall '86

Non-linearly realized $U(1)_{PQ}$ for ALP :

$$U(1)_{PQ} : \Phi \rightarrow \Phi e^{iq_\Phi \alpha}, \quad \frac{a}{f_a} \rightarrow \frac{a}{f_a} + \alpha$$



$\Phi \rightarrow \Phi e^{iq_\Phi a/f_a}$ ALP-dependent field redefinition

$$U(1)_{PQ} : \Phi \rightarrow \Phi, \quad \frac{a}{f_a} \rightarrow \frac{a}{f_a} + \alpha$$

In this field basis (GKR basis), only ALP transforms under $U(1)_{PQ}$.

$$\Rightarrow \mathcal{L}_a = \frac{\partial_\mu a}{f_a} \left[\sum_\psi (\mathbf{c}_\psi)_{ij} \bar{\psi}_i \gamma^\mu \psi_j + \sum_\phi c_{\phi ij} \phi_i^\dagger i \overleftrightarrow{D}^\mu \phi_j \right] + \frac{a}{f_a} \sum_A C_A \frac{g_A^2}{32\pi^2} F^{A\mu\nu} \tilde{F}_{\mu\nu}^A$$

ALP-derivative couplings U(1)_{PQ} anomaly couplings

: manifestly gauge invariant

- Non-derivative ALP couplings (except the anomaly couplings) do not appear due to $U(1)_{PQ}$ even after radiative corrections : easy to control over possible couplings

One loop RG structure

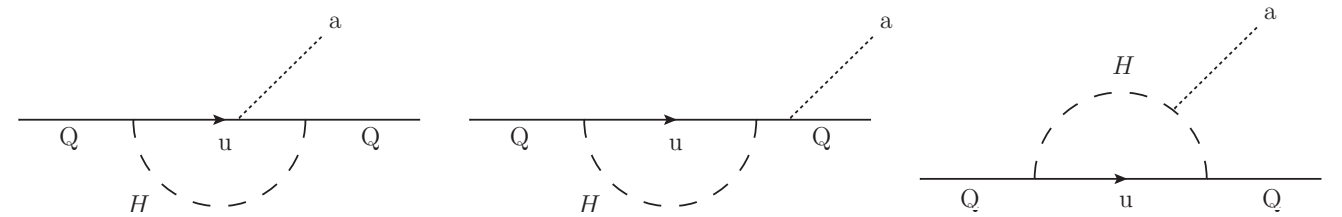
Most general ALP couplings to the SM fermions and Higgs(es) in the GKR field basis

$$: \frac{\partial_\mu a}{f_a} \left[\sum_\psi (\mathbf{c}_\psi)_{ij} \bar{\psi}_i \gamma^\mu \psi_j + \sum_\alpha c_{H_\alpha} H_\alpha^\dagger i \overleftrightarrow{D}^\mu H_\alpha \right] \quad \psi_i = \{Q_i, u_i^c, d_i^c, L_i, e_i^c\} \quad (i = 1, 2, 3)$$

$$(\mathbf{c}_\psi)_{ij}(\mu = \Lambda_a) = c_\psi \delta_{ij} \quad \text{No ALP-FCNC at tree level}$$

$$(\tilde{\mathbf{y}}_u)_{ij} u_i^c Q_j H_u + (\tilde{\mathbf{y}}_d)_{ij} d_i^c Q_j H_d + (\tilde{\mathbf{y}}_e)_{ij} e_i^c L_j H_e$$

Yukawa couplings of the SM fermions



Radiatively induced
flavor off-diagonal elements

$$\xi = \begin{cases} 1, & \text{non-SUSY} \\ 2, & \text{SUSY} \end{cases}$$

$$\frac{d\mathbf{c}_Q}{d \ln \mu} = \frac{\xi}{16\pi^2} \left(n_u \tilde{\mathbf{y}}_u^\dagger \tilde{\mathbf{y}}_u + n_d \tilde{\mathbf{y}}_d^\dagger \tilde{\mathbf{y}}_d \right)$$

$$\frac{d\mathbf{c}_{u^c}^T}{d \ln \mu} = \frac{\xi}{8\pi^2} n_u \tilde{\mathbf{y}}_u \tilde{\mathbf{y}}_u^\dagger, \quad \frac{d\mathbf{c}_{d^c}^T}{d \ln \mu} = \frac{\xi}{8\pi^2} n_d \tilde{\mathbf{y}}_d \tilde{\mathbf{y}}_d^\dagger$$

$$n_u \equiv c_Q + c_{u^c} + c_{H_u}$$

$$n_d \equiv c_Q + c_{d^c} + c_{H_d}$$


The Higgses in the Yukawa couplings \mathbf{H}_u , \mathbf{H}_d , \mathbf{H}_e depend on specific (multi-)Higgs doublet models.

- In the MSSM (Type-II 2HDM), $\mathbf{H}_u \neq \mathbf{H}_d$, $\mathbf{H}_d = \mathbf{H}_e$ \rightarrow $c_{H_d} = c_{H_e}$
- In the SM, $\mathbf{H}_u = \mathbf{H}$, $\mathbf{H}_d = \mathbf{H}_e = \mathbf{H}^*$ \rightarrow $c_{H_u} = -c_{H_d} = -c_{H_e} (\equiv c_H)$

Matching at heavy BSM Higgs mass scale

An important change in the RG running will occur at the scale of heavy BSM Higgs masses. For simplicity, suppose that other BSM particles (e.g. superpartners) lie at the similar scale.

$$\begin{aligned}
 \frac{d\mathbf{c}_Q}{d\ln\mu} &= \frac{\xi}{16\pi^2} \left(\mathbf{n}_u \tilde{\mathbf{y}}_u^\dagger \tilde{\mathbf{y}}_u + \mathbf{n}_d \tilde{\mathbf{y}}_d^\dagger \tilde{\mathbf{y}}_d \right) & \mathbf{n}_u &\equiv c_Q + c_{u^c} + c_{H_u} \\
 \frac{d\mathbf{c}_{u^c}^T}{d\ln\mu} &= \frac{\xi}{8\pi^2} \mathbf{n}_u \tilde{\mathbf{y}}_u \tilde{\mathbf{y}}_u^\dagger, & \mathbf{n}_d &\equiv c_Q + c_{d^c} + c_{H_d} \\
 \frac{d\mathbf{c}_{d^c}^T}{d\ln\mu} &= \frac{\xi}{8\pi^2} \mathbf{n}_d \tilde{\mathbf{y}}_d \tilde{\mathbf{y}}_d^\dagger
 \end{aligned}
 \quad \mu > m_{\text{BSM}}$$


 At $\mu = m_{\text{BSM}}, \left\{ \begin{array}{l} H_u (\equiv H_2) = H \sin \beta, \quad H_1 = H^* \cos \beta \\ \mathbf{y}_u = \tilde{\mathbf{y}}_u \sin \beta \\ \mathbf{y}_d = \tilde{\mathbf{y}}_d \cos \beta \text{ or } \tilde{\mathbf{y}}_d \sin \beta, \quad \mathbf{y}_e = \tilde{\mathbf{y}}_e \cos \beta \text{ or } \tilde{\mathbf{y}}_e \sin \beta \end{array} \right.$

$$\begin{aligned}
 \frac{d\mathbf{c}_Q}{d\ln\mu} &= \frac{1}{16\pi^2} \left(\hat{\mathbf{n}}_u \mathbf{y}_u^\dagger \mathbf{y}_u + \hat{\mathbf{n}}_d \mathbf{y}_d^\dagger \mathbf{y}_d \right) & \hat{\mathbf{n}}_u &= c_Q + c_{u^c} + c_H \\
 \frac{d\mathbf{c}_{u^c}^T}{d\ln\mu} &= \frac{1}{8\pi^2} \hat{\mathbf{n}}_u \mathbf{y}_u \mathbf{y}_u^\dagger, & \hat{\mathbf{n}}_d &= c_Q + c_{d^c} - c_H \\
 \frac{d\mathbf{c}_{d^c}^T}{d\ln\mu} &= \frac{1}{8\pi^2} \hat{\mathbf{n}}_d \mathbf{y}_d \mathbf{y}_d^\dagger
 \end{aligned}
 \quad \mu < m_{\text{BSM}}$$

ALP derivative coupling to the SM Higgs : $c_H = c_{H_2} \sin^2 \beta - c_{H_1} \cos^2 \beta$ (in 2HDMs)



$$\hat{\mathbf{n}}_u = \mathbf{n}_u - \mathbf{n}_H \cos^2 \beta \quad \mathbf{n}_H \equiv c_{H_1} + c_{H_2}$$

One loop induced FCNC couplings to ALP

- down-type quark FCNC couplings to ALP

$$c_{ij}^d \frac{\partial_\mu a}{f_a} \bar{d}_{Li} \gamma^\mu d_{Lj} \rightarrow -i c_{ij}^d \frac{a}{f_a} \bar{d}_i (m_{d_i} P_L - m_{d_j} P_R) d_j \quad \text{equation of motion of the fermions}$$

Unitary rotation matrix
for the mass eigenbasis

diagonalized Yukawa matrix
in the mass eigenbasis

$$\begin{aligned} c_{ij}^d &= (U_{dL}^\dagger \mathbf{c}_Q U_{dL})_{ij} \\ &= -\frac{1}{16\pi^2} \left(V_{\text{CKM}}^\dagger \tilde{\mathbf{y}}_u^{D\dagger} \tilde{\mathbf{y}}_u^D V_{\text{CKM}} \right)_{ij} \left[\xi \mathbf{n}_u \ln \left(\frac{\Lambda_a}{m_{\text{BSM}}} \right) + \sin^2 \beta \hat{n}_u \ln \left(\frac{m_{\text{BSM}}}{\mu} \right) \right] + \dots \\ &\approx -\frac{m_t^2}{16\pi^2 v^2} (V_{\text{CKM}})_{3i}^* (V_{\text{CKM}})_{3j} \left[\frac{\xi}{\sin^2 \beta} \mathbf{n}_u \ln \left(\frac{\Lambda_a}{m_{\text{BSM}}} \right) + \left(\mathbf{n}_u - \underbrace{n_H \cos^2 \beta}_{\approx \frac{n_H}{\tan^2 \beta}} \right) \ln \left(\frac{m_{\text{BSM}}}{m_t} \right) \right] + \dots \end{aligned}$$



$c_{32}^d : B \rightarrow K \ a (b \rightarrow s \ a)$ major constraints on ALP-FCNC
 $c_{21}^d : K \rightarrow \pi \ a (s \rightarrow d \ a)$



For large $\tan \beta$, \mathbf{n}_u (sum of the ALP derivative coupling coefficients of the fields involved in the up-type Yukawa coupling) is the most important factor to induce the down-type quark FCNC couplings to ALP.

One loop induced FCNC couplings to ALP

- up-type quark FCNC couplings to ALP

$$-i \frac{a}{f_a} c_{ij}^u \bar{u}_i (m_{u_i} P_L - m_{u_j} P_R) u_j$$

* Upper entry : $\mathbf{H}_d = \mathbf{H}_1$ (SUSY, type II, Y 2HDMs)

Lower entry : $\mathbf{H}_d = \mathbf{H}_2^*$ (type I, X 2HDMs)

$$\begin{aligned} c_{ij}^u &= (U_{u_L}^\dagger \mathbf{c}_Q U_{u_L})_{ij} \\ &= -\frac{1}{16\pi^2} \left(V_{\text{CKM}}^\dagger \tilde{\mathbf{y}}_d^{D\dagger} \tilde{\mathbf{y}}_d^D V_{\text{CKM}} \right)_{ij} \left[\xi n_d \ln \left(\frac{\Lambda_a}{m_{\text{BSM}}} \right) + \left\{ \frac{\cos^2 \beta}{\sin^2 \beta} \right\} \hat{n}_d \ln \left(\frac{m_{\text{BSM}}}{\mu} \right) \right] + \dots \\ &\approx -\frac{m_b^2}{16\pi^2 v^2} (V_{\text{CKM}})_{i3} (V_{\text{CKM}})^*_{j3} \left[\xi n_d \left\{ \frac{\tan^2 \beta}{1} \right\} \ln \left(\frac{\Lambda_a}{m_{\text{BSM}}} \right) \right. \\ &\quad \left. + \left(n_d - n_H \left\{ \frac{1}{-1/\tan^2 \beta} \right\} \right) \ln \left(\frac{m_{\text{BSM}}}{m_t} \right) \right] + \dots \end{aligned}$$

➡ $c_{21}^u : D \rightarrow \pi a (c \rightarrow u a)$ screened by QCD long-distance effect
: much weaker sensitivity to new physics than the down-type quark sector

➡ Yet, in certain models (SUSY, type II, Y 2HDMs) with large $\tan \beta$, the enhanced bottom-Yukawa coupling with a non-zero n_d may induce a sizable ALP-FCNC.

Implication for UV completed ALP models

- Field theoretic ALP

$$X = \frac{1}{\sqrt{2}} \rho e^{i a / f_a} \quad \text{ALP originating from the phase of PQ-charged complex scalar fields}$$

$$U(1)_{\text{PQ}} : \frac{a}{f_a} \rightarrow \frac{a}{f_a} - \alpha, \quad \Phi \rightarrow \Phi e^{i q_\Phi \alpha}$$

Most general
Lagrangian



$$\begin{aligned} & \left(\frac{X}{M_*} \right)^{q_{u_i} + q_{Q_j} + q_{H_2}} (\lambda_u)_{ij} u_i^c Q_j H_2 + \left(\frac{X}{M_*} \right)^{q_{d_i} + q_{Q_j} + q_{H_d}} (\lambda_d)_{ij} d_i^c Q_j H_d \\ & + \left(\frac{X}{M_*} \right)^{q_{e_i} + q_{L_j} + q_{H_e}} (\lambda_e)_{ij} e_i^c L_j H_e + b_0 \left(\frac{X}{M_*} \right)^{q_{H_1} + q_{H_2}} H_1 H_2 + \text{h.c.} \end{aligned} \quad M_* : \text{cut-off scale} \gg f_a$$



$$\Phi \rightarrow \Phi e^{-i q_\Phi a / f_a} \quad \text{ALP dependent field redefinition (GKR basis)}$$

$$\frac{\partial_\mu a}{f_a} \left[\sum_\psi q_{\psi_i} \delta_{ij} \bar{\psi}_i \gamma^\mu \psi_j + \sum_\alpha q_{H_\alpha} H_\alpha^\dagger i \overleftrightarrow{D}^\mu H_\alpha \right] \quad \psi_i = \{Q_i, u_i^c, d_i^c, L_i, e_i^c\} \quad (i = 1, 2, 3)$$

If $q_{\psi_i} \neq q_{\psi_j}$ ($i \neq j$), there are tree level FCNC couplings to ALP.

For MFV, we are interested in flavor universal $q_{\psi_i} = q_\psi$

Goetz's talk "Axiflavor"
Calibbi, Goertz, Redigolo, Ziegler, Zupan '16
Ema, Hamaguchi, Moroi, Nakayama '16

Flavor-universal PQ charges

$$q_{\psi_i} = q_{\psi} \quad (\psi = Q, u^c, d^c, L, e^c) \quad \longrightarrow \quad q_{\psi} = c_{\psi} \quad \longrightarrow$$

$$n_u = q_Q + q_{u^c} + q_{H_2}$$

$$n_d = q_Q + q_{d^c} + q_{H_d}$$

$$n_H = q_{H_1} + q_{H_2}$$



In terms of the original Lagrangian,

$$\underbrace{\left(\frac{X}{M_*}\right)^{n_u} (\lambda_u)_{ij} u_i^c Q_j H_2}_{\text{down-type quark FCNC to ALP}} + \underbrace{\left(\frac{X}{M_*}\right)^{n_d} (\lambda_d)_{ij} d_i^c Q_j H_d}_{\text{up-type quark FCNC to ALP}} + \underbrace{b_0 \left(\frac{X}{M_*}\right)^{n_H} H_1 H_2 + \text{h.c.}}_{\text{down-type quark FCNC to ALP suppressed by } 1/\tan^2\beta}$$

down-type quark FCNC to ALP

up-type quark FCNC to ALP

down-type quark FCNC to ALP
suppressed by $1/\tan^2\beta$
: Freytsis, Ligeti, Thaler '09



Below the spontaneous PQ breaking scale f_a ,

$$(\tilde{\mathbf{y}}_{\psi})_{ij} = \left(\frac{f_a}{M_*}\right)^{n_{\psi}} (\lambda_{\psi})_{ij} \quad (\psi = u, d, e)$$

$$b = \left(\frac{f_a}{M_*}\right)^{n_H} b_0$$



$$n_u = 0$$

$$\tilde{y}_t \sim \mathcal{O}(1)$$

$$\tilde{y}_b \sim \frac{m_b}{v} \tan \beta \quad (\text{if } H_d = H_1)$$

also, $n_d = 0$ for large $\tan \beta \gtrsim 10$ in the 2HDMs with $H_d = H_1$

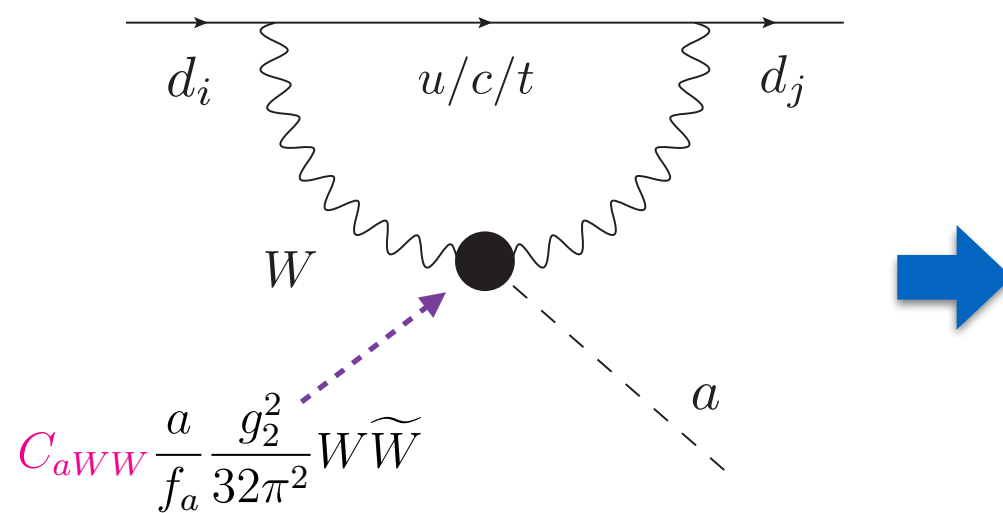
Since $M_* \gg f_a$, $\mathbf{y}_{\psi} \ll \mathcal{O}(1)$ unless $n_{\psi} = 0$

for perturbative $\lambda_{\psi} \lesssim \mathcal{O}(1)$

For a generic field theoretic ALP, the dominant ALP-FCNC at one-loop will come from a non-zero n_H with $1/\tan^2\beta$ suppression and relatively small RG logarithmic factor $\ln(m_{\text{BSM}}/m_t)$.

* Two loop order contribution Izaguirre, Lin, Shuve '16

As the one loop contribution is heavily suppressed by large $\tan \beta$, the radiative correction at two loops can be even more important.



finite radiative correction

$$\mathcal{L}_{d_i \rightarrow d_j} \supset -g_{ad_i d_j} (\partial_\mu a) \bar{d}_j \gamma^\mu \mathcal{P}_L d_i + \text{H.c.},$$

$$g_{ad_i d_j} \equiv -\frac{3\sqrt{2}G_F M_W^2 g_{aW}}{16\pi^2} \sum_{\alpha \in c, t} V_{\alpha i} V_{\alpha j}^* f(M_\alpha^2/M_W^2),$$

$$f(x) \equiv \frac{x[1 + x(\log x - 1)]}{(1 - x)^2},$$

Fig. from Izaguirre, Lin, Shuve '16

We find that this two loop order contribution is dominant over the one loop from n_H for $\tan \beta$ larger than a certain value:

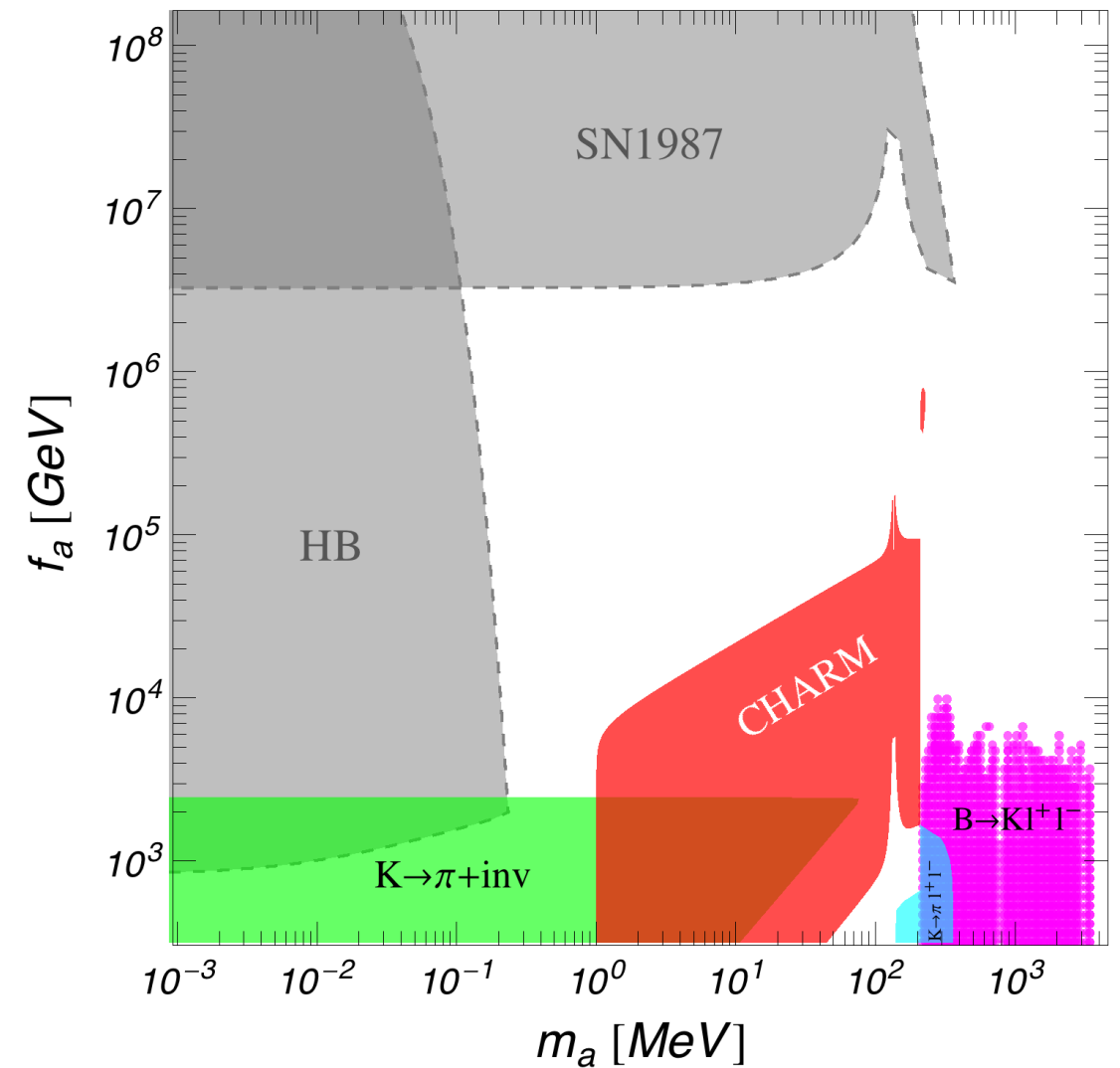
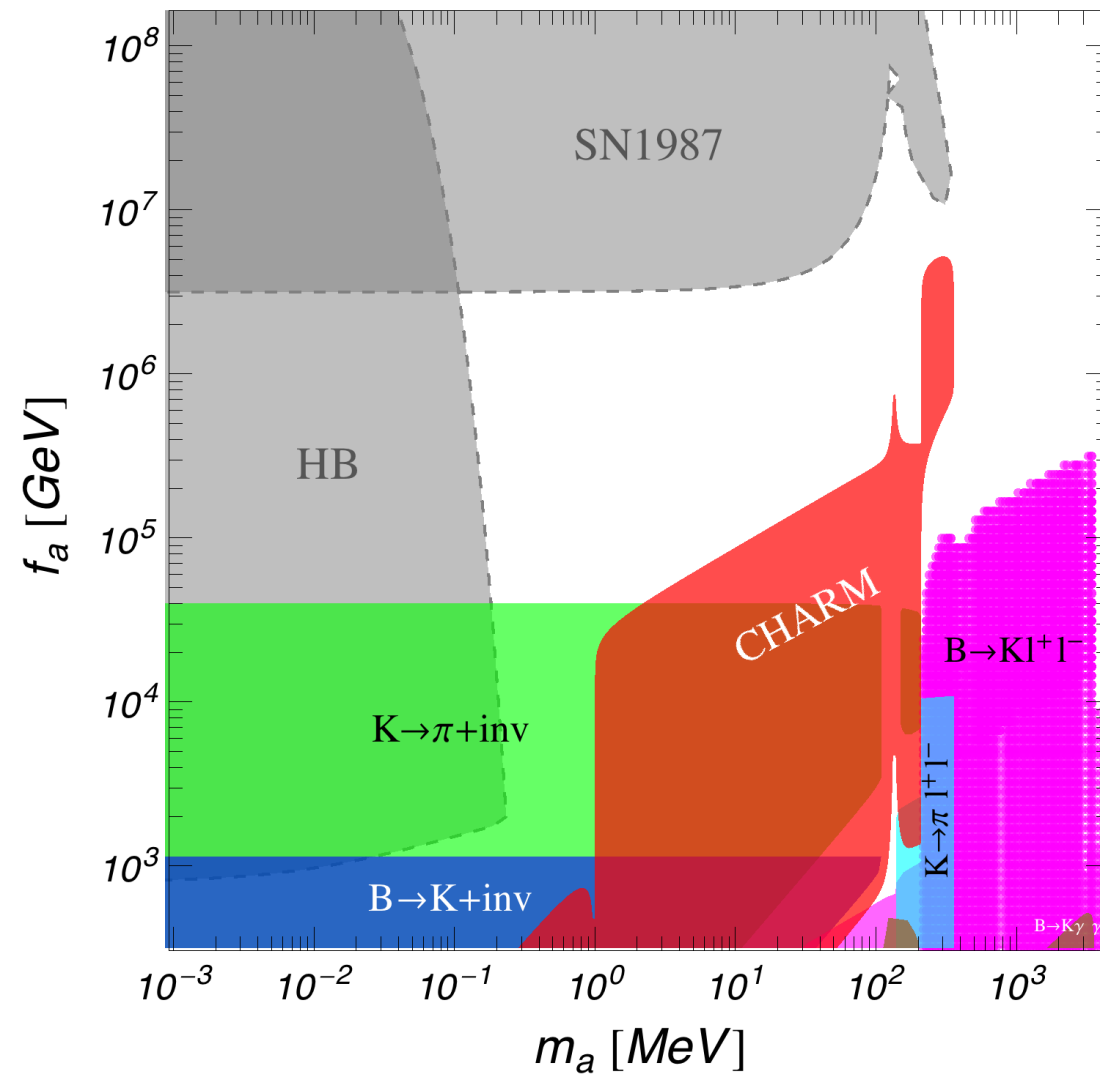
$$\tan \beta \gtrsim 17 \times \sqrt{n_H} \left[\frac{3}{C_{aWW}} \right]^{\frac{1}{2}} \left[\frac{\ln(m_{\text{BSM}}/m_t)}{2} \right]^{\frac{1}{2}}$$

Experimental constraints for the field theoretic ALP

$$n_u = 0$$

$$\tan\beta = 5, m_{H^\pm} = 1 \text{ TeV}$$

$$\tan\beta = 30, C_{aWW} = 1$$




- Even for a moderate $\tan\beta$, the constraint is an order of magnitude weaker than the previous estimation in Dolan, Kahlhoefer, McCabe, Schmidt-Hoberg '14 with “Yukawa-like” ALP couplings ($n_u = n_d \neq 0$).

Implication for UV completed ALP models

- String theoretic ALP


$$C_{[m_1 m_2 \dots m_p]} = \sum_{\alpha} a_{\alpha}(x) \omega_{[m_1 m_2 \dots m_p]}^{\alpha} \quad \text{ALP originating from higher-dimensional p-form gauge field}$$

(ω^{α} : harmonic p-form on the compact internal space)


SUSY-preserving compactification 

$$\left\{ \begin{array}{l} T_{\alpha} = \frac{\tau_{\alpha} + i a_{\alpha}}{\sqrt{2}} \quad (\tau_{\alpha} : \text{volume modulus of p-cycle dual to } \omega^{\alpha}) \\ U(1)_{\text{PQ}} : \quad a_{\alpha} \rightarrow a_{\alpha} + \text{constant}, \quad \Phi_I \rightarrow \Phi_I \quad (\text{GKR basis}) \end{array} \right.$$

: remnant of higher-dimensional gauge symmetry $\delta C_{[m_1 m_2 \dots m_p]} = \partial_{[m_1} \Lambda_{m_2, \dots, m_p]}$

Low energy effective Lagrangian 

$$K = K_0(T_{\alpha} + T_{\alpha}^*) + Z_{IJ}(T_{\alpha} + T_{\alpha}^*)\Phi_I^* \Phi_J \quad \text{4D N=1 SUGRA}$$

Typical scale of the stringy ALP decay constant 

$$f_a \sim M_P / 8\pi^2 \sim 10^{16} \text{ GeV} \quad : \text{too big to be relevant for the ALP-FCNC interactions}$$

With a large internal volume or warp factor, however, f_a can be smaller by many orders of magnitude even around TeV scale.

* The minimal LVS (Large Volume Scenario)

$$T_1 = \frac{\tau_1 + ia_1}{\sqrt{2}}, \quad T_2 = \frac{\tau_2 + ia_2}{\sqrt{2}}$$

τ_1 : volume of big 4-cycle connected to the 6D internal bulk volume

τ_2 : volume of small 4-cycle supporting a hidden non-perturbative dynamics & visible sector

$$K = -3 \ln(T_1 + T_1^*) + \frac{(T_2 + T_2^*)^{3/2}}{(T_1 + T_1^*)^{3/2}} + \frac{(T_2 + T_2^*)^{\omega_N}}{(T_1 + T_1^*)} \Phi_N^* \Phi_N$$

$$\mathcal{F}_A = k_A T_2 \quad (A = 3, 2, 1)$$

The modular weight ω_N turns out to be determined to be flavor-universal with the range [0,1].

: Conlon, Cremades, Quevedo '06

a_1 : too feeble interactions

a_2 : ALP with small f_a

$$\mathcal{L}_a = \frac{\partial_\mu a_1}{\sqrt{6} M_P} \left(\phi_N^* i \overleftrightarrow{D}^\mu \phi_N - \frac{1}{2} \bar{\psi}_N \bar{\sigma}^\mu \psi_N \right) - c_N \frac{\partial_\mu a_2}{f_a} \left(\phi_N^* i \overleftrightarrow{D}^\mu \phi_N + \bar{\psi}_N \gamma^\mu \psi_N \right) - \frac{1}{32\pi^2} \frac{a_2}{f_a} F_{\mu\nu}^A \tilde{F}^{A\mu\nu}$$

$$f_a = \frac{\sqrt{3}}{2\tau_2^{1/4}} \frac{1}{\tau_1^{3/4}} \frac{M_P}{8\pi^2} \sim e^{-a\tau_2/2} \frac{M_P}{8\pi^2}$$

$$c_N = \frac{\sqrt{2}}{16\pi^2 \tau_2} \omega_N$$

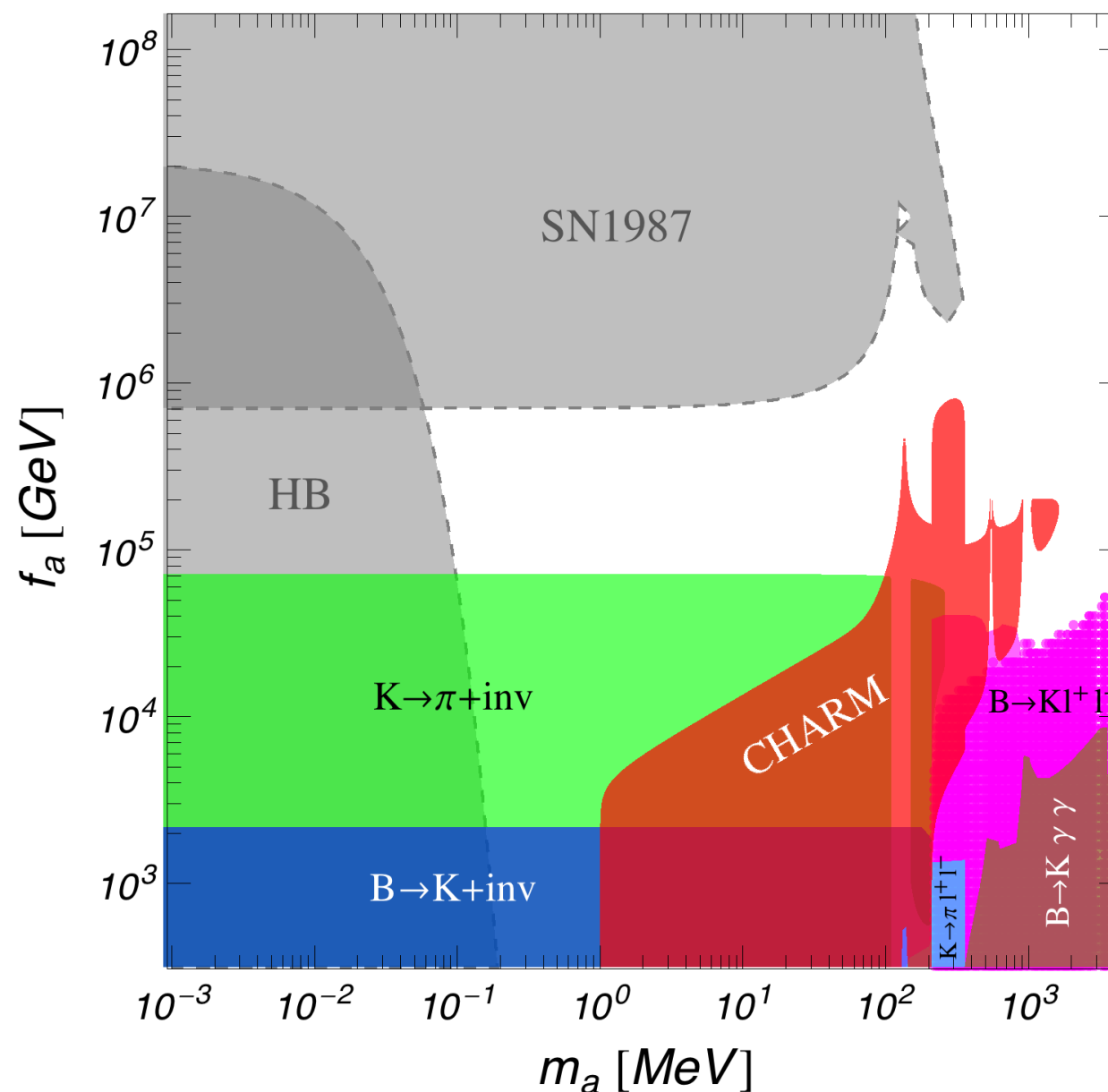
$$\begin{aligned} n_u &= \frac{\sqrt{2}}{16\pi^2 \tau_2} (\omega_Q + \omega_{u^c} + \omega_{H_2}) = \mathcal{O}\left(\frac{1}{16\pi^2}\right) \\ n_d &= \frac{\sqrt{2}}{16\pi^2 \tau_2} (\omega_Q + \omega_{d^c} + \omega_{H_1}) = \mathcal{O}\left(\frac{1}{16\pi^2}\right) \end{aligned}$$

$$\mathcal{L}_{\text{Yukawa}} = \frac{1}{2} y_{LMN} \phi_L \psi_M \psi_N \quad y_{LMN} = \frac{\lambda_{LMN}}{(\sqrt{2}\tau_2)^{(\omega_L + \omega_M + \omega_N)/2}} \quad \tau_2 \sim \mathcal{O}(1)$$

: No problem to obtain the correct top (bottom) Yukawa with a non-zero n_u (n_d).

Experimental constraints for the stringy ALP

Universal modular weights $n_u = n_d = \frac{3}{2} n_H = \frac{1}{16\pi^2}$



- Similar to “Yukawa-like” ALP couplings in Dolan, Kahlhoefer, McCabe, Schmidt-Hoberg ’14, but with smaller ALP coupling by one-loop factor and large RG logarithmic factor $\ln(M_{\text{string}}/m_t)$, which results in an order of magnitude weaker constraints.

$$M_{\text{string}} \sim 16\pi^2 f_a$$

Conclusion

- The minimal flavor violation with ALPs occurs radiatively through the SM Yukawa couplings.
- The down (up)-type quark FCNC couplings to ALP is generated by n_u (n_d) [the sum of the ALP derivative couplings to the fields involved in the up (down)-type Yukawa couplings at UV cut-off], with the subleading contribution from n_H [the sum of the ALP derivative couplings to the Higgses at UV cut-off].
- This means that the field theoretic ALPs have generic suppression of FCNC couplings by $1/\tan^2\beta$, if there is no tree level FCNC.
- The string theoretic ALPs have also weaker FCNC couplings than the usual expectation in the previous literatures, but there is no generic parametric suppression like the field theoretic ALPs.