UV conformal window for asymptotic safety

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Motivation

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- Asymptotic safety: interactive fixed point; residual interactions in the UV
- Critical phenomena: Wilson-Fisher fixed point (ϕ^4 theory in $d = 4 - \epsilon$ dimensions) (Wilson & Fisher, 1972)
- A quantum theory of gravity may be non-perturbatively renormalizable provided there is an interactive UV fixed point. (Weinberg, 1979)
- Necessary conditions and no-go theorems for asymptotic safety in gauge theories have been derived at weak coupling. (Bond & Litim, 2017a)
- There are exact proofs of existence in simple and semi-simple gauge groups, as well as SUSY. (Litim & Sanino, 2014) (Bond & Litim, 2017b) (Bond & Litim, 2018)
- Higher dimensional scalar operators are ok. Fixed points are available away from *4d*. (Buyukbese & Litim, 2017) (Codello, Langæble, Litim, & Sannino,2016)
- Asymptotically safe model building now available. (Bond, Hiller, Kowalska & Litim, 2017)
- IR conformal windows in QCD-like theories have been extensively studied, and are known to extend past the domain of perturbation theory. (Banks & Zaks, 1982) (Appelquist et al., 2008)
- What is the size of the conformal window of asymptotically safe theories? How many fields are required for asymptotic safety? (Del Debbio., 2011)

Interacting fixed points

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- Gauge renormalization group (RG) running at two loop in perturbation theory

$$\beta = \frac{\mathrm{d}\alpha}{\mathrm{d}ln\mu} = -B\alpha^2 + C\alpha^3$$

- The RG flow vanishes at the fixed point

$$\beta^* = 0 \qquad \alpha^* = \frac{B}{C}$$

- Physical coupling: *B* and *C* must have the same sign
- Weak coupling: *B* must be much smaller than *C*

Interacting fixed points

- Give-up asymptotic freedom: $B < 0; \quad C < 0$

- C>0 for any simple or semi simple gauge group, for any matter fields multiplicities and representation

(Bond & Litim, 2017a)

- C<0 can only be achieved with Yukawa interactions

- Necessary ingredients:
  - Gauge group (simple or semi-simple)
  - Fermions
  - Scalars
  - Yukawa interactions

Asymptotically safe theory

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$$L = L_{\rm YM} + L_{\rm kin.} + L_{\rm Yuk.} + L_{\rm pot.}$$

$$\begin{aligned}
L_{\rm YM} &= -\frac{1}{2}{\rm Tr}\, F^{\mu\nu} F_{\mu\nu} \\
L_{\rm kin.} &= {\rm Tr}\left(\overline{Q}\, i \not{D}\, Q\right) + {\rm Tr}\left(\partial_\mu H^\dagger\, \partial^\mu H\right) \\
L_{\rm Yuk.} &= -y\, {\rm Tr}\left(\overline{Q}_L\, H\, Q_R\right) + {\rm h.c.} \\
L_{\rm pot.} &= -u\, {\rm Tr}\left(H^\dagger H\, H^\dagger H\right) - v\left({\rm Tr}\, H^\dagger H\right)^2
\end{aligned}$$

- *4d* gauge theory
- NF fermions in the fundamental representation of SU(NC)
- Meson-like complex scalar NF x NF matrix, uncharged
- Yukawa & scalar quartic interactions

(Litim & Sanino, 2014)

Asymptotically safe theory

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$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}; \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}; \quad \alpha_u = \frac{u N_F}{(4\pi)^2}; \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}$$

- Take the Veneziano limit; field multiplicities are taken to infinity, keeping their ratio fixed
- One free parameter left, which can be taken to be perturbatively small
- The theory has a weakly coupled fixed point

$$N_C \to \infty; \quad N_F \to \infty$$

$$\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$$

$$0 < \epsilon \ll 1$$

Asymptotically safe theory

- Weak coupling: series expansion in a small parameter is justified

$$\alpha_i^* = \lambda_{1i}\epsilon + \lambda_{2i}\epsilon^2 + \lambda_{3i}\epsilon^3 + O\left(\epsilon^n\right)$$

- The beta functions can then be expanded in epsilon. Exactly at the fixed point, all orders must vanish individually. The resulting equations determine the λ coefficients.
- In perturbation theory only the first few loop orders of the beta functions are known.
- Approximations have to be consistent.

Asymptotically safe theory

- The approximation is denoted by the number of loop orders retained:

$$\left.\begin{array}{l} n \text{ loop gauge} \\ m \text{ loop Yukawa} \\ l \text{ loop scalar} \end{array}\right\} \equiv (n, m, l)$$

- The approximation $(n+1, n, n)$ completely determines the coefficient of order $O\left(\epsilon^n\right)$ in the series expansion of the fixed point.

$(2,1,1)$ $(3,2,2)$ $(4,3,3)$ $(n+1,n,n)$

$$\alpha_i^* = \lambda_{1i}\epsilon + \lambda_{2i}\epsilon^2 + \lambda_{3i}\epsilon^3 + O\left(\epsilon^n\right)$$

(Bond, Litim, Medina Vazquez & Steudtner, 2018)

Fixed points and vacuum stability

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- We have computed the running of the scalar self-interactions to two loop. This means our most advanced approximation is (322), an improvement over the original analysis (321).
- The fixed point can be determined analytically to order ϵ^2. Numerically, we find:

$$\alpha_g^* = 0.4561\epsilon + 0.7808\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_y^* = 0.2105\epsilon + 0.5082\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_u^* = 0.1998\epsilon + 0.4403\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_v^* = -0.1373\epsilon - 0.6318\epsilon^2 + O\left(\epsilon^3\right)$$

(Bond et al., 2018)

Fixed points and vacuum stability

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- In order to have a stable vacuum state, we require for the scalar potential to be bounded from below. In the present setting, this means:

$$\alpha_u^* > 0, \quad \alpha_u^* + \alpha_v^* > 0$$

(Litim, Mojaza & Sannino, 2016)

- Comparing with the previous approximations, we detect a change of sign in the subleading term. This implies that the vacuum becomes unstable at a finite value of epsilon.

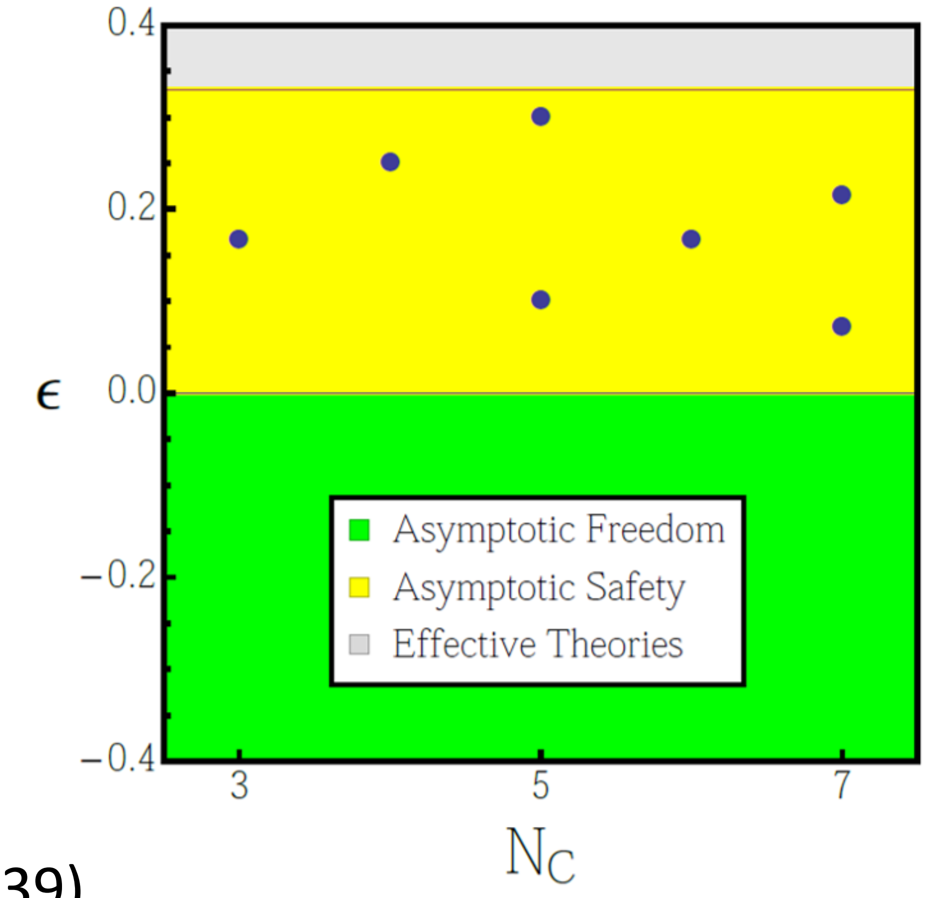
$$\alpha_u^* + \alpha_v^*|_{(211)} = 0.0625\epsilon + O\left(\epsilon^3\right)$$
$$\alpha_u^* + \alpha_v^*|_{(321)} = 0.0625\epsilon + 0.1535\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_u^* + \alpha_v^*|_{(322)} = 0.0625\epsilon - 0.1915\epsilon^2 + O\left(\epsilon^3\right)$$

Fixed points and vacuum stability

- Vacuum instability imposes a physical constraint.

$$0 < \epsilon < 0.326$$

- This effect is driven by the two loop running of the scalar self-interactions.
- The plot shows the UV conformal window. Blue dots correspond to smallest field multiplicities:



(NC,NF) = (3,17), (4,23), (5,28), (5,29), (6,34), (7,39), …

(Bond et al., 2018)

Competition of fluctuations

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- The size of the conformal window can be understood as a competition of fluctuations
- Consider the following qualitative indicator:

$$\beta_g|_{(322)} = 10.24\epsilon^5, \quad \beta_y|_{(322)} = -1.71\epsilon^4, \quad \beta_u|_{(322)} = 1.70\epsilon^4, \quad \beta_v|_{(322)} = 7.24\epsilon^4$$

- This is obtained by inserting the fixed point at order ϵ^2 back into the beta functions
- Positive shifts to the flow equations destabilize the fixed point, while negative ones stabilize it
- The running of the gauge and scalar interactions tend to close the conformal window, while the Yukawa interaction tends to open it

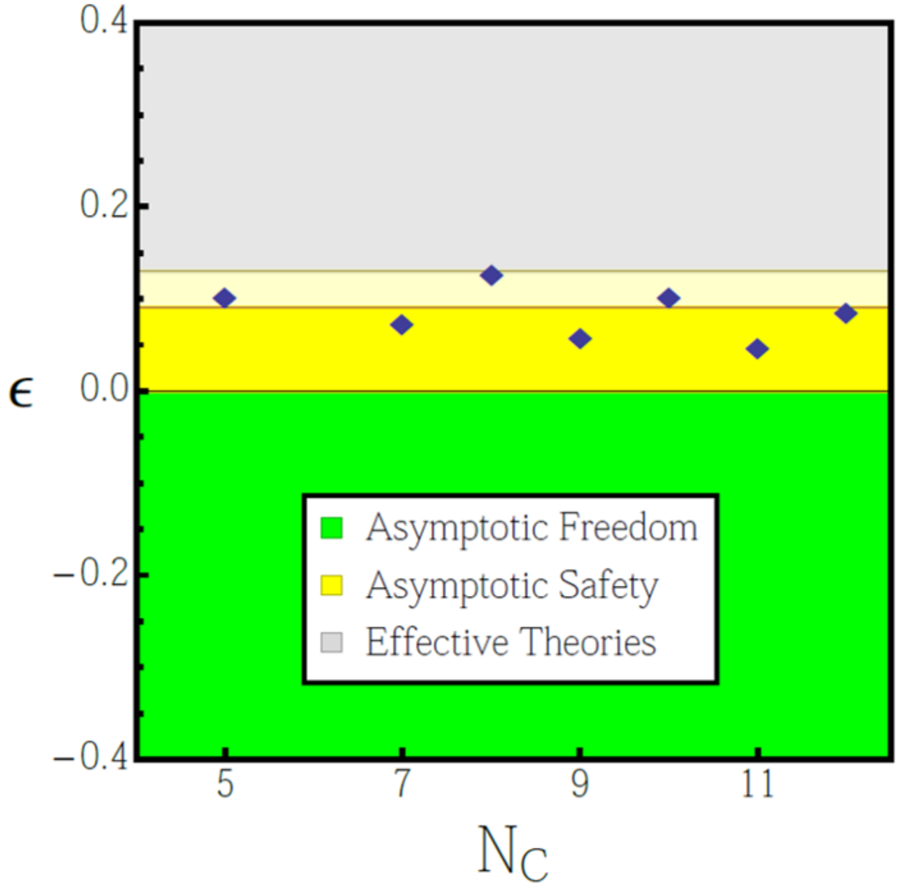
Competition of fluctuations

- We now consider the (incomplete) subleading terms in the beta functions
- The plot shows the conformal window of approximation (321) bounded by a fixed point merger (light yellow), and (322), bounded by vacuum instability (dark yellow).
- Vacuum instability still poses the tightest constraint at a numerically smaller value

$$\epsilon_{max} \approx 0.09 \ \ldots \ 0.13$$

- Blue dots are:

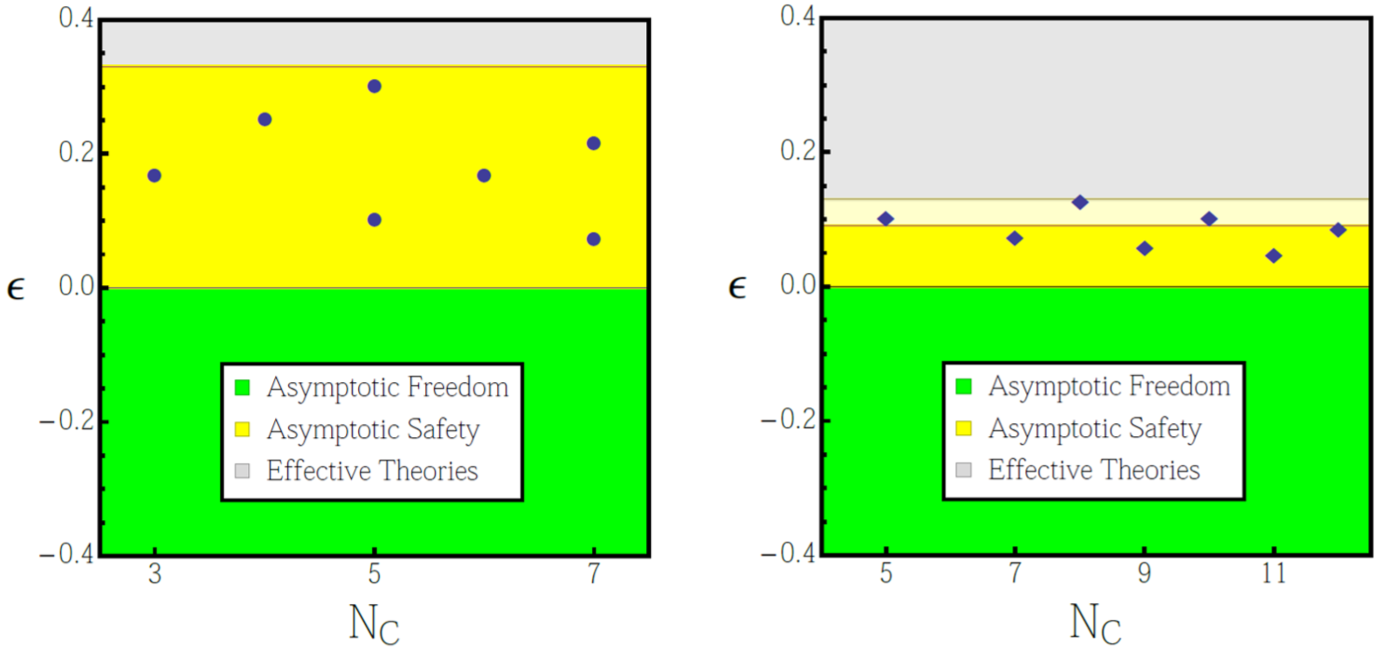
(NC,NF) = (7,39), (9,50), (11,61), (12,67), …



(Bond et al., 2018)

Conclusions

- Vacuum instability imposes the tightest constraint in the theory. This is driven by the two loop running of the scalar self-interactions.
- Two pictures: Which one is right? Both are approximations that we use to understand the trend at higher loop orders.



Conclusions

- The UV conformal window remains roughly within the region of perturbation theory.
- It would be interesting to obtain the next approximation (433), then we can start to say something about the convergence of the boundary of the conformal window.

References

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Appelquist, T., Fleming, G. T., & Neil, E. T. (2007). Lattice Study of the Conformal Window in QCD-like Theories. http://doi.org/10.1103/PhysRevLett.100.171607

Banks, T., & Zaks, A. (1982). On the phase structure of vector-like gauge theories with massless fermions. *Nuclear Physics, Section B*, *196*(2), 189–204. http://doi.org/10.1016/0550-3213(82)90035-9

Bond, A. D., Hiller, G., Kowalska, K., & Litim, D. F. (2017). Directions for model building from asymptotic safety. *Journal of High Energy Physics*, *2017*(8), 4. https://doi.org/10.1007/JHEP08(2017)004

Bond, A. D., & Litim, D. F. (2017). Theorems for asymptotic safety of gauge theories. *The European Physical Journal C*, *77*(6), 429. https://doi.org/10.1140/epjc/s10052-017-4976-5

Bond, A. D., & Litim, D. F. (2017). Asymptotic Safety Guaranteed in Supersymmetry. *Physical Review Letters*, *119*(21), 211601. https://doi.org/10.1103/PhysRevLett.119.211601

Bond, A. D., & Litim, D. F. (2018). More asymptotic safety guaranteed. *Physical Review D*, *97*(8), 085008. https://doi.org/10.1103/PhysRevD.97.085008

Bond, A. D., Litim, D. F., Medina Vazquez, G., & Steudtner, T. (2018). UV conformal window for asymptotic safety. *Physical Review D*, *97*(3), 036019. https://doi.org/10.1103/PhysRevD.97.036019

Buyukbese, T., & Litim, D. (2017). Asymptotic safety of gauge theories beyond marginal interactions. In *Proceedings of 34th annual International Symposium on Lattice Field Theory — PoS(LATTICE2016)* (p. 233). Trieste, Italy: Sissa Medialab. https://doi.org/10.22323/1.256.0233

Codello, A., Langæble, K., Litim, D. F., & Sannino, F. (2016). Conformal gauge-Yukawa theories away from four dimensions. *Journal of High Energy Physics*, *2016*(7), 118. https://doi.org/10.1007/JHEP07(2016)118

Del Debbio, L. (2011). The conformal window on the lattice. Retrieved from http://arxiv.org/abs/1102.4066

Litim, D. F., Mojaza, M., & Sannino, F. (2016). Vacuum stability of asymptotically safe gauge-Yukawa theories. *Journal of High Energy Physics*, *2016*(1), 81. https://doi.org/10.1007/JHEP01(2016)081

Litim, D. F., & Sannino, F. (2014). Asymptotic safety guaranteed. *Journal of High Energy Physics*, *2014*(12), 1–33. http://doi.org/10.1007/JHEP12(2014)178

Weinberg, S. (1979). Ultraviolet divergences in quantum theories of gravitation, . In General Relativity: An Einstein centenary survey, ed. S. W. Hawking and W. Israel, 790- 831.

Wilson, K. G., & Fisher, M. E. (1972). Critical exponents in 3.99 dimensions. *Physical Review Letters*, *28*(4), 240–243. http://doi.org/10.1103/PhysRevLett.28.240