Are Neutrino Masses Modular Forms?

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Indirect Searches for New Physics:
Fifth Forces, Scalars and Massive Neutrinos

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Based on:
- F.F. 1706.08749

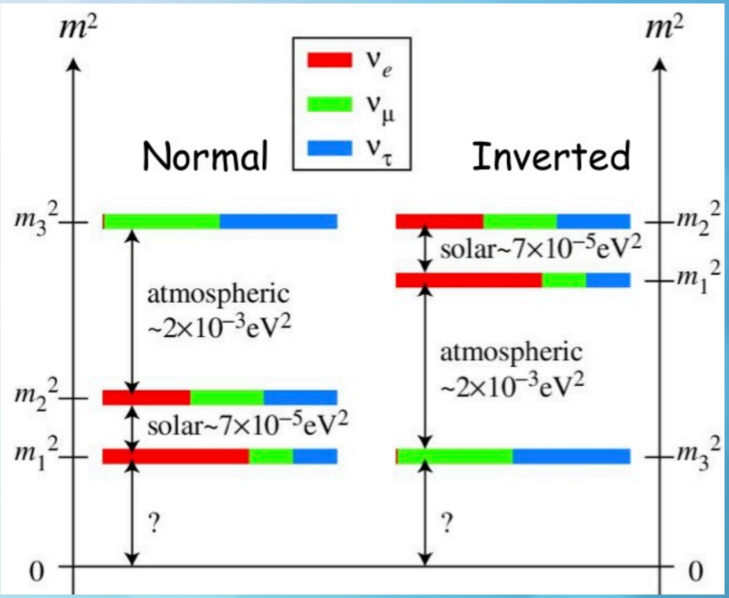
relevant parameters

$$m_1 < m_2 \ [\Delta m_{ij}^2 \equiv m_i^2 - m_j^2]$$

$$\Delta m_{21}^2 < \left|\Delta m_{32}^2\right|, \left|\Delta m_{31}^2\right|$$

i.e. 1 and 2 are, by definition, the closest levels

two possibilities: NO and IO



Mixing matrix U_{PMNS} (Pontecorvo, Maki, Nakagawa, Sakata)

$$L_{CC} = -\frac{g}{\sqrt{2}} W_\mu^- \, \overline{e}_L \gamma^\mu U_{PMNS} \nu_L$$

$0 \le \vartheta_{ij} \le \pi/2$

$0 \le \delta < 2\pi$

Majorana phases

standard parametrization

$$U_{PMNS} = \begin{array}{c|ccc} & \nu_1 & \nu_2 & \nu_3 \\ \hline \nu_e & c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ \nu_\mu & -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ \nu_\tau & -c_{12}s_{13}c_{23}e^{-i\delta} + s_{12}s_{23} & -s_{12}s_{13}c_{23}e^{-i\delta} - c_{12}s_{23} & c_{13}c_{23} \end{array} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Summary of data | Unknowns

$m_\nu < 2.2\ eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

absolute neutrino mass scale is unknown [but well-constrained!]

2% $$\Delta m^2_{atm} \equiv \begin{cases} \Delta m^2_{31} = (2.525^{+0.042}_{-0.030}) \times 10^{-3}\ eV^2 & \text{NO} \\ \Delta m^2_{32} = -(2.505^{+0.034}_{-0.032}) \times 10^{-3}\ eV^2 & \text{IO} \end{cases}$$

sign $[\Delta m^2_{atm}]$ unknown

[complete ordering (either normal or inverted hierarchy) not known]

2% $$\Delta m^2_{sol} \equiv \Delta m^2_{21} = (7.37^{+0.17}_{-0.16}) \times 10^{-5}\ eV^2$$

3% $$\sin^2 \vartheta_{13} = 0.0215 \pm 0.0007$$

$$\delta_{CP} / \pi = 1.38^{+0.23}_{-0.20}$$

[CP violation in lepton sector not yet established]

$$\sin^2 \vartheta_{23} = \begin{cases} 0.425^{+0.021}_{-0.015} & \text{NO} \\ [0.433^{+0.015}_{-0.016}] \oplus [0.589^{+0.016}_{-0.022}] & \text{IO} \end{cases}$$

6% $$\sin^2 \vartheta_{12} = 0.297^{+0.017}_{-0.016}$$

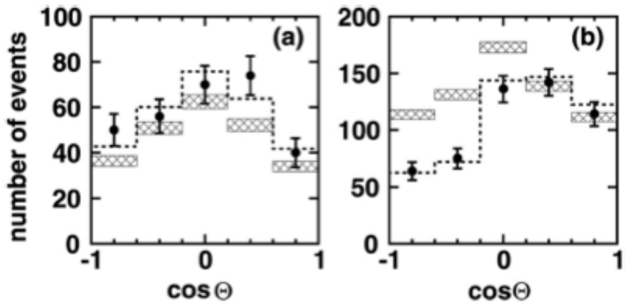
[Capozzi et al. 1703.04471]

violation of individual lepton number implied by neutrino oscillations

violation of total lepton number not yet established

a short history of neutrino oscillations

1998 atmospheric ν data from Superkamiokande shown at Neutrino '98



-- zenith angular distributions of atmospheric ν

-- oscillation solution becomes compelling

-- determination of $(\Delta m^2_{atm}, \sin^2 2\vartheta_{23})$ ≈1 -> maximal mixing

ϑ_{13} compatible with zero at that time!

$$
(|U_{l\nu}|^2) \;=\; \begin{array}{c c}
 & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\
\begin{array}{c} e \\ \mu \\ \tau \end{array} & \left(\begin{array}{ccc} ? & ? & 0 \\ ? & ? & 1/2 \\ ? & ? & 1/2 \end{array} \right)
\end{array}
$$

easily explained in terms of the μ−τ parity symmetry in the flavour basis, require m_ν invariant under U

[Grimus, Lavoura 0110041, 0305046]

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$U^2=1$

$$m_\nu = \begin{pmatrix} x & y & y \\ y & w & z \\ y & z & w \end{pmatrix} \Longleftrightarrow \begin{aligned} \vartheta_{13} &= 0 \\ \vartheta_{23} &= \frac{\pi}{4} \end{aligned}$$

ϑ_{12} undetermined

2002: solution of solar neutrino problem (started in 69! by the R. Davis) by the joint effort of SK, SNO and KamLAND,

MSW LA ⇒ $\sin^2\vartheta_{12} = 0.32^{+0.05}_{-0.06}$ [Bahcall, Gonzalez-Garcia, Pena-Garay 0212147]

TRIBIMAXIMAL MIXING

$$\begin{array}{ccc} & & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ (|U_{l\nu}|^2) = & \begin{array}{c} e \\ \mu \\ \tau \end{array} & \left(\begin{array}{ccc} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{array}\right) \end{array}$$

[Harrison, Perkins, Scott 0202074]

ϑ_{13} compatible with zero at that time!

so "symmetric" and soon derived from a discrete symmetry: A_4

Ma, Rajasekaran 0106291, Babu, Ma, Valle 0206292; Hirsch, Romao, Skadauge, Valle, Villanova del Moral 0312244, Ma 0404199, 0409075]

- TBM is obtained when $x + y = w + z$ now m_ν invariant also under S

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$U^2 = S^2 = 1 \quad [S,U] = 0$$

$Z_2 \times Z_2$ the most general symmetry of m_ν if neutrinos are Majorana

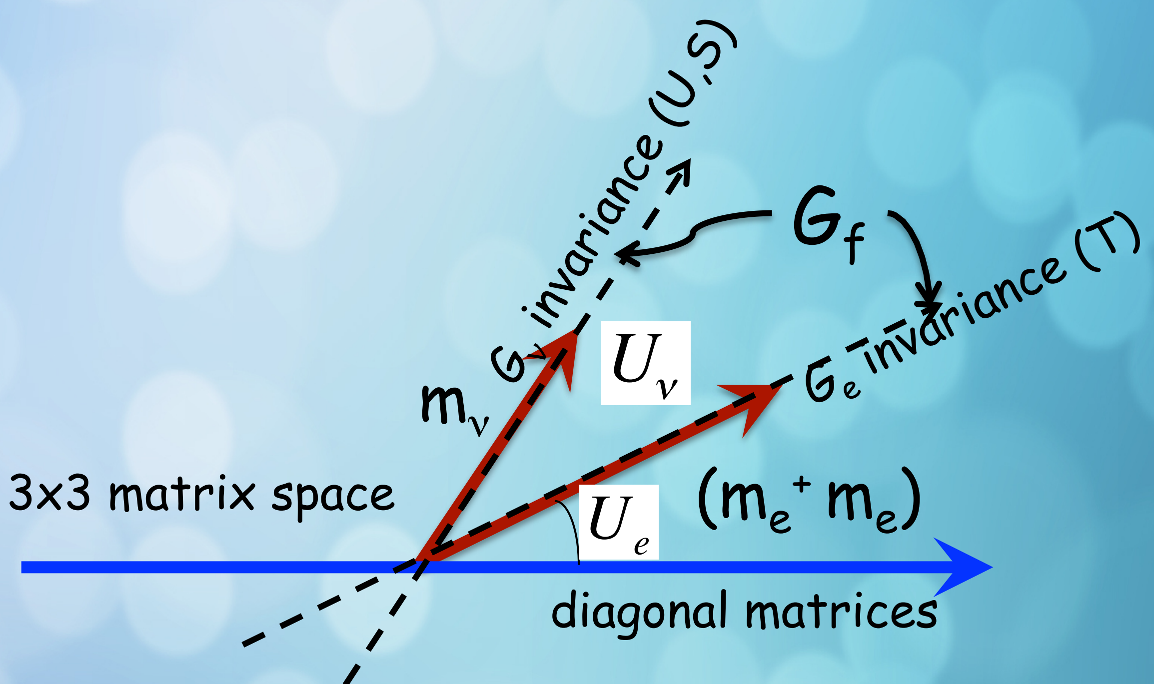
the flavour basis can be guaranteed if $(m_e^+ m_e)$ is invariant under

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{i\frac{2\pi}{3}}$$

[Lam 0708.3665 + 0804.2622]

(S,T) generate A_4 (U can arise as an accidental symmetry)
(S,T,U) generate S_4

geometrical picture of lepton mixing



2011/2012 breakthrough: $\vartheta_{13} \neq 0$

$\vartheta_{13} \approx 0.15$, not a small perturbation: how to cope with it?

-- add "large" corrections $O(\vartheta_{13}) \approx 0.15$ to TBM pattern

-- change discrete group G_f and try to fit lepton mixing

DISCRETE SYMMETRIES

complete classification of $|U_{PMNS}|$

[Fonseca, Grimus 1405.3678]

from any finite group available now!

-- change LO pattern

-- include CP in the SB pattern

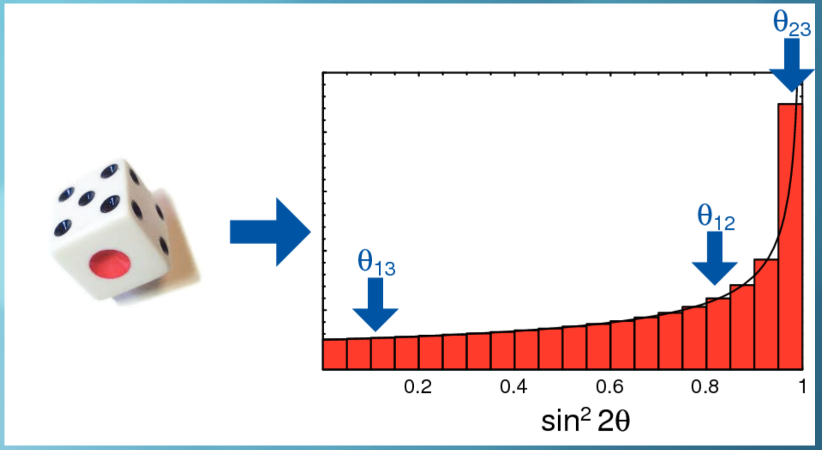
-- relax symmetry requirements

$$U^0_{PMNS} = U_{BM} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & -1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix} + \ldots$$

or no symmetry at all:
ANARCHY

$$|U| = \begin{pmatrix} 0.800 \to 0.844 & 0.515 \to 0.581 & 0.139 \to 0.155 \\ 0.229 \to 0.516 & 0.438 \to 0.699 & 0.614 \to 0.790 \\ 0.249 \to 0.528 & 0.462 \to 0.715 & 0.595 \to 0.776 \end{pmatrix}$$

[Hall, Murayama, Weiner 1999
De Gouvea, Murayama 1204.1249]



This proposal [1706.08749]

perhaps symmetry and anarchy are both needed
but anarchy operates only in the vacuum selection

- neutrino masses and mixings depend on a limited number of fields [ideally a single field τ]

$$m_i(\tau) \quad \vartheta_{ij}(\tau)$$

the functional form of $m_i(\tau)$ $\vartheta_{ij}(\tau)$ is (almost) completely determined by a symmetry

- the VEV $\langle\tau\rangle$ is selected by some unknown mechanism (anthropic, dynamical, statistical,...)

powerful symmetry
+
small number of moduli

$\Rightarrow$ predictive power

- Here: first attempt, adopting modular invariance as flavour symmetry

Modular Invariance as Flavour Symmetry

modular transformations

$$\tau \to \gamma\tau \equiv \frac{a\tau+b}{c\tau+d}$$

a,b,c,d integers
ad-bc=1

τ is a complex field,
Im(τ) > 0

they form the (discrete, infinite) modular group $\overline{\Gamma}$ generated by

$$S: \tau \to -\frac{1}{\tau} \quad , \qquad T: \tau \to \tau + 1$$

$$S^2 = \mathbb{1} \quad , \quad (ST)^3 = \mathbb{1}$$

duality

discrete shift symmetry

τ = i left invariant by S (self-dual point)

τ = (i ∞) left invariant by T

- τ stands for τ/Λ where the scale Λ has been set to 1
- τ promoted to an N=1 chiral superfield τ=τ(x,ϑ)

Action of modular invariance on flavor space

most general transformation on a set of N=1 SUSY chiral multiplets $\varphi^{(I)}$

$$\begin{cases} \tau \to \dfrac{a\tau+b}{c\tau+d} \\ \varphi^{(I)} \to (c\tau+d)^{-k_I}\rho^{(I)}(\gamma)\varphi^{(I)} \end{cases}$$

e.g. $\varphi^{(I)} = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$

the weight, a real number

unitary representation of the finite modular group

Γ_N are finite groups

Γ_2	Γ_3	Γ_4	Γ_5
S_3	A_4	S_4	A_5

$$\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$$

- if all k_I=0, the construction collapses to the well-known models based on linear, unitary flavor symmetries.
- N=1 SUSY modular invariant theories known since late 1980s

focus on Yukawa interactions and $\mathcal{N}$=1 global SUSY

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta}\ K(\Phi,\bar{\Phi}) + \int d^4x d^2\theta\ w(\Phi) + h.c.$$

$$\Phi = (\tau, \varphi)$$

Kahler potential, kinetic terms

superpotential, holomorphic function of Φ Yukawa interactions

S. Ferrara, D. Lust, A. D. Shapere and S. Theisen, Phys. Lett. B **225** (1989) 363.

S. Ferrara, .D. Lust and S. Theisen, Phys. Lett. B **233** (1989) 147.

S invariant if

$$\begin{cases} w(\Phi) \to w(\Phi) \\ K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + f(\Phi) + f(\bar{\Phi}) \end{cases}$$

- invariance of the Kahler potential easy to achieve. For example:

$$K(\Phi,\bar{\Phi}) = -h\log(-i\tau+i\bar{\tau}) + \sum_I (-i\tau+i\bar{\tau})^{-k_I} |\varphi^{(I)}|^2$$

minimal K

- invariance of the superpotential much less trivial. Expand w(Φ) in powers of the matter supermultiplets

$$w(\Phi) = \sum_n Y_{I_1...I_n}(\tau)\, \varphi^{(I_1)}...\varphi^{(I_n)}$$

field-dependent Yukawa couplings

invariance of w(Φ) guaranteed by an holomorphic Y such that

$$Y_{I_1...I_n}(\gamma\tau) = (c\tau+d)^{k_Y(n)} \rho(\gamma)\, Y_{I_1...I_n}(\tau)$$

modular forms of level N and weight k_Y

$$k_Y(n) = k_{I_1} + ... + k_{I_n}$$

The product $\rho \times \rho^{I_1} \times ... \times \rho^{I_n}$ contains an invariant singlet

- extension to $\mathcal{N}$=1 SUGRA straightforward: ask invariance of $G=K+\log|w|^2$

Few facts about (level-N) Modular Forms

- transformation property under the modular group

$$f_i(\gamma\tau) = (c\tau + d)^k \rho(\gamma)_{ij} f_j(\tau)$$

unitary representation of the finite modular group $\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$

- q-expansion

$$f(\tau + N) = f(\tau) \Rightarrow f(\tau) = \sum_{i=0}^{\infty} a_n q_N^n \qquad q_N = e^{\frac{i2\pi\tau}{N}}$$

- $k < 0 \Rightarrow f(\tau) = 0$

  $k = 0 \Rightarrow f(\tau) =$ constant

  $k > 0$ (even integer) $\Rightarrow f(\tau) \in \mathcal{M}_k(\Gamma(N))$ finite-dimensional linear space

- ring of modular forms generated by few elements

$$\mathcal{M}(\Gamma(N)) = \bigoplus_{k=0}^{\infty} \mathcal{M}_{2k}(\Gamma(N))$$

an explicit example in a moment

A minimal example based on Γ_3

Why Γ_3 ? Γ_3 is isomorphic to A_4, smallest group of the Γ_N series possessing a 3-dimensional irreducible representation

focus on the neutrino sector, assuming neutrino masses originate from the operator

$$w_\nu = \frac{1}{\Lambda}(H_u H_u \; LL \; Y)_1$$

weights and representations under Γ_3 of matter multiplets and $Y(\tau)$

	L	H_u	Y
$SU(2)\times U(1)$	$(2,-1/2)$	$(2,+1/2)$	$(1,0)$
$\Gamma_3 \equiv A_4$	3	1	ρ
k_I	k_L	k_u	k_Y

which are the modular forms $Y(\tau)$ of level N=3 we can use in this construction?

dimension of linear space $\mathcal{M}_k(\Gamma(3))$ is (k+1) , k > 0 even integer

3 linearly independent modular forms of level 3 and minimal weight $k_I = 2$

Modular forms of level 3

- 3 linearly independent modular forms of level 3 and minimal weight $k_I = 2$

$$\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}-\frac{27\eta'(3\tau)}{\eta(3\tau)}\right]\\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega^2\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right]\\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega^2\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] .
\end{aligned}$$

can be expressed in terms of the Dedekind eta function

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n) \qquad q \equiv e^{i2\pi\tau}$$

- they transform in a triplet 3 of Γ_3

$$Y(-1/\tau) = \tau^2\,\rho(S)Y(\tau) \qquad Y(\tau+1) = \rho(T)Y(\tau)$$

$$\rho(S) = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad \rho(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \qquad \omega = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

- they satisfy an algebraic constraint

$$Y_2^2 + 2Y_1Y_3 = 0$$

- they generate the whole ring $\mathcal{M}(\Gamma(3))$

any modular form of level 3 and weight 2k can be written as an homogeneous polynomial in Y_i of degree k

	L	H_u	Y
$SU(2)\times U(1)$	$(2,-1/2)$	$(2,+1/2)$	$(1,0)$
$\Gamma_3 \equiv A_4$	3	1	3
k_I	+1	0	+2

if we go minimal

the operator

$$w_\nu = \frac{1}{\Lambda}(H_u H_u\ LL\ Y)_1$$

is completely specified up to an overall constant

we get

$$m_\nu = \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix} \frac{v_u^2}{\Lambda}$$

a familiar matrix but now Y_i are determined by the choice of τ

by scanning τ VEVs the best agreement is obtained for $\tau = 0.0111 + 0.9946i$

	$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$	$\sin^2\vartheta_{12}$	$\sin^2\vartheta_{13}$	$\sin^2\vartheta_{23}$	$\frac{\delta_{CP}}{\pi}$	$\frac{\alpha_{21}}{\pi}$	$\frac{\alpha_{31}}{\pi}$
Exp	0.0292	0.297	0.0215	0.5	1.4	–	–
1σ	0.0008	0.017	0.0007	0.1	0.2	–	–
prediction	0.0292	0.295	0.0447	0.651	1.55	0.22	1.80

many σ away

2-parameter fit to 5 physical quantities

Comments

- results hold under the assumption that, in the charged lepton sector, we have

$$m_l = diag(m_e, m_\tau, m_\mu)$$

i.e. no contribution to the lepton mixing apart from a flip in the 2-3 sector; easy to achieve, an ad-hoc flavon φ is needed

- absolute masses are also predicted, since Λ can be determined by fitting Δm^2_{sol} and Δm^2_{atm} separately

$$m_1 = 4.998 \times 10^{-2}\ eV \qquad m_2 = 5.071 \times 10^{-2}\ eV \qquad m_3 = 7.338 \times 10^{-4}\ eV$$

→ minimal model predicts Inverted mass Ordering

- best value of $\tau = 0.0111 + 0.9946i$ is close to the self-dual point $\tau = i$

at $\tau=i$ the neutrino mass matrix is CP conserving: non-trivial phases are entirely generated by $\tau \neq i$

- couplings of τ to matter multiplets are completely fixed by SUSY and modular invariance to any order in the τ power expansion

corrections (work in progress)

choice of the vacuum (τ,φ)

small contribution from the charged lepton sector <-> φ
to get a better agreement? -> under study

Kahler potential

no contribution to mass/mixing parameters if K minimal
non-minimal K?

SUSY breaking

spurion analysis

$$\langle S \rangle = \vartheta^2 F + \ldots$$

10^{-10} $100\ TeV$

$$\frac{\delta m}{m} \approx \frac{F}{\Lambda^2} \quad \leftrightarrow \quad m_{SUSY} \approx \frac{F}{\Lambda}$$

$\sqrt{F} = 10^{10}\ GeV$ $\Lambda = 10^{15}\ GeV$

1-loop corrections in broken phase

$$\delta y \approx \frac{\alpha}{4\pi} \frac{m_{\tilde{g}} A}{m_{SUSY}^2}$$

can be kept small

Conclusions

1. accuracy

we are entering a precision era in neutrino physics which calls for accurate predictions to match the small experimental errors

2. predictability

predictability in terms of a small set of parameters can be realized by models enjoying strong symmetry properties

3. anarchy

Anarchy can still play a role in these models, but only at the level of vacuum selection [otherwise 1. and 2. are spoiled]

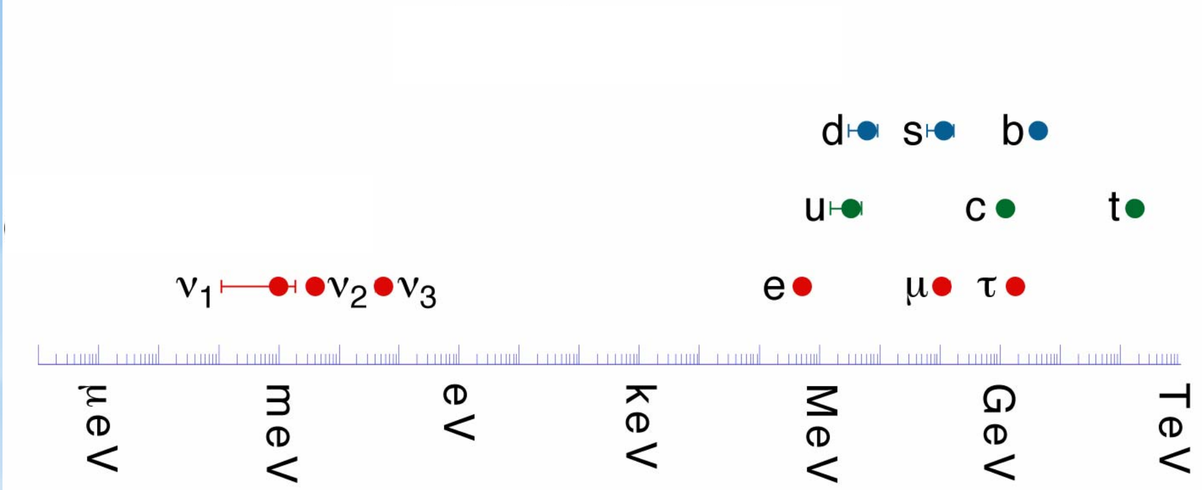
4. Modular invariance?

modular invariant SUSY models seem to naturally incorporate these features and might provide a new framework, still largely unexplored

Backup Slides

General questions

- how to extend the SM in order to accommodate neutrino masses ?
- why neutrino masses are so small, compared with the charged fermion masses ?



- why lepton mixing angles are so different from those of the quark sector ?

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + corrections$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$$\lambda \approx 0.22$$

- can we predict mass/mixings (and match the present experimental accuracy) ?

Discrete Symmetries		Anarchy
DS badly broken in the quark sector	extension to quark sector	Easy: abelian symmetries a la Froggatt-Nielsen; Extra Dimensions,...
quite contrived	inclusion in GUTs	SU(5), SO(10),...
very mild	relevance for fermion masses	coherent picture of masses and mixing angles
the framework can be tested	testable predictions	only order-of-magnitude predictions: the framework cannot be tested

features of models based on DS

large number of flavons needed	$G_f \xrightarrow{\varphi_e} G_e \qquad G_f \xrightarrow{\varphi_\nu} G_\nu$
vacuum alignment	$V(\varphi_e, \varphi_\nu, \ldots) \rightarrow \langle\varphi_e\rangle, \langle\varphi_\nu\rangle, \ldots$
corrections from higher dimensional operators	$\frac{\varphi^n}{\Lambda^n} \bar{e}_R H l_L \qquad \frac{\varphi^n}{\Lambda^{n+1}} H^+ l_L H^+ l_L$

Variants

neutrino masses from see-saw mechanism

$$w_\nu = g\,(N^c H_u L)_1 + \Lambda (N^c N^c Y)_1$$

assignement

	L	N^c	H_u	Y
$SU(2)\times U(1)$	$(2,-1/2)$	$(1,0)$	$(2,+1/2)$	$(1,0)$
$\Gamma_3 \equiv A_4$	3	3	1	3
k_I	k_L	+1	k_u	+2

$1+k_L+k_u=0$

we get the best agreement at $\tau = -0.195 + 1.0636i$

	$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$	$\sin^2\vartheta_{12}$	$\sin^2\vartheta_{13}$	$\sin^2\vartheta_{23}$	$\frac{\delta_{CP}}{\pi}$	$\frac{\alpha_{21}}{\pi}$	$\frac{\alpha_{31}}{\pi}$
Exp	0.0292	0.297	0.0215	0.5	1.4	–	–
1σ	0.0008	0.017	0.0007	0.1	0.2	–	–
prediction	0.0280	0.291	0.0486	0.331	1.47	1.83	1.26

Normal mass ordering is predicted

$$m_1 = 1.096\times 10^{-2}\ eV \qquad m_2 = 1.387\times 10^{-2}\ eV \qquad m_3 = 5.231\times 10^{-2}\ eV$$