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Modelling of locomotion robotic mechanisms using geometric mechanics methods

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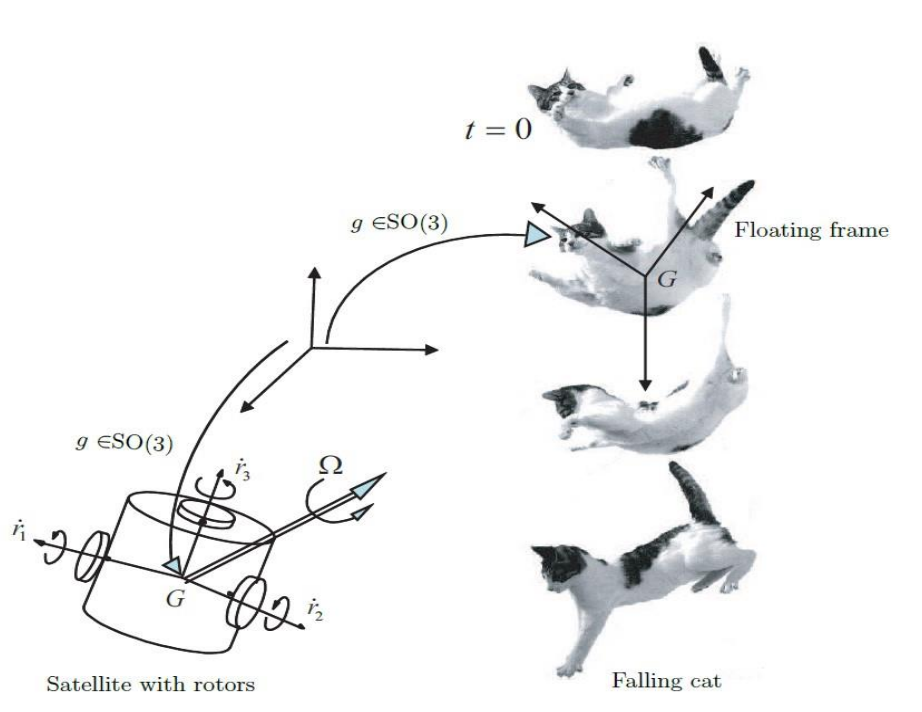
E: 21° 15' 40,5324" (21,92554)

WHAT IS GEOMETRIC MECHANICS ?

- **Geometric mechanics is a branch of mathematics applying particular geometric methods to many areas of mechanics, from mechanics of particles and rigid bodies to fluid mechanics to control theory**.

- Geometric mechanics applies principally to systems for which the configuration space is a Lie group, or a group of diffeomorphisms, or more generally where some aspect of the configuration space has this group structure.
- For example, the configuration space of a rigid body such as a satellite is the group of Euclidean motions (translations and rotations in space).



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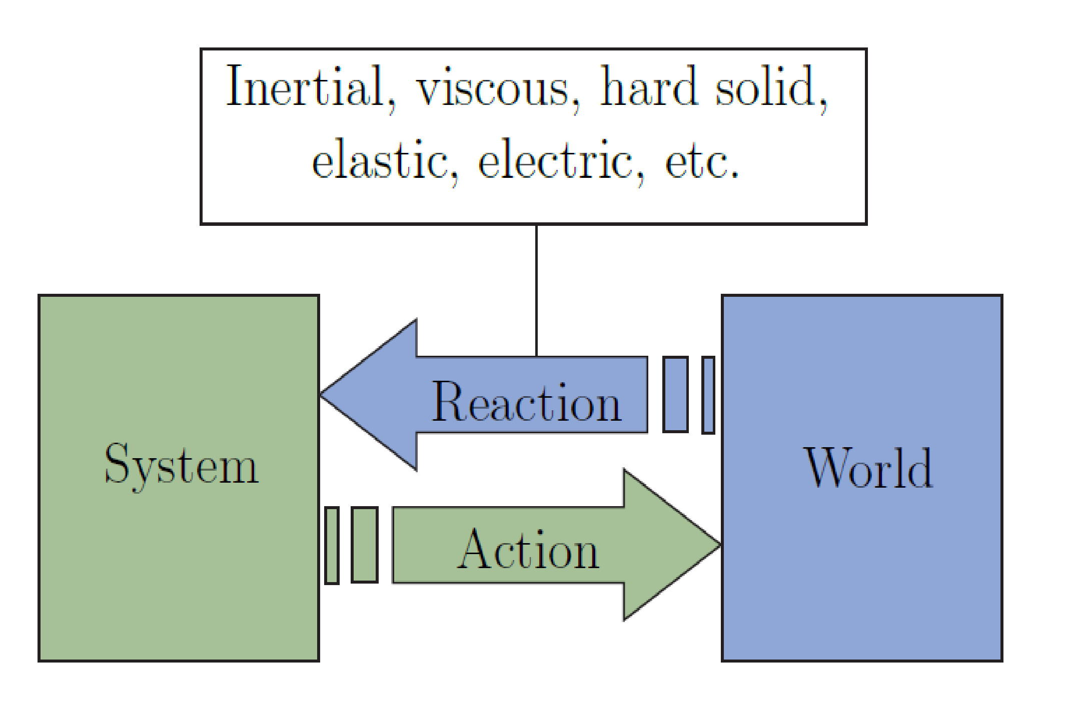
WHAT IS LOCOMOTION ?

- Animal locomotion is the study of how animals move in the world
- Locomotion is the ability to move from place to place in 3D space

For a system, either natural or artificial, the locomotion can be defined more precisely as follows:

- **”The process of producing net (overall) displacement (motion) of a system through internal shape changes (deformations) and interaction with the external world.”**



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PRINCIPLES OF BIOMIMETIC LOCOMOTION

NON-HOLONOMIC CONSTRAINTS

FLUID MECHANICAL CONSTRAINTS

VISCOUS CONSTRAINTS

PIECEWISE HOLONOMIC CONSTRAINTS

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(CONVENTIONAL) LAGRANGIAN MECHANICS

$L(q, \dot{q})$ = System Lagrangian (kinetic – potential energy)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^i}\right) - \frac{\partial L}{\partial q^i} + \lambda_j \omega_i^j = \tau_i$$

$$\omega_j^i(q)\dot{q}^j = 0 \qquad i = 1, \ldots, k$$

velocity constraints

In analytical mechanics, these constraints was input into motion equations using Lagrange multipliers.

- This system contains **n** differential equations of the 2nd order and **k** first order differential equations, **n** is the dimension of the configuration space (number of degrees of freedom) and **k** is the number of non-holonomic conditions.
- Often, physical intuition is lost and that mean we do not see suitable pattern the structure of differential equations inside.

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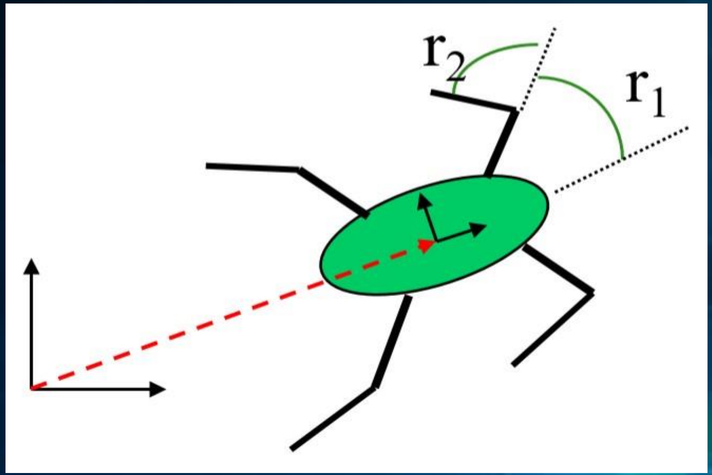
BASIC GEOMETRIC PRINCIPLES

It is the interaction between shape change and constraints that generates motion



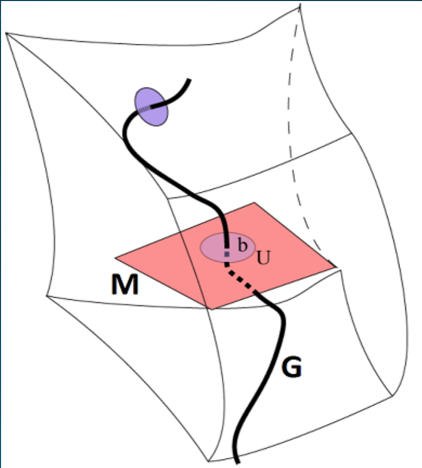
position, shape

$$q = (x, y, \theta, r_1, r_2, \cdots, r_m) = (g, r)$$

$$Q = G \times M$$

M ← Shape Space

G ← Lie Group



The Configuration Space of ***all*** biomimetic locomotors is a (trivial) principal fiber bundle.

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BASIC GEOMETRIC PRINCIPLES

- (Due to trivial fiber bundle structure) Configuration space, can be represented as:

$$Q = G \times M$$

- Constraints (if any) are assumed to be non-holonomic and can be written in Pfaffian form:

$$\omega(q) \cdot \dot{q} = 0,$$

- (Due to triviality of the fiber bundle structure) Constraints can be written as

$$\omega_g(q) \cdot \dot{g} + \omega_r(q) \cdot \dot{r} = 0,$$

- "Triviality" lets us split $q \in Q = (g, r)$ and $\dot{q} \in T_qQ = (\dot{g}, \dot{r})$

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HOW DOES THE FIBER BUNDLE RELATE TO THE BASE SPACE?

- Reconstruction Equation: $$\xi = T_g L_{g^{-1}} \dot{g} = -A(r)\dot{r} + I^{-1}(r) p^{\mathrm{T}}$$
- Connections: $$\mathcal{A}(q) \; : \; T_q Q \rightarrow \mathfrak{g}$$
- Equivalence under left actions: $$\mathcal{A}(\Phi_g q) T_q \Phi_g \dot{q} = \mathrm{Ad}_g \mathcal{A}(g) \dot{q}$$
- Relation between inertia, momentum and connection: $$\mathcal{A}(q) = \mathbb{I}^{-1}(q) J$$

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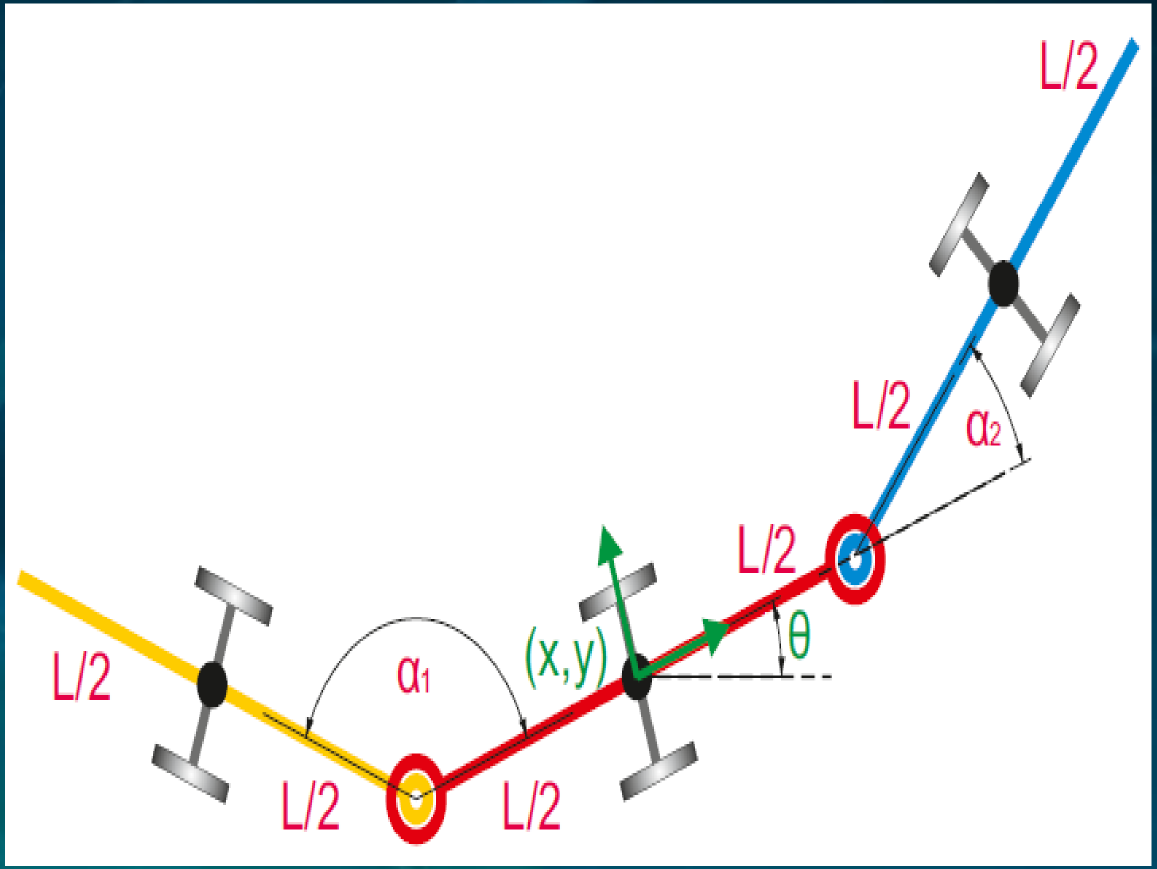
MODEL OF NONHOLONOMIC 3 - LINK SNAKE ROBOT

- The three-link kinematic snake has three degrees of freedom given by variables *(x,y,θ)* and two shape variables *(α_1,α_2)*

- Under-actuated nonholonomic mechanical systems first order

- Principally kinematic systems



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- **Local velocities ξ_{g_1}, ξ_{g_2} and ξ_{g_3}**

$$\xi_{g_1} = \overbrace{\begin{bmatrix} \cos\alpha_1 & -\sin\alpha_1 & L/2\sin\alpha_1 & 0 & 0 \\ \sin\alpha_1 & \cos\alpha_1 & -L/2(1+\cos\alpha_1) & L/2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}}^{J^b_{g_1}} \begin{bmatrix} \xi^x \\ \xi^y \\ \xi^\theta \\ \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix}$$

$$\xi_{g_2} = \overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}}^{J^b_{g_2}} \begin{bmatrix} \xi^x \\ \xi^y \\ \xi^\theta \\ \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix}$$

$$\xi_{g_3} = \overbrace{\begin{bmatrix} \cos\alpha_2 & \sin\alpha_2 & L/2\sin\alpha_2 & 0 & 0 \\ -\sin\alpha_2 & \cos\alpha_2 & L/2(1+\cos\alpha_2) & 0 & L/2 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}}^{J^b_{g_3}} \begin{bmatrix} \xi^x \\ \xi^y \\ \xi^\theta \\ \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix}$$

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- **y– variables of Jacobians $J_{g_i}^{y} = 0$ we will show in Pfaffian form :**

$$\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix} -\sin\alpha_1 & \cos\alpha_1 & \frac{L}{2}(1+\cos\alpha_1) & \frac{L}{2} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \sin\alpha_2 & \cos\alpha_2 & -\frac{L}{2}(1+\cos\alpha_2) & 0 & \frac{L}{2} \end{bmatrix} \begin{bmatrix}\xi^x\\ \xi^y\\ \xi^\theta\\ \dot\alpha_1\\ \dot\alpha_2\end{bmatrix}$$

- **Reconstruction equation of movement :**

$$\xi = \underbrace{\frac{1}{D}\begin{bmatrix} \frac{L}{2}\left(\cos\alpha_2+1\right) & \frac{L}{2}\left(\cos\alpha_1+1\right) \\ 0 & 0 \\ \sin\alpha_2 & \sin\alpha_1 \end{bmatrix}}_{\mathbb{A}} \dot r$$

$$D = \sin\alpha_1 - \sin\alpha_2 + \sin(\alpha_1-\alpha_2)$$

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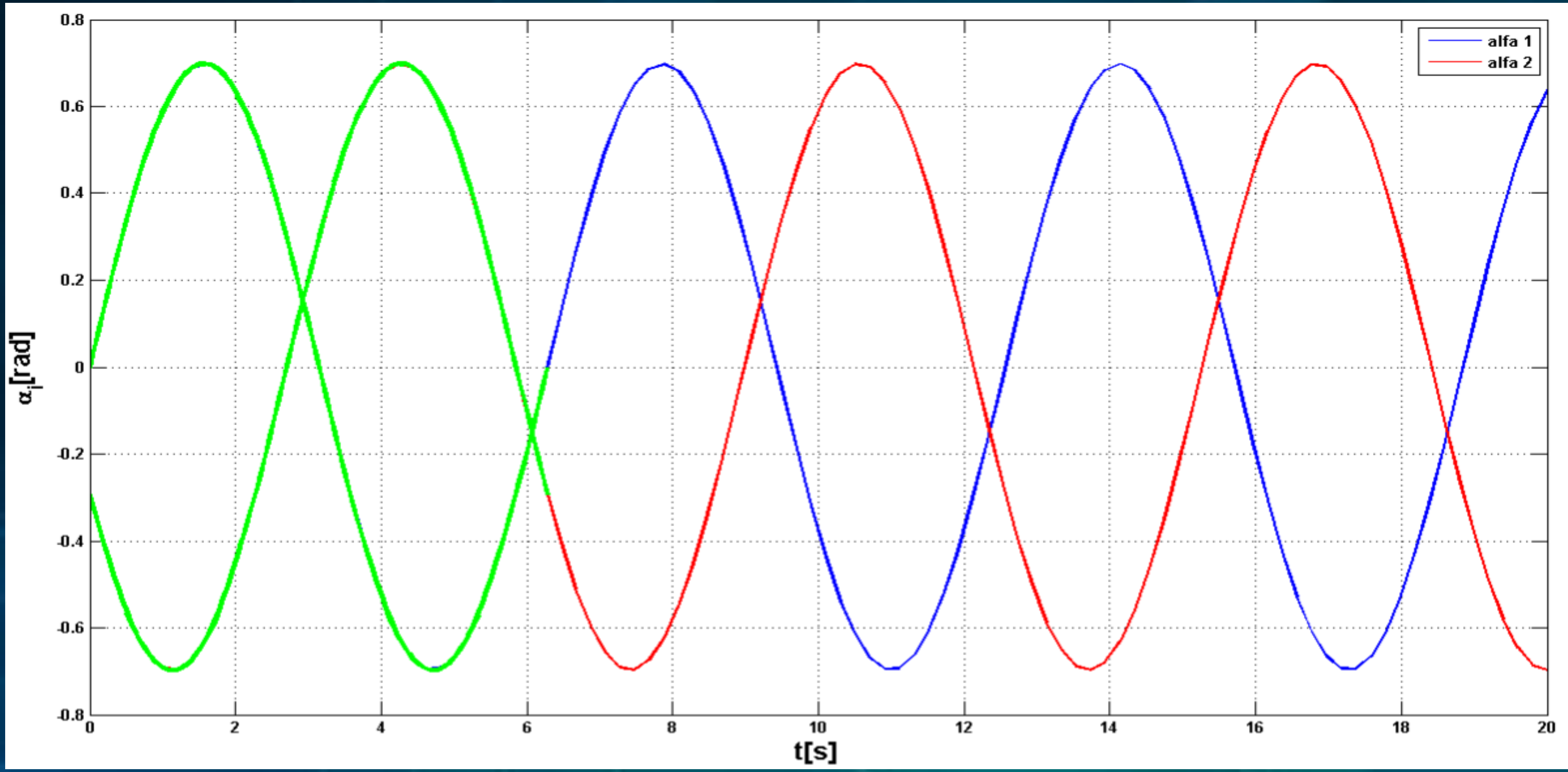
- **Lateral movement of principally kinematic systems**

$$\alpha_1 = 40 \sin t\,, \ \ \alpha_2 = -40 \sin(t + 25)$$



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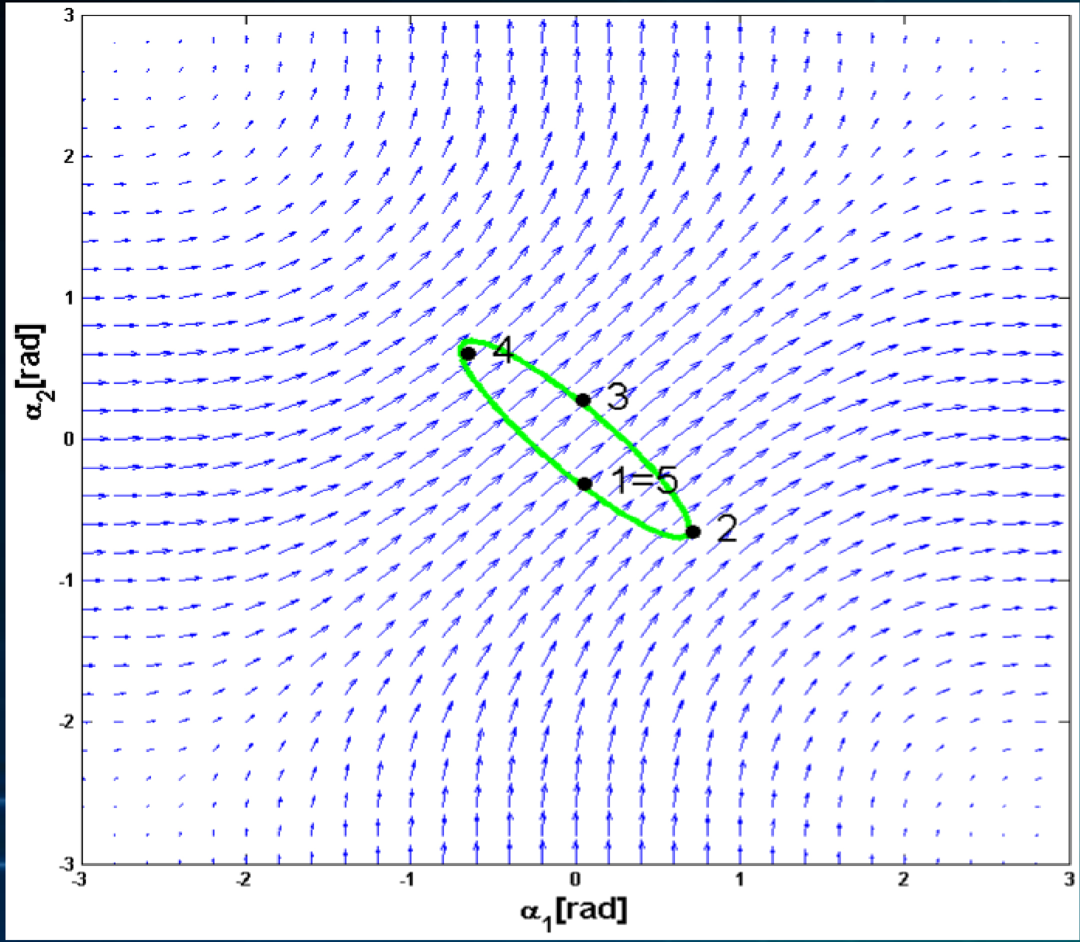
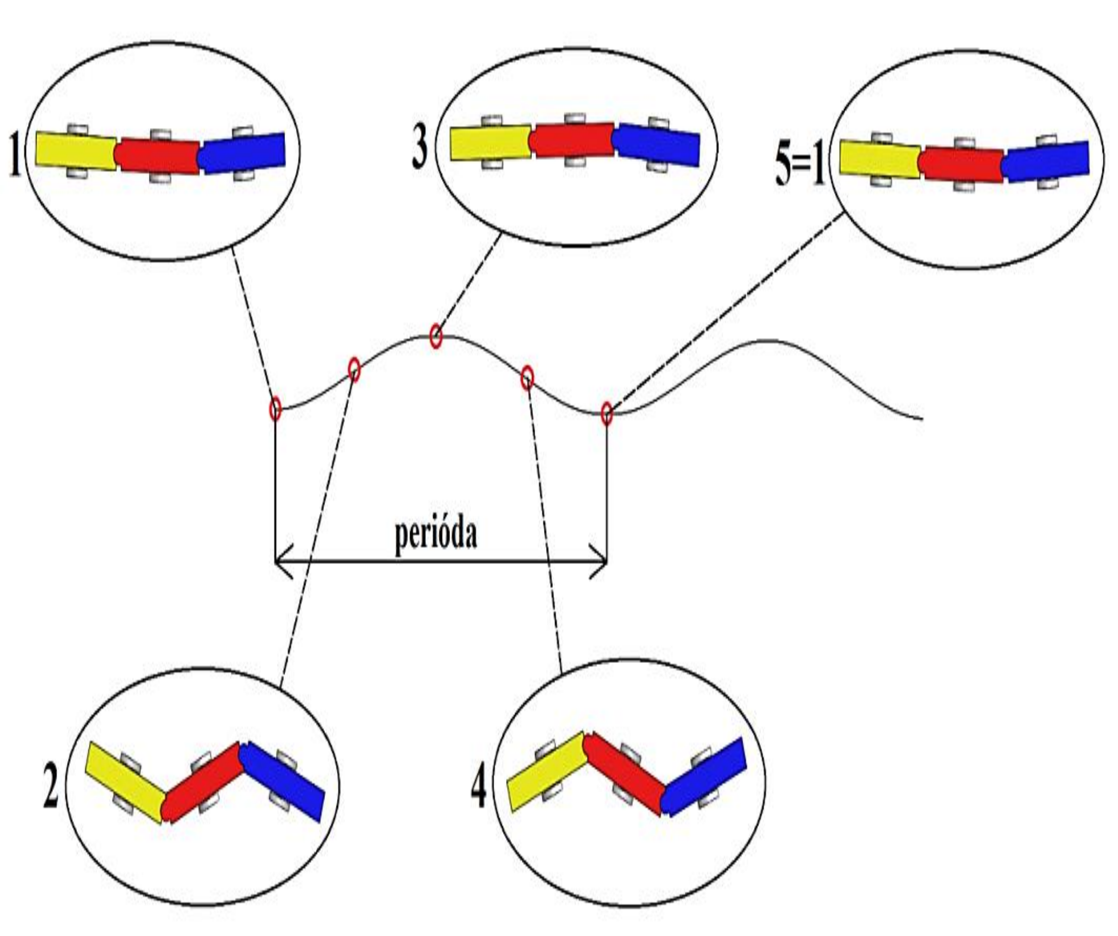
- **Vector field of local conexion $\vec{A}^{\xi_x}$:**





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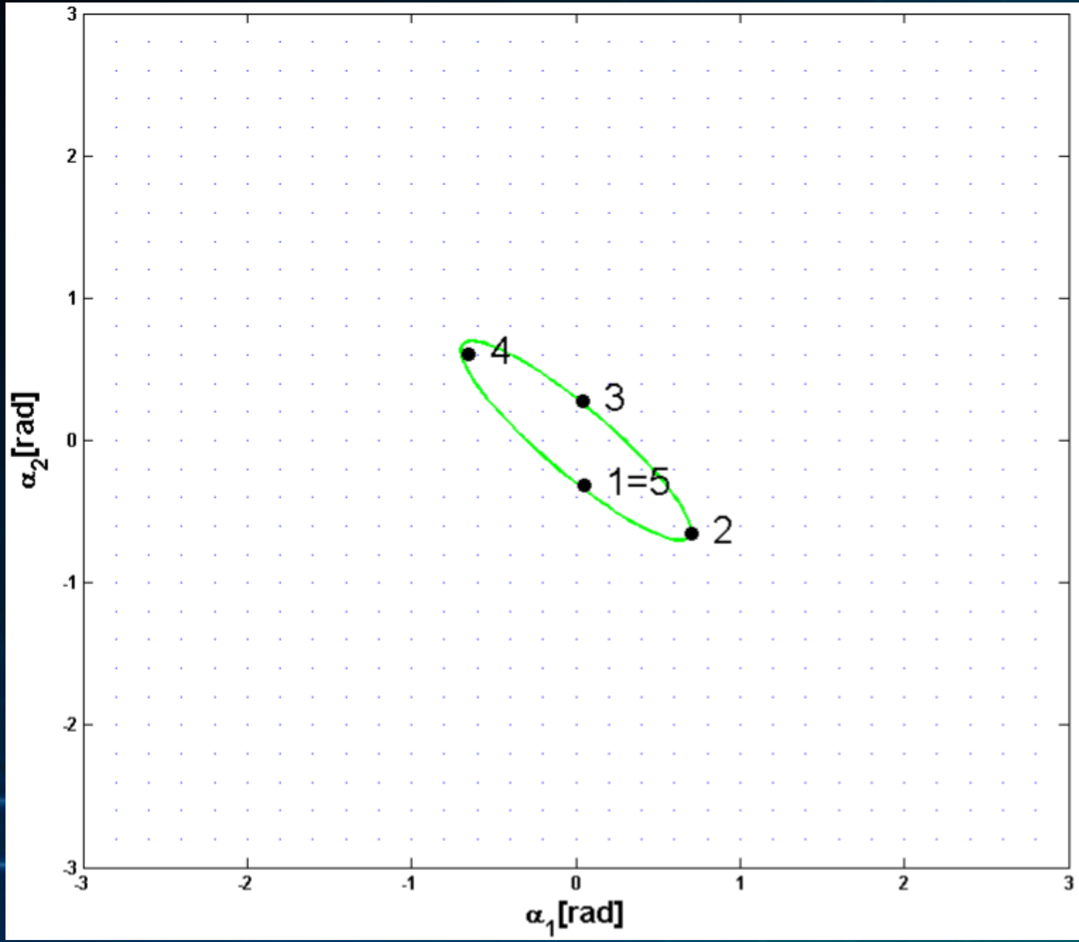
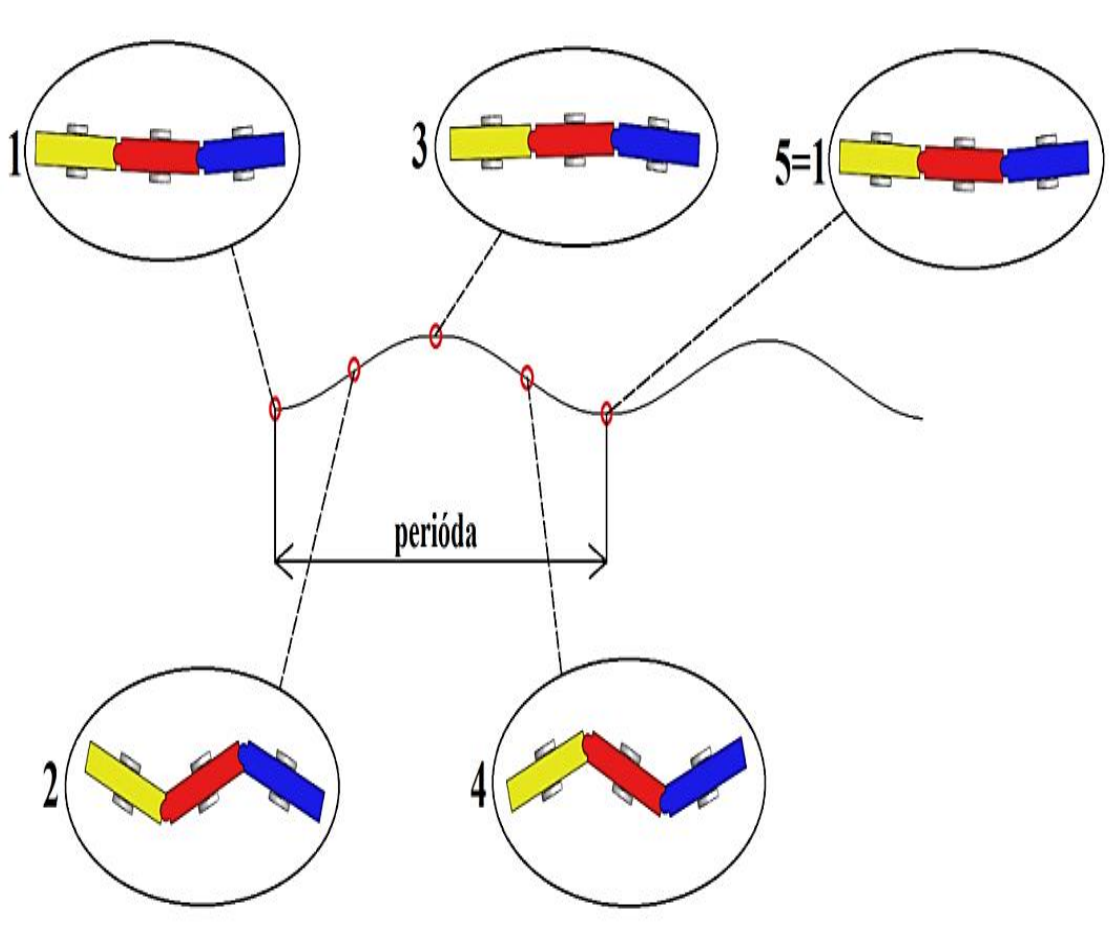
- **Vector field of local conexion $\vec{A}^{\xi y}$:**





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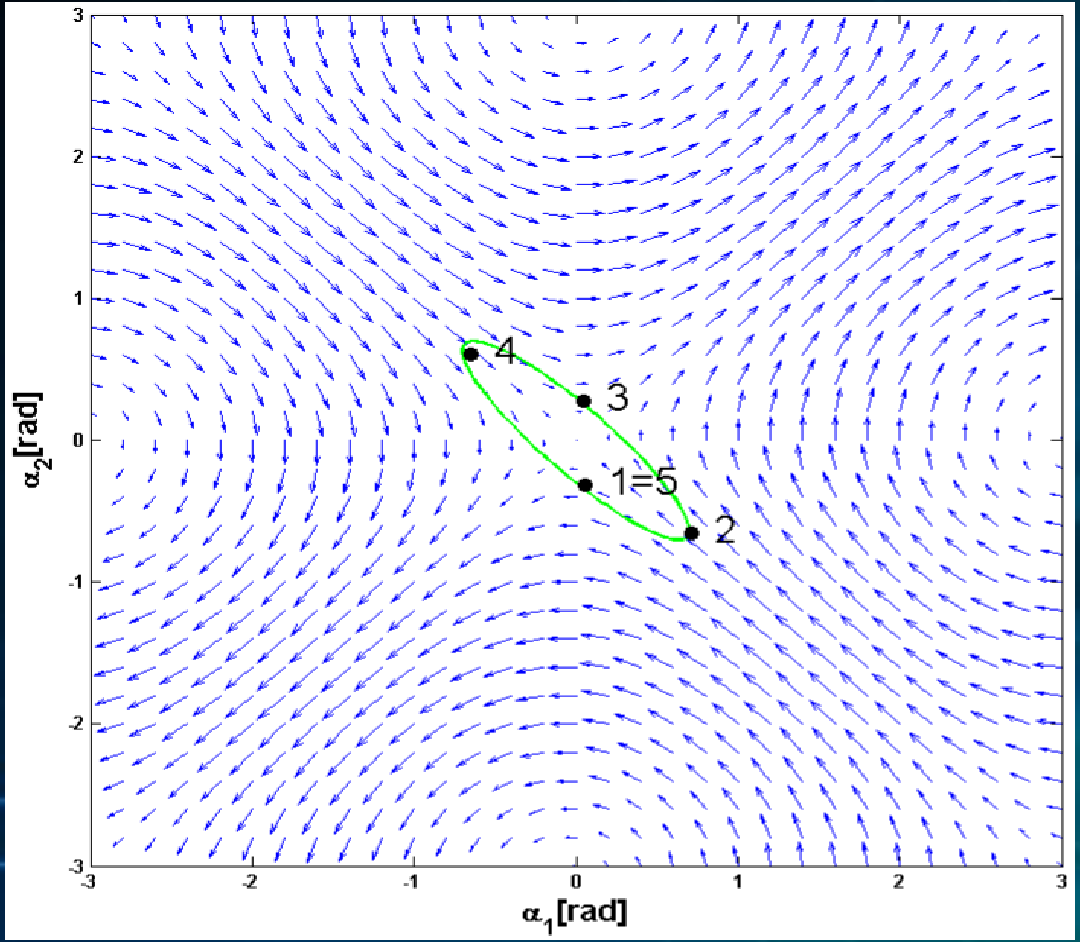
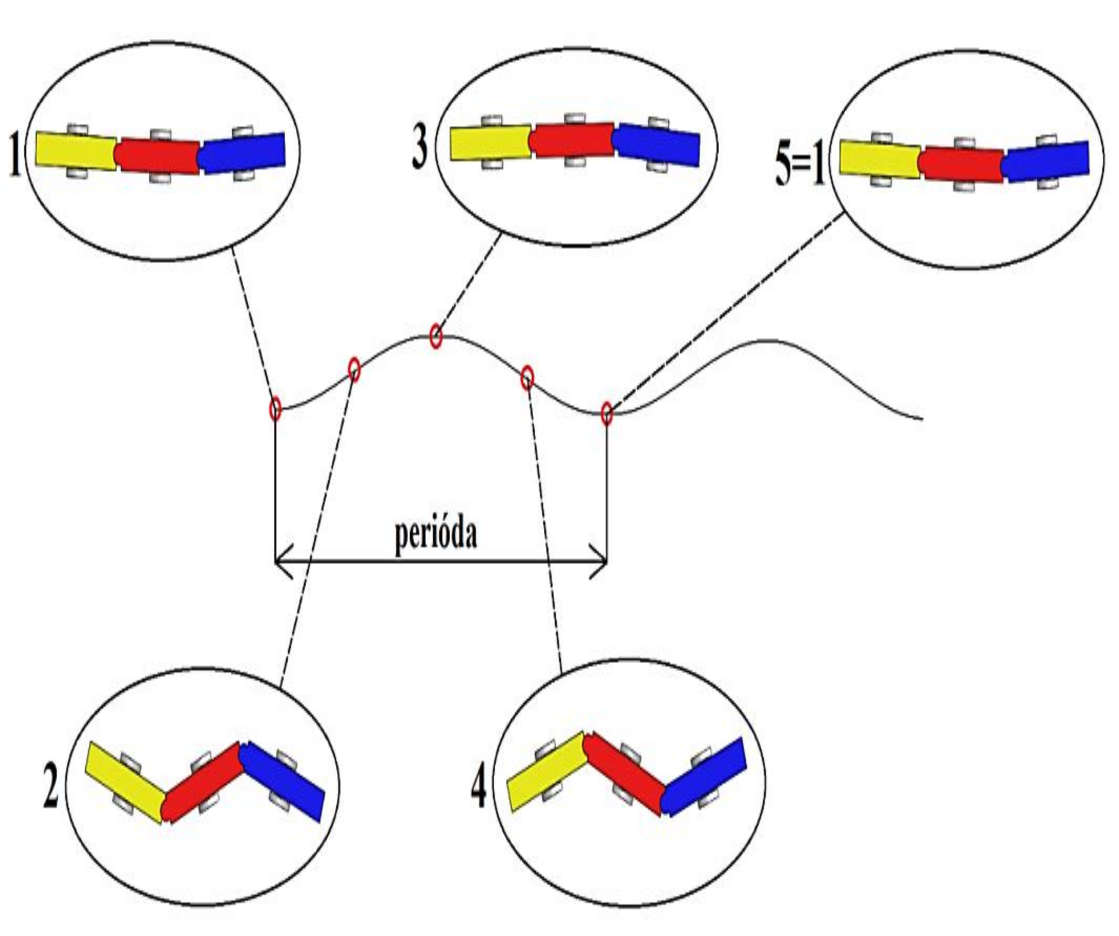
- **Vector field of local conexion $\vec{A}^{\xi\theta}$:**





Thank you for attention !

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