$B_s \to \mu^+\mu^-$ Theory Status

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B-meson or Kaon decays occur at low energies, at scales $\mu \ll M_W$.

We pass from the full theory of electroweak interactions to an effective theory by removing the high-energy degrees of freedom, i.e. integrating out the W-boson and all the other particles with $m \sim M_W$.

$$\mathcal{L}_{\text{(full EW}\times\text{QCD)}} \longrightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}}\left(\begin{array}{c}\text{quarks} \neq t\\ \text{\& leptons}\end{array}\right) + N \sum_n C_n(\mu)\, Q_n$$

Q_n – local interaction terms (operators), $\quad$ C_n – coupling constants (Wilson coefficients).

Operators (dim 6) that matter for $B_s \to \mu^+\mu^-$ read:

$$Q_A = (\bar{b}\gamma^\alpha\gamma_5 s)\,(\bar{\mu}\gamma_\alpha\gamma_5\mu)$$ **– the only relevant one in the SM at the LO in QED**

$$Q_{S(P)} = (\bar{b}\gamma_5 s)\,(\bar{\mu}(\gamma_5)\mu) = \frac{i(\bar{b}\gamma^\alpha\gamma_5 s)\partial_\alpha(\bar{\mu}(\gamma_5)\mu)}{m_b+m_s} + \boxed{E} + \boxed{T}$$

E: vanishing by EOM; T: total derivative

Necessary non-perturbative input: $\quad$ $$\langle 0|\bar{b}\gamma^\alpha\gamma_5 s|B_s(p)\rangle = i p^\alpha f_{B_s}$$ (f_{B_s}: decay constant)

Such a matrix element vanishes for $(\bar{b}\gamma^\alpha s)$ and $(\bar{b}s)$ because B_s is a pseudoscalar.

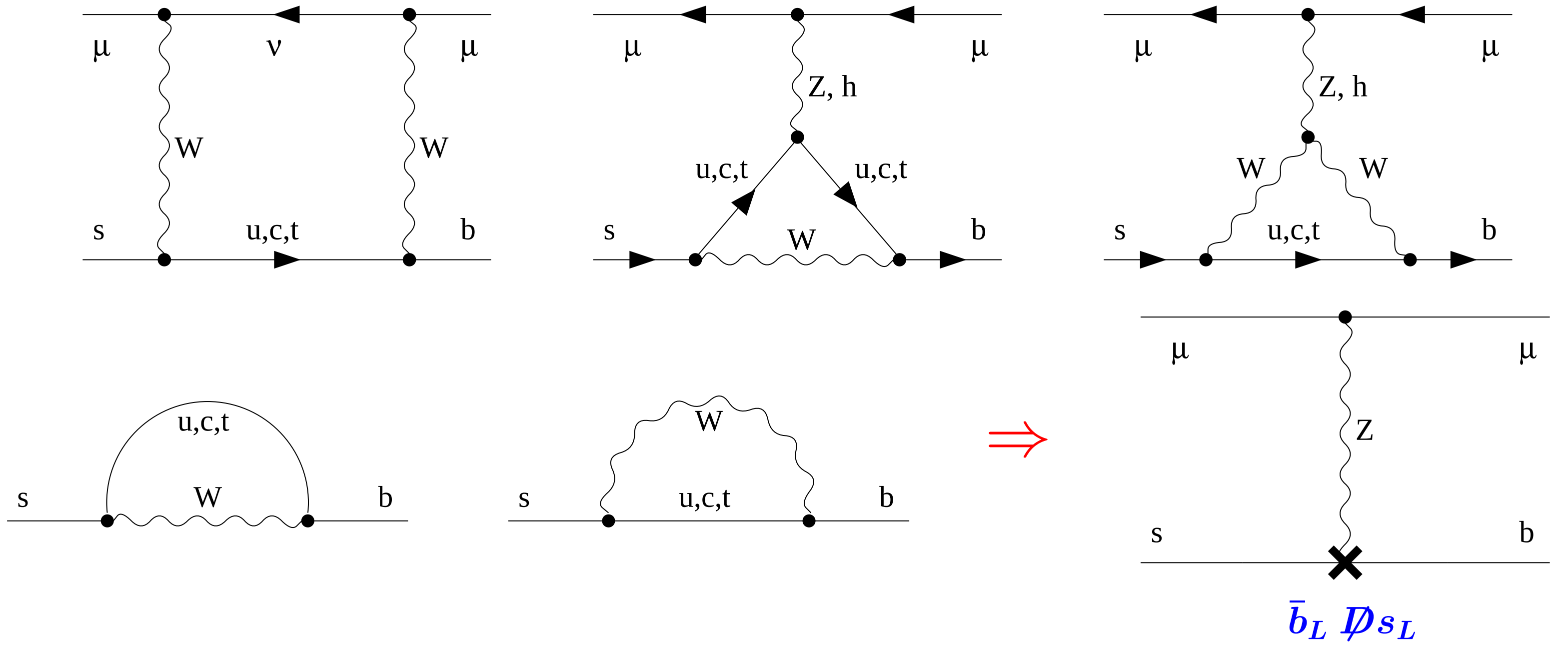
It also vanishes for $(\bar{b}\sigma^{\alpha\beta}s)$ because no antisymmetric tensor can be formed from p^α alone.

$Q_V = (\bar{b}\gamma^\alpha\gamma_5 s)\,(\bar{\mu}\gamma_\alpha\mu)$ **gives no contribution at the LO in QED because**

$$p^\alpha(\bar{\mu}\gamma_\alpha\mu) = \bar{\mu}\not{p}\mu = \bar{\mu}(\not{p}_{\mu^+} + \not{p}_{\mu^-})\mu = \bar{\mu}(-m_\mu + m_\mu)\mu = 0.$$

Q_S gets generated in the SM via the Higgs exchange, but... — see next page.

Evaluation of the LO Wilson coefficients in the SM:



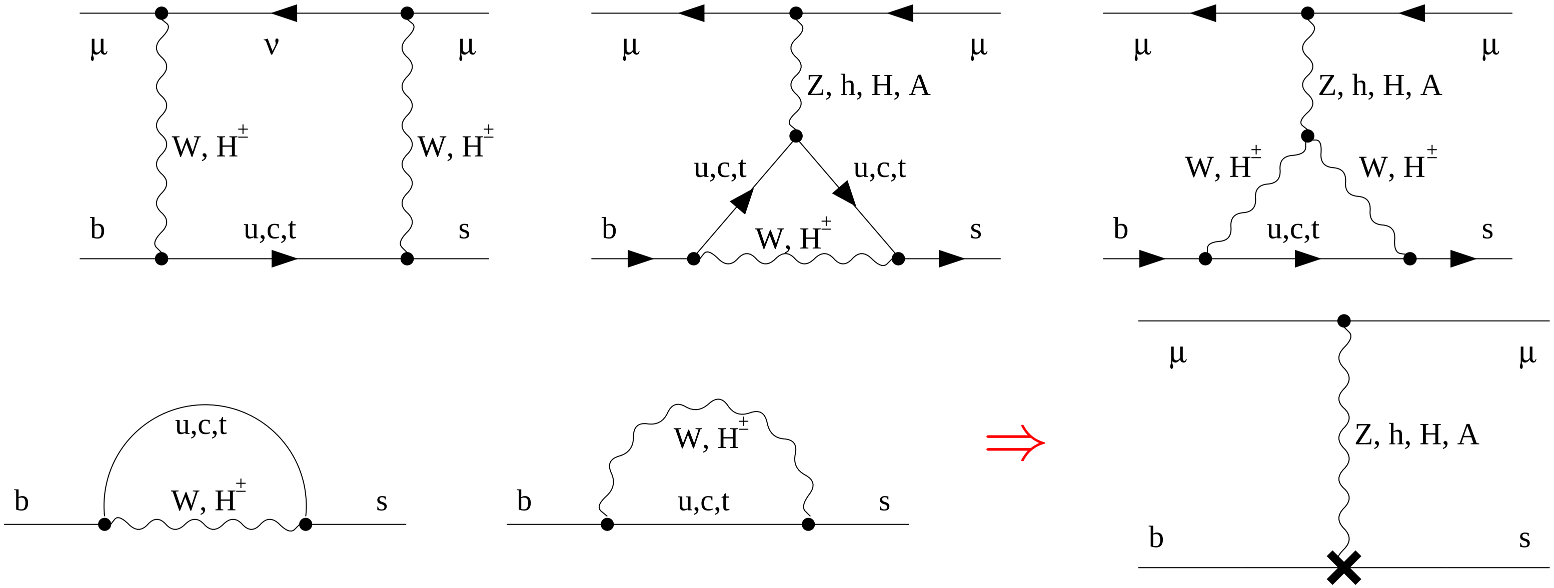
$$C_A^{(0)} = \frac{1}{2} Y_0 \left(m_t^2/M_W^2\right), \qquad Y_0(x) = \frac{3x^2}{8(x-1)^2}\ln x + \frac{x^2-4x}{8(x-1)},$$

$$C_S = \mathcal{O}\left(\frac{m_\mu}{M_W}\right), \qquad C_P = 0.$$

Effects of C_S on the branching ratio are suppressed by $M_{B_s}^2/M_W^2$ $\Rightarrow$ negligible.

Thus, only C_A matters in the SM.

Evaluation of the Wilson coefficients in the Two-Higgs-Doublet Model II



$$\tan\beta = v_2/v_1, \qquad z = M_{H^\pm}^2/m_t^2,$$

$$C_S \simeq C_P \simeq \frac{m_\mu m_b \tan^2\beta}{4M_W^2}\,\frac{\ln z}{z-1} > 0,$$

H.E. Logan and U. Nierste,
NPB 586 (2000) 39
($\mathcal{O}(\tan\beta)$ neglected)

$$\mathcal{B}(B_s \to \mu^+\mu^-) \simeq (\text{const.}) \left[\left| \frac{2m_\mu}{M_{B_s}} C_A - C_P \right|^2 + |C_S|^2 \right]$$

$$C_A = \underset{\text{positive}}{C_A^{\text{SM}}} + \underset{\text{small}}{\Delta C_A} \qquad \Rightarrow \begin{cases} \text{suppression for moderate } C_{S,P} \\ \text{enhancement for huge } \tan\beta \text{ only} \end{cases}$$

Average time-integrated branching ratio:

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi\Gamma_H^s}\beta\left(|rC_A - uC_P|^2 F_P + |u\beta C_S|^2 F_S\right) + \mathcal{O}(\alpha_{em}),$$

where $N = \frac{V_{tb}^* V_{ts} G_F^2 M_W^2}{\pi^2}$, $r = \frac{2m_\mu}{M_{B_s}}$, $\beta = \sqrt{1-r^2}$, $u = \frac{M_{B_s}}{m_b+m_s}$,

$$F_P = 1 - \frac{\Delta\Gamma^s}{\Gamma_L^s}\sin^2\left[\tfrac{1}{2}\phi_s^{\rm NP} + \arg(rC_A - uC_P)\right] \xrightarrow{\text{SM CP}} 1 ,$$

$$F_S = 1 - \frac{\Delta\Gamma^s}{\Gamma_L^s}\cos^2\left[\tfrac{1}{2}\phi_s^{\rm NP} + \arg C_S\right] \xrightarrow{\text{SM CP}} \frac{\Gamma_H^s}{\Gamma_L^s}$$

derived following [K. de Bruyn *et al.*, Phys. Rev. Lett. 109 (2012) 041801]

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derived following [K. de Bruyn *et al.*, Phys. Rev. Lett. 109 (2012) 041801]

In the limit of no CP-violation, mass eigenstates are CP eigenstates:

Heavier, CP-odd: $B_s^H = \frac{1}{\sqrt{2}}(B_s + \bar{B}_s)$, annihilated by $\bar{b}\gamma_5 s + \bar{s}\gamma_5 b$, $(\tau_H = 1.619(9)\,\text{ps})$

Lighter, CP-even: $B_s^L = \frac{1}{\sqrt{2}}(B_s - \bar{B}_s)$, annihilated by $\bar{b}\gamma_5 s - \bar{s}\gamma_5 b$, $(\tau_L = 1.518(4)\,\text{ps})$

Our interactions in this limit are all CP-even:

$$Q_A + Q_A^\dagger = \left[(\bar{b}\gamma^\alpha\gamma_5 s) + (\bar{s}\gamma^\alpha\gamma_5 b)\right](\bar{\mu}\gamma_\alpha\gamma_5\mu)$$

$$Q_P + Q_P^\dagger = \left[(\bar{b}\gamma_5 s) + (\bar{s}\gamma_5 b)\right](\bar{\mu}\gamma_5\mu)$$

} annihilate B_s^H, produce CP-odd dimuons

$$Q_S + Q_S^\dagger = \left[(\bar{b}\gamma_5 s) - (\bar{s}\gamma_5 b)\right](\bar{\mu}\mu)$$

} annihilates B_s^L, produces CP-even dimuons

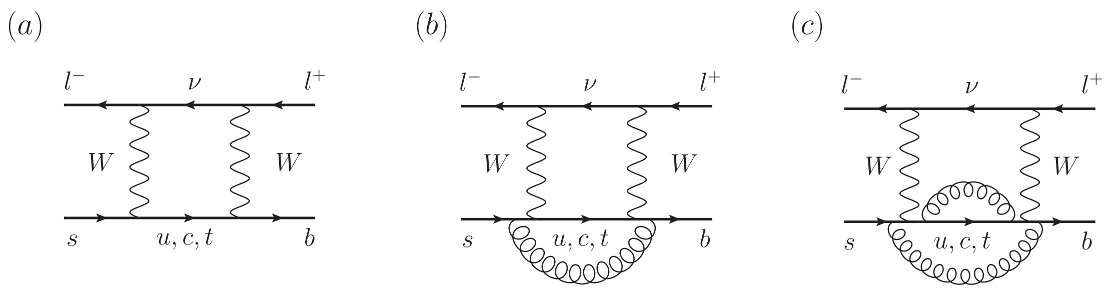
With SM-like CP-violation – still $Q_{A,P}$ annihilate B_s^H and Q_S annihilates B_s^L.

Beyond SM – interesting time-dependent observables, see arXiv:1303.3820, 1407.2771.

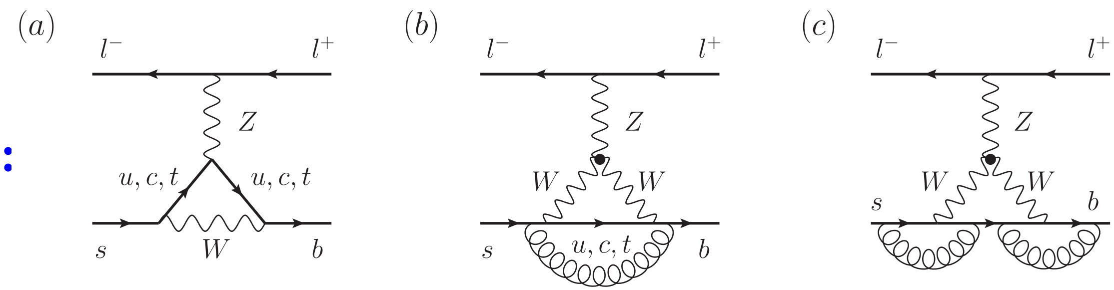
Evaluation of the NNLO QCD matching corrections in the SM

[T. Hermann, MM, M. Steinhauser, JHEP 1312 (2013) 097]

W-boxes:
(1LPI)



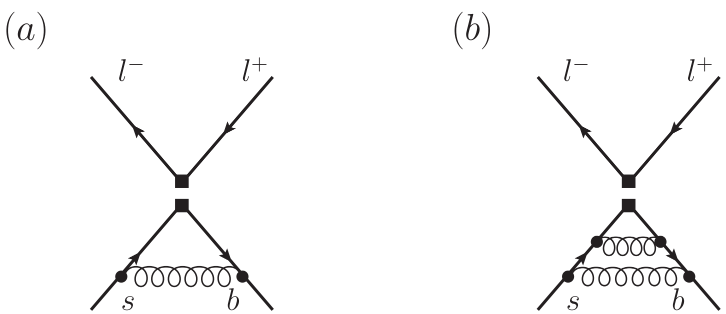
Z-penguins:
(1LPI)



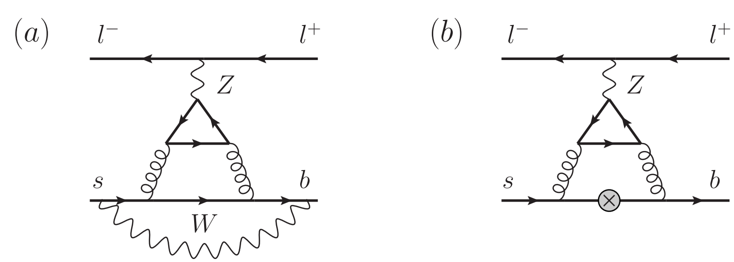
Subtleties: (i) counterterms with finite parts $\sim \bar{b}_L \not{D} s_L$

(ii) evanescent operators: $E_B = (\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5 s)(\bar{\mu}\gamma^\sigma\gamma^\rho\gamma^\nu\gamma_5\mu) - 4(\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu)$

$E_T = \text{Tr}\,(\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\alpha\gamma_5)(\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma s)(\bar{\mu}\gamma_\alpha\gamma_5\mu) + 24(\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu)$



Renormalization of E_B

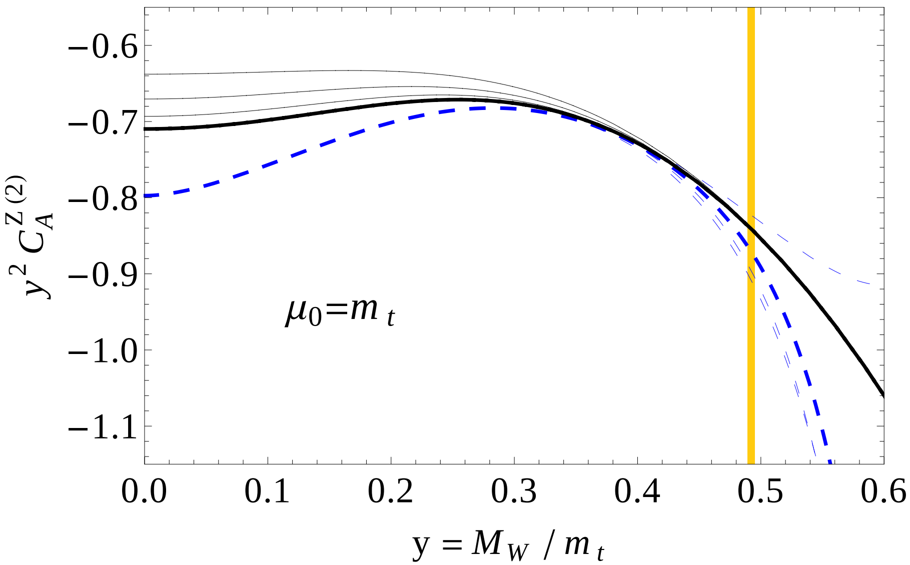


Diagrams generating E_T

Perturbative series for the Wilson coefficient at $\mu = \mu_0 \sim m_t, M_W$:

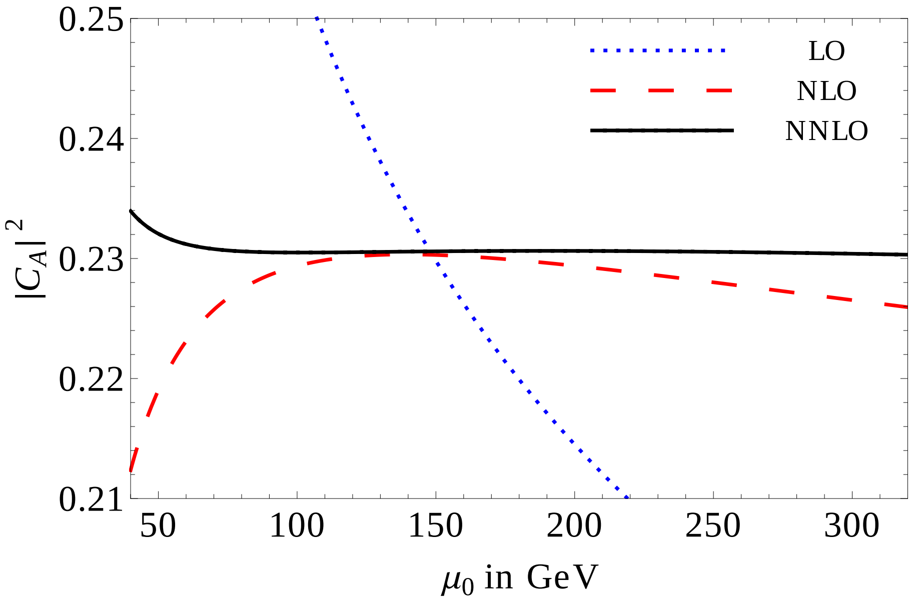
$$C_A(\mu_0) = C_A^{(0)}(\mu_0) + \frac{\alpha_s}{4\pi} C_A^{(1)}(\mu_0) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A^{(2)}(\mu_0) + \frac{\alpha_{em}}{4\pi} \Delta_{\rm EW} C_A(\mu_0) + \ldots$$

The top quark mass is $\overline{\rm MS}$-renormalized at μ_0 with respect to QCD, and on shell with respect to the EW interactions. Both α_s and α_{em} are $\overline{\rm MS}$-renormalized at μ_0 in the effective theory.



$$C_A^{(n)} = C_A^{W,(n)} + C_A^{Z,(n)}$$

To deal with single-scale tadpole integrals, we expand around $y = 1$ (solid lines) and around $y = 0$ (dashed lines), where $y = M_W/m_t$. The expansions reach $(1-y^2)^{16}$ and y^{12}, respectively. The blue band indicates the physical region.



Matching scale dependence of $|C_A|^2$ gets significantly reduced. The plot corresponds to $\Delta_{\rm EW} C_A(\mu_0) = 0$. However, with our conventions for m_t and the global normalization, μ_0-dependence is due to QCD only.

NNLO fit (with $\Delta_{\rm EW} C_A(\mu_0) = 0$):

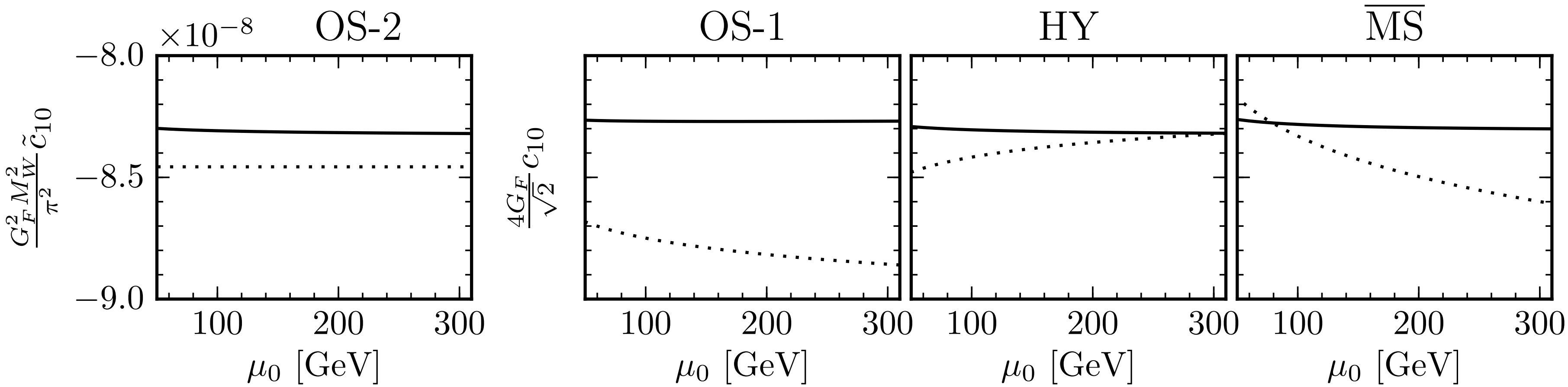
$$C_A = 0.4802 \left(\frac{M_t}{173.1}\right)^{1.52} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.09} + \mathcal{O}(\alpha_{em})$$

Evaluation of the NLO EW matching corrections in the SM

[C. Bobeth, M. Gorbahn, E. Stamou, Phys. Rev. D 89 (2014) 034023]

Method: similar to the NNLO QCD case. Two-loop integrals with three mass scales are present.

Dependence of the final result on μ_0 in various renormalization schemes (dotted – LO, solid – NLO):



In all the four plots: no QCD corrections to C_A included, $m_t(m_t)$ w.r.t. QCD used.

OS-2 scheme: Global normalization factor in $\mathcal{L}_{\text{eff}}$ set to $N = V_{tb}^* V_{ts}\, G_F^2 M_W^2/\pi^2$

Masses at the LO renormalized on-shell w.r.t. EW interactions (including M_W in N)

Plotted quantity: $-2C_A\, G_F^2 M_W^2/\pi^2$ in GeV^{-2}

NLO EW matching correction to the BR: -3.7%

other schemes: Global normalization factor in $\mathcal{L}_{\text{eff}}$ set to $4V_{tb}^* V_{ts}\, G_F/\sqrt{2}$

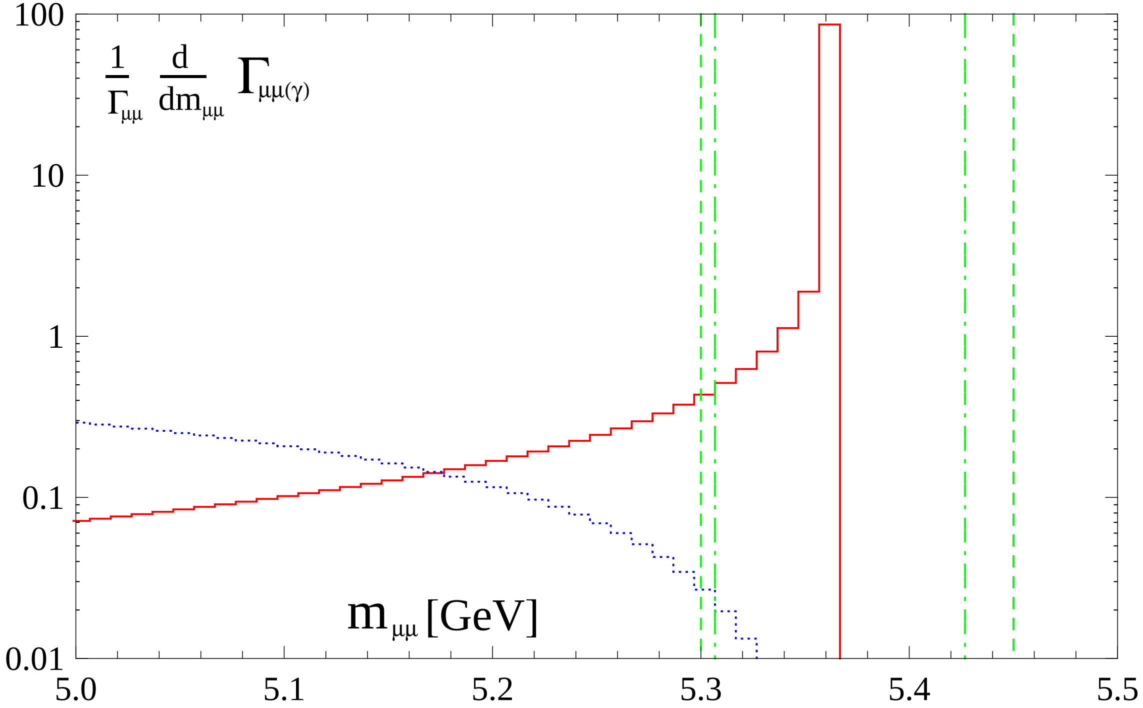
At the LO, $\alpha_{em}(\mu_0)$ used

$\overline{\text{MS}}$: Masses and $\sin^2\theta_W$ renormalized at μ_0

OS-1: Masses as in OS-2, $\sin^2\theta_W$ on-shell

HY (hybrid): Masses as in OS-2, $\sin^2\theta_W$ as in $\overline{\text{MS}}$.

Radiative tail in the dimuon invariant mass spectrum



Ggreen vertical lines – experimental "blinded" windows [CMS and LHCb, Nature 522 (2015) 68]

Red line – no real photon and/or radiation only from the muons. It vanishes when $m_\mu \to 0$.

[A.J. Buras, J. Girrbach, D. Guadagnoli, G. Isidori, Eur.Phys.J. C72 (2012) 2172]

[S. Jadach, B.F.L. Ward, Z. Was, Phys.Rev. D63 (2001) 113009], Eq. (204) as in PHOTOS

Blue line – remainder due to radiation from the quarks. IR-safe because B_s is neutral.

Phase-space suppressed but survives in the $m_\mu \to 0$ limit.

[Y.G. Aditya, K.J. Healey, A.A. Petrov, Phys.Rev. D87 (2013) 074028]

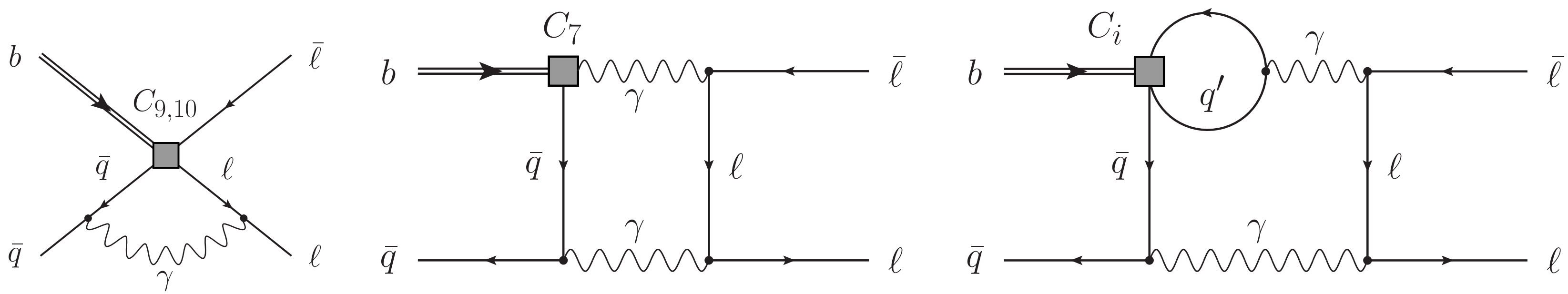
[D. Melikhov, N. Nikitin, Phys.Rev. D70 (2004) 114028]

Interference between the two contributions is negligible – suppressed both by phase-space and $m_\mu^2/M_{B_s}^2$.

Enhanced QED effects in $B_q \to \ell^+\ell^-$

The leading contribution to the decay rate is proportonal to $f_{B_q}^2 \sim \frac{\Lambda^3}{M_{B_q}}$.

As observed by M. Beneke, C. Bobeth and R. Szafron in arXiv:1708.09152, some of the QED corrections scale like Λ^2:



Consequently, the relative QED correction scales like $\frac{\alpha_{em}}{\pi} \frac{M_{B_q}}{\Lambda}$.

Their explicit calculation implies that the previous results for all the $B_q \to \ell^+\ell^-$ branching ratios need to be multiplied by

$$0.993 \pm 0.004.$$

Thus, despite the $\frac{M_{B_q}}{\Lambda}$-enhancement, the effect is well within the previously estimated $\pm 1.5\%$ non-parametric uncertainty.

However, it is larger than $\pm 0.3\%$ stemming from scale-variation of the Wilson coefficient $C_A(\mu_b)$.

SM predictions for all the branching ratios $\overline{\mathcal{B}}_{q\ell} \equiv \overline{\mathcal{B}}(B_q^0 \to \ell^+\ell^-)$

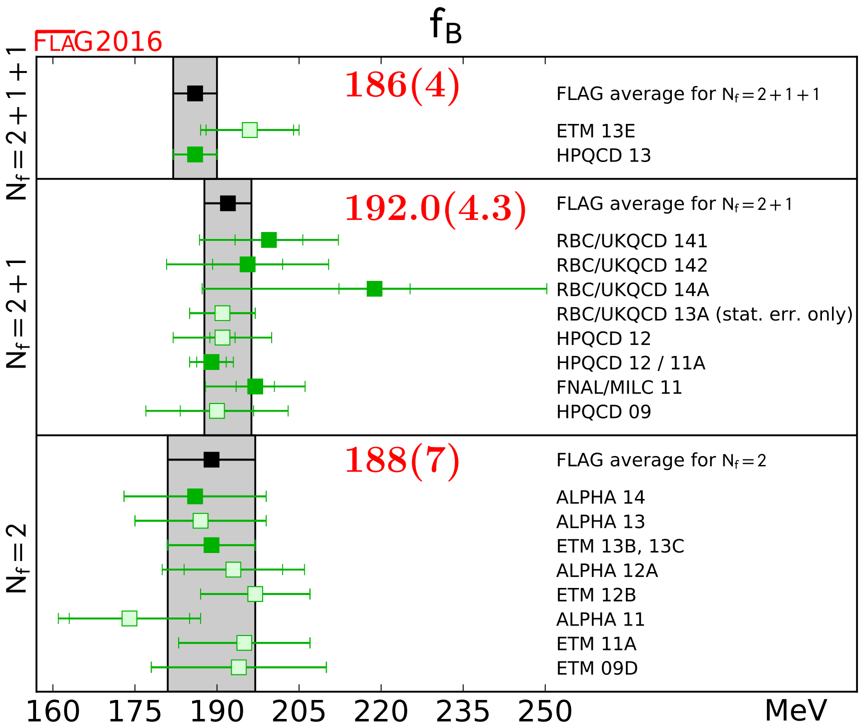
[C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou, M. Steinhauser, PRL 112 (2014) 101801]

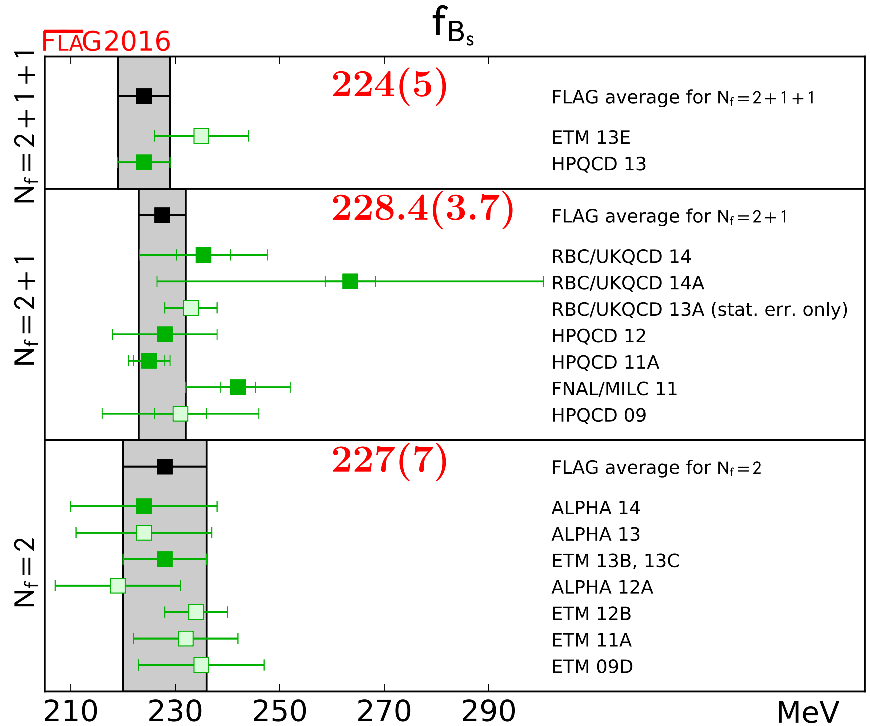
$$
\begin{aligned}
\overline{\mathcal{B}}_{se} \times 10^{14} &= (8.54 \pm 0.13)\, R_{t\alpha}\, R_s, \\
\overline{\mathcal{B}}_{s\mu} \times 10^{9} &= (3.65 \pm 0.06)\, R_{t\alpha}\, R_s, \\
\overline{\mathcal{B}}_{s\tau} \times 10^{7} &= (7.73 \pm 0.12)\, R_{t\alpha}\, R_s, \\
\overline{\mathcal{B}}_{de} \times 10^{15} &= (2.48 \pm 0.04)\, R_{t\alpha}\, R_d, \\
\overline{\mathcal{B}}_{d\mu} \times 10^{10} &= (1.06 \pm 0.02)\, R_{t\alpha}\, R_d, \\
\overline{\mathcal{B}}_{d\tau} \times 10^{8} &= (2.22 \pm 0.04)\, R_{t\alpha}\, R_d,
\end{aligned}
$$

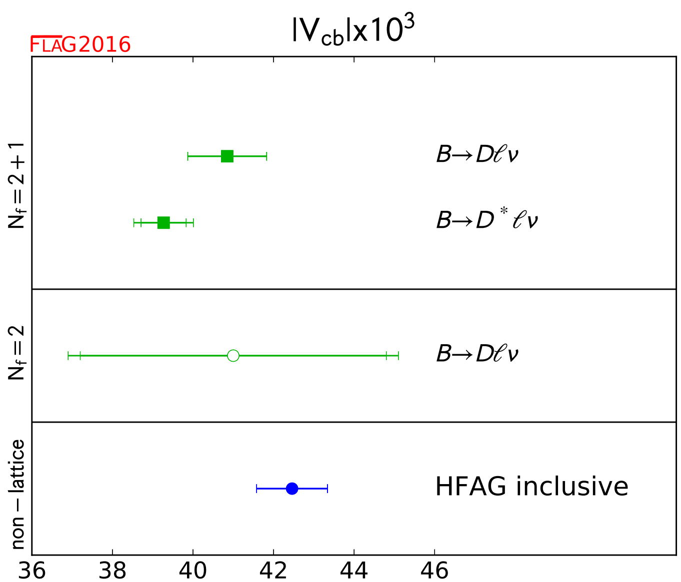
where

$$
\begin{aligned}
R_{t\alpha} &= \left(\frac{M_t}{173.1\ \mathrm{GeV}}\right)^{3.06} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.18}, \\
R_s &= \left(\frac{f_{B_s}[\mathrm{MeV}]}{227.7}\right)^2 \left(\frac{|V_{cb}|}{0.0424}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s\,[\mathrm{ps}]}{1.615}, \\
R_d &= \left(\frac{f_{B_d}[\mathrm{MeV}]}{190.5}\right)^2 \left(\frac{|V_{tb}^\star V_{td}|}{0.0088}\right)^2 \frac{\tau_d^{\mathrm{av}}\,[\mathrm{ps}]}{1.519}.
\end{aligned}
$$

Inputs from FLAG, arXiv:1607.00299, Figs. 20 and 30 (+ web page update)







0.041(1)

0.03927(76) **(2.7σ tension with the inclusive)**

$\longrightarrow$ **0.04200(64)** **from P. Gambino, K. J. Healey and S. Turczyk Phys.Lett.B 763 (2016) 60.**

Update of the input parameters

	2014 paper	this talk	source
M_t [GeV]	173.1(9)	174.30(65)	CDF & D0, arXiv:1608.01881
$\alpha_s(M_Z)$	0.1184(7)	0.1182(12)	PDG 2016
f_{B_s} [GeV]	0.2277(45)	0.2240(50)	FLAG 2016
f_{B_d} [GeV]	0.1905(42)	0.1860(40)	FLAG 2016
$\|V_{cb}\|$	0.04240(90)	0.04089(44)	naive average excl. & incl.
$\|V_{tb}^* V_{ts}\|/\|V_{cb}\|$	0.9800(10)	0.9819(4)	derived from CKMfitter 2016
$\|V_{tb}^* V_{td}\|$	0.0088(3)	0.0087(2)	derived from CKMfitter 2016
τ_H^s [ps]	1.615(21)	1.619(9)	HFLAV 2017
τ_H^d [ps]	1.519(7)	1.518(4)	HFLAV 2017
$\overline{\mathcal{B}}_{s\mu} \times 10^9$	3.65(23)	3.35(18)	
$\overline{\mathcal{B}}_{d\mu} \times 10^{10}$	1.06(9)	1.00(7)	

Sources of uncertainties	f_{B_q}	CKM	τ_H^q	M_t	α_s	other parametric	non-parametric	$\sum$
$\overline{\mathcal{B}}_{s\ell}$	4.5%	2.2%	0.6%	1.2%	0.1%	$< 0.1\%$	1.5%	5.4%
$\overline{\mathcal{B}}_{d\ell}$	4.3%	4.6%	0.3%	1.2%	0.1%	$< 0.1\%$	1.5%	6.7%

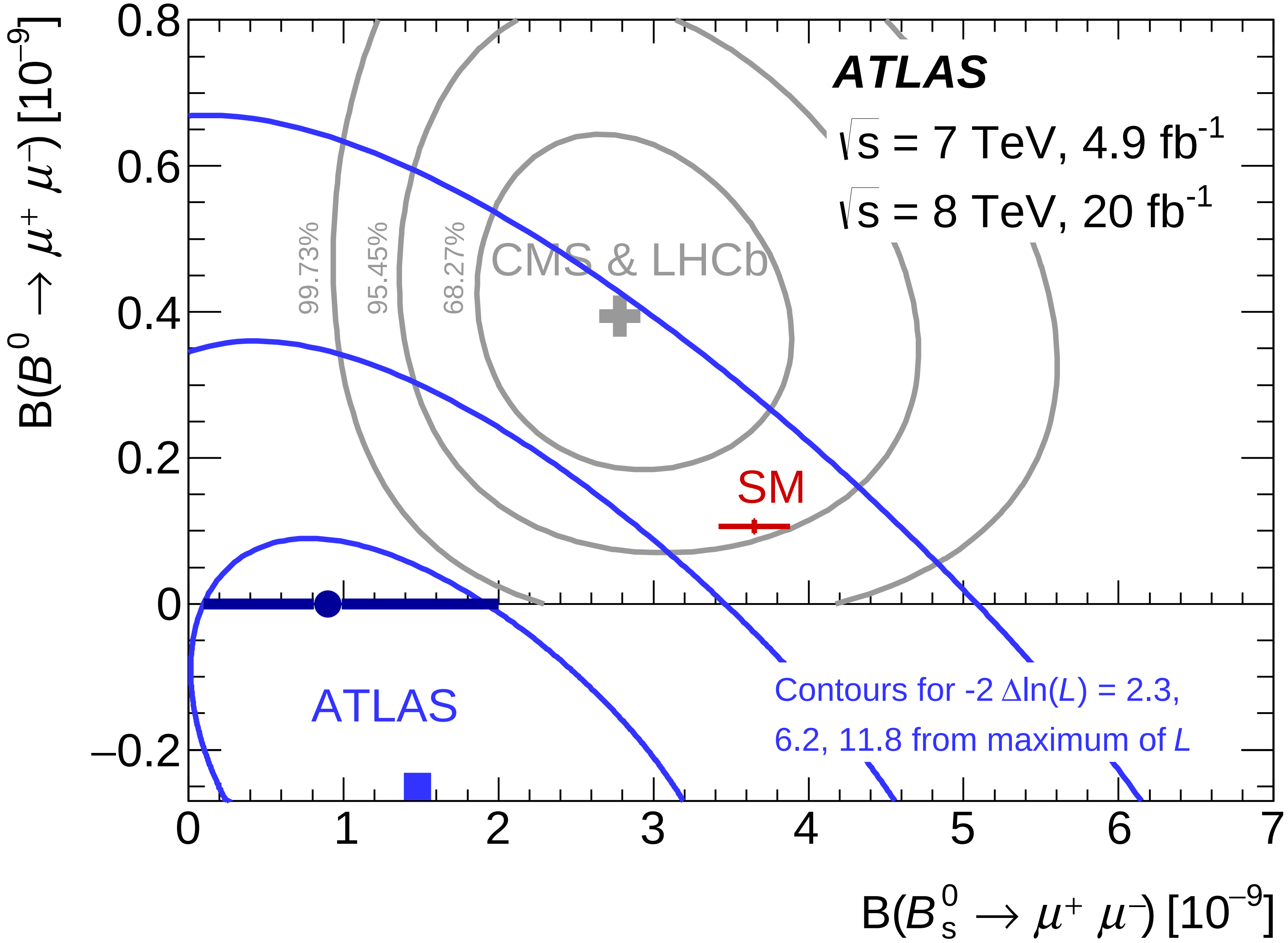
If the inclusive $|V_{cb}| = 0.04200(64)$ alone is used instead of the naive average, then $\overline{\mathcal{B}}_{s\mu} \times 10^9 = 3.54(21)$.

Comparison with the measurements

Previous averages, CMS and LHCb, Nature 522 (2015) 68: $\overline{\mathcal{B}}_{s\mu} = \left(2.8^{+0.7}_{-0.6}\right) \times 10^{-9}$, $\overline{\mathcal{B}}_{d\mu} = \left(3.9^{+1.6}_{-1.4}\right) \times 10^{-10}$.

New results of LHCb, PRL 118 (2017) 191801: $\overline{\mathcal{B}}_{s\mu} = \left(3.0 \pm 0.6^{+0.3}_{-0.2}\right) \times 10^{-9}$, $\overline{\mathcal{B}}_{d\mu} = \left(1.5^{+1.2+0.2}_{-1.0-0.1}\right) \times 10^{-10}$.

ATLAS in EPJC 76 (2016) 513 gives 95% C.L. bounds: $\overline{\mathcal{B}}_{s\mu} < 3.0 \times 10^{-9}$ and $\overline{\mathcal{B}}_{d\mu} < 4.2 \times 10^{-10}$.



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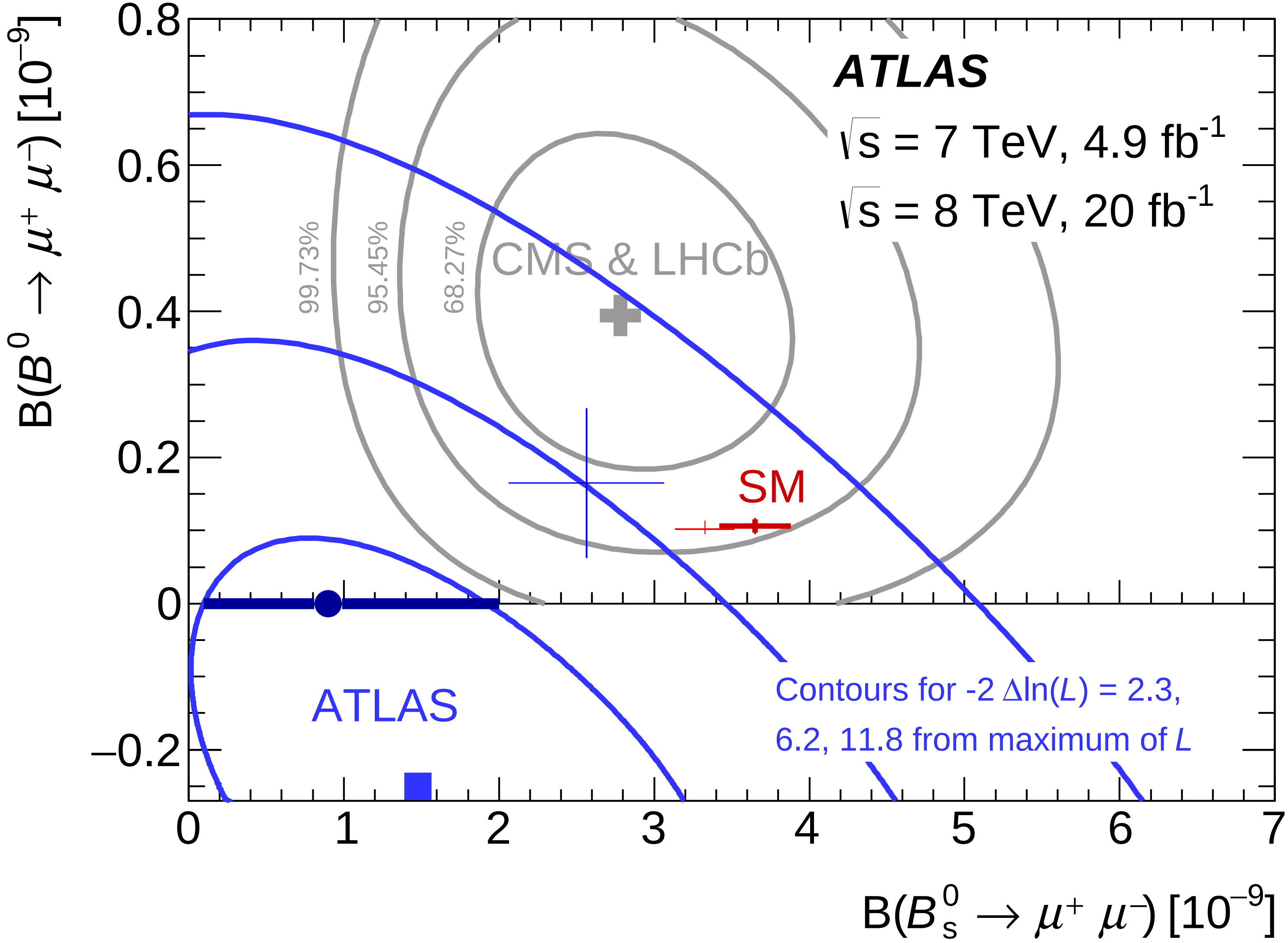

Summary

- Uncertainties in the SM predictions for $\overline{\mathcal{B}}_{q\ell}$ are dominated by the parametric ones, mainly due to the decay constants and CKM factors.

- In the $\overline{\mathcal{B}}_{s\ell}$ case, resolving the inclusive-exclusive tension in $|V_{cb}|$ would help a lot.

- The central values of the SM predictions for $\overline{\mathcal{B}}_{s\mu}$ and $\overline{\mathcal{B}}_{d\mu}$ are in good agreement with the data from LHCb, CMS and ATLAS.

- Some of the QED corrections involve non-perturbative physics beyond what is contained in the decay constants. Despite the recently found unexpected enhancement factors in such corrections, the non-parametric uncertainty can be retained at the $\pm 1.5\%$ level.