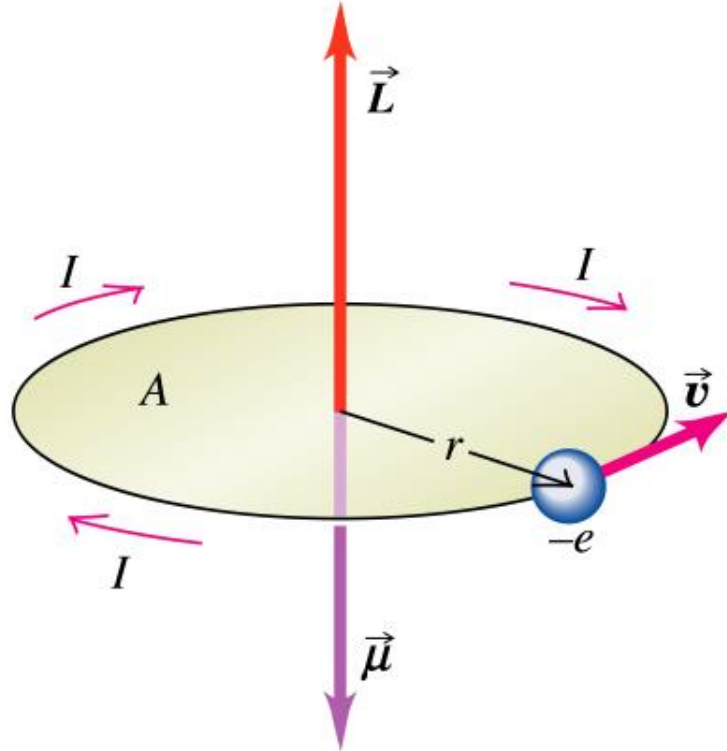


# Bohr magneton

Orbiting electrons form a current loop which give rise to a magnetic field.



$$i = -\frac{e}{T} = -\frac{e v}{2\pi r} \quad A = \pi r^2$$

$$\mu = -\frac{e v}{2\pi} \pi r^2 = -\frac{e v r m_e}{2 m_e} \quad \hbar = v m_e r$$

For the electron:  $\mu_b = \frac{e\hbar}{2m_e} = 9.27 \cdot 10^{-24} \frac{J}{T}$

Magnetic moment of a current loop:

$$|\mu| = iA$$

current  $\nearrow$  area enclosed by current loop

*For the electron, the Bohr magneton is the simplest model possible to the smallest possible current to the smallest possible area closed by the current loop*

# Rutherford (1911) - Borh (1913) model of atom

- 1) Borh use the classical mechanics
- 2) The only permitted orbital are those for with  $L_{orb} = n\hbar$
- 3) For these orbitals the electrons don't radiate electromagnetic waves
- 4) The energy of photon is :  $E = hf$

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{m_e v^2}{r} \quad \Rightarrow \quad L = n\hbar \quad \Rightarrow \quad v = \frac{n\hbar}{m_e r}$$

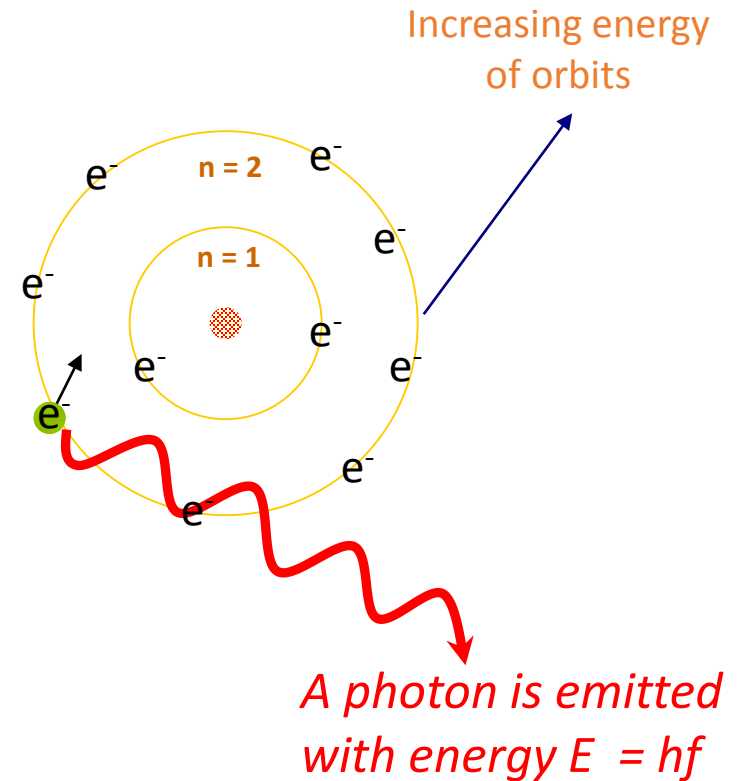
*Electrostatitc force*      *Centrifugal force*

$$k = \frac{1}{4\pi\epsilon_0}$$

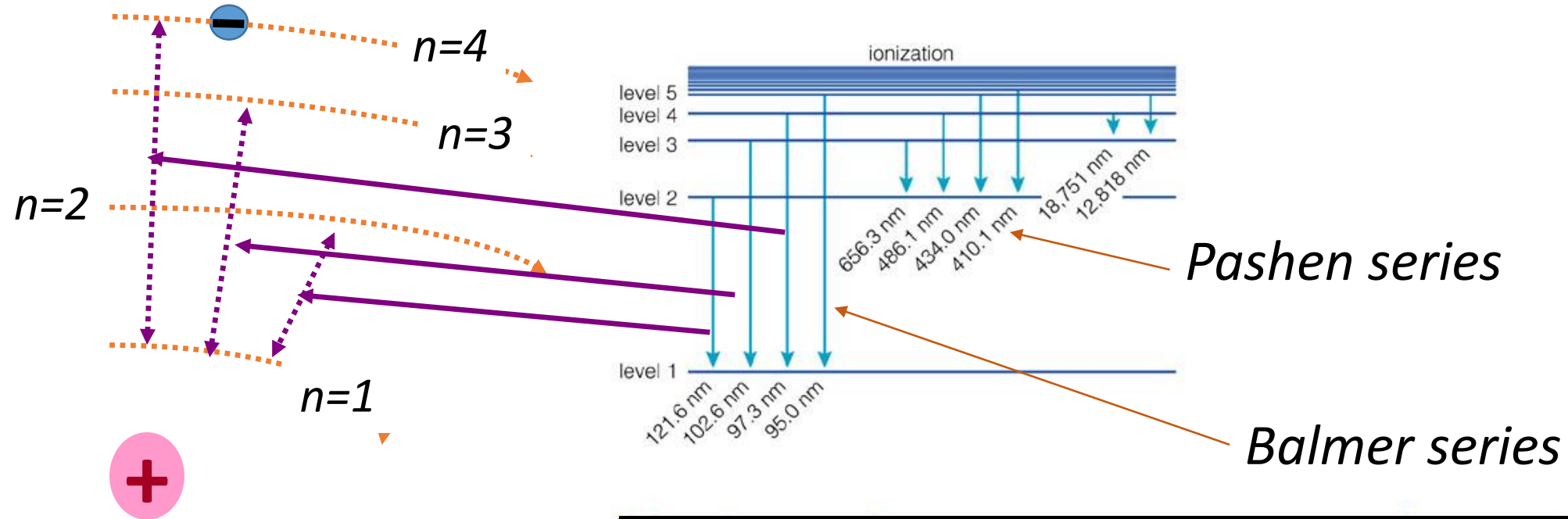
$$r_n = \frac{\hbar^2 n^2}{m_e k e^2}$$

$$r_n = a_0 n^2$$

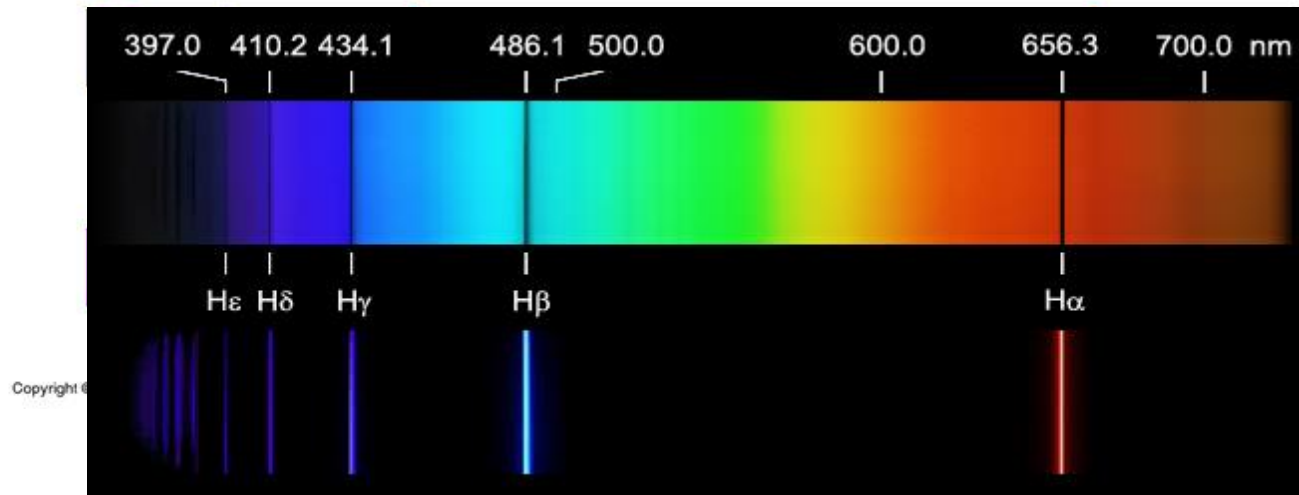
*Borh radius*  $a_0 \equiv \frac{\hbar^2}{m_e k e^2} = 0.05297 \text{ nm}$



# Hydrogen energy levels (Borh- Rutherford Model)

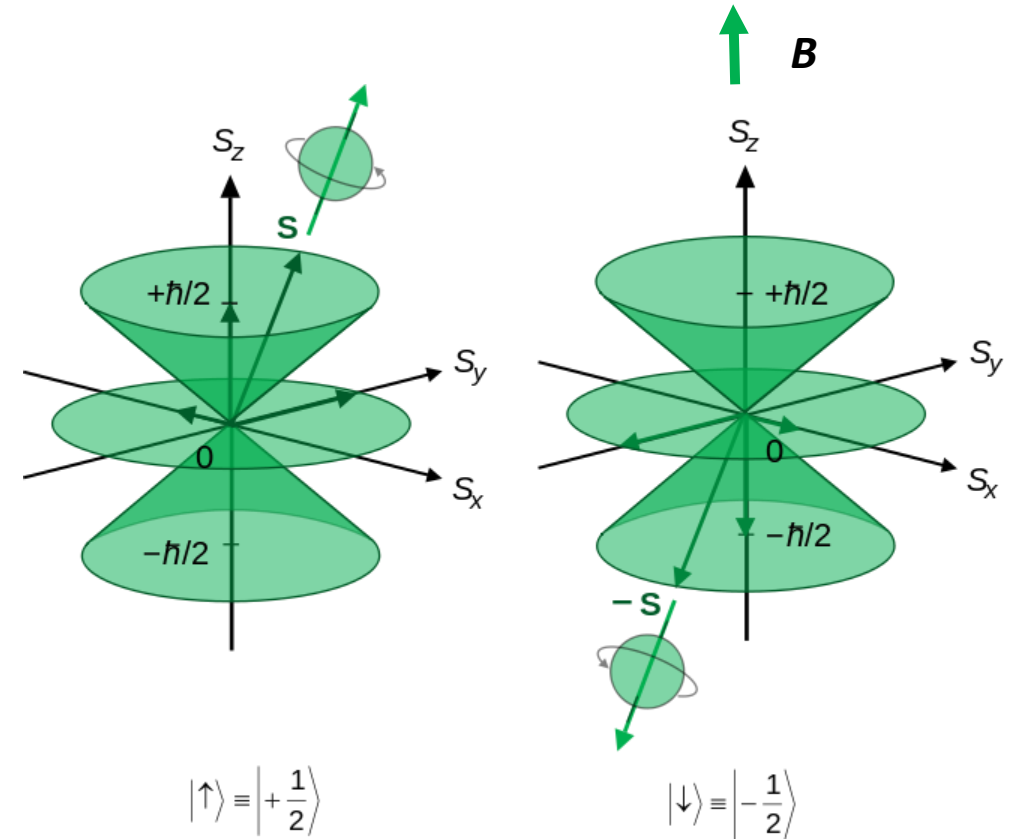
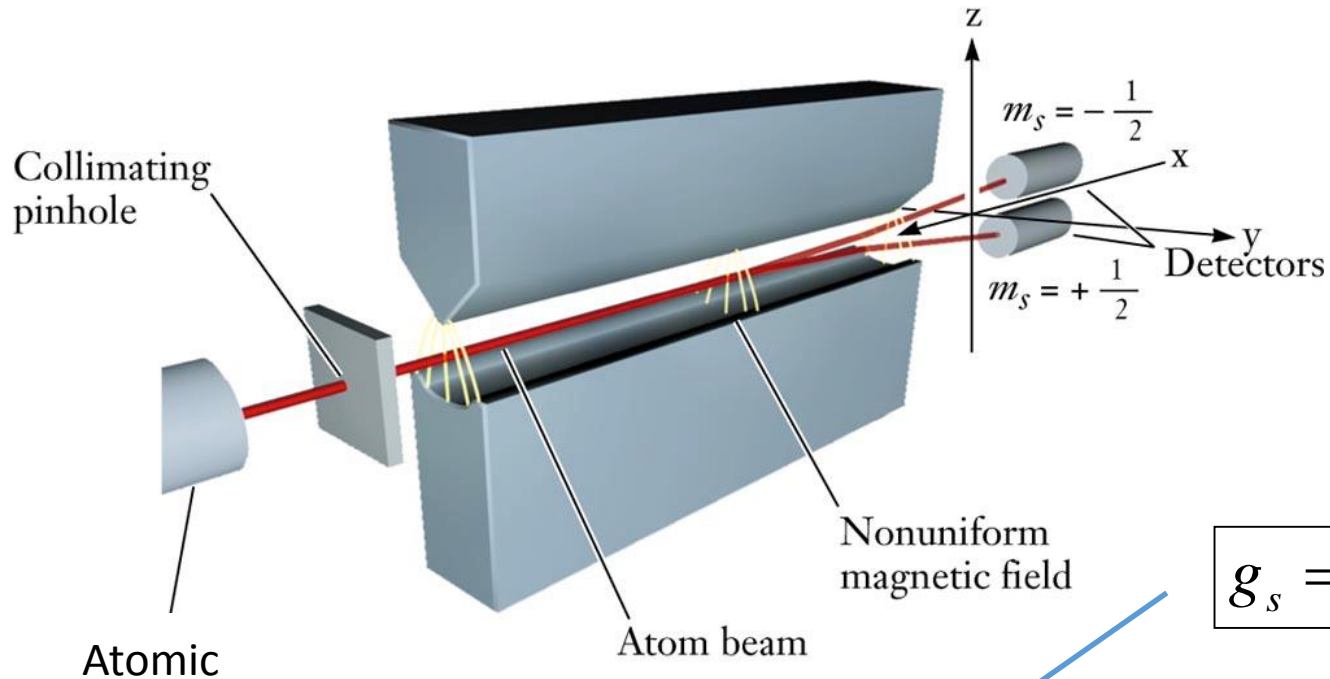


$$E_n = -\frac{m_e e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2} = -\frac{13.6 \text{ eV}}{n^2},$$



# Stern Gerlach experiment (1922)

A classic experiment that shows a difference between quantum and classical mechanics. Only two discrete deflections show up, corresponding to "spin up" and "spin down" for neutral particles.



$$g_s = 2.0023$$

© 2003 Thomson-Brooks/Cole

Spin magnetic moment

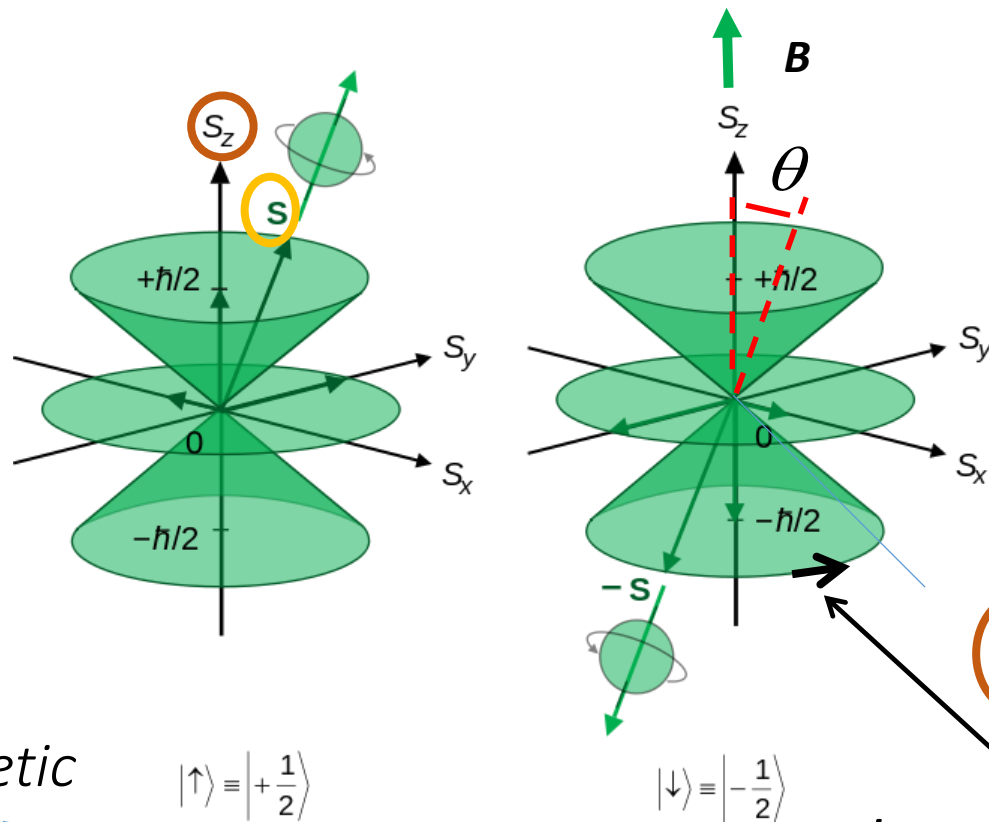


$$\boldsymbol{\mu}_s = -g_s \frac{e}{2m_e} \mathbf{S}_z = -g_s \frac{e}{2m_e} m_s \hbar = -g_s \mu_b m_s$$

$m_s = +1/2$  (spin up)

$m_s = -1/2$  (spin down)

# The spin orientation



*Because of the uncertainty principle, it is impossible to orient the spin  $S$  parallel to  $B$*

$$\cos(\theta)S \geq \frac{\hbar}{2}$$

$$m_s = +1/2 \quad (\text{spin up})$$

$$m_s = -1/2 \quad (\text{spin down})$$

$$S_z = m_s \hbar$$

Larmor precession

Gyromagnetic ratio

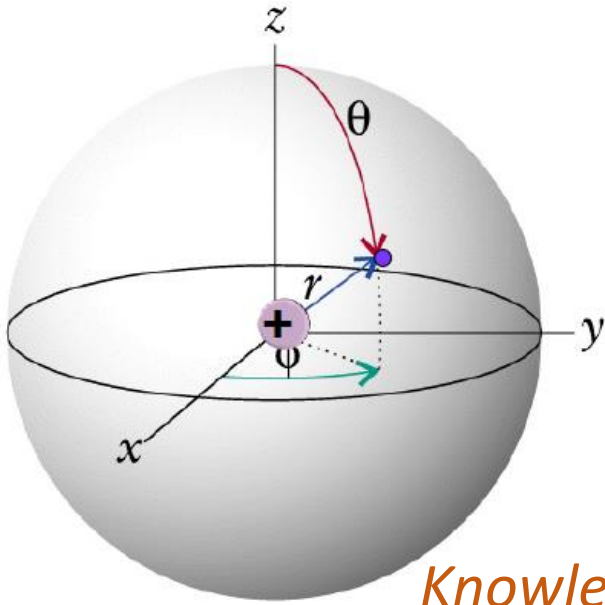
$$|\uparrow\rangle = \left|+\frac{1}{2}\right\rangle$$

$$|\downarrow\rangle = \left|-\frac{1}{2}\right\rangle$$

Larmor frequency  $\Rightarrow \omega = \frac{g_l \mu_B}{\hbar} B = 28 \text{GHz} * B$

$$S = \sqrt{s(s+1)}\hbar = \sqrt{1/2(1/2+1)}\hbar = \frac{\sqrt{3}}{2}\hbar$$

# The Schrödinger equation (1922) to the hydrogen atom (without spin)



*Kinetics energy*

*Potential energy*

*Total energy*

$$-\frac{\hbar^2}{2\mu} \nabla^2 \psi(x, y, z) + V \psi(x, y, z) = E \psi(x, y, z)$$

*Wave function represent the probability amplitude.*

*It is a deterministic function (M. Born).*

*Knowledge of  $\Psi(r, t)$  then enables (in the Copenhagen interpretation) to know the dynamic of the wave function (its evolution in the space-time).*

*For the relativist case see Dirac equation.*

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Psi}{\partial r} \right) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial \phi^2} + \frac{2\mu}{\hbar^2} (E - V) \Psi = 0$$

*Kinetics energy*

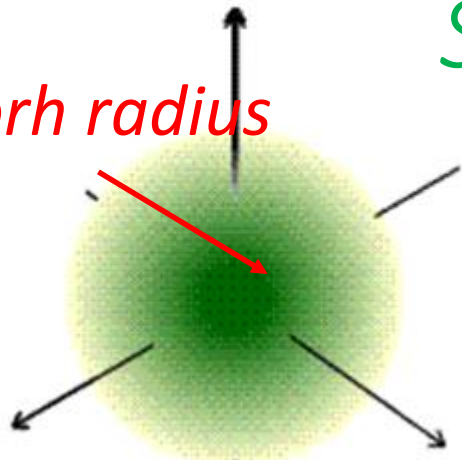
*Total energy*

*Potential energy*

# The Schrödinger equation to the hydrogen atom

## Solutions of radial equation 1s, 2s.

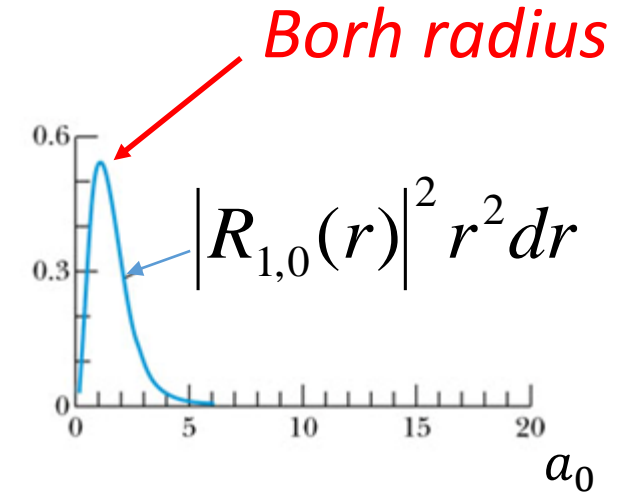
Borh radius



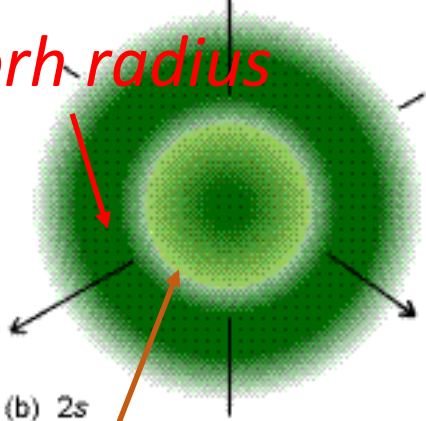
Orbital 1s ( $n=1$ ,  $l=0$ ), volume probability density for the ground state of the hydrogen atom

$$E_1 = -\frac{m_e e^4}{(4\pi\epsilon_0)^2 2\hbar^2} = -13.6 \text{ eV}$$

2 electrons maximum



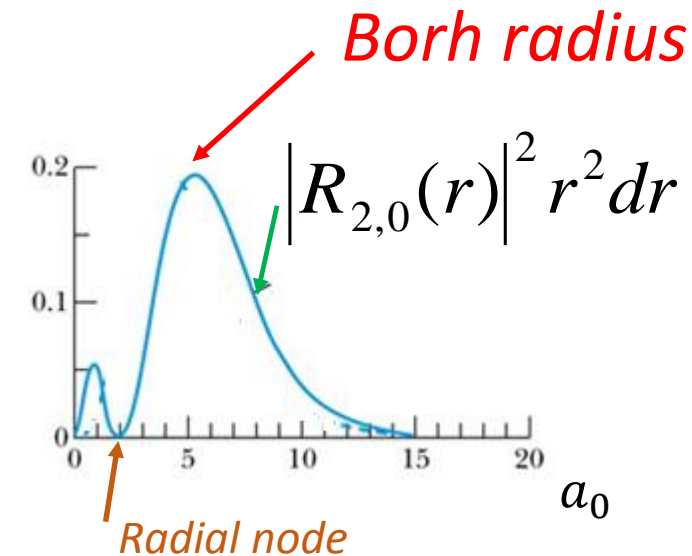
Borh radius



Orbital 2s ( $n=2$ ,  $l=0$ ), volume probability density for the hydrogen atom in the quantum state, the gap in the dot density pattern marks a spherical surface over which the radial wave function is zero

$$E_2 = -\frac{m_e e^4}{(4\pi\epsilon_0)^2 2\hbar^2} \frac{1}{4} = -\frac{13.6 \text{ eV}}{4} = -3.4 \text{ eV}$$

$$a_0 = 5.29 \times 10^{-11} \text{ m}$$



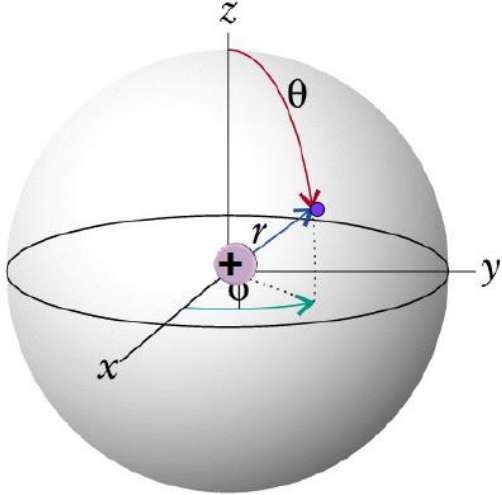


# The Schrödinger equation to the hydrogen atom

## Solutions of harmonic equation

$$\frac{1}{\sin(\theta)} \frac{d}{d\theta} \left( \sin(\theta) \frac{d}{d\theta} P_l^{m_l}(\theta) \right) + \left[ l(l+1) - \frac{m_l^2}{\sin^2(\theta)} \right] P_l^{m_l}(\theta) = 0$$

Legendre polynomials



$$[P_l^{m_l}(\theta)]^2 d\theta$$

The probability density to find an electron in an angle  $\theta$  with an apperture angle  $d\theta$

Quantum number  $l$  called the **orbital quantum number**, is a measure of the magnitude of the angular momentum associated with the quantum state  $l = 0, 1, 2, 3, \dots, n-1$

$$[\Phi(\varphi)]^2 d\varphi$$

The probability density to find an electron in an angle  $\varphi$  with an apperture angle  $d\varphi$

$$\frac{d^2 \Phi(\varphi)}{d\varphi^2} = -m_l^2 \Phi(\varphi)$$

Quantum number  $m_l$ , called the **orbital magnetic quantum number** is related to the orientation in space of the angular momentum vector  
 $-l \leq m_l \leq +l$



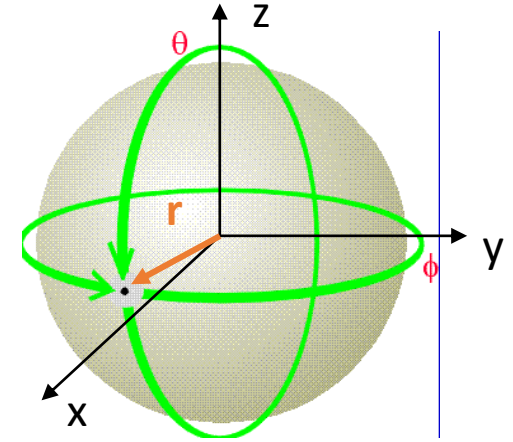
# The Schrödinger equation to the hydrogen atom: orbital 2p (n=2, l=1)

$$p_0 = R_{2,1}(r)P_1^0(\theta)\Phi(\phi) = \frac{1}{4(2\pi)^{1/2}} \left(\frac{1}{a_0}\right)^{5/2} r \cos \theta e^{-\frac{r}{2a_0}} = \underline{r \cos \theta} \cdot f(r) = z \cdot f(r) \equiv p_z$$

$$p_{\pm 1} = R_{2,1}(r)P_1^{\pm 1}(\theta)\Phi(\phi) = \mp \frac{1}{8\pi^{1/2}} \left(\frac{1}{a_0}\right)^{5/2} r e^{-\frac{r}{2a_0}} \sin \theta e^{\pm i\phi}$$

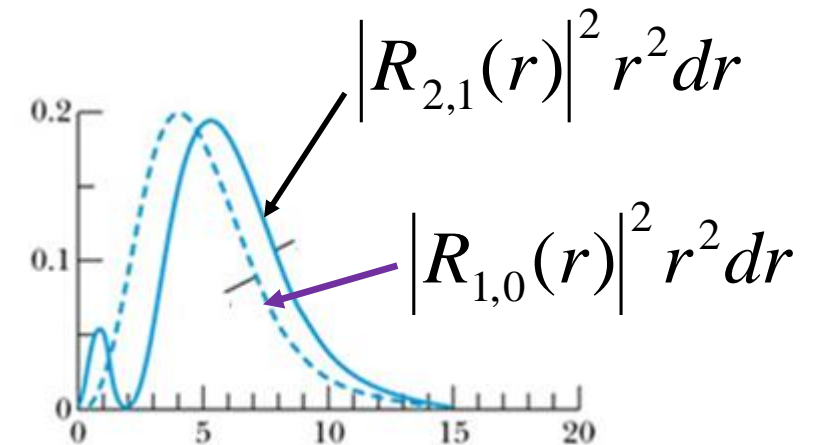
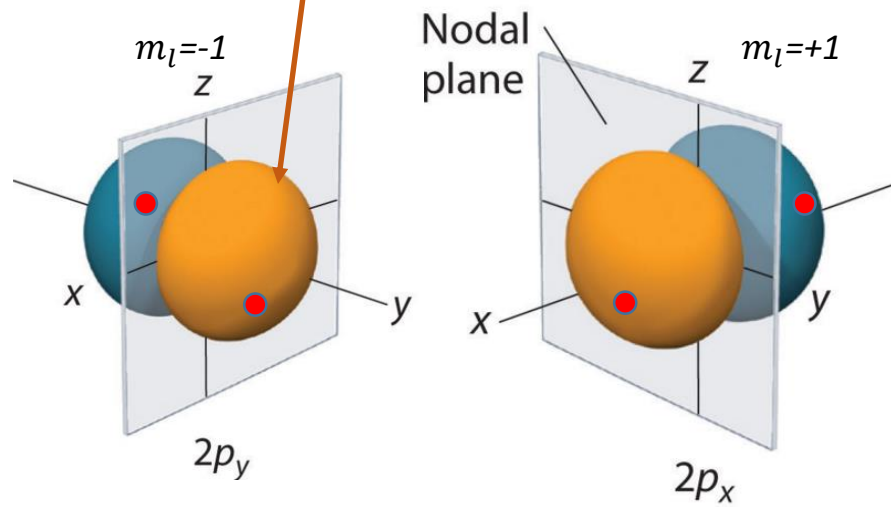
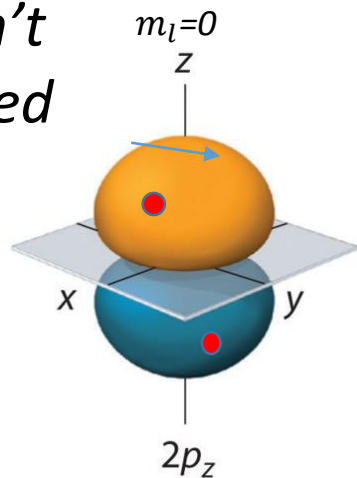
$$= \mp \frac{1}{2^{1/2}} r \sin \theta e^{\pm i\phi} f(r)$$

Different rotations: clockwise, counter-clockwise



$$\begin{cases} p_x = -\frac{1}{2^{1/2}}(p_{+1} - p_{-1}) = \underline{r \sin \theta \cos \phi} f(r) = x f(r) \\ p_y = \frac{i}{2^{1/2}}(p_{+1} + p_{-1}) = \underline{r \sin \theta \sin \phi} f(r) = y f(r) \end{cases}$$

We haven't represented  $f(r)$

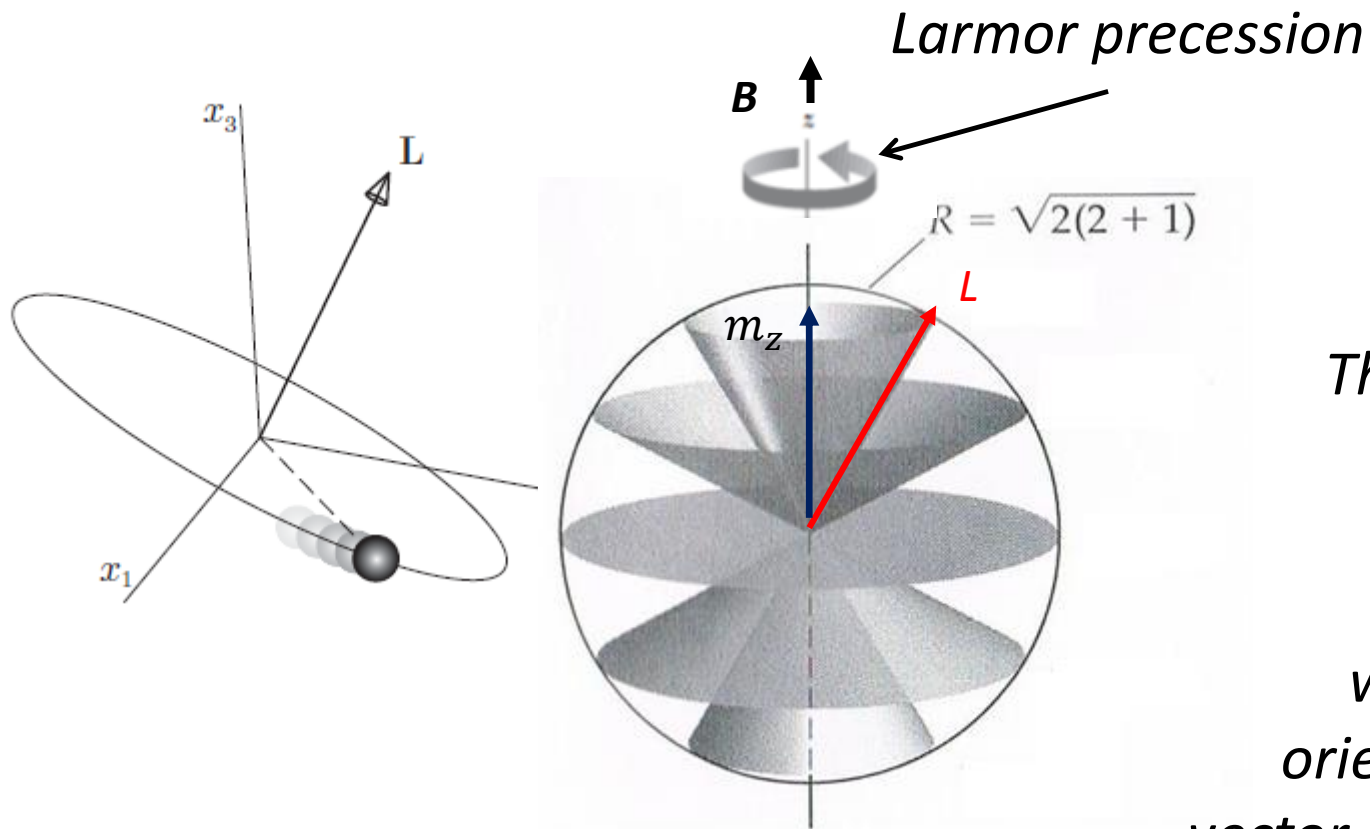


6 electrons maximum

# The orbital angular momentum for $l=2$

$$-l \leq m_z \leq +l$$

Classical equation  $\vec{L} = \vec{r} \times \vec{p}$



Because of the uncertainty principle, it is impossible to orient the orbital angular momentum  $\vec{L}$  parallel to  $B$

This figure shows the five quantized components  $L_z$  of the orbital angular momentum for an electron with  $l=2$ , as well as the associated orientations of the angular momentum vector (however, *we should not take the figure literally* as we cannot detect  $\vec{L}$  this way)

$$L_z = m_\ell \hbar \quad L = \hbar \sqrt{\ell(\ell+1)} = \sqrt{6} \hbar$$

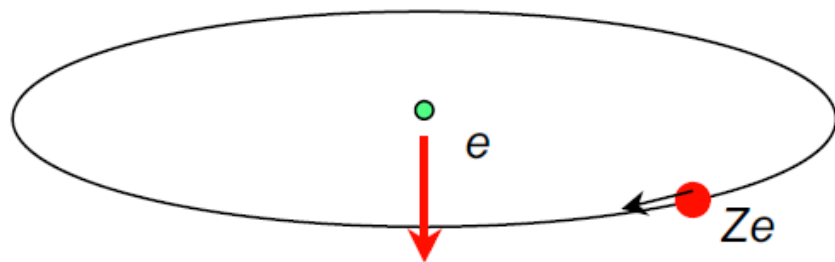
# Total angular momentum in a magnetic field

The total angular momentum can be visualized as precessing about any externally applied magnetic field. We don't consider the spin-orbit interaction.

*From the electron's point of view, the nucleus revolves round it. It is a current loop*

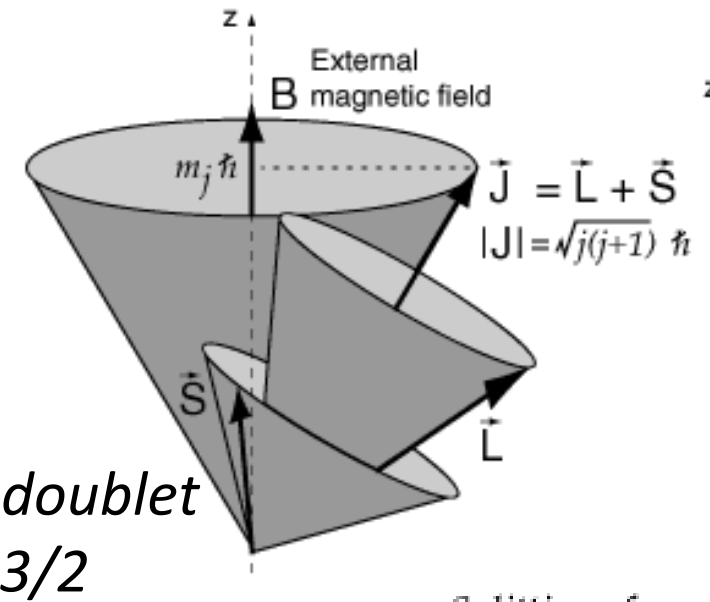
It is a current loop  $I = Ze v / 2\pi r$  Which produces a magnetic field  $\mu_0 I / 2r$  at the centre  $B_{so} = \mu_0 Ze v / 2\pi r^2$

$E_{so} = -\mu_b B_{so}$ . Since  $r \approx a_0 / Z$  and  $m_e v r = \hbar$

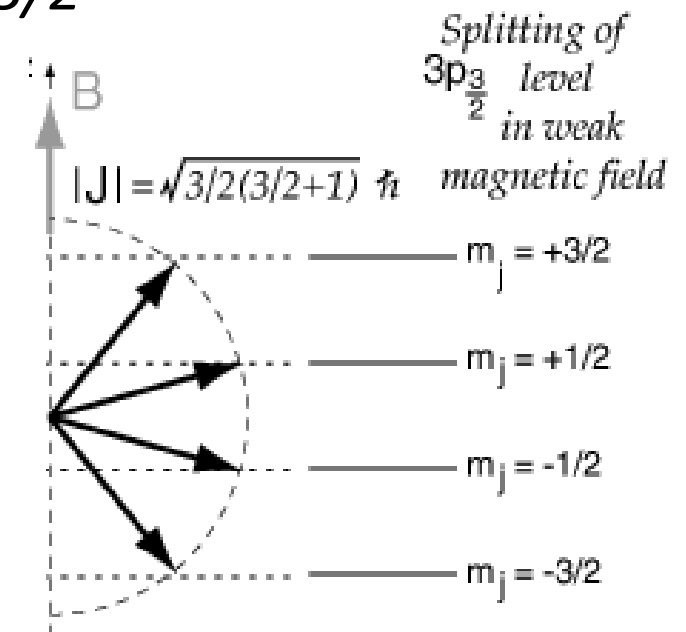


$$E_{so} \approx -\mu_0 \mu_B^2 Z^4 / 4\pi a_0^3$$

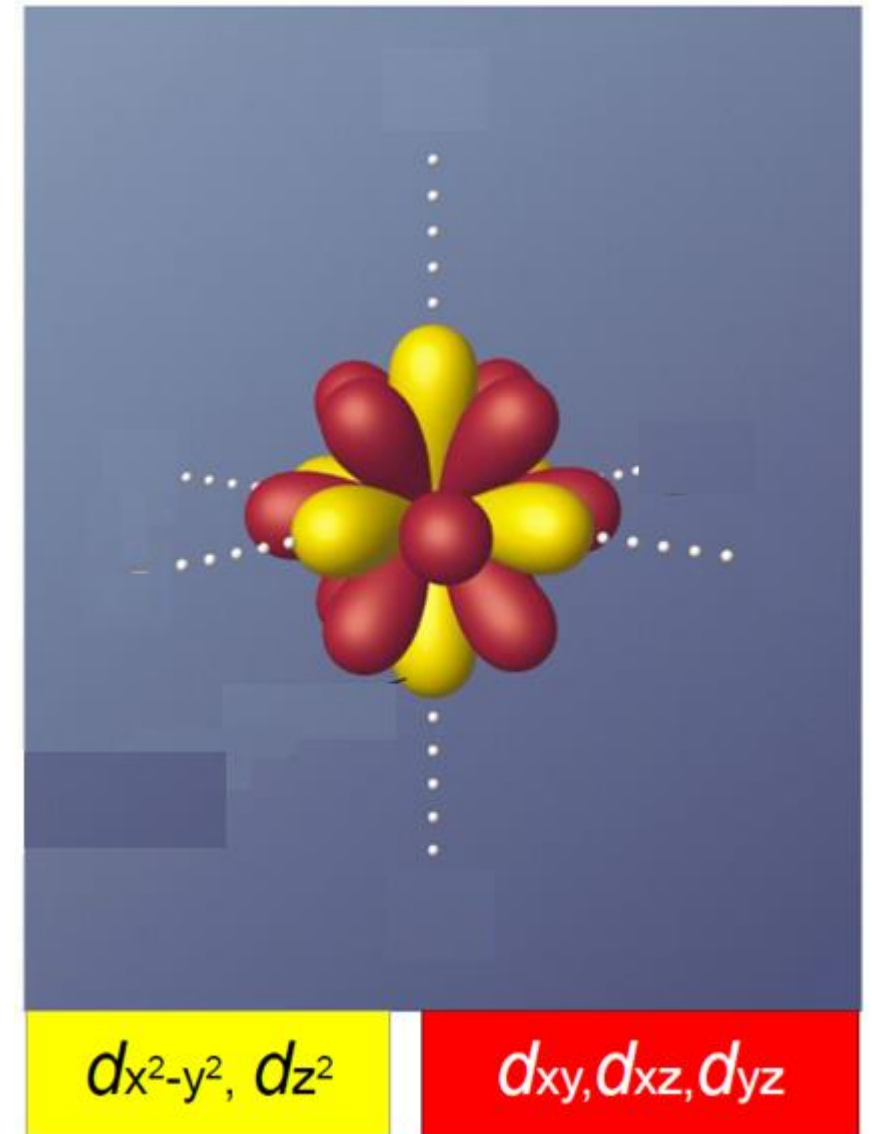
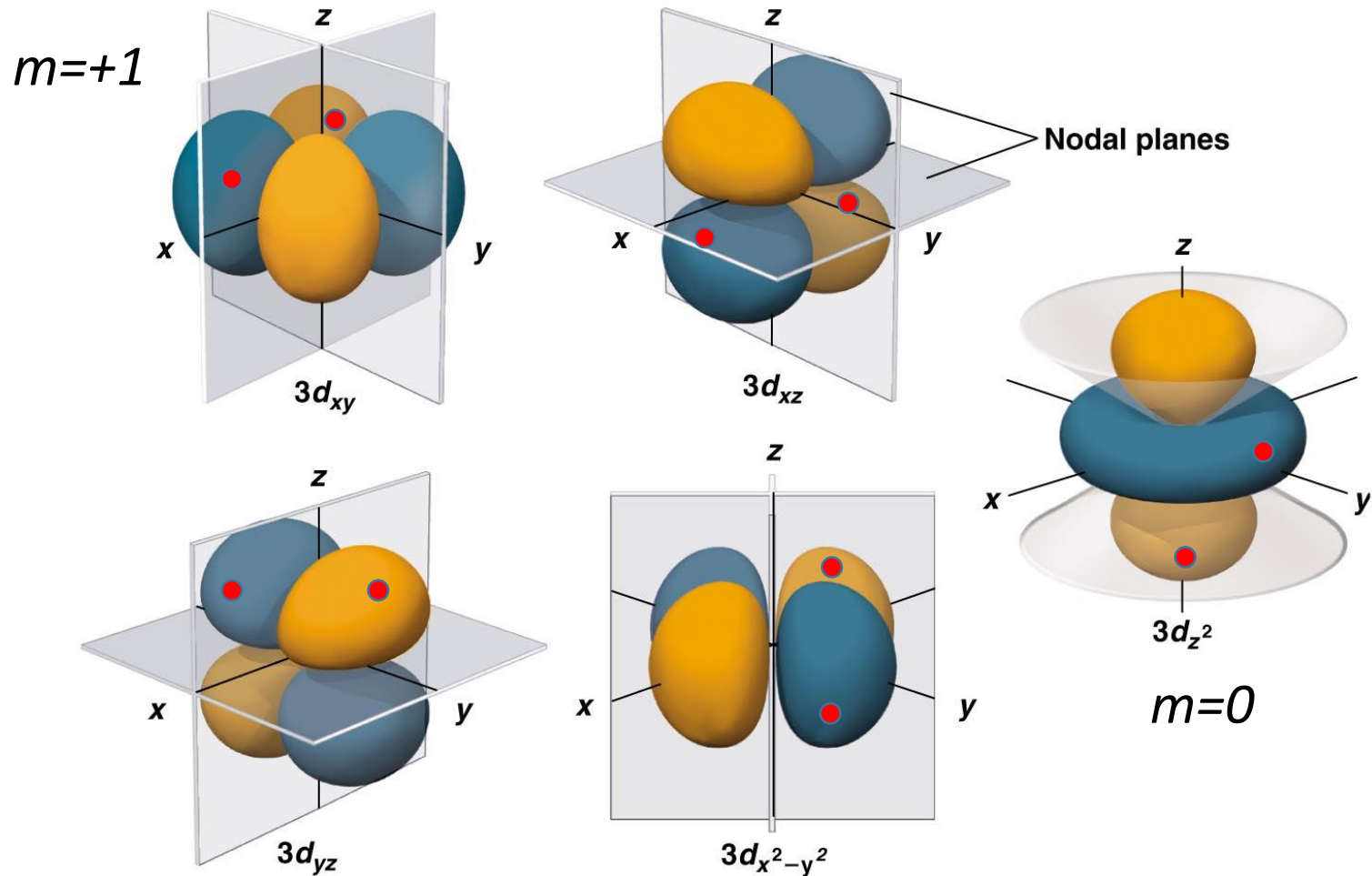
Note if we consider that the spin-orbit, the formula indicates that spin orbit-coupling interactions are significantly larger for atoms that are further down a particular column of the periodic table.



the sodium doublet with  $j=3/2$

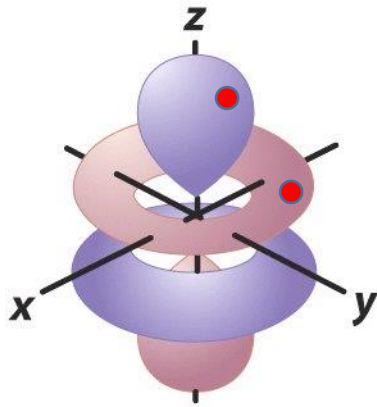


# The Schrödinger equation to the hydrogen atom: orbital 3d ( $n=3, l=2$ )

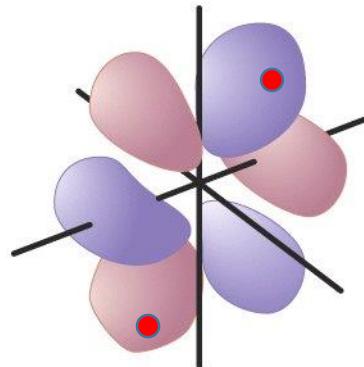


10 electrons maximum

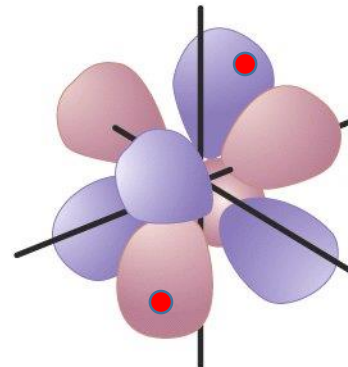
*The Schrödinger equation to the hydrogen atom: orbital 4d  
( $n=4, l=3$ )*



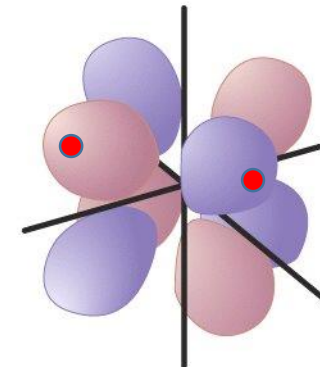
$$5z^3 - 3zr^2$$



$$5xz^2 - xr^2$$

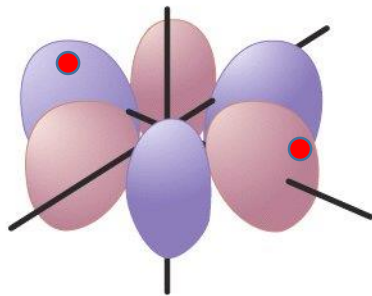


$$zx^2 - zy^2$$

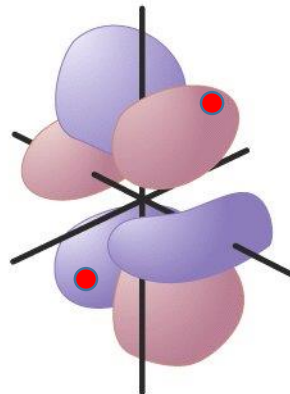


$$xyz$$

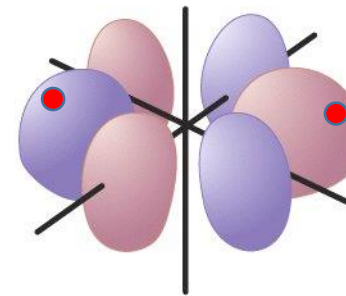
*14 electrons maximum*



$$y^3 - 3yx^2$$



$$5yz^2 - yr^2$$



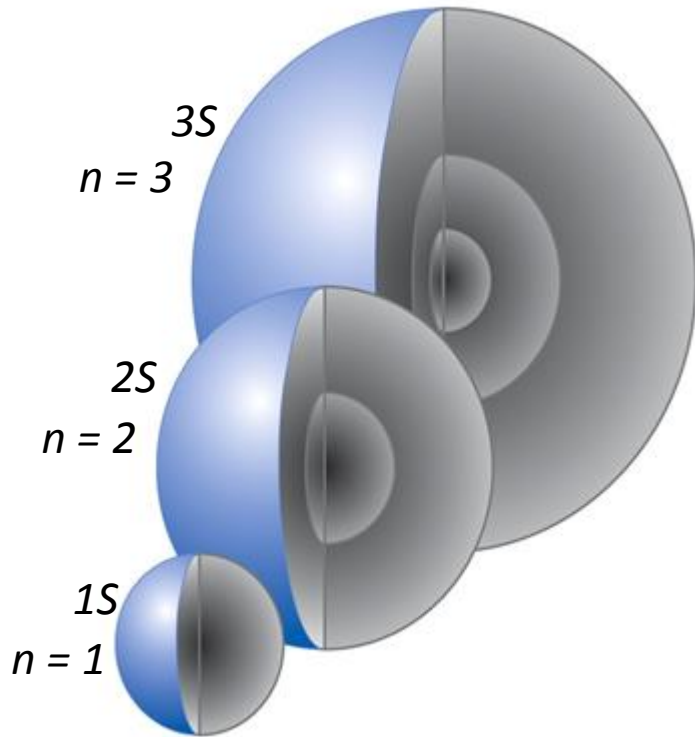
$$x^3 - 3xy^2$$



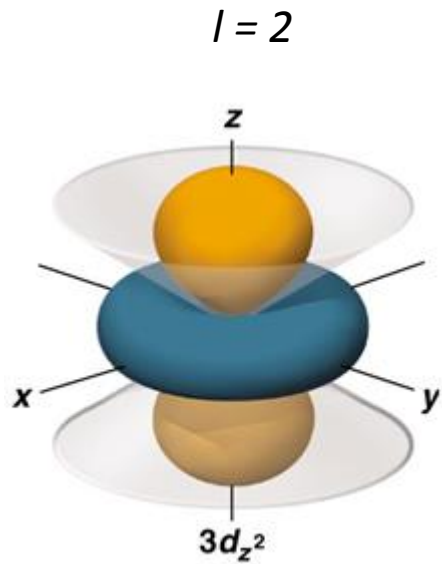
# Quantum Numbers

❖ The *principal quantum number* ( $n$ ) has possible values of:

$$n = 1, 2, 3, \dots \infty$$



*It describes the relative **size** of the orbital*

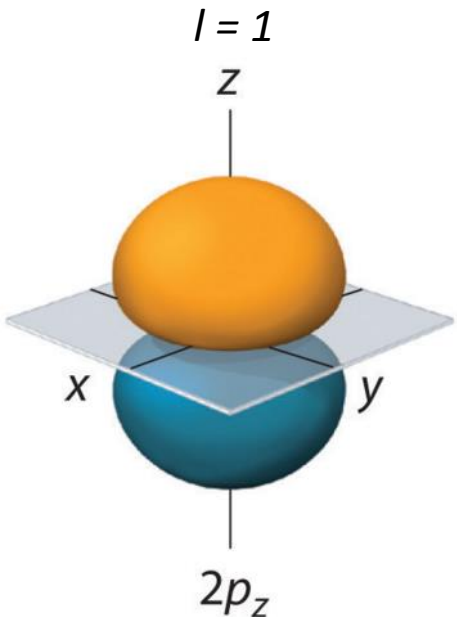


## Quantum Numbers

❖ The *angular momentum quantum number* ( $\ell$ )

has possible values of:

$$\ell = 0, 1, 2, \dots n-1$$

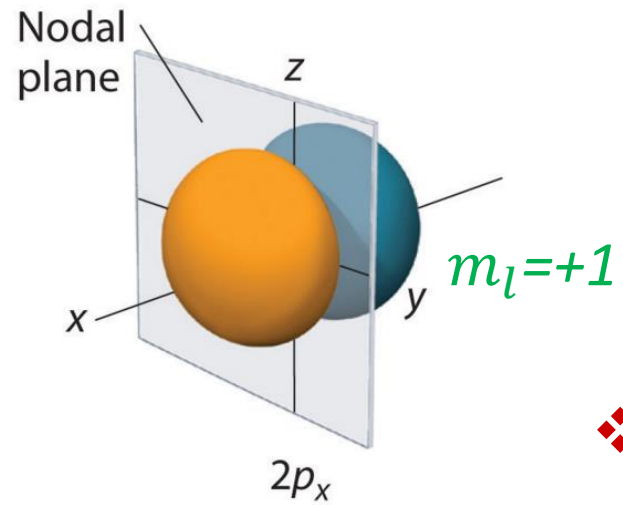


It describes the *shape* of the orbital.

☆ The value of  $\ell$  is often referred to by a letter equivalent;

$0 = s, 1 = p, 2 = d, 3 = f, \dots$  (the rest are alphabetical)

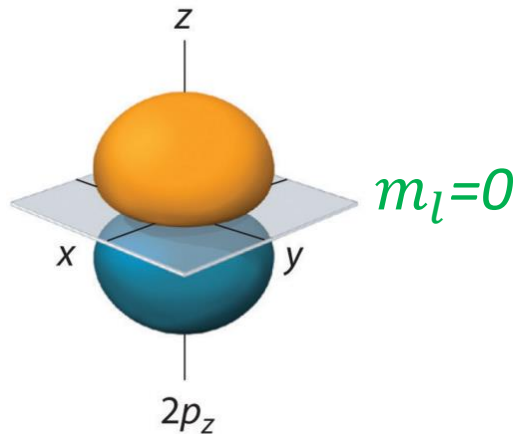
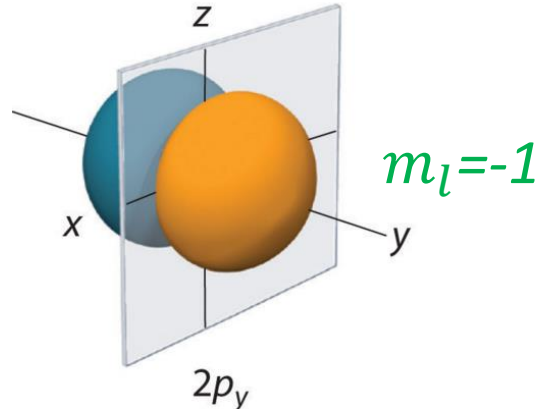




## Quantum Numbers

❖ *The magnetic quantum number ( $m_l$ ) has values:*

$$m_l = -l, \dots -1, 0, 1, \dots l$$



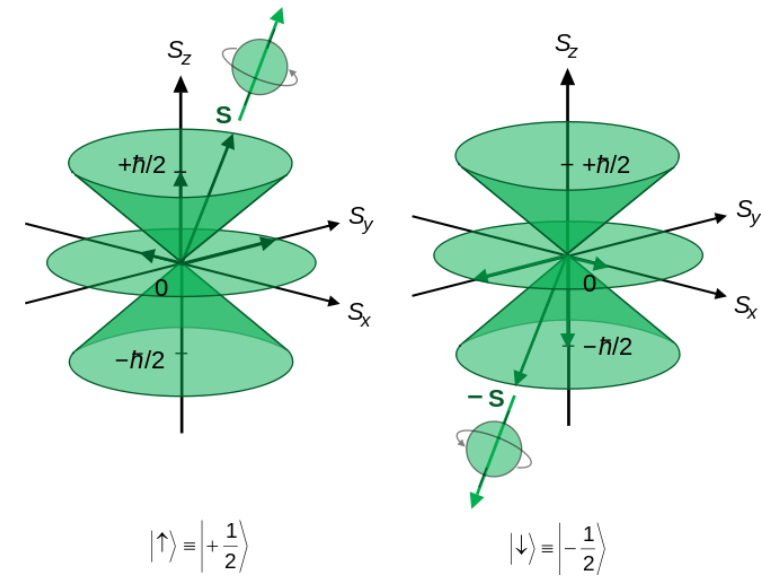
*It describes the **orientation** of the orbital in space.*

# Quantum Numbers

❖ *The spin angular momentum number ( $m_s$ )*  
has possible values of:

$$m_s = \pm \frac{1}{2}$$

*It describes the **orientation** of the spin.  
The vertical axis is used as reference.*



## *Pauli exclusion principle (1925)*

*Wolfgang Pauli postulated the (Pauli) exclusion principle, which states that no two electrons in one atom can exist in the same quantum state.*

*"State" refers to the four quantum numbers  $n$ ,  $\ell$ ,  $m_\ell$ ,  $m_s$ . Obviously, all electrons have the same  $s$ .*

# Subshells Schrödinger (1922)

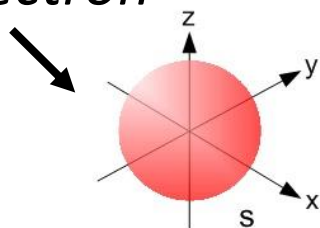


Subshell very important to capture or loss an electron

Subshell very important for the ligands

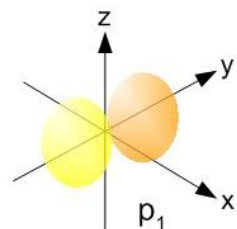
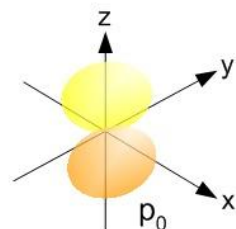
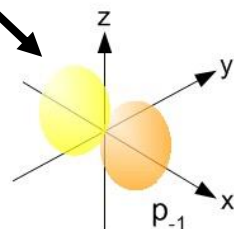
Subshell very important for the metals

Subshell very important for  
rare earths

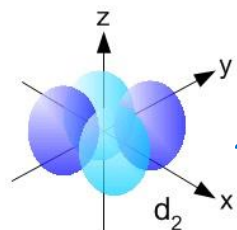
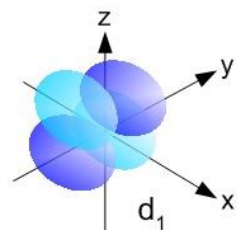
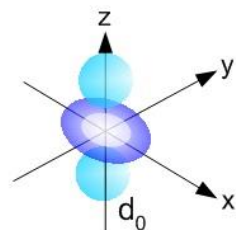
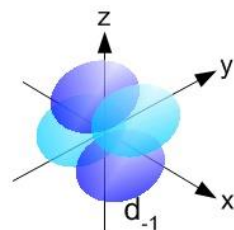
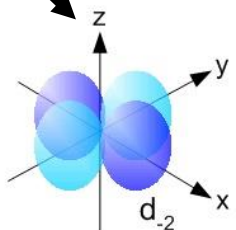


2 electrons max

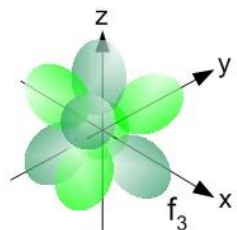
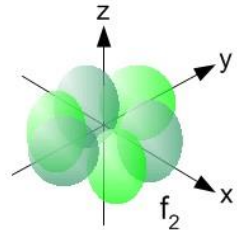
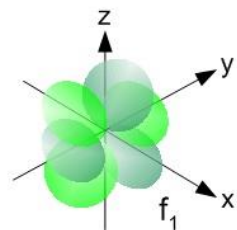
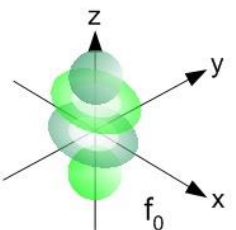
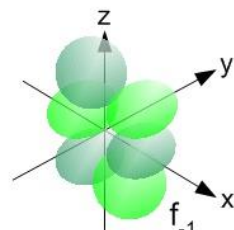
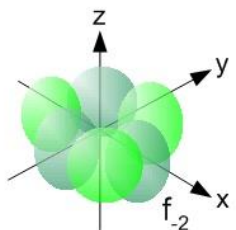
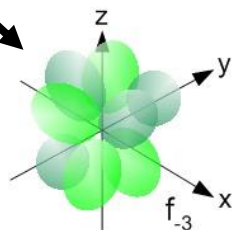
Pauli exclusion principle (1925)



6 electrons max

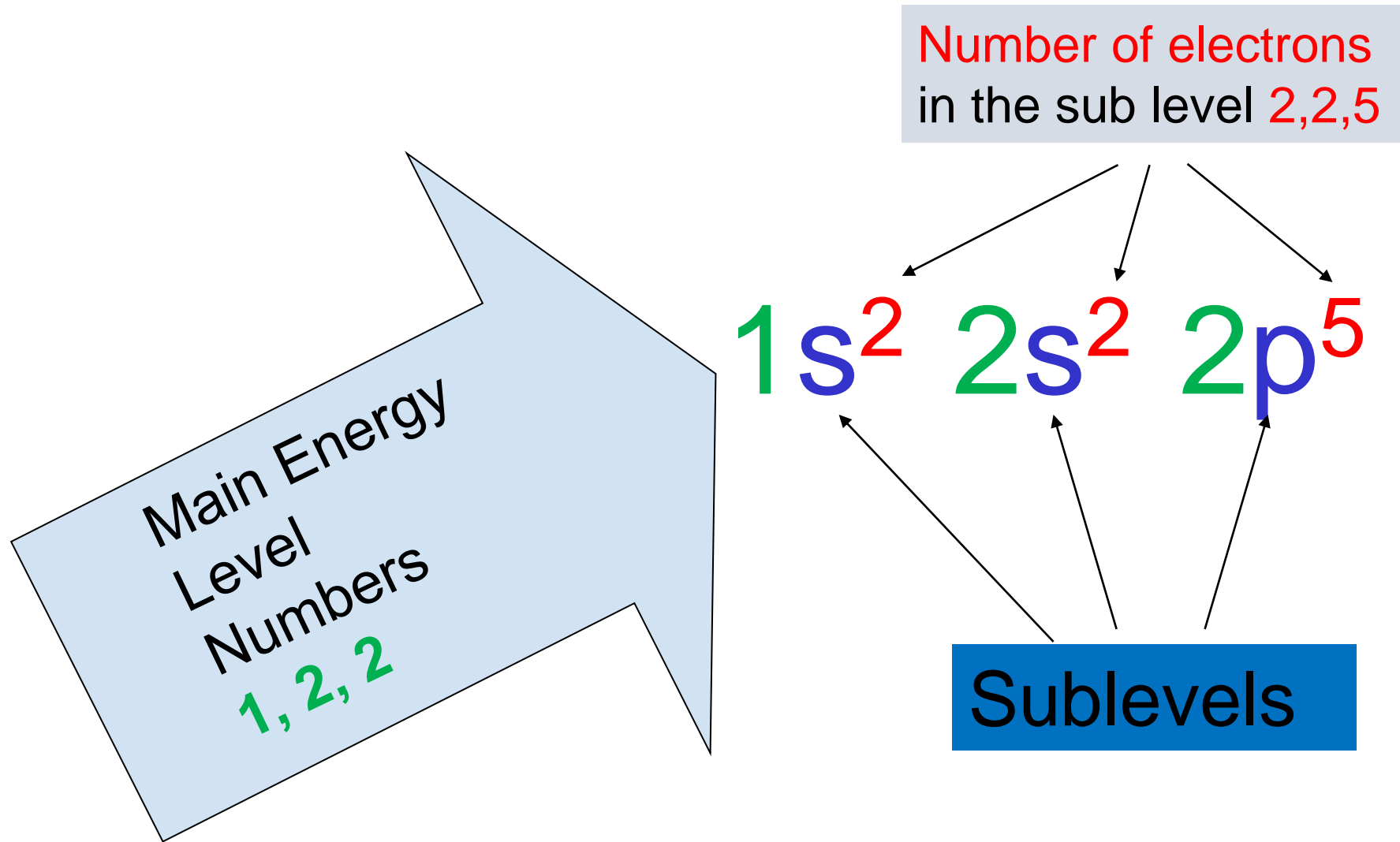


10 electrons max



14 electrons max

## Standard Notation of Fluorine

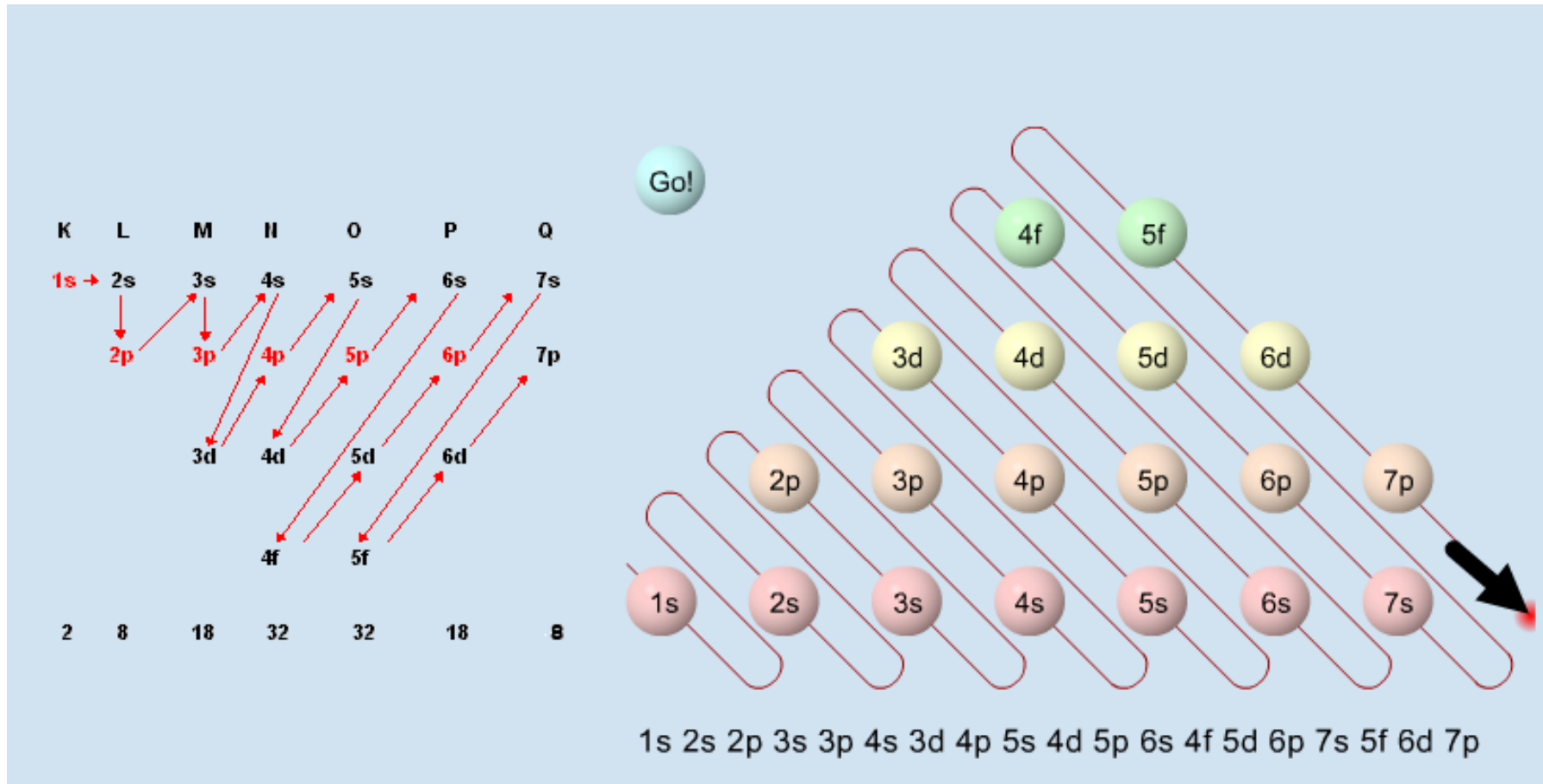


# Hund's rule

Orbital Filling Order (Diagonal Rule).

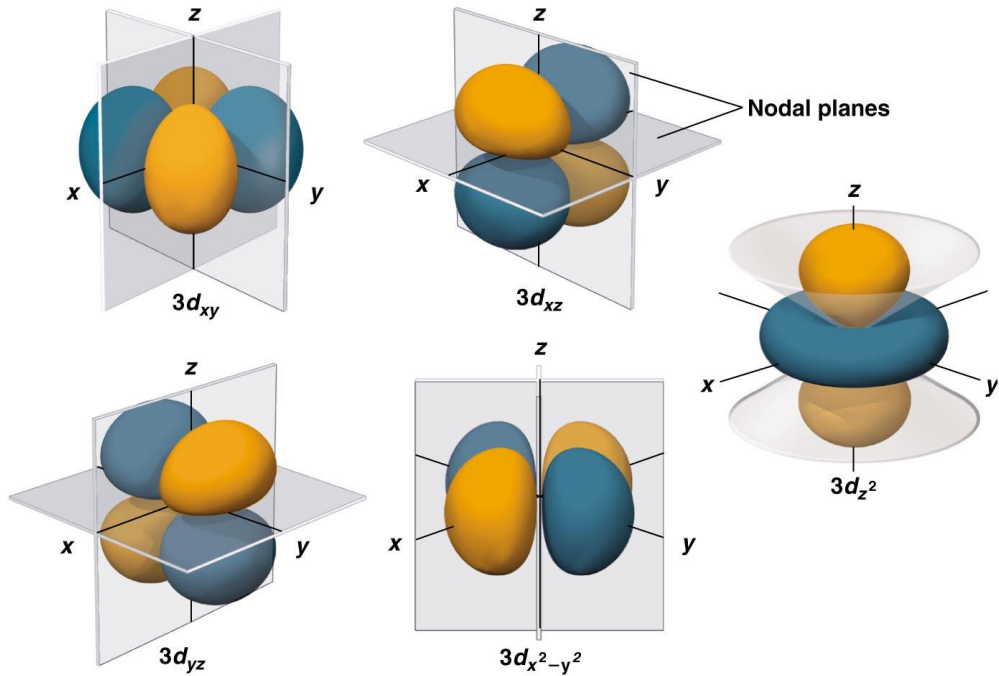
*It is a semi-empirical law with exceptions case*

*Minimize the coulomb interaction + Pauli exclusion principle.*



# The transition elements (d block)

orbital  $3d$  ( $n=3, l=2$ )



Periodic table showing the  $d$ -block transition elements (shaded blue). The  $d$ -block is labeled "TRANSITION ELEMENTS  $d$  block".

Electron configurations for the highlighted elements:

- Ti:  $[Ar] 3d^2 4s^2$  (represented as  $\uparrow \uparrow$ )
- Fe:  $[Ar] 3d^6 4s^2$  (represented as  $\uparrow\downarrow \uparrow \uparrow \uparrow \uparrow$ )
- Zn:  $[Ar] 3d^{10} 4s^2$  (represented as  $\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow \uparrow\downarrow \uparrow\downarrow$ )

10 electrons maximum



# Orbital occupancy for the transition metals

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Element	Partial Orbital Diagram									Unpaired Electrons
	4s	3d					4p			
Sc	↑↓	↑								1
Ti	↑↓	↑	↑							2
V	↑↓	↑	↑	↑						3
Cr	↑	↑	↑	↑	↑	↑				6
Mn	↑↓	↑	↑	↑	↑	↑				5
Fe	↑↓	↑↓	↑	↑	↑	↑				4
Co	↑↓	↑↓	↑↓	↑	↑	↑				3
Ni	↑↓	↑↓	↑↓	↑↓	↑	↑				2
Cu	↑	↑↓	↑↓	↑↓	↑↓	↑↓				1
Zn	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓				0

*Paramagnetic*

*Antiferromagnetic !!!!*

*Néel temperature*

*Antiferromagnetic*

*or paramagnetic !!!!*

*Néel temperature*

*Ferromagnetic*

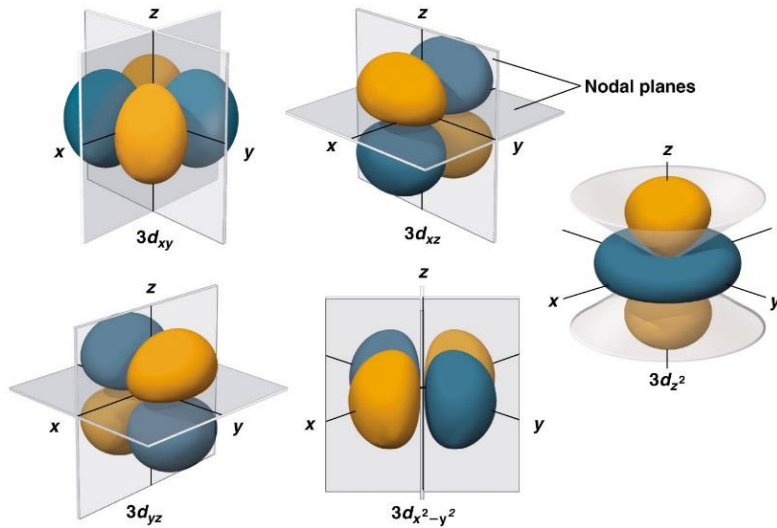
*Diamagnetic*

**3d ferromagnetism**

- 1) When the sub shell is not full.
- 2) Depend of the distance between the atoms and the radius shell 3d.
- 3) Exchange energy (Heisenberg 1927)

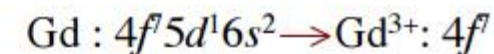
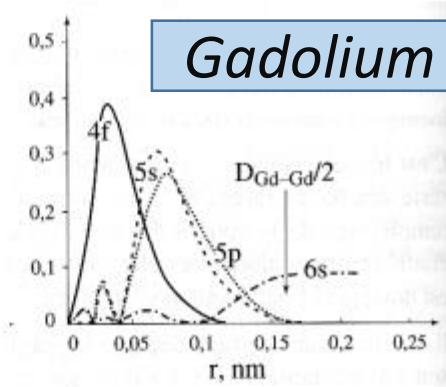
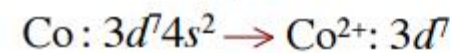
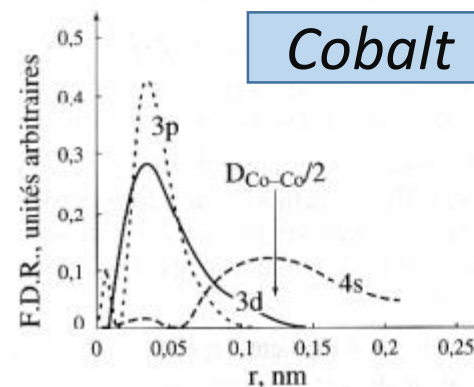
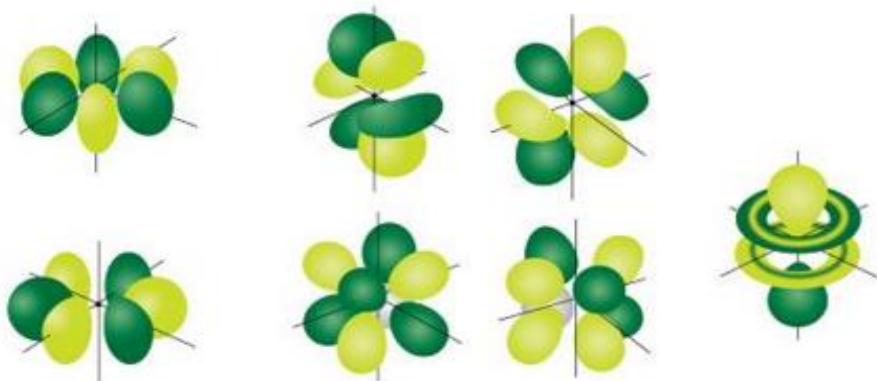
*It is preferable to use the exchange model*

## 3d subshell



H																	He
Li	Be											B	C	N	O	F	Ne
para	dia											dia	dia	dia	AF	dia	dia
Na	Mg											Al	Si	P	S	Cl	Ar
para	para											para	dia	dia	dia	dia	dia
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
para	para	para	para	para	AF	AF	Ferro	Ferro	Ferro	dia	dia	dia	dia	dia	dia	dia	dia
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
para	para	para	para	para	para		para	para	para	dia	dia	dia	*	dia	dia	dia	dia
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
para			para	para	para	para	para	para	para	dia	dia	dia	dia	dia			dia
Fr	Ra	Ac															
			Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	
			*	para	AF		AF	Ferri	Ferro	Ferro	Ferro	Ferri	Ferri	Ferri	para	para	
			Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lw	
			para		para												

## 4f subshell

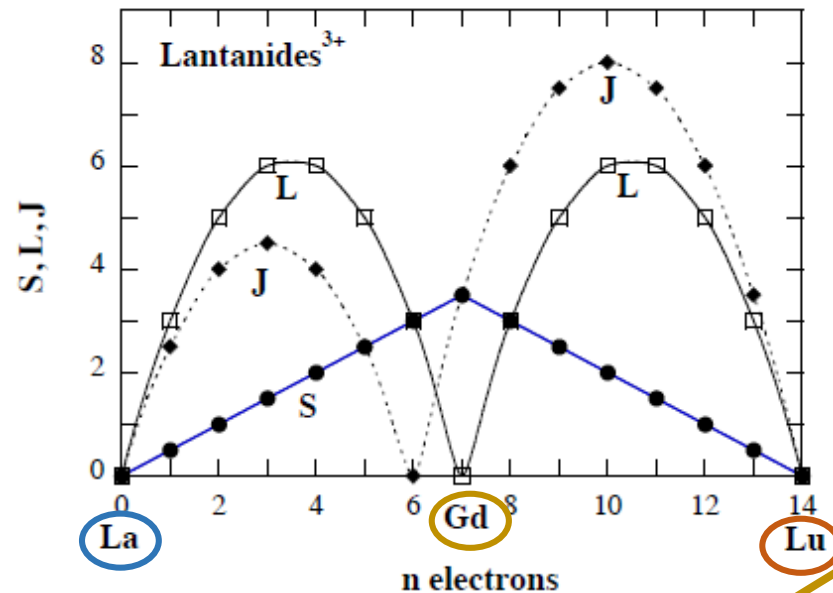


**3d magnetism**  
created by  
itinerant electrons

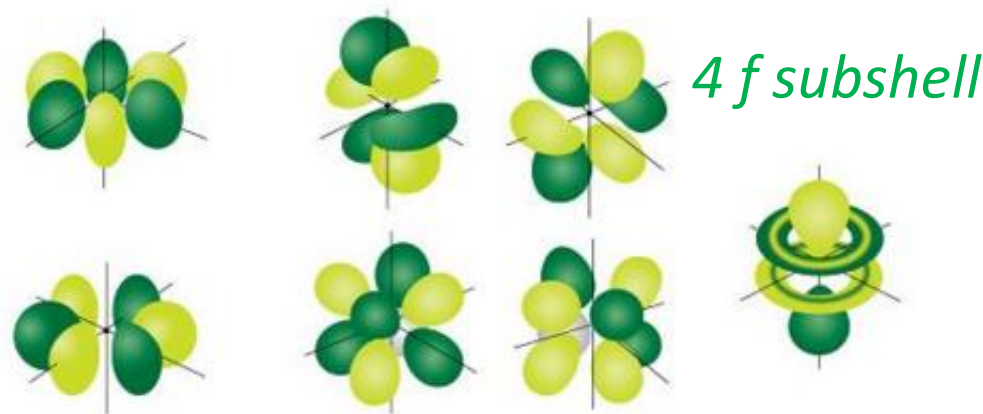
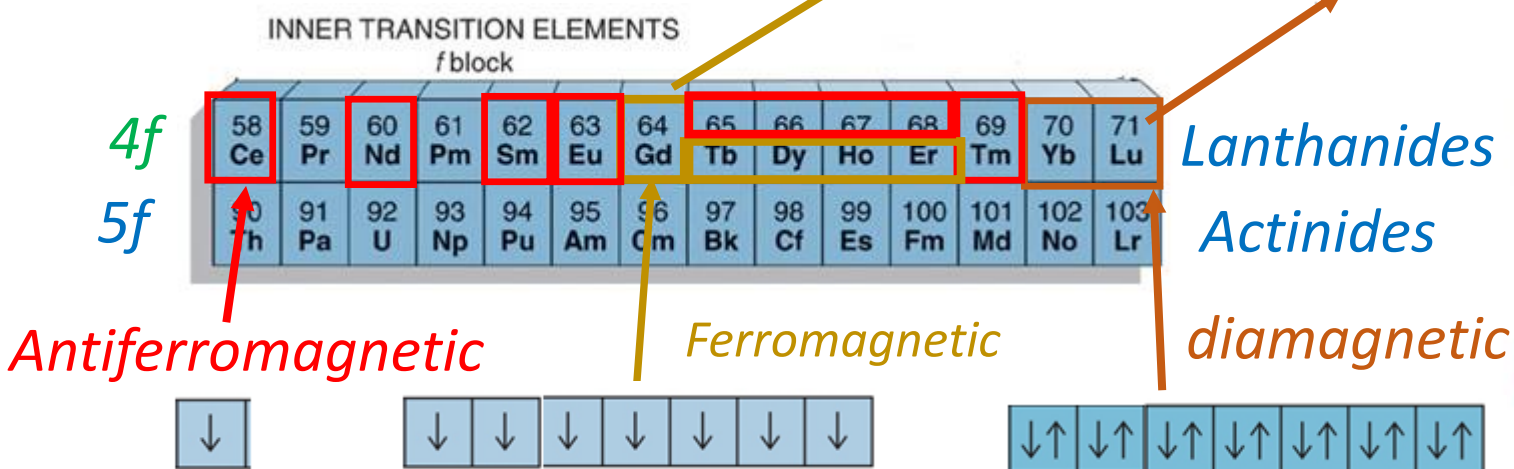
**4f local magnetism**  
created by  
the atoms

# Lanthanides (4f block)

Paramagnetic properties depend temperature !

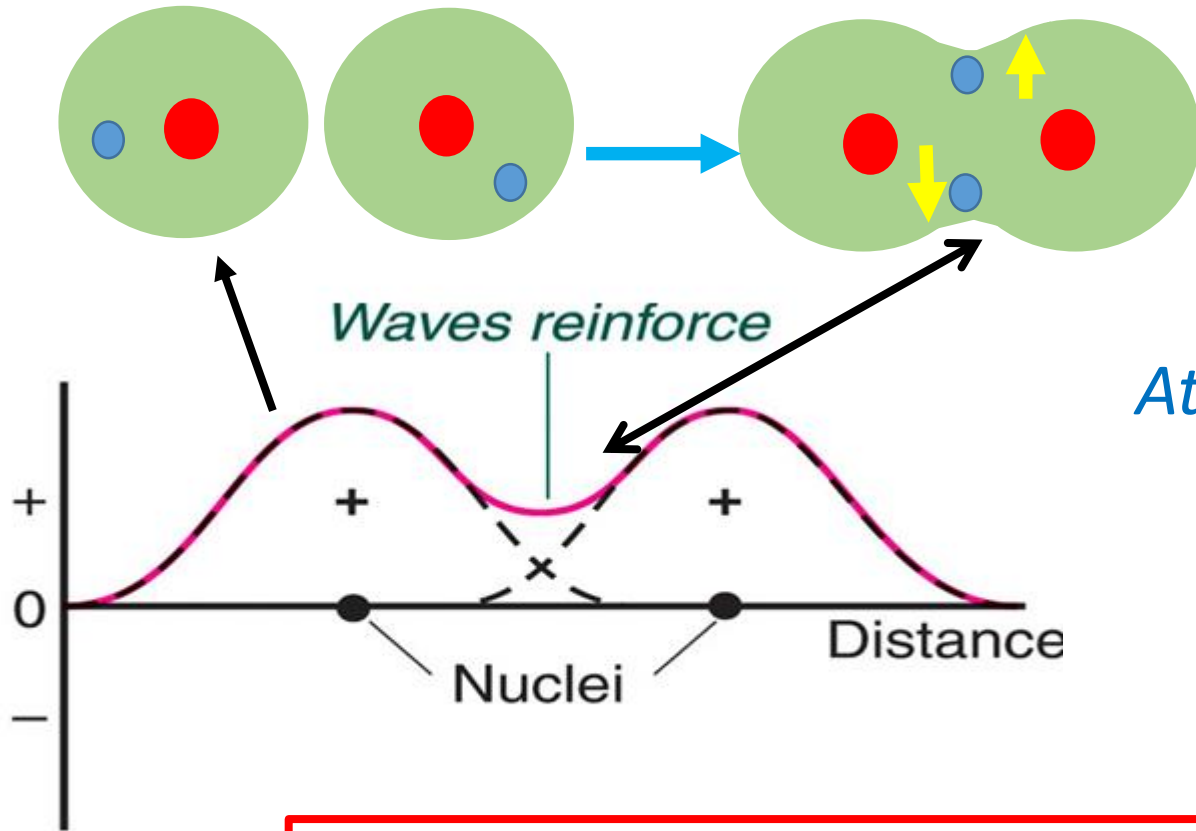


ion	shell	$m_l$							$S$	$L$	$J$	term
		+3	+2	+1	0	-1	-2	-3				
Ce $^{3+}$	4f $^1$	↓							$\frac{1}{2}$	3	$\frac{5}{2}$	$^2F_{5/2}$
Pr $^{3+}$	4f $^2$	↓	↓						1	5	4	$^3H_4$
Nd $^{3+}$	4f $^3$	↓	↓	↓					$\frac{3}{2}$	6	$\frac{9}{2}$	$^4I_{9/2}$
Pm $^{3+}$	4f $^4$	↓	↓	↓	↓				2	6	4	$^5I_4$
Sm $^{3+}$	4f $^5$	↓	↓	↓	↓	↓			$\frac{5}{2}$	5	$\frac{5}{2}$	$^6I_{5/2}$
Eu $^{3+}$	4f $^6$	↓	↓	↓	↓	↓	↓		3	3	0	$^7F_0$
Gd $^{3+}$	4f $^7$	↓	↓	↓	↓	↓	↓	↓	$\frac{7}{2}$	0	$\frac{7}{2}$	$^8S_{7/2}$
Tb $^{3+}$	4f $^8$	↑↓	↑	↑	↑	↑	↑	↑	3	3	6	$^7F_6$
Dy $^{3+}$	4f $^9$	↑↓	↑↓	↑	↑	↑	↑	↑	$\frac{5}{2}$	5	$\frac{15}{2}$	$^6H_{15/2}$
Ho $^{3+}$	4f $^{10}$	↑↓	↑↓	↑↓	↑	↑	↑	↑	2	6	8	$^5I_8$
Er $^{3+}$	4f $^{11}$	↑↓	↑↓	↑↓	↑↓	↑	↑	↑	$\frac{3}{2}$	6	$\frac{15}{2}$	$^4I_{15/2}$
Tm $^{3+}$	4f $^{12}$	↑↓	↑↓	↑↓	↑↓	↑↓	↑	↑	1	5	6	$^3H_6$
Yb $^{3+}$	4f $^{13}$	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑	$\frac{1}{2}$	3	$\frac{7}{2}$	$^2F_{7/2}$
Lu $^{3+}$	4f $^{14}$	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	0	0	0	$^1S_0$



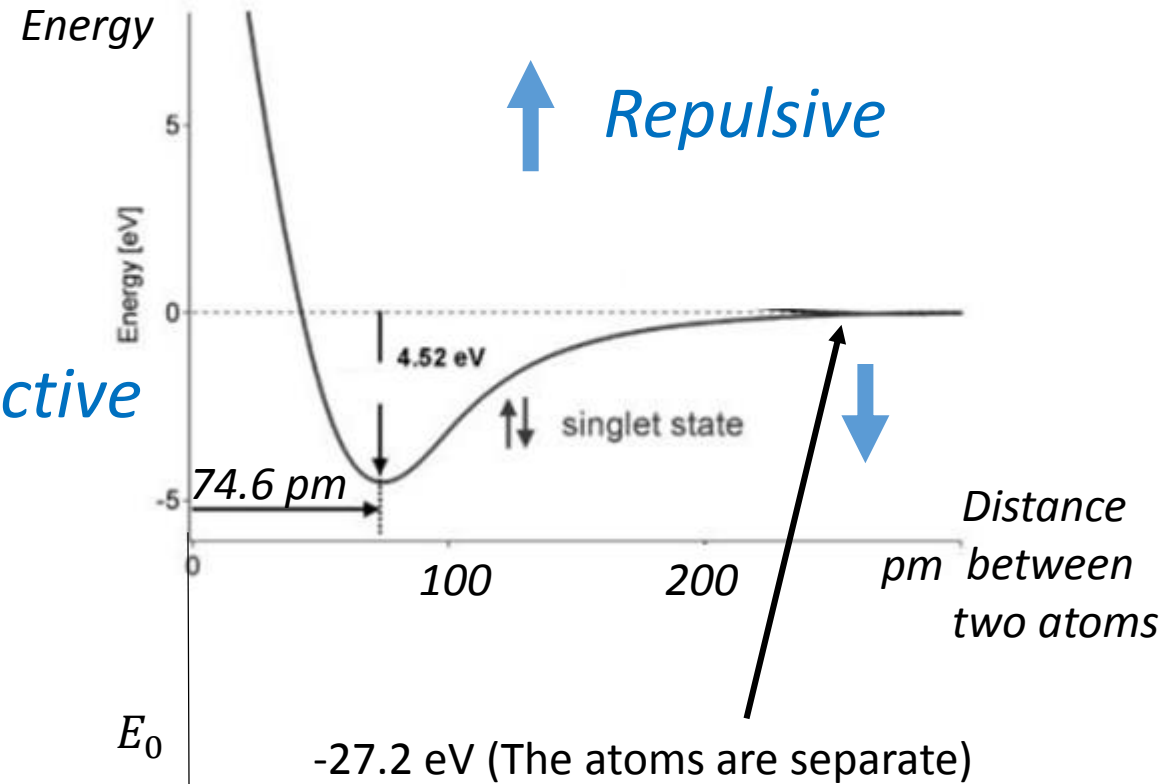
## Orbital model for $H_2$

*Bonding*



*Pauli exclusion principle forbid the same spin orientation.*

*Attractive*

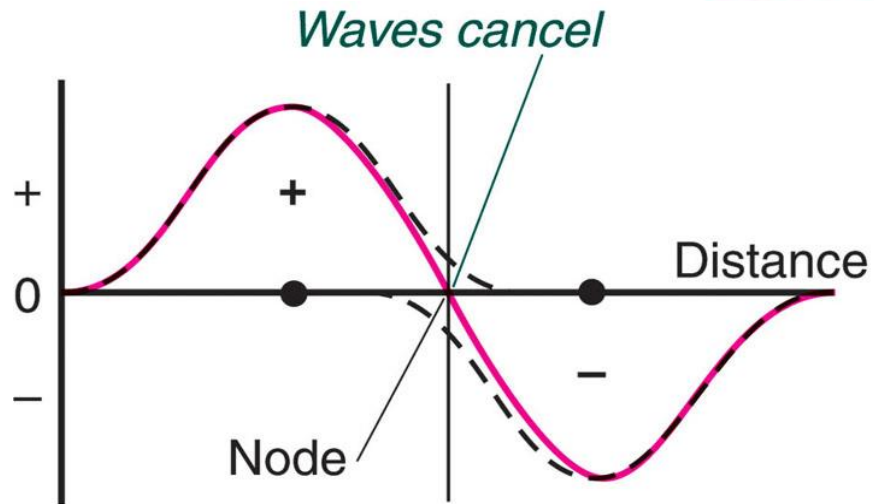


*symmetric for the waves,  
antisymmetric for the spin*



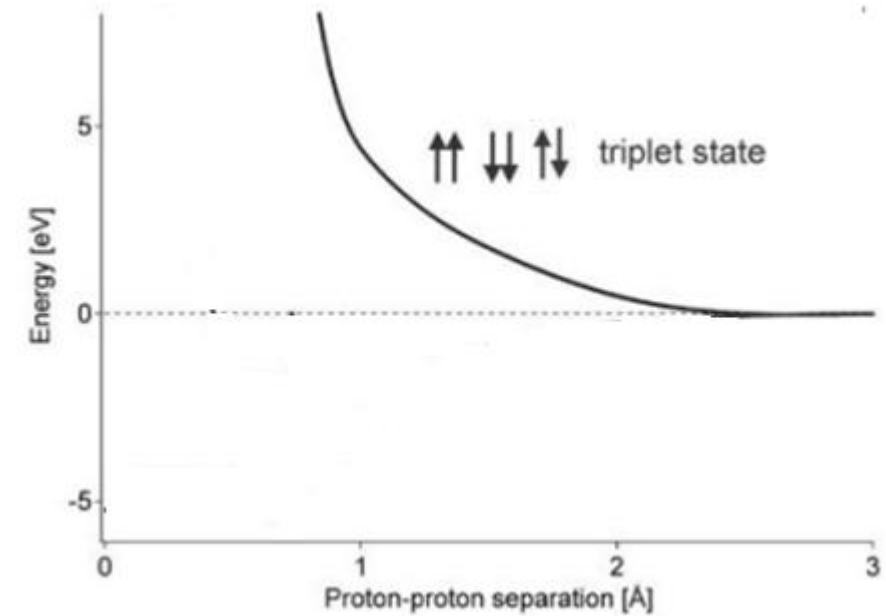
## Orbital model for $H_2$

### Antibonding



Antibonding

Not stable (Coulomb  
repulsion between protons)



Two different orbits  $1s$  allow the same spin  
orientation.

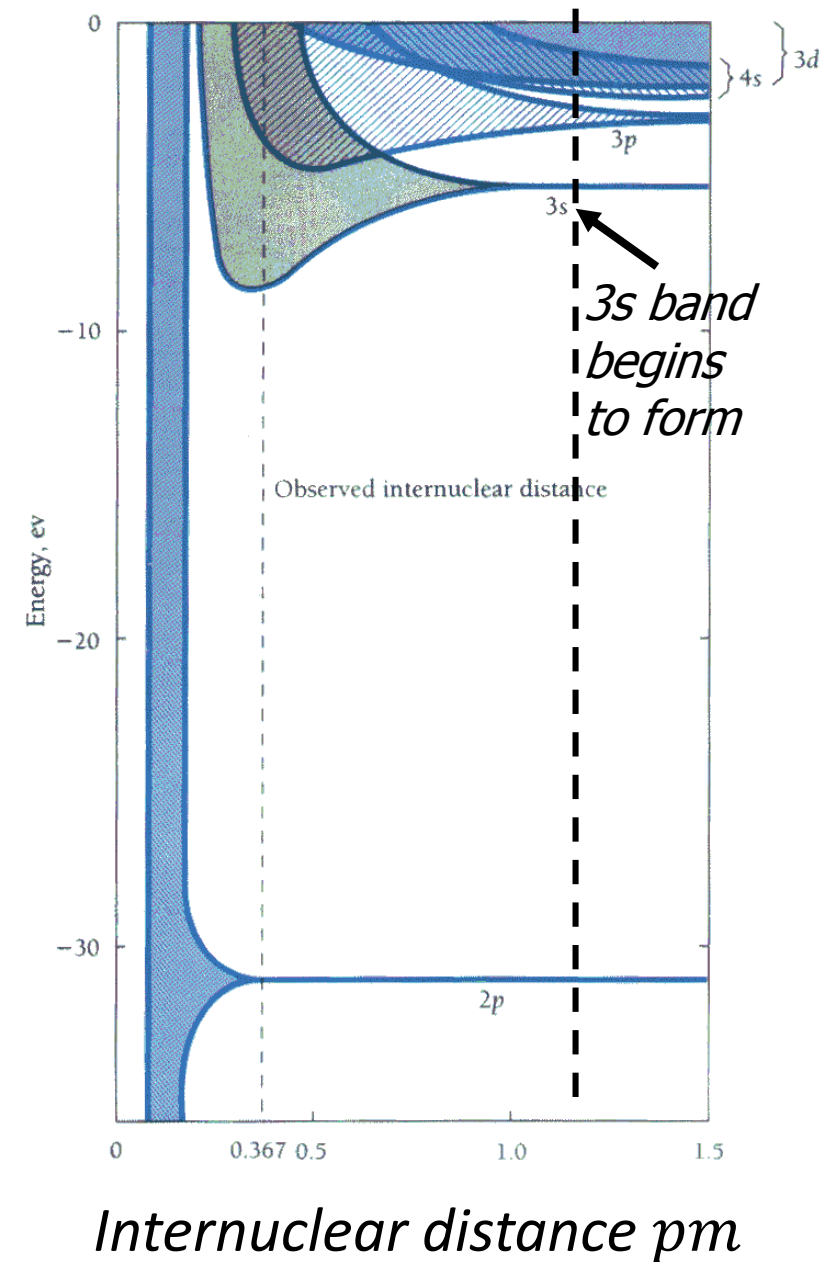
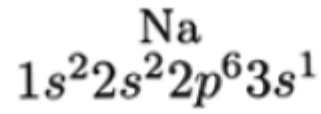
symmetric for the spin,  
antisymmetric for the waves

## Band theory (Solid state physics)

*Now let's take a closer look at the energy levels in solid sodium. Remember, the 3s is the outermost occupied level*

*When sodium atoms are brought within about 1 nm of each other, the 3s levels in the individual atoms overlap enough to begin the formation of the 3s band.*

*The 3s band broadens as the separation further decreases*



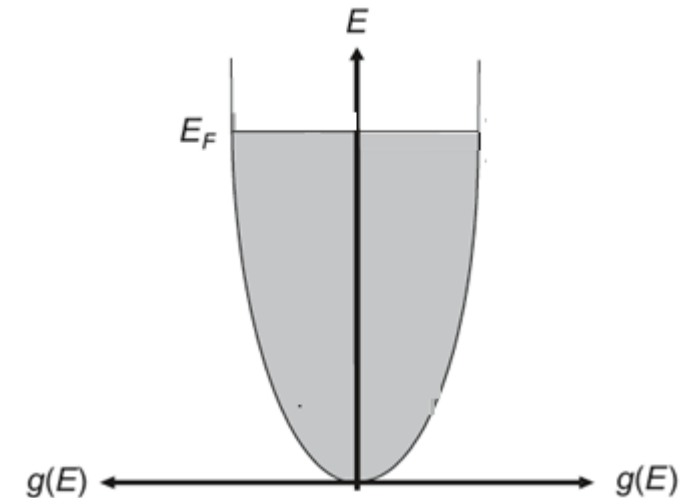
# Sommerfeld model of free electrons (1928)



- 1) We are in the non relativist case.
- 2) We don't consider the full subshells  $\Sigma L_i=0$  and  $\Sigma S_i=0$  until the subshell 3p (included). It is a have a positive ion.
- 3) We consider the itinerant electrons as a gas (the electrons inside the subshell 3d and the last subshell 4s).
- 4) The itinerant electrons have a kinetic energy only.
- 5) It is a first approximation.

## Heisenberg uncertainty principle (1927)

$$\Delta p * \Delta r \geq \hbar/2$$



- 1) The Pauli exclusion principle and the uncertainty principle limit the number of electrons with a low velocity.
- 2) If you increase the number of electrons, you must increase their velocity because all the states with a lower energy are busy...



# Fermi-Dirac distribution (Sommerfeld model)

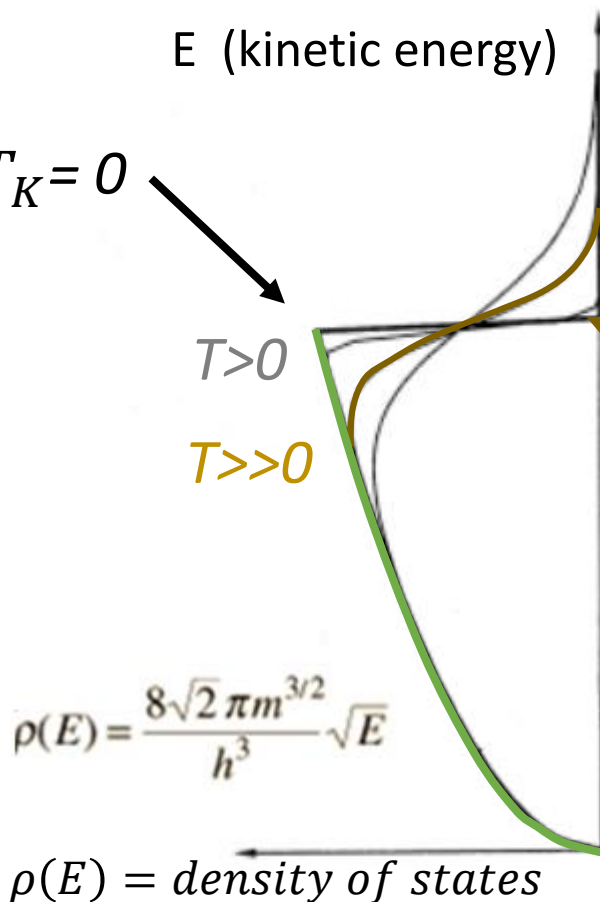
- Each state can hold 2 electrons of opposite spin (Pauli's principle).
- Near zero degree Kelvin the free electrons have a kinetic energy.

For a transition metal, the augmentation of temperature is created by the nucleus vibrations . These vibrations obey to a Bose-Einstein distribution.

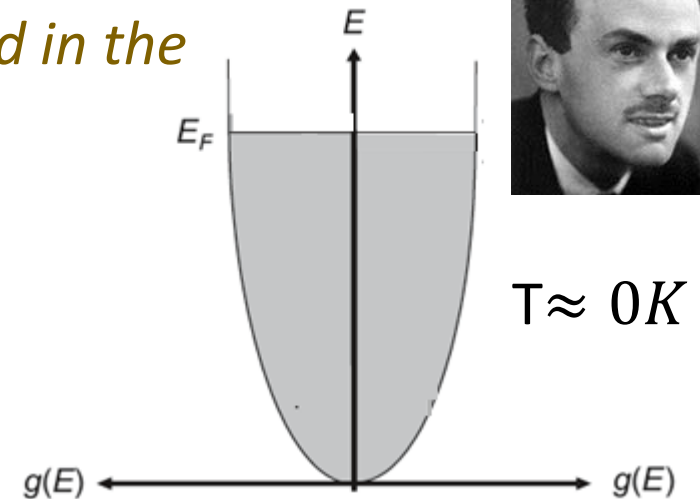
Only the free electrons near the Fermi are sensitive to the temperature (Fermi-Dirac distribution).

The spin of these particles is oriented in the sense that the external field.

$E_F$  (Fermi energy)



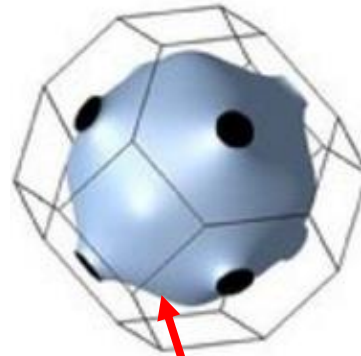
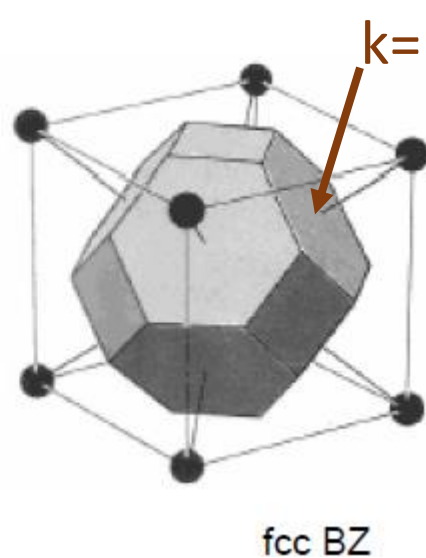
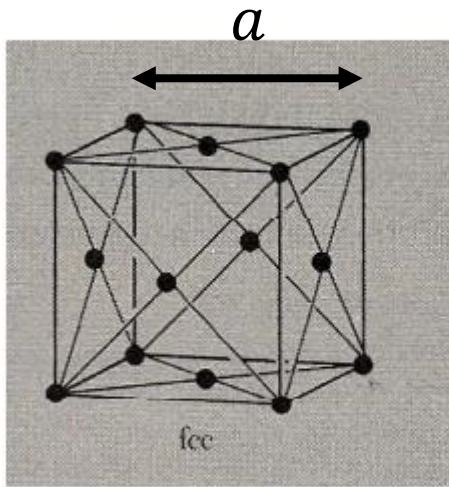
$$E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2}{2m} (3\pi^2 \eta_e)^{2/3}$$



# Fermi Parameters for some metals (Sommerfeld model)

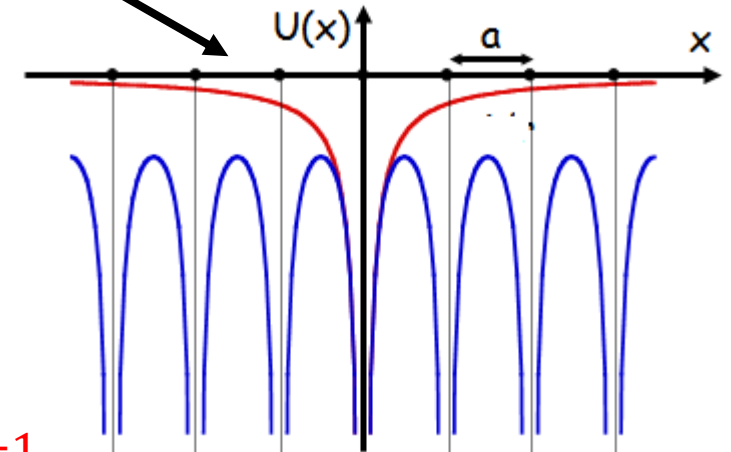
Element	Molar volume	electrons	density	Fermi energy calculated	Fermi temperature	Fermi velocity	electrons cm-3
	$m^3/\text{mol}$		$\text{g}/\text{cm}^3$	$E_F[\text{eV}]$	$T_F[10^3\text{K}]$	$10^8 \text{ cm}^{-1}$	
Scandium	15,00×10-6	21	2.985				4.01 10^22
Titanium	10,64×10-6	22	4.506				5.67 10^22
Vanadium	8,32×10-6	23	6.01				7.20 10^22
Chromium	7,23×10-6	24	7.19				8.33 10^22
manganese	7,35×10-6	25	7.31	10.9	12.7	1.96	8.19 10^22
Fe	7,09×10-6	26	7.874	11.15	12.94	1.98	8.49 10^22
Co	6,67×10-6	27	8.9	11.7	13.58	2.03	9.01 10^22
Ni	6,59×10-6	28	8.902	11.74	13.62	2.03	9.14 10^22
Cu	7,11×10-6	29	8.96	7.04	8.17	1.57	8.47 10^22
Zn	9,16×10-6	30	7.14	9.47	10.9	1.82	6.57 10^22

# Fermi surface for Ag Bloch model 1946 (crystallography)



Atomic nucleus

$$k = \frac{4\pi}{a} = 39.2 \cdot 10^9 \text{ m}^{-1}$$

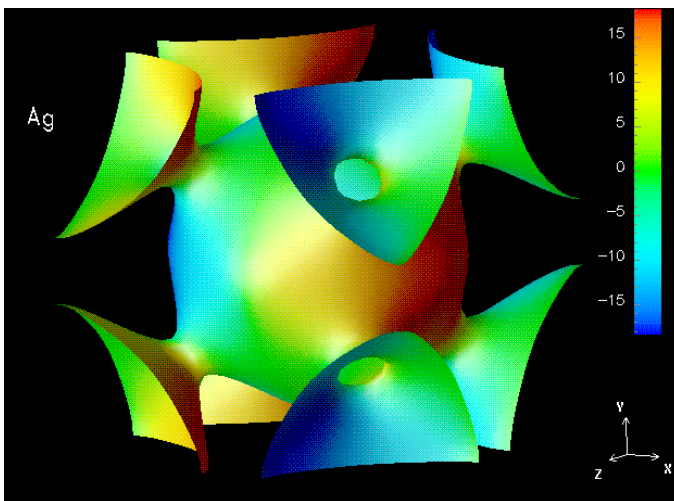


Fermi surface

$$k_F = 12 \cdot 10^9 \text{ m}^{-1}$$

Crystalline structure for Ag

The reference for  $k$  is the radius of the atom



$$p \cdot \lambda = h$$

$$p = \hbar \cdot k$$

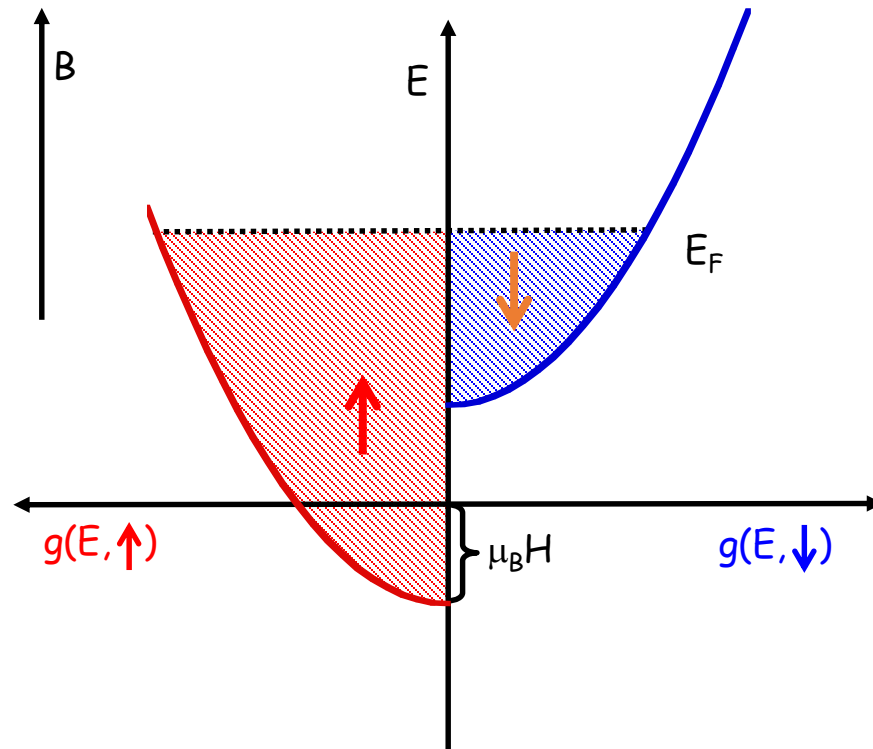
A velocity in a direction  
isn't possible if the wavelength is  
already occupied by a distance  
between two atoms.

$$\frac{p}{\hbar} = k < 4\pi/a$$



# Free Electrons for the metals in a Magnetic Field (Pauli paramagnetism)

*“The difference between paramagnetism and Pauli paramagnetism is that the latter applies to a metal because it describes the tendency of free electrons in an electron gas to align with an applied magnetic field.” [Inna Vishik](#) (Stanford)*



Magnetic Spin – Susceptibility

Low temperature

$$\chi_P = \frac{M}{H} = \frac{3\eta_e \mu_0 \mu_B^2}{2E_F}$$

(Pauli Paramagnetism)



W. Pauli

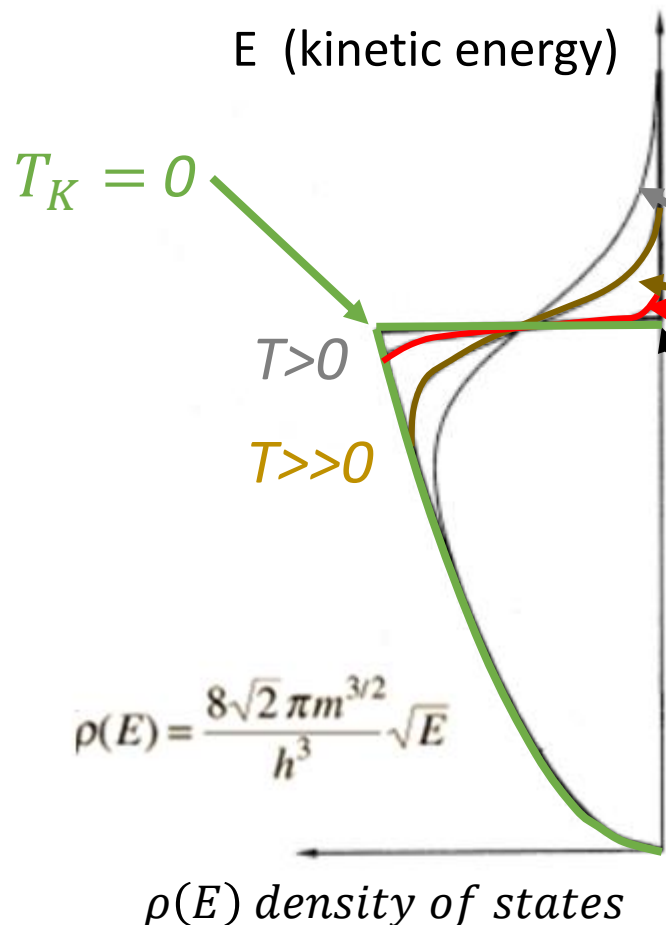
Nobel Price 1945

Titanium	<a href="#">Pauli magnetic susceptibility (<math>\chi_{mol}</math>)</a>	$33.4 \cdot 10^{-6} \text{ cm}^3 / \text{mol}$
Vanadium	<a href="#">Pauli magnetic susceptibility (<math>\chi_{mol}</math>)</a>	$84.2 \cdot 10^{-6} \text{ cm}^3 / \text{mol}$

# Paramagnetism for free electrons.

Fermi gas of electrons without  
an external magnetic field.

The electrons are free (no subshell)



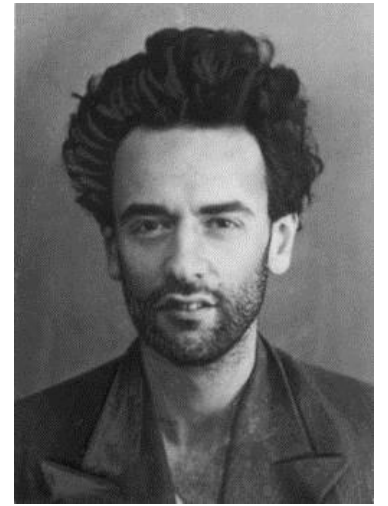
- 1) The order of the Fermi kinetic energy is high  $T \approx 10\,000\text{ K}$ .
- 2) The reference temperature is  $\approx 300\text{ K}$ .
- 3) The thermal energy can increase the kinetic energy only for a very small number of particles.
- 4) The spin of these particles is oriented in the sense that the external field.
- 5) The paramagnetism is created by these electrons...

$$\chi_{\text{Pauli}} \sim \chi_{\text{curie}} \left( \frac{T}{T_F} \right) \sim \chi_{\text{Pauli}} 10^{-2}$$

$T \approx 0\text{ K}$

$T \approx 300\text{ K}$

## Landau diamagnetism (1930) $T \approx 0K$



- 1) A gas of free electrons in a magnetic field.
- 2) Free electrons move along spiral trajectories.
- 3) Lenz's law.
- 4) Diamagnetic effect.
- 5) The energy of the free electrons depend of
  - A) The kinetic energy is principally limited by the energy of Fermi.
  - B) The quantification of the energy created by the circular movement of the electrons

$$E_l = \left(l + \frac{1}{2}\right) \hbar \omega_c. \quad M_{Landau} = -\frac{N \mu^2}{2k_B T_F} B \quad \chi_{Landau} = -\frac{\chi_{Pauli}}{3}$$

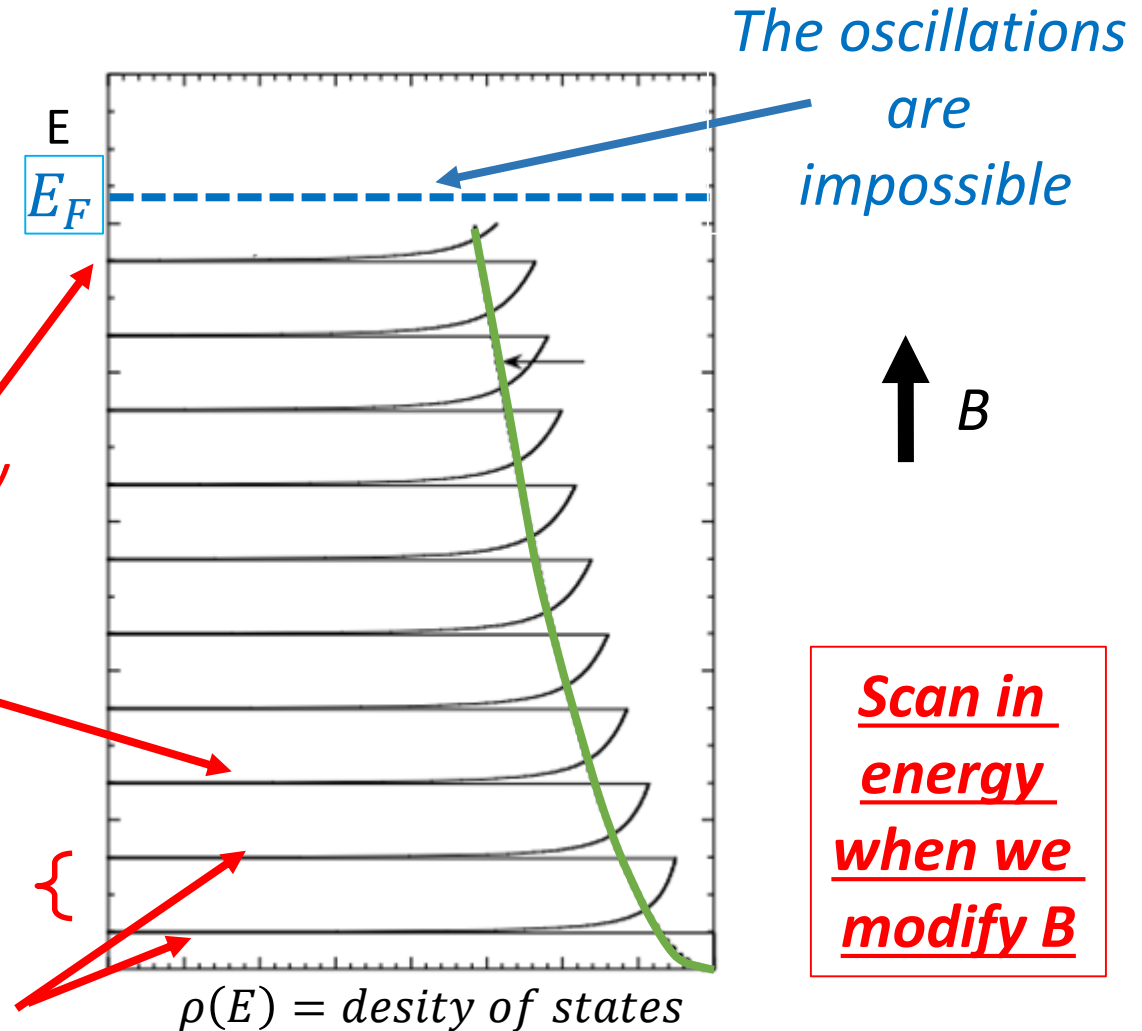
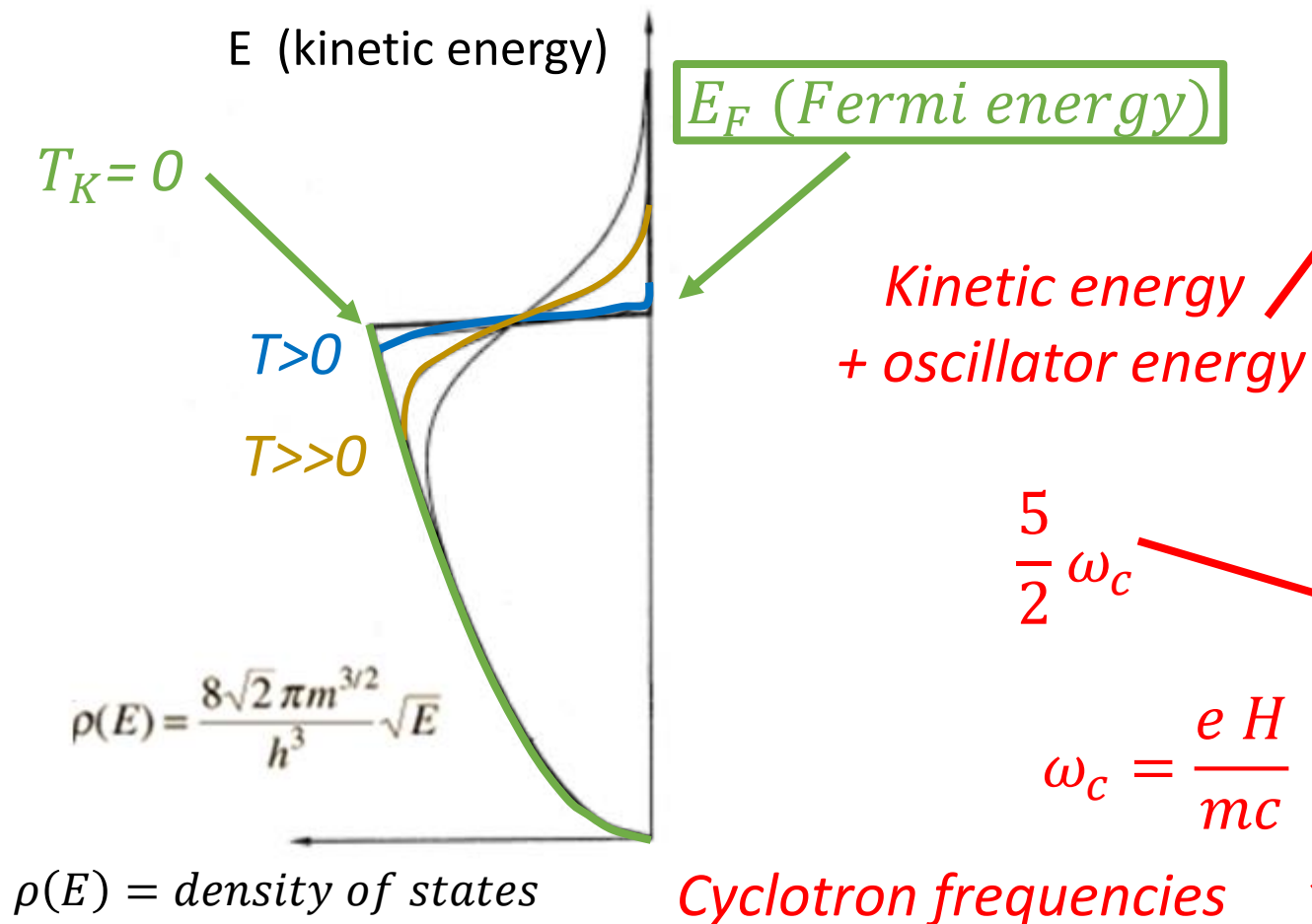
Titanium	<u>Landau diamagnetic susceptibility (<math>\chi_{mol}</math>)</u>	$-11.1 \cdot 10^{-6} \text{ cm}^3/\text{mol}$
Vanadium	<u>Landau diamagnetic susceptibility (<math>\chi_{mol}</math>)</u>	$-28.6 \cdot 10^{-6} \text{ cm}^3/\text{mol}$



*Landau diamagnetism  $T \approx 0K$*

*Fermi gas of electrons without  
an external magnetic field.*

*Fermi gas of electrons with  
an external magnetic field.*





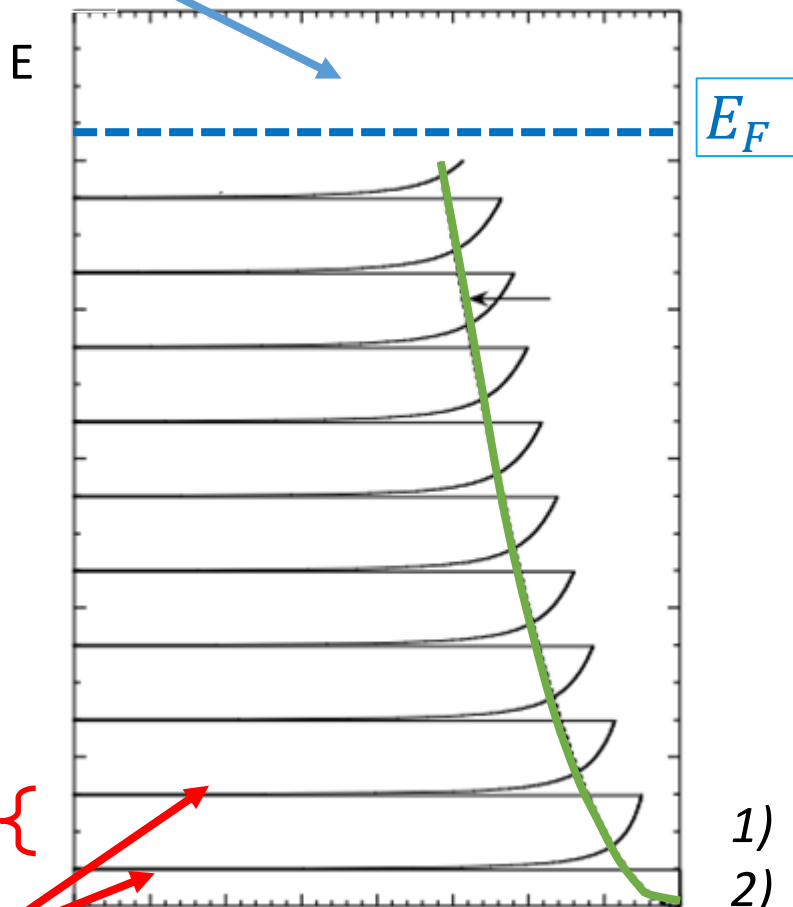
# Haas-van Halphen effect (1930) $T \approx 0K$

The oscillations  
are  
impossible

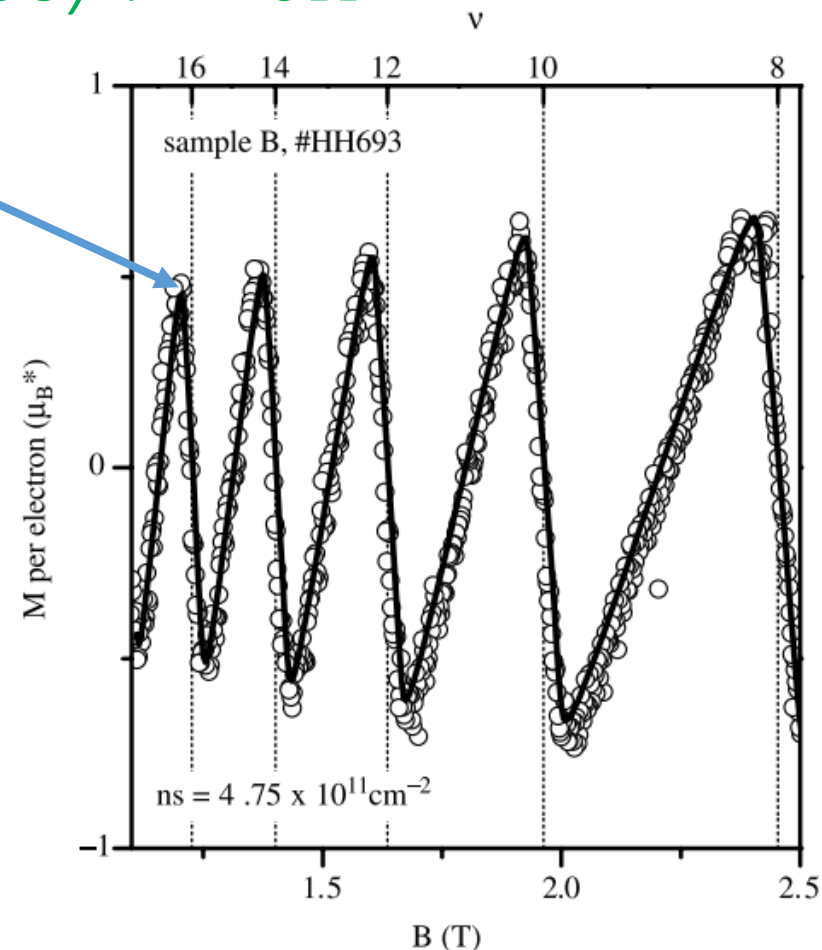
$\omega_c \uparrow$  if  $H \uparrow$

$$\omega_c = \frac{e H}{mc}$$

$\rho(E)$  = density of states



$E_F$

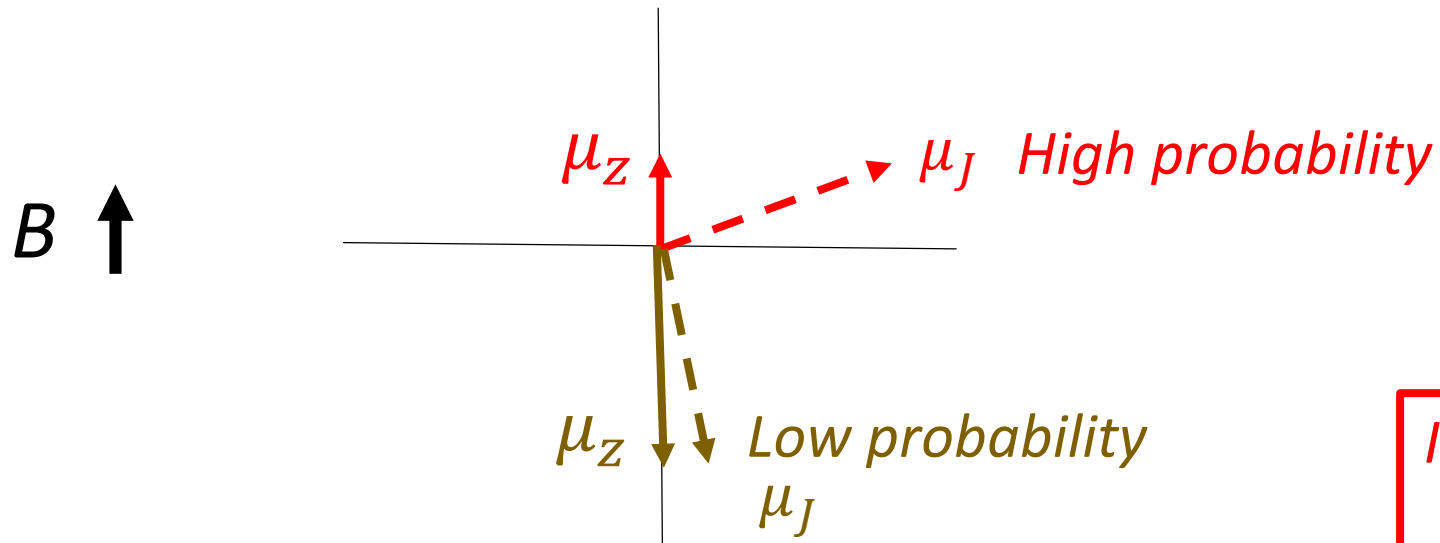


- 1) The cyclotron frequencies increase with the magnetic field.
- 2) The diamagnetic field increase with the energy.
- 3) The diamagnetic field stopped with the  $E_F$ .
- 4) The diamagnetic field increase again with the next cyclotron frequency.

Cyclotron frequencies

# Brillouin-Langevin paramagnetism (statistical physics)

- 1) Each atom is independent.
- 2) For each atom, the total magnetic moment  $\mu_J$  (orbitals + spin) is the same for each atom. It is calculated with quantum mechanics.
- 3) The distribution of the magnetic moments obeys the Boltzmann distribution.
- 4) We calculate  $\langle \text{total magnetic moment} \rangle$  for one atom and we multiply by  $N$ .
- 5) We use the  $Z$  axis as reference.



$$\mu_J = g_J J \mu_b$$

$$\mu_z = g_J m \mu_b$$

*In quantum mechanics the number of orientations is limited*

# Paramagnetism (quantum mechanics)

Scandium	<u>Electrical resistivity</u>	$55 \times 10^{-8} \Omega \text{ m}$ (at 20 °C)
	<u>Magnetic susceptibility (<math>\chi_{\text{mol}}</math>)</u>	$+ 3.956 \cdot 10^{-9} \text{ m}^3/\text{mol}$
	<u>Electronegativity</u>	Pauling scale: 1.36
Titanium	<u>Electrical resistivity</u>	$40 \times 10^{-8} \Omega \text{ m}$ (at 20 °C)
	<u>Magnetic susceptibility (<math>\chi_{\text{mol}}</math>)</u>	$+1.919 \cdot 10^{-9} \text{ m}^3/\text{mol}$
	<u>Electronegativity</u>	Pauling scale: 1.54
Vanadium	<u>Electrical resistivity</u>	$20 \times 10^{-8} \Omega \text{ m}$ (at 20 °C)
	<u>Magnetic susceptibility (<math>\chi_{\text{mol}}</math>)</u>	$+3.199 \cdot 10^{-9} \text{ m}^3/\text{mol}$
	<u>Electronegativity</u>	Pauling scale: 1.63

1) The valence electrons are more located in the 3d subshell 4s.

2) The number of electrons with the same spin orientation is limited.

Remark:

**Electronegativity** is a chemical property that describes the tendency of an atom to attract electrons.

Element	Partial Orbital Diagram										Unpaired Electrons
	4s	3d					4p				
Sc	$\uparrow\downarrow$	$\uparrow$									1
Ti	$\uparrow\downarrow$	$\uparrow$	$\uparrow$								2
V	$\uparrow\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$							3

← Paramagnetic

# Langevin diamagnetism

$$\langle r^2 \rangle \approx 1$$

$$\chi \approx -0.99 \cdot 10^{-5} \cdot Z \left( \sum_{i=1}^Z \frac{1}{Z} \langle 0 \left[ \frac{r_i^2}{a_0^2} \right] 0 \rangle \right)$$

$\approx$  Borh radius

Atom radius

Copper

<a href="#">Electrical resistivity</a>	16.78 nΩ·m (at 20 °C)
<a href="#">Electronegativity</a>	Pauling scale: 1.90
<a href="#">Magnetic susceptibility</a>	-6.86×10 <sup>-11</sup> m <sup>3</sup> /mol

Zinc

<a href="#">Electrical resistivity</a>	59.0 nΩ·m (at 20 °C)
<a href="#">Electronegativity</a>	Pauling scale: 1.65
<a href="#">Magnetic susceptibility</a> (χ <sub>mol</sub> )	- 1.45×10 <sup>-10</sup> m <sup>3</sup> /mol

- 1) The number of electrons with the same spin orientation is limited.
- 2) The external field modify the external orbit.
- 3) By the law Lenz the electrons create a magnetic field in opposition wit the external field.
- 4) The diamagnetism is localized in the atoms.

Element	Partial Orbital Diagram			Unpaired Electrons
	4s	3d	4p	
Cu	↑	↑↓ ↑↓ ↑↓ ↑↓ ↑↓	□ □ □	1
Zn	↑↓	↑↓ ↑↓ ↑↓ ↑↓ ↑↓	□ □ □	0

Diamagnetic

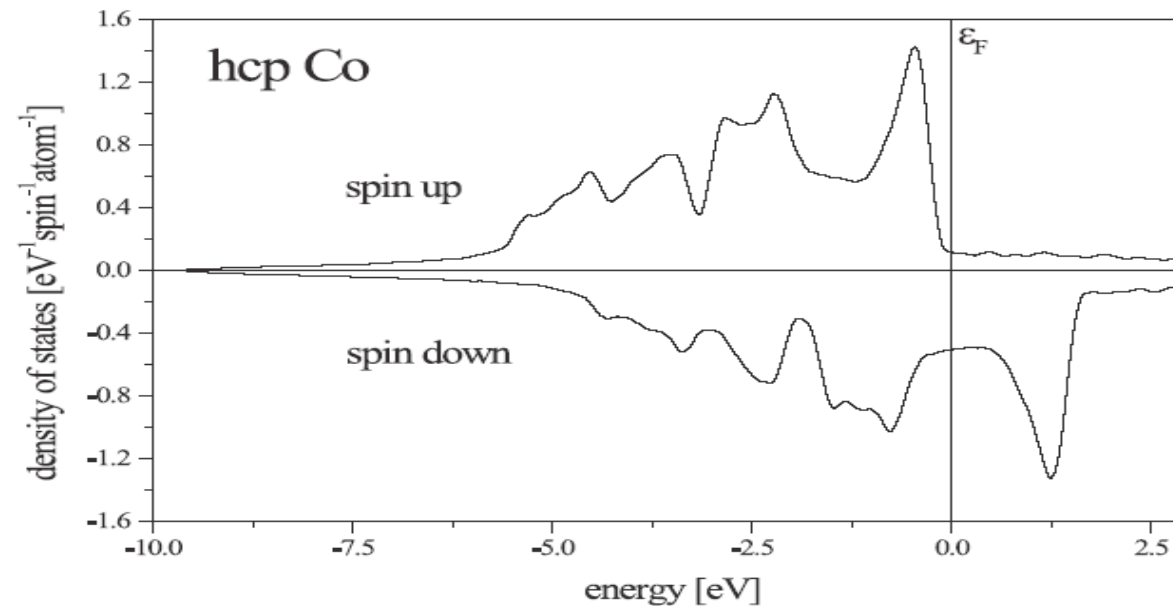
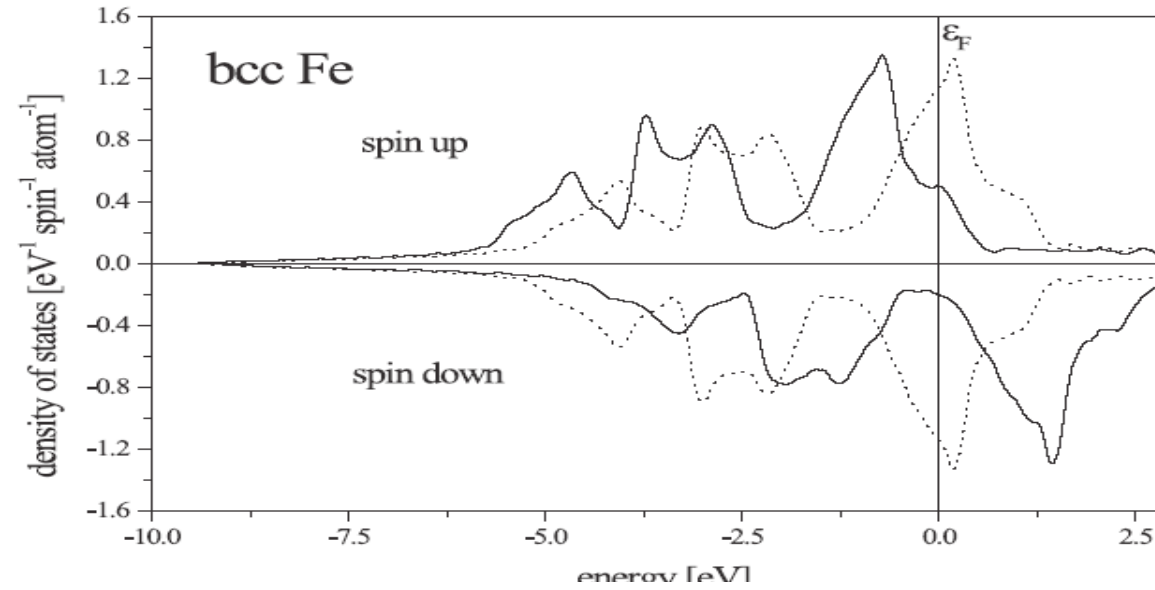
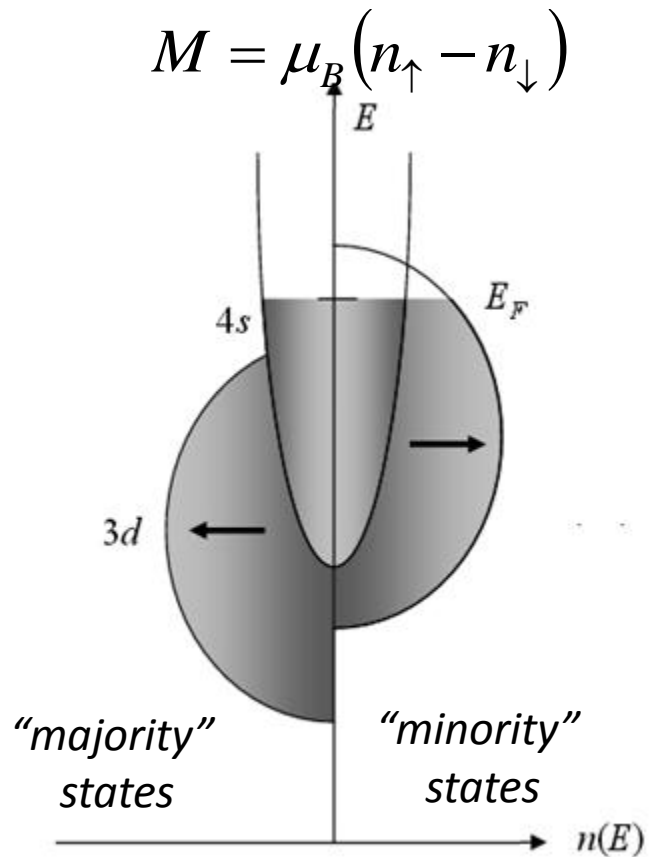
## *Landau diamagnetism (1930)*

- 1) A gas of free electrons in a magnetic field.*
- 2) Free electrons move along spiral trajectories*
- 3) Lenz's law*
- 4) Diamagnetic effect*

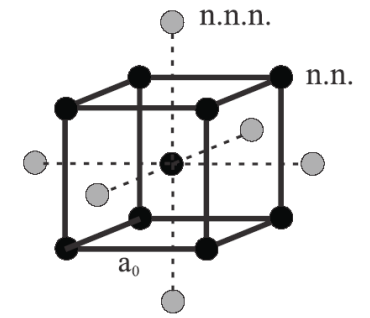
$$M_{Landau} = -\frac{N\mu^2}{2k_B T_F} B$$

$$M = M_{Pauli} + M_{Landau} = \frac{N\mu^2}{2k_B T_F} B$$

# Ferromagnetism (Solid state physics)



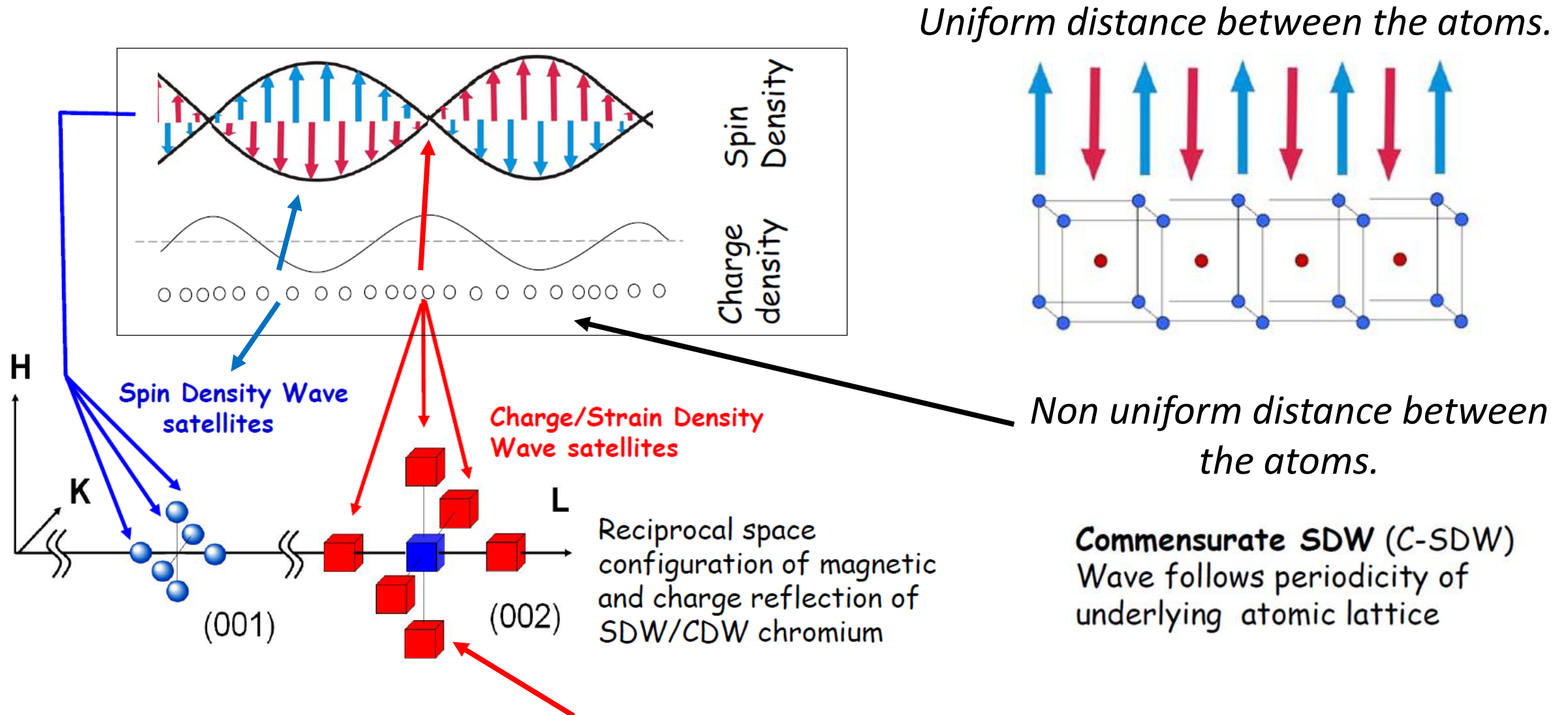
*Fe: weak ferromagnet (almost)*



*Co: strong ferromagnet*



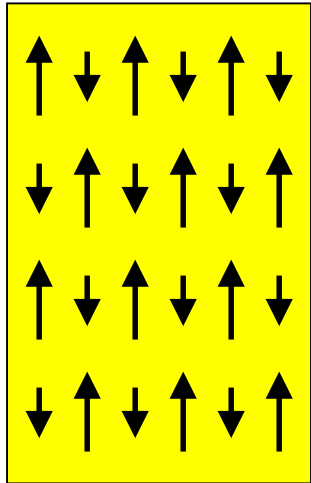
# Antiferromagnetism chromium (molecular physic)



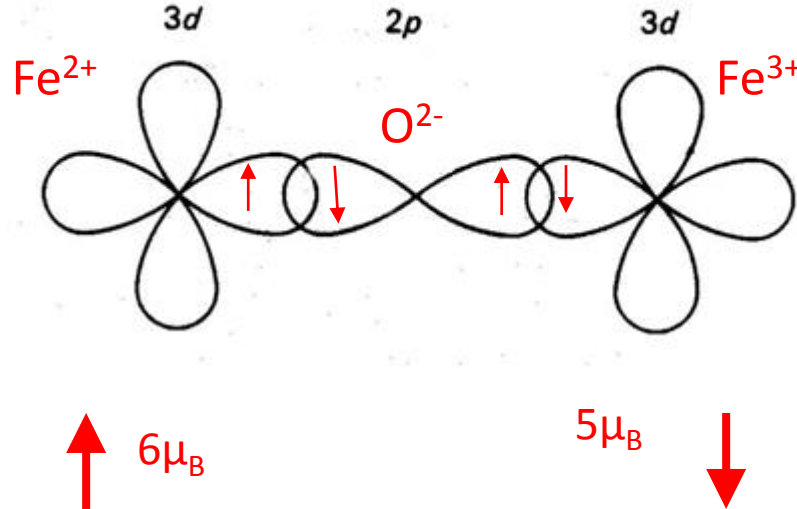
To reduce the energy, we have a alternation of the total magnetic moment (Pauli principle)

# Ferrimagnetism (molecular quantum physic)

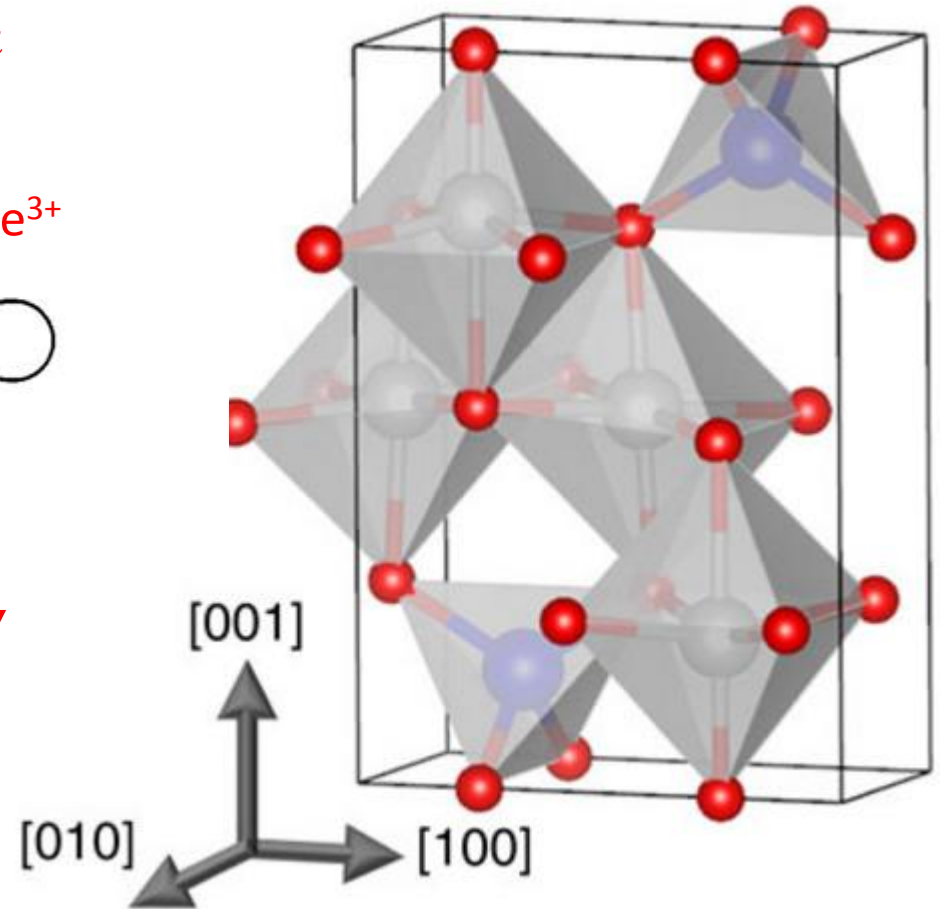
## Super-exchange interaction



*Ferrimagnets (ferrites)  
behave similar to  
ferromagnets*



*Magnetite (Fe<sub>3</sub>O<sub>4</sub>)*



*Oxygen atoms (small sphere in red)*

*Fe<sup>2+</sup> (tetrahedral sphere in blue) , Fe<sup>3+</sup> (octahedral sphere grey)*