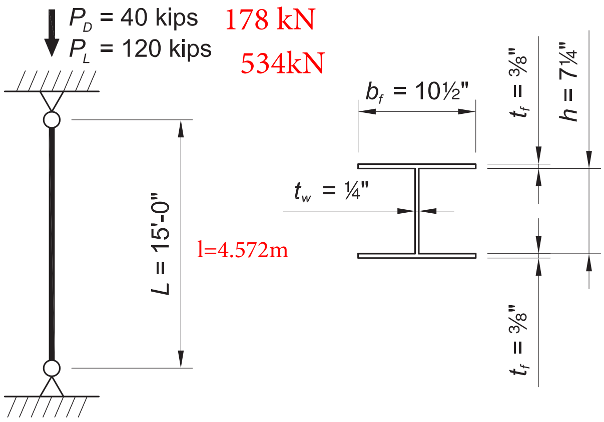
EXAMPLE E.3 BUILT-UP COLUMN WITH SLENDER FLANGES

Given:

Determine if a built-up, ASTM A572 Grade 50 column with PL⅜ in. × 10½ in. flanges and a PL¼ in. × 7¼ in. web has sufficient available strength to carry a dead load of 40 kips and a live load of 120 kips in axial compression. The column's unbraced length is 15.0 ft in both axes and the ends are pinned.



Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Built-Up Column
ASTM A572 Grade 50
F_y = 50 ksi   fy=340MPa   Yield strenght
F_u = 65 ksi   fu=445MPa   Tensile strenght

The geometric properties are as follows:

Built-Up Column
d = 8.00 in.   h=203.2mm   section depth
b_f = 10½ in.   b=266.7mm   section width
t_f = ⅜ in. t_f=9.5mm   flange thickness
h = 7¼ in.   d=184.15mm   clear distance between flanges
t_w = ¼ in. t_w=6.35mm   web thickness

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

load combination as EC3
P_u- required axial strenght using LRFD combinations

LRFD Load & resistance factor design	ASD Allowable strenght design
P_u = 1.2(40 kips) + 1.6(120 kips) N_{Sd}=1067kN = 240 kips 1067kN	P_a = 40 kips + 120 kips N_{Sd}=712kN = 160 kips 712kN

load combination as EC3
P_a- required axial strenght using ASD combinations

Note: according to C1 global stabilty shall be assessed with LRFD or 1.6*ASD

Built-Up Section Properties (ignoring fillet welds)

A_g = 2(10½ in.)(⅜ in.) + 7¼ in.(¼ in.)
= 9.69 in.^{2} A=6251.6mm2   Cross-section area

Because the unbraced length is the same for both axes, the weak axis will govern.

$$I_y = 2\left[\frac{(\tfrac{3}{8}\text{ in.})(10\tfrac{1}{2}\text{ in.})^3}{12}\right] + \frac{(7\tfrac{1}{4}\text{ in.})(\tfrac{1}{4}\text{ in.})^3}{12}$$
$$= 72.4\text{ in.}^4$$

I_z=3014cm^4 Moment of inertia

$$r_y = \sqrt{\frac{I_y}{A}}$$
$$= \sqrt{\frac{72.4\text{ in.}^4}{9.69\text{ in.}^2}}$$
$$= 2.73\text{ in.}$$

i_z=69.342mm Radius of gyration

$$I_x = 2(10\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.})(3.81\text{ in.})^2 + \frac{(\tfrac{1}{4}\text{ in.})(7\tfrac{1}{4}\text{ in.})^3}{12} + \frac{2(10\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.})^3}{12}$$
$$= 122\text{ in.}^4$$

I_y=5078cm^4 Moment of inertia

Web Slenderness

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a case 5:

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$
$$= 1.49\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 35.9$$

EC 3 classifies cross sections based on its geometry and stresses.
Subsquent strenght calculations are a function of this classification.
A reduction factor χ (used to determine the buckling resistance) is based on a slenderness factor λ

To conclude both standards classify the section for local buckling effects.

$$\lambda = \frac{h}{t_w}$$
$$= \frac{7\tfrac{1}{4}\text{ in.}}{\tfrac{1}{4}\text{ in.}}$$
$$= 29.0$$

$\lambda < \lambda_r$; therefore, the web is not slender.

Note that the fillet welds are ignored in the calculation of h for built up sections.

Flange Slenderness

Calculate k_c.

$$k_c = \frac{4}{\sqrt{h/t_w}}$$ from AISC *Specification* Table B4.1b note [a]

$$= \frac{4}{\sqrt{7\tfrac{1}{4}\text{ in.}/\tfrac{1}{4}\text{ in.}}}$$
$$= 0.743\text{, where } 0.35 \le k_c \le 0.76 \quad \textbf{o.k.}$$

Use $k_c = 0.743$

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a case 2.

$$\lambda_r = 0.64\sqrt{\frac{k_c E}{F_y}}$$
$$= 0.64\sqrt{\frac{29{,}000 \text{ ksi}(0.743)}{50 \text{ ksi}}}$$
$$= 13.3$$

$$\lambda = \frac{b}{t}$$
$$= \frac{5.25 \text{ in.}}{3/8 \text{ in.}}$$
$$= 14.0$$

$\lambda > \lambda_r$; therefore, the flanges are slender

For compression members with slender elements, Section E7 of the AISC *Specification* applies. The nominal compressive strength, P_n, shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling. Depending on the slenderness of the column, AISC *Specification* Equation E7-2 or E7-3 applies. F_e is used in both equations and is calculated as the lesser of AISC *Specification* Equations E3-4 and E4-4.

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same for both axes, the weak axis will govern.

$$\frac{K_y L_y}{r_y} = \frac{1.0(15.0 \text{ ft})}{2.73 \text{ in.}}\left(\frac{12 \text{ in.}}{\text{ft}}\right)$$
$$= 65.9$$

Elastic Critical Stress, F_e, for Flexural Buckling

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(Spec. Eq. E3-4)}$$

N_{cr} - elastic critical buckling

EC 3 requires the calculation of flexural buckling (Euler), torsional and torsional-flexural buckling. The N_{cr} is the minimum of the relevant buckling modes.

$$= \frac{\pi^2(29{,}000 \text{ ksi})}{(65.9)^2}$$
$$= 65.9 \text{ ksi}$$ 454.4 MPa

Elastic Critical Stress, F_e, for Torsional Buckling

Note: This limit state is not likely to govern, but the check is included here for completeness.

From the User Note in AISC *Specification* Section E4,

$$C_w = \frac{I_y h_o^2}{4}$$
$$= \frac{72.4 \text{ in.}^4 (7.63 \text{ in.})^2}{4}$$
$$= 1{,}050 \text{ in.}^6$$

From AISC Design Guide 9, Equation 3.4,

$$J = \sum \frac{bt^3}{3}$$
$$= \frac{2(10\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.})^3 + 7\tfrac{1}{4}\text{ in.}(\tfrac{1}{4}\text{ in.})^3}{3}$$
$$= 0.407\text{ in.}^4$$

$$F_e = \left[\frac{\pi^2 EC_w}{(K_zL)^2} + GJ\right]\frac{1}{I_x + I_y}$$ (*Spec*. Eq. E4-4)

$$= \left[\frac{\pi^2(29{,}000\text{ ksi})(1{,}050\text{ in.}^6)}{(180\text{ in.})^2} + (11{,}200\text{ ksi})(0.407\text{ in.}^4)\right]\left(\frac{1}{122\text{ in.}^4 + 72.4\text{ in.}^4}\right)$$
$$= 71.2\text{ ksi} > 65.9\text{ ksi}$$ Minimum value as EC3

490.9 MPa

Therefore, use F_e = 65.9 ksi. 454.4MPa

Slenderness Reduction Factor, Q

$Q = Q_sQ_a$ from AISC *Specification* Section E7, where Q_a = 1.0 because the web is not slender.

Calculate Q_s, the unstiffened element (flange) reduction factor from AISC *Specification* Section E7.1(b).

Determine the proper equation for Q_s by checking limits for AISC *Specification* Equations E7-7 to E7-9.

$$\frac{b}{t} = 14.0 \text{ as previously calculated}$$

These a general provisions for members for compression.

$$0.64\sqrt{\frac{Ek_c}{F_y}} = 0.64\sqrt{\frac{29{,}000\text{ ksi}(0.743)}{50\text{ ksi}}}$$
$$= 13.3$$

$$1.17\sqrt{\frac{Ek_c}{F_y}} = 1.17\sqrt{\frac{29{,}000\text{ ksi}(0.743)}{50\text{ ksi}}}$$
$$= 24.3$$

$$0.64\sqrt{\frac{Ek_c}{F_y}} < \frac{b}{t} \le 1.17\sqrt{\frac{Ek_c}{F_y}}$$ therefore, AISC *Specification* Equation E7-8 applies

$$Q_s = 1.415 - 0.65\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{Ek_c}}$$ (*Spec*. Eq. E7-8)

$$= 1.415 - 0.65(14.0)\sqrt{\frac{50\text{ ksi}}{(29{,}000\text{ ksi})(0.743)}}$$
$$= 0.977$$

$$Q = Q_sQ_a$$
$$= 0.977(1.0)$$
$$= 0.977$$

Nominal Compressive Strength

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{0.977(50\text{ ksi})}}$$

= 115 > 65.9, therefore, AISC *Specification* Equation E7-2 applies

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y$$ (*Spec*. Eq. E7-2)

The critical stress is based on the elastic buckling stress F_e. F_e is also reduced.

$$= 0.977\left[0.658^{\frac{0.977(50\text{ ksi})}{65.9\text{ ksi}}}\right](50\text{ ksi})$$

$$= 35.8\text{ ksi}$$

$P_n = F_{cr}A_g$ (*Spec*. Eq. E7-1)

$N_{b.Rd}$=1543.5kN

= 35.8 ksi(9.69 in.^{2})
= 347 kips

From AISC *Specification* Section E1, the available compressive strength is:

Note that the LRFD method meets results of EC3

LRFD	ASD
$\phi_c = 0.90$ γ_{M1}=1.1....1/1.1=0.91 $\phi_c P_n = 0.90(347\text{ kips})$ = 312 kips > 240 kips **o.k.** 1387.85kN	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{347\text{ kips}}{1.67}$ = 208 kips > 160 kips **o.k.**

925kN

Note: Built-up sections are generally more expensive than standard rolled shapes; therefore, a standard compact shape, such as a W8×35 might be a better choice even if the weight is somewhat higher. This selection could be taken directly from AISC *Manual* Table 4-1.