



Dartmouth



TRIUMF

The lecture will begin shortly. Please mute your microphone until you are ready to speak.



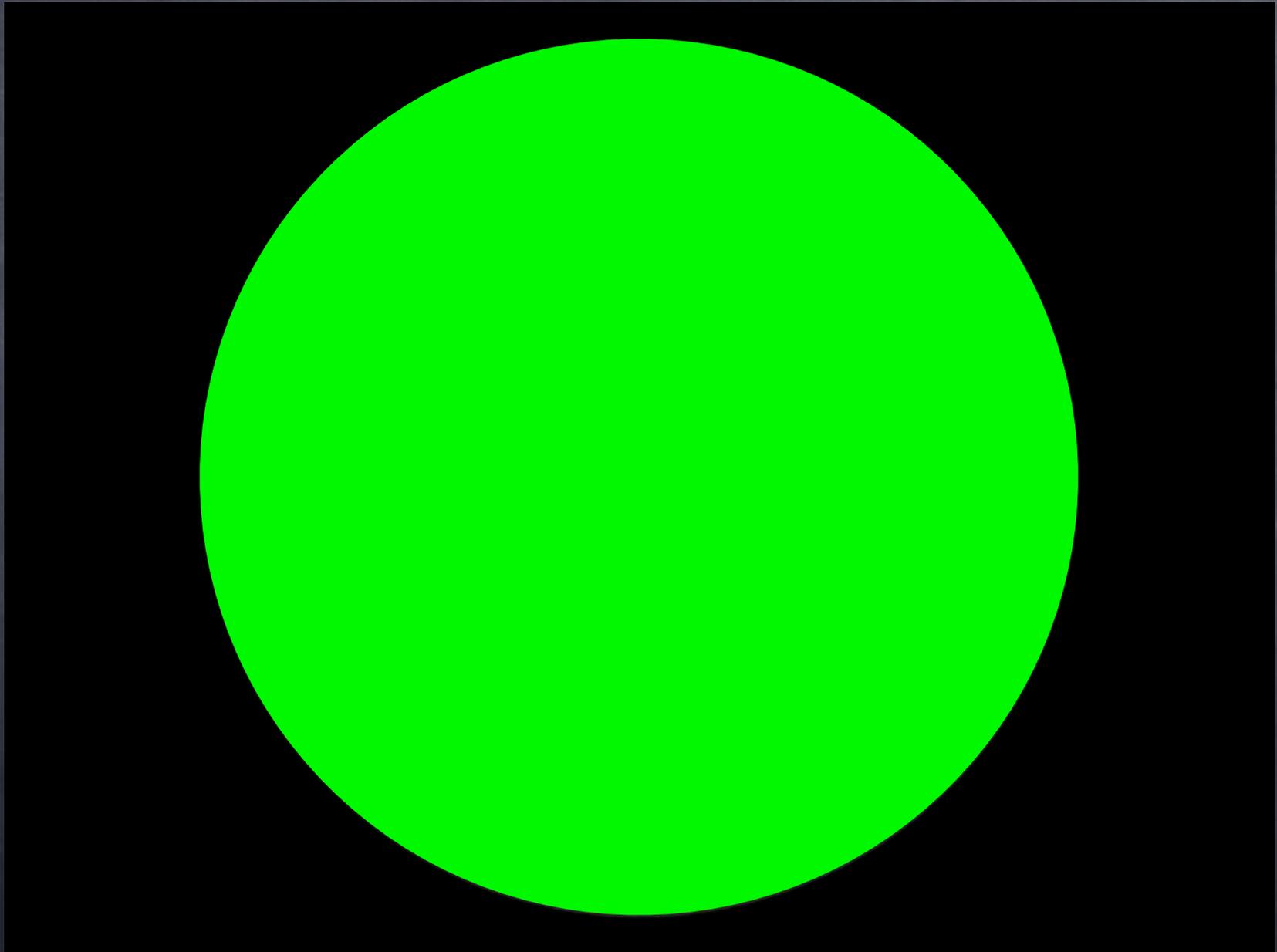
Cosmological Recombination

Douglas Scott

Summary

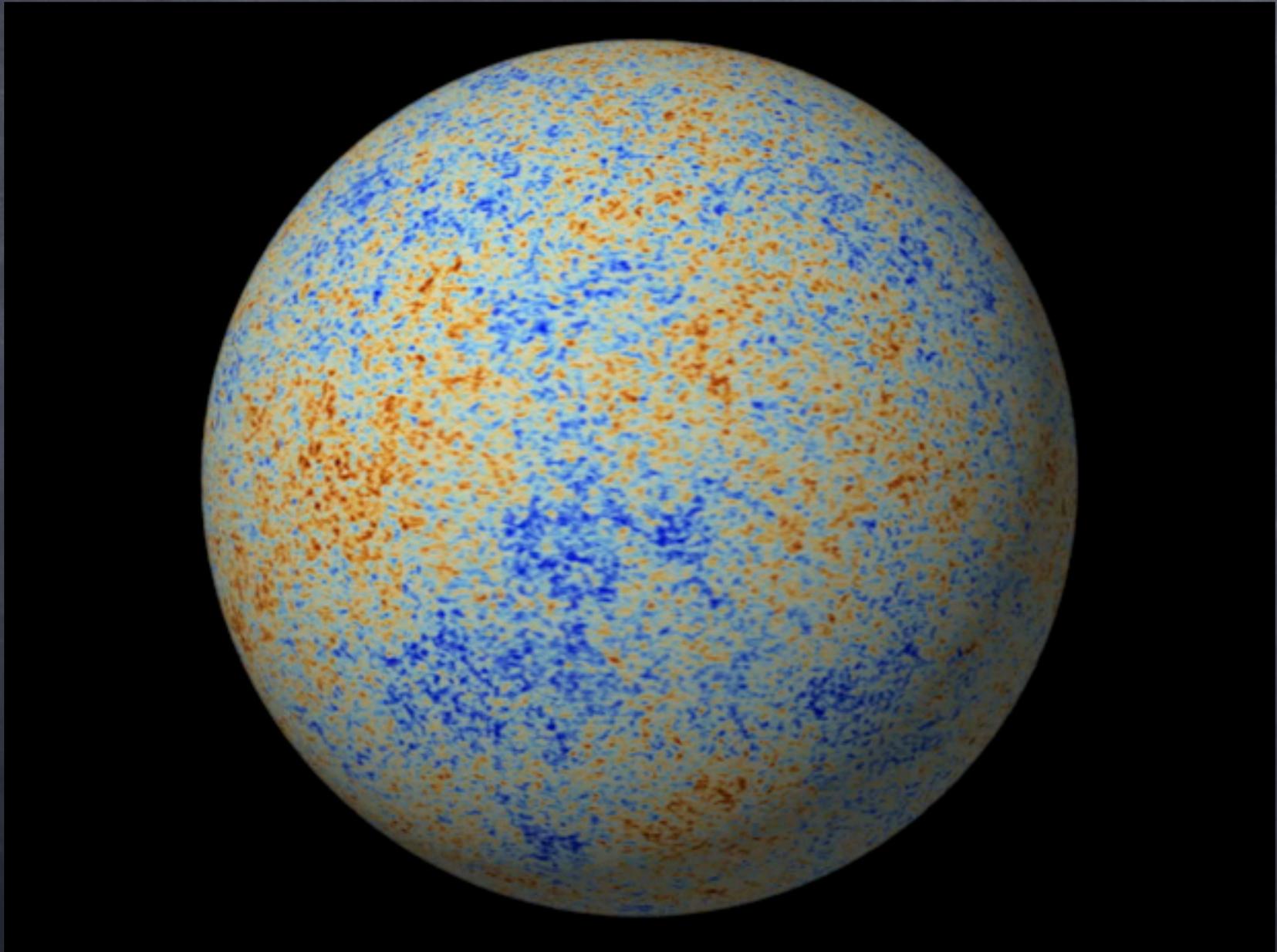
- ✿ Universe underwent a phase transition at about 400,000 years
- ✿ Precise evolution defines the “visibility function”
- ✿ Shape of $V(z)$ affects CMB anisotropies, which help answer Big Questions
- ✿ Need to get the eV physics right to infer the correct 10^{15} GeV physics!
- ✿ Reaching $\sim 0.1\%$ accuracy is hard!
- ✿ But gory details can be worked out \rightarrow fix simple ODE code \rightarrow recfast (with fudges!)

CMB Sky



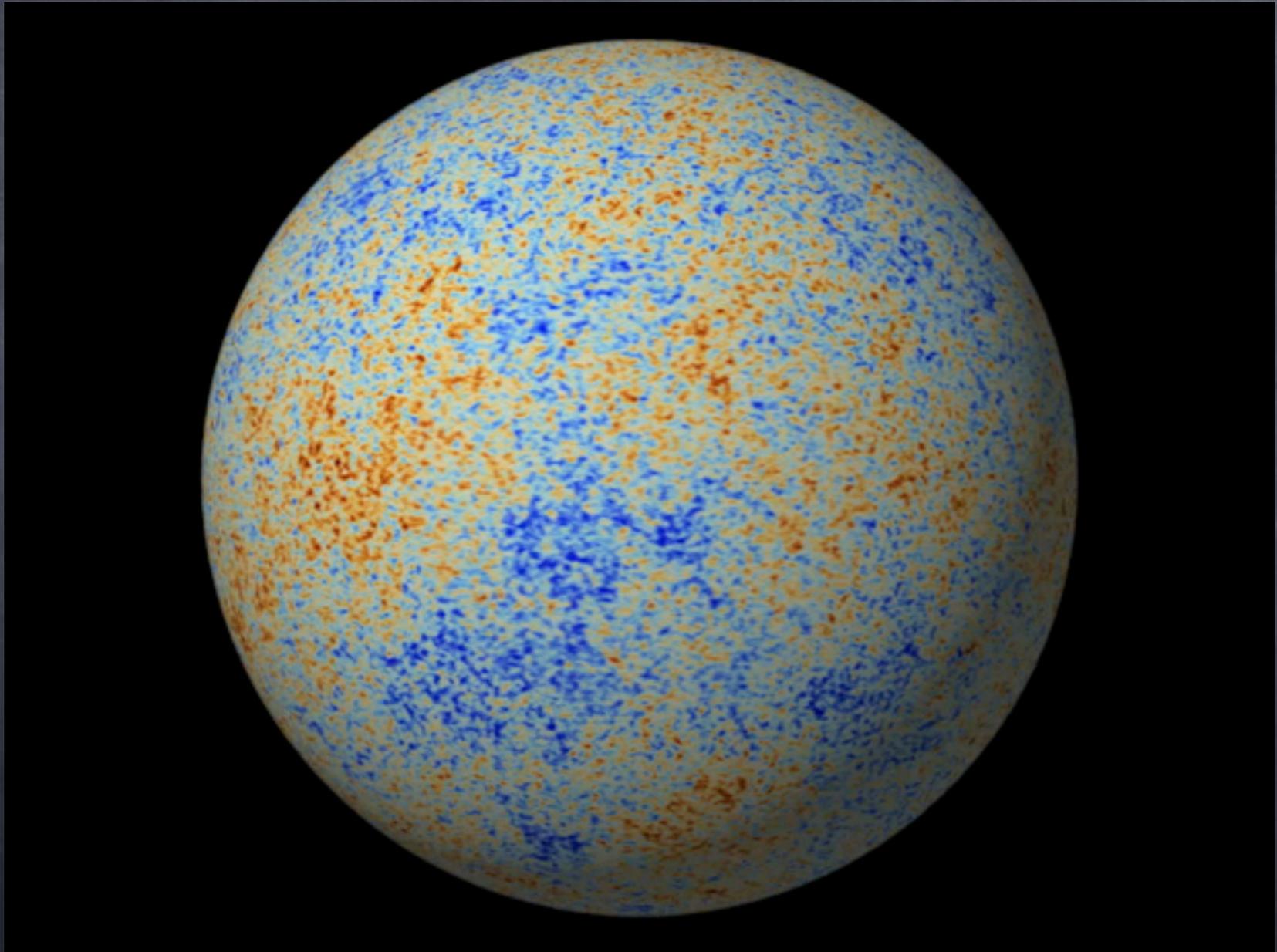
Temperature anisotropies at 400,000 years

CMB Sky



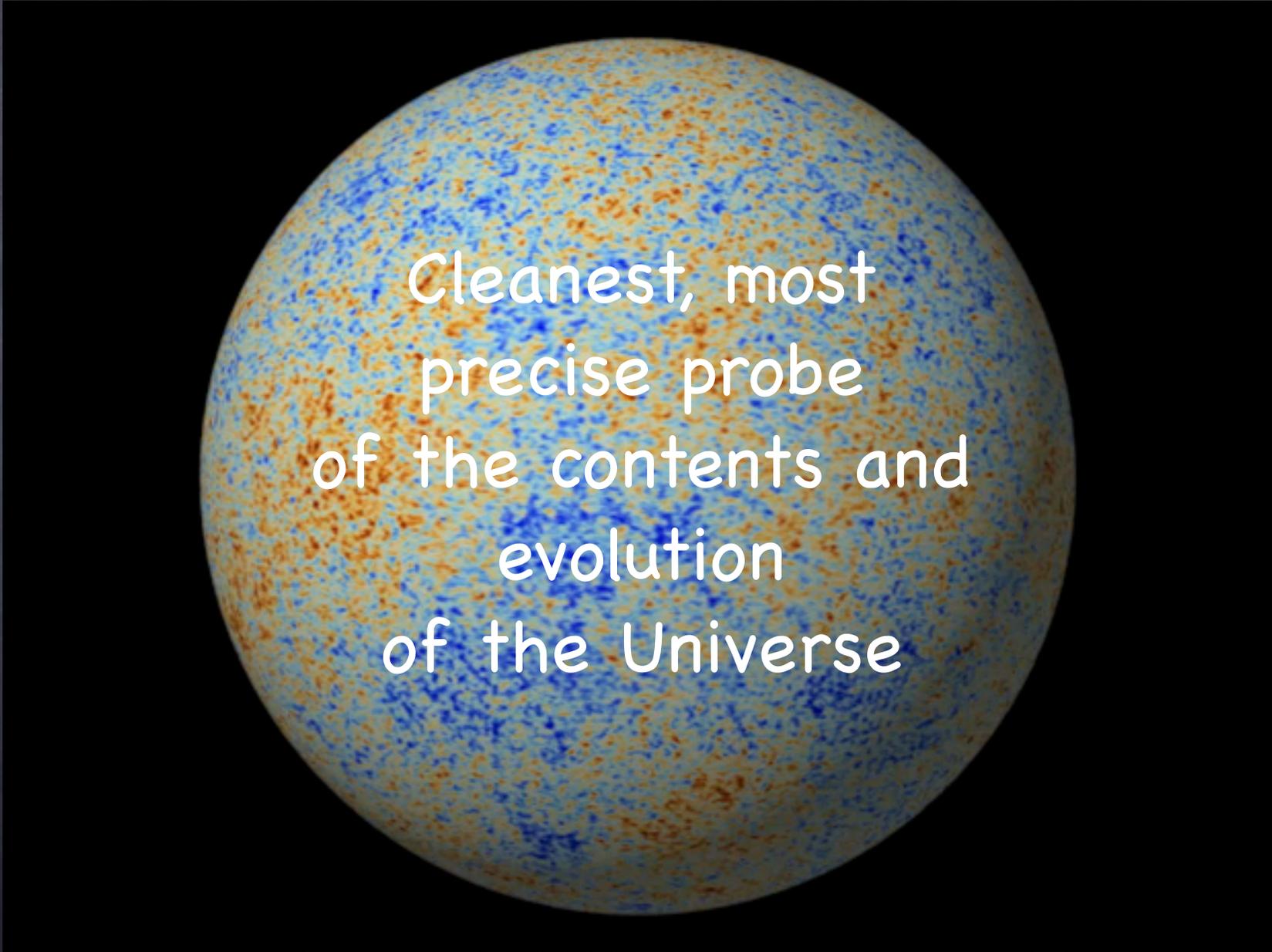
Temperature anisotropies at 400,000 years

CMB Sky



Temperature anisotropies at 400,000 years

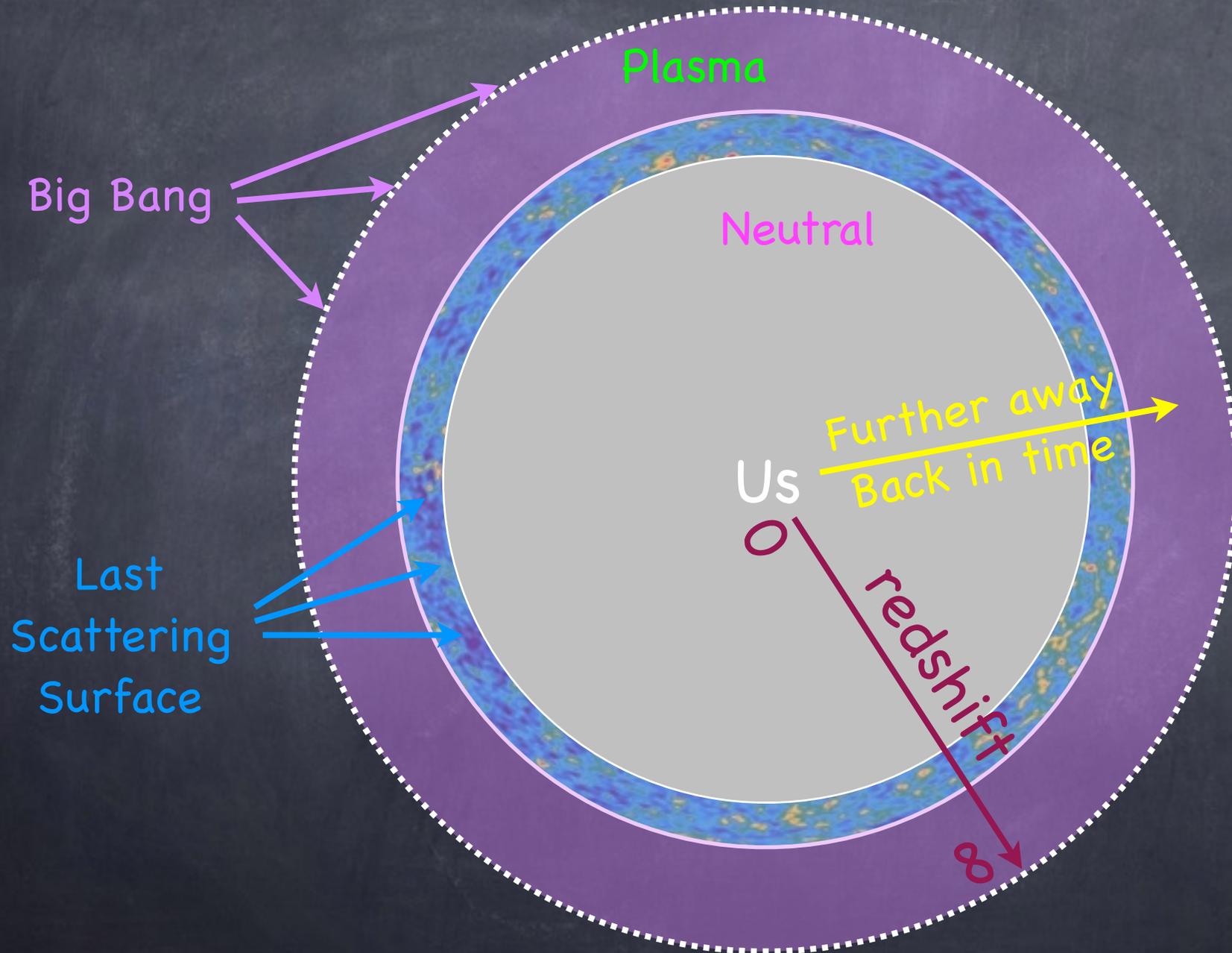
CMB Sky



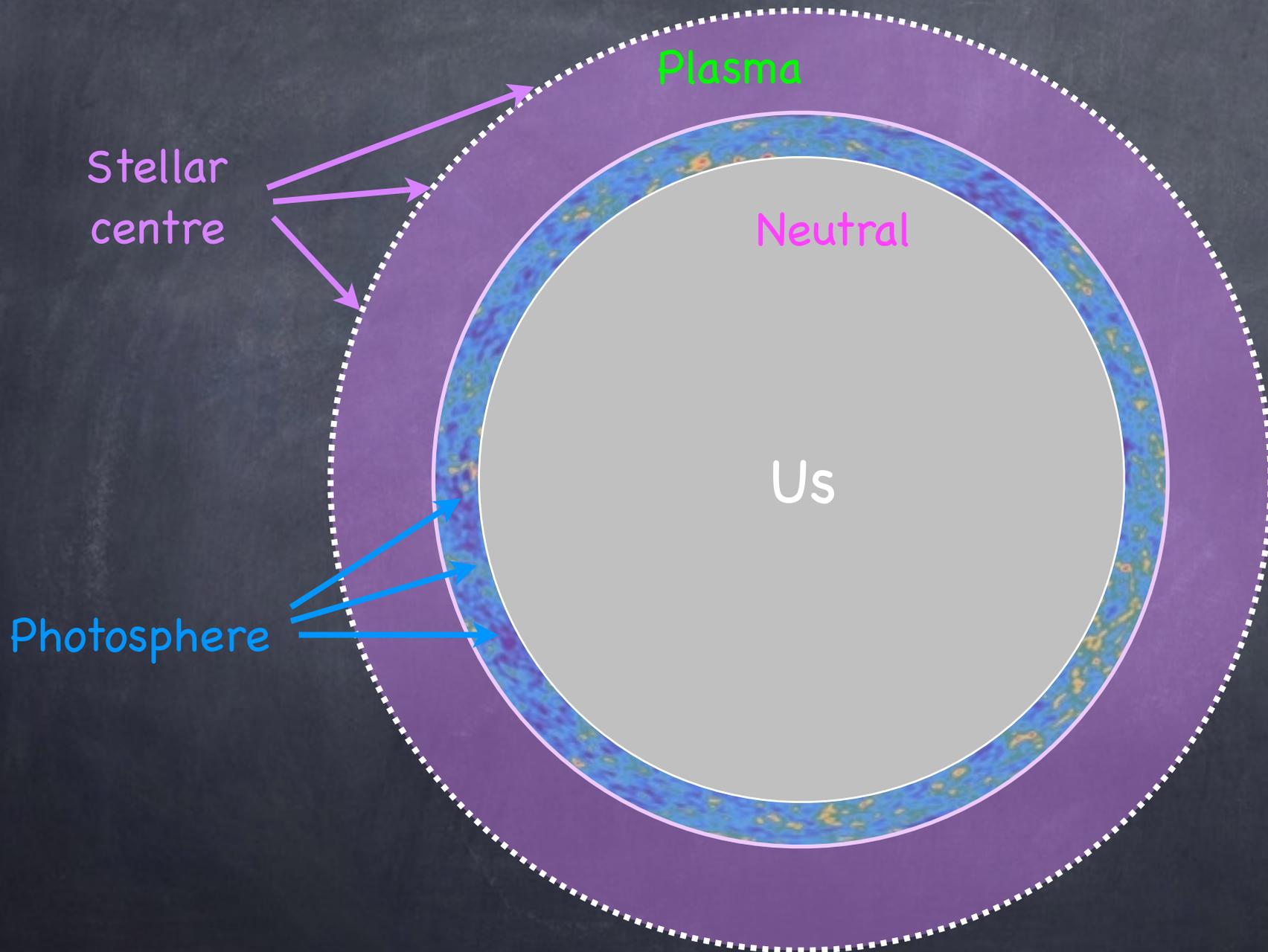
Cleanest, most
precise probe
of the contents and
evolution
of the Universe

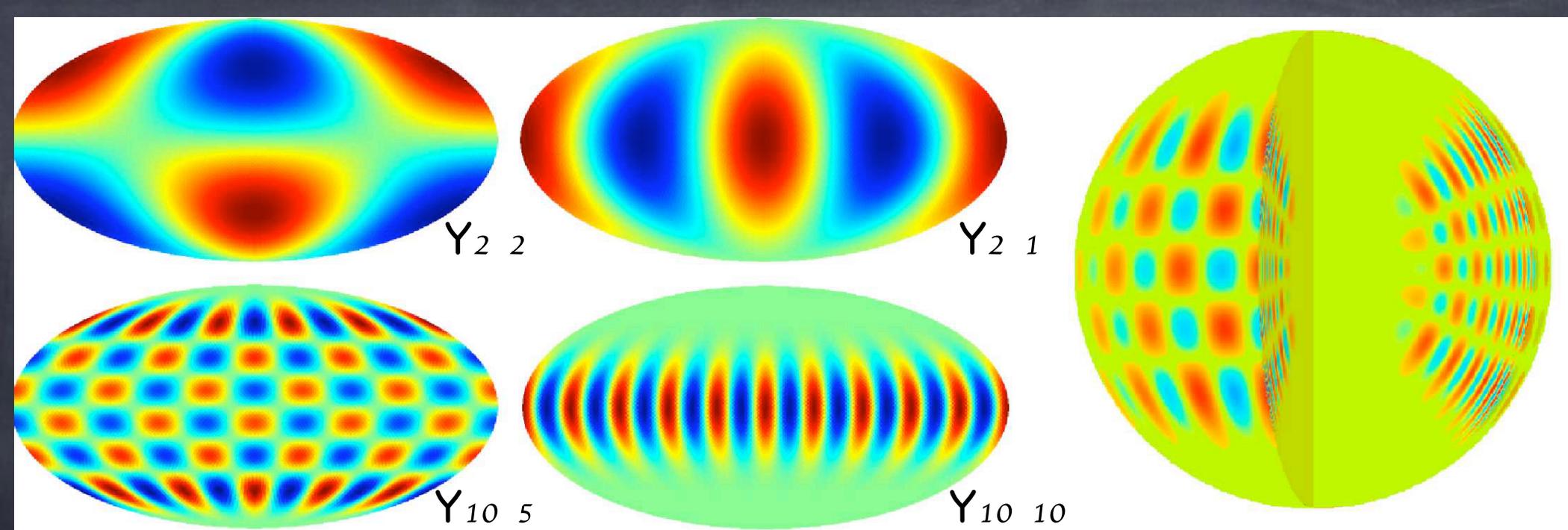
Temperature anisotropies at 400,000 years

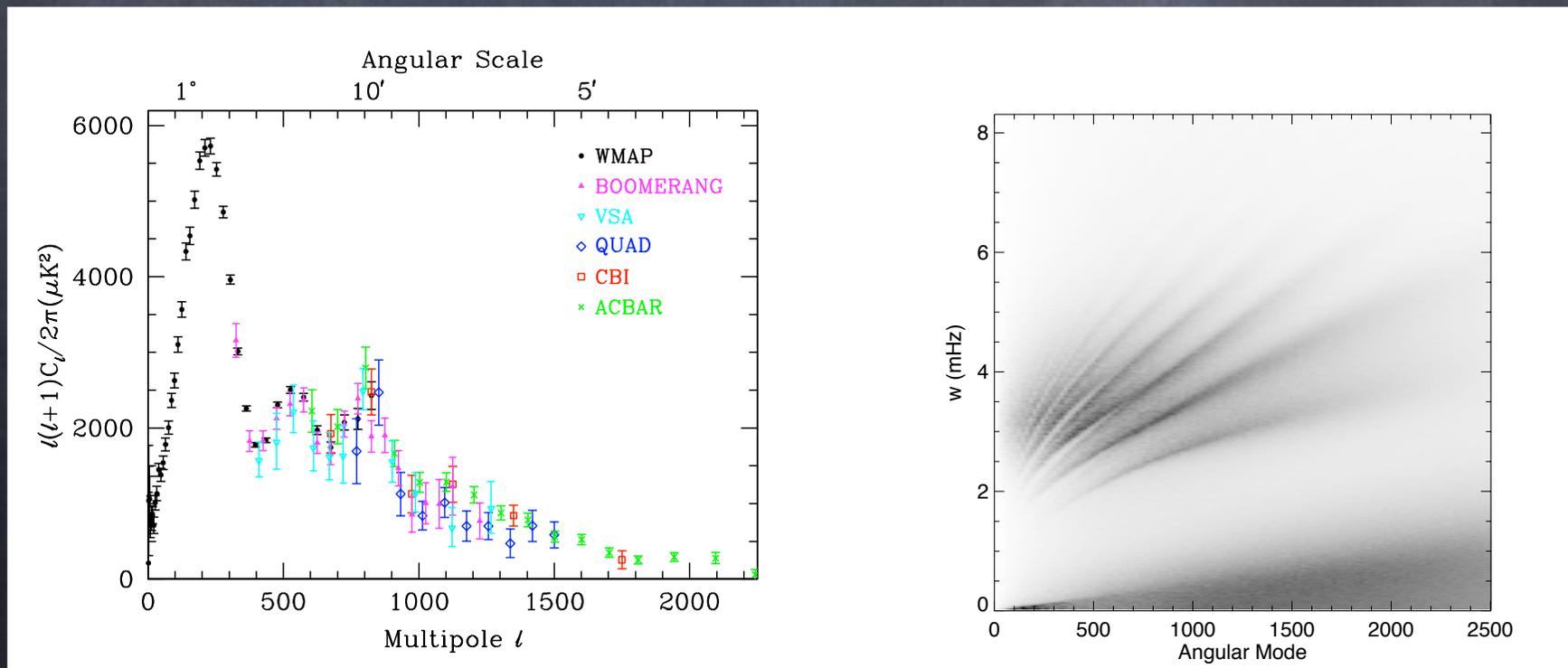
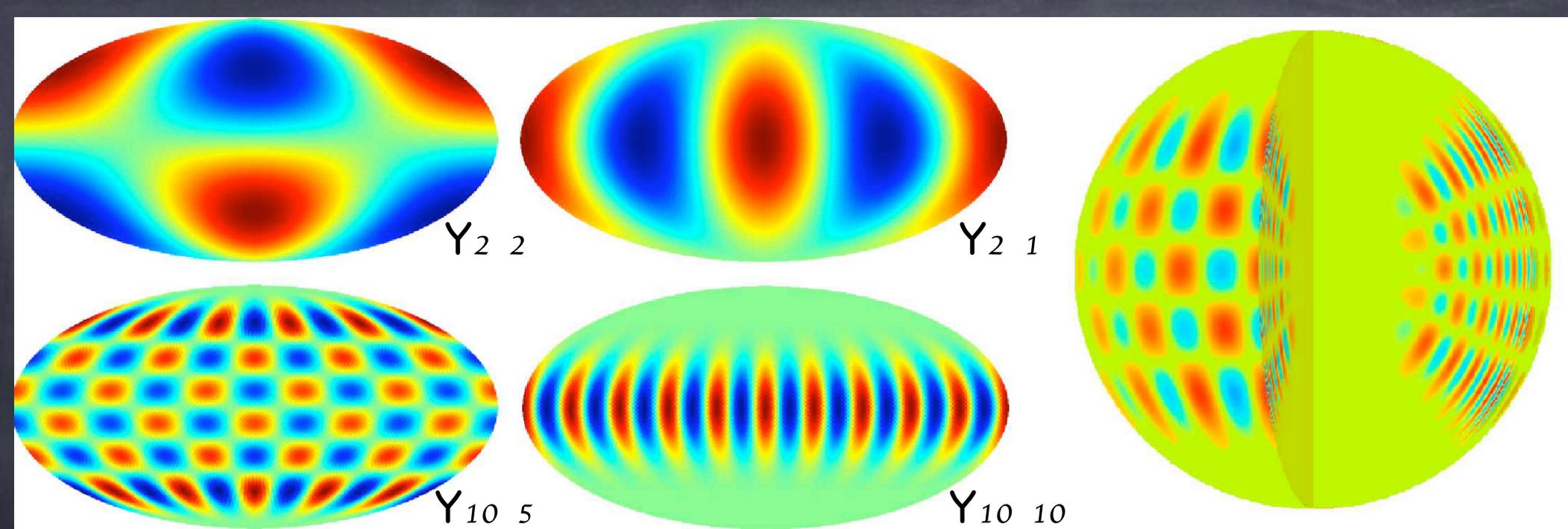
The Universe is an inside-out star!

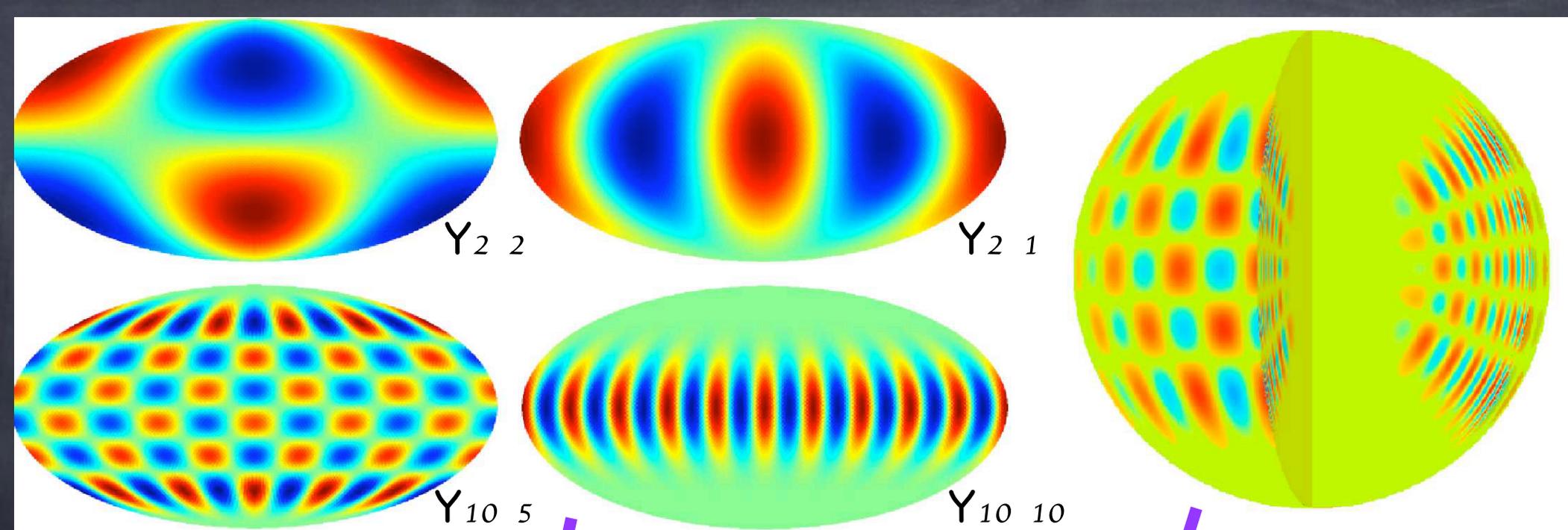


The Universe is an inside-out star!

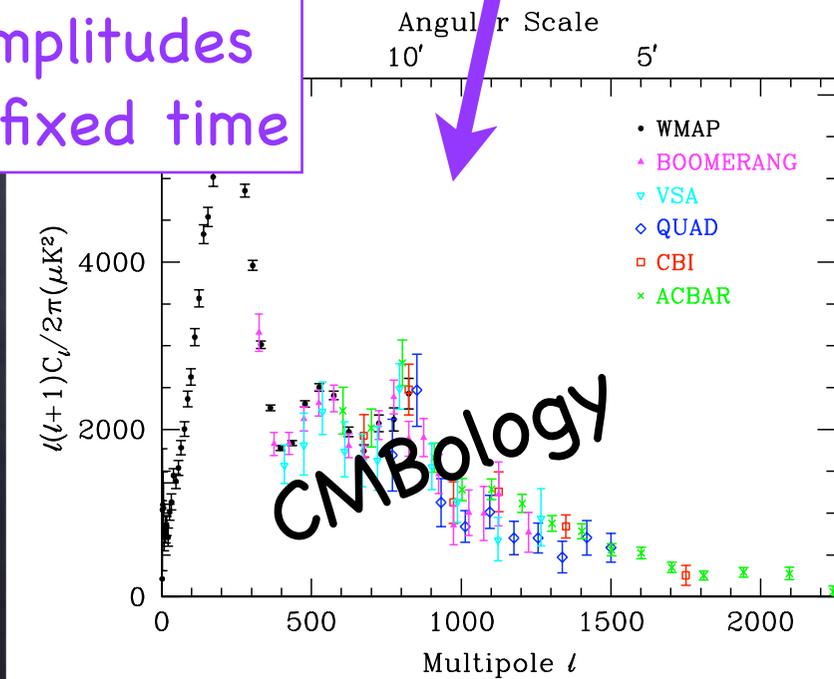




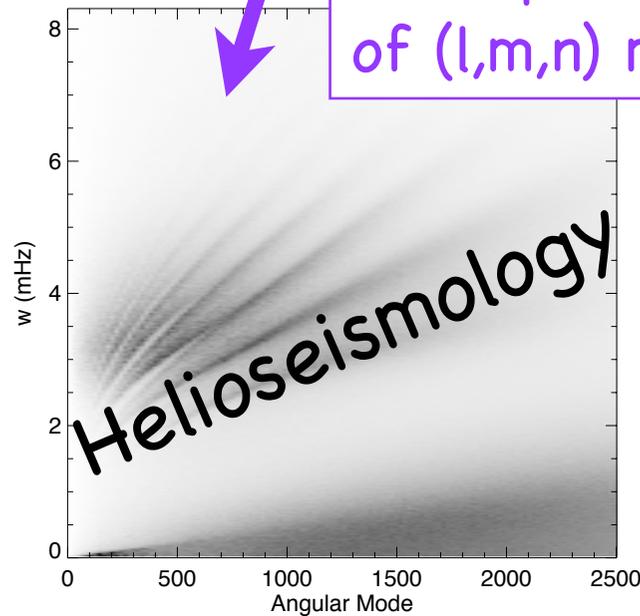


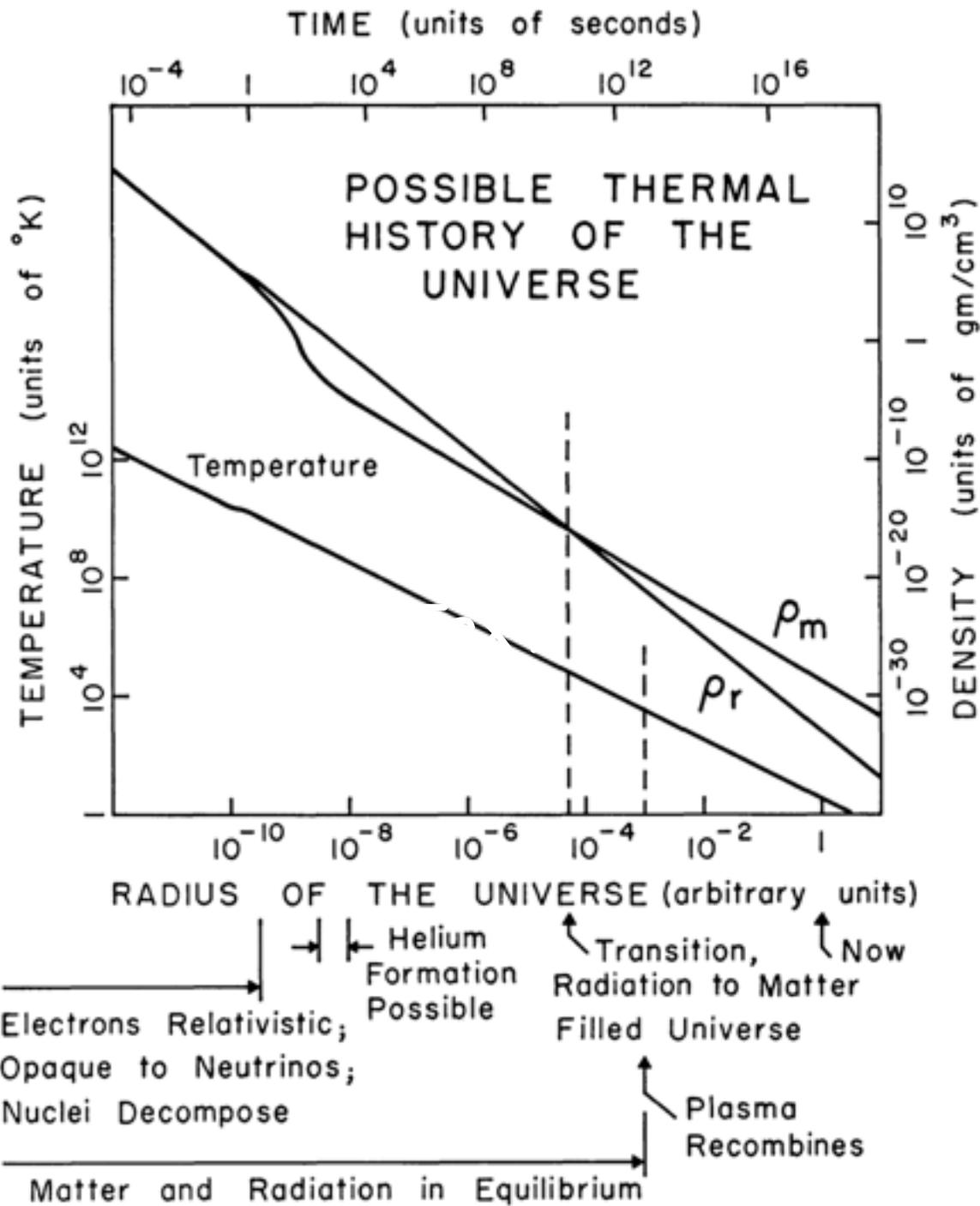


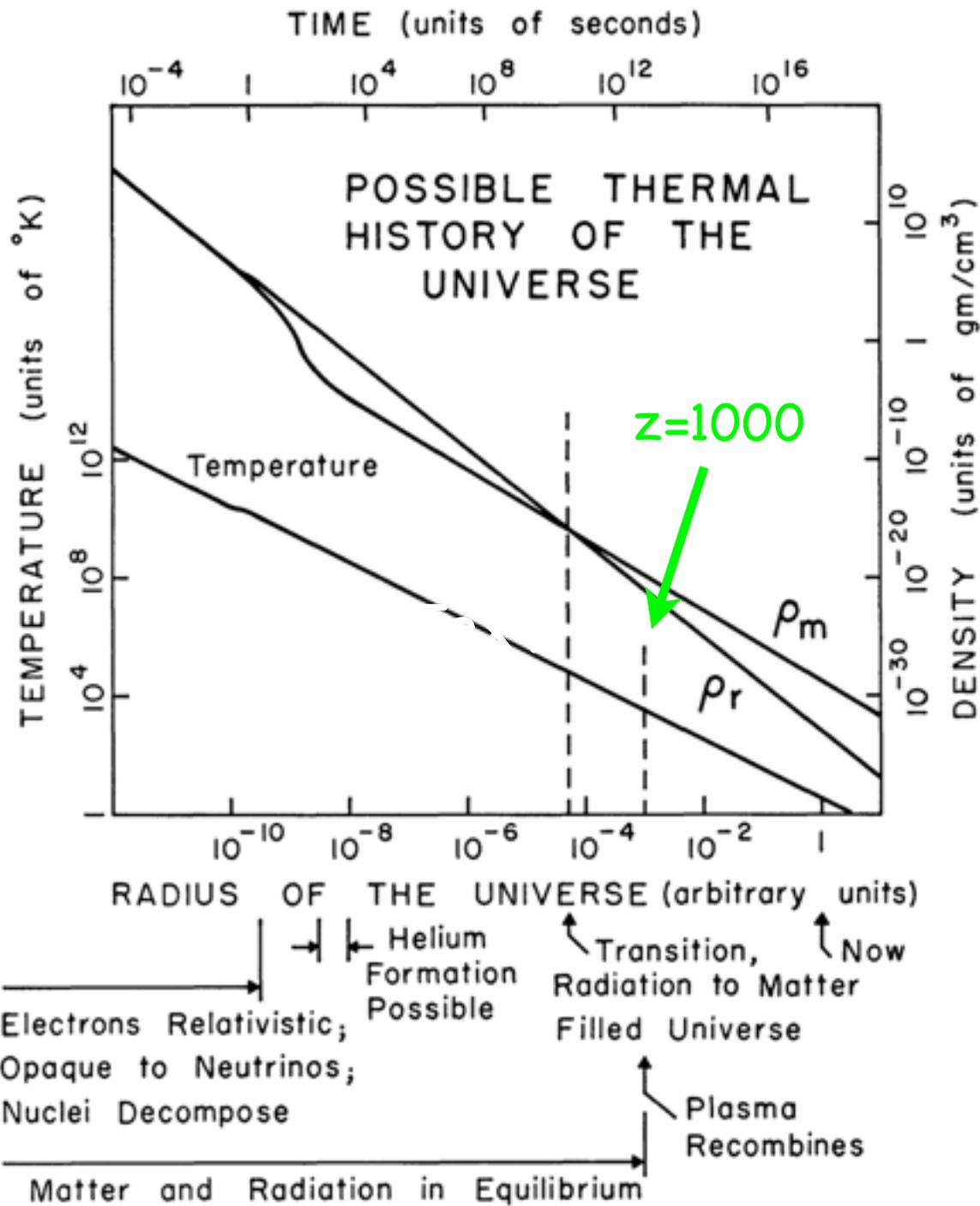
Use squared amplitudes at fixed time



Use temporal frequencies of (l,m,n) modes







RECOMBINATION OF HYDROGEN IN THE HOT MODEL OF THE UNIVERSE

Ya. B. ZEL'DOVICH, V. G. KURT and R. A. SYUNYAEV

Applied Mathematics Institute, U.S.S.R. Academy of Sciences

Submitted December 27, 1967

Zh. Eksp. Teor. Fiz. 55, 278–286 (July, 1968)

The considerable emission of energetic quanta during recombination of hydrogen in an expanding universe leads to a slowing down of the recombination and to a distortion of the relict radiation spectrum in the Wien region. The energy exchange between electrons and radiation in the Compton effect maintains the temperature of matter equal to that of the radiation up to a time corresponding to a red shift of $z \sim 150$, and this leads, in particular, to a change in the time dependence of the Jeans wavelength of the gravitational instability of a homogeneous medium in an expanding universe.

THE ASTROPHYSICAL JOURNAL, Vol. 153, July 1968

RECOMBINATION OF THE PRIMEVAL PLASMA*

P. J. E. PEEBLES

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

Received December 11, 1967

ABSTRACT

A theory is presented for the plasma recombination that would have taken place when the Universe had expanded and cooled to a Primeval Fireball temperature of about 4000°K . The computed residual ionization of the hydrogen following this recombination is in the range of 2×10^{-5} to 2×10^{-4} , depending on the assumed cosmological model. In the closed cosmological model the matter temperature would have effectively decoupled from the radiation at a temperature of 1200°K , while in the lowest density model the matter temperature would not have fallen much below the radiation temperature before the galaxies formed. Also computed is the effect of the recombination radiation on the spectrum of the Primeval Fireball.

Basic calculations
completed by 1968
in 2 seminal papers



Basic calculations
completed by 1968
in 2 seminal papers



Basic calculations
completed by 1968
in 2 seminal papers



Basic calculations
completed by 1968
in 2 seminal papers



Basic calculations
completed by 1968
in 2 seminal papers



Zeldovich



Dicke



Peebles



Kurt



Basic calculations
completed by 1968
in 2 seminal papers



Zeldovich



Dicke



Peebles



Kurt



Sunyaev

When did the Universe recombine?

✿ Temperature cools with expansion: $T(z)=T_0(1+z)$

✿ $3kT=13.6\text{eV} \Rightarrow z \approx 23,000$

But $\sim 10^9$ photons per baryon

✿ Saha in a cooling bath $\Rightarrow z \approx 1500$ (for $x=0.5$)

But $\text{Ly}\alpha$ optically thick, 2-photon slow, and
"freeze-out" of free electron fraction

✿ $V(z) \equiv \exp(-\tau) d\tau$ peaks at $z \approx 1100$

But what about low and high z tails?

✿ Historically, $\Delta z/z \approx 0.1$ led to ~ 10 reduction in
 $\Delta T/T$ estimates

Saha (equilibrium) equation

$$\left(\frac{n_i}{n_e n_c} \right) = \frac{g_i}{2g_c} \left(\frac{h_P^2}{2\pi m_e k_B T_M} \right)^{3/2} e^{E_i/k_B T_M}$$

Diagram illustrating the Saha (equilibrium) equation with labels for its components:

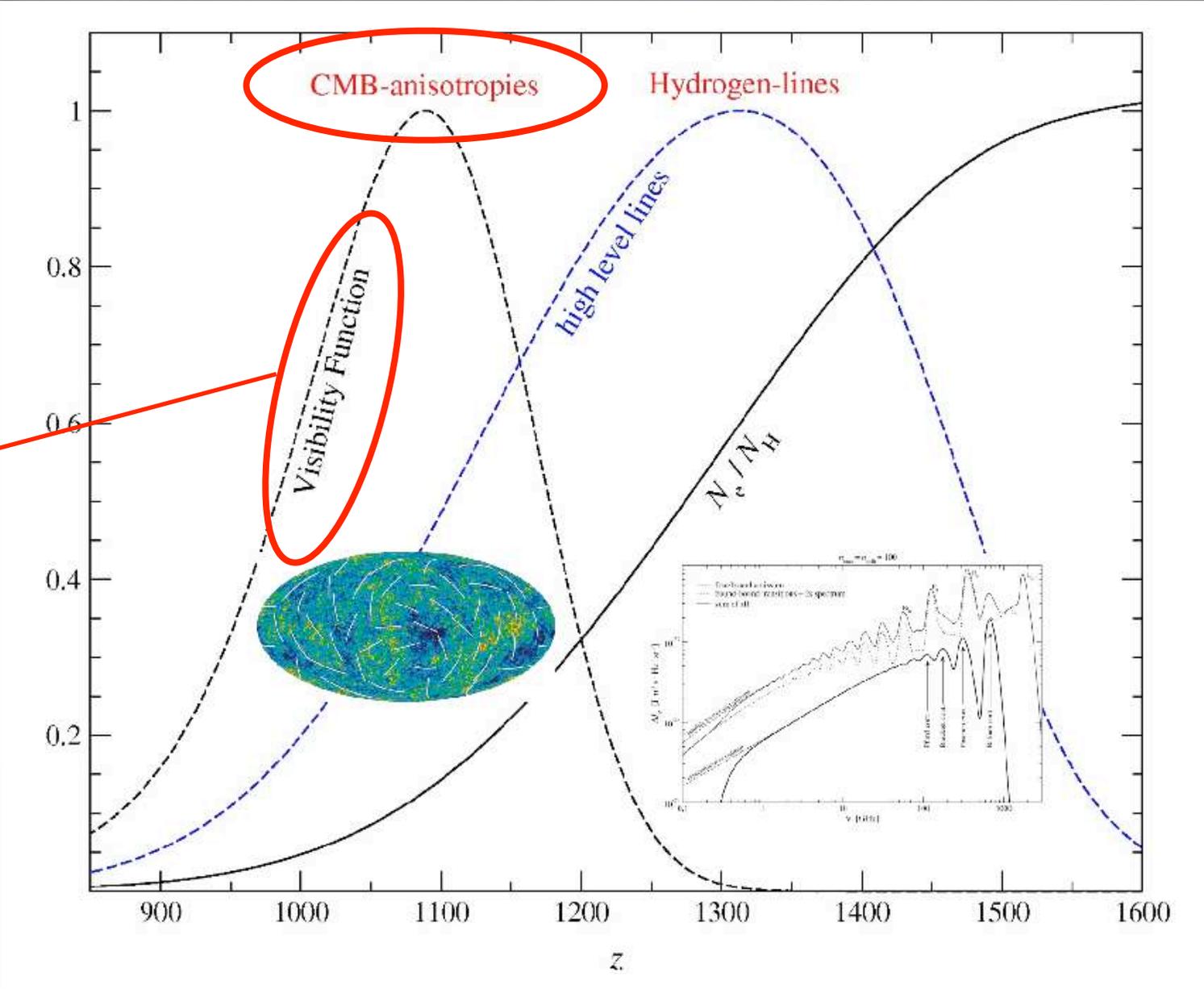
- $\left(\frac{n_i}{n_e n_c} \right)$: bound state (points to n_i), electrons (points to n_e), continuum (points to n_c)
- $\frac{g_i}{2g_c}$: statistical weights (points to g_i)
- $\left(\frac{h_P^2}{2\pi m_e k_B T_M} \right)^{3/2}$: kinetic temperature of matter (points to T_M)
- $e^{E_i/k_B T_M}$: bound energy level (points to E_i)

Easy to solve for $x_e \equiv \left(\frac{n_e}{n_H} \right)$

with $T_M \propto (1+z)$, $n \propto (1+z)^3$

Profile of the Cosmic Photosphere

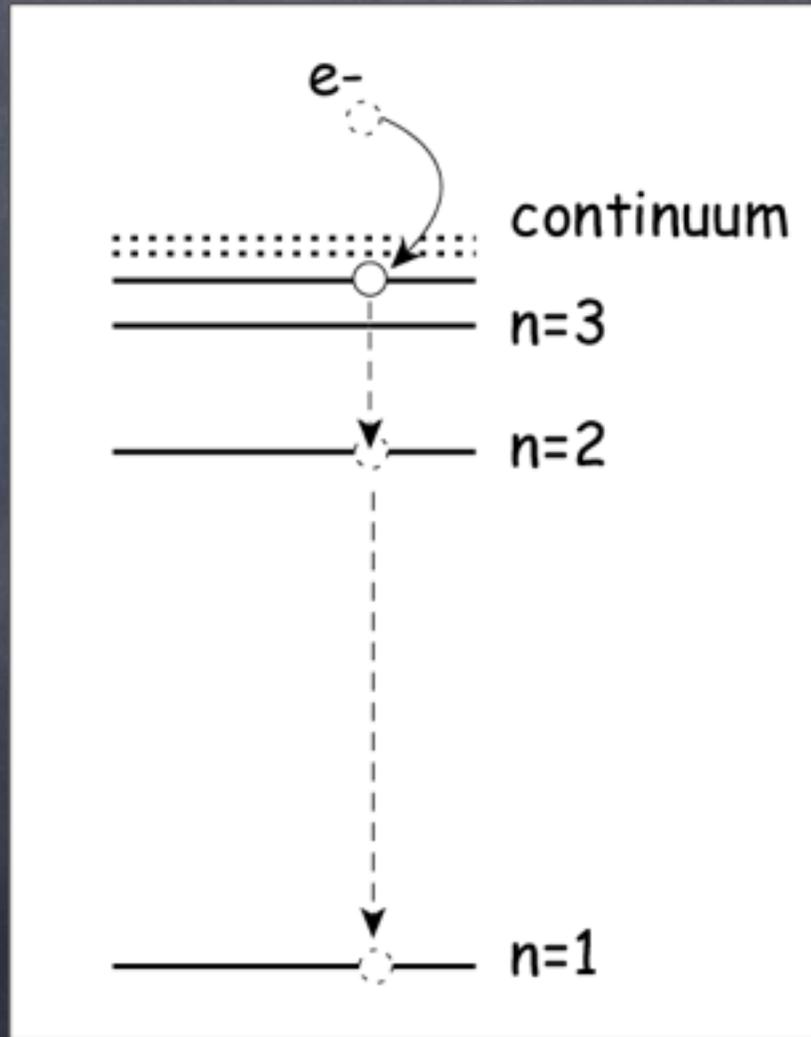
$$V(z) \equiv e^{-\tau} d\tau/dz$$



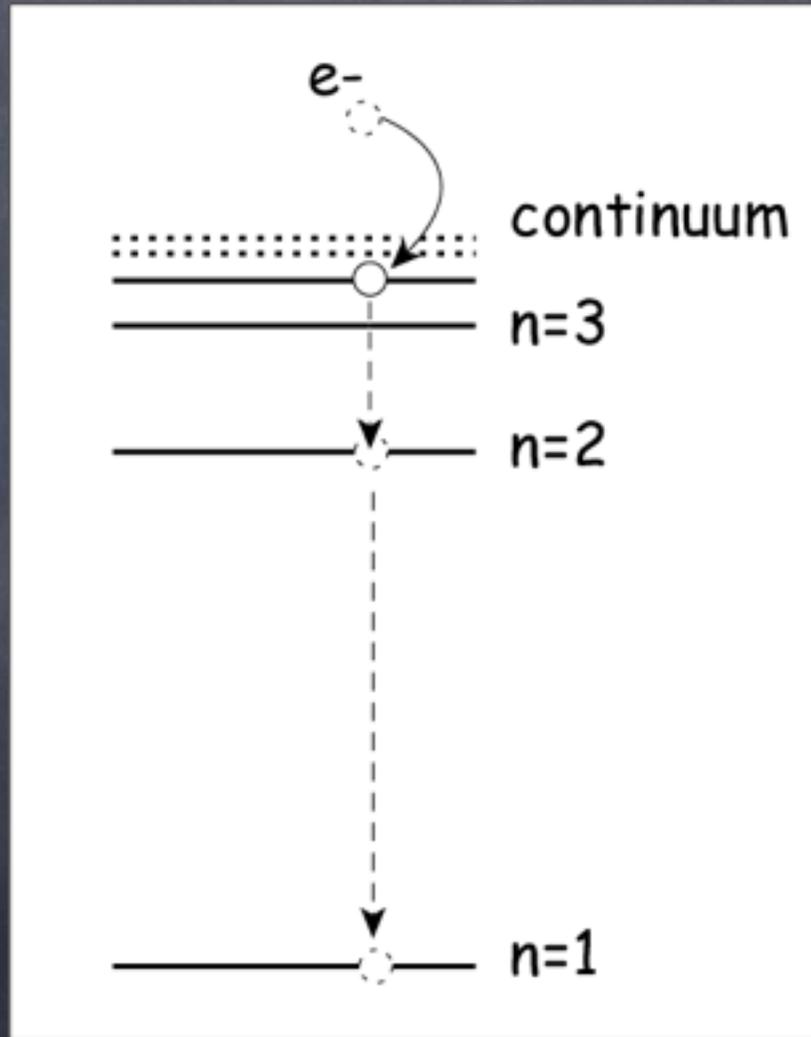
How did the Universe recombine?

- ✿ Unlike in the Lab and most other astro systems!
- ✿ Radiation determines everything (can ignore collisional rates)
- ✿ Wien tail means that there are zillions of $n \rightarrow 2$ photons compared with $2 \rightarrow 1$ photons
- ✿ So once an electron gets to ground state it's safe
- ✿ Primary effects are those involving $2 \rightarrow 1$
- ✿ More than $\frac{1}{2}$ H atoms used the 2-photon channel (not so forbidden!)

Effective 3-level atom



Effective 3-level atom

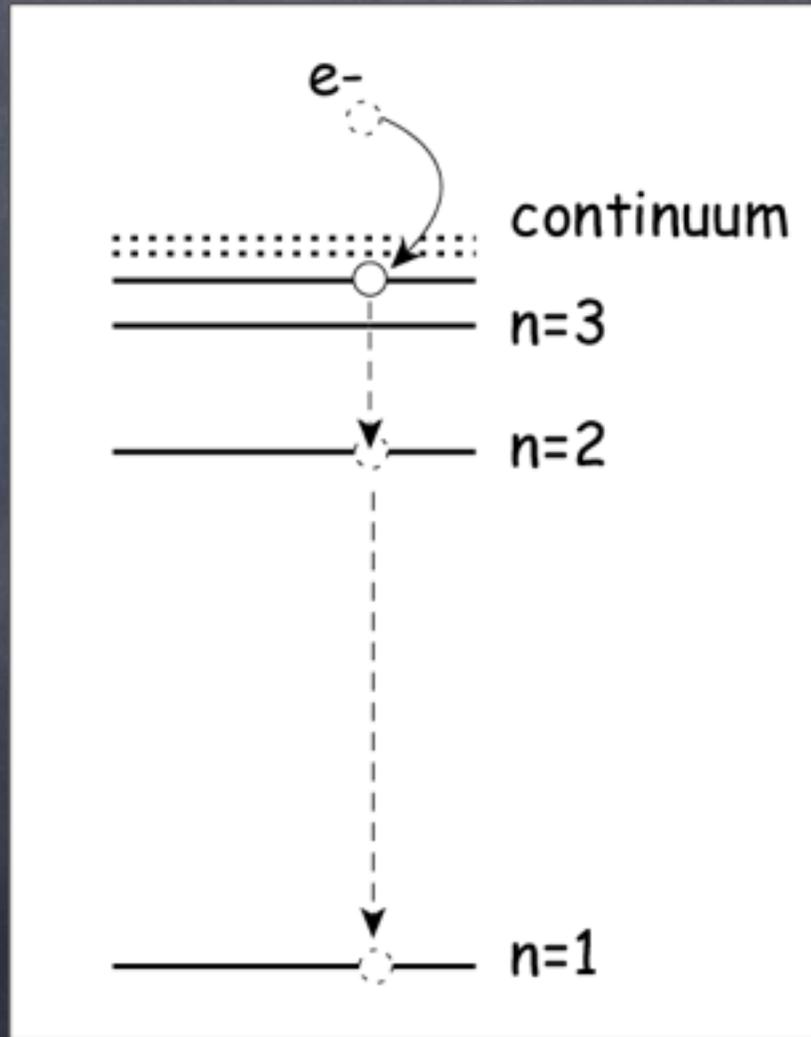


Ground state

Effective 3-level atom

1st excited state

Ground state

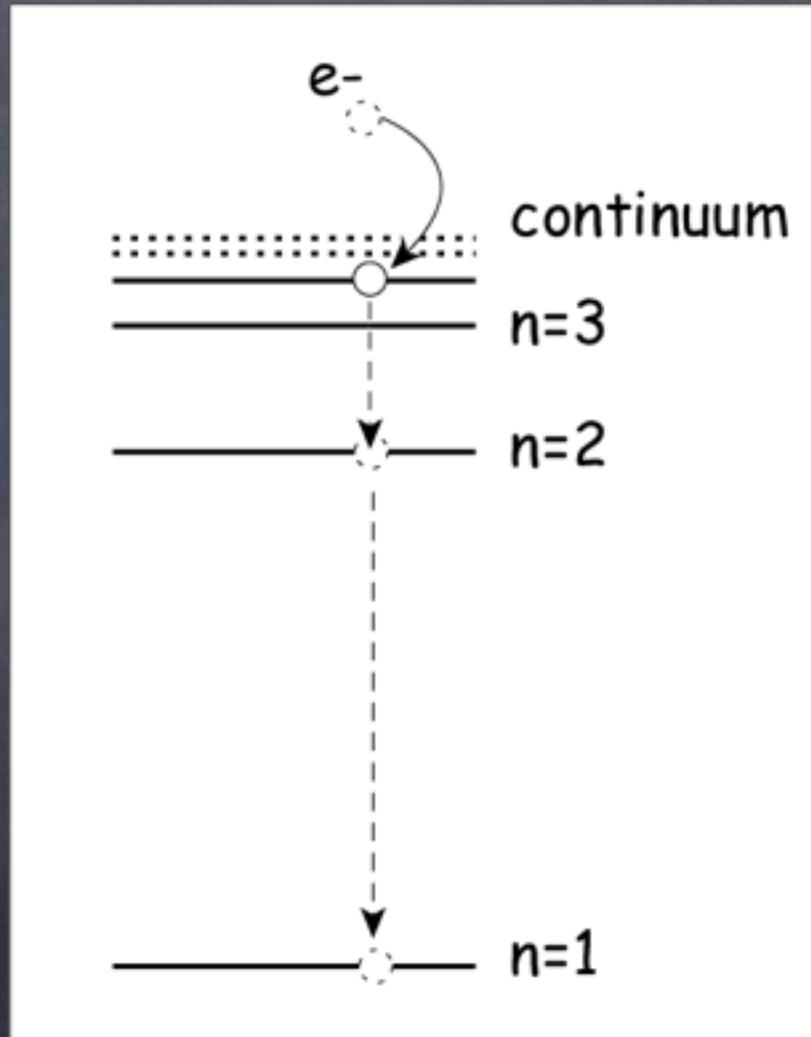


Effective 3-level atom

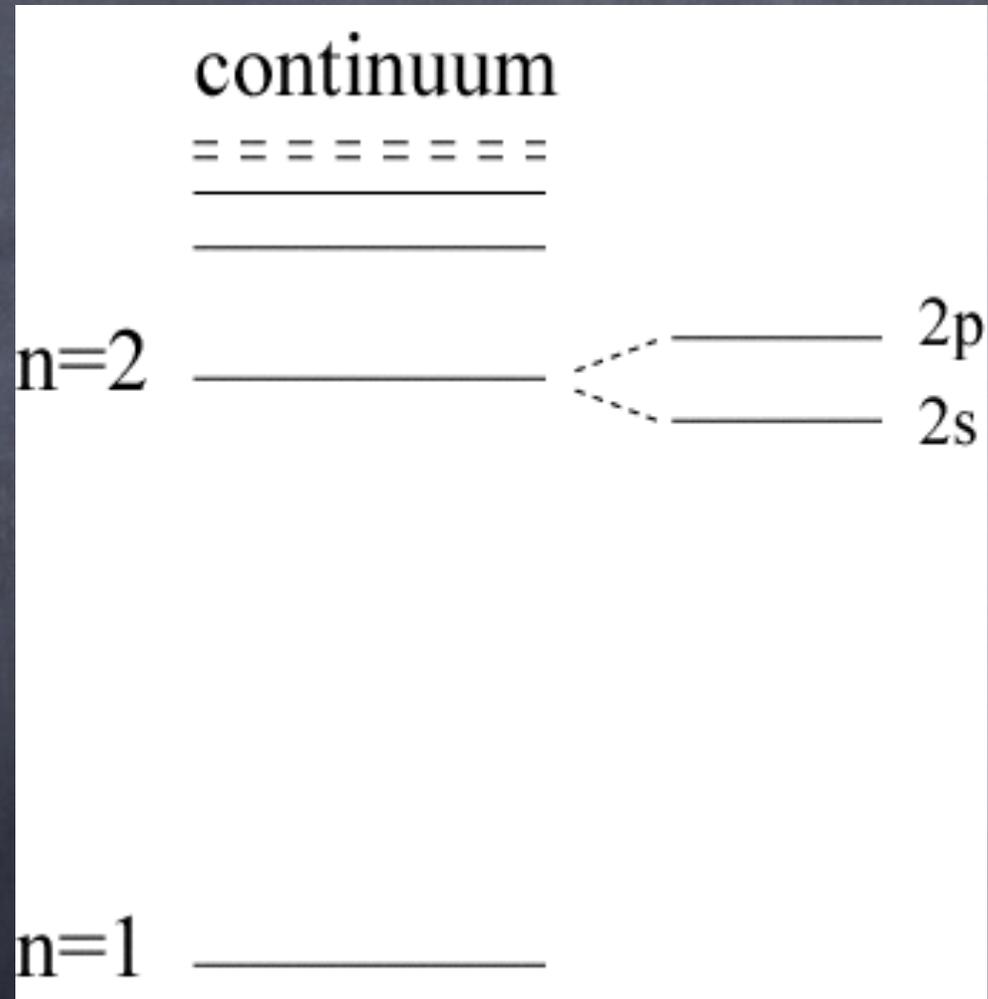
All higher states
in equilibrium
with continuum

1st excited state

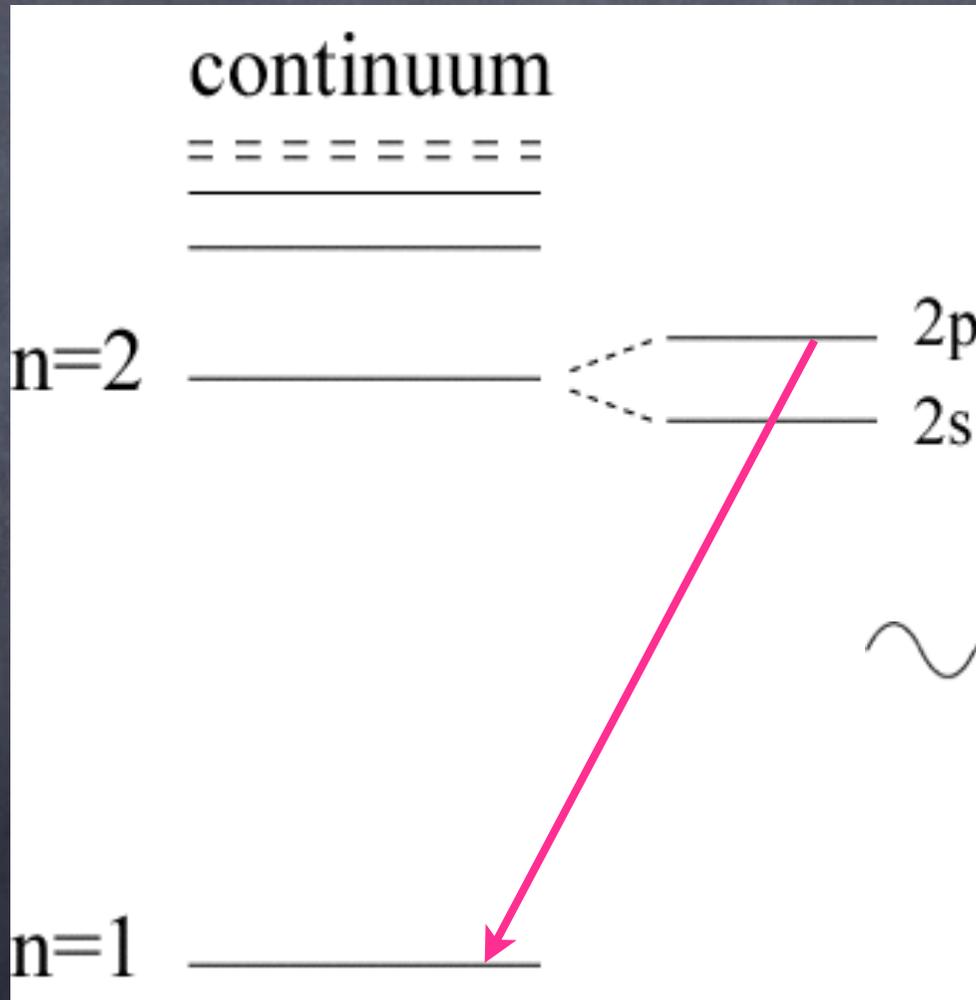
Ground state



How to get to the ground state in H



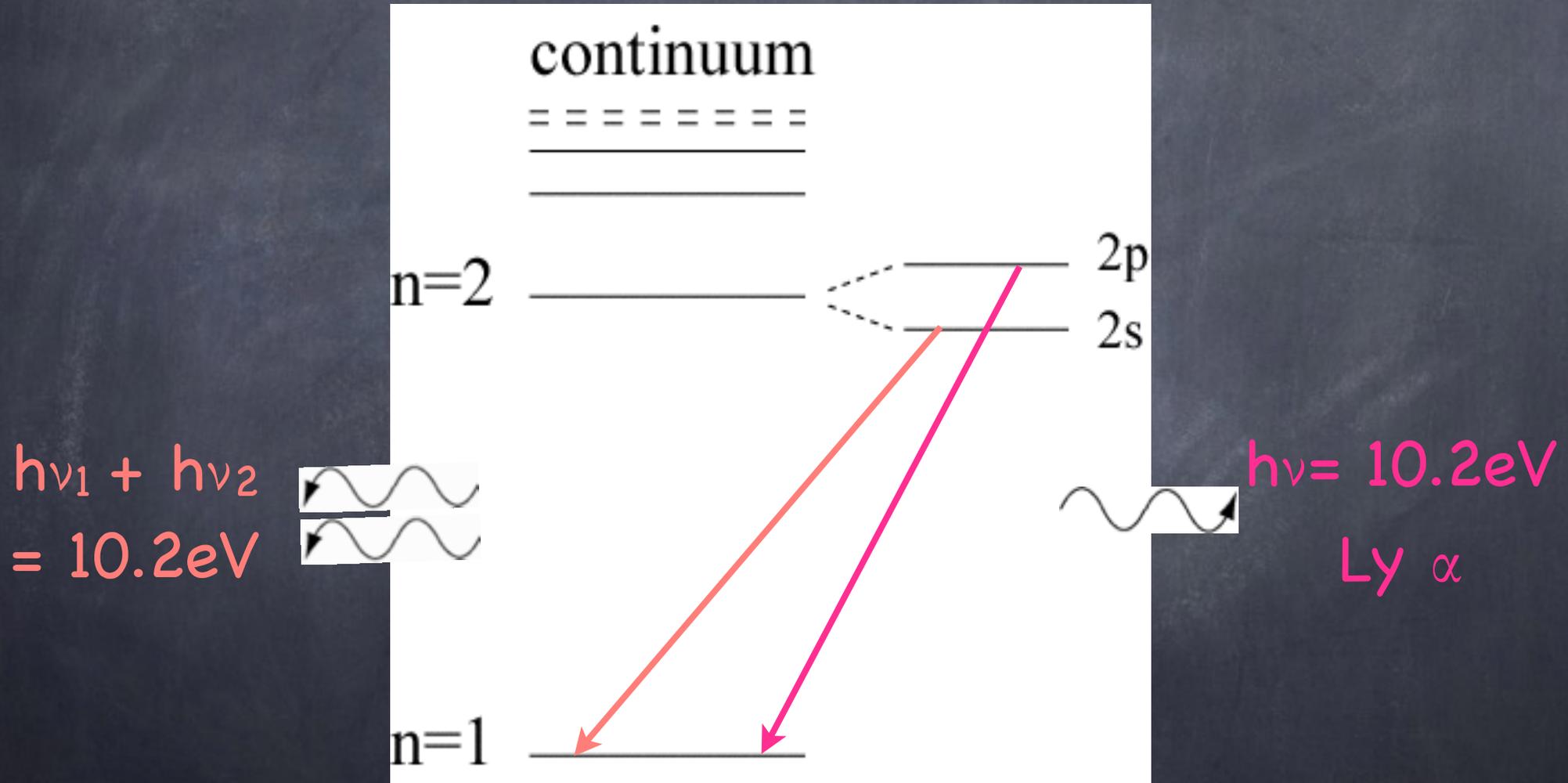
How to get to the ground state in H



$$h\nu = 10.2\text{eV}$$

$\text{Ly } \alpha$

How to get to the ground state in H



recast

Solve ODE for level population

recast

Solve ODE for level population
using public code "dverk"

recast

Solve ODE for level population

using public code "dverk"

written in fortran

recast

Solve ODE for level population

using public code "dverk"

written in fortran

with clunky initial condition generator

recast

Solve ODE for level population

using public code "dverk"

written in fortran

with clunky initial condition generator

add helium deionization

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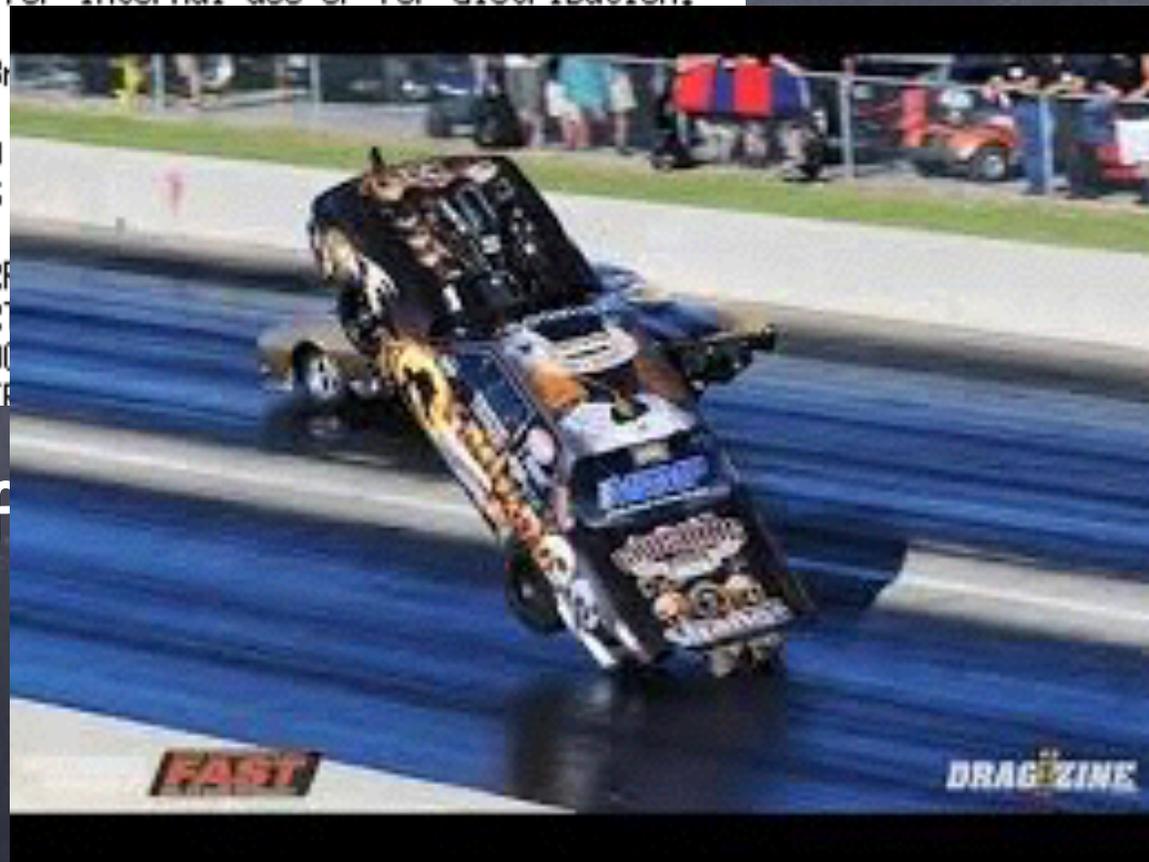
and later "fudge factors"

```
#####  
C Integrator for Cosmic Recombination of Hydrogen and Helium,  
C developed by Douglas Scott (dscott@astro.ubc.ca)  
C based on calculations in the papers Seager, Sasselov & Scott  
C (ApJ, 523, L1, 1999; ApJS, 128, 407, 2000)  
C and "fudge" updates in Wong, Moss & Scott (2008).  
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recfast

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ref



a fast wreck

recfast: basic equations

$$H(z)(1+z)\frac{dx_p}{dz} = \left(x_e x_p n_H \alpha_H F_H - \beta_H (1-x_p) e^{-h\nu_{H2s}/kT_M} \right) C_H$$

$$H(z)(1+z)\frac{dx_{\text{HeII}}}{dz} = \left(x_{\text{HeII}} x_e n_H \alpha_{\text{HeI}} - \beta_{\text{HeI}} (f_{\text{He}} - x_{\text{HeII}}) e^{-h\nu_{\text{HeI},2^1s}/kT_M} \right) C_{\text{HeI}} \\ + \left(x_{\text{HeII}} x_e n_H \alpha_{\text{HeI}}^t - \frac{g_{\text{HeI},2^3s}}{g_{\text{HeI},1^1s}} \beta_{\text{HeI}}^t (f_{\text{He}} - x_{\text{HeII}}) e^{-h\nu_{\text{HeI},2^3s}/kT_M} \right) C_{\text{HeI}}^t$$

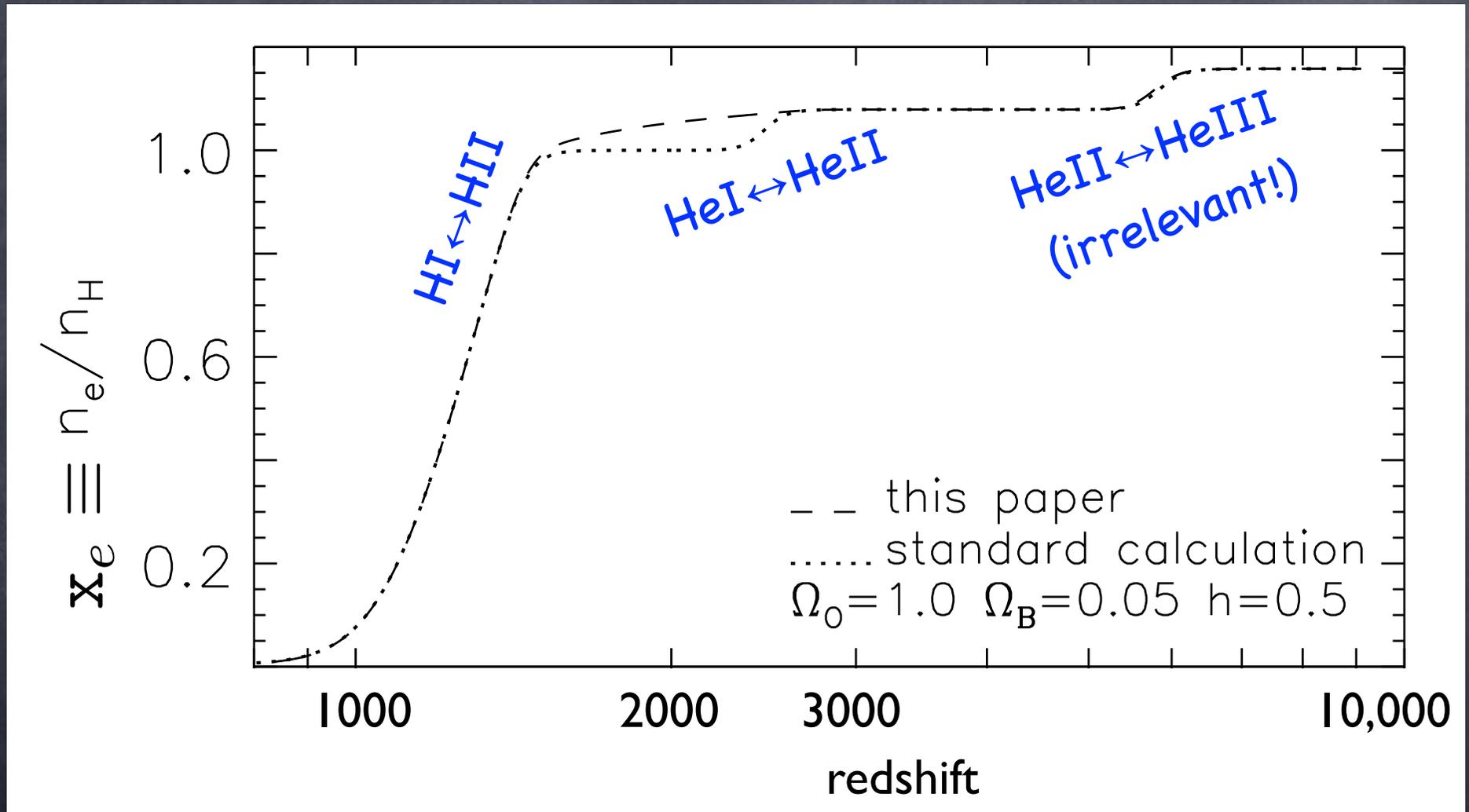
with $C_H = \frac{1 + K_H \Lambda_H n_H (1-x_p)}{1 + K_H (\Lambda_H + \beta_H) n_H (1-x_p)}$

$$C_{\text{HeI}} = \frac{1 + K_{\text{HeI}} \Lambda_{\text{He}} n_H (f_{\text{He}} - x_{\text{HeII}}) e^{h\nu_{ps}/kT_M}}{1 + K_{\text{HeI}} (\Lambda_{\text{He}} + \beta_{\text{HeI}}) n_H (f_{\text{He}} - x_{\text{HeII}}) e^{h\nu_{ps}/kT_M}}$$

$$C_{\text{HeI}}^t = \frac{1}{1 + K_{\text{HeI}}^t \beta_{\text{HeI}}^t n_H (f_{\text{He}} - x_{\text{HeII}}) e^{h\nu_{ps}^t/kT_M}}$$

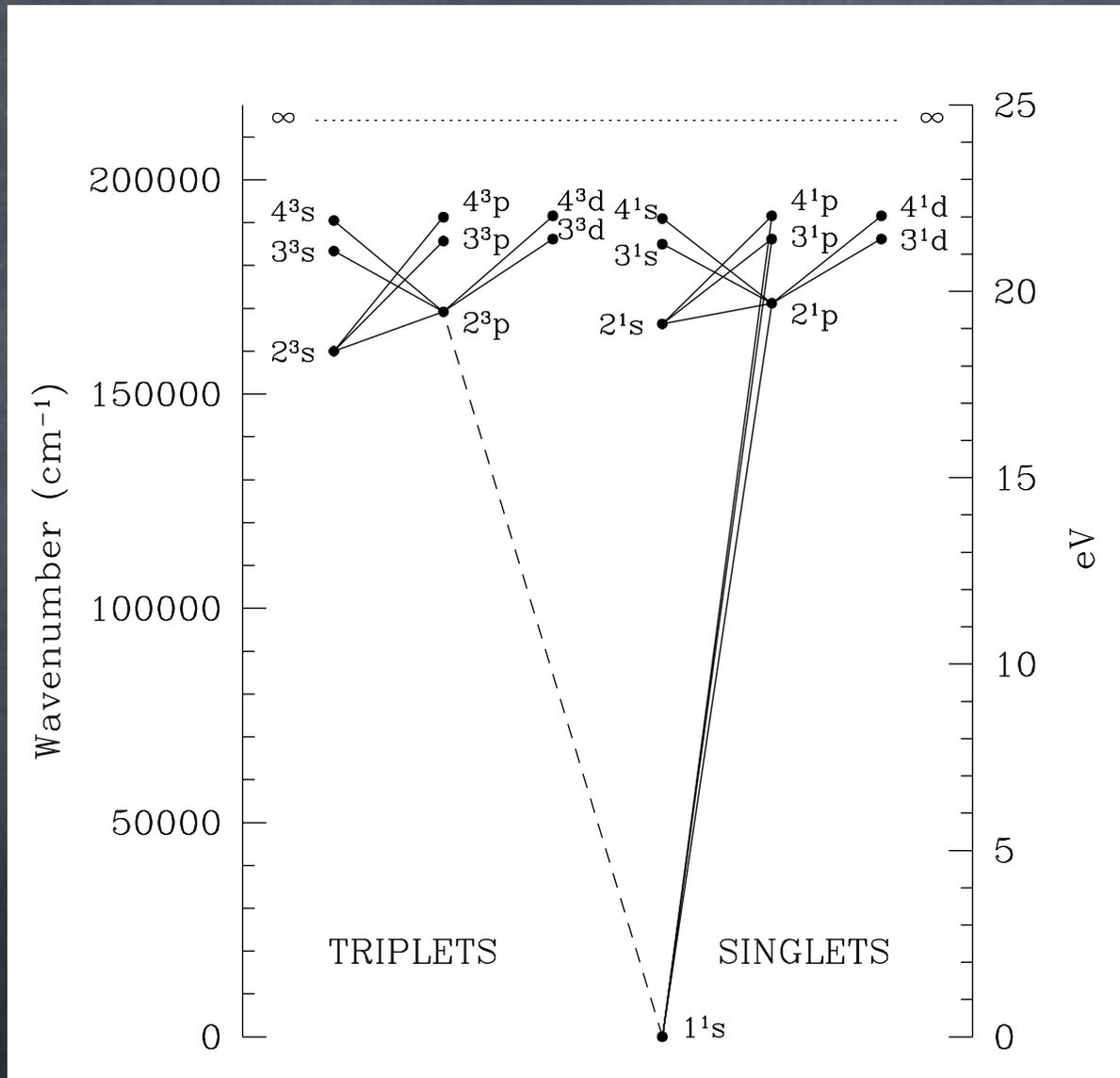
$$F_H = 1.14 \quad F_{\text{He}} = 0.86$$

3 main recombination transitions



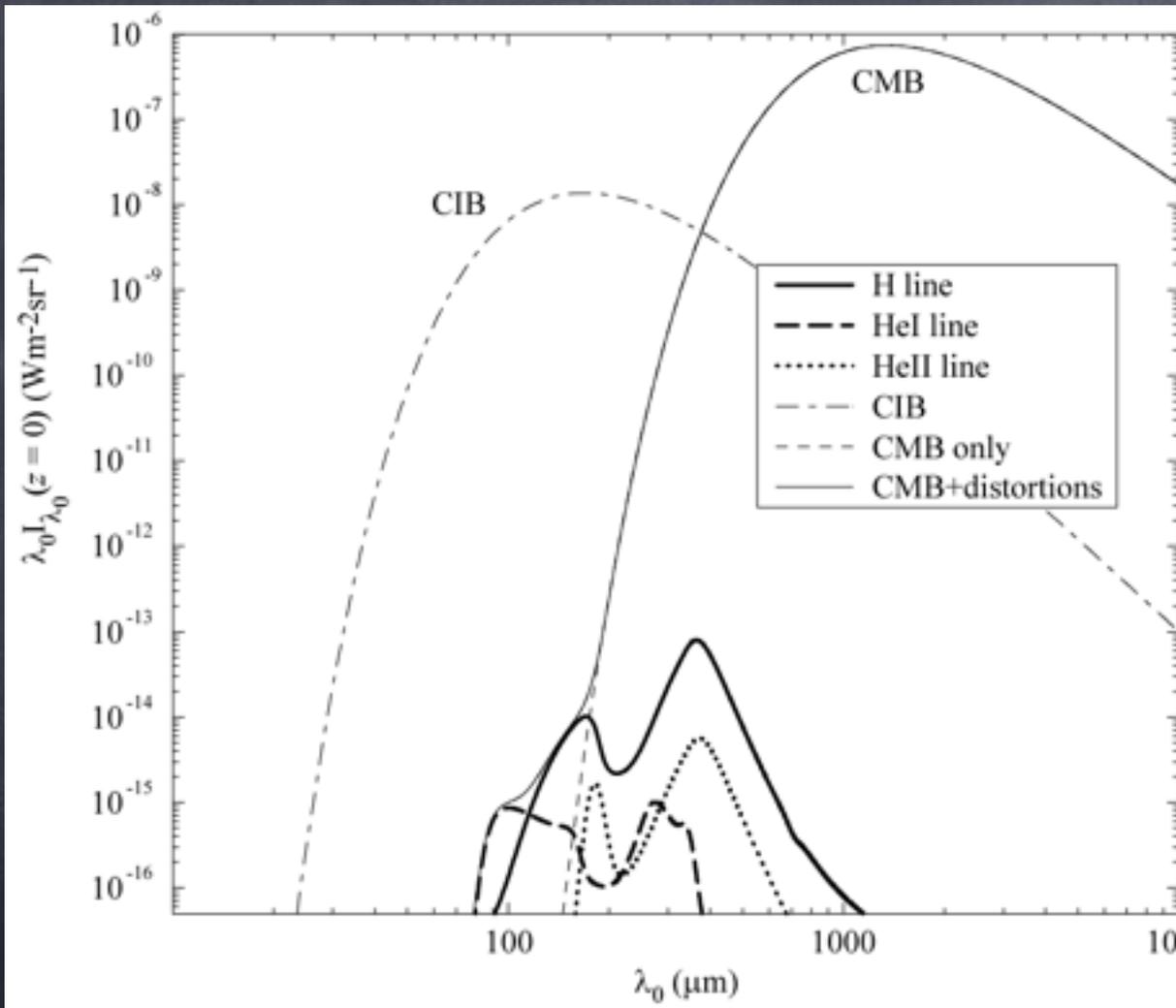
Seager, Sasselov & Scott (2000)
[but not correct!]

Grotrian diagram for Helium



Complicated!

What happens to the recombination photons?



For Ly α and
2-photon here

Other lines
can also be
calculated

Definite prediction but measurable??

recfast

<http://www.astro.ubc.ca/people/scott/recfast.html>

latest version 1.5.2

good enough for Planck

essentially solves a "3-level atom"
simultaneously for H and for He
(it's ZKS (1968) with attention to accuracy)

includes "fudge factors" to approximate the
results of more accurate calculations

the corrections are cosmology-independent
because standard cosmology is so well constrained
(but not true for far-out theories)

The devil is in the details

- How big an effect on the anisotropy power spectra is implied by uncertainty in $V(z)$?
- Hu, Scott, Sugiyama & White (1995):
1% effects are not fully under control
- Seager, Sasselov & Scott (1999,2000): recfast
- Seljak, Sugiyama, White & Zaldarriaga (2003):
0.1% accuracy if you use same recombination
- Lots of new detailed effects in last ~10 years
- Moss, Scott & Wong (2008):
Most effects can be "fudged" in 3-level code
- 0.1% is very challenging, but now done

Why should I care?!

Shape of $V(z)$ affects CMB anisotropies

Why should I care?!

Shape of $V(z)$ affects CMB anisotropies

CMB used to estimate cosmological parameters

Why should I care?!

Shape of $V(z)$ affects CMB anisotropies

CMB used to estimate cosmological parameters

So wrong $V(z)$ can bias cosmological parameters

Why should I care?!

Shape of $V(z)$ affects CMB anisotropies

CMB used to estimate cosmological parameters

So wrong $V(z)$ can bias cosmological parameters

Subtly wrong $V(z)$ can bias the parameters

Why should I care?!

Shape of $V(z)$ affects CMB anisotropies

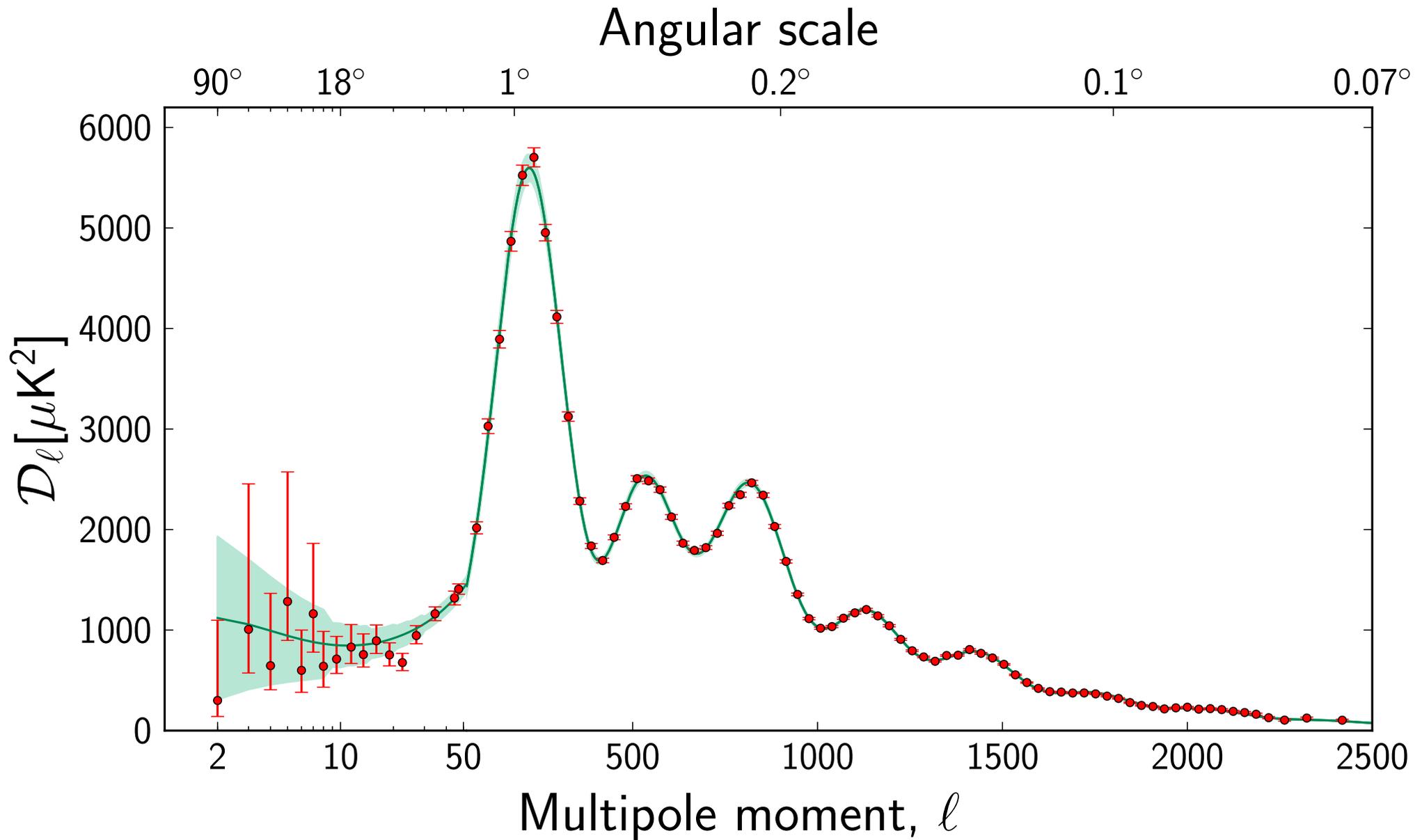
CMB used to estimate cosmological parameters

So wrong $V(z)$ can bias cosmological parameters

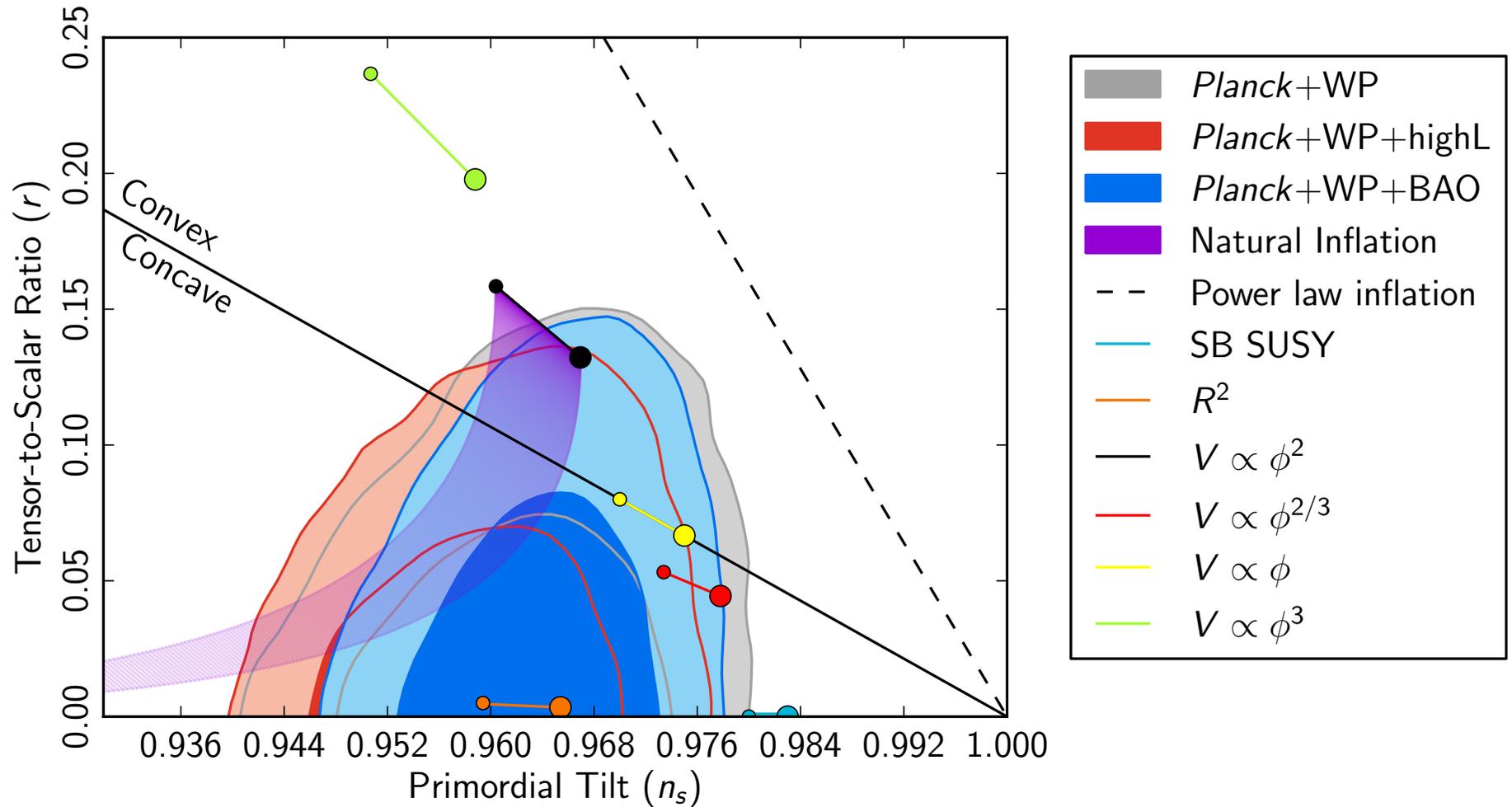
Subtly wrong $V(z)$ can bias the parameters

Need to get the $\sim eV$ physics insanely precise
in order not to screw up the $\sim 10^{15}$ GeV physics!

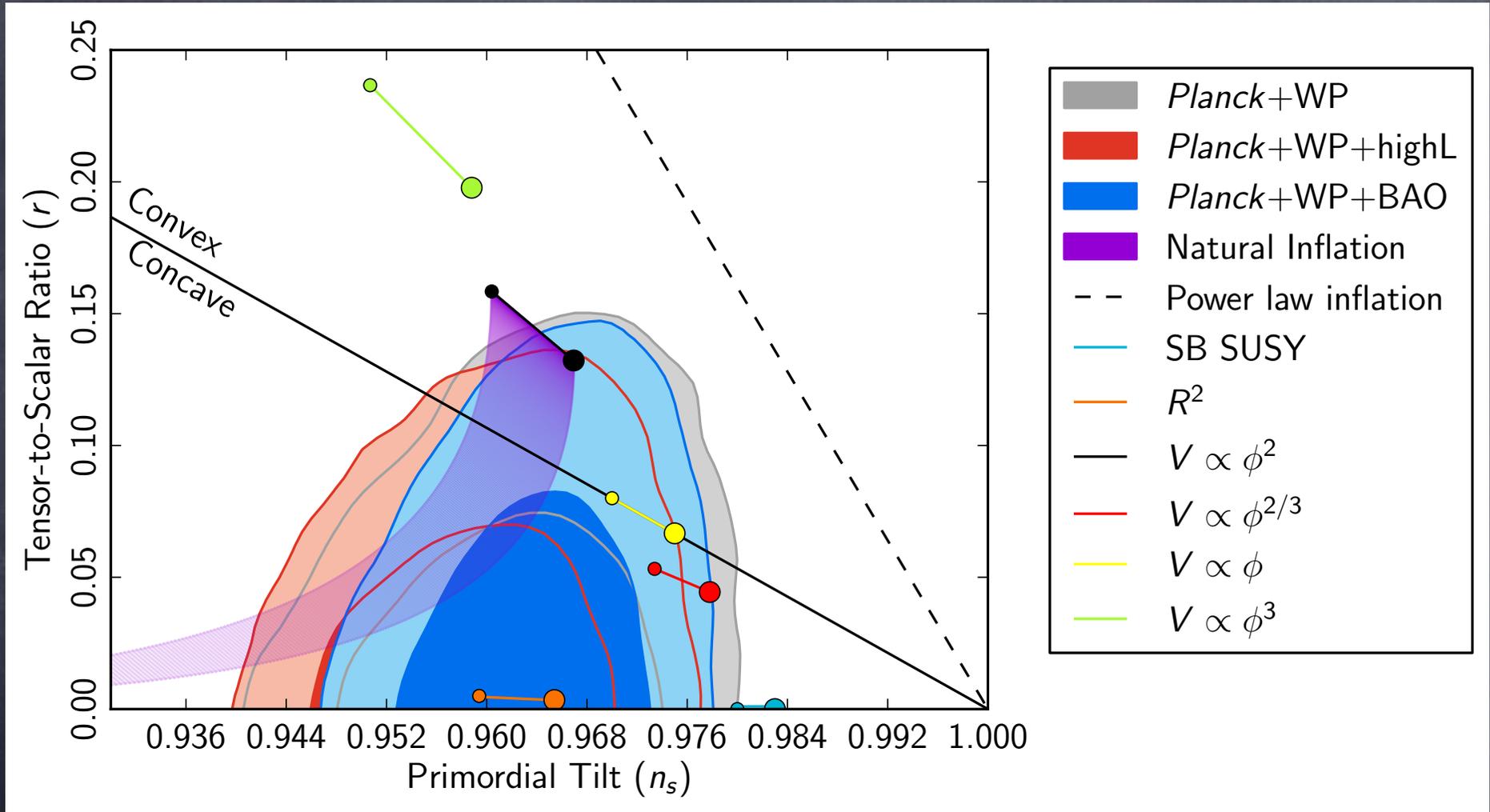
Planck's exquisite power spectrum!



Planck (+others) constraining inflation

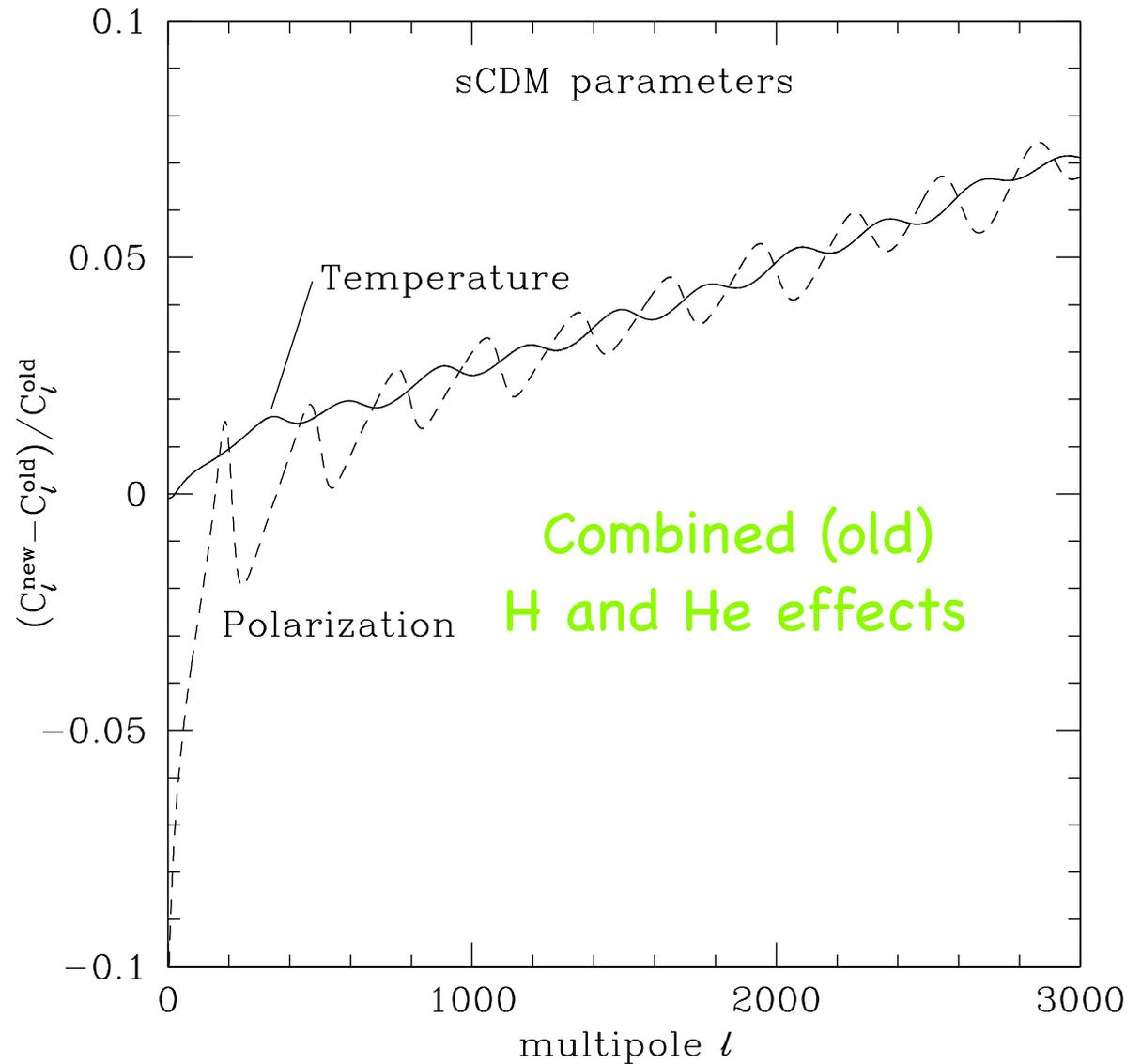


Planck (+others) constraining inflation



But this can be biased if we don't get high- z $V(z)$ right

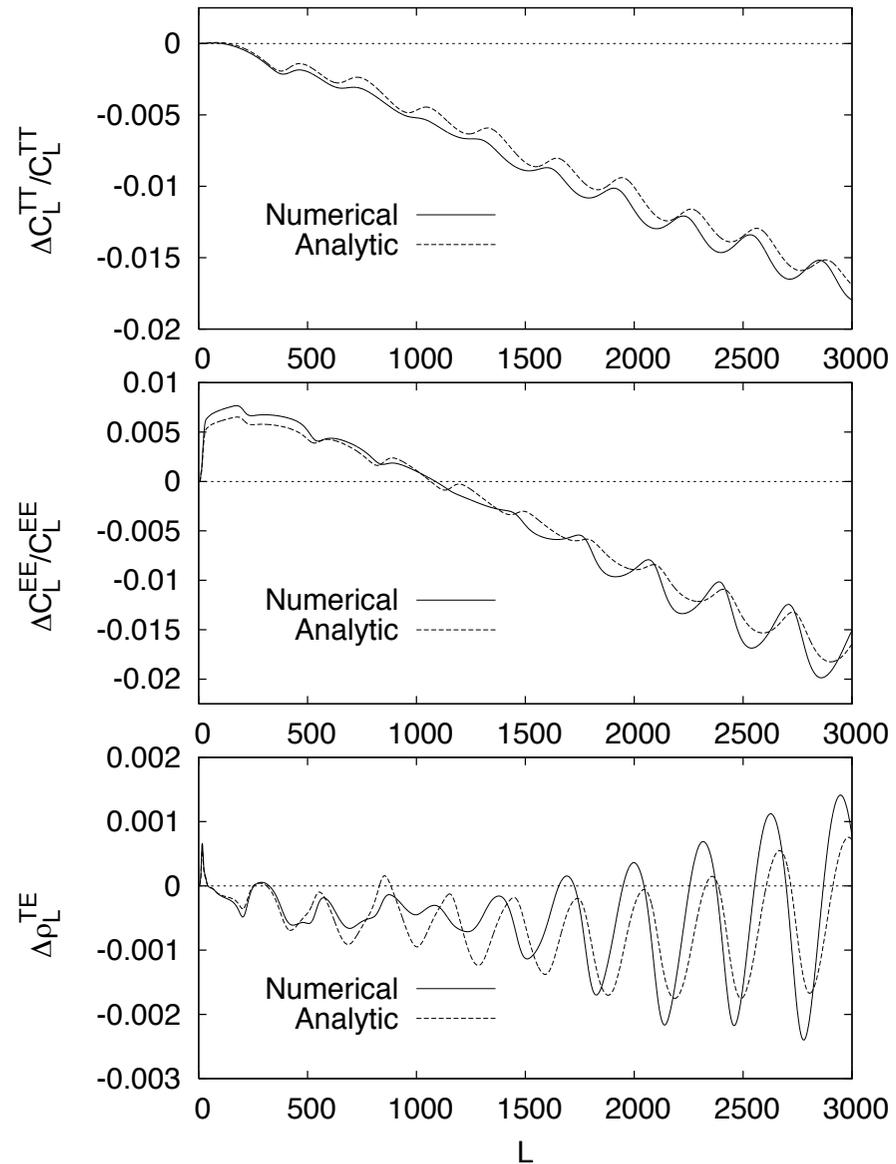
With a given recombination history, can calculate $V(z)$ and hence effect on anisotropies



Seager, Sasselov & Scott (2000)

New H effects

Change in power spectra



Better treatment
of 2-photon
transitions (including
intermediate
resonant states)
plus Raman scattering

Burst of activity, motivated by CMB anisotropies

Burst of activity, motivated by CMB anisotropies

About a paper a year for 30 years!
Then lots of activity (now slowing?)

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About a paper a year for 30 years!
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The 0.1% goal is extremely hard for any
calculation of anything

Burst of activity, motivated by CMB anisotropies

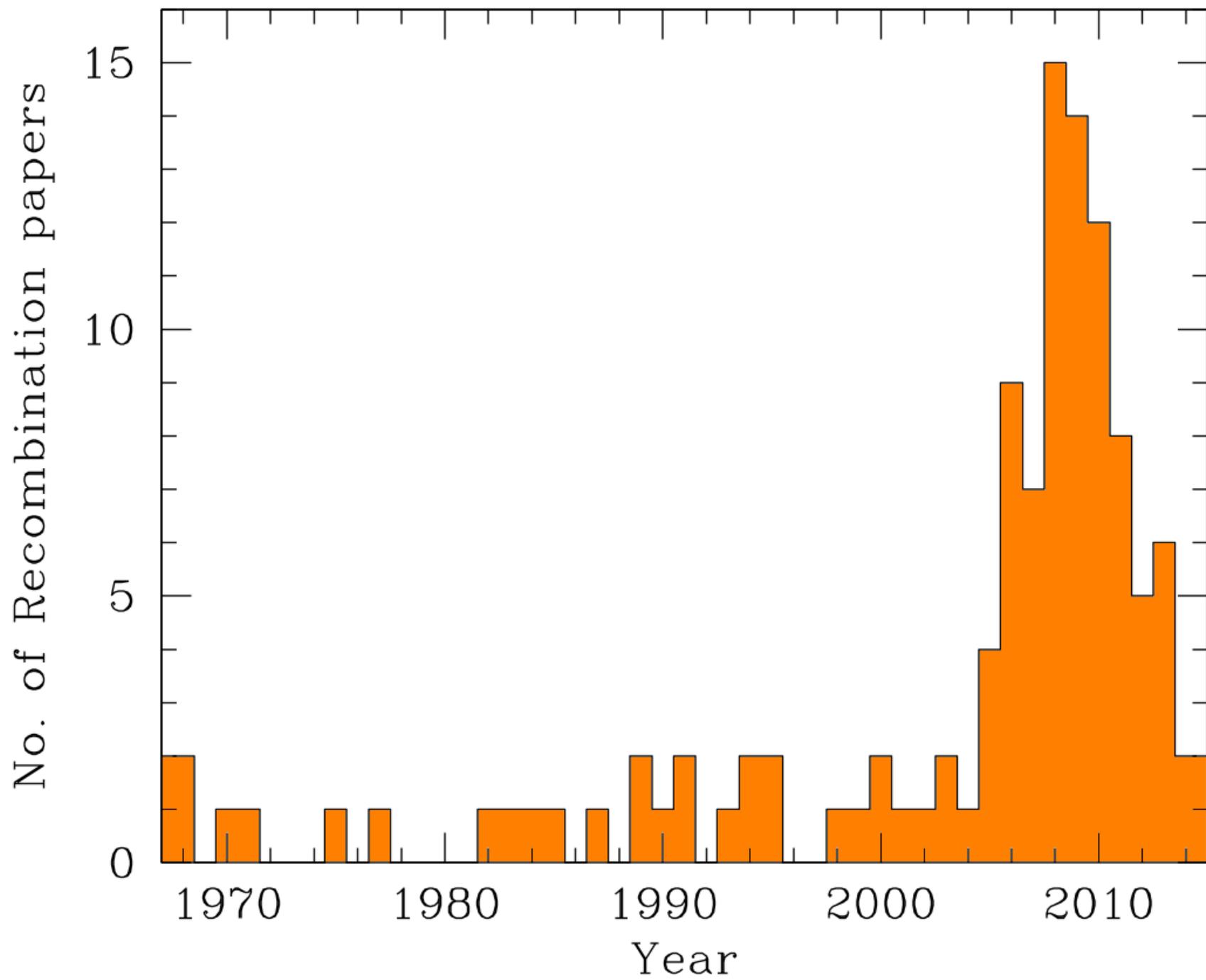
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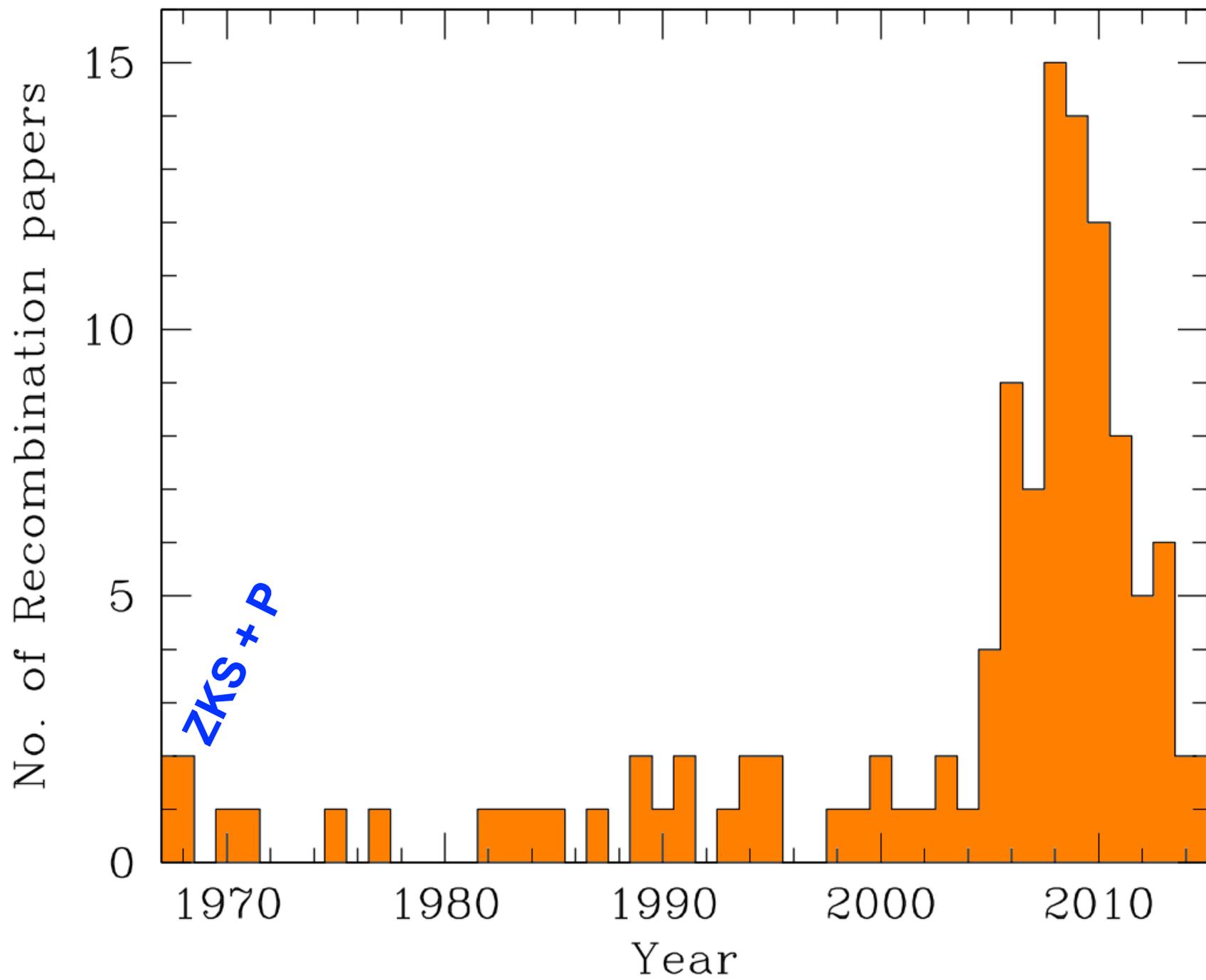
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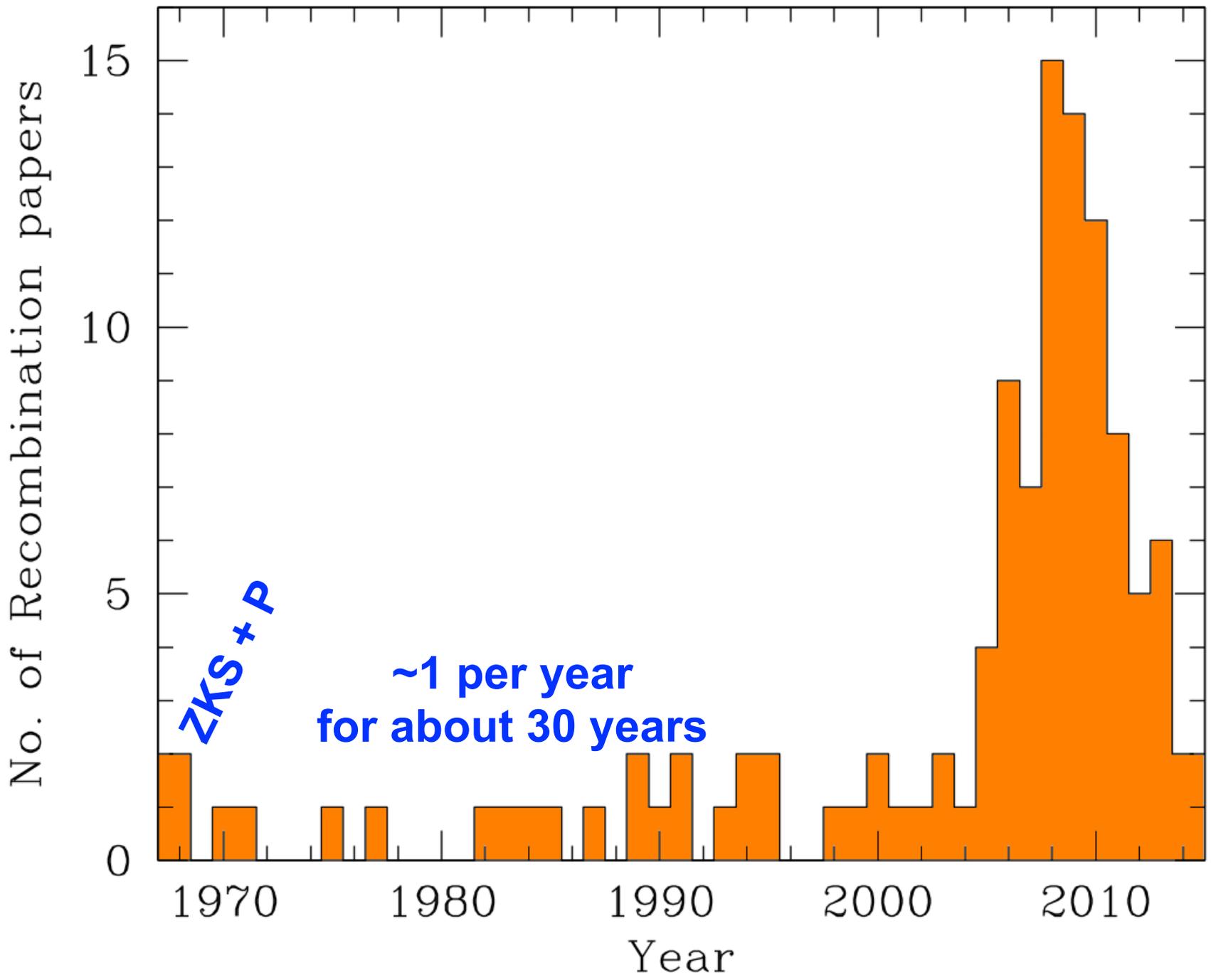
It won't get any easier

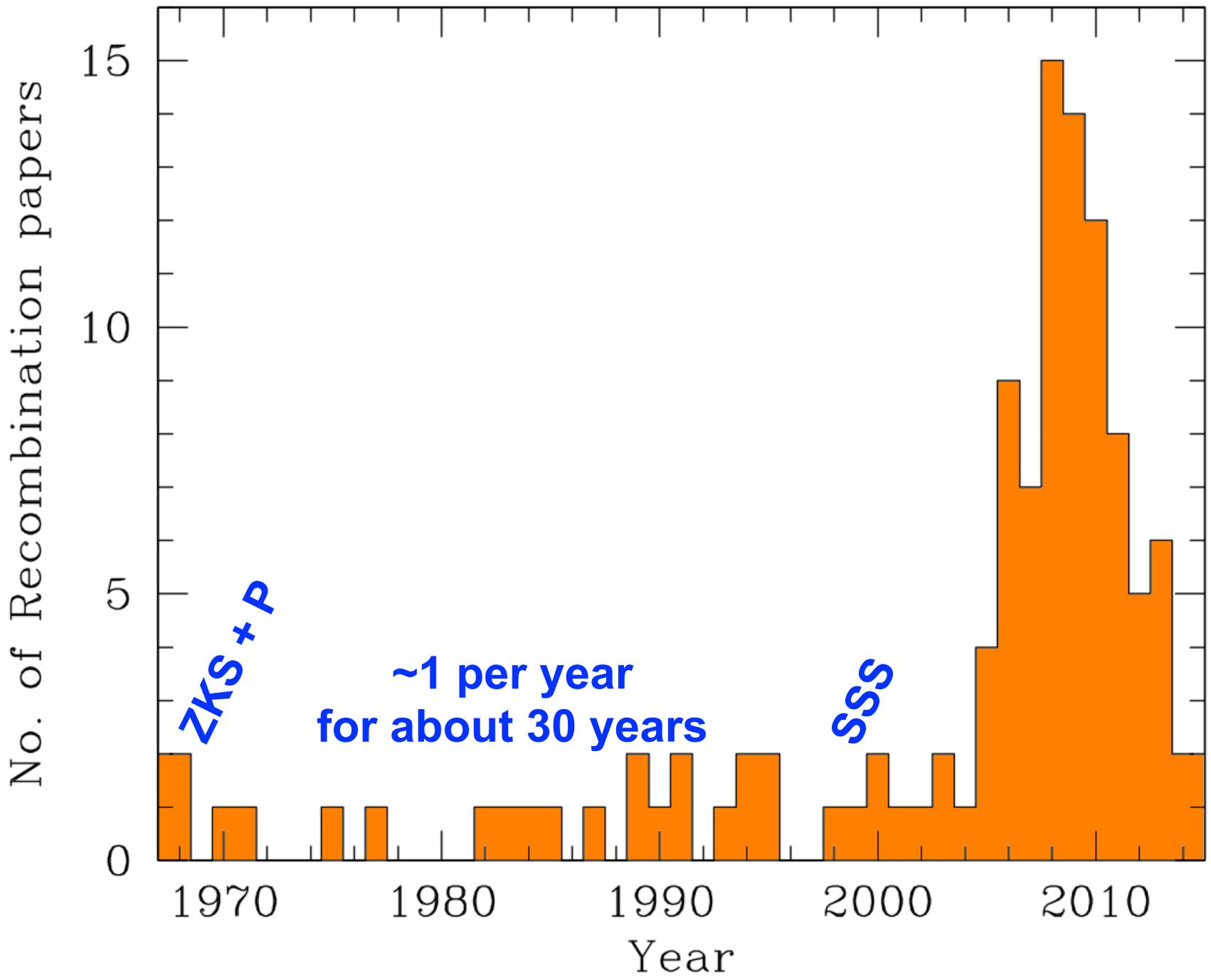
Future recombination calculations are not
for the faint of heart!

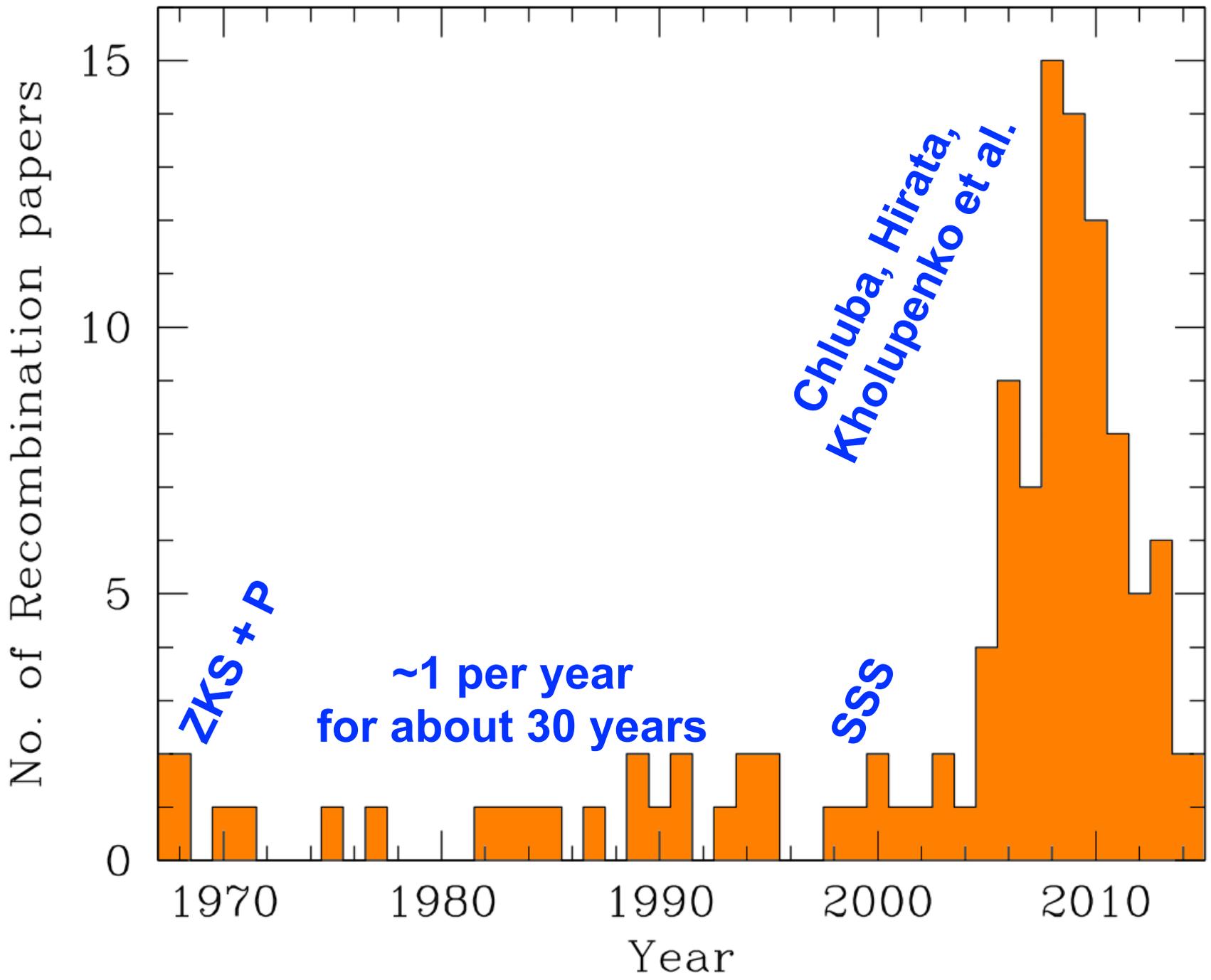
But they will eventually be necessary!









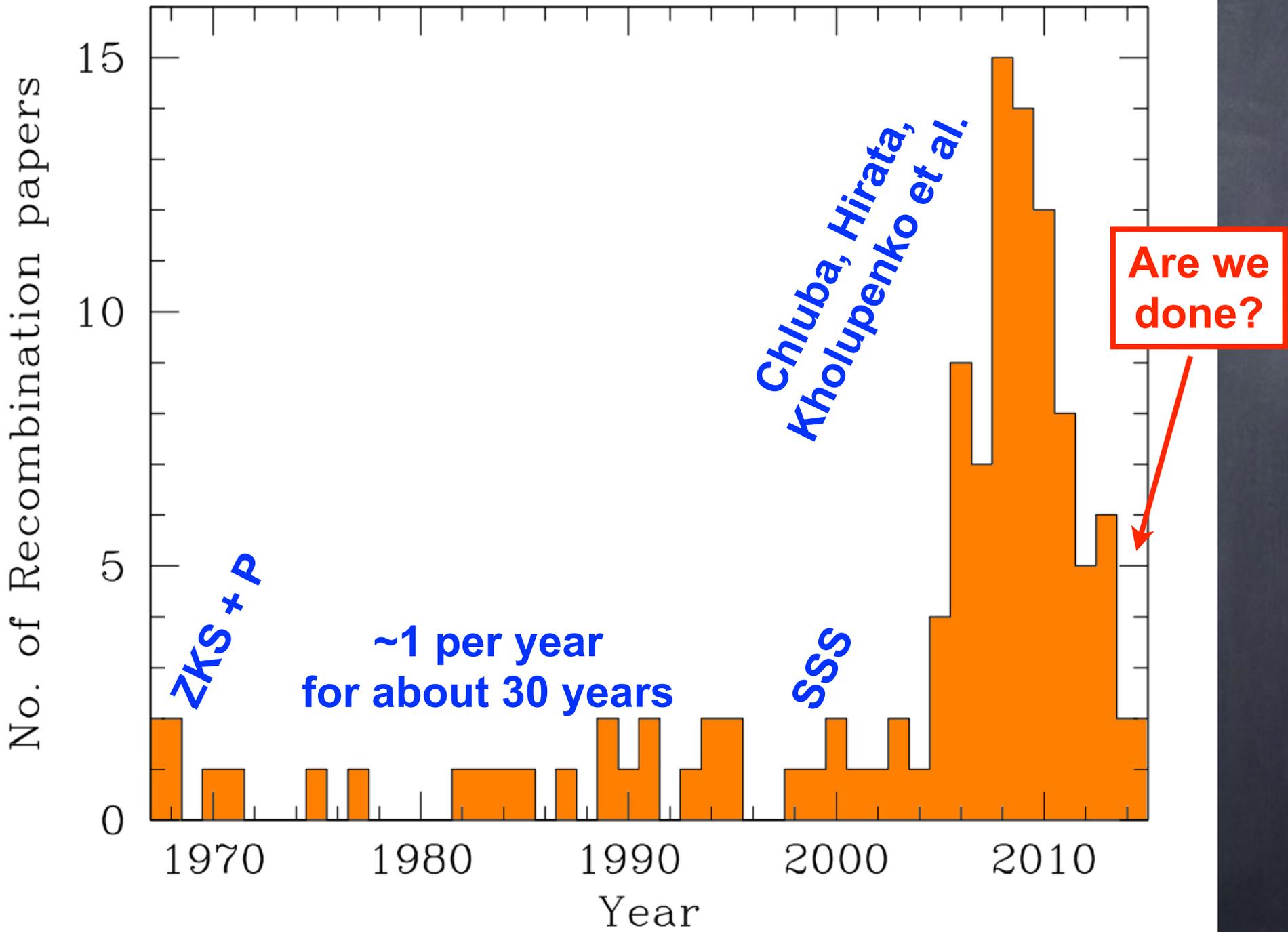


ZKS + P

**~1 per year
for about 30 years**

**Chluba, Hirata,
Kholupenko et al.**

SSS



List of
some
issues to
address in
order to
achieve
precision
+ there
are many
more
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List of
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Effect	Max. Δx_e	z	References
Energy level			
Seperate L -state in H I atom	-0.7% +1%	1090 ≤ 900	[6, 50]
Bound-bound transition			
Inclusion of He I $2^3P_1-1^1S_0$	-1.1% -0.3%*	1750 1900	[19, 64] [62]
Inclusion of He I $n^3P_1-1^1S_0$ ($n \geq 3$)	$-4 \times 10^{-3}\%$ *	2000	[62]
Inclusion of H I $ns, nd-1s$ ($n \geq 3$):			
- effective rate only	-0.4%	1200	[9, 64]
- with feedback	-1.2%	1250	[26]
- with feedback and Raman scattering	+1.3%	900	[26]
Inclusion of He I $n^1S, n^1D-1^1S_0$ ($n \geq 3$):			
- effective rate only	-0.5%	1800	[19, 64]
- with feedback and Raman scattering	-0.05%	2000	[27]
Bound-free transition			
Direct recombination for H I	$-6 \times 10^{-4}\%$	1280	[8]
Direct recombination for He I	-0.02%	1900	[28, 61]
Radiative transfer			
Continuum opacity of H I in He I $2^1P_1-1^1S_0$	-2.5%*	1800	[61, 62]
Feedback between He I $2^3P-1^1S_0$ and $2^1P_1-1^1S_0$	+1.5%*	1800 -2600	[61, 62]
Stimulated and induced H I $2s-1s$	+0.6%	900	[7, 26, 32]
Diffusion of Ly α line profile (with recoil of H atoms)	$\sim 1\%$	900	[23, 24, 35, 36]
Continuum opacity of H I modified to feedback in He I lines	-0.5%*	1800	[61, 62]
Continuum opacity of H I in He I $n^1P-1^1S_0, n^3P-1^1S_0$ ($n \geq 3$), $n^1D-1^1S_0$	-0.05%*	1900	[61, 62]
Coherent scattering in $n^1P-1^1S_0$	-0.02%*	2000	[61]
Evolution of T_M	$\pm 10^{-3}\%$	-	[28, 56, 61]
Secondary distortions from He I & H I in H I recombination	+0.1%	-	[56, 60]
Others			
He I $2^3P_1-1^1S_0$ spontaneous rate	$\pm 0.1\%$	1900	[62]
CMB monopole temp. $T_{CMB} \pm 1$ mK	$\pm 0.5\%$	900	[10]
Formation of hydrogen molecule	-1%	< 150	[56]

Table 2.1: Summary of the improvements and uncertainties in numerical recombination calculation. Here Δx_e is the difference from the value given by RECFAST [55]. *Note: This is the relative change compared with the full radiative model in Switzer & Hirata (2007) [62].

Wan Yan
Wong's
thesis
(2008)

Progress

- ✿ New codes produced – more physics, but more computing time:
 - **Hyrec** – Ali-Haïmoud & Hirata (2011)
 - **Cosmorec** – Chluba & Thomas (2011)
- ✿ Useful code comparisons carried out
- ✿ General agreement that calculations accurate enough for primary CMB anisotropies
- ✿ But there's always more to do!

Are we done?

- Have we included all the levels needed for H?
- Are all rates and cross-sections for He known?
- Included all interactions between H and He?
- Radiative transfer good enough? Line redistribution?
- Will we eventually need ray tracing?
- Is the line spectrum accurate (for measurement?)
- Are any other scattering effects important?
- Do we ever need to think about perturbations?
- Other things we haven't thought of yet?
- Can we constrain unknown effects by constraining phenomenological parameters in MCMC fits to data?

recfast

recfast

Started in 1989!

recfast

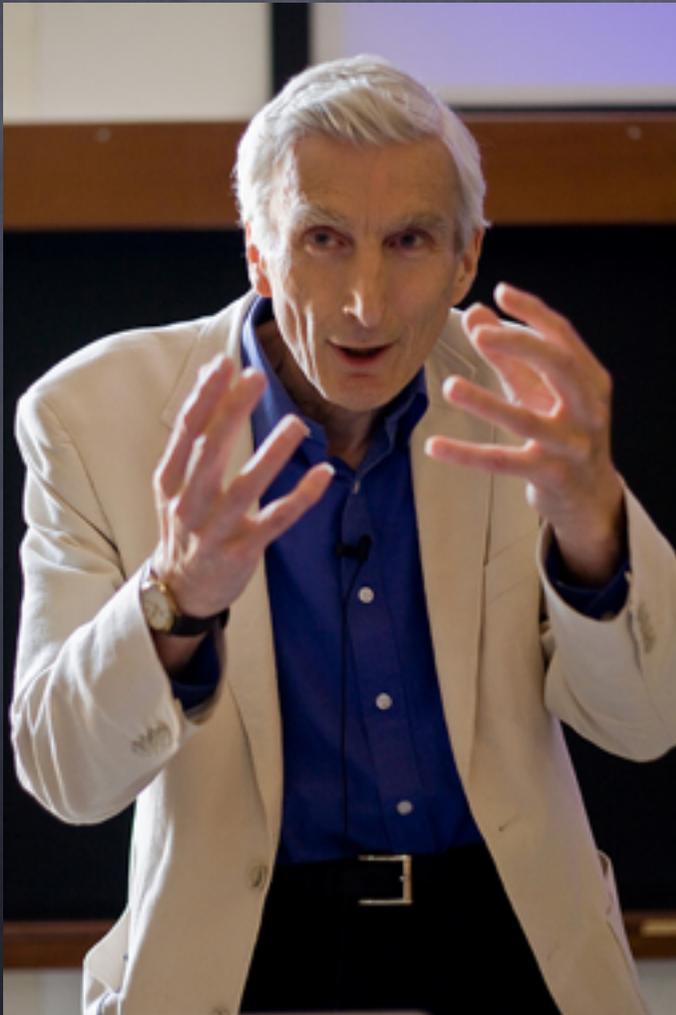
Started in 1989!

Adapted recombination history to include decaying particles

recfast

Started in 1989!

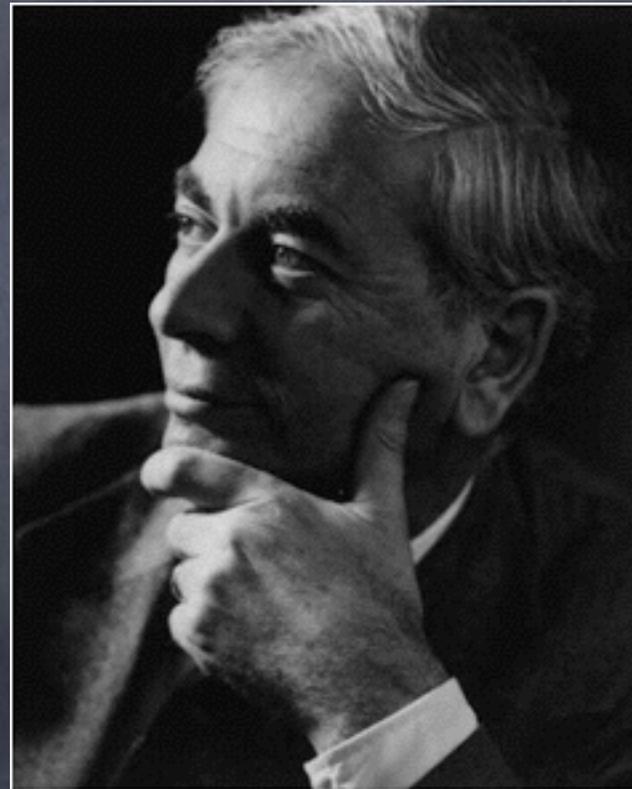
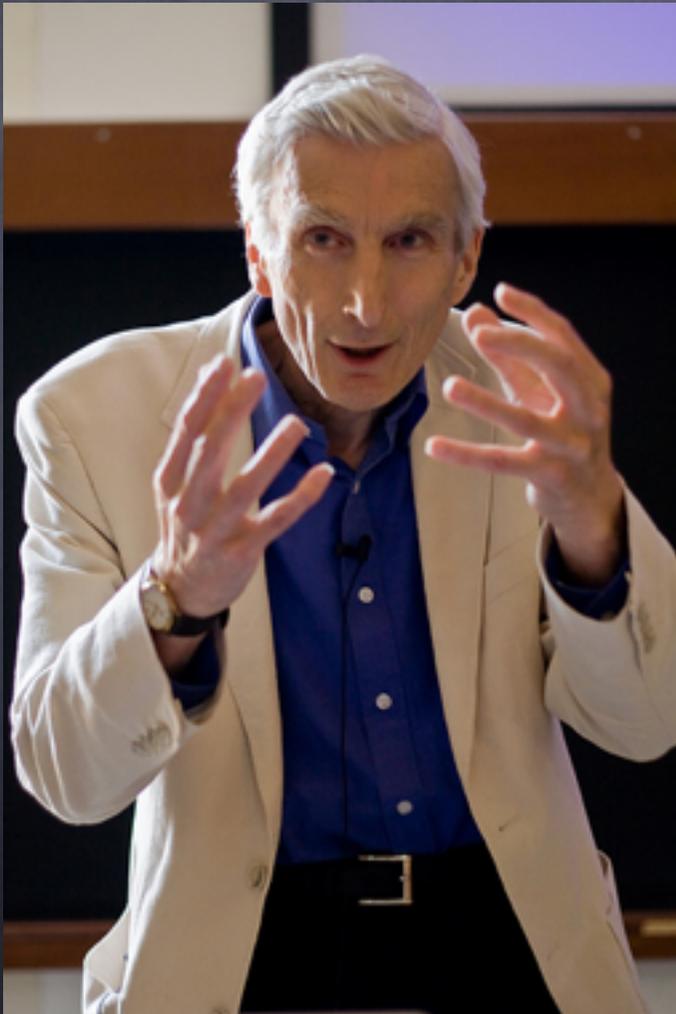
Adapted recombination history to include decaying particles



recfast

Started in 1989!

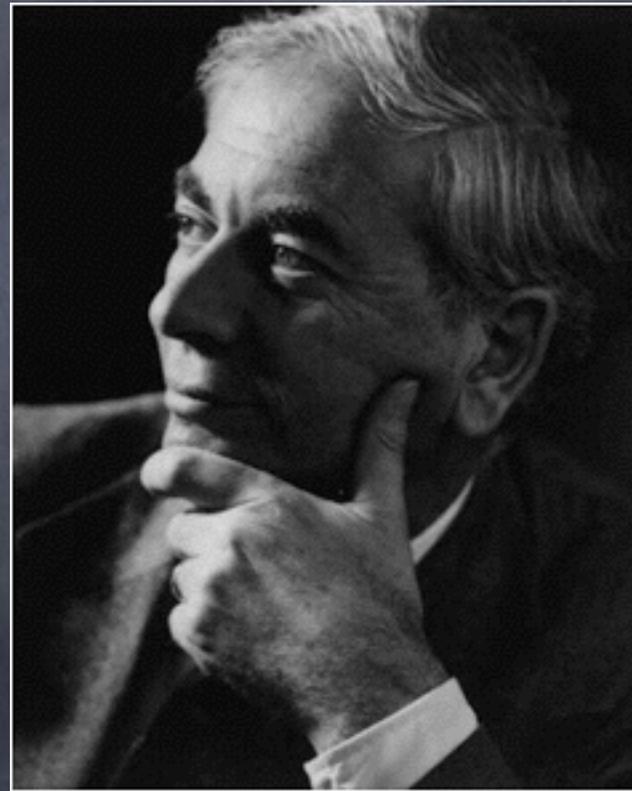
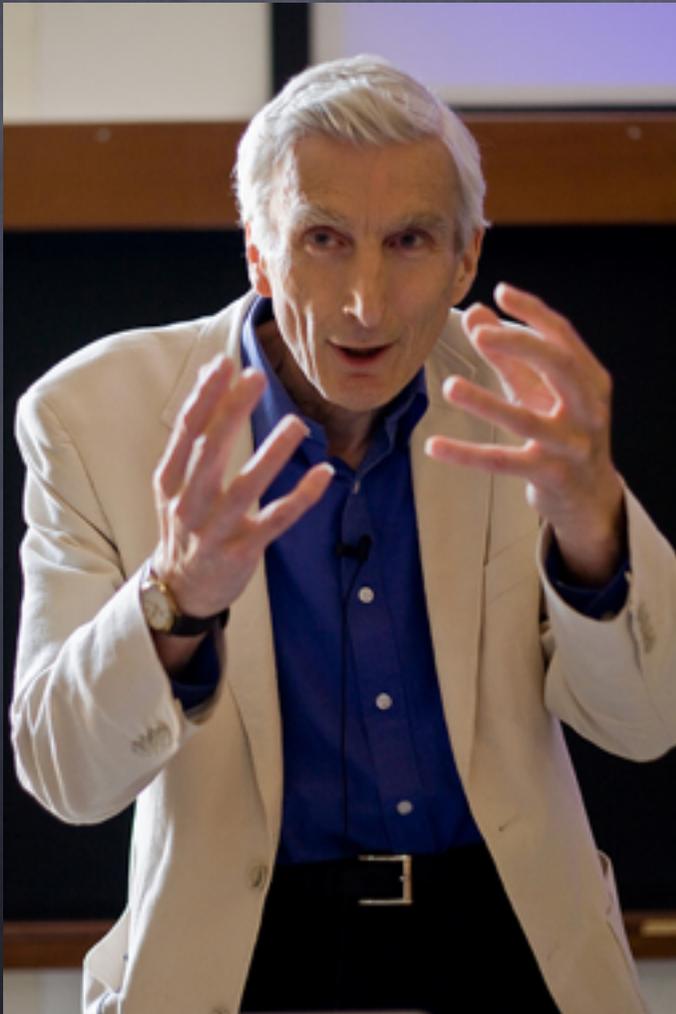
Adapted recombination history to include decaying particles



recfast

Started in 1989!

Adapted recombination history to include decaying particles



Scott, Rees & Sciama 1991

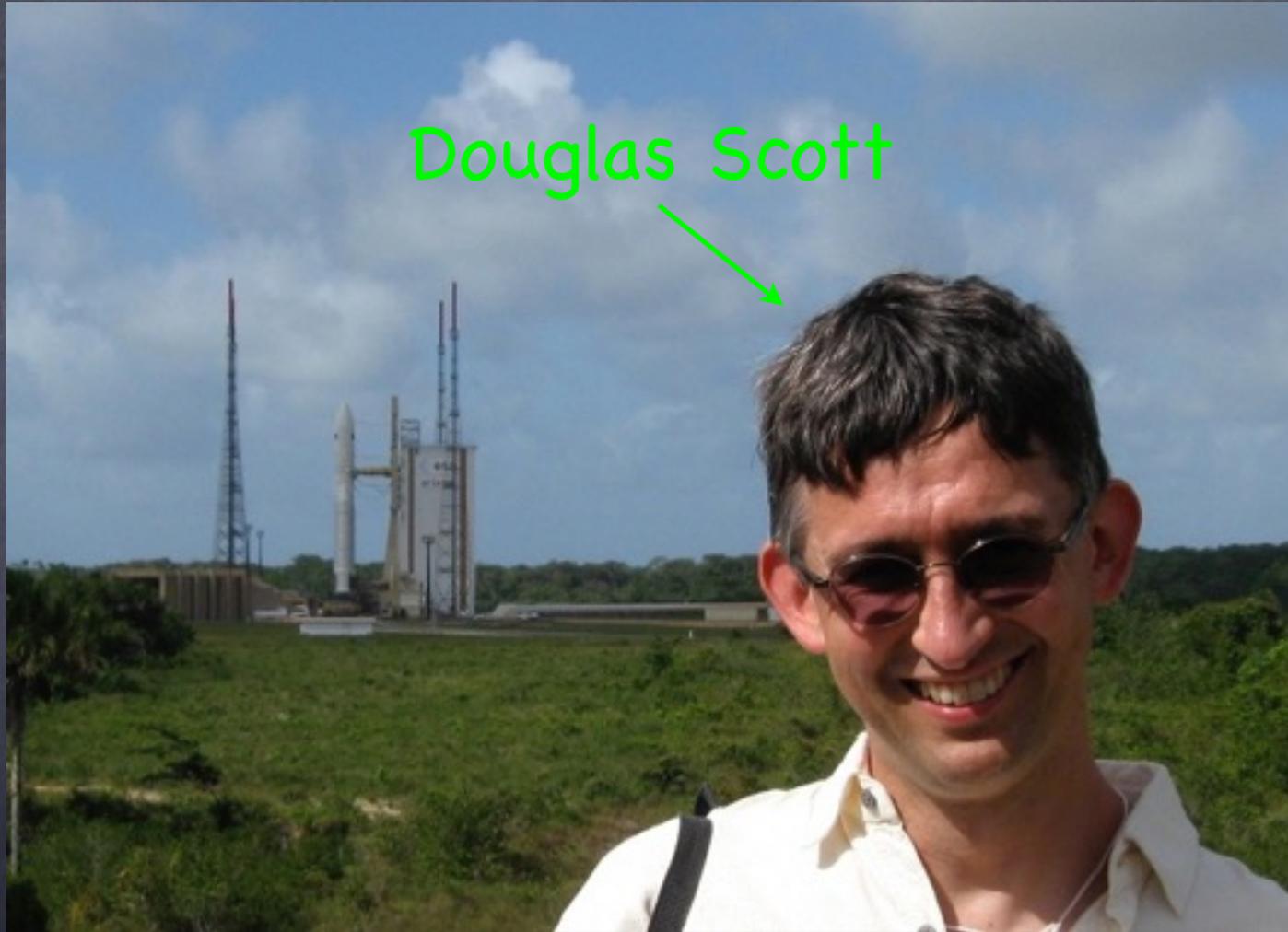
(see Appendix A of Ph.D. thesis)

recfast

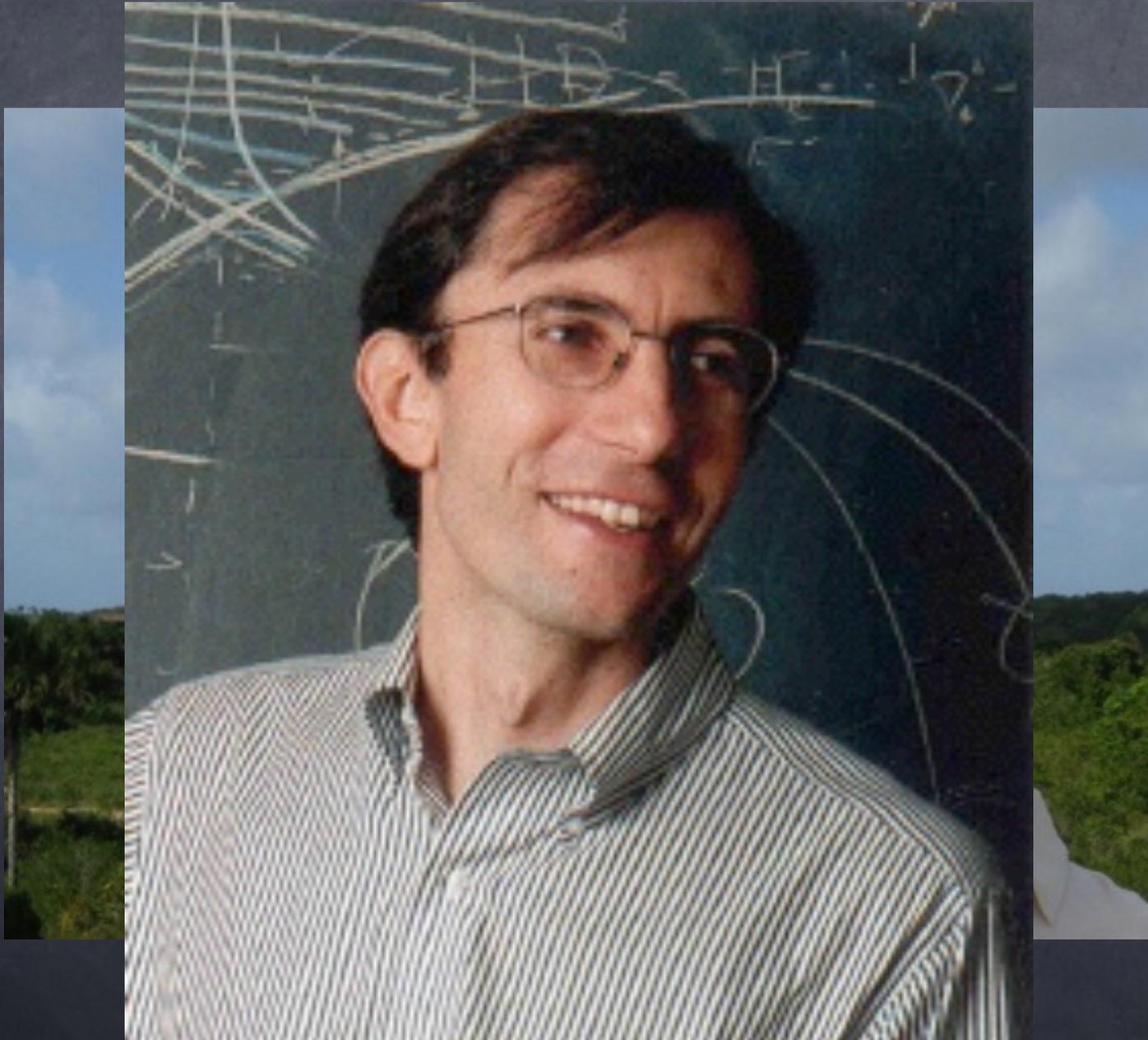
recfast



recfast

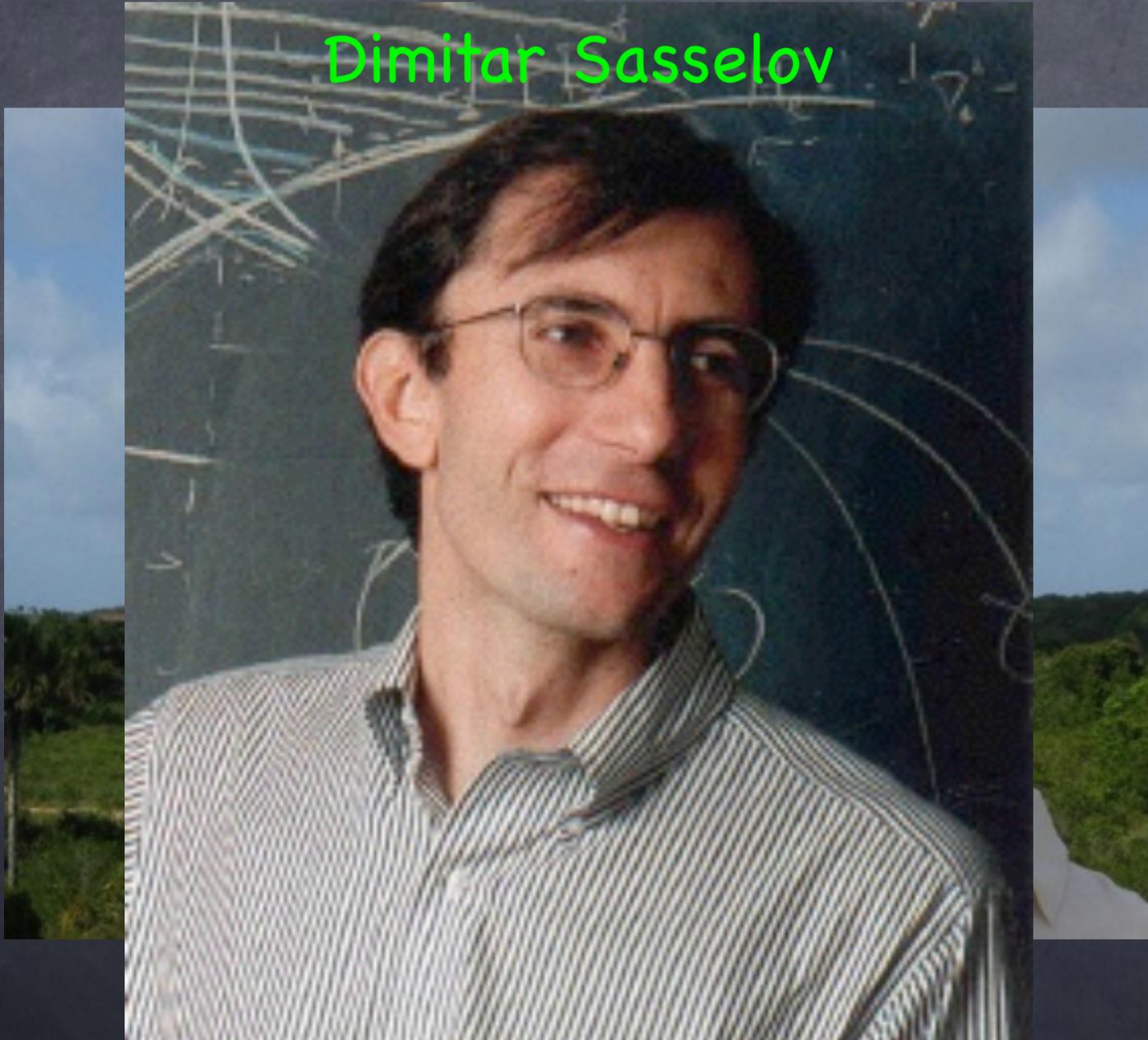


recfast



recfast

Dimitar Sasselov



recfast



Origins PUBLIC LECTURE
INSTITUTE

Sara SEAGER

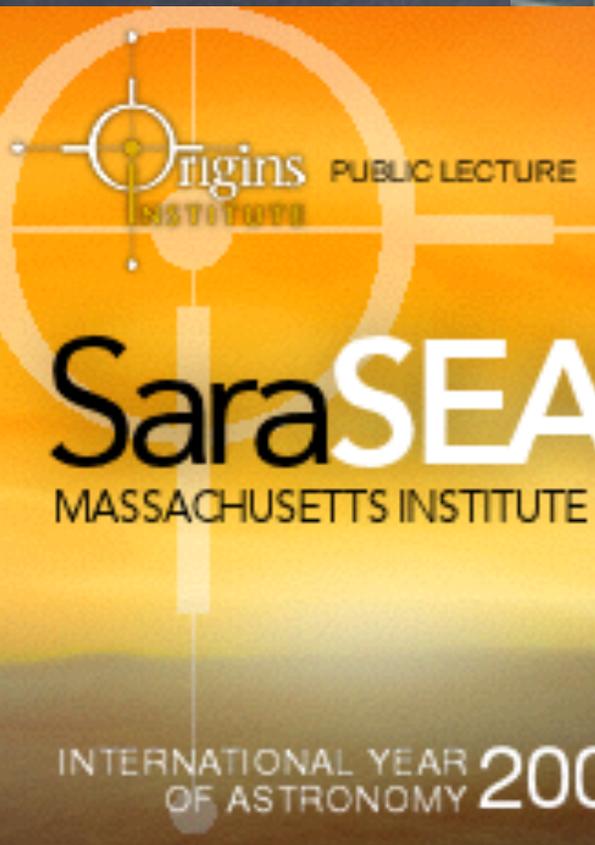
MASSACHUSETTS INSTITUTE OF TECHNOLOGY



INTERNATIONAL YEAR OF ASTRONOMY 2009 PUBLIC LECTURE



recfast



RECFAST

A code to calculate the recombination history of the Universe

This code is meant to reproduce the calculations described in the paper "A new calculation of the recombination epoch", Seager, S., Sasselov, D. & Scott, D., 1999, ApJ, 523, L1 ([astro-ph/9909275](#)), which is a fast approximation to the detailed calculations described in "How exactly did the Universe become neutral?" Seager, S., Sasselov, D. & Scott, D., 2000, ApJS, 128, 407 ([astro-ph/9912182](#)). And with some updates (to He etc.) which are described in Wong, W.Y., Moss, A. & Scott, D., 2008, MNRAS, 386, 1023 ([arXiv:0711.1357](#)), together with the Compton coupling treatment in Scott, D. & Moss A., 2009, MNRAS, 397, 445 ([arXiv:0902.3438](#)).

It solves a modified 3-level atom for each of hydrogen and helium, along with some corrections motivated by more detailed calculations. It is thus *fast*, while being accurate enough for current CMB experiments.

Here are things that the code treats:

- Solves H and He simultaneously
- Kinetic temperature of matter evolved separately, including Compton and adiabatic cooling
- Careful consideration of dependency of quantities on T_M and T_R
- Accurate values of all relevant physical constants
- Full treatment of background cosmology, including radiation, Lambda and curvature (but no explicit "w")
- Saha assumed for ionized helium
- Accurate look-up table for recombination coefficients
- Approximation for H recombination at low redshift, to account for out of equilibrium
- Singlet and triplet states considered for neutral He
- Additional "fudge factor" for He recombination
- Solves for temperature difference between matter and radiation, giving smoother transition in T_M evolution
- Includes an extra "fudge" function to correct $K(z)$ to approximate the H physics corrections from detailed codes (see Rubino-Martin et al. 2010)
- Extra "fudge factor" Gaussians, designed to make correction for additional He physics for cosmologies near the standard one

This results in $x_e(z)$ producing (it is believed) better than 0.1 percent accuracy for CMB anisotropy C_l .

There are 2 separate versions of the code, one in *fortran* (developed by DS), and one in *c* (developed by SS).

Grab code recfast.for from web page

To compile with a modern compiler (because of old fortran dialect that uses tabs for line continuation):

```
gfortran -ffixed-line-length-0 -std=legacy recfast.for -o recfast
```

(There are probably other solutions, and other compilers that could be used)

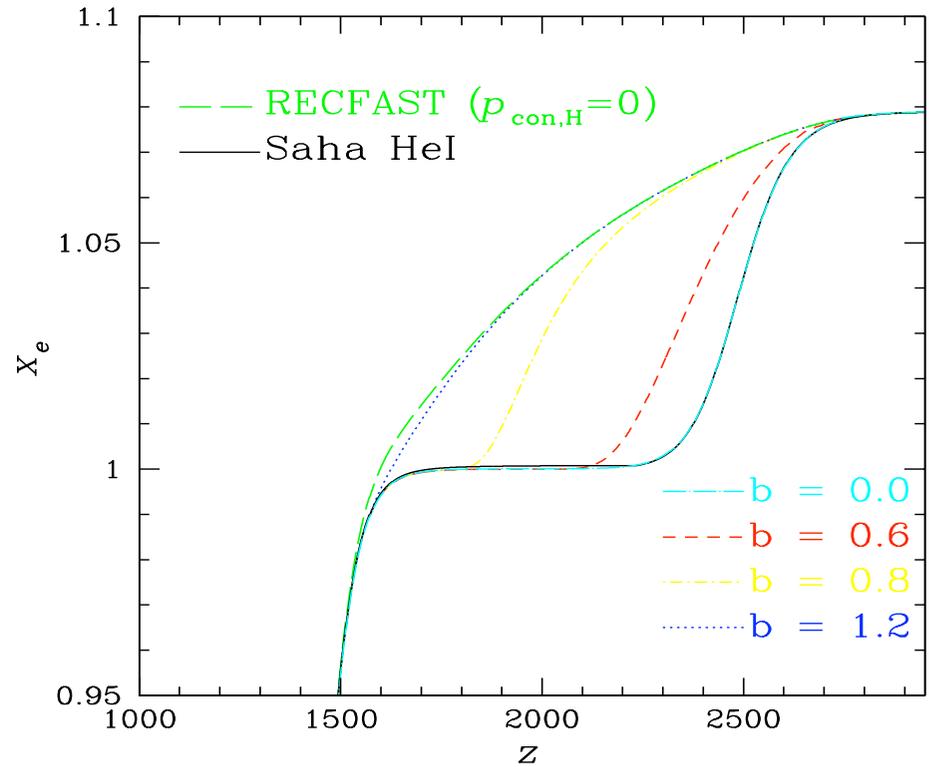
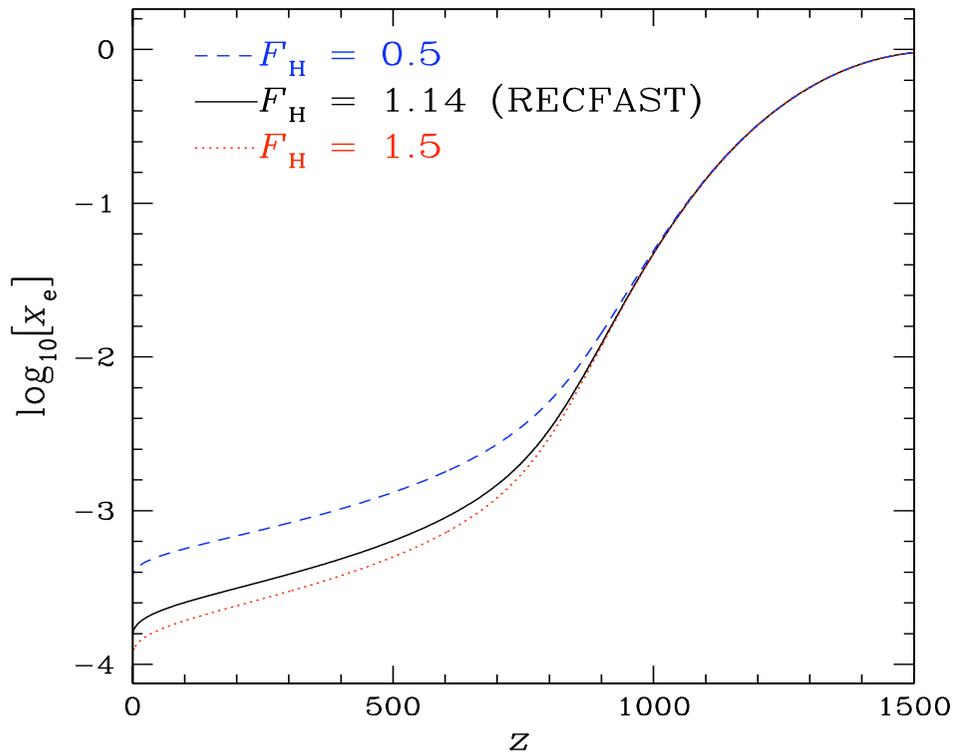
Then run with ./recfast

Answer questions about parameters etc., e.g.

```
filename.out  
0.04 0.20 0.76  
70 2.725 0.25  
1  
6
```


Old
fudges!

The devil is in the details



Fudge factor to approximate
out-of-equilibrium H
at lower redshifts

Fudge factor to approximate
several Helium effects at
higher redshifts

Wong, Moss & Scott (2008)

The devil is in the details

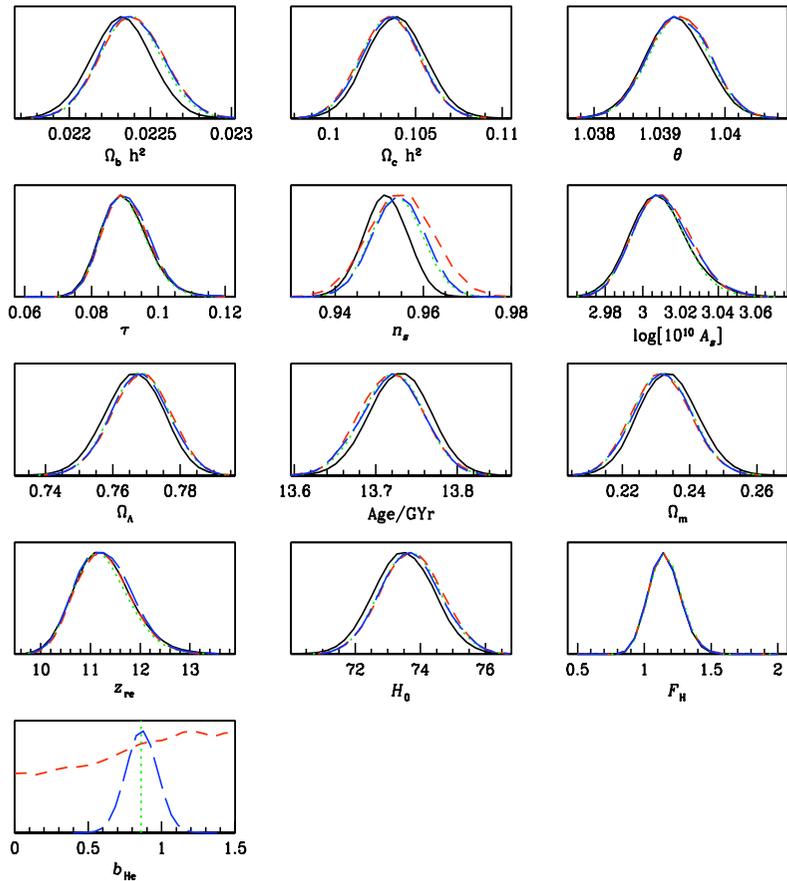


Figure 5. Marginalized posterior distributions for forecast *Planck* data with hydrogen and helium phenomenological parameters both allowed to vary. The solid (black) curve shows the constraints using the original RECFAST code and allowing F_H to be a free parameter. The other curves also allow for the variation of F_H and use the fitting function for HeI recombination described in Section 2: the dotted (green) line sets b_{He} equal to 0.86; the dashed (red) one is with a flat prior for b_{He} from 0 to 1.5; and the long-dashed (blue) one is with a narrow prior for b_{He} , consisting of a Gaussian centred at 0.86 and with $\sigma = 0.1$.

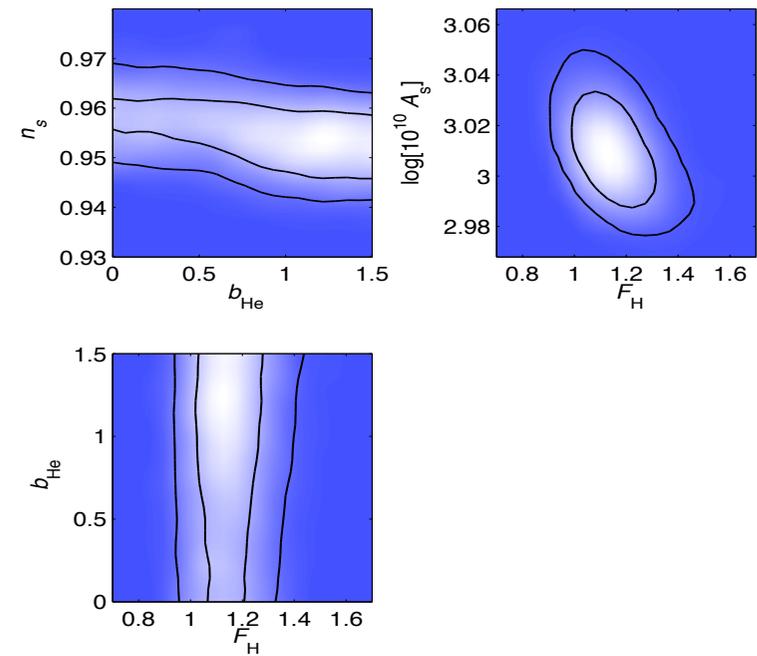
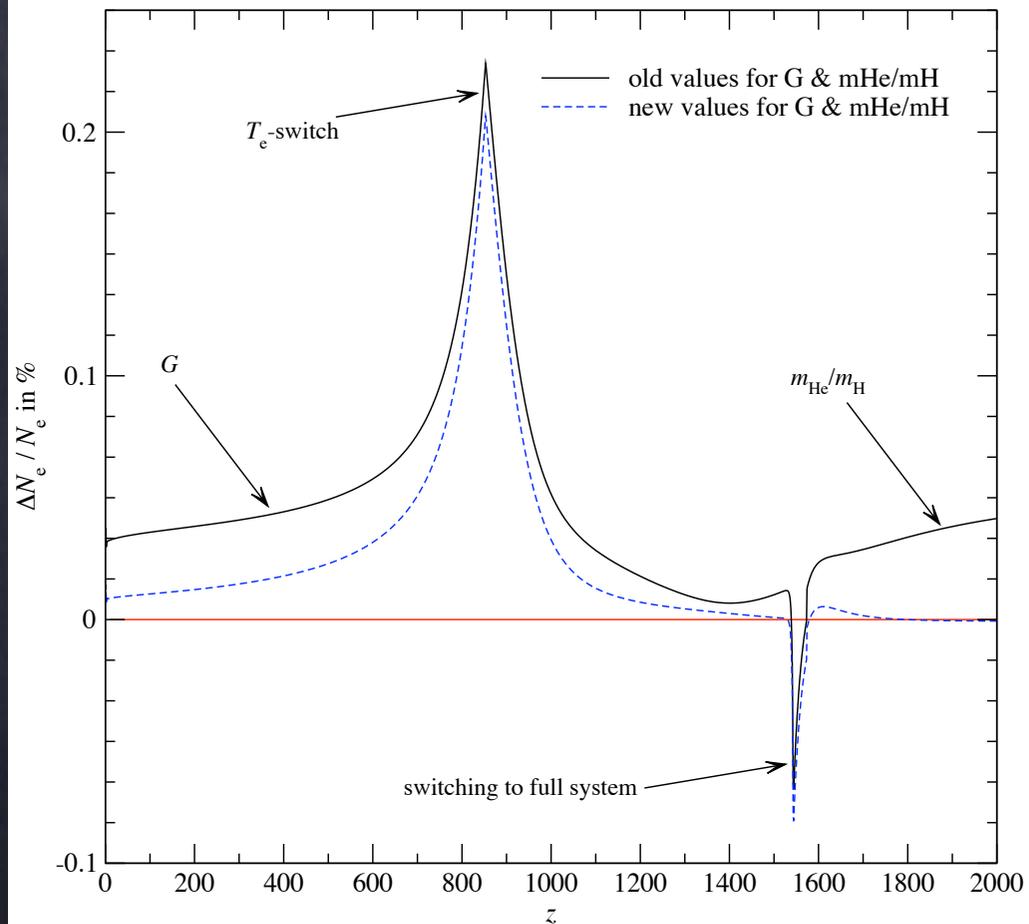


Figure 6. Projected 2D likelihood for the four parameters n_s , A_s , F_H and b_{He} . Shading corresponds to the marginalized probabilities with contours at 68 per cent and 95 per cent confidence.

Look at expected constraints
on cosmological parameters plus
extra fudge parameters
- biggest effect on n
- also some correlations

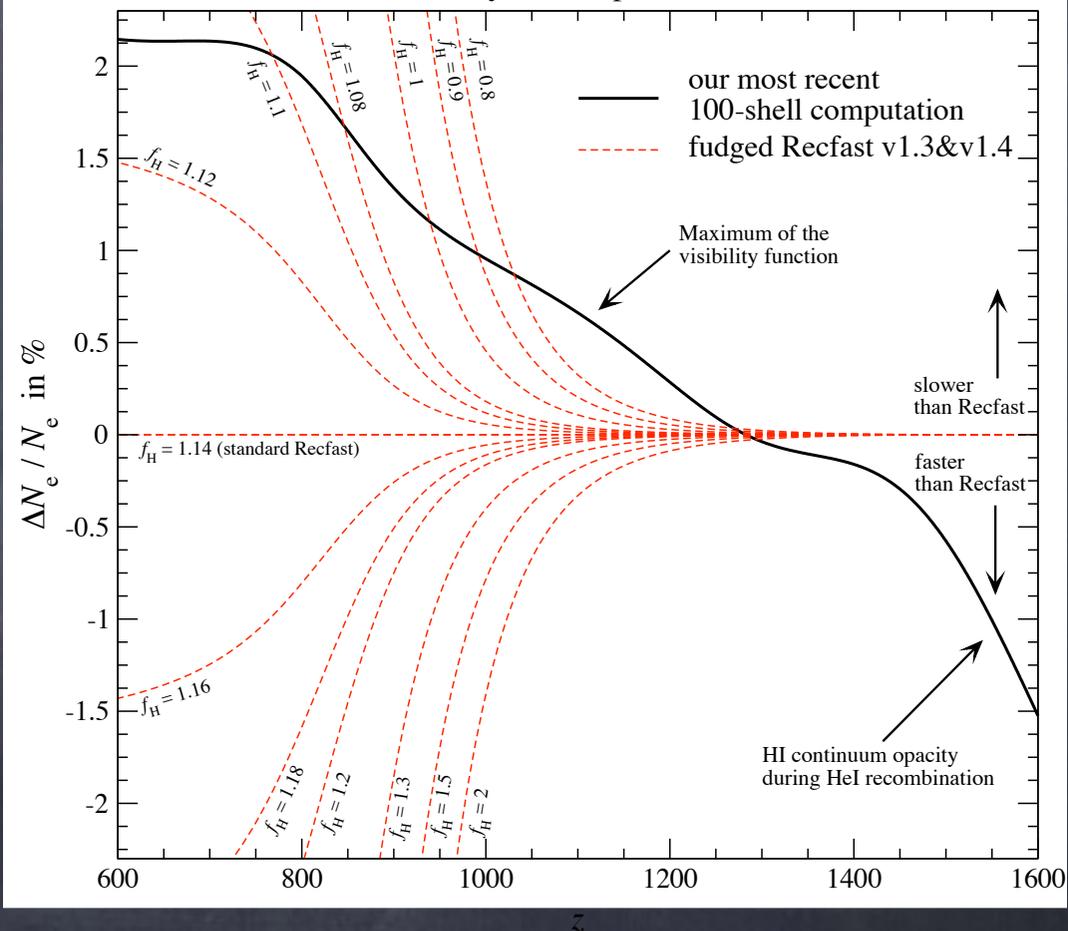
Some problems with RECFAST

Effect of Switches in Recfast v1.3 & v1.4



Switches in code

Ionization History in Comparison with Recfast



Low z fudge

Matter temperature

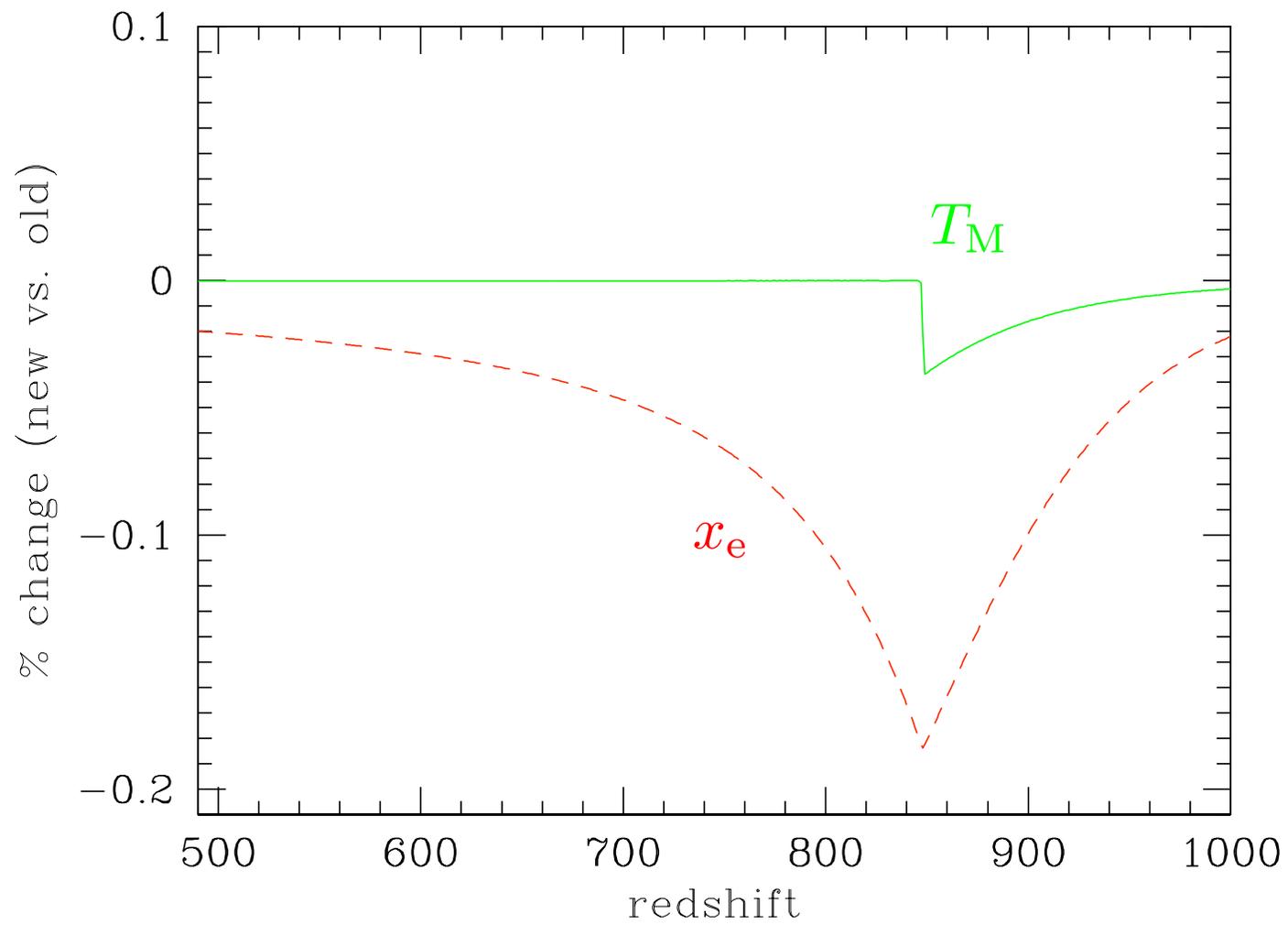
Basic equation: $(1+z) \frac{dT_M}{dz} = \frac{(T_M - T_R)}{H t_C} + 2T_M$

with $t_C = \frac{3m_e c}{8\sigma_T a_R T_R^4} \frac{1 + f_{\text{He}} + x_e}{x_e}$

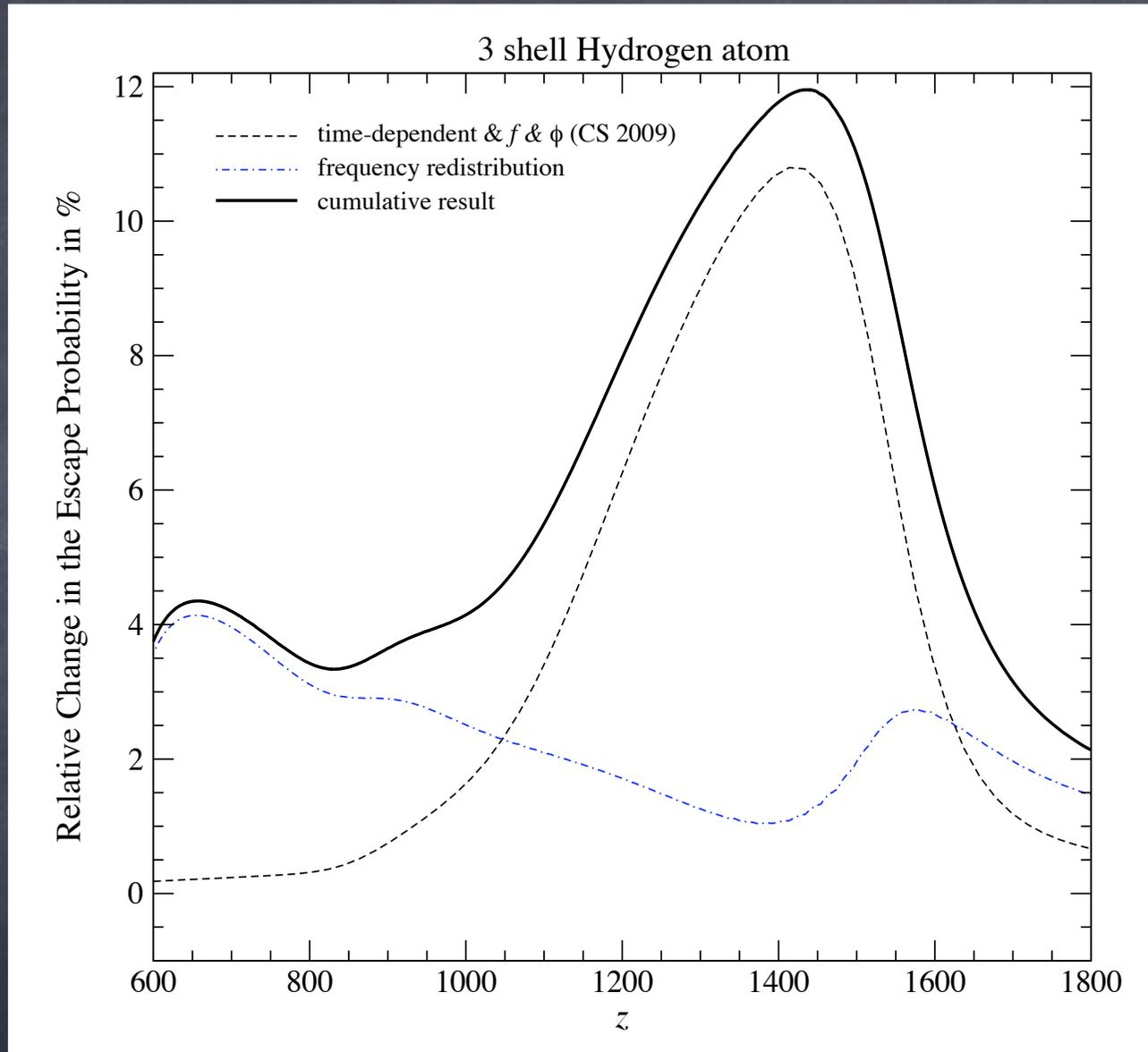
Solve with $T_M = T_R - \epsilon$ and $T_R \propto (1+z)$

$$\rightarrow \frac{dT_M}{dz} = \frac{T_R}{(1+z)} + \epsilon \left\{ \frac{1 + f_{\text{He}}}{1 + f_{\text{He}} + x_e} \frac{dx_e/dz}{x_e} + \left[\frac{3}{(1+z)} - \frac{dH/dz}{H} \right] \right\}$$

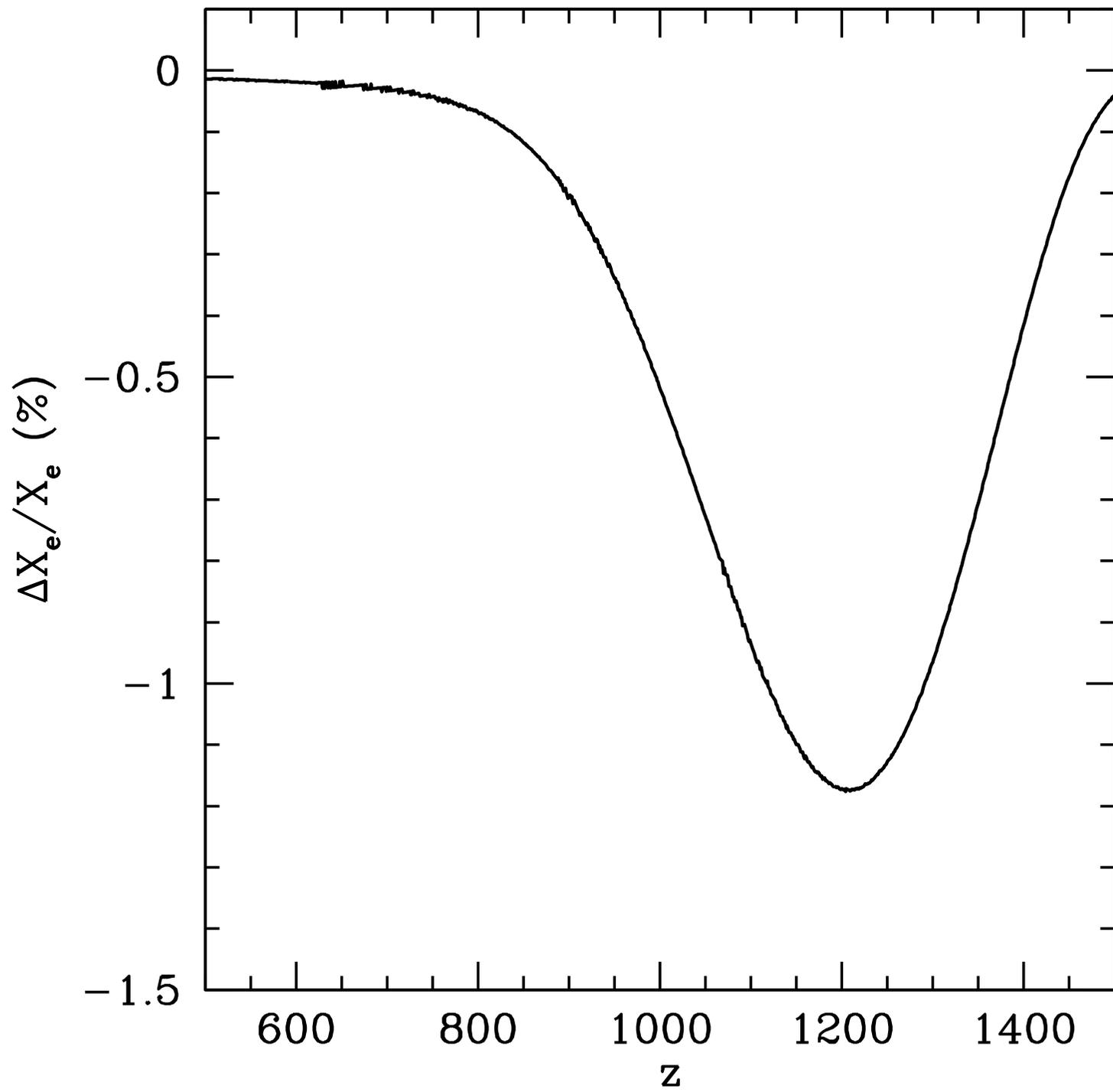
which can be used to follow matter until ϵ not small



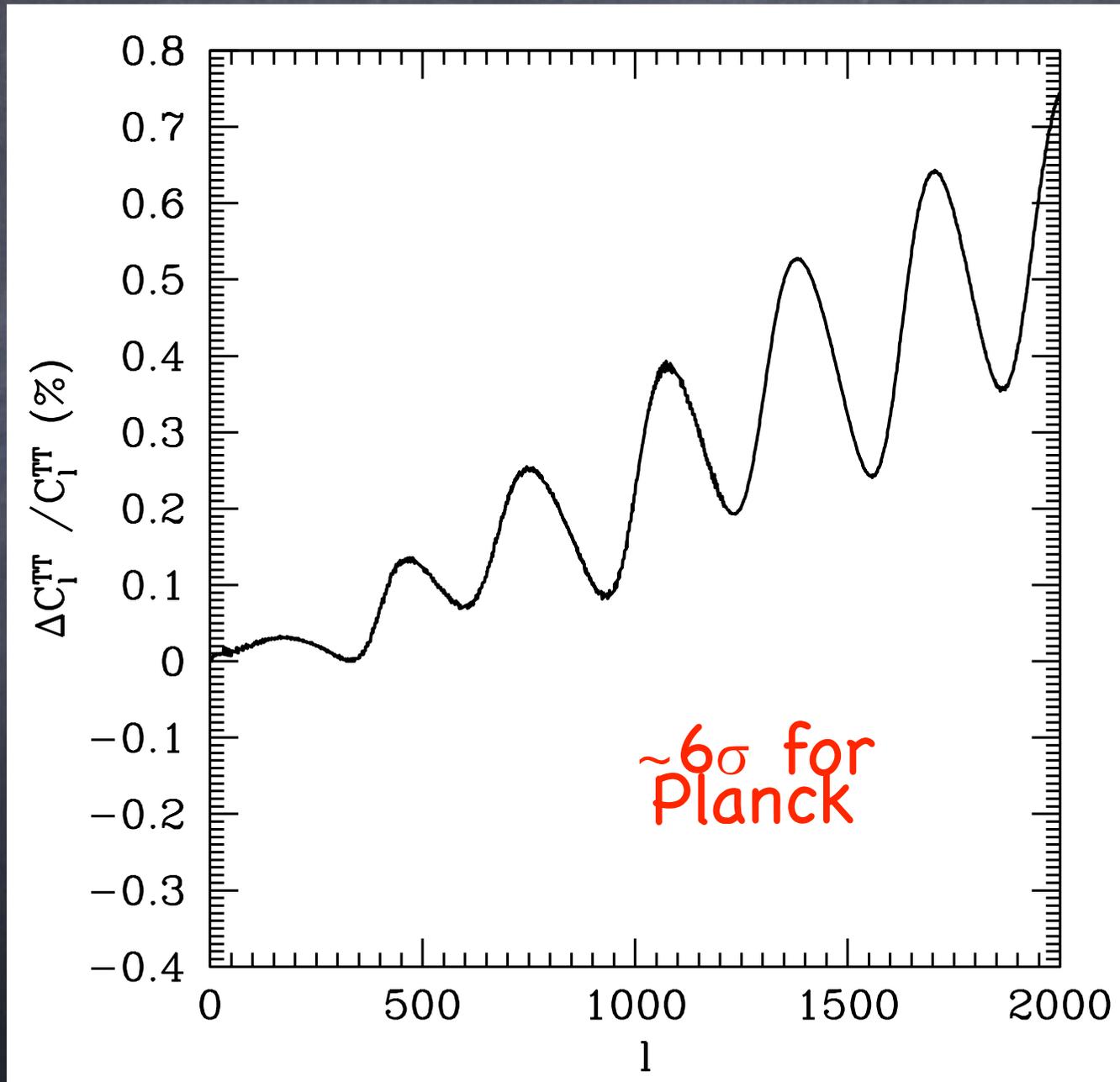
New
fudge
example



Gaussian: mean=1450; width=200; A=0.1



Variation in CMB power spectrum



recfast 2.0?

- ✿ Include corrections for all new effects
 - Changes to Sobolev escape probability
 - More elaborate H fudge (function) for low z
- ✿ Improve switches for He
- ✿ Check cosmology dependence
- ✿ Rewrite in C/python