



Kaon multiplicity of SIDIS off the deuteron



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Kobe, Japan



Collaboration and Reference

In Collaboration with

Nam Seung-il (KIAS)

Dong-Jing Yang, Fu-Jiun Jiang (NTNU, Taiwan)

Wen-Chen Chang (AS, Taiwan)

This talk is based on the following works:

(1) Nam Seung-il, CWK, Phys. Rev. D85, 034023(2012)

(2) Nam Seung-il, CWK, : Phys.Rev.D85, 094023(2012)

(3) Dong Jing Yang, Fu-Jiun Jaing, CWK, Nam Seung-il:
Phys. Rev D87,097004. (2013)

(4) Dong Jing Yang, Fu-Jiun Jaing, CWK, Nam Seung-il: arXiv 1407.4453

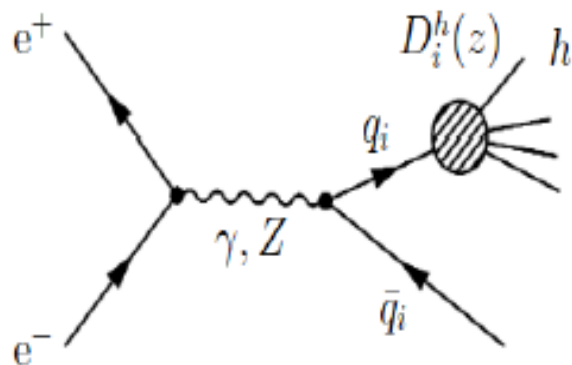
(5) Dong Jing Yang, Fu-Jiun Jaing, Wen-Chen Chang, CWK,
Nam Seung-il: Phys. Lett. B 755(2016) 393

How to extract Fragmentation functions?

e^+e^- annihilation (into hadrons)

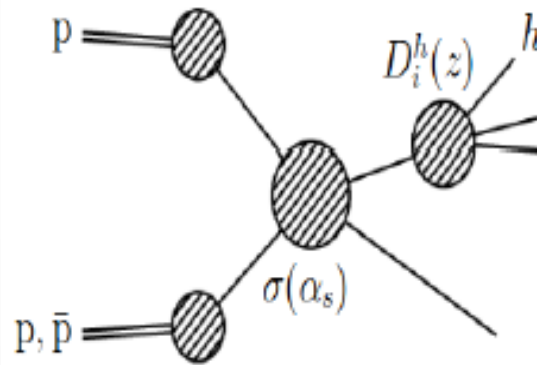
(Belle & BABAR)

- High precision data
- No dependence on PDF
- Access to singlet combination only
($D_\Sigma = D_u^h + D_d^h + D_s^h + \dots$)



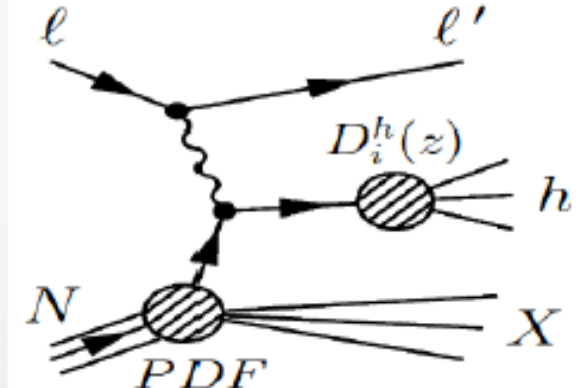
Hadron-hadron collision (RHIC, Fermi Lab, ..)

- High precision data
- Flavor/charge separation
- Sensitive to gluon FF
- Dependence on PDF



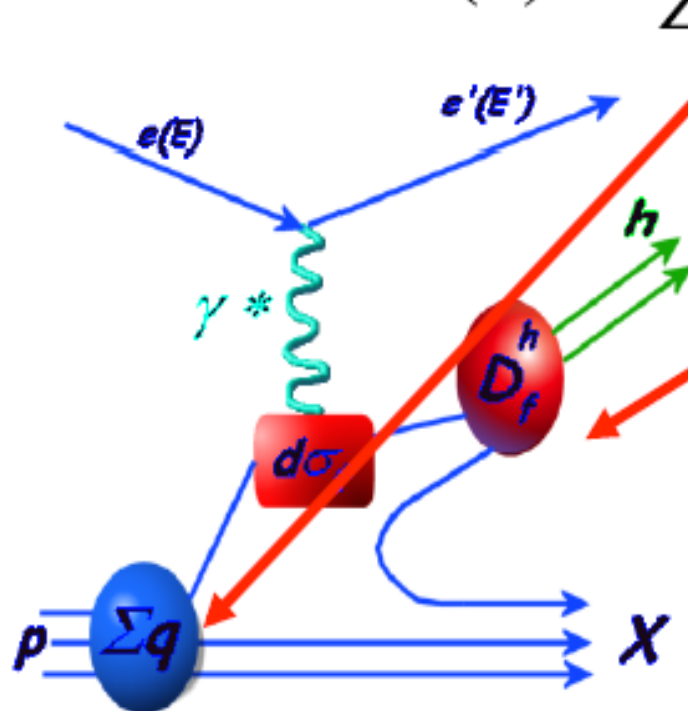
Lepton-hadron collision (COMPASS, HERMES, JLab)

- High precision data
- Flavor/charge separation
- Access larger z
- Study of hadronization process
- Dependence on PDF



SIDIS on nucleon

$$d\sigma^h(z) \propto \sum q_f(x) \otimes d\sigma_f \otimes D_f^h(z)$$



Under the QPM and (LO) assumption:

$$\frac{1}{N_e} \frac{d^2 N_h(x, z)}{dx dz} = \frac{\sum_q e_q^2 q_f(x) \overset{\text{DF}}{\boxed{D_f^h(z)}}}{\sum_q e_q^2 q_f(x)}$$

FF

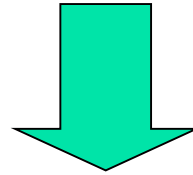
SIDIS hadron multiplicity open access to parton Fragmentation Function



SIDIS using deuteron target

Charge Conjugation of fragmentation functions

$$D_q^{\pi^+}(z, Q^2) = D_{\bar{q}}^{\pi^-}(z, Q^2), \quad D_q^{\pi^-}(z, Q^2) = D_{\bar{q}}^{\pi^+}(z, Q^2).$$



$$\begin{aligned} M_D^\pi(x, Q^2) &\equiv M_D^{\pi^+}(x, Q^2) + M_D^{\pi^-}(x, Q^2) = \frac{dN^\pi(x, Q^2)}{dN^{DIS}(x, Q^2)} \\ &= \frac{\sum_q e_q^2 (q^p(x, Q^2) + \bar{q}^p(x, Q^2) + q^n(x, Q^2) + \bar{q}^n(x, Q^2)) \int_{z_{min}}^{z_{max}} D_q^{\pi^+ + \pi^-}(z, Q^2) dz}{\sum_q e_q^2 (q^p(x, Q^2) + \bar{q}^p(x, Q^2) + q^n(x, Q^2) + \bar{q}^n(x, Q^2))} \end{aligned}$$

$$q = u, d, s.$$



SIDIS off deuteron target

$$\begin{aligned} M_D^\pi(x, Q^2) &\equiv M_D^{\pi^+}(x, Q^2) + M_D^{\pi^-}(x, Q^2) = \frac{dN^\pi(x, Q^2)}{dN^{DIS}(x, Q^2)} \\ &= \frac{\sum_q e_q^2 (q^p(x, Q^2) + q^n(x, Q^2)) \int_{z_{min}}^{z_{max}} D_q^{\pi^+ + \pi^-}(z, Q^2) dz}{\sum_q e_q^2 (q^p(x, Q^2) + q^n(x, Q^2))} + \mathcal{O}(\alpha_s). \end{aligned}$$

$$q = u, d, s, \bar{u}, \bar{d}, \bar{s}.$$

p: proton, n: neutron

$$D_q^{\pi^+ + \pi^-}(z, Q^2) = D_q^{\pi^+}(z, Q^2) + D_q^{\pi^-}(z, Q^2).$$



SIDIS using deuteron target

Assume PDFs are charge symmetric:

$$\begin{aligned}u^p(x, Q^2) &= d^n(x, Q^2), \quad d^p(x, Q^2) = u^n(x, Q^2), \quad s^p(x, Q^2) = s^n(x, Q^2), \\ \bar{u}^p(x, Q^2) &= \bar{d}^n(x, Q^2), \quad \bar{d}^p(x, Q^2) = \bar{u}^n(x, Q^2), \quad \bar{s}^p(x, Q^2) = s^n(x, Q^2).\end{aligned}$$

$$M_D^\pi(x, Q^2) = M_D^{\pi^+}(x, Q^2) + M_D^{\pi^-}(x, Q^2) = \frac{dN^\pi(x, Q^2)}{dN^{DIS}(x, Q^2)} = \frac{Q(x, Q^2)D_Q^\pi(Q^2) + S(x, Q^2)D_S^\pi(Q^2)}{5Q(x, Q^2) + 2S(x, Q^2)}.$$

$$S(x, Q^2) = s^p(x, Q^2) + \bar{s}^p(x, Q^2)$$

$$Q(x) = u^p(x, Q^2) + d^p(x, Q^2) + \bar{u}^p(x, Q^2) + \bar{d}^p(x, Q^2).$$



SIDIS using deuteron target

$$\begin{aligned} D_Q^\pi(Q^2) &= 4 \int_{z_{\min}}^{z_{\max}} D_u^{\pi^+}(z, Q^2) dz + \int_{z_{\min}}^{z_{\max}} D_d^{\pi^+}(z, Q^2) dz \\ &\quad + 4 \int_{z_{\min}}^{z_{\max}} D_u^{\pi^-}(z, Q^2) dz + \int_{z_{\min}}^{z_{\max}} D_d^{\pi^-}(z, Q^2) dz \\ &= 4D_u^{\pi^+}(Q^2) + D_d^{\pi^+}(Q^2) + 4D_u^{\pi^-}(Q^2) + D_d^{\pi^-}(Q^2). \end{aligned}$$

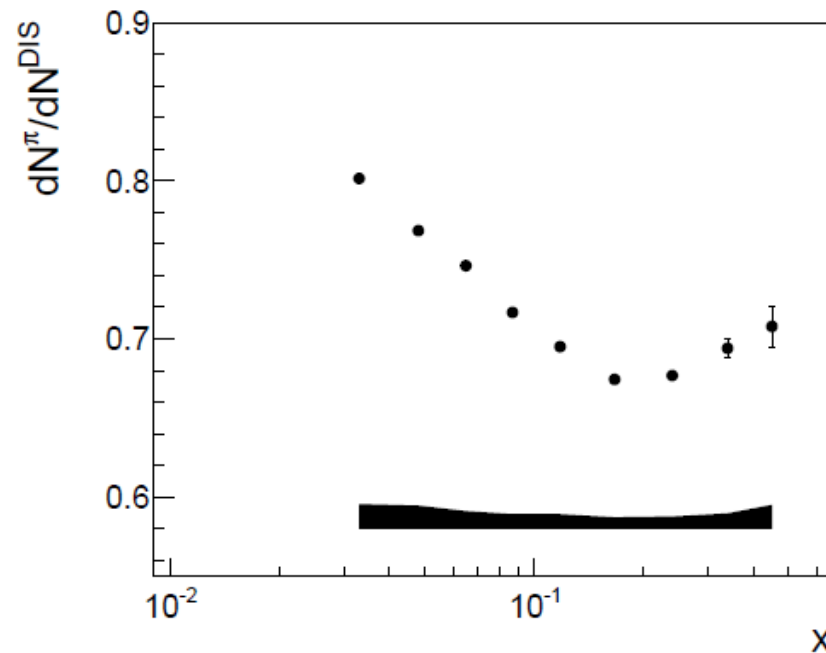
$$\begin{aligned} D_S^\pi(Q^2) &= 2 \int_{z_{\min}}^{z_{\max}} D_s^{\pi^+}(z, Q^2) dz + 2 \int_{z_{\min}}^{z_{\max}} D_s^{\pi^-}(z, Q^2) dz \\ &= 2D_s^{\pi^+}(Q^2) + 2D_s^{\pi^-}(Q^2) \end{aligned}$$

$D_Q \gg D_S$

HERMES Data of Pion multiplicity

A. Airapetian *et al.* (HERMES Collaboration), Phys. Rev. D **87**, 074029 (2013).

Data	x	Q^2	M_D^π
A	0.03349	1.1931	0.800
B	0.04767	1.3822	0.777
C	0.06495	1.5553	0.748
D	0.08729	1.7278	0.717
E	0.11756	2.1732	0.701
F	0.16562	3.1672	0.680
G	0.23975	4.8779	0.676
H	0.3397	7.4768	0.694
J	0.45147	10.2355	0.709



$$0.67 \leq M_d^\pi(\text{HERMES}) \leq 0.81.$$

$$0.034 \leq x \leq 0.452.$$



Kaon Multiplicity

$$M_D^K(x, Q^2) \equiv M_D^{K^+}(x, Q^2) + M_D^{K^-}(x, Q^2) : \\ = \frac{dN^K(x, Q^2)}{dN^{DIS}(x, Q^2)} = \frac{Q(x, Q^2)D_Q^K(Q^2) + S(x, Q^2)D_S^K(Q^2)}{5Q(x, Q^2) + 2S(x, Q^2)}.$$

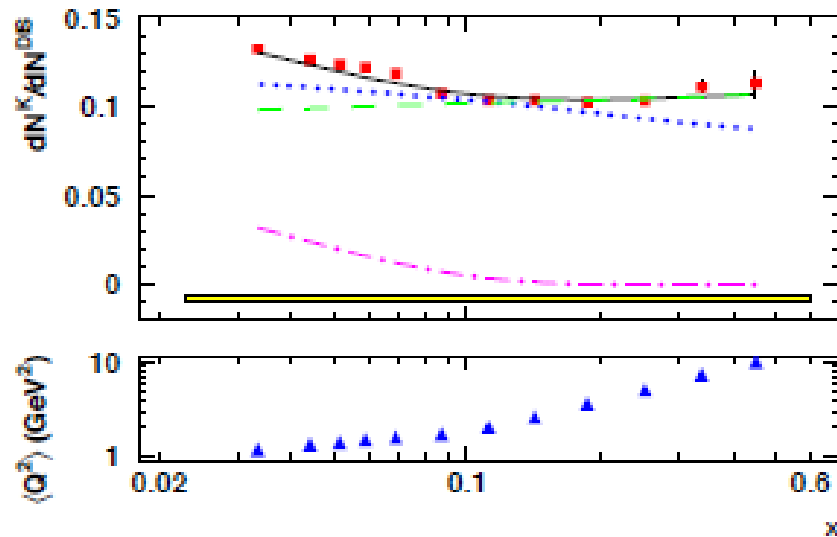
$$D_Q^K(Q^2) = 4 \int_{z_{min}}^{z_{max}} D_u^{K^+}(z, Q^2) dz + \int_{z_{min}}^{z_{max}} D_d^{K^+}(z, Q^2) dz \\ + 4 \int_{z_{min}}^{z_{max}} D_u^{K^-}(z, Q^2) dz + \int_{z_{min}}^{z_{max}} D_d^{K^-}(z, Q^2) dz.$$

$$D_S^K(Q^2) = 2 \int_{z_{min}}^{z_{max}} D_s^{K^+}(z, Q^2) dz + 2 \int_{z_{min}}^{z_{max}} D_s^{K^-}(z, Q^2) dz$$

HERMES Kaon Data

A. Airapetian et al. (HERMES Collaboration), Phys. Rev. D 448 89, 097101 (2014).

Data	x	Q^2	M_D^K
A	0.03349	1.1931	0.132
B	0.04767	1.3822	0.126
C	0.06495	1.5553	0.121
D	0.08729	1.7278	0.107
E	0.11756	2.1732	0.106
F	0.16562	3.1672	0.104
G	0.23975	4.8779	0.103
H	0.3397	7.4768	0.111
J	0.45147	10.2355	0.113



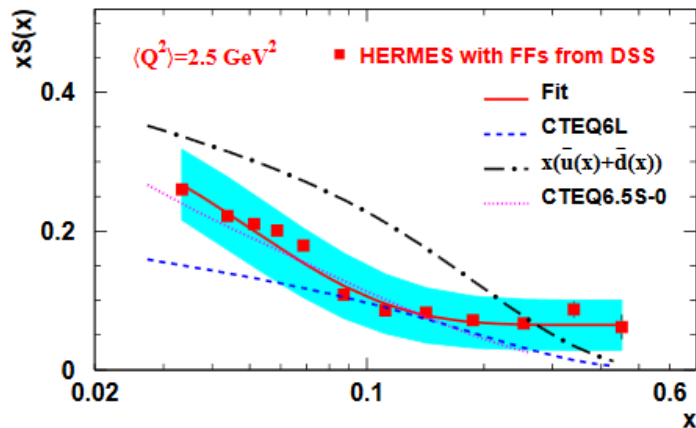
$$0.1 \leq M_d^K (\text{HERMES}) \leq 0.13.$$

$$0.034 \leq x \leq 0.452.$$

Pion and kaon multiplicities

For pion and kaon multiplicities of SIDIS off the deuteron target:

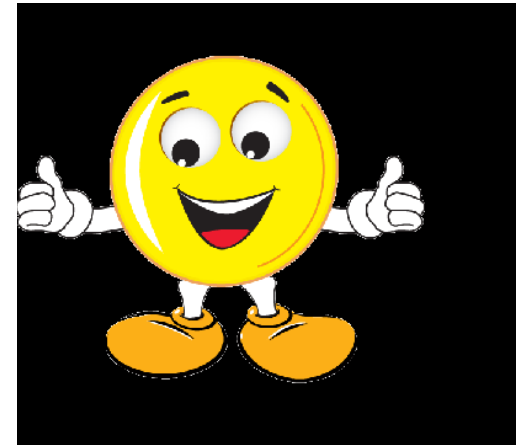
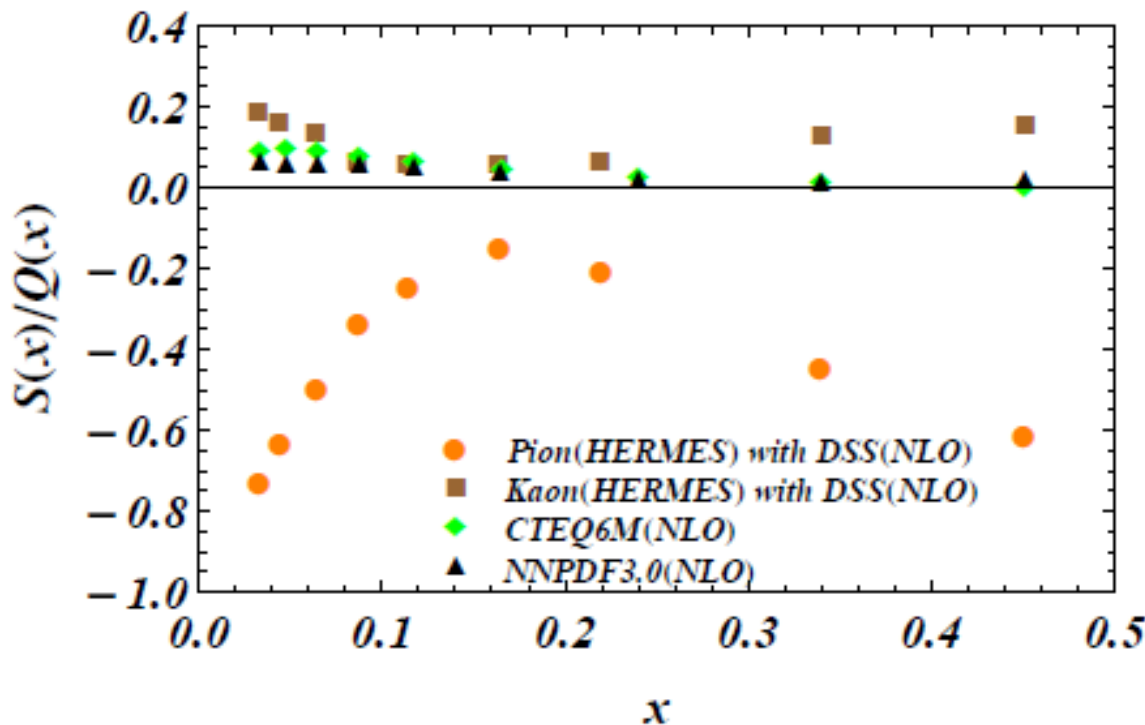
$$\frac{S(x, Q^2)}{Q(x, Q^2)} = \frac{5M_D^\pi(x, Q^2) - D_Q^\pi(Q^2)}{D_S^\pi(Q^2) - 2M_D^\pi(x, Q^2)} = \frac{5M_D^K(x, Q^2) - D_Q^K(Q^2)}{D_S^K(Q^2) - 2M_D^K(x, Q^2)}.$$



Combine the HERMES data of kaon and DSS Fragmentation functions, the strange quark PDF was extracted.

Phys. Rev. D 89, 097101 (2014)

$$\frac{S(x, Q^2)}{Q(x, Q^2)} = \frac{5M_D^\pi(x, Q^2) - D_Q^\pi(Q^2)}{D_S^\pi(Q^2) - 2M_D^\pi(x, Q^2)} = \frac{5M_D^K(x, Q^2) - D_Q^K(Q^2)}{D_S^K(Q^2) - 2M_D^K(x, Q^2)}.$$





Constraints on A and B

$$S(x, Q^2) = \left[\frac{5M_D^\pi(x, Q^2) - D_Q^\pi(Q^2)}{D_S^\pi(Q^2) - 2M_D^\pi(x, Q^2)} \right] Q(x, Q^2).$$

Since $S(x) > 0$, $Q(x) > 0$, then $S(x)/Q(x)$ is positive!

At fixed x and Q^2 :

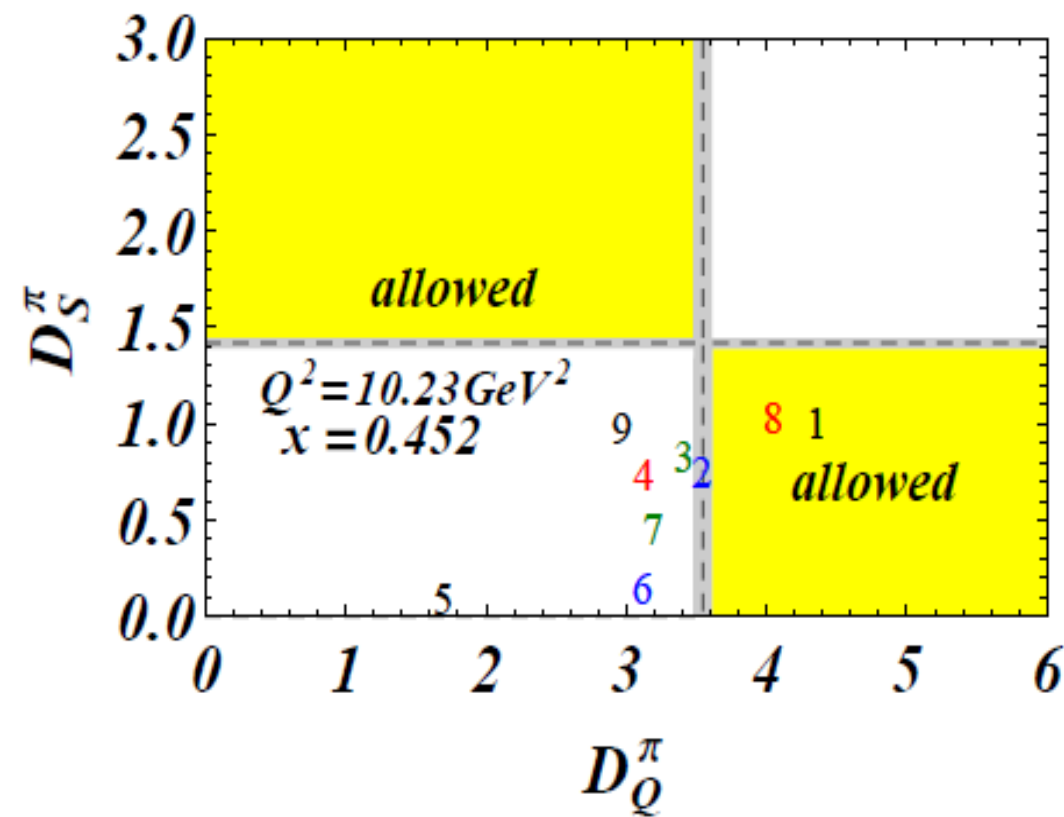
$$\frac{D_S^\pi(Q^2)}{2} < M_D^\pi(x, Q^2) \leq \frac{D_Q^\pi(Q^2)}{5}, \text{ or } \frac{D_Q^\pi(Q^2)}{5} \leq M_D^\pi(x, Q^2) < \frac{D_S^\pi(Q^2)}{2}.$$

Note that the values of D_S^π and D_Q^π are independent of x !



Choices of FFs

Number	Name	Order	Category	Data	Reference
1	HKNS	LO	Parameterization	e^+e^- annihilation	[7]
2	HKNS	NLO	Parameterization	e^+e^- annihilation	[7]
3	FSS	LO	Parameterization	e^+e^- annihilation, SIDIS	[9]
4	FSS	NLO	Parameterization	e^+e^- annihilation, SIDIS	[9]
5	NJL	—	Model	—	[21, 22]
6	NL χ QM	—	Model	—	[23–25]
7	AKK	NLO	Parameterization	e^+e^- annihilation	[8]
8	SKMNA	NLO	Parameterization	e^+e^- annihilation	[10]
9	FSEHPS	NLO	Parameterization	e^+e^- annihilation, SIDIS	[11]



$$D_Q^\pi(Q^2) \leq 5M_D^\pi(x, Q^2),$$

$$D_S^\pi(Q^2) > 2M_D^\pi(x, Q^2).$$

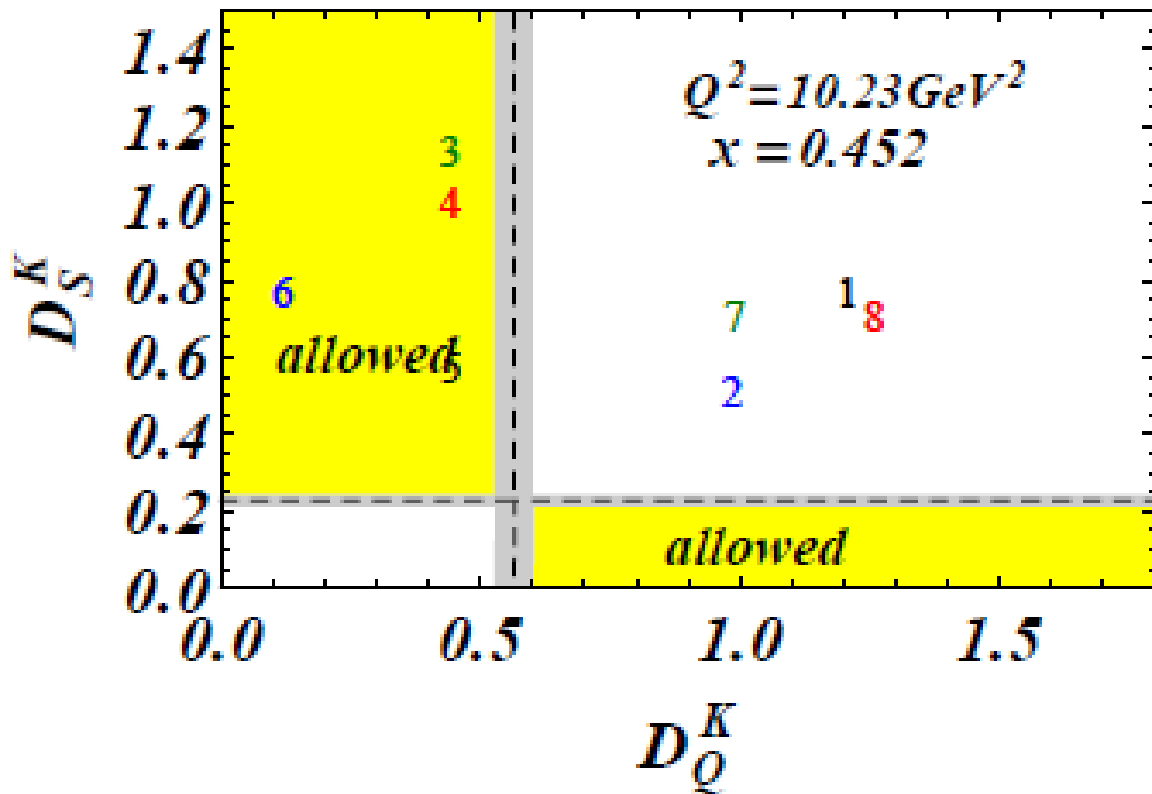
$$D_Q^\pi(Q^2) \geq 5M_D^\pi(x, Q^2),$$

$$D_S^\pi(Q^2) < 2M_D^\pi(x, Q^2)$$

$$S(x, Q^2) = \left[\frac{5M_D^\pi(x, Q^2) - D_Q^\pi(Q^2)}{D_S^\pi(Q^2) - 2M_D^\pi(x, Q^2)} \right] Q(x, Q^2).$$

- 1. HKNS (LO)
- 2. HKNS (NLO)
- 3. DSS (LO)
- 4. DSS (NLO)

- 5. NJL-Jet Model
- 6. Nonlocal Chiral Quark Model
- 7. AKK08
- 8. SMKA
- 9. DSEHS



$$D_Q^K(Q^2) \leq 5M_D^K(x, Q^2),$$

$$D_S^K(Q^2) > 2M_D^K(x, Q^2)$$

$$D_Q^K(Q^2) \geq 5M_D^K(x, Q^2)$$

$$D_S^K(Q^2) < 2M_D^K(x, Q^2).$$

1. HKNS (LO)

2. HKNS (NLO)

3. DSS (LO)

4. DSS (NLO)

5. NJL-Jet Model

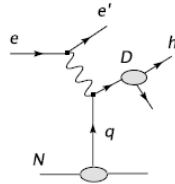
6. Nonlocal Chiral Quark Model

7. AKK08

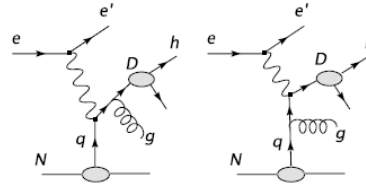
$$S(x, Q^2) = \left[\frac{5M_D^K(x, Q^2) - D_Q^K(Q^2)}{D_S^K(Q^2) - 2M_D^K(x, Q^2)} \right] Q(x, Q^2).$$

NLO fitting of SIDIS

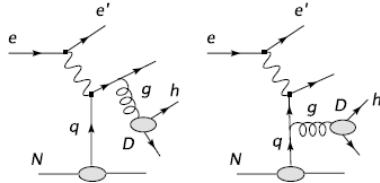
LO:



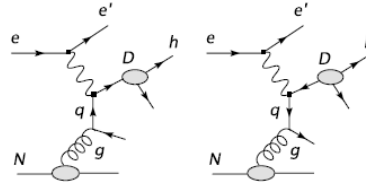
NLO-qq:



NLO-qg:



NLO-gg:



$$\sigma^h(x, z) = \sum_f e_f^2 q_f \left[1 + \otimes \frac{\alpha_s}{2\pi} \mathcal{C}_{qq} \otimes \right] D_{qf}^h$$

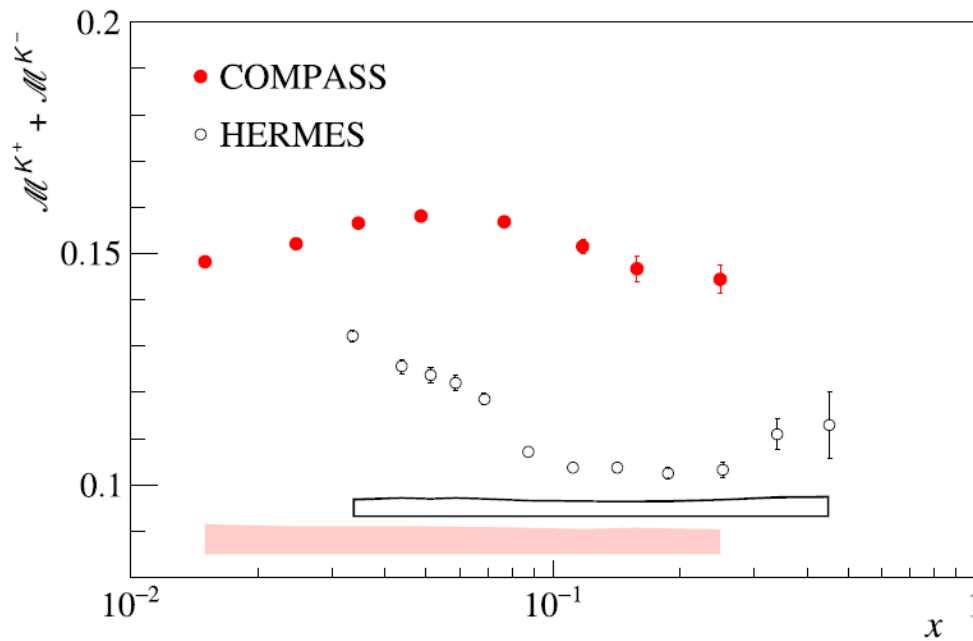
$$+ \left(\sum_f e_f^2 q_f \right) \otimes \frac{\alpha_s}{2\pi} \mathcal{C}_{qg} \otimes D_G^h + G \otimes \frac{\alpha_s}{2\pi} \mathcal{C}_{gq} \otimes \left(\sum_f e_f^2 D_{qf}^h \right),$$

$$[q \otimes C \otimes D](x, z) = \int_x^1 \frac{dx'}{x'} \int_z^1 \frac{dz'}{z'} q\left(\frac{x}{x'}\right) C(x', z') D\left(\frac{z}{z'}\right).$$

A game need two players....



COMPASS and HERMES



The data points are taken at Different Q^2 values compared with HERMES data.

Fig. 8. Sum of z -integrated multiplicities, $\mathcal{M}^{K^+} + \mathcal{M}^{K^-}$. COMPASS data (160 GeV, full points) are compared to HERMES data [13] (27.5 GeV, open points) (see text). The bands show the total systematic uncertainties.

Table 1: Bin limits for the three-dimensional (x, y, z) representation.

	bin limits											
x	0.004	0.01	0.02	0.03	0.04	0.06	0.1	0.14	0.18	0.4		
y	0.1	0.15	0.2	0.3	0.5	0.7						
z	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75 0.85



COMPASS and HERMES

HERMES

Data	x	Q ²
A	0.003349	1.1931
B	0.04767	1.3822
C	0.06495	1.5553
D	0.08769	1.7278
E	0.11756	2.1732
F	0.16562	3.1672
G	0.23975	4.8779
H	0.3397	7.4768
I	0.45147	10.2355

COMPASS

Data	x	Q ²
A	0.0085	1.1709
B	0.0157	1.454
C	0.0247	2.1489
D	0.0345	3.0170
E	0.0487	4.2476
F	0.0765	6.6756
G	0.1176	10.2629
H	0.1581	11.8938
I	0.2502	20.0857

COMPASS and HERMES

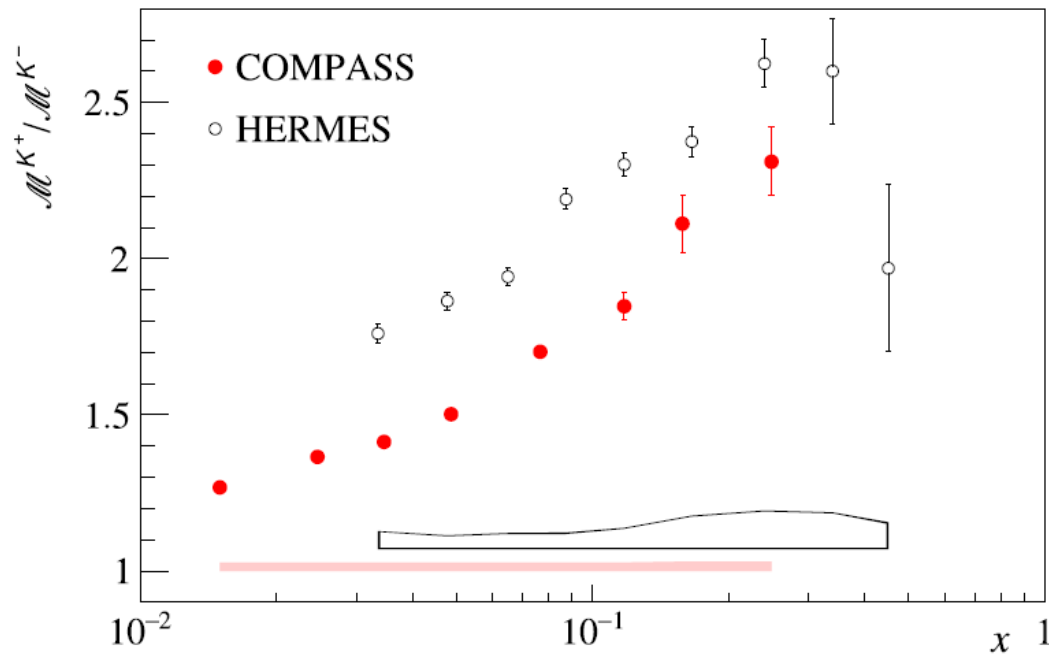


Fig. 9. Ratio of z-integrated multiplicities, $\mathcal{M}^{K^+}/\mathcal{M}^{K^-}$. COMPASS data (160 GeV, full points) are compared to HERMES data [26] (27.5 GeV, open points). The bands show the total systematic uncertainties.

Phys.Rev. D95 (2017) no.9, 094019 , already have 21 citations.

DSS 2017 fragmentation functions

Parton-to-Kaon Fragmentation Revisited

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We revisit the global QCD analysis of parton-to-kaon fragmentation functions at next-to-leading order accuracy using the latest experimental information on single-inclusive kaon production in electron-positron annihilation, lepton-nucleon deep-inelastic scattering, and proton-proton collisions. An excellent description of all data sets is achieved, and the remaining uncertainties in parton-to-kaon fragmentation functions are estimated and discussed based on the Hessian method. Extensive comparisons to the results from our previous global analysis are made.

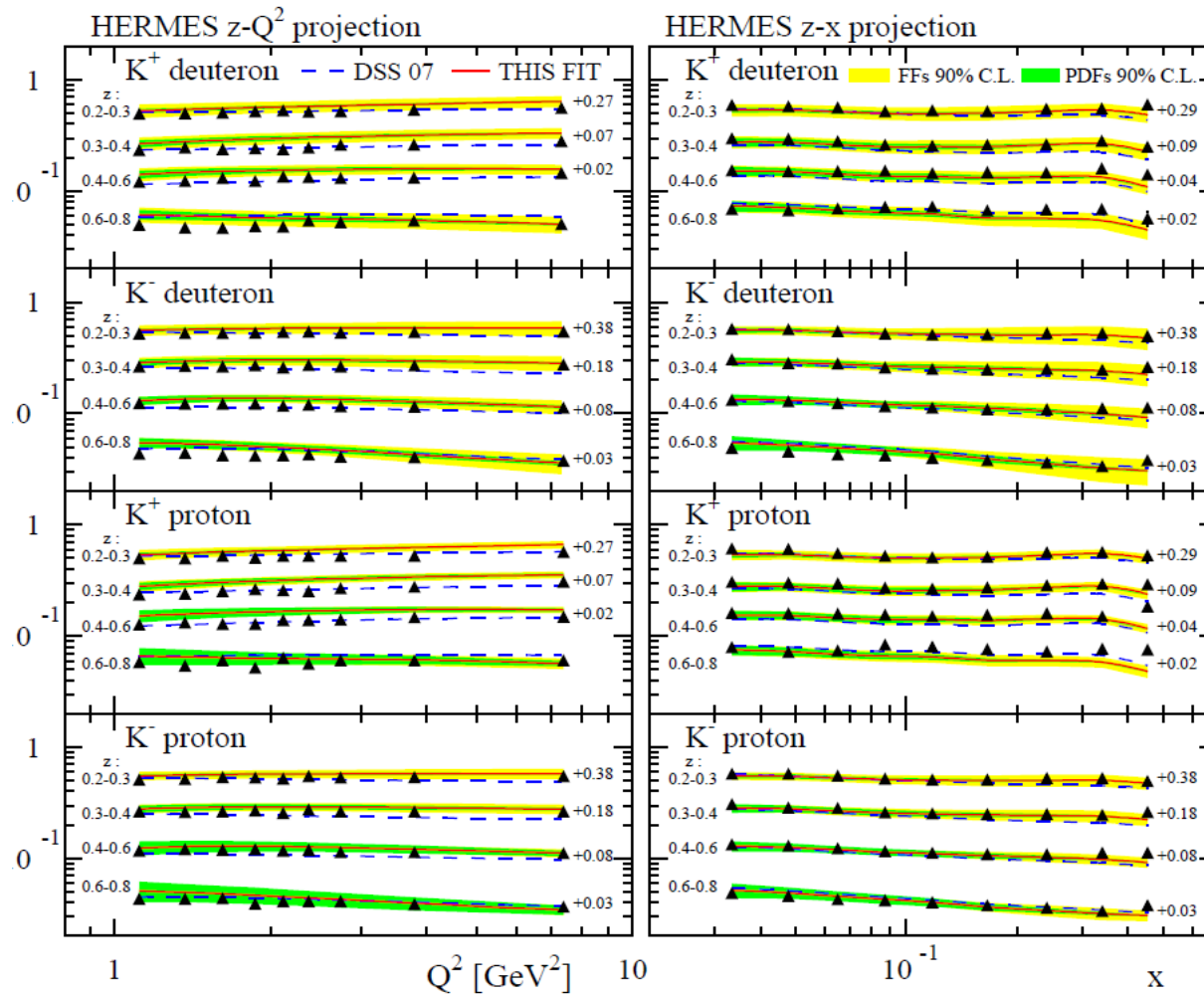
[hep-ph] 21 Feb 2017



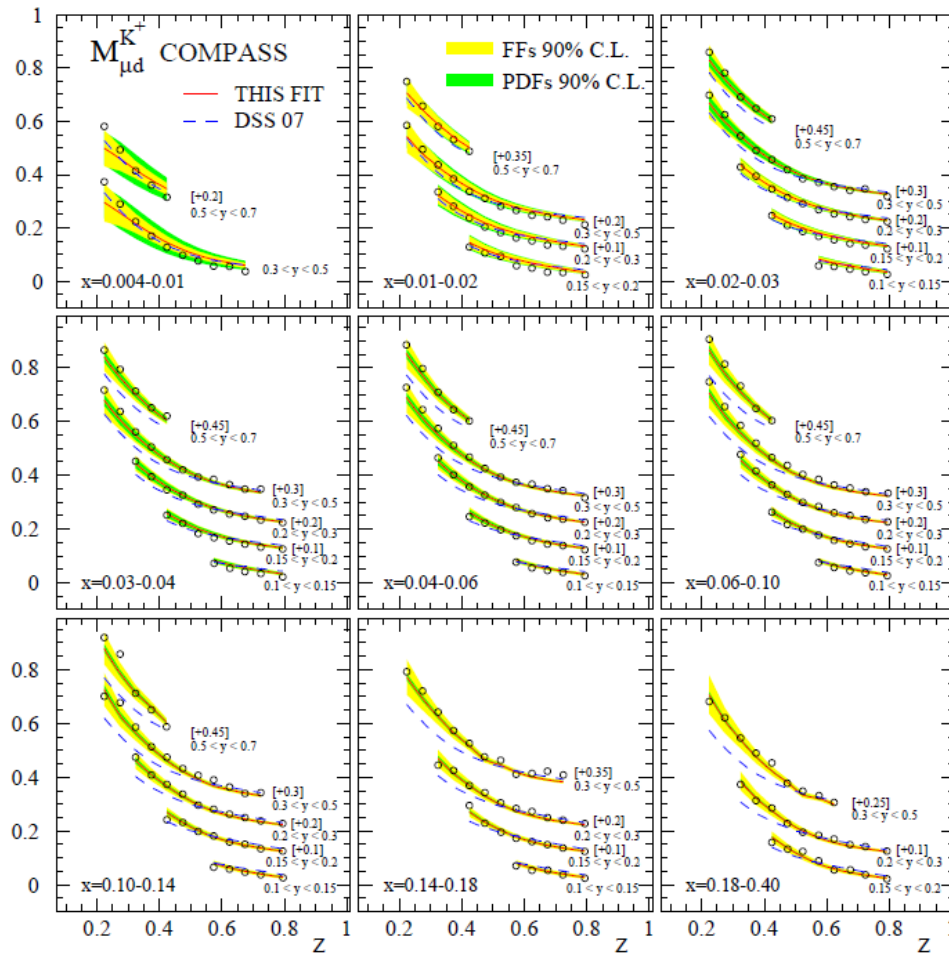
DSS 2017 fragmentation functions

Another crucial addition to the available suite of data on identified charged kaons are first multiplicity results in SIDIS from the COMPASS experiment at CERN [22]. These data are very precise despite exhibiting a rather fine binning in the relevant kinematic variables. Most importantly, COMPASS multiplicities reach much higher values of momentum transfer $Q^2 \lesssim 60 \text{ GeV}^2$ than HERMES $Q^2 \lesssim 30 \text{ GeV}^2$ and, therefore, combining them in a global fit not only allows us to test and quantify their level of consistency, but should, in principle, also lead to a considerably better flavor separation of the obtained parton-to-kaon FFs. Not surprisingly, the largest differences with respect to the original DSS 07 analysis are found mainly at the higher Q^2 values not covered by the HERMES data.

DSS 2017 fragmentation functions



DSS 2017 fragmentation functions



DSS 2017 fragmentation functions

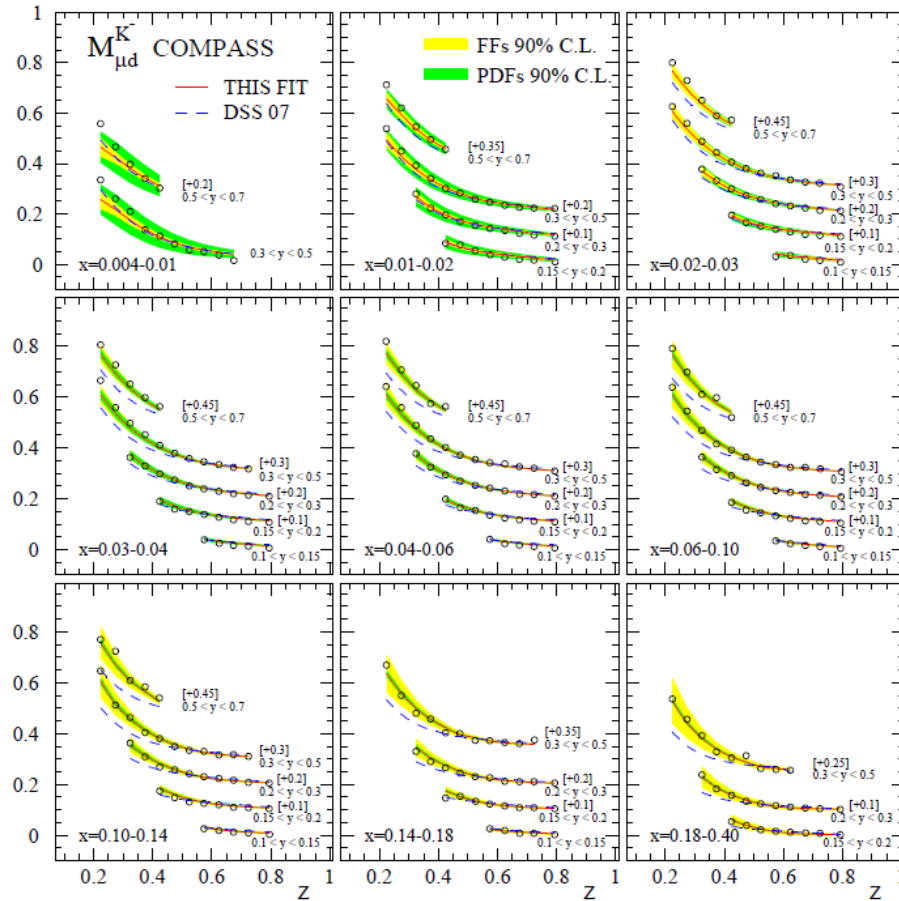


FIG. 7: Same as in Fig. 6 but now for the K^- multiplicities $M_{\mu d}^{K^-}$.

DSS 2017 fragmentation functions

First and foremost, these results demonstrate that the low-energy HERMES [19] and the new COMPASS [22] charged kaon multiplicity data can be described simultaneously and, equally important, without spoiling the agreement with SIA results discussed before. This is, to a somewhat lesser extent, even the case when one adopts the old DSS 07 set of kaon FFs. As can be seen from Figs. [6] and [7], they lead to a fair agreement with the COMPASS data without any re-fitting except for some of the bins corresponding to the highest Q^2 values; for the $z - Q^2$ projections of the HERMES data, shown in the left panel of Fig. [5], the DSS 07 FFs even lead to a slightly better description of the data than the new, updated global fit. The bottom line is, that the new COMPASS data mainly correct the charge and flavor separation provided by the DSS 07 set of FFs at higher Q^2 values, an information that was beyond the reach of the HERMES data adopted in the DSS 07 fit.

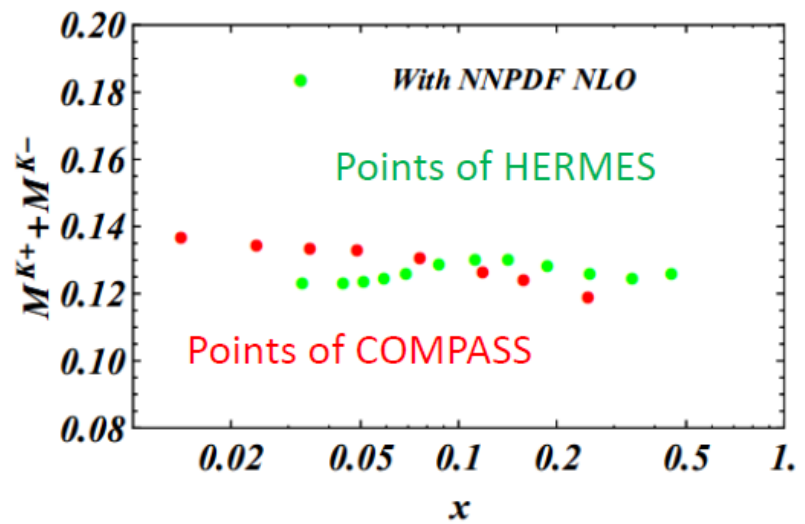
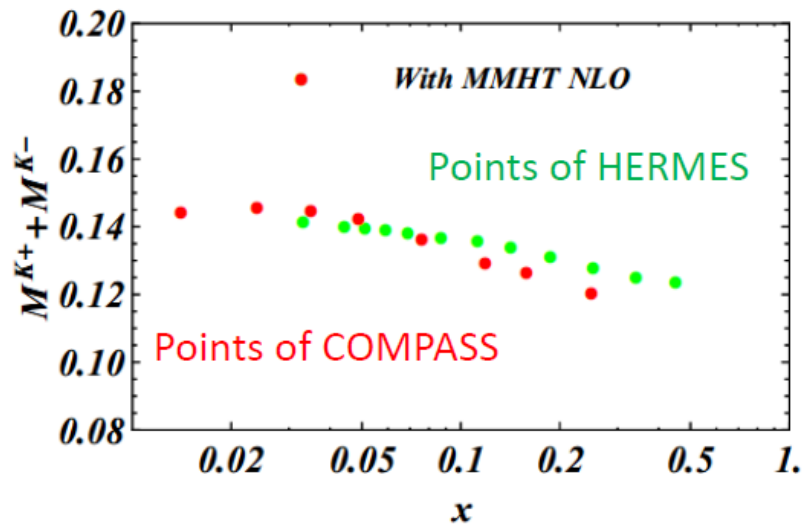




Our Puzzle over DSS17

- If DSS17 can describe both of HERMES and COMPASS kaon multiplicities well, then the two sets of data surely are compatible.
- However, the simple plot shows otherwise.
- So what's going on?
- Maybe the difference of two sets of data is indeed due to the different Q^2 ?

Test of DSS17 at HERMES and COMPASS kinematics

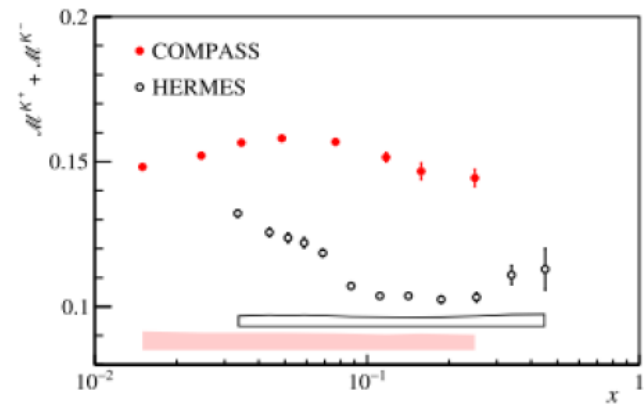
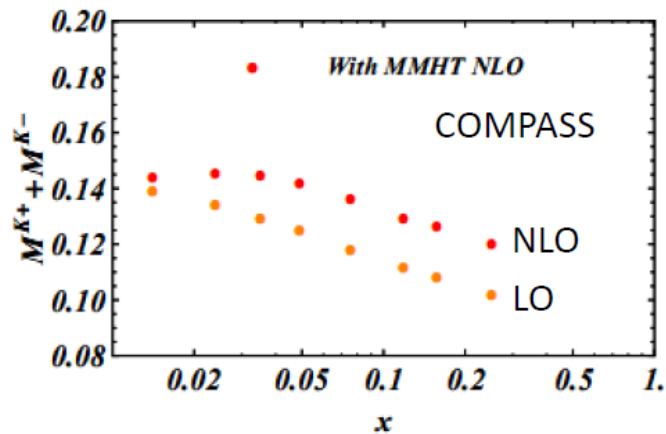
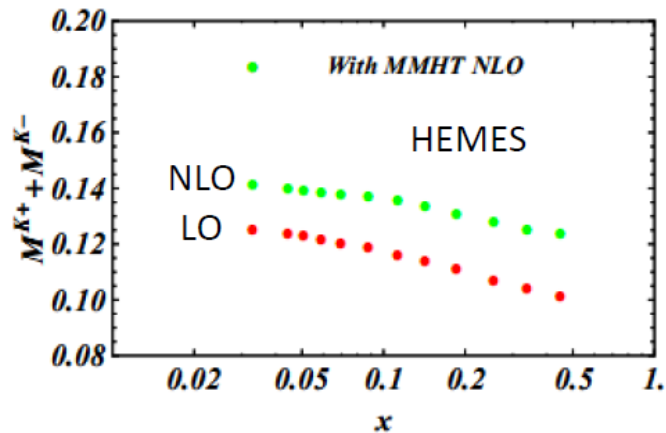


Apply DSS17 with MMHT and NNPDF at (x, Q^2) of HERMES data points and COMPASS, we find the theoretical results of both should be close.

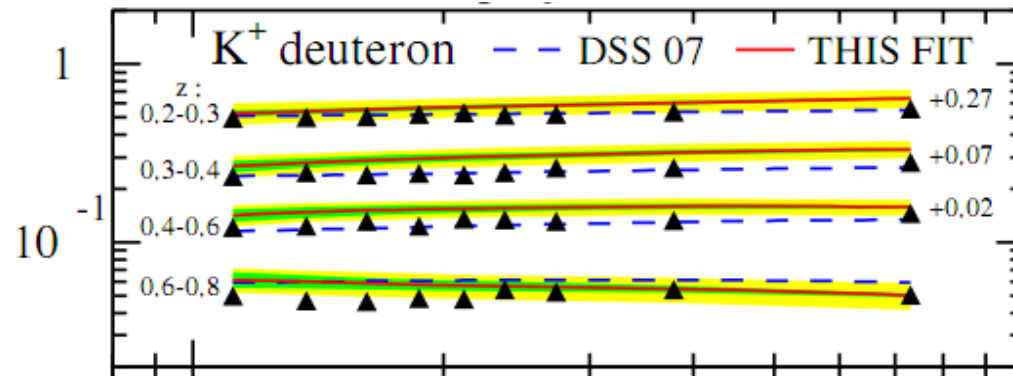
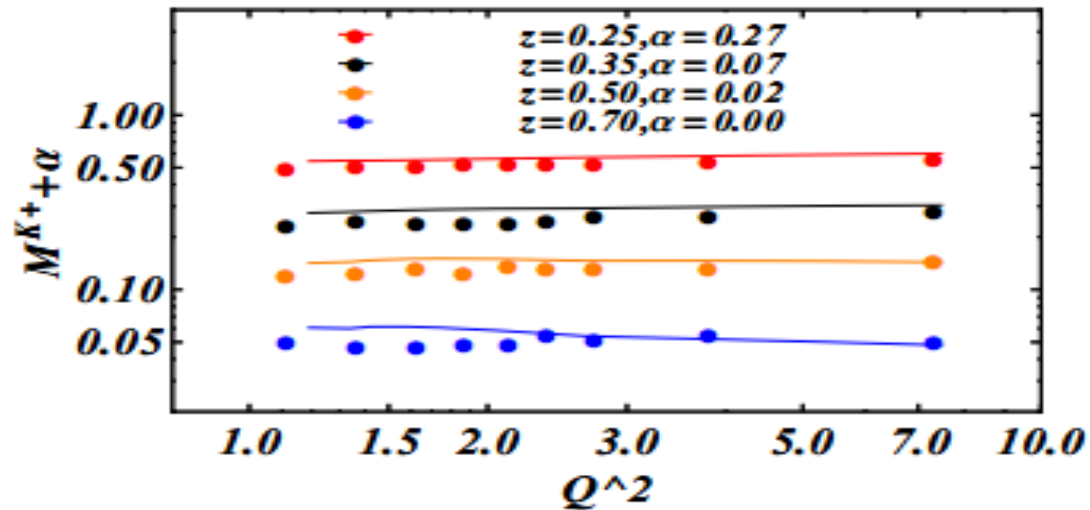
The effect of Q^2 evolution is not enough to explain the difference between two data sets.

Kaon multiplicity

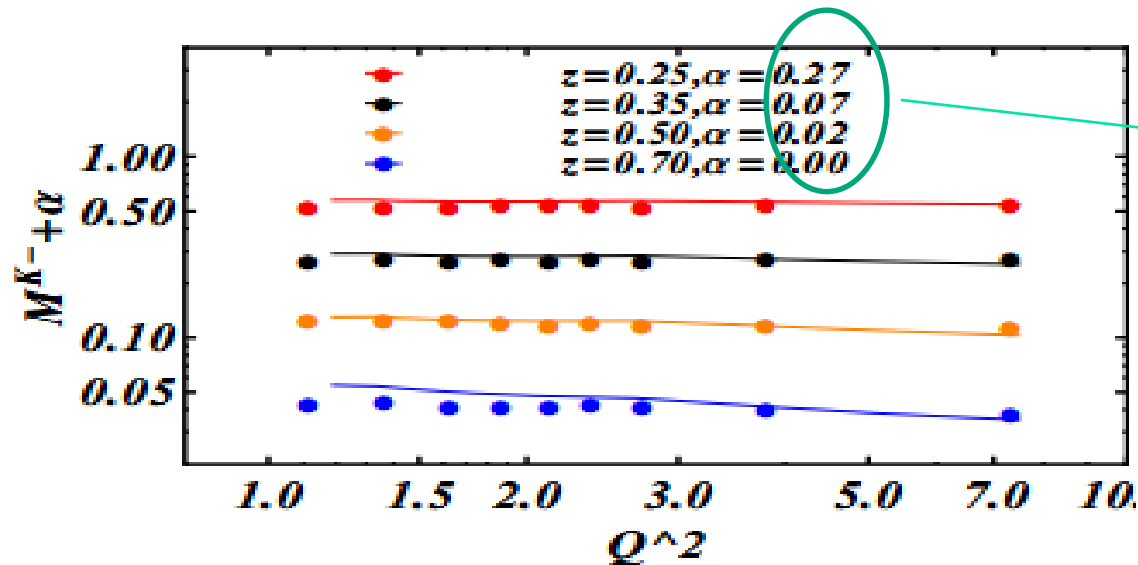
Kaon multiplicity



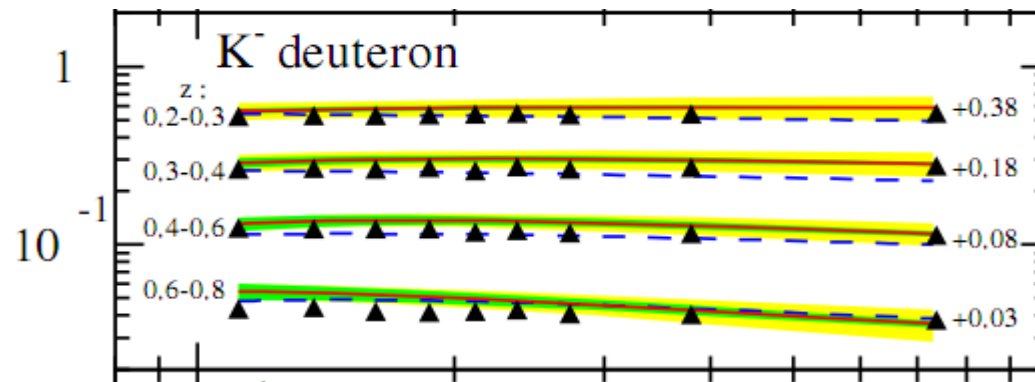
Check HERMES data K^+



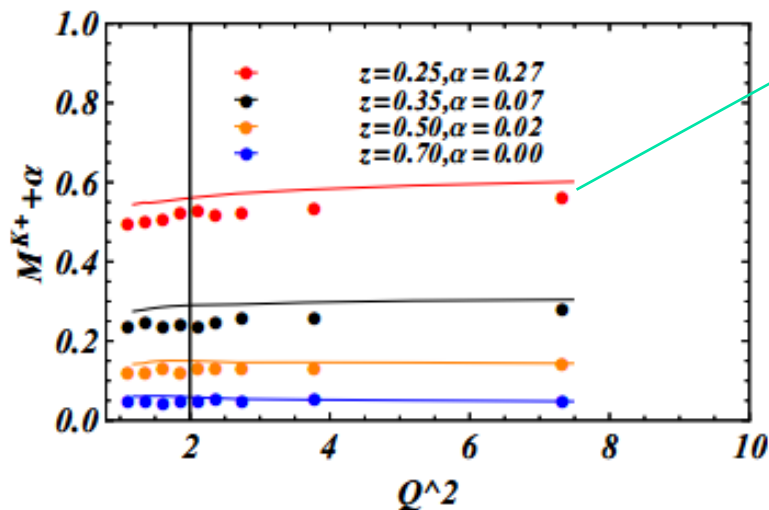
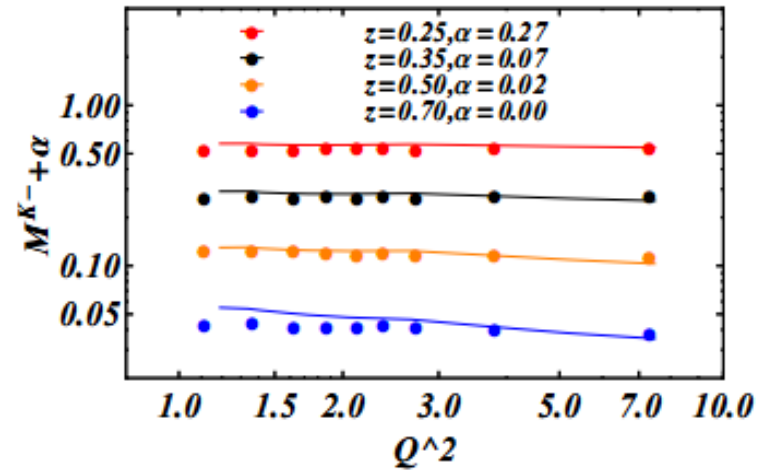
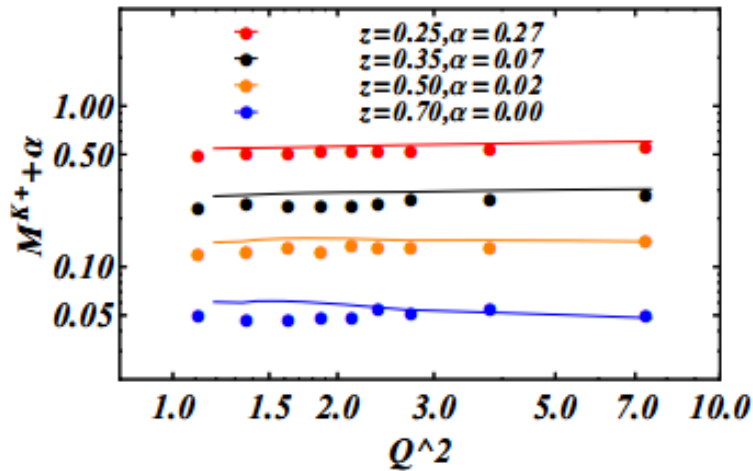
Check HERMES data K^-



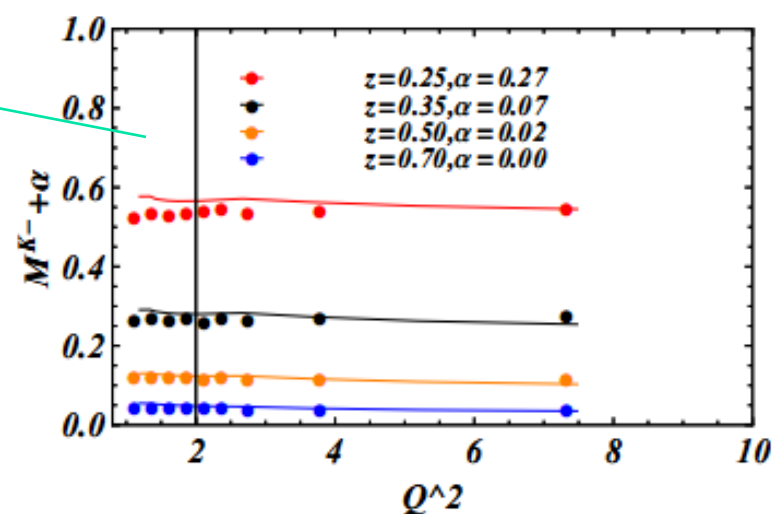
Error!
It should be
 $\alpha = 0.38$
 $\alpha = 0.07$
 $\alpha = 0.02$
 $\alpha = 0.00$



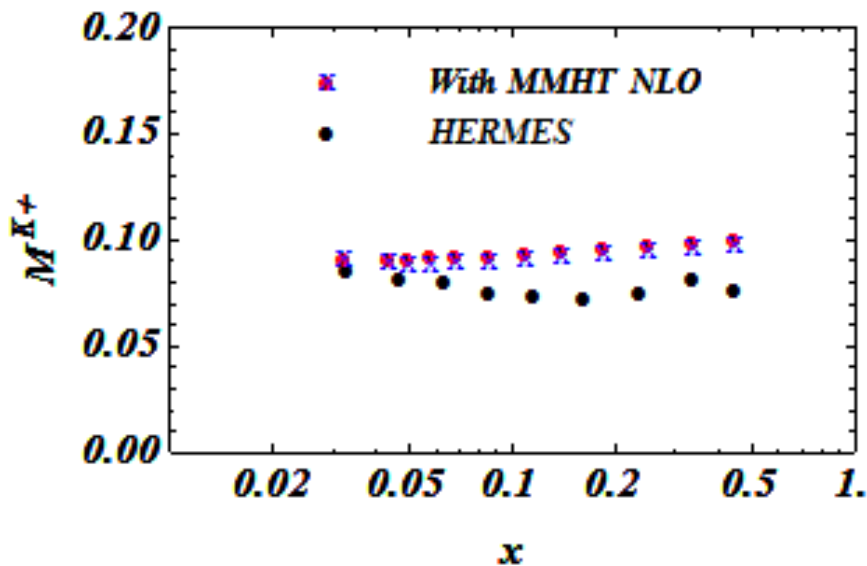
Log to Log & Linear to Linear:



Fit
well?

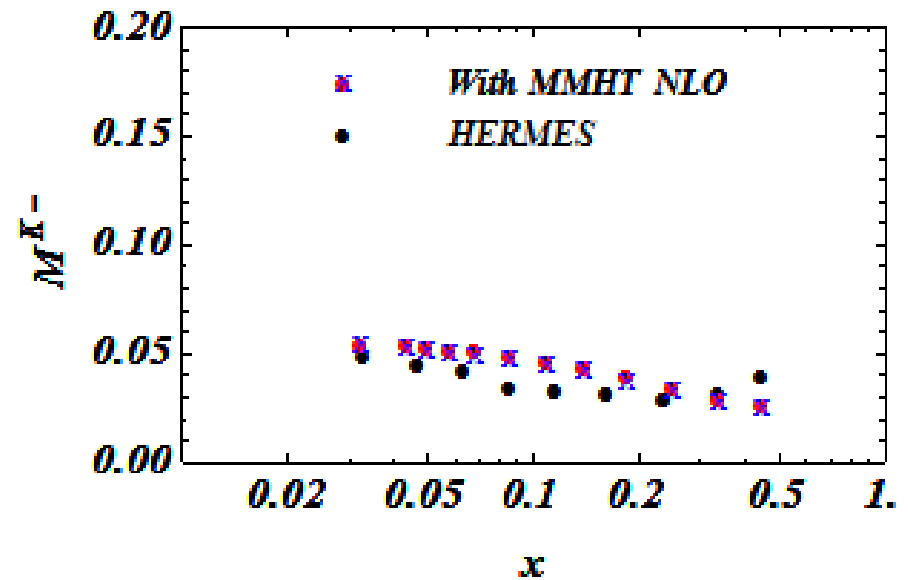


$M^{K^+}(x, Q^2)$ and $M^{K^-}(x, Q^2)$

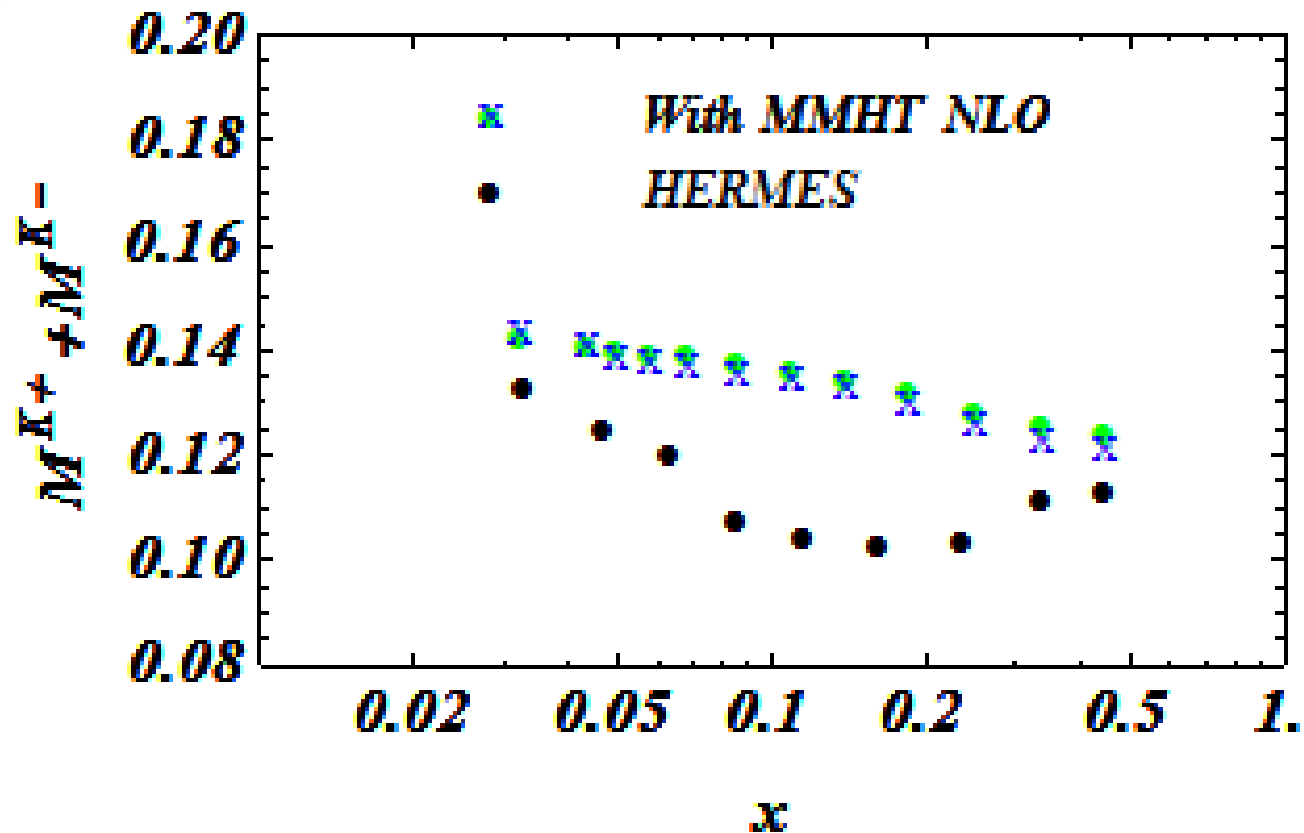


For positive and negative kaon,
The PDF_xFF values are higher than
The experimental values.

Red point: usual integral method
Blue cross: integrate z by
HERMES method
Black point: HERMES

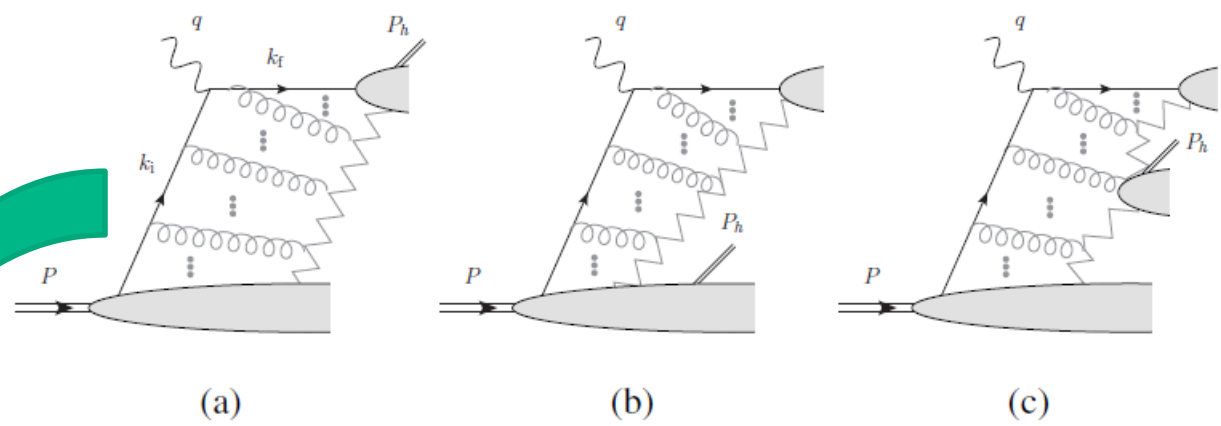


Charged Kaon Multiplicity



Similar work on COMPASS kinematics is going on.....

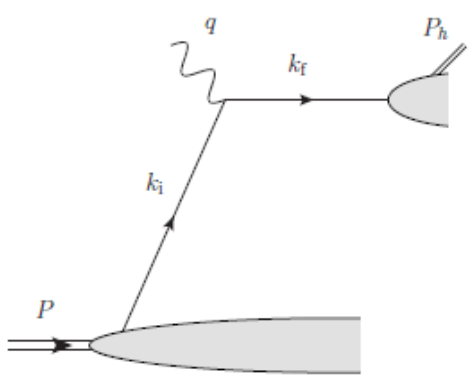
Three regions



$$Q^2 \gg \Lambda_{\text{QCD}}^2$$

P_{hT} is small

(a) the current region (b) the target region and (c) the central (soft) region.

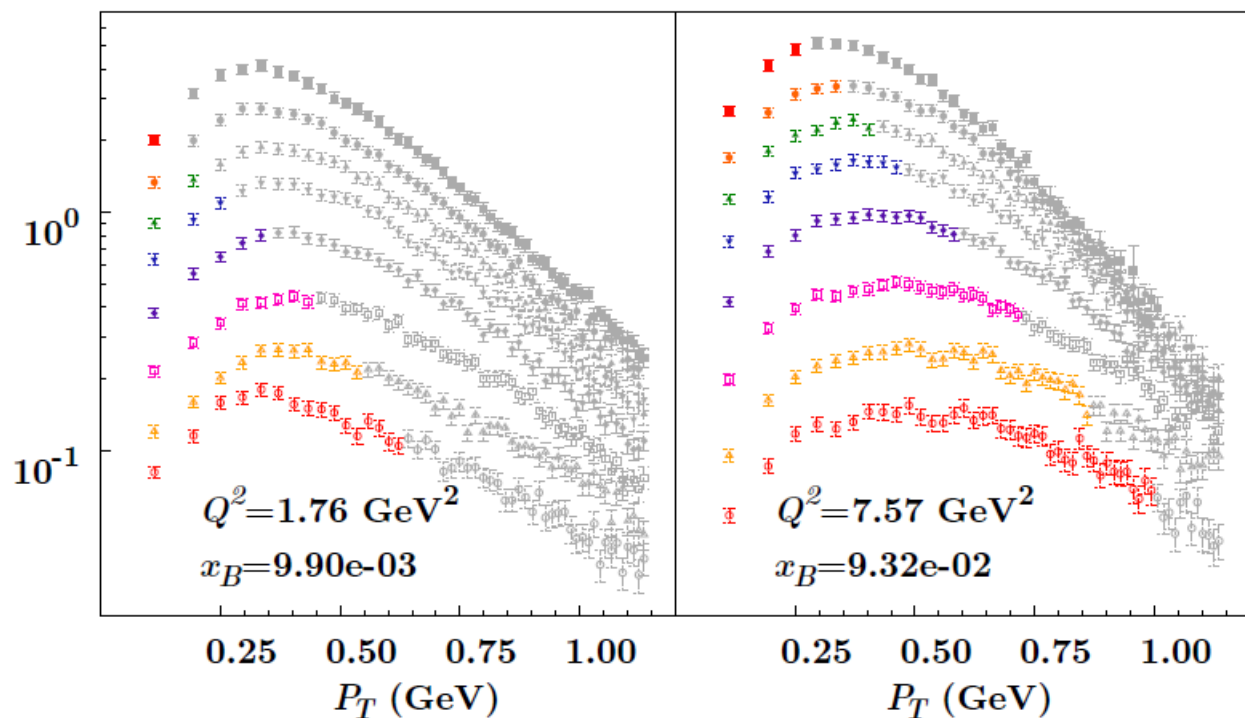


LO cross section: PDF X FF

A selection of COMPASS data

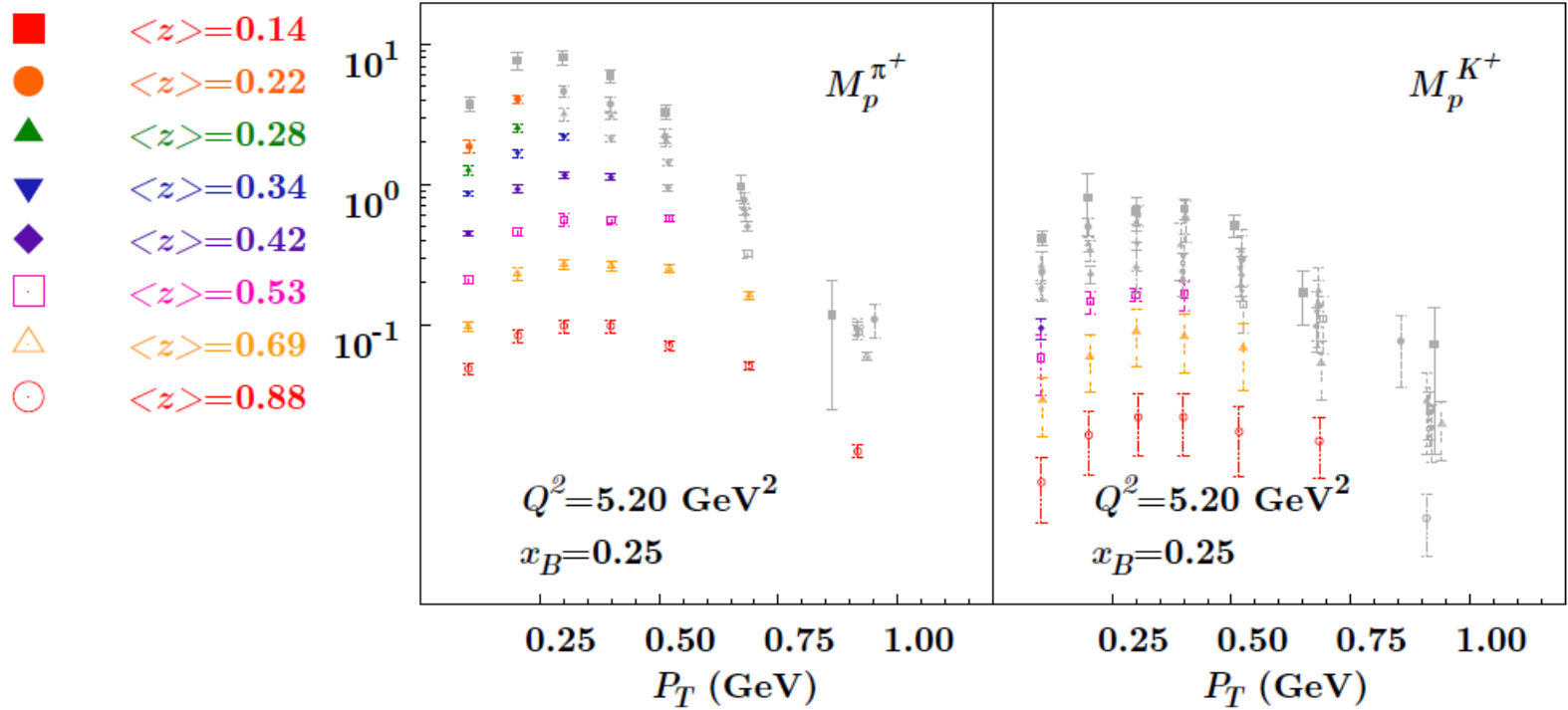
COMPASS $M_D^{h^+}$

- $\langle z \rangle = 0.23$
- $\langle z \rangle = 0.28$
- ▲ $\langle z \rangle = 0.33$
- ▼ $\langle z \rangle = 0.38$
- ◆ $\langle z \rangle = 0.45$
- $\langle z \rangle = 0.55$
- △ $\langle z \rangle = 0.65$
- $\langle z \rangle = 0.75$



grey points are likely to receive important contributions from non-current regions.

HERMES



grey points are likely to receive important contributions from non-current regions.



Summary and outlook

- Is DSS17 good as they claimed?
- Can one extract the strange PDFs from the current SIDIS data?
- Is there tension between COMPASS and HERMES data of SIDIS?
- Is the contribution of the central region able to explain the difference between HERMES and COMPASS data?

Two data sets of SIDIS cause many confusions,
but two cups of ice cream are just perfect!

