Heavy quark mass effects in associated production

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$a \times b = c$
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Example: Factorisation (not the only one!)

\[ a \times b = c \]

\[ \sum_{ab} \int dx_1 \int dx_2 \left( f_a(x_1) f_b(x_2) \right) \times d\sigma_{ab} = \sigma \]
\[ \sum_{ab} \int \, dx_1 \int \, dx_2 \, f_a(x_1) f_b(x_2) \times d\sigma_{ab} = \sigma \]

- Def of \( a \) and \( b \), is arbitrary as long as it is compensated in \( d\sigma \)

- Extreme ex: only gluons in the proton, compute Drell-Yan
\[ \int \! dx_1 \int \! dx_2 \, f_g(x_1) \, f_g(x_2) \times \]
\[ \int dx_1 \int dx_2 \, f_g(x_1) \, f_g(x_2) \times \frac{1}{\varepsilon^2} \]
\[
\int \! dx_1 \int \! dx_2 \, f_g(x_1) \, f_g(x_2) \times \ldots
\]

What if they’re not massless?

\[
\frac{1}{\varepsilon^2}
\]

\[
\alpha_s^2 \log^2 \left( \frac{\eta^2}{m^2} \right)
\]

\[
\eta = p_T \sim 10 \text{ GeV}; \quad m = 2 \times 10^{-6} \text{ GeV}; \quad \log \sim 16 \sim \frac{1}{\alpha_s}
\]
What happened?

When logs are dominant over mass effects we have that:

\[
\frac{\alpha_s \log \frac{\eta^2}{m_b^2}}{m_b^2} \times \sum_{b} \bar{b}b
\]

DGLAP equations:

\[
\begin{align*}
\frac{df_b(x, \mu^2)}{d \log \mu^2} &= \alpha_s P_{qg} \otimes f_g \\
&\rightarrow f_b(x, \eta^2) = \alpha_s \log \frac{\eta^2}{m_b^2} P_{qg} \otimes f_g
\end{align*}
\]

at LL…
\[ \int dx_1 \int dx_2 \, f_a(x_1, \mu^2) \, f_b(x_1, \mu^2) \, d\sigma_{ab}(\mu^2) = \sigma \]

- Varying the scale simply shuffles terms around

- expansion in coupling makes everything more complicated
4F Scheme:

- LO more complicated
- possible log problems
- exact mass dep

5F Scheme:

- LO and HO easy, but not much info
- no log problems
- no mass dep...
Problem!

In the kinematic region $\hat{s} m^2 b \log \hat{s} m^2 b \ll O(1)$

Solution

5 flavours scheme, re-sums such logs via DGLAP eqs in \( b \)-PDF.

\( m_b = 0 \)

\( g \)

\( \bar{b} \)

\( b \)

\( H \)

\( / \)

\( 2 \)

\( S \log \hat{s} m^2 b \)

\( b \bar{b} \)

\( H \)

\( / \)

\{z\}

Absorbed into a \( b \)-PDF!

4F Scheme:

- LO more complicated
- possible log problems
- exact mass dep

Better for differential observables

5F Scheme:

- LO and HO easy, but not much info
- no log problems
- no mass dep...

Better for inclusive ones
In the kinematic region $\hat{s}$

\[
S(\hat{s}) \log \hat{s} m^2 b \overset{\text{absorbed}}{\sim} O(1)
\]

Solution

5 flavours scheme, re-sums such logs via DGLAP eqs in $b$-PDF.

$mb = 0$

\[
\alpha_s^2 \left[ c^{(2)} \left( \frac{m^2}{\eta^2} \right) L^2 + c^{(1)} \left( \frac{m^2}{\eta^2} \right) L + \frac{m^2}{\eta^2} c^{(0)} + K \right]
\]

\[
\left( 1 - e^{-\alpha_s^2 \left[ c^{(2)} L^2 + c^{(1)} L + K \right]} \right) + O(\alpha_s^3)
\]

4F Scheme:

$\begin{align*}
\alpha_s^2 \left[ c^{(2)} \left( \frac{m^2}{\eta^2} \right) L^2 + c^{(1)} \left( \frac{m^2}{\eta^2} \right) L + \frac{m^2}{\eta^2} c^{(0)} + K \right]
\end{align*}$

5F Scheme:
4F and 5F have actually the same contributions…
\[ \sigma^{\text{matched}} = \sigma^{(5F)} + \sigma^{(4F)} - \text{d.c.} \]
$$\sigma^{\text{FONLL}} - \sigma^{5F} = A \frac{m^2}{m_A^2} + K$$
• Inclusive XS, it does seem like 5F better approximation…

• Can we something on more differential obs?

• What to do for more complicated procs?
Massive 5F (5FMS):

\[ \mathcal{B} + \mathcal{V} = \left| \begin{array}{c} b \\ \bar{b} \end{array} \right| H + \left| \begin{array}{c} b \\ \bar{b} \end{array} \right| H^2 = \left| \begin{array}{c} b \\ \bar{b} \end{array} \right| H^2 \times \left( 1 + 2 \text{Re}(\delta_g) \right) \]

\[ \mathcal{R} = \left| \begin{array}{c} b \\ \bar{b} \end{array} \right| g - \left| \begin{array}{c} b \\ \bar{b} \end{array} \right| g^2 + \left| \begin{array}{c} g \\ b \end{array} \right|^2 \]
Differences can be huge in some critical regions of phase-space
Not so much in more inclusive observables (now CKKW merging)
• 4F vs 5F scheme a fight on the rise again

• Neutral boson + HF seem to prefer the 5F

• 5F massive scheme to include mass effects

• Need further theoretical study…