Accessing Generalized TMDs through Double Drell-Yan and Double Quarkonium Production Processes

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OUTLINE

- **Generalized TMDs (GTMDs)**
- **Quark GTMDs in Exclusive Double Drell-Yan Process**

  (S. Bhattacharya, A. Metz, J. Zhou / arXiv: 1702.04387)

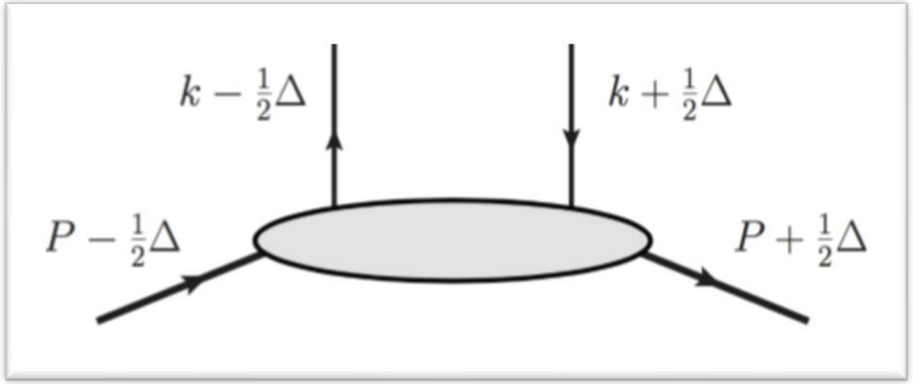
  - **Old + New results on accessing quark GTMDs**
- **Gluon GTMDs in Exclusive Double Quarkonia Production Process**

  (S. Bhattacharya, A. Metz, V. K. Ojha, J. Tsai, J. Zhou / arXiv: 1802.10550)

  - **Define the process**
  - **Kinematics**
  - **Scattering Amplitude**
  - **Gluon GTMDs**
  - **Polarization Observables**
- **Summary and Outlook**

Generalized Transverse Momentum Dependent distributions

GTMD matrix element- graphical depiction



$$P = \frac{(p+p')}{2} \qquad \Delta = p' - p \qquad t = \Delta^2$$

GTMD matrix element definition

$$W^{q\,[\Gamma]}_{\lambda,\lambda'}(P,\Delta,x,\vec{k}_\perp) =$$
$$\int \frac{dz^-\, d^2\vec{z}_\perp}{2(2\pi)^3}\, e^{ik\cdot z}\, \langle p',\lambda'|\, \bar{q}(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\, q(\tfrac{z}{2})\, |p,\lambda\rangle \Big|_{z^+=0}$$

Matrix elements parameterized through GTMDs

$$X(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

$$x = \frac{k^+}{P^+} \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$$

GTMDs as 'mother distributions'

GTMD $(x,\vec{k}_\perp,\Delta)$

$\xrightarrow{\int d^2\vec{k}_\perp}$ **GPD** (x,Δ)

$\xrightarrow{\Delta=0}$ **TMD** $(x,\vec{k}_\perp)$

GPD (x,Δ)

$\xrightarrow{\int dx}$ **FFs** (Δ)

$\xrightarrow{\Delta=0}$ **PDFs** (x)

In particular, $F_{1,4}$ **and** $G_{1,1}$ **have a direct role in understanding spin-structure of hadrons**

(Lorcé, Pasquini, 2011 / Hatta, 2011 / Lorcé, 2014).

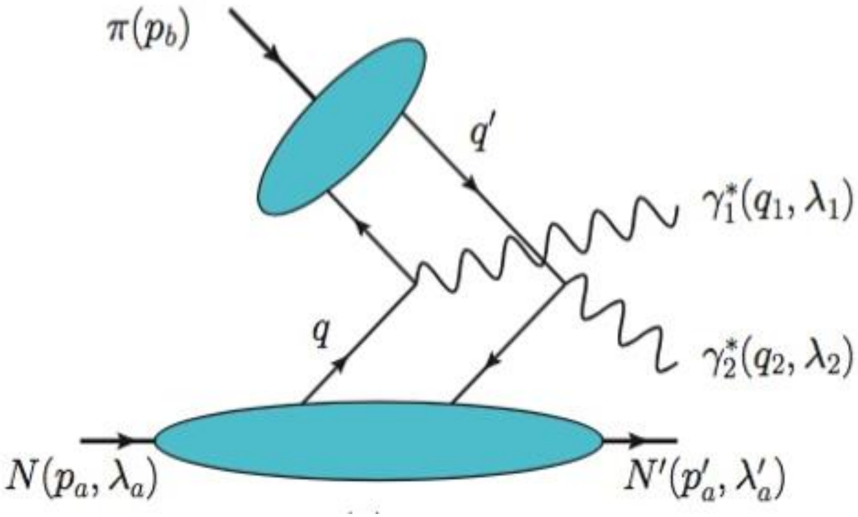
But, how would we measure them?

Exclusive Double Drell-Yan Process

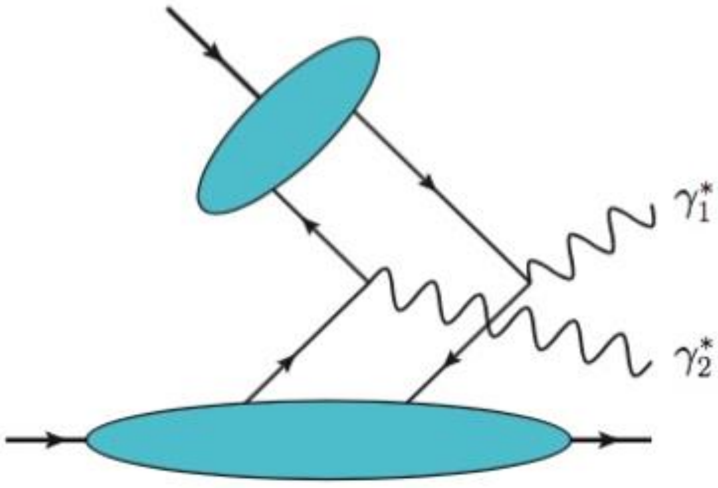
(SB, Metz, Zhou / arXiv: 1702.04387)

Process $\pi(p_b)\, N(p_a, \lambda_a) \to (l_1^- l_1^+)(l_2^- l_2^+)\, N'(p_a, \lambda_a)$

Leading-order diagrams



Graph (a)



Graph (b)

Kinematics (TMD type)

$s = (p_a + p_b)^2 \approx 2p_a^+ p_b^-$ large

$q_1^2,\ q_2^2$ large $\quad |\vec{q}_{i\perp}^{\,2}| \ll q_i^2$

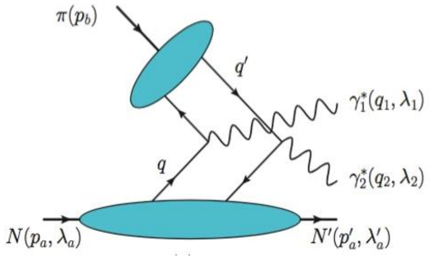
Kinematical variables

$$x_a = \frac{k_a^+}{P_a^+} \qquad x_b = \frac{k_b^-}{p_b^-} \qquad \xi_a = \frac{(q_1^+ + q_2^+)}{2P_a^+} \qquad \vec{\Delta}_{a\perp} = -(\vec{q}_{1\perp} + \vec{q}_{2\perp})$$

Scattering Amplitude

$$\mathcal{T}^{\mu\nu}_{\lambda_a,\lambda'_a} = i\sum_{q,q'} e_q e'_q e^2 \frac{1}{N_c}\int d^2\vec{k}_{a\perp}\int d^2\vec{k}_{b\perp}\,\delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2}-\vec{k}_{a\perp}-\vec{k}_{b\perp}\right)\Phi_\pi^{q'q}(x_b,\vec{k}^2_{b\perp})$$
$$\left[-i\varepsilon_\perp^{\mu\nu}\left(W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp})-W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda'_a}(-x_a,-\vec{k}_{a\perp})\right)\right.$$
$$\left.-g_\perp^{\mu\nu}\left(W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp})+W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda'_a}(-x_a,-\vec{k}_{a\perp})\right)\right]$$

Graph (a)

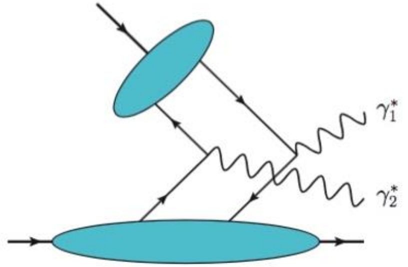


Longitudinal parton momenta fixed as:

$$x_a = \frac{(q_1^+ - q_2^+)}{2P_a^+}$$

$$x_b = 1-\frac{q_1^-}{p_b^-}=\frac{q_2^-}{p_b^-}$$

Graph (b)



Longitudinal parton momenta fixed as:

$$x_a = -\frac{(q_1^+ - q_2^+)}{2P_a^+}$$

$$x_b = \frac{q_1^-}{p_b^-}$$

ERBL region

$$\xi_a = \frac{(q_1^+ + q_2^+)}{2P_a^+}$$

$$-\xi_a \le x_a \le \xi_a$$

Independent transverse vectors

$$\vec{\Delta}_{a\perp} = -(\vec{q}_{1\perp}+\vec{q}_{2\perp})$$

$$\Delta\vec{q}_\perp = (\vec{q}_{1\perp}-\vec{q}_{2\perp})$$

Quark GTMDs (Meissner, Metz, Schlegel / arXiv: 0906.5323)

$$W^{q\,[\gamma^+]}_{\lambda,\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F^q_{1,1} + \frac{i\sigma^{i+}k^i_\perp}{P^+}F^q_{1,2} + \frac{i\,\sigma^{i+}\Delta^i_\perp}{P^+}F^q_{1,3} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}F^q_{1,4}\right]u(p,\lambda)$$

$$W^{q\,[\gamma^+\gamma_5]}_{\lambda,\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[-\frac{i\varepsilon^{ij}_\perp k^i_\perp\Delta^j_\perp}{M^2}G^q_{1,1} + \frac{i\sigma^{i+}\gamma_5 k^i_\perp}{P^+}G^q_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta^i_\perp}{P^+}G^q_{1,3} + i\sigma^{+-}\gamma_5 G^q_{1,4}\right]u(p,\lambda)$$

GTMDs have real and imaginary part

Polarization Observables

Unpolarized

$$\tau_{UU} = \frac{1}{2}\sum_{\lambda,\lambda'} |\mathcal{T}_{\lambda,\lambda'}|^2$$

SSA

$$\tau_{LU} = \frac{1}{2}\sum_{\lambda'} \left(|\mathcal{T}_{+,\lambda'}|^2 - |\mathcal{T}_{-,\lambda'}|^2\right)$$

DSA

$$\tau_{LL} = \frac{1}{2}\Big(\big(|\mathcal{T}_{+,+}|^2 - |\mathcal{T}_{+,-}|^2\big) - \big(|\mathcal{T}_{-,+}|^2 - |\mathcal{T}_{-,-}|^2\big)\Big)$$

Key point is to realize that if we ~
Define 'Polarization Observables' in a certain fashion
Take proper linear combination of those observables
$\downarrow$
'Single out' what you want!

Remarks:

- Summation over photon polarization $(\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2})$ is implied
- If photon polarization $(\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2})$ is summed over, there is no interference between the objects $W^{q[\gamma^+]}_{\lambda,\lambda'}$ and $W^{q[\gamma^+\gamma^5]}_{\lambda,\lambda'}$

Simple Case: Addressing GTMDs at a specific kinematical point $\vec{\Delta}_{a\perp} = 0$

Not discussed in arXiv: 1702.04387

Surviving Candidates: $F_{1,1}$ $F_{1,2}$ $G_{1,2}$ $G_{1,4}$

Direct Access

- $$\frac{1}{4}(\tau_{UU}+\tau_{LL}+\tau_{XX}+\tau_{YY}) = 2C^{(-)}\left[F_{1,1}(x_a,\xi_a,\vec{k}^2_{a\perp})\,\phi_\pi(x_b,\vec{k}^2_{b\perp})\right]C^{(-)}\left[F^*_{1,1}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp})\,\phi^*_\pi(x_b,\vec{p}^{\,2}_{b\perp})\right]$$
- $$\frac{1}{4}(\tau_{UU}+\tau_{LL}-\tau_{XX}-\tau_{YY}) = 2C^{(+)}\left[G_{1,4}(x_a,\xi_a,\vec{k}^2_{a\perp})\,\phi_\pi(x_b,\vec{k}^2_{b\perp})\right]C^{(+)}\left[G^*_{1,4}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp})\,\phi^*_\pi(x_b,\vec{p}^{\,2}_{b\perp})\right]$$
- $$\tau_{XY} = \tau_{YX}$$
$$= -4\frac{(1-\xi_a^2)^2}{M_a^2}\Delta q^1_\perp \Delta q^2_\perp \Big\{C^{(+)}\left[\frac{\Delta\vec{q}_\perp.\vec{k}_{a\perp}}{\Delta\vec{q}^{\,2}_\perp}F_{1,2}(x_a,\xi_a,\vec{k}^{\,2}_{a\perp})\,\phi_\pi(x_b,\vec{k}^2_{b\perp})\right]C^{(+)}\left[\frac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}^{\,2}_\perp}F^*_{1,2}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp})\,\phi^*_\pi(x_b,\vec{p}^{\,2}_{b\perp})\right]$$
$$- C^{(-)}\left[\frac{\Delta\vec{q}_\perp.\vec{k}_{a\perp}}{\Delta\vec{q}^{\,2}_\perp}G_{1,2}(x_a,\xi_a,\vec{k}^{\,2}_{a\perp})\,\phi_\pi(x_b,\vec{k}^2_{b\perp})\right]C^{(-)}\left[\frac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}^{\,2}_\perp}G^*_{1,2}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp})\,\phi^*_\pi(x_b,\vec{p}^{\,2}_{b\perp})\right]\Big\}$$

Convolution integral

$$C^{(\pm)}\left[w(\vec{k}_{a\perp},\vec{k}_{b\perp})\,X\,\Phi_\pi\right] = \frac{e^2}{\sqrt{1-\xi_a^2}N_c}\sum_{q,q'}e_q e'_q\int d^2\vec{k}_{a\perp}\int d^2\vec{k}_{b\perp}\,\delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2}-\vec{k}_{a\perp}-\vec{k}_{b\perp}\right)w(\vec{k}_{a\perp},\vec{k}_{b\perp})$$
$$\times\left[X^{qq'}(x_a,\vec{k}_{a\perp}) \pm X^{qq'}(-x_a,-\vec{k}_{a\perp})\right]\Phi_\pi^{q'q}(x_b,\vec{k}^2_{b\perp})$$

$$\varepsilon^{\mu\nu}_\perp\,\epsilon^*_\mu\,\epsilon^*_\nu W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp})$$
$$g^{\mu\nu}_\perp\,\epsilon^*_\mu\,\epsilon^*_\nu W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp})$$

Access through Interference ($\vec{\Delta}_{a\perp} = 0$)

Not discussed in arXiv: 1702.04387

- $$\tau_{XU} = 4\frac{(1-\xi_a^2)}{M_a}\Delta q_\perp^2 \boxed{Im.}\Big\{C^{(-)}\Big[F_{1,1}(x_a,\xi_a,\vec{k}^{\,2}_{a\perp})\,\phi_\pi(x_b,\vec{k}^{\,2}_{b\perp})\Big]C^{(+)}\Big[\frac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}^{\,2}_\perp}F^*_{1,2}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp})\,\phi^*_\pi(x_b,\vec{p}^{\,2}_{b\perp})\Big] - C^{(+)}\Big[G_{1,4}(x_a,\xi_a,\vec{k}^{\,2}_{a\perp})\,\phi_\pi(x_b,\vec{k}^{2}_{b\perp})\Big]C^{(-)}\Big[\frac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}^{\,2}_\perp}G^*_{1,2}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp})\,\phi^*_\pi(x_b,\vec{p}^{\,2}_{b\perp})\Big]\Big\}$$

- $$\tau_{XL} = -4\frac{(1-\xi_a^2)}{M_a}\Delta q_\perp^1 \boxed{Re.}\Big\{C^{(-)}\Big[F_{1,1}(x_a,\xi_a,\vec{k}^{\,2}_{a\perp})\,\phi_\pi(x_b,\vec{k}^{\,2}_{b\perp})\Big]C^{(+)}\Big[\frac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}^{\,2}_\perp}F^*_{1,2}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp})\,\phi^*_\pi(x_b,\vec{p}^{\,2}_{b\perp})\Big] - C^{(+)}\Big[G_{1,4}(x_a,\xi_a,\vec{k}^{\,2}_{a\perp})\,\phi_\pi(x_b,\vec{k}^{2}_{b\perp})\Big]C^{(-)}\Big[\frac{\Delta\vec{q}_\perp.\vec{p}_{a\perp}}{\Delta\vec{q}^{\,2}_\perp}G^*_{1,2}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp})\,\phi^*_\pi(x_b,\vec{p}^{\,2}_{b\perp})\Big]\Big\}$$

Remark:
$Re.F_{1,1}$ **and** $Re.G_{1,4}$ **treated as \`key' candidates to get access to real and imaginary part of other GTMDs since they are presumably large**

✓ $Re.(F_{1,1})\Big|_{\Delta=0} = f_1(x,\vec{k}^{\,2}_\perp)$

✓ $Re.(G_{1,4})\Big|_{\Delta=0} = g_1(x,\vec{k}^{\,2}_\perp)$

$$\boxed{Im.(F_{1,2})\Big|_{\Delta=0} = -f_{1T}^\perp}$$

$$\boxed{Re.(G_{1,2})\Big|_{\Delta=0} = g_{1T}}$$

General Case: Keep $\vec{\Delta}_{a\perp}$!

Direct Access

Old piece! But now,

$$G_{1,4}(x,\xi,\vec{k}_\perp^{\,2},\vec{\Delta}_\perp^{\,2},\vec{k}_\perp.\vec{\Delta}_\perp)$$

- $$\frac{1}{4}\left(\tau_{UU}+\tau_{LL}-\tau_{XX}-\tau_{YY}\right) = 2\,C^{(+)}\left[G_{1,4}\,\phi_\pi\right]C^{(+)}\left[G^*_{1,4}\,\phi^*_\pi\right]+ \boxed{\frac{2}{M_a^4}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i\,\Delta_{a\perp}^j\right)^2 C^{(+)}\left[\vec{\beta}_\perp\cdot\vec{k}_{a\perp}F_{1,4}\phi_\pi\right]C^{(+)}\left[\vec{\beta}_\perp\cdot\vec{p}_{a\perp}F^*_{1,4}\phi^*_\pi\right]}$$

New piece!

Old piece!

- $$\frac{1}{4}\left(\tau_{UU}+\tau_{LL}+\tau_{XX}+\tau_{YY}\right) = 2\,C^{(-)}\left[F_{1,1}\,\phi_\pi\right]C^{(-)}\left[F^*_{1,1}\,\phi^*_\pi\right]+ \boxed{\frac{2}{M_a^4}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i\,\Delta_{a\perp}^j\right)^2 C^{(-)}\left[\vec{\beta}_\perp\cdot\vec{k}_{a\perp}G_{1,1}\phi_\pi\right]C^{(-)}\left[\vec{\beta}_\perp\cdot\vec{p}_{a\perp}G^*_{1,1}\phi^*_\pi\right]}$$

New piece!

weight factor

$$\vec{\beta}_\perp = \frac{\vec{\Delta}_{a\perp}^2\,\Delta\vec{q}_\perp-(\vec{\Delta}_{a\perp}\cdot\Delta\vec{q}_\perp)\,\vec{\Delta}_{a\perp}}{\vec{\Delta}_{a\perp}^2\,\Delta\vec{q}_\perp^{\,2}-(\vec{\Delta}_{a\perp}\cdot\Delta\vec{q}_\perp)^2} = \frac{\vec{\Delta}_{a\perp}\times(\Delta\vec{q}_\perp\times\vec{\Delta}_{a\perp})}{\left(\varepsilon_\perp^{ij}\Delta q_\perp^i\,\Delta_{a\perp}^j\right)^2}$$

Access through Interference ($\vec{\Delta}_{a\perp} \neq 0$)

$$\frac{1}{2}\left(\tau_{XY} - \tau_{YX}\right) = \frac{4}{M_a^2}\left(\varepsilon_{\perp}^{ij}\Delta q_{\perp}^{i}\Delta_{a\perp}^{j}\right)\boxed{Re.}\Big\{C^{(-)}\left[F_{1,1}\,\phi_{\pi}\right]C^{(+)}\left[\vec{\beta}_{\perp}\cdot\vec{k}_{a\perp}F_{1,4}^{*}\phi_{\pi}^{*}\right] - C^{(+)}\left[G_{1,4}\,\phi_{\pi}\right]C^{(-)}\left[\vec{\beta}_{\perp}\cdot\vec{p}_{a\perp}G_{1,1}^{*}\phi_{\pi}^{*}\right]\Big\}$$

Remark:

This linear combination is sensitive to $Re.F_{1,4}$ **and** $Re.G_{1,1}$ **(sensitivity to strength of spin-orbit correlation)**

What about $F_{1,2}$ $F_{1,3}$ $G_{1,2}$ $G_{1,3}$ **in the general case,** $\vec{\Delta}_{a\perp} \neq 0$ **?**

Not discussed in arXiv: 1702.04387

Example: $F_{1,2}$

Direct Access — **May not be possible in this case!**

$$\boxed{\varepsilon_{\perp}^{\mu\nu}}\, \epsilon_{\mu}^{*}\, \epsilon_{\nu}^{*} W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda_a'}(x_a,\vec{k}_{a\perp})$$

$$\boxed{g_{\perp}^{\mu\nu}}\, \epsilon_{\mu}^{*}\, \epsilon_{\nu}^{*} W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda_a'}(x_a,\vec{k}_{a\perp})$$

Access through Interference — **Challenging! But possible!**

- $$\frac{1}{4}\left[\Delta^1_{a\perp}(\tau_{XU}+\tau_{UX}) + \frac{\xi_a \vec{\Delta}^2_{a\perp}}{2M_a}(\tau_{LU}+\tau_{UL})\right]$$
$$= -2\frac{(1-\xi_a^2)}{M_a}(\varepsilon_{\perp}^{ij}\Delta q_{\perp}^i \Delta_{a\perp}^j)\, \boxed{Im.}\left\{C^{(-)}\left[F_{1,1}\,\phi_\pi\right]C^{(+)}\left[\vec{\beta}_{a\perp}.\vec{p}_{a\perp} F^*_{1,2}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp},\vec{\Delta}^{\,2}_{a\perp},\vec{p}_{a\perp}.\vec{\Delta}_{a\perp})\,\phi^*_\pi\right]\right\} + \ldots$$

- $$\frac{1}{4}\left[\Delta^1_{a\perp}(\tau_{LY}-\tau_{YL}) - \frac{\xi_a \vec{\Delta}^2_{a\perp}}{2M_a}(\tau_{XY}-\tau_{YX})\right]$$
$$= -2\frac{(1-\xi_a^2)}{M_a}(\varepsilon_{\perp}^{ij}\Delta q_{\perp}^i \Delta_{a\perp}^j)\, \boxed{Re.}\left\{C^{(-)}\left[F_{1,1}\,\phi_\pi\right]C^{(+)}\left[\vec{\beta}_{a\perp}.\vec{p}_{a\perp} F^*_{1,2}(x_a,\xi_a,\vec{p}^{\,2}_{a\perp},\vec{\Delta}^{\,2}_{a\perp},\vec{p}_{a\perp}.\vec{\Delta}_{a\perp})\,\phi^*_\pi\right]\right\} + \ldots$$

Remark:
Similar consideration can be made for $F_{1,3}$ **,** $G_{1,2}$ **and** $G_{1,3}$

Check-list

✓	$F_{1,1}$	$\vec{\Delta}_{a\perp} = 0\ ,\ \vec{\Delta}_{a\perp} \neq 0$	✓	$G_{1,4}$
✓	$F_{1,2}$	$\vec{\Delta}_{a\perp} = 0\ ,\ \vec{\Delta}_{a\perp} \neq 0$	✓	$G_{1,2}$
✓	$F_{1,3}$	$\vec{\Delta}_{a\perp} \neq 0$	✓	$G_{1,3}$
✓	$F_{1,4}$	$\vec{\Delta}_{a\perp} \neq 0$	✓	$G_{1,1}$

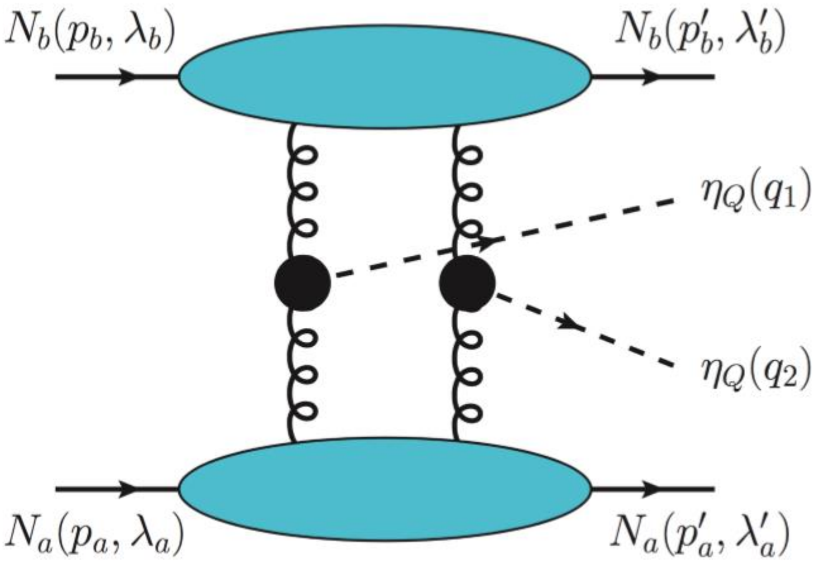
Exclusive Double Quarkonium Production and GTMDs of gluon

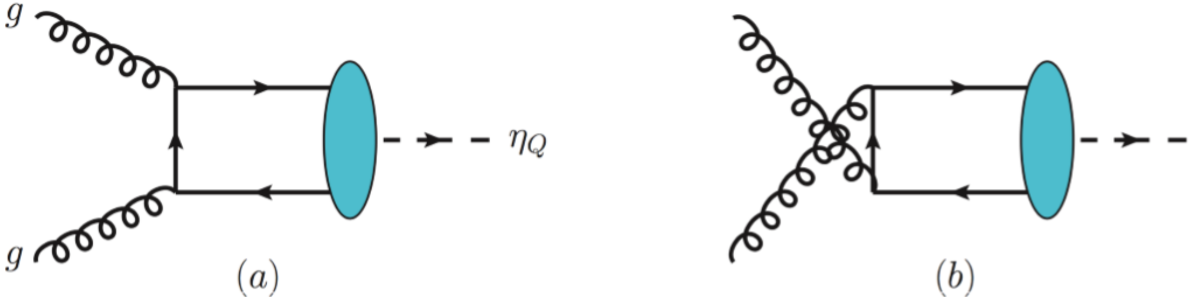
(SB, Metz, Ojha, Tsai, Zhou / arXiv: 1802.10550)

Process

$$N_a(p_a,\lambda_a) + N_b(p_b,\lambda_b) \to \eta_Q(q_1) + \eta_Q(q_2) + N_a(p'_a,\lambda'_a) + N_b(p'_b,\lambda'_b)$$

Leading-order diagrams





Kinematics (TMD type)

$$s = (p_a + p_b)^2 \approx 2p_a^+ p_b^- \text{ large} \qquad |\vec{q}_{i\perp}| << m_\eta \qquad q_1^2 = q_2^2 = m_\eta^2$$

$$\xi_a = \frac{(q_1^+ + q_2^+)}{2P_a^+} \qquad \xi_b = \frac{(q_1^- + q_2^-)}{2P_b^-}$$

Scattering Amplitude

$\Delta\vec{q}_\perp = (\vec{q}_{1\perp} - \vec{q}_{2\perp})$

$$\mathcal{T}_{\lambda_a,\lambda'_a;\lambda_b,\lambda'_b} = -i\,A \int d^2\vec{k}_{a\perp} \int d^2\vec{k}_{b\perp}\, \delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right)$$
$$\times \Big[W^g_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp})\, W^g_{\lambda_b,\lambda'_b}(x_b,\vec{k}_{b\perp}) + W^g_{\lambda_a,\lambda'_a}(-x_a,-\vec{k}_{a\perp})\, W^g_{\lambda_b,\lambda'_b}(-x_b,-\vec{k}_{b\perp})$$
$$+ \widetilde{W}^g_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp})\, \widetilde{W}^g_{\lambda_b,\lambda'_b}(x_b,\vec{k}_{b\perp}) + \widetilde{W}^g_{\lambda_a,\lambda'_a}(-x_a,-\vec{k}_{a\perp})\, \widetilde{W}^g_{\lambda_b,\lambda'_b}(-x_b,-\vec{k}_{b\perp}) \Big]$$

Radial wavefunction of η_Q at origin

$$A = \frac{g_s^4\, R_0^2(0)\, s}{N_c(N_c^2-1)\,\pi\, m_\eta^5\,(1+\xi_a)(1+\xi_b)}$$

Longitudinal parton momenta fixed:

$$x_a = \frac{(q_1^+ - q_2^+)}{2P_a^+} \quad x_b = \frac{(q_1^- - q_2^-)}{2P_b^-} \quad -\xi \leq x \leq \xi$$ **ERBL region**

Gluon GTMDs

$$W^{g\,[ij]}_{\lambda,\lambda'}(P,\Delta,x,\vec{k}_\perp) = \frac{1}{P^+}\int \frac{dz^-\, d^2\vec{z}_\perp}{(2\pi)^3}\, e^{ik\cdot z}\, \langle p',\lambda'|\, F_a^{+i}(-\tfrac{z}{2})\, \mathcal{W}_{ab}(-\tfrac{z}{2},\tfrac{z}{2})\, F_b^{+j}(\tfrac{z}{2})\,|p,\lambda\rangle\Big|_{z^+=0}$$

$$W^g_{\lambda,\lambda'} = \delta_\perp^{ij} W^{g[ij]}_{\lambda,\lambda'} \rightarrow W^{q[\gamma^+]}_{\lambda,\lambda'}$$

$$\widetilde{W}^g_{\lambda,\lambda'} = -i\epsilon_\perp^{ij} W^{g[ij]}_{\lambda,\lambda'} \rightarrow W^{q[\gamma^+\gamma^5]}_{\lambda,\lambda'}$$

Observables

Polarization Observables

$$\tau_{UU} = \frac{1}{2}\sum_{\lambda_b,\lambda'_b}\frac{1}{2}\sum_{\lambda_a,\lambda'_a}|\mathcal{T}_{\lambda_a,\lambda'_a;\lambda_b,\lambda'_b}|^2$$

$$\tau_{LU} = \frac{1}{2}\sum_{\lambda_b,\lambda'_b}\frac{1}{2}\sum_{\lambda'_a}\left(|\mathcal{T}_{+,\lambda'_a;\lambda_b,\lambda'_b}|^2 - |\mathcal{T}_{-,\lambda'_a;\lambda_b,\lambda'_b}|^2\right)$$

$$\tau_{LL} = \frac{1}{2}\sum_{\lambda_b,\lambda'_b}\frac{1}{2}\left(\left(|\mathcal{T}_{+,+;\lambda_b,\lambda'_b}|^2 - |\mathcal{T}_{+,-;\lambda_b,\lambda'_b}|^2\right) - \left(|\mathcal{T}_{-,+;\lambda_b,\lambda'_b}|^2 - |\mathcal{T}_{-,-;\lambda_b,\lambda'_b}|^2\right)\right)$$

We sum / average over polarizations of Nucleon `b'

Accessing $F_{1,4}$ ($\vec{\Delta}_{a\perp} \neq 0, \vec{\Delta}_{b\perp} = 0$)

Assumed hierarchy of magnitude of GTMDs

$F_{1,1} > G_{1,4} >$ others

Direct Access

$$\frac{1}{4}(\tau_{UU} + \tau_{LL} - \tau_{XX} - \tau_{YY})$$

$$\approx \quad \frac{1}{M^4}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right)^2 C\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} F_{1,4}(x_a, \vec{k}_{a\perp})\, F_{1,1}(x_b, \vec{k}_{b\perp})\right] C\left[\vec{\beta}_\perp \cdot \vec{p}_{a\perp} F_{1,4}^*(x_a, \vec{p}_{a\perp})\, F_{1,1}^*(x_b, \vec{p}_{b\perp})\right]$$

$$+ \quad C\left[G_{1,4}(x_a, \vec{k}_{a\perp})\, G_{1,4}(x_b, \vec{k}_{b\perp})\right] C\left[G_{1,4}^*(x_a, \vec{p}_{a\perp})\, G_{1,4}^*(x_b, \vec{p}_{b\perp})\right]$$

convolution integral

$$C\left[w(\vec{k}_{a\perp}, \vec{k}_{b\perp})\, X\, Y\right] \quad = \quad \frac{2A}{\sqrt{1-\xi_a^2}\,\sqrt{1-\xi_b^2}} \int d^2\vec{k}_{a\perp} \int d^2\vec{k}_{b\perp}\, \delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right)$$

$$\times\, w(\vec{k}_{a\perp}, \vec{k}_{b\perp})\, X(x_a, \vec{k}_{a\perp})\, Y(x_b, \vec{k}_{b\perp})$$

weight factor

$$\vec{\beta}_\perp = \frac{\vec{\Delta}_{a\perp}^2\, \Delta\vec{q}_\perp - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)\, \vec{\Delta}_{a\perp}}{\vec{\Delta}_{a\perp}^2\, \Delta\vec{q}_\perp^{\,2} - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)^2} = \frac{\vec{\Delta}_{a\perp} \times (\Delta\vec{q}_\perp \times \vec{\Delta}_{a\perp})}{\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right)^2}$$

Accessing $G_{1,1}$ ($\vec{\Delta}_{a\perp} \neq 0, \vec{\Delta}_{b\perp} = 0$)

Direct Access

Assumed hierarchy of magnitude of GTMDs
$F_{1,1} > G_{1,4} >$ others

$$\frac{1}{4}(\tau_{UU} + \tau_{LL} + \tau_{XX} + \tau_{YY})$$
$$\approx \frac{\varepsilon_{\perp}^{ij}\Delta_{a\perp}^{j}}{M}\frac{\varepsilon_{\perp}^{kl}\Delta_{a\perp}^{l}}{M} C\left[\frac{k_{a\perp}^{i}}{M} G_{1,1}(x_a, \vec{k}_{a\perp})\, G_{1,4}(x_b, \vec{k}_{b\perp})\right] C\left[\frac{p_{a\perp}^{k}}{M} G_{1,1}^{*}(x_a, \vec{p}_{a\perp})\, G_{1,4}^{*}(x_b, \vec{p}_{b\perp})\right]$$
$$+\ C\left[F_{1,1}(x_a, \vec{k}_{a\perp})\, F_{1,1}(x_b, \vec{k}_{b\perp})\right] C\left[F_{1,1}^{*}(x_a, \vec{p}_{a\perp})\, F_{1,1}^{*}(x_b, \vec{p}_{b\perp})\right]$$

Takes over $\left|G_{1,1}\right|^2$ contribution. Hence, $G_{1,1}$ cannot probably be accessed directly.

Access through Interference ($\vec{\Delta}_{a\perp} \neq 0, \vec{\Delta}_{b\perp} = 0$)

Assumed hierarchy of magnitude of GTMDs

$F_{1,1} > G_{1,4} >$ others

- $$\frac{1}{2}(\tau_{UL} + \tau_{LU})$$

$$\approx 2\boxed{\mathrm{Im}}\Big\{ -\frac{1}{M^2}(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\, C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} F_{1,4}(x_a, \vec{k}_{a\perp})\, F_{1,1}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[F_{1,1}^*(x_a, \vec{p}_{a\perp})\, F_{1,1}^*(x_b, \vec{p}_{b\perp})\Big]$$

$$+ \frac{1}{M^2}(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\, C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} G_{1,1}(x_a, \vec{k}_{a\perp})\, G_{1,4}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[G_{1,4}^*(x_a, \vec{p}_{a\perp})\, G_{1,4}^*(x_b, \vec{p}_{b\perp})\Big]\Big\}$$

$$\approx 2\,\mathrm{Im}\Big\{ -\frac{1}{M^2}(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\, C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} F_{1,4}(x_a, \vec{k}_{a\perp})\, F_{1,1}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[F_{1,1}^*(x_a, \vec{p}_{a\perp})\, F_{1,1}^*(x_b, \vec{p}_{b\perp})\Big]\Big\}$$

$Re.G_{1,1}$ $Im.G_{1,1}$ **overwhelmed by three powers of** $F_{1,1}$

- $$\frac{1}{2}(\tau_{XY} - \tau_{YX})$$

$$\approx 2\boxed{\mathrm{Re}}\Big\{ -\frac{1}{M^2}(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\, C\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} F_{1,4}(x_a, \vec{k}_{a\perp})\, F_{1,1}(x_b, \vec{k}_{b\perp})\Big]\, C\Big[F_{1,1}^*(x_a, \vec{p}_{a\perp})\, F_{1,1}^*(x_b, \vec{p}_{b\perp})\Big]\Big\}$$

Remark:
The second linear combination is sensitive to gluon Orbital Angular Momentum

Summary and Outlook

- **G**eneralized **T**ransverse **M**omentum **D**ependent distributions are important-
  - ‘mother’ distributions to TMDs, GPDs and other non-perturbative functions
  - direct link to partonic orbital angular momentum ($F_{1,4}$ and $G_{1,1}$)
- **Proposal:** Processes that are sensitive to quark / gluon **GTMDs** are the **Exclusive Double Drell-Yan and Exclusive Double Quarkonia production**

  1) Quark / Gluon **GTMDs** can be accessed in the ERBL region

  2) **GTMDs** can be accessed through **P**olarization **O**bservables
- **What else can be done?**
  - include effect of linearly polarized gluons in exclusive double quarkonia production
  - search for process that is sensitive to GTMDs in DGLAP region
  - numerical estimate for observables