Production of transversely polarized $\Lambda$ hyperon from unpolarized quark fragmentation in the diquark model

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XXVI International Workshop on Deep Inelastic Scattering and Related Subjects
16-20 April 2018, Kobe University

Outline

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- The model calculation of $D_1$ and $G_1$
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- Summary
The production of a polarized $\Lambda$ date from the 1970s in $pp$ collisions.


The production of transversely polarized $\Lambda$ hyperon can provide further information about the spin structure of the $\Lambda$ hyperon and the non-perturbative hadronization mechanism.

A class of the so-called time-reversal-odd (T-odd) fragmentation functions has been the main focus on understanding the fragmentation mechanism behind the $\Lambda$ polarization, such as $D_{1T}^\perp$.

Since the experimental information on $D_{1T}^\perp$ of the $\Lambda$ hyperon still remains unknown, the model calculations will provide an approach to acquire knowledge of this quantity.

The spectator model has been applied to calculate the Collins fragmentation function of pion and kaon mesons.

The calculation of $D_1$

Lowest order diagram describing the fragmentation of a quark to a hyperon (taking $u$ quark as an example), as $u \rightarrow \Lambda(uds) + D(\bar{d}s)$:

The unpolarized TMD FF $D_1^A(z, k_T)$ is obtained from

$$D_1^A(z, k_T) = \frac{1}{4} \text{Tr}[(\Delta(z, k_T; S_\Lambda) + \Delta(z, k_T; -S_\Lambda))\gamma^-].$$
The calculation of $D_1$

The quark-quark correlation function:

$$\Delta(z, k_T; S_\Lambda) = \frac{1}{2z} \int dk^+ \Delta(k, P_\Lambda; S_\Lambda)$$

$$\equiv \sum_X \int \frac{d^2 \xi}{2z(2\pi)^3}$$

$$\times e^{ik\cdot\xi} \langle 0 | \mathcal{U}^{++}_{(\xi)} | P_\Lambda, S_\Lambda ; X \rangle$$

$$\times \langle P_\Lambda, S_\Lambda ; X | \bar{\psi}(0) \mathcal{U}^{++}_{(0, +\infty)} | 0 \rangle |_{\xi^- = 0}.$$ 

Where the matrix element:

$$\langle P_\Lambda, S_\Lambda ; X | \bar{\psi}(0) | 0 \rangle$$

$$= \left\{ \begin{array}{ll} \bar{U}(P_\Lambda, S_\Lambda) \gamma_s \frac{i}{k-m_q} & \text{scalar diquark}, \\
\bar{U}(P_\Lambda, S_\Lambda) \gamma_{\mu} v \frac{i}{k-m_q} \varepsilon_{\mu} & \text{axial-vector diquark}. \end{array} \right.$$ 

The vertex structures of $\hat{\gamma}$:

$$\gamma_s = 1 g_s,$$

$$\gamma_{\mu} v = \frac{g_v}{\sqrt{3}} \gamma_5 \left( \gamma^\mu + \frac{P_\Lambda^\mu}{M_\Lambda} \right).$$
Applying the diquark model, the TMD FF $D_1$ is derived as

\[
D_1^{(s)}(z, z^2 k_T^2) = D_1^{(v)}(z, z^2 k_T^2) = \frac{g_D^2}{2(2\pi)^3} \frac{(1-z)[z^2 k_T^2 + (M_\Lambda + zm_q)^2]}{z^4 (k_T^2 + L^2)^2},
\]

Assuming SU(6) spin-flavor symmetry, the FFs of $\Lambda$ hyperon for light flavors satisfy the following relations between the different quark flavors and diquark types

\[
D^{u\rightarrow\Lambda} = D^{d\rightarrow\Lambda} = \frac{1}{4} D^{(s)} + \frac{3}{4} D^{(v)}, \quad D^{s\rightarrow\Lambda} = D^{(s)}.
\]

And we also choose $k^2$-dependent Gaussian form factor for coupling $g_D \rightarrow \frac{g_D}{z} e^{-k^2/\Lambda^2}$. 
The calculation of $D_1$

The analytic result for $D_1^\Lambda(z)$ is

$$D_1^\Lambda(z) = \frac{g_s^2}{4(2\pi)^2} \frac{e^{\frac{2m_D^2}{\Lambda^2}}}{z^4L^2} \left\{ z(1-z)((m_q + M_\Lambda)^2 - m_D^2) \right.$$  

$$\times \exp \left( \frac{-2zL^2}{(1-z)\Lambda^2} \right)$$  

$$+ ((1-z)\Lambda^2 - 2((m_q + M_\Lambda)^2 - m_D^2))$$  

$$\times \frac{z^2L^2}{\Lambda^2} \Gamma \left( 0, \frac{2zL^2}{(1-z)\Lambda^2} \right) \right\},$$

The incomplete gamma function has the form:

$$\Gamma(0, z) \equiv \int_z^\infty \frac{e^{-t}}{t} \, dt.$$  

And the $\Lambda^2$ has the general form $\Lambda^2 = \lambda^2 \, z^\alpha (1-z)^\beta$. 
The calculation of $D_1$

The fitted parameters are

$$g_D = 1.983, \quad m_D = 0.745 \text{ GeV}, \quad \lambda = 5.967 \text{ GeV},$$
$$\alpha = 0.5(\text{fixed}), \quad \beta = 0(\text{fixed}).$$

The DSV parameterization and model result for $D_1(z)$

The calculation of $G_1$

The polarized FF $G^\Lambda_1$ can be obtained from the following trace:

$$\frac{1}{4} \text{Tr}[(\Delta(z, k_T; S_\Lambda) - \Delta(z, k_T; -S_\Lambda))\gamma^5]$$

$$= S_\Lambda G_{1L} - \frac{k_T \cdot S_\Lambda T}{M_\Lambda} G_{1T},$$

We give at the following expression for $G_{1L}^D$ in diquark model:

$$G_{1L}^{(D)}(z, k_T^2) = -a_D \frac{g_D^2}{2(2\pi)^3} \frac{(1 - z)[z^2 k_T^2 - (zm_D + M_\Lambda)^2]}{z^4 (k_T^2 + L^2)^2}$$

The spin factor $a_D$ takes the values $a_s = 1$ and $a_v = -\frac{1}{3}$. So we can obtained the light flavors fragmentation function $G_{1L}$ as follows

$$G_{1L}^{u\rightarrow \Lambda}(z, k_T^2) = 0, \quad G_{1L}^{d\rightarrow \Lambda}(z, k_T^2) = 0,$$

$$G_{1L}^{s\rightarrow \Lambda}(z, k_T^2) = G_{1L}^{(s)}(z, k_T^2).$$
The calculation of $G_1$

The integrated fragmentation function $G_1^\Lambda(z)$ is defined as

$$G_1^\Lambda(z) = \pi z^2 \int_0^\infty dk_T^2 G_{1L}^\Lambda(z, z^2 k_T^2).$$

We take the same choice for the form factor as in the calculation of $D_1$ and give the expression of $G_1(z)$

$$G_1^\Lambda(z) = \frac{g_s^2}{4(2\pi)^2} \frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4 L^2} \left\{ (1 - z) \left[ M^2_{\Lambda}(2 - z) + 2zm_q M_{\Lambda} + z(m_D^2 + m_q^2(2z - 1)) \right] \exp \left( \frac{-2zL^2}{(1 - z)\Lambda^2} \right) 
- z \left[ 2M^2_{\Lambda}(2 - z) + 4zm_q M_{\Lambda} + z((1 - z)\Lambda^2 + 2(m_D^2 + m_q^2(2z - 1))) \right] \frac{L^2}{\Lambda^2} \Gamma \left( 0, \frac{2zL^2}{(1 - z)\Lambda^2} \right) \right\}.$$
The calculation of $G_1$

The comparison result between model result and scenario 1 of DSV
The T-odd TMD FF $D_{1T}^{\perp}$ describes the number density of a transversely polarized $\Lambda$ hyperon fragmented from an unpolarized quark and can be obtained from the following trace:

$$
\frac{\epsilon_T^\rho k_T^\rho S_{\Lambda T}^\sigma}{M_\Lambda} D_{1T}^{\perp}(z, k_T) = \frac{1}{4} \text{Tr}[(\Delta(z, k_T; S_{\Lambda T}) - \Delta(z, k_T; -S_{\Lambda T}))\gamma^\perp].
$$

As is well known, the nonzero contribution comes from the loop corrections. (The tree-level calculation cannot provide a contribution to T-odd fragmentation functions, because of the lack of final state interactions to produce imaginary phases in the scattering amplitude.)
At one loop level, there are four diagrams that can generate imaginary phases (the fragmentation of a quark into a Λ hyperon in the spectator model):

(a)

(b)

(c)

(d)
The gluon diquark vertex $\Gamma$:

$$\Gamma_s^{\rho,a} = igT^a (2k - 2P_\Lambda - l)^\rho,$$

$$\Gamma_v^{\rho,\mu\nu,a} = -igT^a [(2k - 2P_\Lambda - l)^\rho g_{\mu\nu} - (k - P_\Lambda - l)^\nu g^{\rho\mu} - (k - P_\Lambda)^\mu g^{\rho\nu}].$$

In the calculation of T-odd functions, we utilize the following Cutkosky cut rules to put the gluon and quark lines inside the loop on the mass shell to obtain the result. This corresponds to the following replacements

$$\frac{1}{l^2 + i\varepsilon} \rightarrow -2\pi i\delta(l^2), \quad \frac{1}{(k - l)^2 + i\varepsilon} \rightarrow -2\pi i\delta((k - l)^2).$$
We give the expressions by diquark model:

\[
D_{1T}^{±}(z, k_T^2) = \frac{\alpha_s g_s^2 C_F}{(2\pi)^4} \left( \frac{e^{-\frac{2k^2}{\Lambda^2}}}{z^2(1-z)} \right) \frac{1}{(k^2 - m^2)} \left( D_{1T(a)}^{±}(z, k_T^2) + D_{1T(b)}^{±}(z, k_T^2) + D_{1T(c)}^{±}(z, k_T^2) + D_{1T(d)}^{±}(z, k_T^2) \right),
\]

\[
D_{1T(a)}^{±}(z, k_T^2) = \frac{m_q M_\Lambda}{(k^2 - m^2)} \left( 3 - \frac{m_q^2}{k^2} \right) I_1,
\]

\[
D_{1T(b)}^{±}(z, k_T^2) = M_\Lambda \left\{ m_q (2I_2 - A) - M_\Lambda (B - 2I_2 + 2A) \right\},
\]

\[
D_{1T(c)}^{±}(z, k_T^2) = 0,
\]

\[
D_{1T(d)}^{±}(z, k_T^2) = \frac{M_\Lambda}{z} \left\{ 2(1-z)(m_q C P_\Lambda - M_\Lambda D P_\Lambda) - z(M_\Lambda B - m_q A) \right\}.
\]
T-odd FF $D_{1T}^\perp$:

$$D_{1T}^{\perp(v)}(z, k_T^2) = \frac{2\alpha s g_s^2 C_F}{(2\pi)^4} \frac{e^{-2k^2}}{z^2(1-z)} \frac{1}{M_A(k^2 - m_q^2)} \left( D_{1T}^{\perp(v)}(z, k_T^2) + D_{1T}^{\perp(v)}(z, k_T^2) + D_{1T}^{\perp(v)}(z, k_T^2) + D_{1T}^{\perp(v)}(z, k_T^2) \right),$$

$$D_{1T}^{\perp(a)}(z, k_T^2) = \frac{-m_q M_A}{(1-z)(k^2 - m_q^2)} \left( 1 - \frac{m_q^2}{3k^2} \right) I_1,$$

$$D_{1T}^{\perp(b)}(z, k_T^2) = \frac{1}{3(k^2 - m_q^2)} \left\{ 2M_A [m_q(I_2 - A) + M_A(A - I_2 - B)] + k \cdot P_A (4I_2 - 6A) 
- (Ak \cdot P_A + BP_A^2) + \frac{3}{2} \left( \frac{k^2 - m_q^2}{2k^2} I_1 + (k^2 - m_q^2)A \right) \right\},$$

$$D_{1T}^{\perp(c)}(z, k_T^2) = 0,$$

$$D_{1T}^{\perp(d)}(z, k_T^2) = \frac{-1}{3M_A(k^2 - m_q^2)} \left\{ [M_A((k^2 - m_q)^2)CP_A + 2M_A^2 DP_A 
- 2m_q M_A CP_A^-] + 2k \cdot P_A (m_q CP_A^- - M_A DP_A^-) + z \frac{m_q}{2} I_1 + \frac{(k^2 - m_q^2)}{2} (M_A DP_A^- - m_q CP_A^-) 
- M_A(m_q M_A A + 2k \cdot P_A A + M_A^2 B) - \frac{2M_A}{z} (m_q M_A CP_A^- + k \cdot P_A CP_A^-) \right\}. $$

Axial-vector diquark component
Here, $A, B, C$ and $D$ are functions of $k^2, m_q, m_D$ and $M_\Lambda$.

\[
A = \frac{I_1}{\lambda(M_\Lambda, m_D)} \times \left( 2k^2(k^2 - m_D^2) - M_\Lambda^2 \right) \frac{I_2}{\pi} + (k^2 + M_\Lambda^2 - m_D^2),
\]
\[
B = -\frac{2k^2}{\lambda(M_\Lambda, m_D)} I_1 \left( 1 + \frac{k^2 + m_D^2 - M_\Lambda^2}{\pi} I_2 \right),
\]
\[
CP^-_\Lambda = \frac{I_{34}}{2k_T^2} + \frac{1}{2z k_T^2} (-z k^2 + (2 - z) M_\Lambda^2 + z m_D^2) I_2,
\]
\[
DP^-_\Lambda = \frac{-I_{34}}{2z k_T^2} - \frac{1}{2z k_T^2} ((1 - 2z) k^2 + M_\Lambda^2 - m_D^2) I_2.
\]
The FF $D_{1T}^{1/2}$ follows the SU(6) spin-flavor symmetry

$$D_{1T}^{1/2} = D_{1T}^{d} = \frac{1}{4} D_{1T}^{(s)} + \frac{3}{4} D_{1T}^{(v)}, \quad D_{1T}^{s} = D_{1T}^{(s)}.$$

We also check the positivity bound in our model

$$2D_{1T}^{(1/2)}(z) \leq D_{1}(z).$$

The half $k_T$-moment of $D_{1T}^{1/2}$ is defined as:

$$D_{1T}^{(1/2)}(z) = z^2 \int dk_T^2 \frac{|k_T|}{2M_A} D_{1T}^{1/2}(z, k_T^2).$$
T-odd FF $D_{1T}^{\perp}$

We plot the numerical results of the half $k_T$-moment of $D_{1T}^{\perp}$ (multiplied by a factor of 2)
In our model, the axial-vector diquark model is the dominant contribution to $D_{1T}^\perp$

- The size of ud quark is negative and around several percent.
- The size of s quark is consistent with zero.

The model calculation of $D_{1T}^\perp$ for ud quarks does not always satisfy the positivity bound

- The bound is violated at the large z region ($z \geq 0.75$).
- The violation may arise from the fact that T-odd TMD FFs are evaluated to $O(\alpha_s)$, while the T-even TMD functions are usually truncated at the lowest order.
We apply the model result to predict the transverse $\Lambda$ polarization:

$$P_T^\Lambda = \frac{d\Delta \sigma}{d\sigma} = \frac{[d\sigma(S_{\Lambda T}) - d\sigma(-S_{\Lambda T})]}{[d\sigma(S_{\Lambda T}) + d\sigma(-S_{\Lambda T})]}.$$ 

In SIDIS:

$$P_T^\Lambda(x, y, z, P_T)|_{DIS} = \frac{\sum q e_q^2 f_{q/p}(x)[d\sigma^{q \ell q}/dy] \Delta D_{\Lambda^+/q}(z, P_T^2)}{\sum q e_q^2 f_{q/p}(x)[d\sigma^{q \ell q}/dy] D_1^{q \rightarrow \Lambda}(z, P_T^2)},$$

In SIA (single inclusive $e^+e^-$ annihilation):

$$P_T^\Lambda(y, z, P_T)|_{SIA} = \frac{\sum q e_q^2 [d\sigma^{\ell q}/dy] \Delta D_{\Lambda^+/q}(z, P_T^2)}{\sum q e_q^2 [d\sigma^{\ell q}/dy] D_1^{q \rightarrow \Lambda}(z, P_T^2)}.$$
Transverse $\Lambda$ polarization

Using the SU(3) flavor symmetry for unpolarized $\Lambda$ FFs and ignoring the sea quarks’ contribution, we obtain the approximate result of $P_T$

\[
P_T^{\Lambda}|_{\text{DIS}} \approx \frac{\Delta D_{\Lambda^u/u}}{D_1^{u\to\Lambda}}.
\]

\[
P_T^{\Lambda}|_{\text{SIA}} = \frac{5\Delta D_{\Lambda^u/u} + \Delta D_{\Lambda^s/s}}{5D_1^{u\to\Lambda} + D_1^{s\to\Lambda}}.
\]

- We present the result in the region $z \leq 0.75$.
- The numerical results show that $P_T$ is negative in both SIDIS and SIA.

Approximate result of $P_T$
In this work, we studied the T-odd transversely polarized fragmentation function $D_{1T}^\perp$ for the process $q \to \Lambda^\uparrow + X$

- We performed the calculation of $D_1$, $G_1$, $D_{1T}^\perp$ of $\Lambda$ hyperon in the diquark spectator model, and we used the relation between the quark flavors and diquark types for fragmentation functions.

- We obtained the values of the model parameters, by fitting the model result $D_1(z)$ to the DSV parametrization at the initial scale $\mu_0^2 = 0.23$ GeV$^2$.

- Using our numerical result of $D_{1T}^\perp$, we estimated the transverse polarization $P_T$ in both SIDIS and SIA, and found that in these two processes the polarizations are negative and substantial in the large $z$ region.
Summary

Some comments

- In our model the flavor dependence of the fragmentation functions was obtained based on the assumption of SU(6) symmetry of the octet baryons.

- In the calculation of $P_T$, we only considered the leading order result, and assumed that the evolution for $D_{1T}^\perp$ is equal to that of $D_1$.

- We note that SU(6) symmetry breaking, higher order corrections and evolution effects for $D_{1T}^\perp$ may alter the results only quantitatively, but we fully expect that they will not change qualitatively.
Thanks for your attention!