Production of transversely polarized Λ hyperon from unpolarized quark fragmentation in the diquark model

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Y.L. Yang, Z. Lu and I. Schmidt, Phys. Rev. D ***96,*** *034010(2017)*

Outline

- Introduction
- The model calculation of D_1 and G_1
- The model calculation of $D_{1T}^{\perp}$ and transverse polarization P_T
- Summary

Introduction

- The production of a polarized Λ date from the 1970s in *pp* collisions.

  A. Lesnik et al., Phys. Rev. Lett. 35, 770 (1975).
  G. Bunce et al., Phys. Rev413. Lett. 36, 1113 (1976).

- The production of transversely polarized Λ hyperon can provide further information about the spin structure of the Λ hyperon and the non-perturbative hadronization mechanism.

- A class of the so-called time-reversal-odd (T-odd) fragmentation functions has been the main focus on understanding the fragmentation mechanism behind the Λ polarization, such as $D_{1T}^{\perp}$.

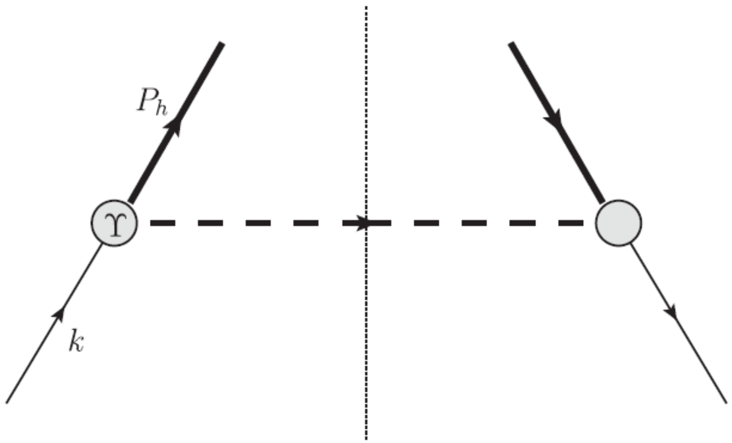
- Since the experimental information on $D_{1T}^{\perp}$ of the Λ hyperon still remains unknown, the model calculations will provide an approach to acquire knowledge of this quantity.

- The spectator model has been applied to calculate the Collins fragmentation function of pion and kaon mesons.

  A. Bacchetta et al., Phys. Lett. B 659, 234(2008)

The calculation of D_1

Lowest order diagram describing the fragmentation of a quark to a hyperon (taking u quark as an example), as $u \rightarrow \Lambda(uds) + D(\bar{d}\bar{s})$:



The unpolarized TMD FF $D_1^{\Lambda}(z, k_T)$ is obtained from

$$D_1^{\Lambda}(z, k_T) = \frac{1}{4}\mathrm{Tr}[(\Delta(z, k_T; S_\Lambda) + \Delta(z, k_T; -S_\Lambda))\gamma^-],$$

The calculation of D_1

The quark-quark correlation function:

$$\Delta(z, k_T; S_\Lambda) = \frac{1}{2z}\int dk^+ \Delta(k, P_\Lambda; S_\Lambda)$$

$$\equiv \sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{2z(2\pi)^3}$$

$$\times\, e^{ik\cdot\xi}\langle 0|\mathcal{U}^{n^+}_{(+\infty,\xi)}\psi(\xi)|P_\Lambda, S_\Lambda; X\rangle$$

$$\times\, \langle P_\Lambda, S_\Lambda; X|\bar{\psi}(0)\mathcal{U}^{n^+}_{(0,+\infty)}|0\rangle|_{\xi^-=0}.$$

Where the matrix element:

$$\langle P_\Lambda, S_\Lambda; X|\bar{\psi}(0)|0\rangle$$

$$= \begin{cases} \bar{U}(P_\Lambda, S_\Lambda)\Upsilon_s \frac{i}{\not{k}-m_q} & \text{scalar diquark,} \\ \bar{U}(P_\Lambda, S_\Lambda)\Upsilon_v^\mu \frac{i}{\not{k}-m_q}\varepsilon_\mu & \text{axial-vector diquark.} \end{cases}$$

The vertex structures of Υ:

$$\Upsilon_s = \mathbf{1} g_s, \qquad \Upsilon_v^\mu = \frac{g_v}{\sqrt{3}}\gamma_5\left(\gamma^\mu + \frac{P_\Lambda^\mu}{M_\Lambda}\right)$$

Diquark model

The calculation of D_1

Applying the diquark model, the TMD FF D_1 is derived as

$$D_1^{(s)}(z, z^2\boldsymbol{k}_T^2) = D_1^{(v)}(z, z^2\boldsymbol{k}_T^2) = \frac{g_D^2}{2(2\pi)^3}\frac{(1-z)[z^2\boldsymbol{k}_T^2 + (M_\Lambda + zm_q)^2]}{z^4(k_T^2 + L^2)^2},$$

Assuming SU(6) spin-flavor symmetry, the FFs of Λ hyperon for light flavors satisfy the following relations between the different quark flavors and diquark types

$$D^{\mathrm{u}\to\Lambda} = D^{\mathrm{d}\to\Lambda} = \frac{1}{4}D^{(s)} + \frac{3}{4}D^{(v)}, \qquad D^{\mathrm{s}\to\Lambda} = D^{(s)},$$

And we also choose k^2-dependent Gaussian form factor for coupling $g_D \to \frac{g_D}{z} e^{-k^2/\Lambda^2}$

Model result

The calculation of D_1

The analytic result for $D_1^\Lambda(z)$ is

$$D_1^\Lambda(z) = \frac{g_s^2}{4(2\pi)^2} \frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4 L^2} \Big\{ z(1-z)((m_q + M_\Lambda)^2 - m_D^2) \\ \times \exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right) \\ + ((1-z)\Lambda^2 - 2((m_q + M_\Lambda)^2 - m_D^2)) \\ \times \frac{z^2 L^2}{\Lambda^2} \Gamma\left(0, \frac{2zL^2}{(1-z)\Lambda^2}\right) \Big\},$$

The incomplete gamma function has the form:

$$\Gamma(0, z) \equiv \int_z^\infty \frac{e^{-t}}{t} dt.$$

And the Λ^2 has the general form $\Lambda^2 = \lambda^2 \, z^\alpha (1-z)^\beta$.

Model result

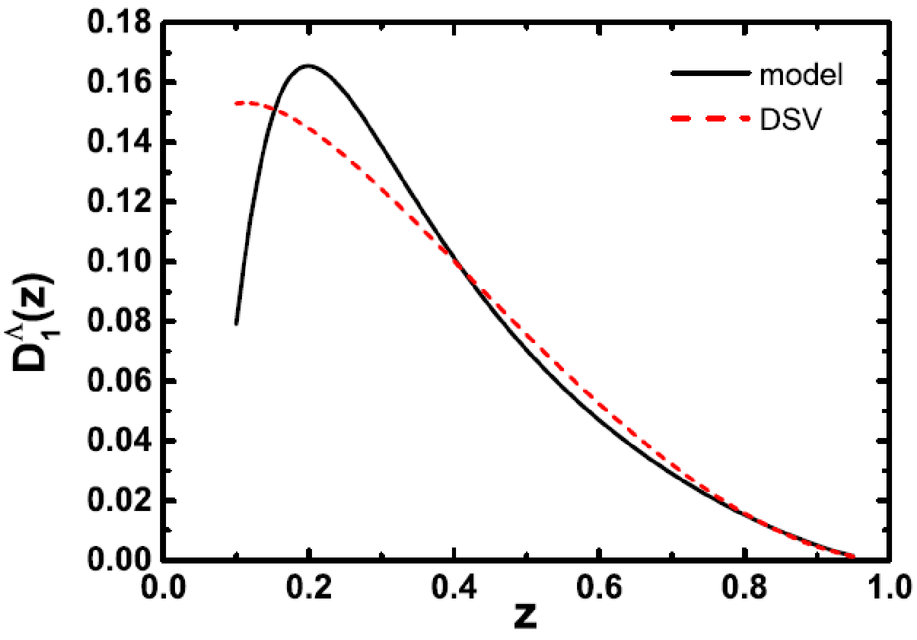
The calculation of D_1

The fitted parameters are

$$g_D = 1.983, \qquad m_D = 0.745 \text{ GeV}, \qquad \lambda = 5.967 \text{ GeV},$$
$$\alpha = 0.5(\text{fixed}), \qquad \beta = 0(\text{fixed}).$$

The DSV parameterization and model result for $D_1(z)$

D. de Florian et al., Phys. Rev. D 57, 5811(1998).



Fitted parameters

The calculation of G_1

The polarized FF G_1^Λ can be obtained from the following trace:

$$\frac{1}{4}\mathrm{Tr}[(\Delta(z,k_T;S_\Lambda)-\Delta(z,k_T;-S_\Lambda))\gamma^-\gamma_5]$$
$$=S_{\Lambda L}G_{1L}-\frac{k_T\cdot S_{\Lambda T}}{M_\Lambda}G_{1T},$$

We give at the following expression for G_{1L} in diquark model:

$$G_{1L}^{(D)}(z,k_T^2)=-a_D\frac{g_D^2}{2(2\pi)^3}\frac{(1-z)[z^2k_T^2-(zm_D+M_\Lambda)^2]}{z^4(k_T^2+L^2)^2}$$

The spin factor a_D takes the values $a_s=1$ and $a_v=-\frac{1}{3}$. So we can obtained the light flavors fragmentation function G_{1L} as follows

$$G_{1L}^{\mathrm{u}\to\Lambda}(z,k_T^2)=0,\qquad G_{1L}^{\mathrm{d}\to\Lambda}(z,k_T^2)=0,$$
$$G_{1L}^{\mathrm{s}\to\Lambda}(z,k_T^2)=G_{1L}^{(s)}(z,k_T^2).$$

The result of G_1

The calculation of G_1

The integrated fragmentation function $G_1^{\Lambda}(z)$ is defined as

$$G_1^{\Lambda}(z) = \pi z^2 \int_0^{\infty} dk_T^2 G_{1L}^{s\to\Lambda}(z, z^2\boldsymbol{k}_T^2).$$

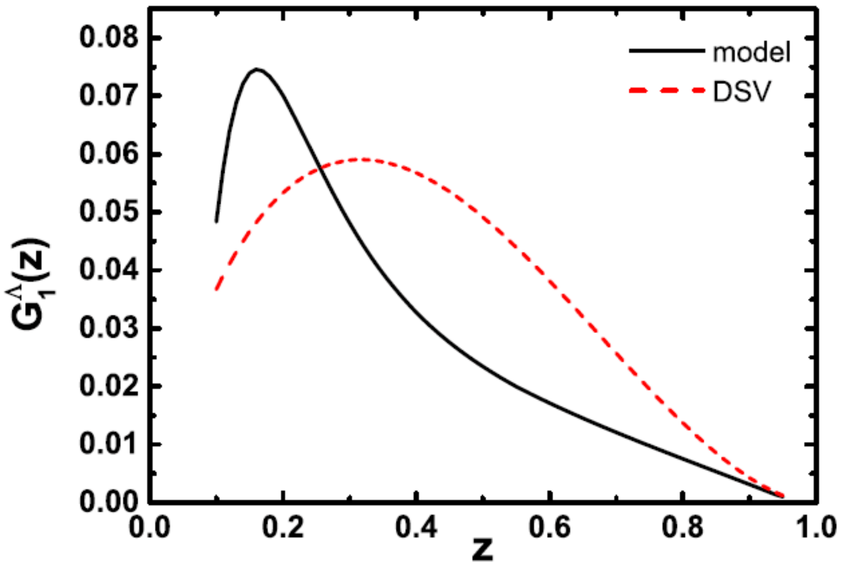
We take the same choice for the form factor as in the calculation of D_1 and give the expression of $G_1(z)$

$$\begin{aligned} G_1^{\Lambda}(z) = &\frac{g_s^2}{4(2\pi)^2}\frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4L^2}\Big\{(1-z)\big[M_\Lambda^2(2-z)+2z\,m_qM_\Lambda+z(m_D^2+m_q^2(2z-1))\big]\exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right)\\ &-z[2M_\Lambda^2(2-z)+4z\,m_qM_\Lambda+z((1-z)\Lambda^2+2(m_D^2+m_q^2(2z-1)))]\frac{L^2}{\Lambda^2}\Gamma\left(0,\frac{2zL^2}{(1-z)\Lambda^2}\right)\Big\}. \end{aligned}$$

The result of G_1

The calculation of G_1

The comparison result between model result and scenario 1 of DSV



The result of G_1

T-odd FF $D_{1T}^{\perp}$

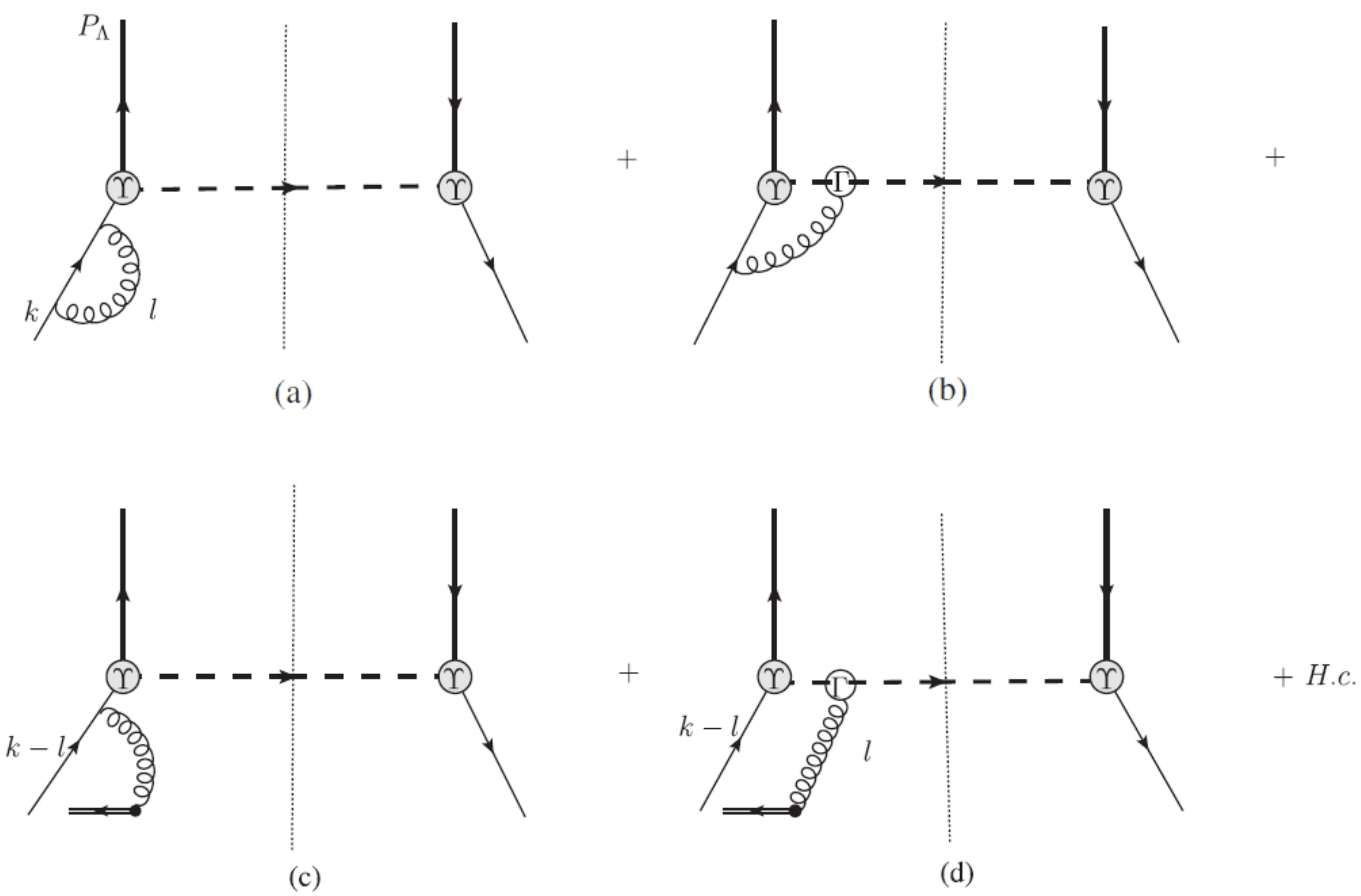
The T-odd TMD FF $D_{1T}^{\perp}$ describes the number density of a transversely polarized Λ hyperon fragmented from an unpolarized quark and can be obtained from the following trace:

$$\frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{\Lambda T\sigma}}{M_\Lambda} D_{1T}^{\perp}(z, k_T) = \frac{1}{4} \mathrm{Tr}[(\Delta(z, k_T; S_{\Lambda T}) - \Delta(z, k_T; -S_{\Lambda T}))\gamma^-].$$

As is well known, the nonzero contribution comes from the loop corrections. (The tree-level calculation cannot provide a contribution to T-odd fragmentation functions, because of the lack of final state interactions to produce imaginary phases in the scattering amplitude.)

T-odd FF $D_{1T}^{\perp}$

At one loop level, there are four diagrams that can generate imaginary phases (the fragmentation of a quark into a Λ hyperon in the spectator model):



T-odd FF $D_{1T}^{\perp}$

The gluon diquark vertex Γ:

$$\Gamma_s^{\rho,a} = igT^a(2k - 2P_\Lambda - l)^\rho,$$

$$\begin{aligned}\Gamma_v^{\rho,\mu\nu,a} = &-igT^a[(2k - 2P_\Lambda - l)^\rho g^{\mu\nu} \\ &- (k - P_\Lambda - l)^\nu g^{\rho\mu} - (k - P_\Lambda)^\mu g^{\rho\nu}].\end{aligned}$$

In the calculation of T-odd functions, we utilize the following Cutkosky cut rules to put the gluon and quark lines inside the loop on the mass shell to obtain the result. This corresponds the following replacements

$$\frac{1}{l^2 + i\varepsilon} \to -2\pi i\delta(l^2), \qquad \frac{1}{(k-l)^2 + i\varepsilon} \to -2\pi i\delta((k-l)^2)\,.$$

T-odd FF $D_{1T}^{\perp}$

We give the expressions by diquark model:

$$D_{1T}^{\perp(s)}(z,k_T^2) = \frac{\alpha_s g_s^2 C_F}{(2\pi)^4} \frac{e^{\frac{-2k^2}{\Lambda^2}}}{z^2(1-z)} \frac{1}{(k^2-m^2)} \left(D_{1T(a)}^{\perp(s)}(z,k_T^2) + D_{1T(b)}^{\perp(s)}(z,k_T^2) + D_{1T(c)}^{\perp(s)}(z,k_T^2) + D_{1T(d)}^{\perp(s)}(z,k_T^2) \right) ,$$

$$D_{1T(a)}^{\perp(s)}(z,k_T^2) = \frac{m_q M_\Lambda}{(k^2-m_q^2)}(3-\frac{m_q^2}{k^2})\, I_1 \,,$$

$$D_{1T(b)}^{\perp(s)}(z,k_T^2) = M_\Lambda \Big\{ m_q(2I_2-\mathcal{A}) - M_\Lambda(\mathcal{B}-2I_2+2\mathcal{A}) \Big\} ,$$

$$D_{1T(c)}^{\perp(s)}(z,k_T^2) = 0 \,,$$

$$D_{1T(d)}^{\perp(s)}(z,k_T^2) = \frac{M_\Lambda}{z} \Big\{ 2(1-z)(m_q \mathcal{C} P_\Lambda^- - M_\Lambda \mathcal{D} P_\Lambda^-) - z(M_\Lambda \mathcal{B} - m_q \mathcal{A}) \Big\} .$$

Scalar diquark component

T-odd FF $D_{1T}^{\perp}$

$$D_{1T}^{\perp(v)}(z,k_T^2) = \frac{2\alpha_s g_s^2 C_F}{(2\pi)^4}\,\frac{e^{\frac{-2k^2}{\Lambda^2}}}{z^2(1-z)}\,\frac{1}{M_\Lambda(k^2-m_q^2)}\left(D_{1T(v)}^{\perp(v)}(z,k_T^2)+D_{1T(b)}^{\perp(v)}(z,k_T^2)+D_{1T(c)}^{\perp(v)}(z,k_T^2)+D_{1T(d)}^{\perp(v)}(z,k_T^2)\right),$$

$$D_{1T(a)}^{\perp(v)}(z,k_T^2) = \frac{-m_q M_\Lambda}{(1-z)(k^2-m_q^2)}\left(1-\frac{m_q^2}{3k^2}\right) I_1\,,$$

$$D_{1T(b)}^{\perp(v)}(z,k_T^2) = \frac{1}{3(k^2-m_q^2)}\Big\{2M_\Lambda[m_q(I_2-\mathcal{A})+M_\Lambda(\mathcal{A}-I_2-\mathcal{B})]+k\cdot P_\Lambda(4I_2-6\mathcal{A})$$
$$-(\mathcal{A}k\cdot P_\Lambda+\mathcal{B}P_\Lambda^2)+\frac{3}{2}\left(\frac{k^2-m_q^2}{2k^2}I_1+(k^2-m_q^2)\mathcal{A}\right)\Big\},$$

$$D_{1T(c)}^{\perp(v)}(z,k_T^2) = 0\,,$$

$$D_{1T(d)}^{\perp(v)}(z,k_T^2) = \frac{-1}{3M_\Lambda(k^2-m_q^2)}\Big\{[M_\Lambda\big((k^2-m_q^2)\mathcal{C}P_\Lambda^- + 2M_\Lambda^2\mathcal{D}P_\Lambda^-$$
$$-2m_qM_\Lambda\mathcal{C}P_\Lambda^-\big)+2k\cdot P_\Lambda(m_q\,\mathcal{C}P_\Lambda^- - M_\Lambda\,\mathcal{D}P_\Lambda^-)+z\frac{m_q}{2}I_1+\frac{(k^2-m_q^2)}{2}(M_\Lambda\mathcal{D}P_\Lambda^- - m_q\mathcal{C}P_\Lambda^-)]$$
$$-M_\Lambda(m_qM_\Lambda\,\mathcal{A}+2k\cdot P_\Lambda\mathcal{A}+M_\Lambda^2\mathcal{B})-\frac{2M_\Lambda}{z}(m_q\,M_\Lambda\mathcal{C}P_\Lambda^- + k\cdot P_\Lambda\,\mathcal{C}P_\Lambda^-)\Big\}.$$

Axial-vector diquark component

T-odd FF $D_{1T}^{\perp}$

Here, $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are functions of k^2, m_q, m_D and M_Λ.

$$\mathcal{A} = \frac{I_1}{\lambda(M_\Lambda, m_D)} \times \left(2k^2(k^2 - m_D^2 - M_\Lambda^2)\frac{I_2}{\pi} + (k^2 + M_\Lambda^2 - m_D^2)\right),$$

$$\mathcal{B} = -\frac{2k^2}{\lambda(M_\Lambda, m_D)} I_1 \left(1 + \frac{k^2 + m_D^2 - M_\Lambda^2}{\pi} I_2\right),$$

$$\mathcal{C}P_\Lambda^- = \frac{I_{34}}{2k_T^2} + \frac{1}{2zk_T^2}(-zk^2 + (2 - z)M_\Lambda^2 + zm_D^2)I_2,$$

$$\mathcal{D}P_\Lambda^- = \frac{-I_{34}}{2zk_T^2} - \frac{1}{2zk_T^2}((1 - 2z)k^2 + M_\Lambda^2 - m_D^2)I_2.$$

Axial-vector diquark component

T-odd FF $D_{1T}^{\perp}$

The FF $D_{1T}^{\perp}$ follows the SU(6) spin-flavor symmetry

$$D_{1T}^{\perp u} = D_{1T}^{\perp d} = \frac{1}{4} D_{1T}^{\perp(s)} + \frac{3}{4} D_{1T}^{\perp(v)}, \qquad D_{1T}^{\perp s} = D_{1T}^{\perp(s)}.$$

We also check the positivity bound in our model

$$2D_{1T}^{\perp(1/2)}(z) \leq D_1(z).$$

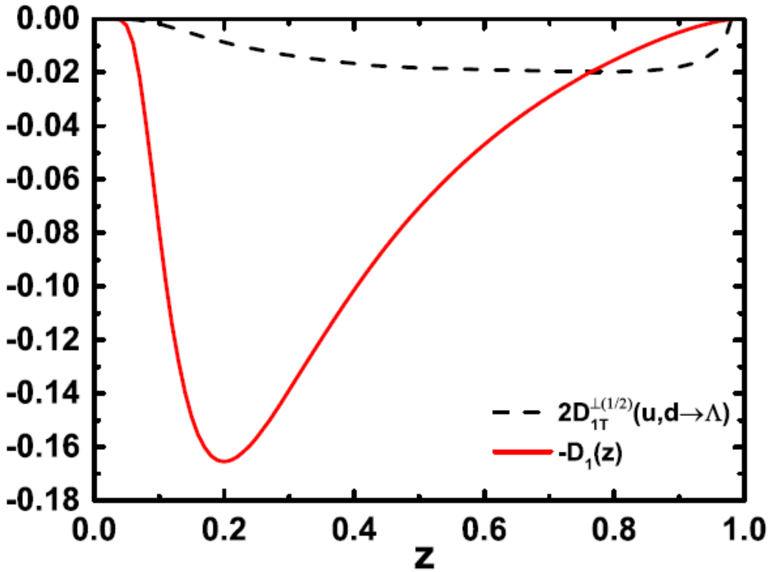
The half k_T-moment of $D_{1T}^{\perp}$ is defined as:

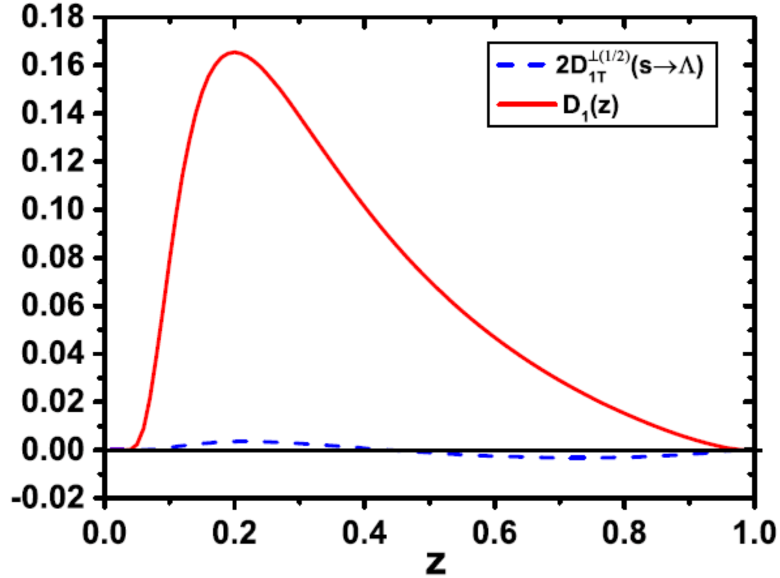
$$D_{1T}^{\perp(1/2)}(z) = z^2 \int dk_T^2 \frac{|\boldsymbol{k}_T|}{2M_\Lambda} D_{1T}^{\perp}(z, \boldsymbol{k}_T^2).$$

Positivity bound

T-odd FF $D_{1T}^{\perp}$

We plot the numerical results of the half k_T-moment of $D_{1T}^{\perp}$ (multiplied by a factor of 2)





Positivity bound

T-odd FF $D_{1T}^{\perp}$

In our model, the axial-vector diquark model is the dominant contribution to $D_{1T}^{\perp}$

- The size of ud quark is negative and around several percent.
- The size of s quark is consistent with zero.

The model calculation of $D_{1T}^{\perp}$ for ud quarks does not always satisfy the positivity bound

- The bound is violated at the large z region ($z \geq 0.75$).
- The violation may arise from the fact that T-odd TMD FFs are evaluated to $O(\alpha_s)$, while the T-even TMD functions are usually truncated at the lowest order.

Comparison

Transverse Λ polarization

We apply the model result to predict the transverse Λ polarization:

M. Anselmino et al., Phys. Rev. D 65, 114014 (2002).
D. Boer et al., Phys. Rev. Lett. 105, 202001 (2010).

$$P_T^\Lambda = \frac{d\Delta\sigma}{d\sigma} = \frac{[d\sigma(S_{\Lambda T}) - d\sigma(-S_{\Lambda T})]}{[d\sigma(S_{\Lambda T}) + d\sigma(-S_{\Lambda T})]}.$$

In SIDIS:

$$P_T^\Lambda(x,y,z,P_T)|_{\mathrm{DIS}} = \frac{\sum_q e_q^2 f_{q/p}(x)[d\sigma^{\ell q}/dy]\Delta D_{\Lambda^\uparrow/q}(z,\boldsymbol{P}_T^2)}{\sum_q e_q^2 f_{q/p}(x)[d\sigma^{\ell q}/dy] D_1^{q\to\Lambda}(z,\boldsymbol{P}_T^2)},$$

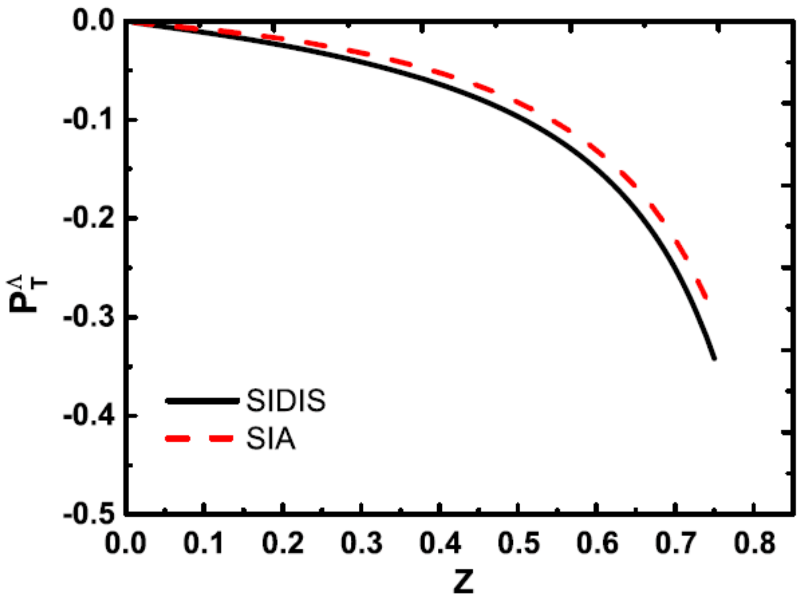
In SIA(single inclusive e^+e^- annihilation):

$$P_T^\Lambda(y,z,P_T)|_{\mathrm{SIA}} = \frac{\sum_q e_q^2 [d\sigma^{e^+e^-}/dy]\Delta D_{\Lambda^\uparrow/q}(z,\boldsymbol{P}_T^2)}{\sum_q e_q^2 [d\sigma^{e^+e^-}/dy] D_1^{q\to\Lambda}(z,\boldsymbol{P}_T^2)}.$$

Transverse Λ polarization

Using the SU(3) flavor symmetry for unpolarized Λ FFs and ignoring the sea quarks' contribution, we obtain the approximate result of P_T

$$P_T^\Lambda|_{\mathrm{DIS}} \approx \frac{\Delta D_{\Lambda^\uparrow/\mathrm{u}}}{D_1^{\mathrm{u}\to\Lambda}}. \qquad P_T^\Lambda|_{\mathrm{SIA}} = \frac{5\Delta D_{\Lambda^\uparrow/\mathrm{u}} + \Delta D_{\Lambda^\uparrow/\mathrm{s}}}{5D_1^{\mathrm{u}\to\Lambda} + D_1^{\mathrm{s}\to\Lambda}}.$$



- We present the result in the region $z \leq 0.75$.
- The numerical results show that P_T is negative in both SIDIS and SIA.

Approximate result of P_T

Summary

In this work, we studied the T-odd transversely polarized fragmentation function $D_{1T}^{\perp}$ for the process q $\longrightarrow \Lambda^{\uparrow}$ +X

- We performed the calculation of D_1, G_1, $D_{1T}^{\perp}$ of Λ hyperon in the diquark spectator model, and we used the relation between the quark flavors and diquark types for fragmentation functions.

- We obtained the values of the model parameters, by fitting the model result $D_1(z)$ to the DSV parametrization at the initial scale $\mu_0^2 = 0.23\ \mathrm{GeV}^2$.

- Using our numerical result of $D_{1T}^{\perp}$, we estimated the transverse polarization P_T in both SIDIS and SIA, and found that in these two processes the polarizations are negative and substantial in the large z region.

Summary

Some comments

- In our model the flavor dependence of the fragmentation functions was obtained based on the assumption of SU(6) symmetry of the octet baryons
- In the calculation of P_T, we only considered the leading order result, and assumed that the evolution for $D_{1T}^{\perp}$ is equal to that of D_1.
- We note that SU(6) symmetry breaking, higher order corrections and evolution effects for $D_{1T}^{\perp}$ may alter the results only quantitatively, but we fully expect that they will not change qualitatively.

Thanks for your attention!