

α_s review (2018)

DIS 2018

Kobe (Japan), 18th April 2018

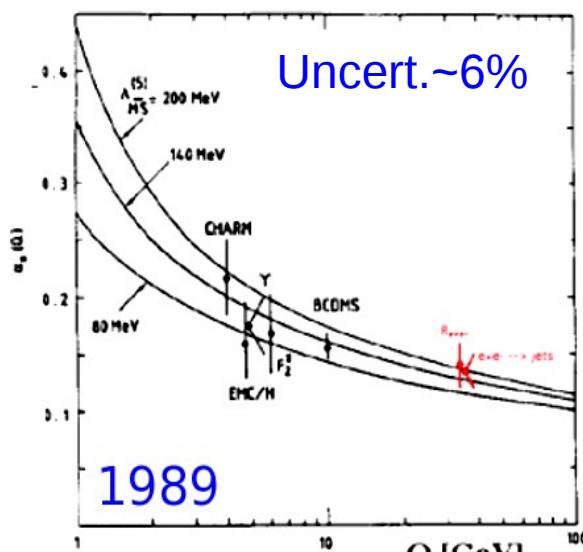
David d'Enterria

CERN

Mostly based on: *D. d'Enterria, P.Z. Skands (eds.), Proceedings.
"High-precision α_s from LHC to FCC-ee", CERN Oct. 2015;
[arXiv:1512.05194](https://arxiv.org/abs/1512.05194) (plus 2017, 2018 updates)*

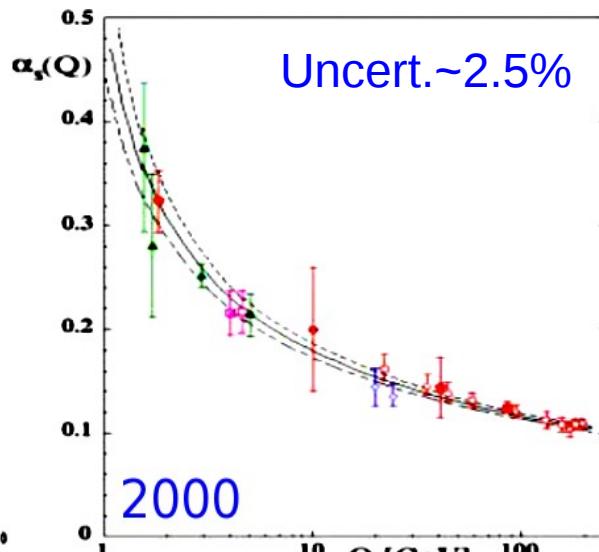
QCD coupling α_s

- Determines **strength of the strong interaction** between quarks & gluons.
- Single free parameter in QCD in the $m_q \rightarrow 0$ limit.
- Determined at a ref. scale ($Q=m_Z$), decreases as $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$, $\Lambda \sim 0.2$ GeV



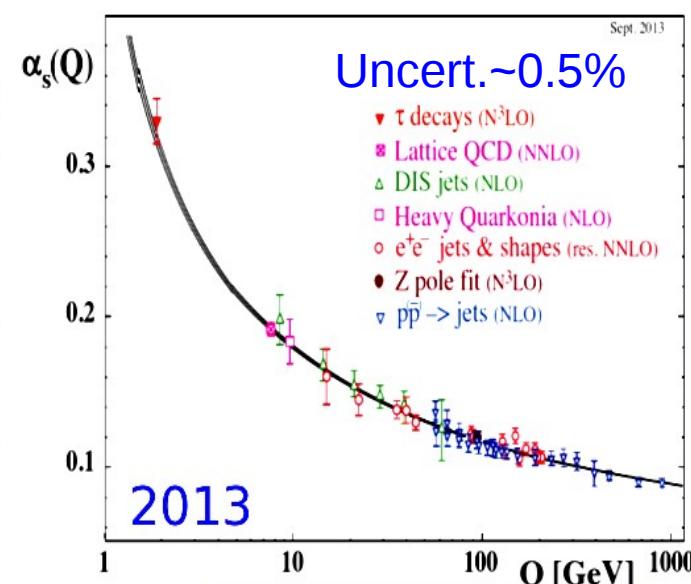
$$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008} \text{ (NLO)}$$

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$$\alpha_s(M_Z) = 0.1184 \pm 0.0031 \text{ (NNLO)}$$

S. B., J. Phys. G 26, 2000

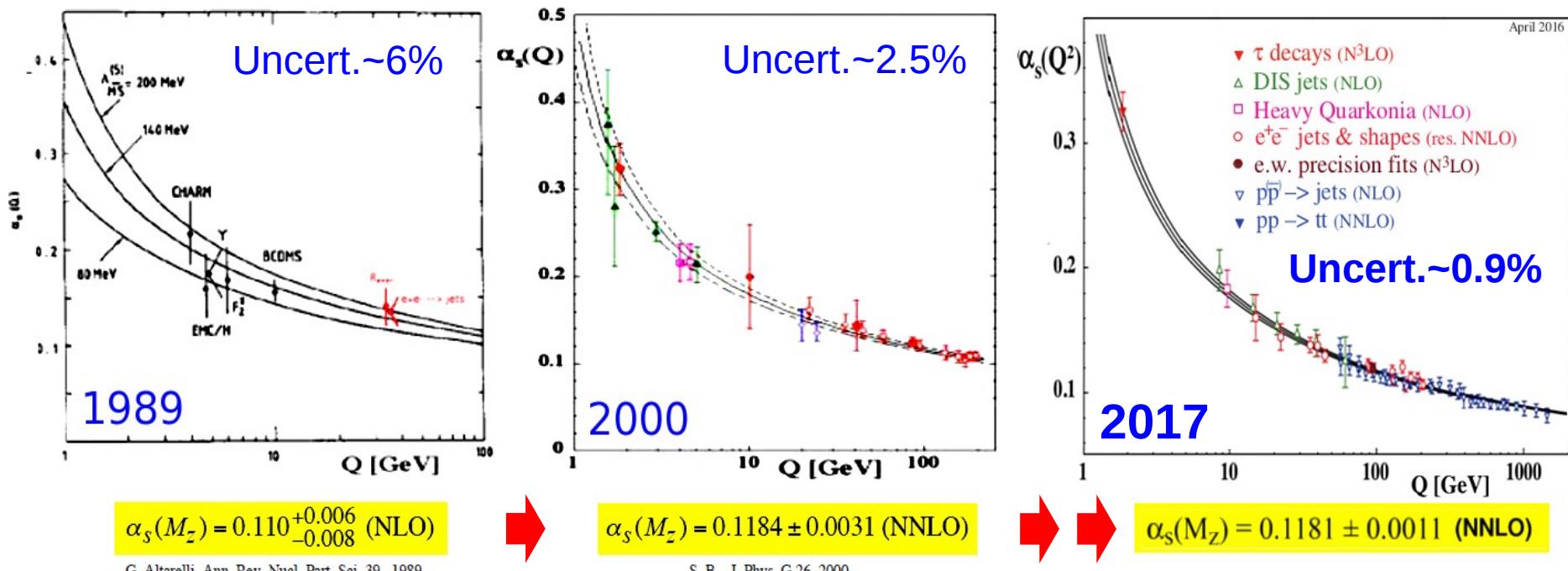


$$\alpha_s(M_Z) = 0.1185 \pm 0.0006 \text{ (NNLO)}$$

David d'Enterria (CERN)

QCD coupling α_s

- Determines **strength of the strong interaction** between quarks & gluons.
- Single free parameter in QCD in the $m_q \rightarrow 0$ limit.
- Determined at a ref. scale ($Q=m_Z$), decreases as $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$, $\Lambda \sim 0.2$ GeV



► Least precisely known of all interaction **couplings** !

$$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$$

Importance of the QCD coupling α_s

→ Impacts all QCD x-sections & decays (H), precision top & parametric EWPO:

Process	σ (pb)	$\delta\alpha_s$ (%)	PDF + α_s (%)	Scale(%)
ggH	49.87	± 3.7	-6.2 +7.4	-2.61 + 0.32
tH	0.611	± 3.0	± 8.9	-9.3 + 5.9

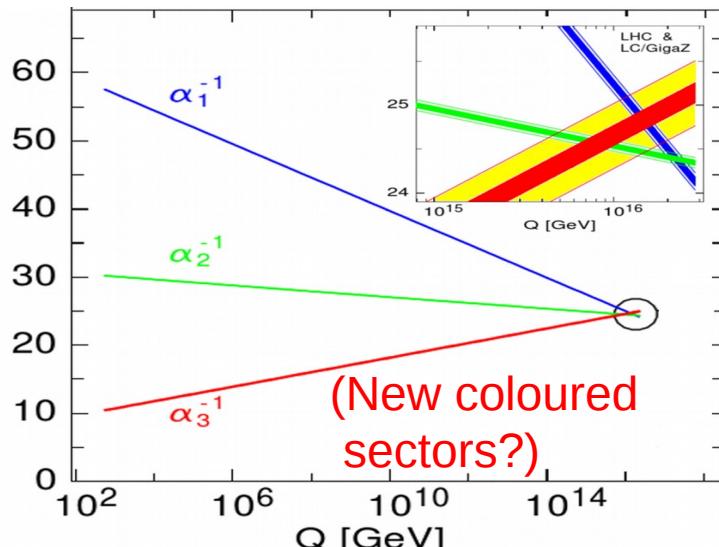
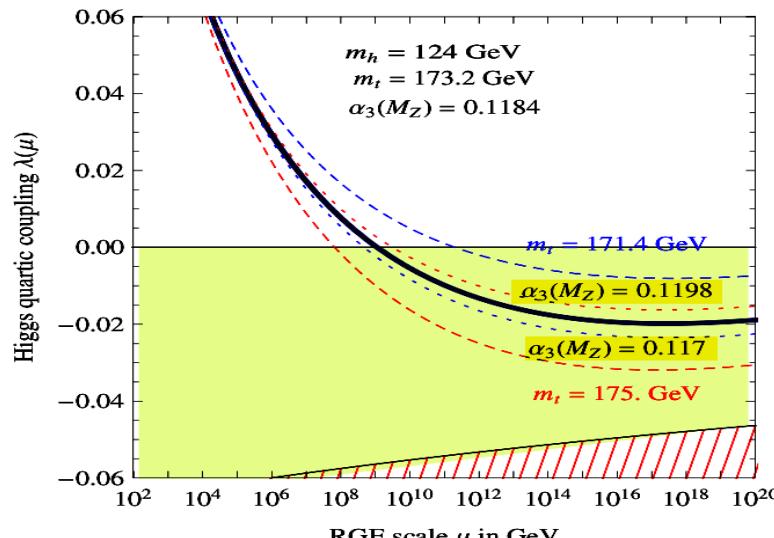
Channel	M_H [GeV]	$\delta\alpha_s$ (%)	Δm_b	Δm_c
$H \rightarrow c\bar{c}$	126	± 7.1	$\pm 0.1\%$	$\pm 2.3\%$
$H \rightarrow gg$	126	± 4.1	$\pm 0.1\%$	$\pm 0\%$

$(\delta M_t^{\text{SD-low}})^{\text{exp}}$	$(\delta M_t^{\text{SD-low}})^{\text{theo}}$	$(\delta \bar{m}_t(\bar{m}_t))^{\text{conversion}}$	$(\delta \bar{m}_t(\bar{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV
⇒ improvement in α_s crucial			$\delta\alpha_s(M_z) = 0.001$

Quantity	FCC-ee	future param.unc.	Main source
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$
R_b [10^{-5}]	6	< 1	$\delta\alpha_s$
R_ℓ [10^{-3}]	1	1.3	$\delta\alpha_s$

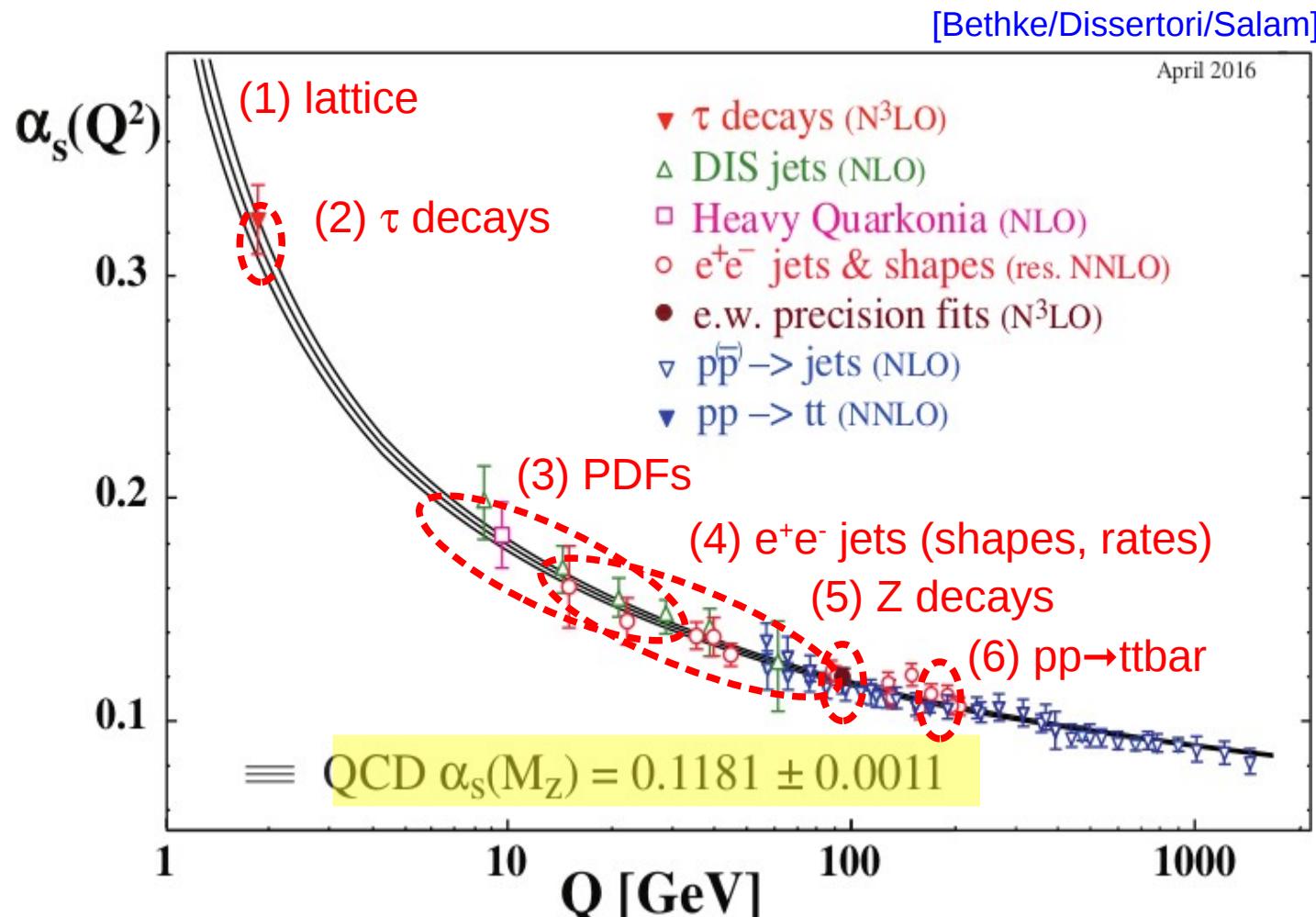
Sven Heinemeyer – 1st FCC physics workshop, CERN, 17.01.2017

→ Impacts physics approaching Planck scale: EW vacuum stability, GUT



World α_s determination (PDG 2017)

- Determined today by comparing 6 experimental observables to pQCD NNLO,N³LO predictions, plus global average at the Z pole scale:



(1) α_s from lattice QCD

- Comparison of short-distance quantities (Wilson loops, q static potential, vacuum polariz,...) computed at NNLO in pQCD, to lattice QCD “data”:

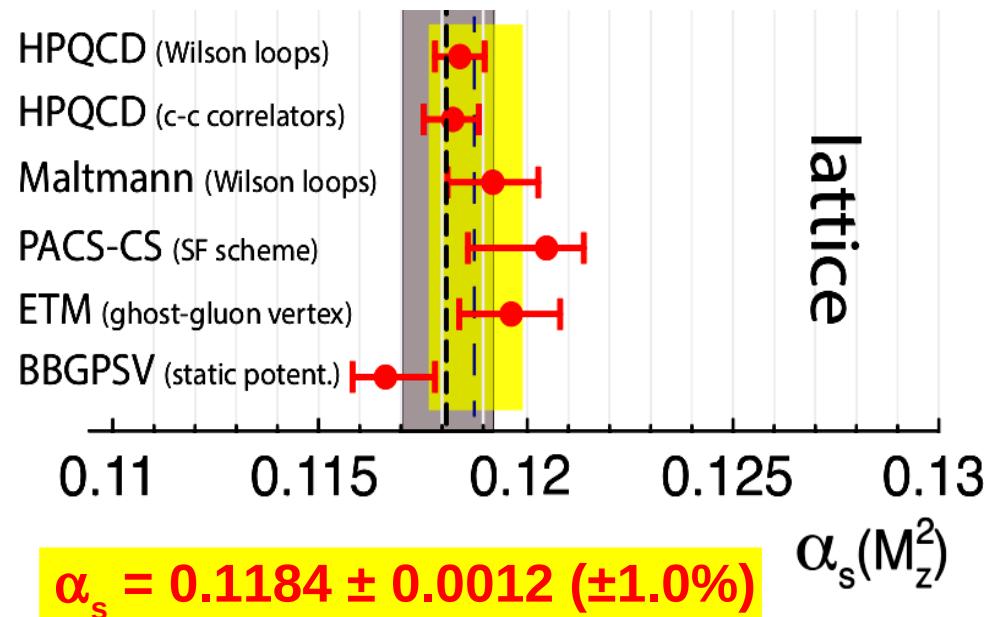
$$K^{\text{NP}} = K^{\text{PT}} = \sum_{i=0}^n c_i \alpha_s^i$$

[FLAG Collab. <http://itpwiki.unibe.ch/flag>]

- Currently, it's extraction with smallest uncertainties: ±1% (lattice spacing & statistics).

Extracted value depends on observables:

Uncertainty increased:
2013 ($\pm 0.4\%$) → 2017 ($\pm 1.0\%$)



- Future prospects:

- Uncertainty in α_s could be halved with (much) better numerical data.
- Reaching ±0.1% requires 4th-loop perturbation theory (~10 years?)

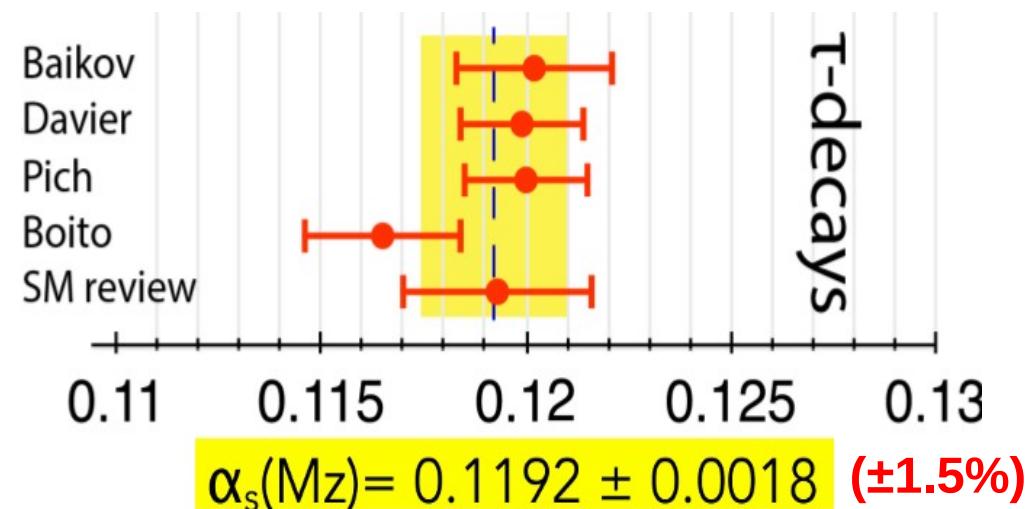
(2) α_s from hadronic τ -lepton decays

→ Computed at **N³LO**: $R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = S_{\text{EW}} N_C (1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5)) + \delta_{\text{np}}$

→ Experimentally: $R_{\tau, \text{exp}} = 3.4697 \pm 0.0080 (\pm 0.23\%)$

→ Various pQCD approaches (**FOPT vs CIPT**) & treatment of non-pQCD corrections (note: $(\Lambda/m_\tau)^2 \sim 2\%$), yield different results.

Uncertainty slightly increased:
2013 ($\pm 1.3\%$) → 2017 ($\pm 1.5\%$)

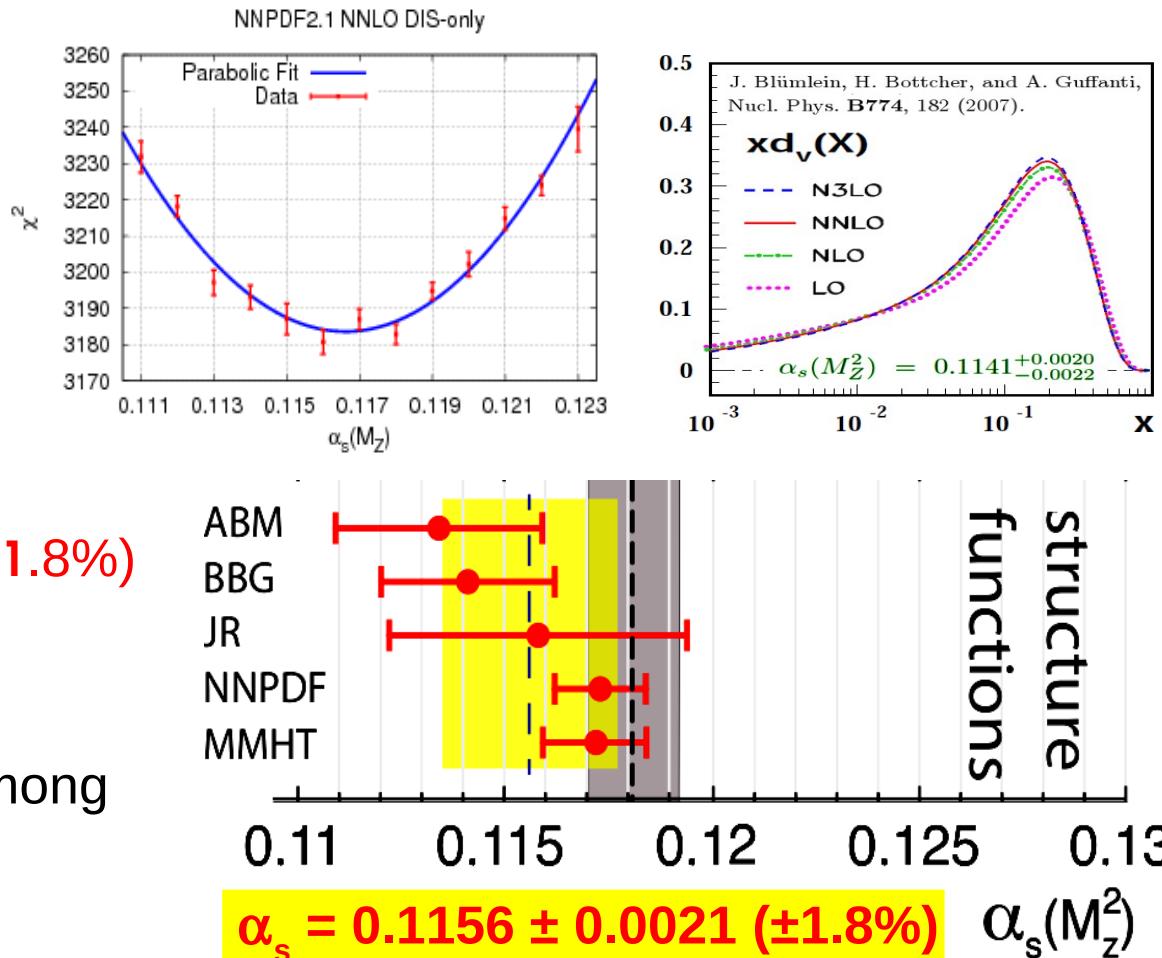


- Future prospects:
- Better understanding of **FOPT vs CIPT differences**.
 - **Better spectral functions** needed (high stats & better precision): B-factories (BELLE-II)
 - High-stats: $\mathcal{O}(10^{11})$ from $Z(\tau\tau)$ at FCC-ee(90) : $\delta\alpha_s/\alpha_s < 1\%$

(3) α_s from proton structure functions

- Computed at **N^{2,3}LO**: $F_2(x, Q^2) = x \sum_{n=0}^{\infty} \frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n} \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_{2,i}^{(n)}(z, Q^2, \mu_R^2, \mu_F^2) f_{i/p}\left(\frac{x}{z}, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$
- Experimentally: Multiple $F_2(x, Q^2)$, $F_2^c(x, Q^2)$, $F_L(x, Q^2)$, PDFs(x, Q^2)

- Different approaches:
Non-singlet fits,
singlet+non-singlet fits,
global fits of PDFs, ...

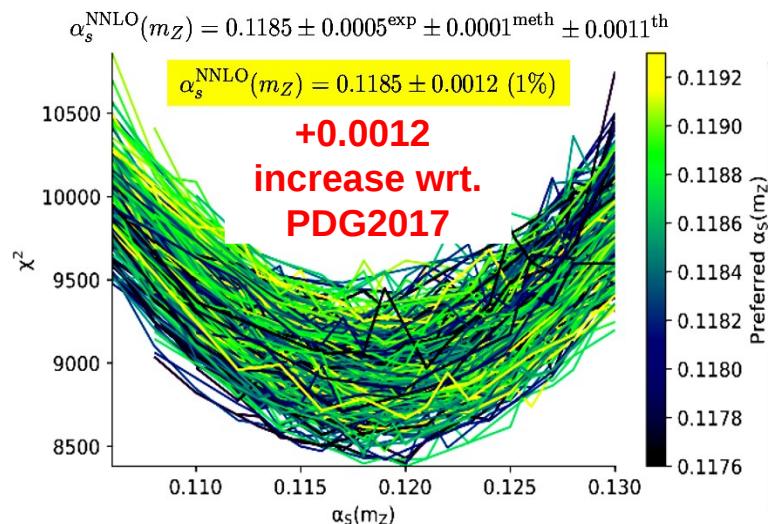
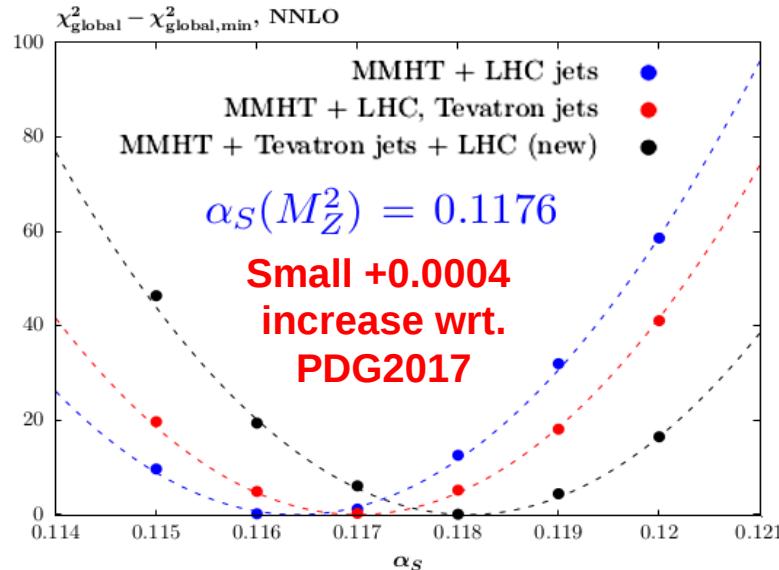


Uncertainty ~stable:
2013 ($\pm 1.7\%$) → 2015 ($\pm 1.8\%$)

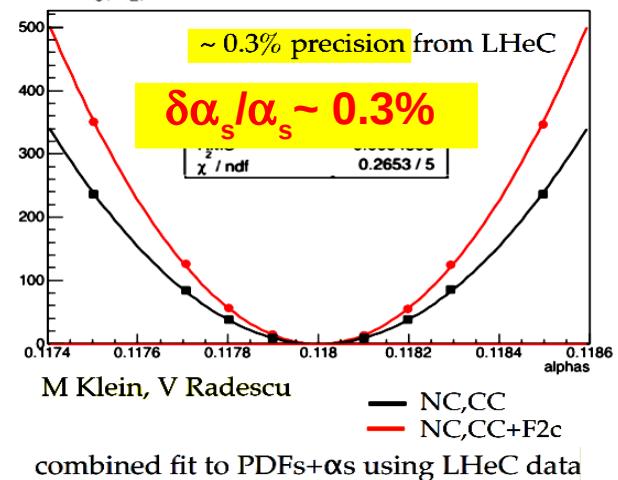
- Lowest central value among all extractions methods.

(3) α_s from proton structure functions (updates)

- Computed at **N^{2,3}LO**: $F_2(x, Q^2) = x \sum_{n=0}^{\infty} \frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n} \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_{2,i}^{(n)}(z, Q^2, \mu_R^2, \mu_F^2) f_{i/p}\left(\frac{x}{z}, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$
- Updates by **MMHT** (R.Thorne, DIS'18) & **NNPDF3.1** (N.Hartland, DIS'18)

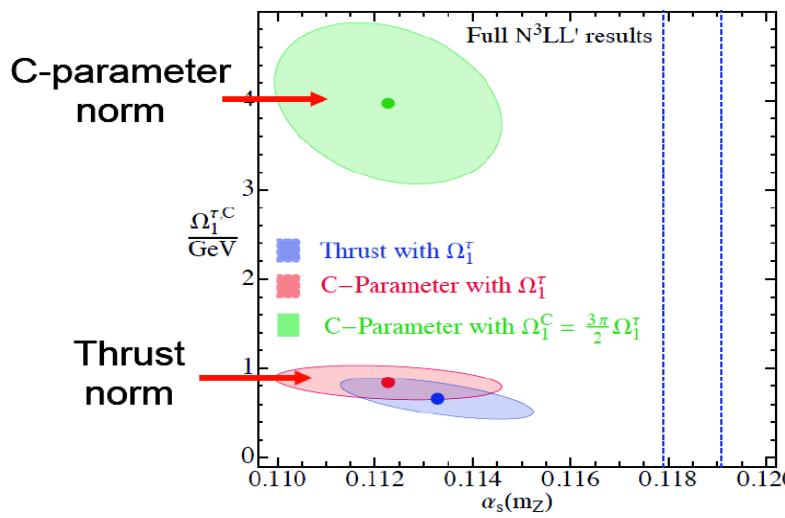


- Jets at NNLO included for the first time.
- Small central α_s value increase towards world average.
- Future: LHeC/FCC-eh stats. should lead to 3-permille uncertainty.



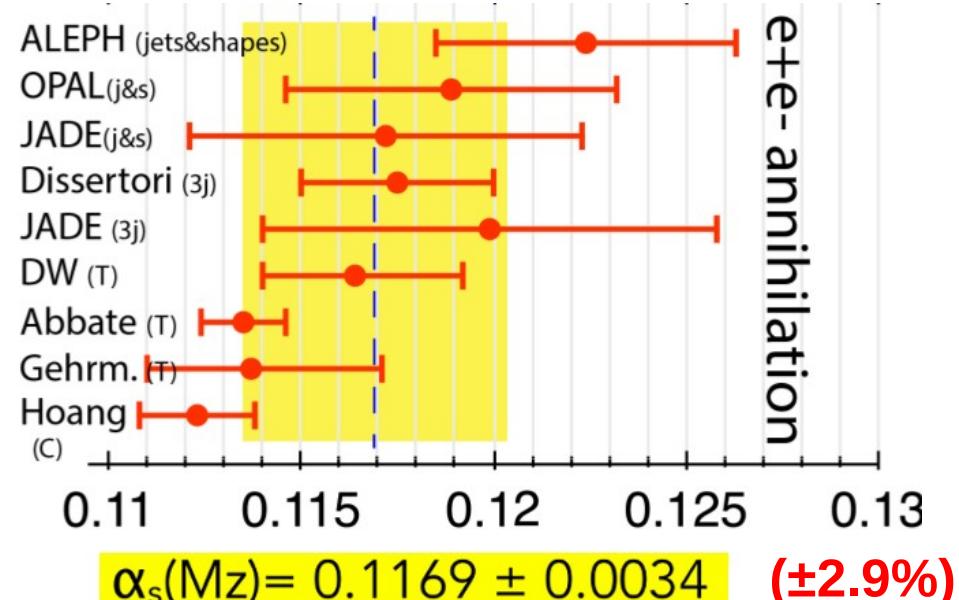
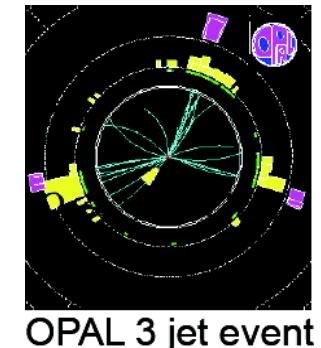
(4) α_s from e^+e^- event shapes & jet rates

- Computed at $N^{2,3}\text{LO} + N^{(2)}\text{LL}$ accuracy.
- Experimentally (LEP):
 - Thrust, C-parameter, jet shapes
 - 3-jet x-sections
- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:



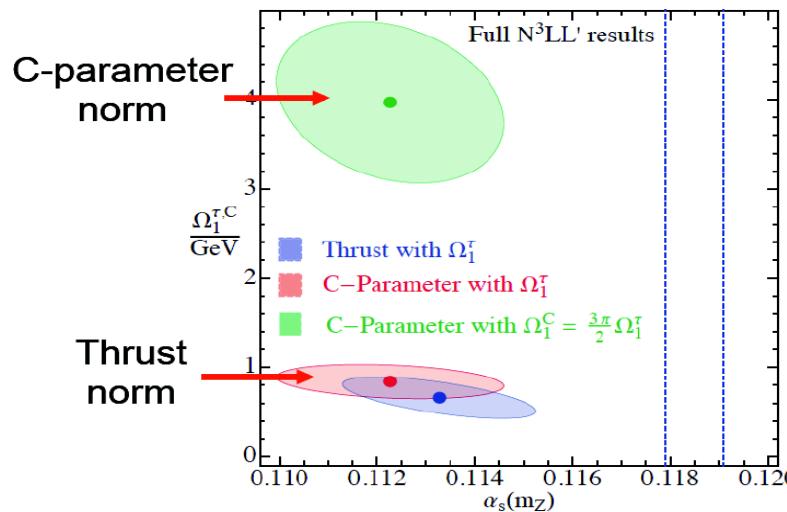
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$



(4) α_s from e^+e^- event shapes & jet rates (2018)

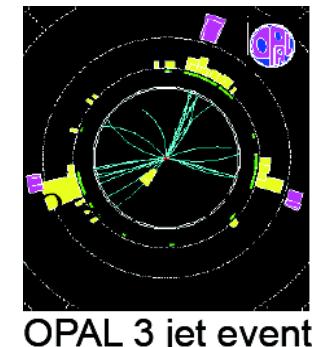
- Computed at $N^{2,3}\text{LO} + N^{(2)}\text{LL}$ accuracy.
- Experimentally (LEP):
 - Thrust, C-parameter, jet shapes
 - 3-jet x-sections
- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:



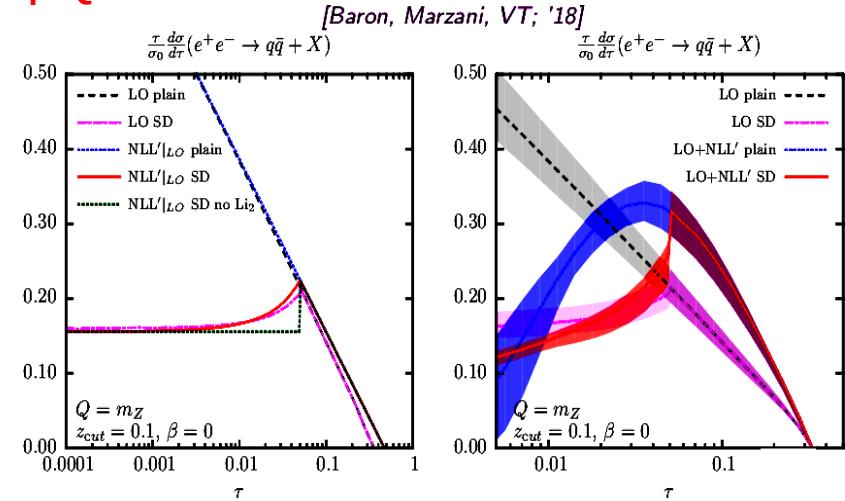
- Future: $\delta\alpha_s/\alpha_s < 1\%$
 - New e^+e^- : lower- \sqrt{s} (Belle-II) for evt shapes, higher- \sqrt{s} (FCC-ee) for rates
 - TH: Improved ($N^{2,3}\text{LL}$) resummation for rates, hadroniz. for shapes

$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$



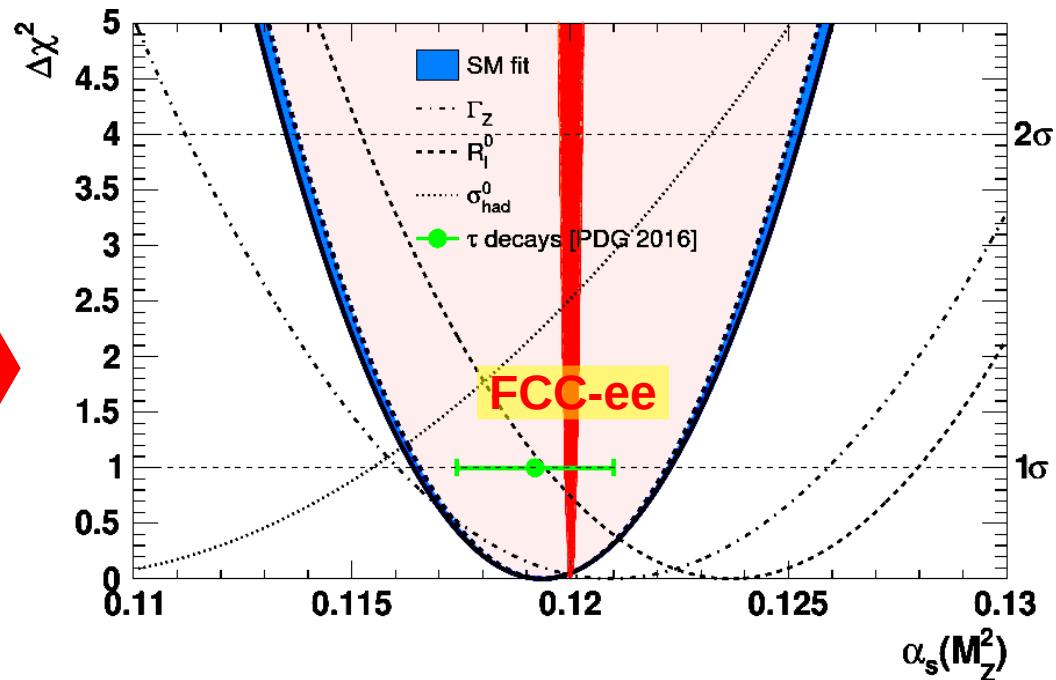
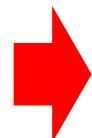
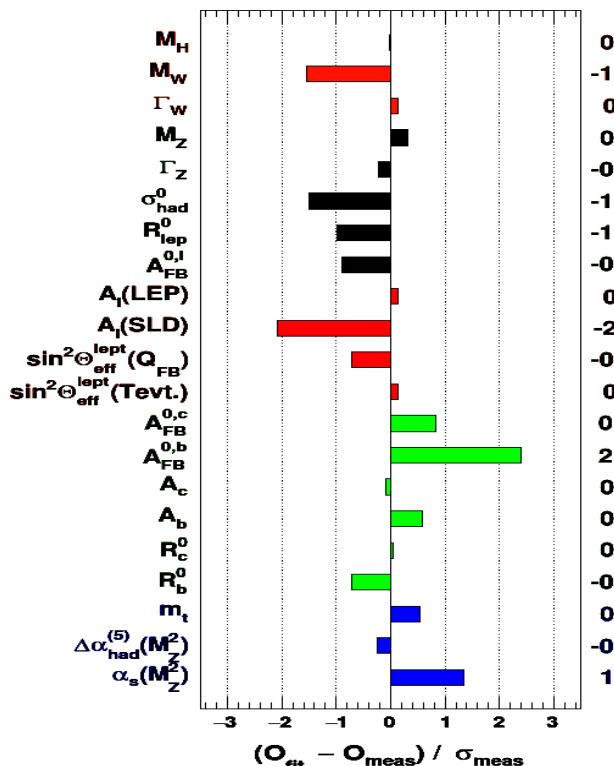
- Modern jet substructure techniques:
“Soft drop” can help reduce non-pQCD corrections for thrust:



(5) α_s from hadronic Z decays

- Computed at N³LO: $R_Z \equiv \frac{\Gamma(Z \rightarrow h)}{\Gamma(Z \rightarrow l)} = R_Z^{\text{EW}} N_C (1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5)) + \delta_m + \delta_{\text{np}}$
- LEP: $\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV } (\pm 0.1\%)$, $R_\ell^0 = \frac{\Gamma_{\text{had}}}{\Gamma_\ell}$, $\sigma_{\text{had}}^0 = \frac{12\pi}{m_Z} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}$, $\sigma_\ell^0 = \frac{12\pi}{m_Z} \frac{\Gamma_\ell^2}{\Gamma_Z^2}$

After Higgs discovery, α_s can be directly determined from full fit of SM:



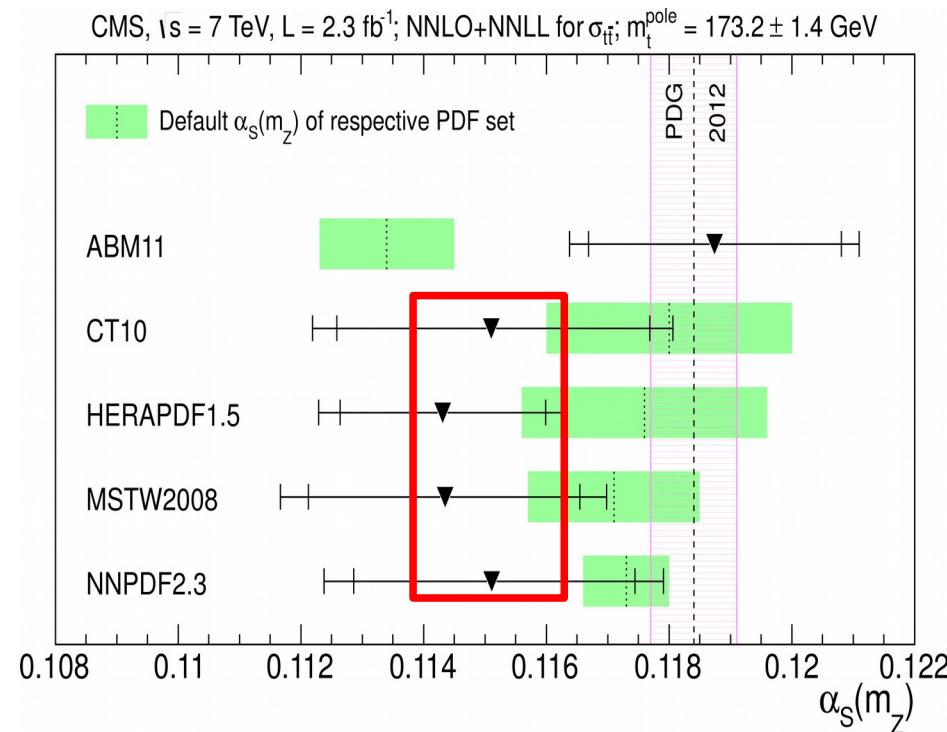
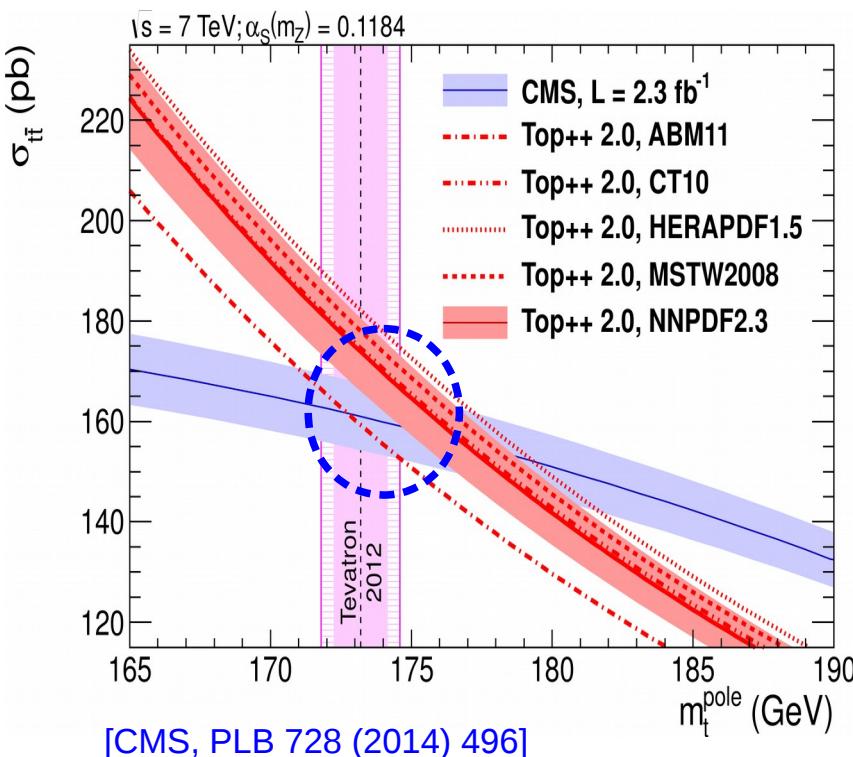
$$\alpha_s(M_Z) = 0.1196 \pm 0.0030 \quad (\pm 2.5\%)$$

- FCC-ee:
 - Z stats ($\times 10^5$ LEP) will lead to: $\delta \alpha_s / \alpha_s < 0.2\%$
 - TH (parametric) uncertainties: $\sin^2 \theta_{\text{eff}}, m_W, m_{\text{top}}$

(6) α_s from top-pair p-p cross sections

- Total top-antitop cross section (known at NNLO+NNLL) is the 1st p-p collider observable to constrain α_s at NNLO accuracy:

Data-theory x-section comparison for varying PDF+ α_s as a function of m_{top} :



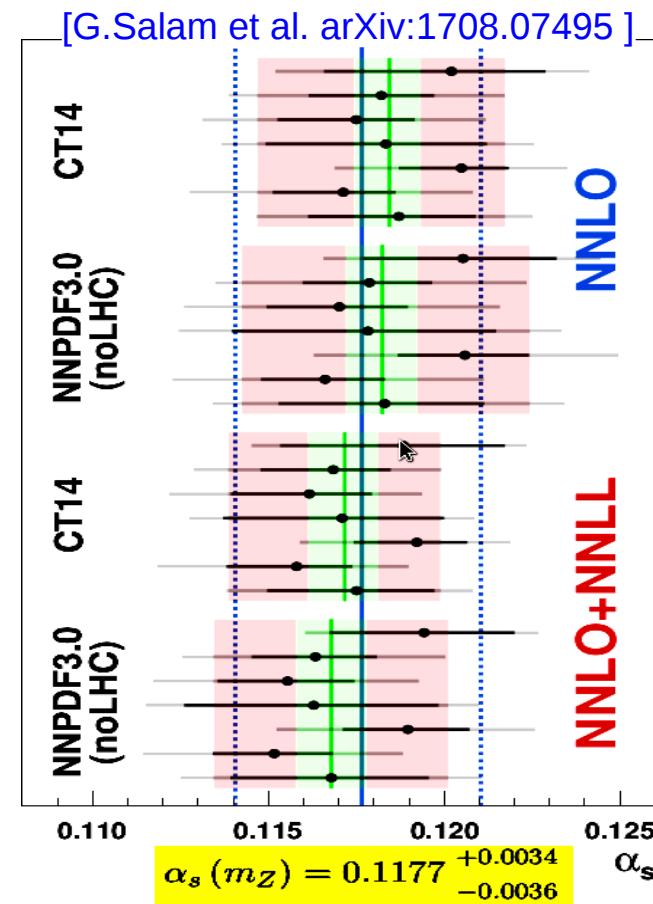
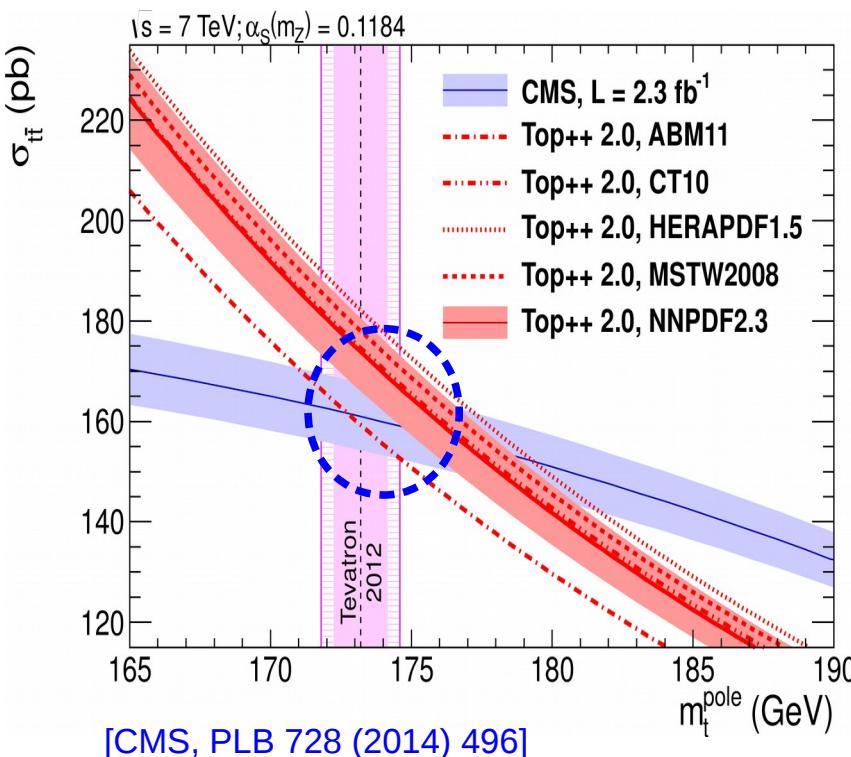
Precise measurement dominated by associated PDF uncertainty ($\pm 2.5\%$)

$$\alpha_s(M_Z^2) = 0.1151^{+0.0028}_{-0.0027}$$

(6) α_s from top-pair p-p cross sections (update)

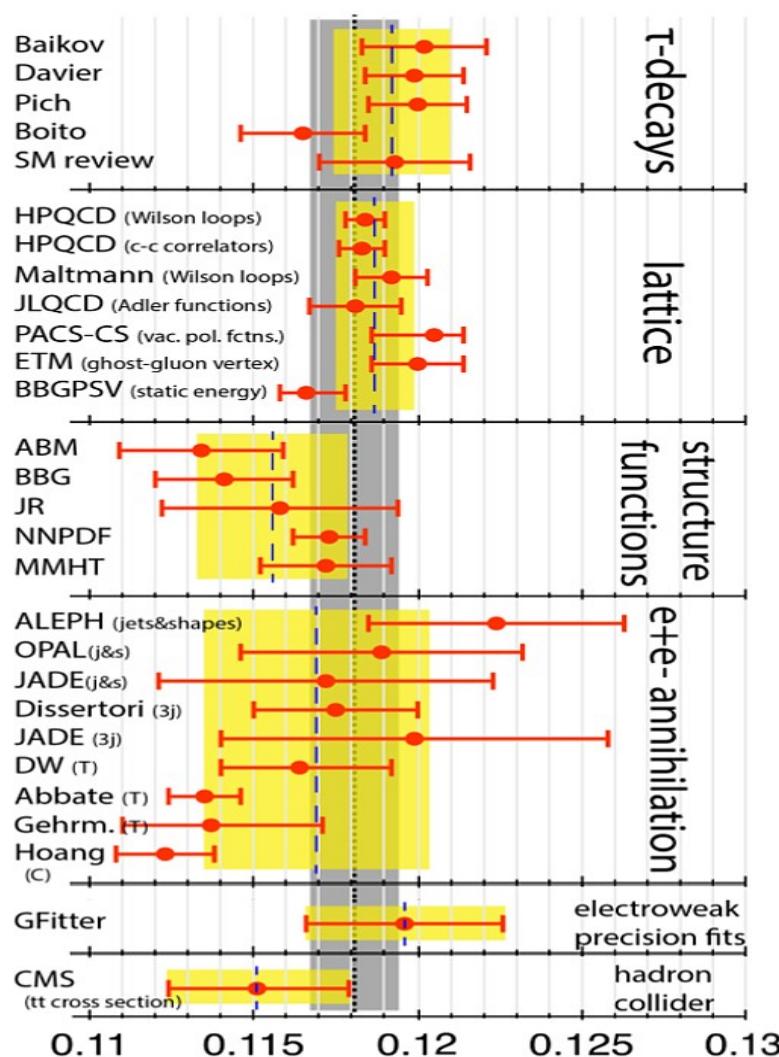
- Total top-antitop cross section (known at NNLO+NNLL) is the 1st p-p collider observable to constrain α_s at NNLO accuracy:

Data-theory x-section comparison for varying PDF+ α_s as a function of m_{top} :



Inclusion of full set of t-tbar data increases $\alpha_s(m_Z)$ & uncertainty: $\pm 2.9\%$

PDG 2017 α_s world average (NNLO)



class averages:

$$\alpha_s(M_Z) = 0.1192 \pm 0.0018 \quad (\pm 1.5\%)$$

$$\alpha_s(M_Z) = 0.1184 \pm 0.0012 \quad (\pm 1.0\%)$$

$$\alpha_s(M_Z) = 0.1156 \pm 0.0021 \quad (\pm 1.8\%)$$

$$\alpha_s(M_Z) = 0.1169 \pm 0.0034 \quad (\pm 2.9\%)$$

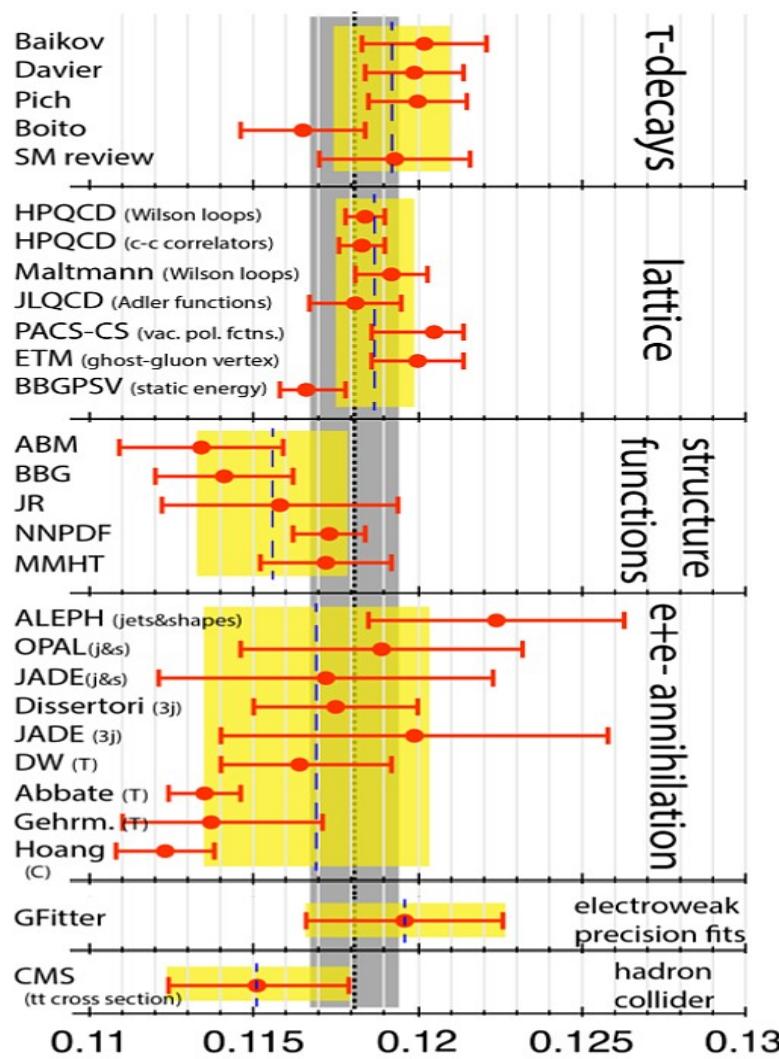
$$\alpha_s(M_Z) = 0.1196 \pm 0.0030 \quad (\pm 2.5\%)$$

$$\alpha_s(M_Z) = 0.1151 \pm 0.0028 \quad (\pm 2.5\%)$$

unweighted χ^2 average:

$$\alpha_s(M_Z) = 0.1181 \pm 0.0011 \quad (\pm 0.9\%)$$

2018 “updated” α_s world average (NNLO)



unweighted χ^2 average:

class averages:

$$\alpha_s(M_z) = 0.1192 \pm 0.0018 \quad (\pm 1.5\%)$$

$$\alpha_s(M_z) = 0.1184 \pm 0.0012 \quad (\pm 1.0\%)$$

$$\alpha_s(M_z) = 0.1156 \pm 0.0021 \quad (\pm 1.8\%)$$

$$\alpha_s(M_z) = 0.1157 \pm 0.0020 \quad (\pm 1.8\%)$$

$$\alpha_s(M_z) = 0.1169 \pm 0.0034 \quad (\pm 2.9\%)$$

$$\alpha_s(M_z) = 0.1196 \pm 0.0030 \quad (\pm 2.5\%)$$

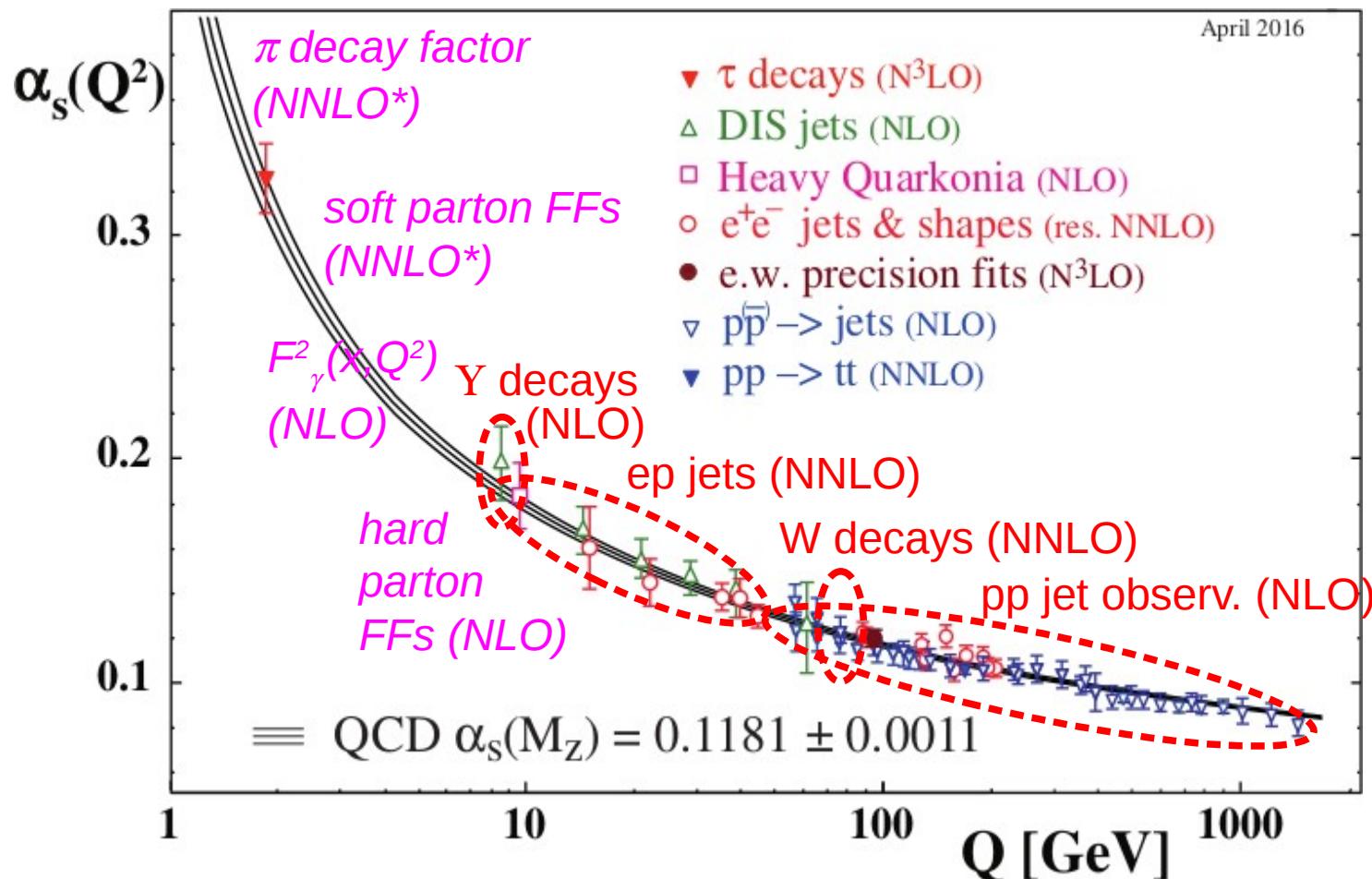
$$\alpha_s(M_z) = 0.1151 \pm 0.0028 \quad (\pm 2.5\%)$$

$$\alpha_s(M_z) = 0.1177 \pm 0.0035 \quad (\pm 2.9\%)$$

$$\alpha_s(M_z) = 0.1181 \pm 0.0011 \quad (\pm 0.9\%)$$

Other α_s extractions (not yet in world average)

- There exist at least 8 other classes of observables, computed at lower accuracy (NLO, NNLO*), used to extract the QCD coupling:

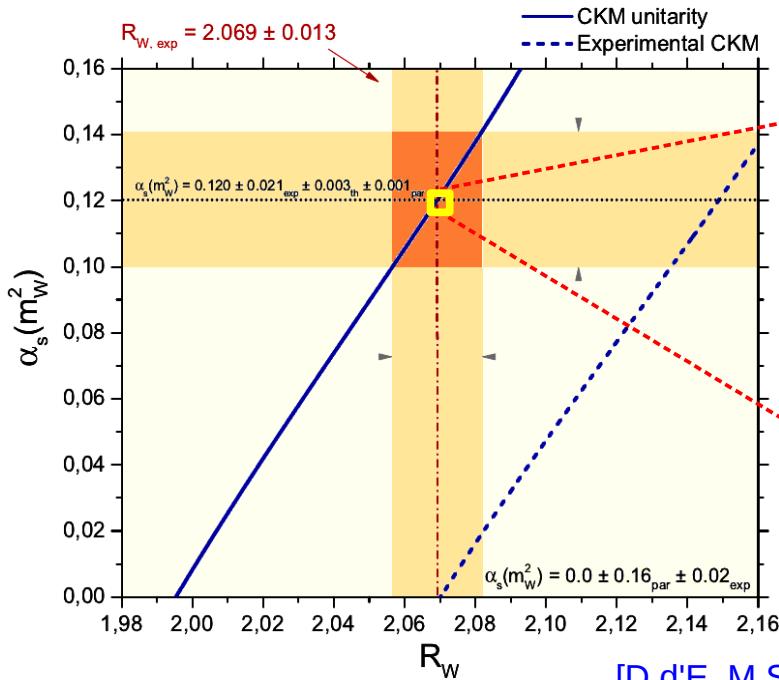


α_s from hadronic W decays (NNLO)

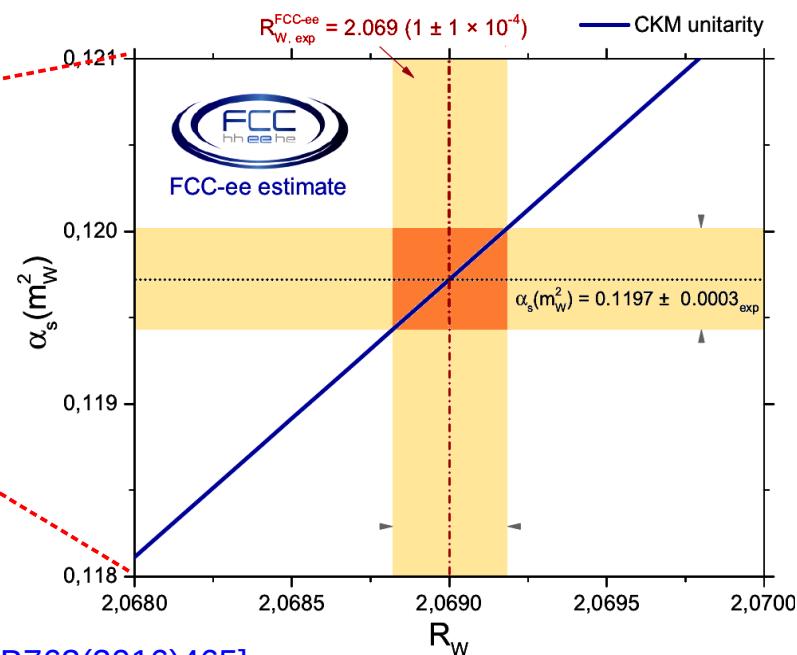
- Computed at N^{2,3}LO: $\Gamma_{W,\text{had}} = \frac{\sqrt{2}}{4\pi} G_F m_W^3 \sum_{\text{quarks } i,j} |V_{i,j}|^2 \left[1 + \sum_{k=1}^4 \left(\frac{\alpha_s}{\pi} \right)^k + \delta_{\text{electroweak}}(\alpha) + \delta_{\text{mixed}}(\alpha \alpha_s) \right]$
- LEP: $\Gamma_W = 1405 \pm 29 \text{ MeV} (\pm 2\%)$, $\text{BR}_W = 0.6741 \pm 0.0027 (\pm 0.4\%)$

Extraction with large exp. & parametric
(CKM V_{cs}) uncertainties today:

$$\alpha_s(M_z) = 0.117 \pm 0.040 \quad (\pm 35\%)$$



[D.d'E, M.Srebre, PLB763(2016)465]

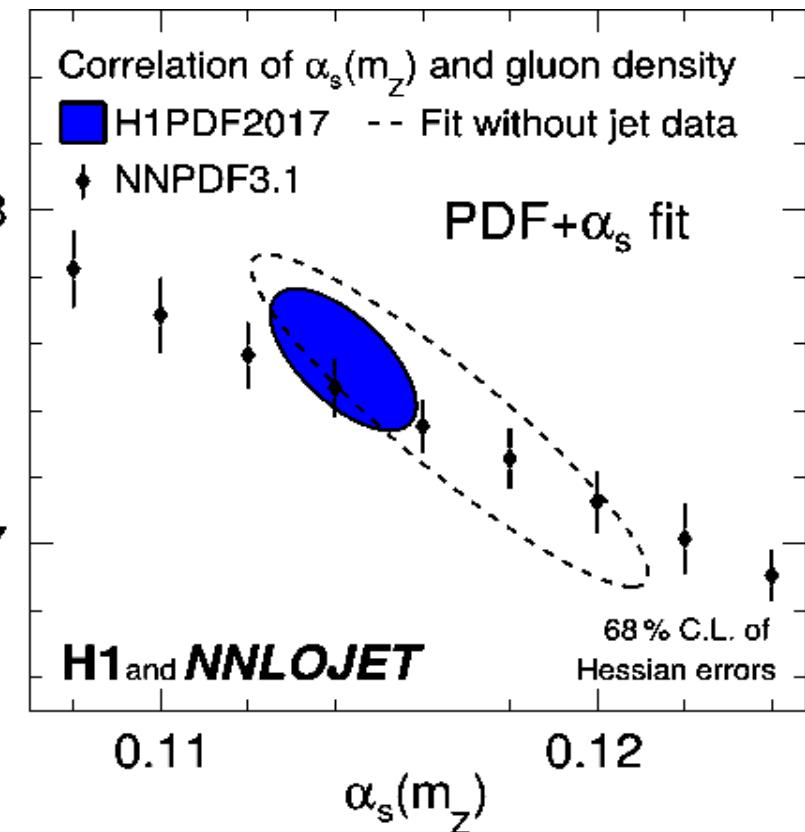
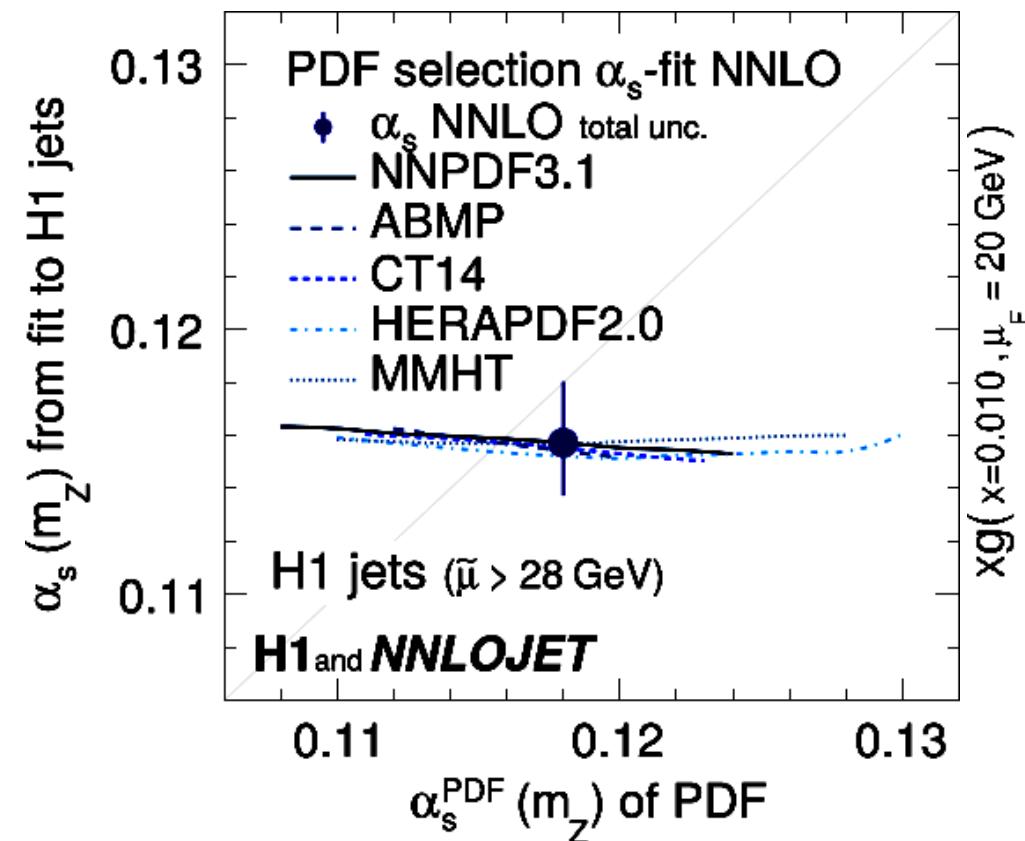


- FCC-ee:
 - Huge W stats ($\times 10^4$ LEP) will lead to: $\delta \alpha_s / \alpha_s < 0.3\%$
 - TH (param.) uncertainty: $|\delta V_{cs}|$ to be significantly improved (10^{-4})

α_s coupling from e-p \rightarrow jets (NNLO)

- DIS H1 jet x-sections and jets+PDF-fit compared for the 1st time to NNLOjet calculations:

[Radek Žlebčík, H1, arxiv:1709.07251]

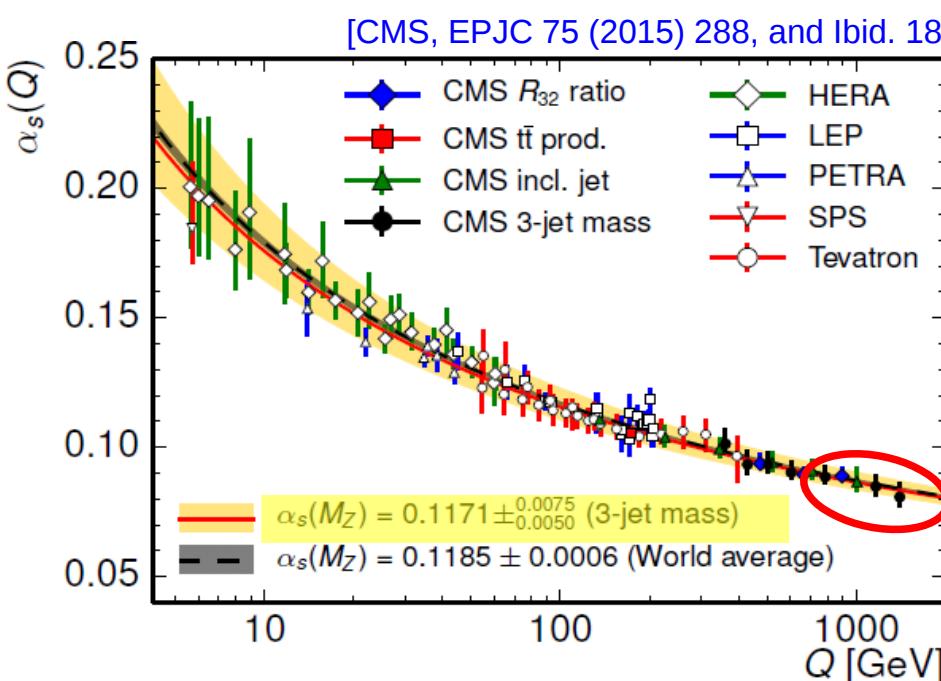


$$\alpha_S^{\text{H1jets}, \tilde{\mu} > 28 \text{ GeV}}(m_Z) = 0.1157(20)_{\text{exp}}(28)_{\text{theor.}} \quad (\pm 3.0\%)$$

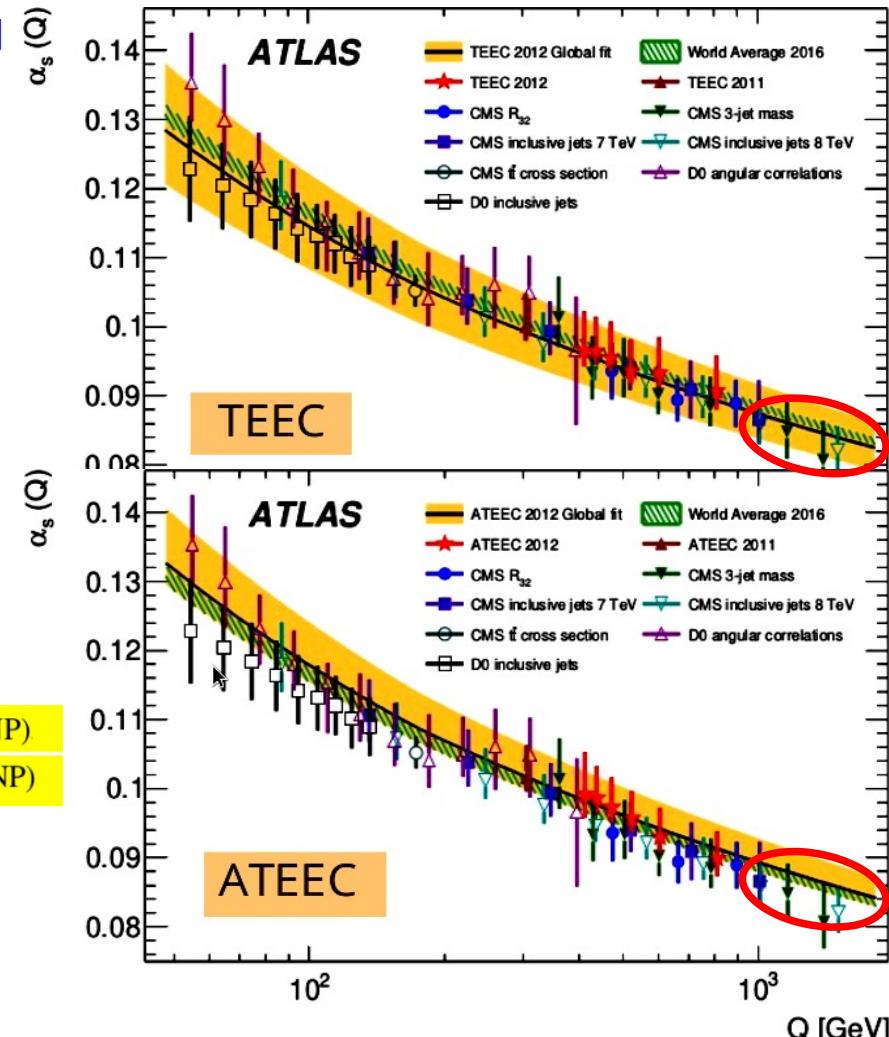
$$\alpha_S^{\text{H1PDF2017}}(m_Z) = 0.1142(11)_{\text{exp}}(26)_{\text{theor.}} \quad (\pm 2.4\%)$$

α_s coupling from other LHC jet results (NLO)

- Ratio of 3-jets to 2-jets, 3-jet mass x-sections & energy-energy correl. test running α_s (NLO only) up to so-far unprobed scales $Q \sim 1.5$ TeV:



$$\begin{aligned} \alpha_s(m_Z) &= 0.1162 \pm 0.0011 \text{ (exp.)} {}^{+0.0076}_{-0.0061} \text{ (scale)} \pm 0.0018 \text{ (PDF)} \pm 0.0003 \text{ (NP)} \\ \alpha_s(m_Z) &= 0.1196 \pm 0.0013 \text{ (exp.)} {}^{+0.0061}_{-0.0013} \text{ (scale)} \pm 0.0017 \text{ (PDF)} \pm 0.0004 \text{ (NP)} \end{aligned}$$



→ 1st time asymptotic freedom
is probed at the TeV scale!

α_s from γ QCD structure function (NNLO)

→ Computed at NNLO: $\int_0^1 dx F_2^\gamma(x, Q^2, P^2) = \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \left\{ \frac{4\pi}{\alpha_s(Q^2)} c_{LO} + c_{NLO} + \frac{\alpha_s(Q^2)}{4\pi} c_{NNLO} + \mathcal{O}(\alpha_s^2) \right\}$

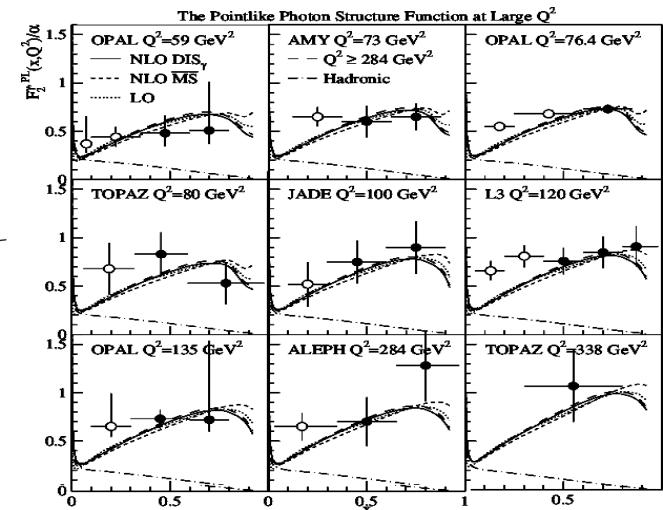
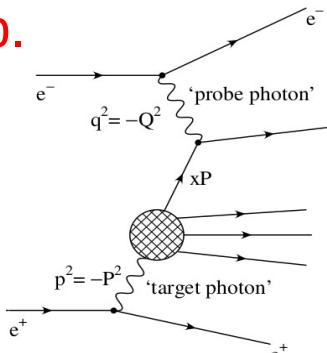
→ Poor $F_\gamma^2(x, Q^2)$ experimental measurements:

→ Extraction (NLO) with large exp. uncertainties today:

$$\alpha_s(M_Z) = 0.1198 \pm 0.0054$$

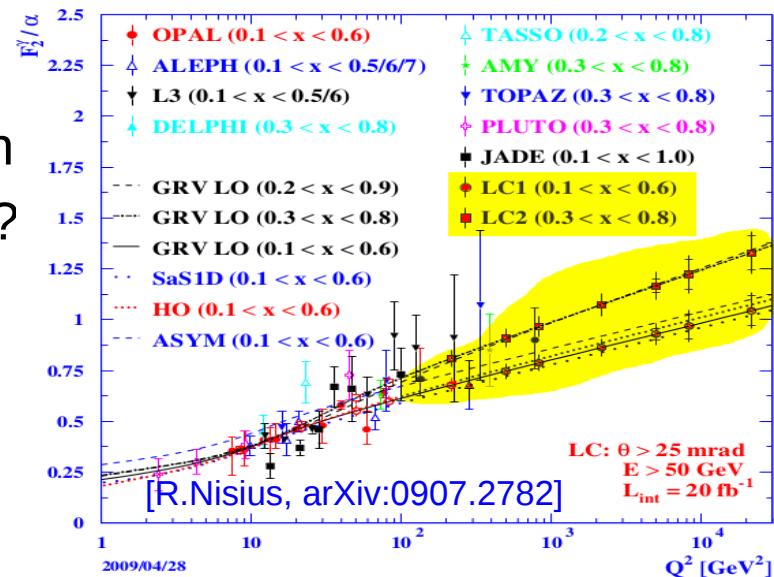
(±4.5%)

[M.Klasen et al. PRL89 (2002)122004]



→ Future prospects:

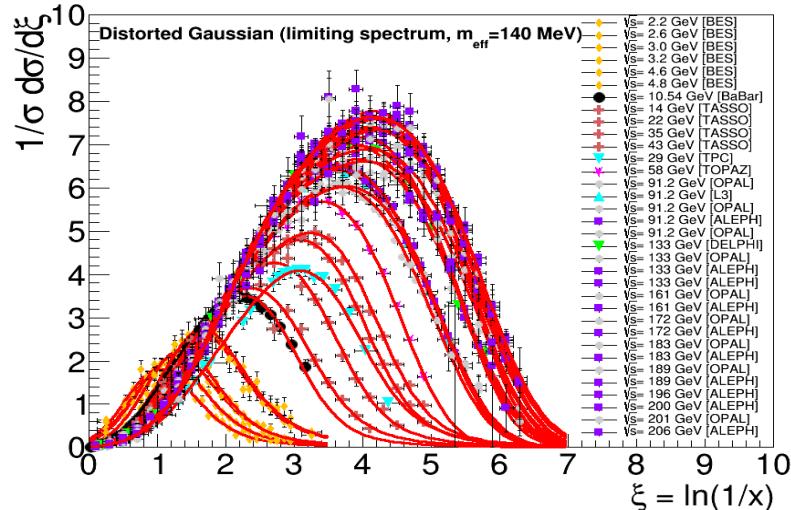
- Fit with NNLO F_γ^2 evolution (ongoing)
- Better data badly needed: Belle-II ?
- Dedicated studies at ILC exist:
- Huge $\gamma\gamma$ (EPA) stats at FCC-ee will lead to: $\delta\alpha_s/\alpha_s < 1\%$



Other α_s extractions (NLO, NNLO*)

→ Soft parton-to-hadron FFs (NNLO*+NNLL):

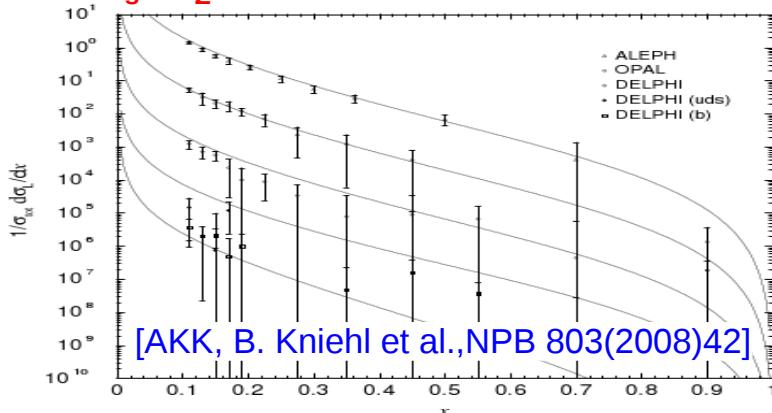
$$\alpha_s(m_z) = 0.1205 \pm 0.0022 (\pm 2\%)$$



[D.d'E., R.Perez-Ramos, arXiv:0505.02624]

→ Hard parton-to-hadron FFs (NLO):

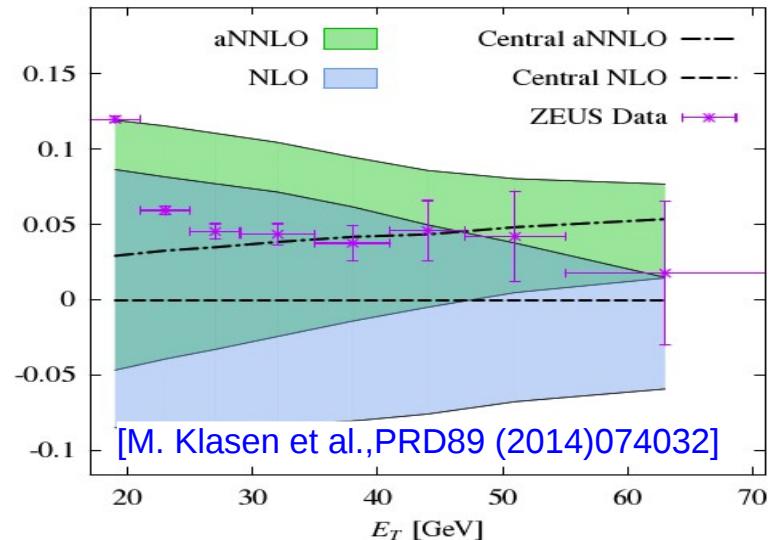
$$\alpha_s(m_z) = 0.1176 \pm 0.0055 (\pm 4.7\%)$$



Dip 2010, ROME, April 2010
22/23

→ Jet x-sections in γ -p (NNLO*):

$$\alpha_s(m_z) = 0.112 \pm 0.002 \pm 0.003 (\pm 4\%)$$



→ π decay factor (N³LO, RGOPT):

$$\alpha_s(m_z) = 0.1174 \pm 0.0017 (\pm 1.5\%)$$

[Kneur&Neveu, PRD81(2010)125012]

→ Y decay (NLO): [Mambrilla et al. PRD75(07)074014]

$$\alpha_s(m_z) = 0.1190 \pm 0.007 (\pm 6\%)$$

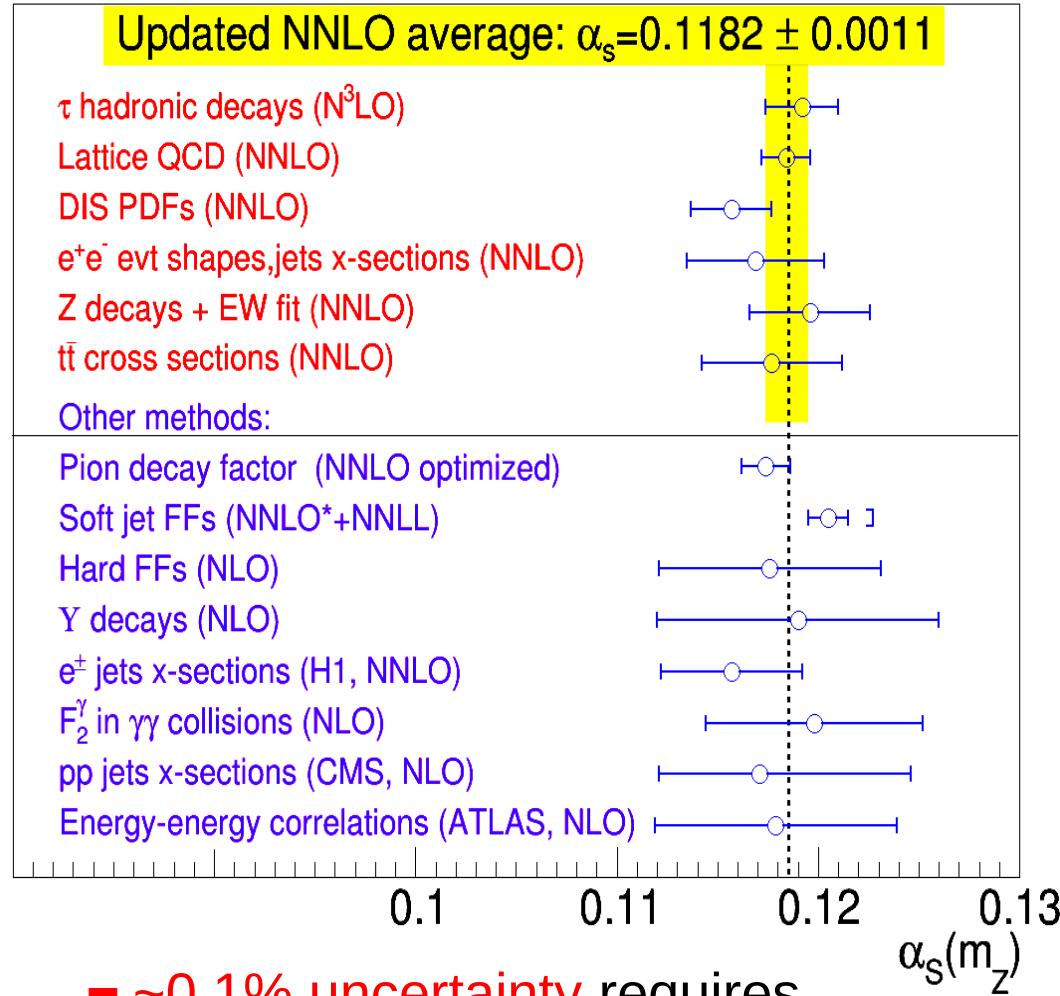
$$R_\gamma \equiv \frac{\Gamma(\Upsilon(1S) \rightarrow \gamma X)}{\Gamma(\Upsilon(1S) \rightarrow X)} = \frac{36 e_b^2 \alpha}{5 \alpha_s} \frac{N}{D},$$

$$N, D = 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(v^2) + \mathcal{O}\left(\frac{v^4}{\alpha_s}\right) \\ + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s v^2) + \mathcal{O}\left(\frac{v^4}{\alpha_s}\right) + \mathcal{O}(v^4) + \mathcal{O}\left(\frac{v^6}{\alpha_s}\right)$$

Summary: α_s status (2018)

- World-average QCD coupling at **NNLO**:
 - Determined from **6 observables** with **1% uncertainty** (**least well-known coupling**)
 - Impacts all **LHC QCD x-sections & decays**.
 - Role **beyond SM**: GUT, EWK vacuum stability New colored sectors?
- **4 new extractions/updates**:
 - PDF fits (with NNLO pp jets)
 - e-p jets at NNLO
 - Full $pp \rightarrow t\bar{t}$ data
 - W hadronic BR at NNLO
- **8 other extraction methods** proposed. Work towards NNLO accuracy.
- LHC: Running up to $Q \sim 1.5$ TeV

(Simple updated average gives +0.001 increase)



■ ~0.1% uncertainty requires high-luminosity e^+e^- collider.

Backup slides

α_s from pion and Y decays

[J.L.Kneur]

$$F_\pi^2(\text{pert})_{\overline{\text{MS}}} = N_c \frac{m^2}{2\pi^2} \left[-L + \frac{\alpha_s}{4\pi} (8L^2 + \frac{4}{3}L + \frac{1}{6}) + (\frac{\alpha_s}{4\pi})^2 [f_{30}(n_f)L^3 + f_{31}(n_f)L + f_{32}(n_f)L + f_{33}(n_f)] + \mathcal{O}(\alpha_s^3) \right]$$

$$L \equiv \ln \frac{m}{\mu}, n_f = 2(3)$$

$$\bar{\alpha}_S(m_Z) = 0.1174^{+0.0010}_{-0.0005}(\text{rgopt th}) \pm .0010|_{(F_\pi/F_0)} \pm .0005_{\text{evol}}$$

Issues:

- Too low scale for pQCD?
- Optimization approach,...
- Intriguing agreement with world average.

$$\alpha_s = 0.1174 \pm 0.0017 \text{ } (\pm 1.5\%)$$

$$R_\gamma \equiv \frac{\Gamma(\Upsilon(1S) \rightarrow \gamma X)}{\Gamma(\Upsilon(1S) \rightarrow X)} = \frac{36}{5} \frac{e_b^2 \alpha}{\alpha_s} \frac{N}{D},$$

[J. Soto]

$$\begin{aligned} N, D &= 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(v^2) + \mathcal{O}\left(\frac{v^4}{\alpha_s}\right) \\ &\quad + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s v^2) + \mathcal{O}\left(\alpha_s \frac{v^4}{\alpha_s}\right) + \mathcal{O}(v^4) + \mathcal{O}\left(\frac{v^6}{\alpha_s}\right) \end{aligned}$$

$$\alpha_s(\text{NLO}) = 0.1190 \pm 0.007 \text{ } (\pm 6\%)$$

- A NNLO extraction of α_s appears feasible in the coming years, the key ingredients being:
 - More precise data for the $\Upsilon(1S)$ photon spectrum (and total hadronic width)
 - Non-trivial higher order perturbative calculations