DIS 2018

16-20 April 2018 Kobe, Japan

“Accessing Quark Helicity in e^+e^- and SIDIS via Dihadron Correlations.

P.R.D97, 074019 (2018); arXiv:1712.06384.

Hrayr Matevosyan

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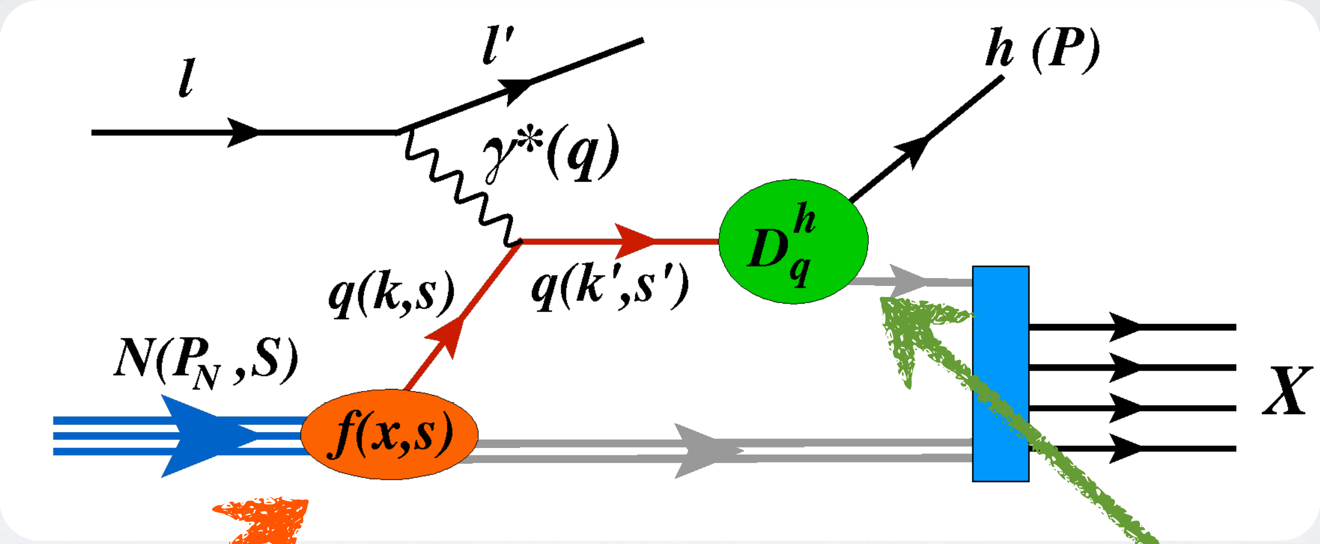
MEASURING PDFS WITH TRANSVERSE MOMENTUM DEPENDENCE

- ***Measurement of the transverse momentum of the produced hadron in SIDIS provides access to TMD PDFs/FFs.***

  - *SIDIS Process with TM of hadron measured.*



- *TMD PDFs*

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\ h_{1T}^\perp$

- *TMD FFs*

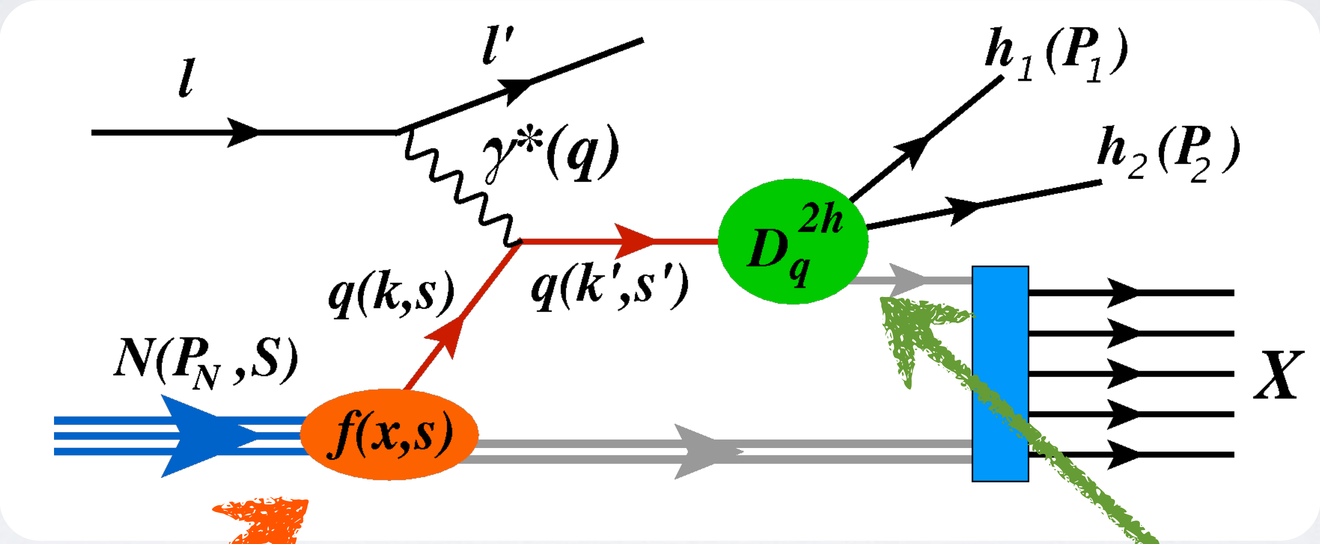
q/h	U
U	D_1
L	
T	$H_1^\perp$

*unpol/spinless h!

TMD PDFs with Two-Hadron FFs

- ***Measuring <u>two-hadron</u> semi-inclusive DIS: an additional method for accessing TMD PDFs.***

  - *SIDIS Process with TM of hadrons measured.*



- *TMD PDFs*

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1 \; h_{1T}^\perp$

- *TMD DiFFs*

q/h_1h_2	U
U	D_1
L	$G_1^\perp$
T	$H_1^\perp \quad H_1^\sphericalangle$

✶unpol/spinless h!

SYSTEMATICS OF DIHADRON FRAGMENTATION FUNCTIONS

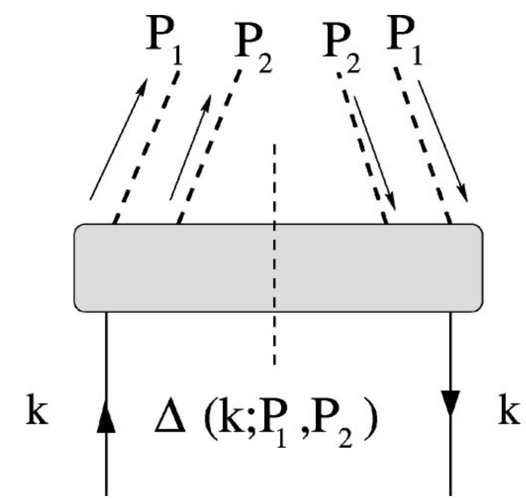
Two-Hadron Kinematics

A. Bianconi et al: PRD 62, 034008 (2000).

✦ Total and Relative TM of hadron pair.

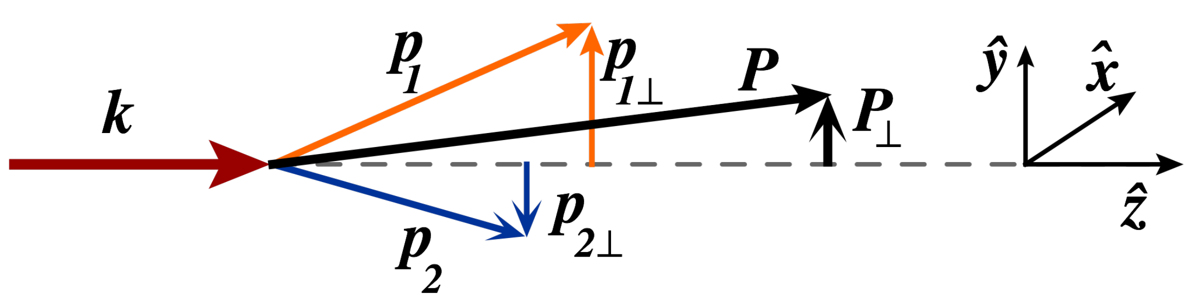
$$P = P_1 + P_2 \qquad z = z_1 + z_2$$

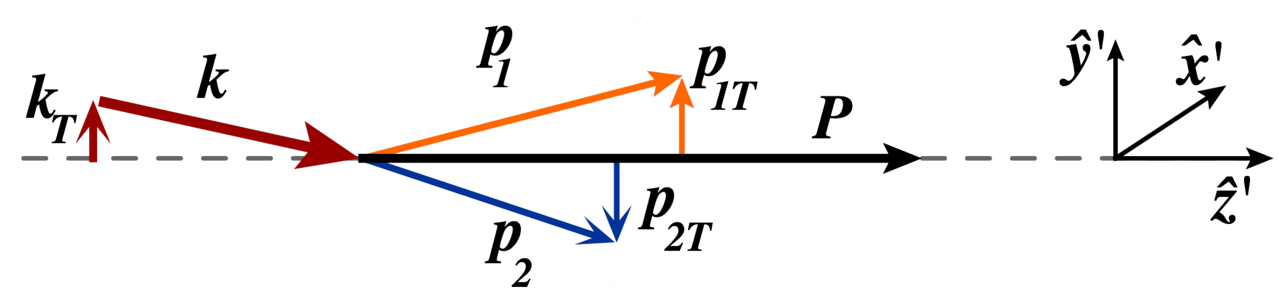
$$R = \frac{1}{2}(P_1 - P_2) \quad \xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}$$



✦ Two Coordinate systems:

- $\perp$: modelling hadronization
- T: field-theoretical definition of DiFFs





✦ Lorentz Boost:

$$\boldsymbol{P}_{1T} = \boldsymbol{P}_{1\perp} + z_1 \boldsymbol{k}_T$$

$$\boldsymbol{P}_{2T} = \boldsymbol{P}_{2\perp} + z_2 \boldsymbol{k}_T$$

$$\boldsymbol{k}_T = -\frac{\boldsymbol{P}_\perp}{z}$$

✤ Relative TM in two systems

$$\boldsymbol{R}_\perp = \frac{1}{2}(\boldsymbol{P}_{1\perp} - \boldsymbol{P}_{2\perp})$$

$$\boldsymbol{R}_T = \frac{z_2 \boldsymbol{P}_{1\perp} - z_1 \boldsymbol{P}_{2\perp}}{z}$$

Field-Theoretical Definitions

- ***The quark-quark correlator.***

$$\Delta_{ij}(k;P_1,P_2)=\sum_X\int d^4\zeta e^{ik\cdot\zeta}\langle 0|\psi_i(\zeta)|P_1P_2,X\rangle\langle P_1P_2,X|\bar{\psi}_j(0)|0\rangle$$

- ***The definitions of DiFFs from the correlator.***

Quark Polarization

$$\Delta^{[\gamma^-]}=D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Unpolarised

$$\Delta^{\gamma^-\gamma_5]}=\frac{\epsilon_T^{ij}R_{Ti}k_{Tj}}{M_h^2}G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Longitudinal

$$\Delta^{[i\sigma^{i-}\gamma_5]}=\frac{\epsilon_T^{ij}R_{Tj}}{M_h}H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

$$+\frac{\epsilon_T^{ij}k_{Tj}}{M_h}H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Transverse

Field-Theoretical Definitions

- ***The quark-quark correlator.***

$$\Delta_{ij}(k;P_1,P_2)=\sum_X\int d^4\zeta e^{ik\cdot\zeta}\langle 0|\psi_i(\zeta)|P_1P_2,X\rangle\langle P_1P_2,X|\bar{\psi}_j(0)|0\rangle$$

- ***The definitions of DiFFs from the correlator.***

Quark Polarization

$$\Delta^{[\gamma^-]}=D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Unpolarised

related to "jet handedness"

$$\Delta^{\gamma^-\gamma_5]}=\frac{\epsilon_T^{ij}R_{Ti}k_{Tj}}{M_h^2}G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Longitudinal

$$\Delta^{[i\sigma^{i-}\gamma_5]}=\frac{\epsilon_T^{ij}R_{Tj}}{M_h}H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)+\frac{\epsilon_T^{ij}k_{Tj}}{M_h}H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Transverse

Fourier Moments of DiFFs

- *Expanded dependence on $\varphi_{RK} \equiv \varphi_R - \varphi_k$ in cos series*

$$D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{KR})) = \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(n\cdot\varphi_{KR})}{1+\delta_{0,n}}\; D_1^{[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|),$$

$$F^{[n]} = \int d\varphi_{KR}\;\cos(n\varphi_{KR})\;F\Big(\cos(\varphi_{KR})\Big)$$

- *Integrated DiFFs and Fourier moments*

$$D_1^a(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\; D_1^{a,[0]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2)$$

$$G_1^{\perp a,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\left(\frac{\boldsymbol{k}_T^2}{2M_h^2}\right)\frac{|\boldsymbol{R}_T|}{M_h}\;G_1^{\perp a,[n]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2).$$

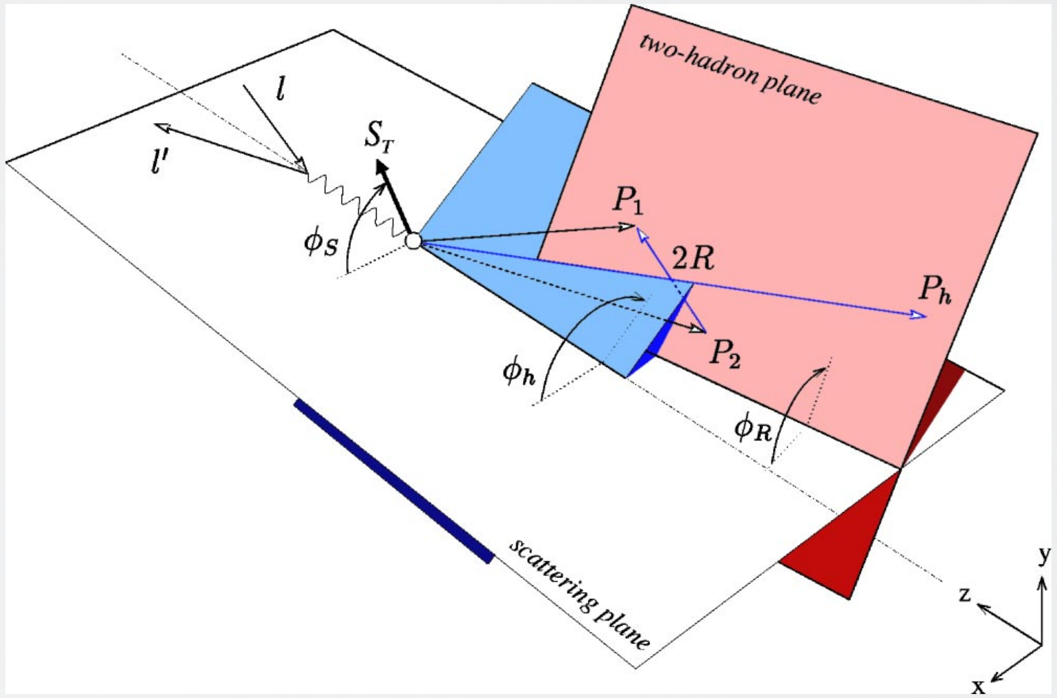
$$H_1^{\sphericalangle,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\;\frac{|\boldsymbol{R}_T|}{M_h}H_1^{\sphericalangle,[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

$$H_1^{\perp,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\;\frac{|\boldsymbol{k}_T|}{M_h}H_1^{\perp,[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

ACCESS TO TRANSVERSITY PDF From DiFF

M. Radici, et al: PRD 65, 074031 (2002).

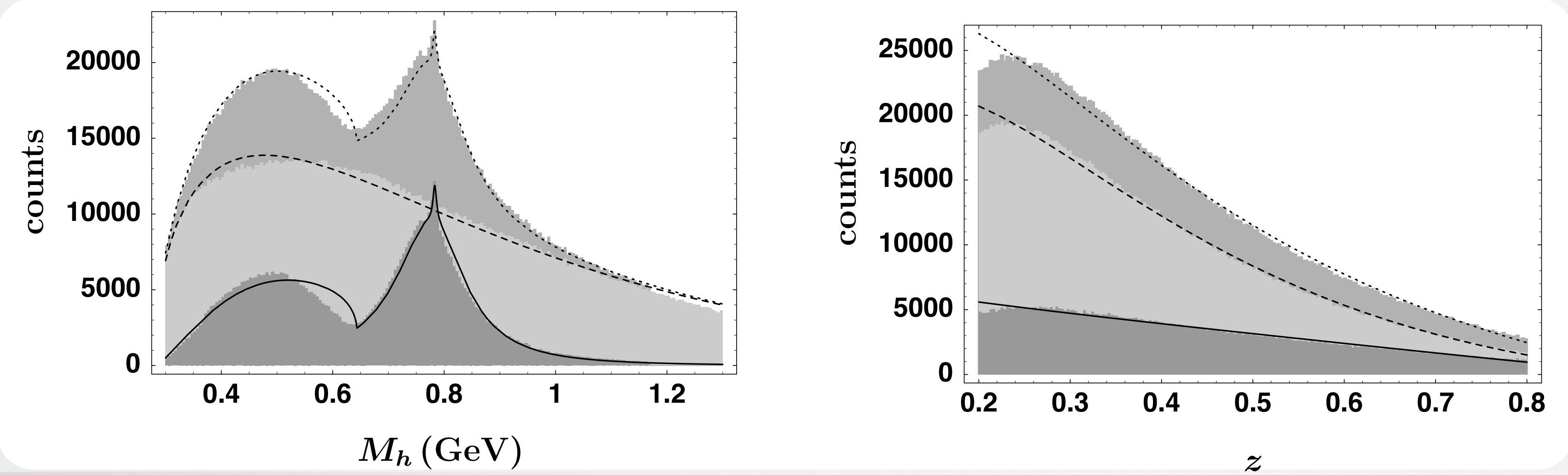
- In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!
- Allows clean access to *transversity*.
- *Unpolarized* and *Interference* Dihadron FFs are needed!



$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 \; h_1^q(x)/x \;\; H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2 \; f_1^q(x)/x \;\; D_1^q(z, M_h^2)}$$

- **Empirical Model for D_1^q has been fitted to PYTHIA simulations.**

A. Bacchetta and M. Radici, PRD 74, 114007 (2006).



Experiments:
BELLE,
HERMES,
COMPASS.

Moments of DiFFs in SIDIS

A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

- *Here transversely polarised DiFFs are **admixture of cos Fourier moments** of both unintegrated DiFFs:*

$$H_{1,SIDIS}^{\sphericalangle}(z, M_H^2) = \left[H_1^{\sphericalangle[0]} + H_1^{\perp[1]}\right]$$

$$H_{1,SIDIS}^{\perp}(z, M_H^2) = \left[H_1^{\perp[0]} + H_1^{\sphericalangle[1]}\right]$$

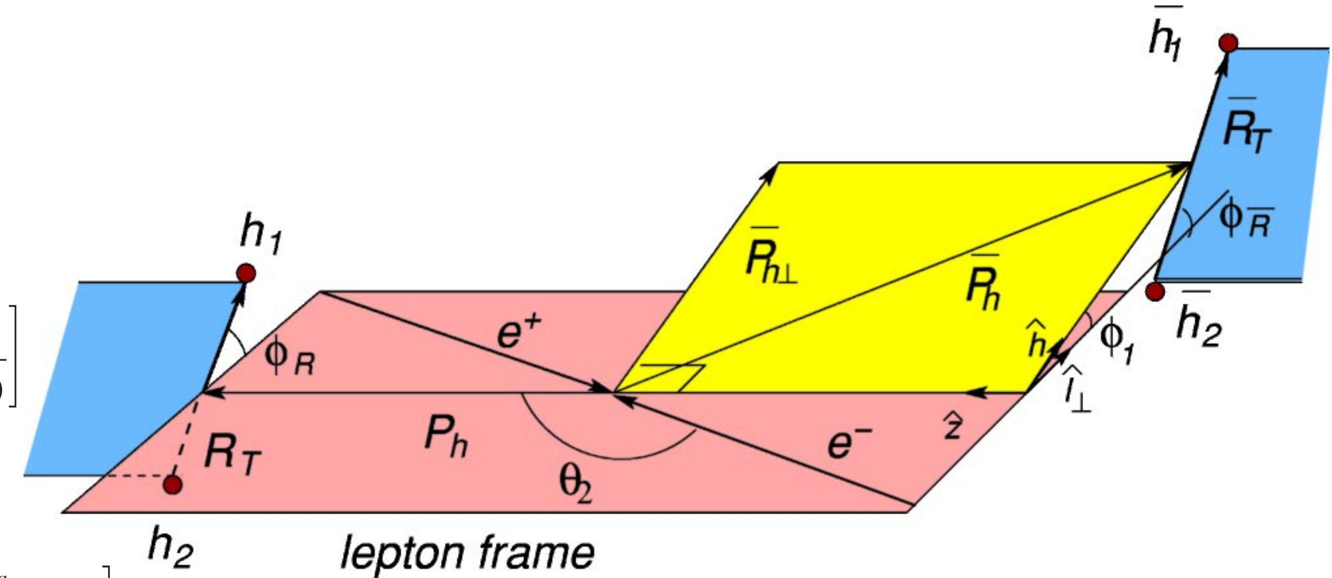
- *Generated by $\cos(\varphi_{RK})$ dependences of unintegrated DiFFs:*

$\varphi_{RK} \equiv \varphi_R - \varphi_k$

$$d\sigma_{UT} \sim \sin(\varphi_R + \varphi_S)\mathcal{C}\left[h_1^{\perp} H^{\sphericalangle}(\cos(\varphi_{RK}))\right]$$
$$+ \sin(\varphi_k + \varphi_S)\mathcal{C}\left[h_1^{\perp} H^{\perp}(\cos(\varphi_{RK}))\right] + ..$$

Back-to-back *two* hadron pairs in e^+e^-

D. Boer et al: PRD 67, 094003 (2003).



$$
\begin{aligned}
&\frac{d\sigma(e^+e^-\to(h_1h_2)(\bar h_1\bar h_2)X)}{d\boldsymbol{q}_T dz\, d\xi\, dM_h^2 d\phi_R\, d\bar z\, d\bar\xi\, d\bar M_h^2 d\phi_{\bar R}\, dy\, d\phi^l} \\
&= \sum_{a,\bar a} e_a^2 \frac{6\alpha^2}{Q^2} z^2\bar z^2 \Big\{ A(y)\mathcal{F}[D_1^a \bar D_1^a] + \cos(2\phi_1)B(y)\mathcal{F}\Big[(2\hat{\boldsymbol h}\cdot\boldsymbol k_T\,\hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T - \boldsymbol k_T\cdot\bar{\boldsymbol k}_T)\frac{H_1^{\perp a}\bar H_1^{\perp a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\Big] \\
&\quad -\sin(2\phi_1)B(y)\mathcal{F}\Big[(\hat{\boldsymbol h}\cdot\boldsymbol k_T\,\hat{\boldsymbol g}\cdot\bar{\boldsymbol k}_T + \hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T\,\hat{\boldsymbol g}\cdot\boldsymbol k_T)\frac{H_1^{\perp a}\bar H_1^{\perp a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\Big] + \cos(\phi_R+\phi_{\bar R}-2\phi^l) \\
&\quad \times B(y)|\boldsymbol R_T||\bar{\boldsymbol R}_T|\mathcal{F}\Big[\frac{H_1^{\sphericalangle a}\bar H_1^{\sphericalangle a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\Big] + \cos(\phi_1+\phi_R-\phi^l)B(y)|\boldsymbol R_T|\mathcal{F}\Big[\hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T\frac{H_1^{\sphericalangle a}\bar H_1^{\perp a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\Big] \\
&\quad -\sin(\phi_1+\phi_R-\phi^l)B(y)|\boldsymbol R_T|\mathcal{F}\Big[\hat{\boldsymbol g}\cdot\bar{\boldsymbol k}_T\frac{H_1^{\sphericalangle a}\bar H_1^{\perp a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\Big] + \cos(\phi_1+\phi_{\bar R}-\phi^l)B(y)|\bar{\boldsymbol R}_T| \\
&\quad \times\mathcal{F}\Big[\hat{\boldsymbol h}\cdot\boldsymbol k_T\frac{H_1^{\perp a}\bar H_1^{\sphericalangle a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\Big] - \sin(\phi_1+\phi_{\bar R}-\phi^l)B(y)|\bar{\boldsymbol R}_T|\mathcal{F}\Big[\hat{\boldsymbol g}\cdot\boldsymbol k_T\frac{H_1^{\perp a}\bar H_1^{\sphericalangle a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\Big] + A(y)|\boldsymbol R_T||\bar{\boldsymbol R}_T| \\
&\quad \times\Big(\sin(\phi_1-\phi_R+\phi^l)\sin(\phi_1-\phi_{\bar R}+\phi^l)\mathcal{F}\Big[\hat{\boldsymbol h}\cdot\boldsymbol k_T\,\hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T\frac{G_1^{\perp a}\bar G_1^{\perp a}}{M_1M_2\bar M_1\bar M_2}\Big] + \sin(\phi_1-\phi_R+\phi^l)\cos(\phi_1-\phi_{\bar R}+\phi^l) \\
&\quad \times\mathcal{F}\Big[\hat{\boldsymbol h}\cdot\boldsymbol k_T\,\hat{\boldsymbol g}\cdot\bar{\boldsymbol k}_T\frac{G_1^{\perp a}\bar G_1^{\perp a}}{M_1M_2\bar M_1\bar M_2}\Big] + \cos(\phi_1-\phi_R+\phi^l)\sin(\phi_1-\phi_{\bar R}+\phi^l)\mathcal{F}\Big[\hat{\boldsymbol g}\cdot\boldsymbol k_T\,\hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T\frac{G_1^{\perp a}\bar G_1^{\perp a}}{M_1M_2\bar M_1\bar M_2}\Big] + \cos(\phi_1-\phi_R+\phi^l) \\
&\quad \times\cos(\phi_1-\phi_{\bar R}+\phi^l)\mathcal{F}\Big[\hat{\boldsymbol g}\cdot\boldsymbol k_T\,\hat{\boldsymbol g}\cdot\bar{\boldsymbol k}_T\frac{G_1^{\perp a}\bar G_1^{\perp a}}{M_1M_2\bar M_1\bar M_2}\Big]\Big)\Big\}, \qquad (19)
\end{aligned}
$$

- ***Can access both helicity and transverse pol. dependent DiFFs:***

$$
\boxed{A^{\cos(\varphi_R+\varphi_{\bar R})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar H_1^{\sphericalangle}(\bar z,M_{\bar h}^2)}{D_1(z,M_h^2)\ \bar D_1(\bar z,M_{\bar h}^2)}}
\qquad
\boxed{A^{\cos(2(\varphi_R-\varphi_{\bar R}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar G_1^{\perp}(\bar z,M_{\bar h}^2)}{D_1(z,M_h^2)\ \bar D_1(\bar z,M_{\bar h}^2)}}
$$

Moments of DiFFs in e$^+$e$^-$

D. Boer et al: PRD 67, 094003 (2003).

- *Entering the integrated cross-section expressions.*

$\cos(\varphi_R - \varphi_k)$ **moment**

$$G_1^{\perp}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; \boxed{(\boldsymbol{k}_T \cdot \boldsymbol{R}_T)}\, G_1^{\perp}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

- ***Differ from SIDIS ! Might affect combined analysis.***

$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; |\boldsymbol{R}_T| H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = H_1^{\sphericalangle,[0]}$$

$$H_{1,e^+e^-}^{\perp}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; |\boldsymbol{k}_T| H_1^{\perp}(z_h, \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

$$H_{1,e^+e^-}^{\perp}(z, M_h^2) = H_1^{\perp,[0]}$$

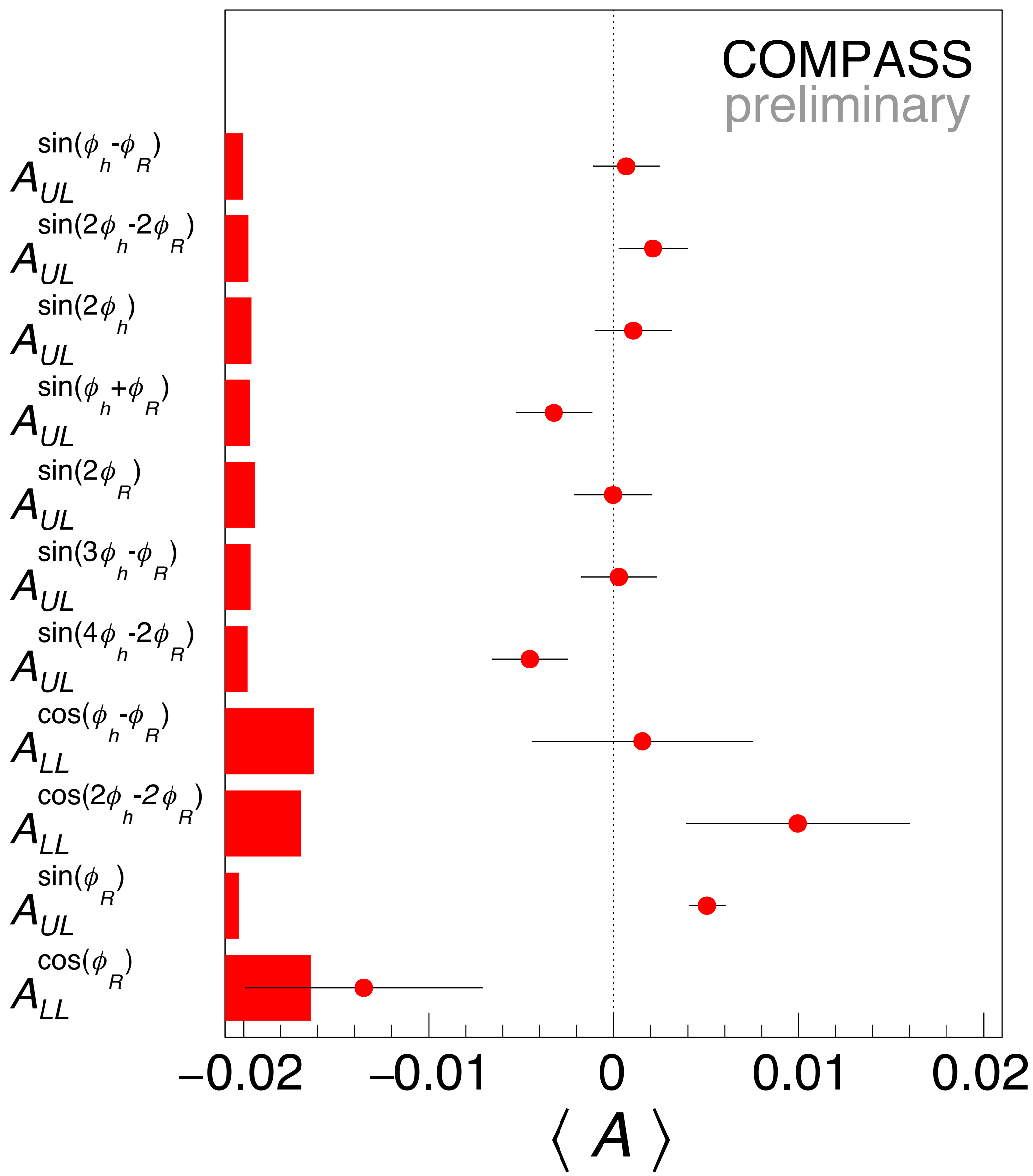
Helicity DiFFs in SIDIS

‣ *SIDIS extraction in COMPASS*

$$d\sigma_{UL} \sim -A(y)\mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2}\; g_{1L}^a\; G_1^{\perp a}\Big]$$

$$+B(y)\mathcal{G}\Big[\frac{p_T k_T \sin(\varphi_p + \varphi_k)}{M M_h}\; h_{1L}^{\perp a}\; H_1^{\perp a}\Big]$$

$$+B(y)\mathcal{G}\Big[\frac{p_T R_T \sin(\varphi_p + \varphi_R)}{M M_h} h_{1L}^{\perp a}\; H_1^{\sphericalangle a}\Big]$$



$$\mathcal{G}[w f^q D^q] \equiv \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T \delta^2\Big(\boldsymbol{k}_T - \boldsymbol{p}_T + \frac{\boldsymbol{P}_{h\perp}}{z}\Big)$$

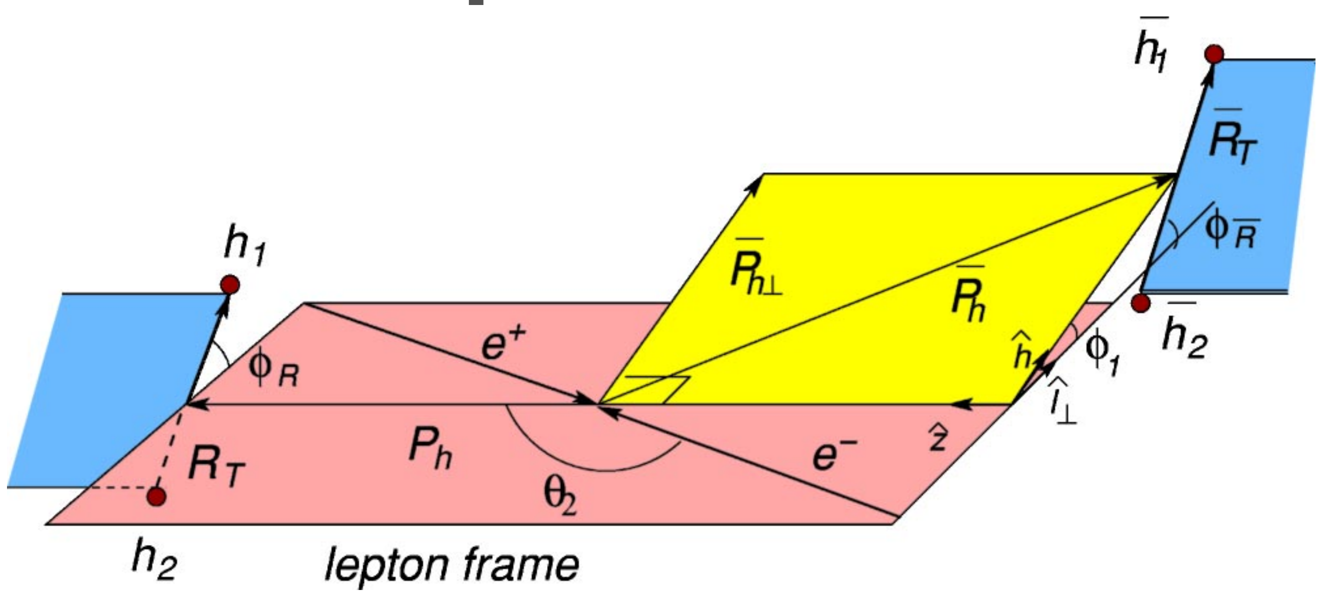
$$\times\, w(\boldsymbol{p}_T, \boldsymbol{k}_T, \boldsymbol{R}_T) f^q(x, \boldsymbol{p}_T^2) D^q(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

✦ $A^{\sin(n(\varphi_h - \varphi_R))}$ **are** ***convolutions*** **of** g_{1L} **and** $G_1^{\perp}$ **!**

Back-to-back *two* hadron pairs in e^+e^-

D. Boer et al: PRD 67, 094003 (2003).

- *Can access both helicity and transverse pol. dependent DiFFs:*

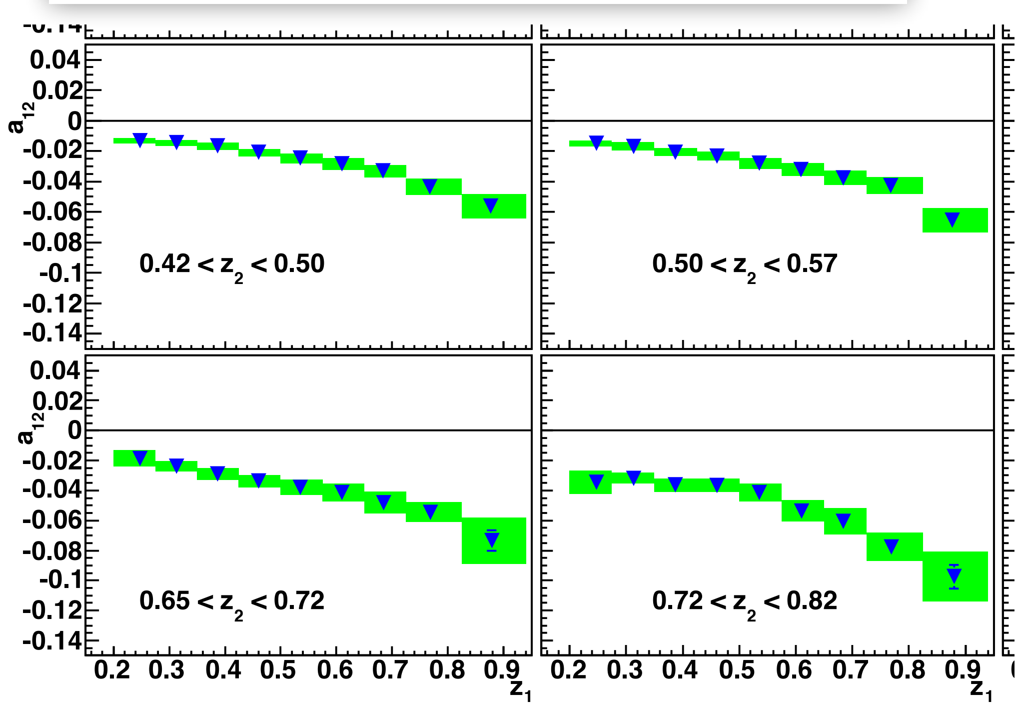


$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z, M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z}, M_{\bar{h}}^2)}{D_1(z, M_h^2)\ \bar{D}_1(\bar{z}, M_{\bar{h}}^2)}$$

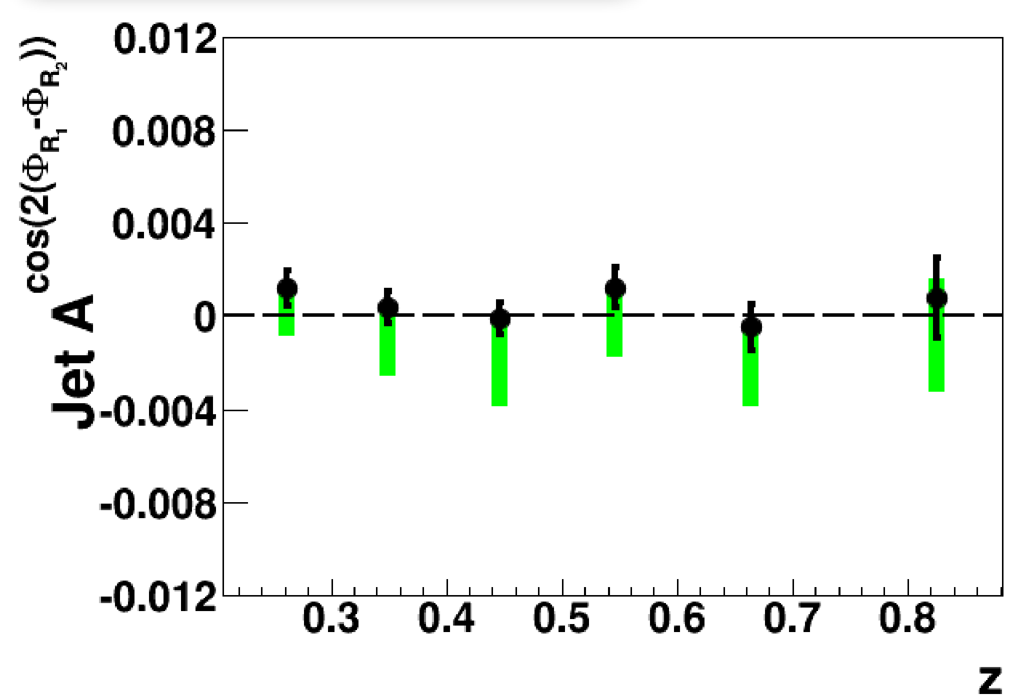
$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z, M_h^2)\bar{G}_1^{\perp}(\bar{z}, M_{\bar{h}}^2)}{D_1(z, M_h^2)\ \bar{D}_1(\bar{z}, M_{\bar{h}}^2)}$$

✦ ***BELLE results.***

Phys.Rev.Lett. 107 (2011) 072004



PoS DIS2015 (2015) 216



Back-to-back *two* hadron pairs in e^+e^-

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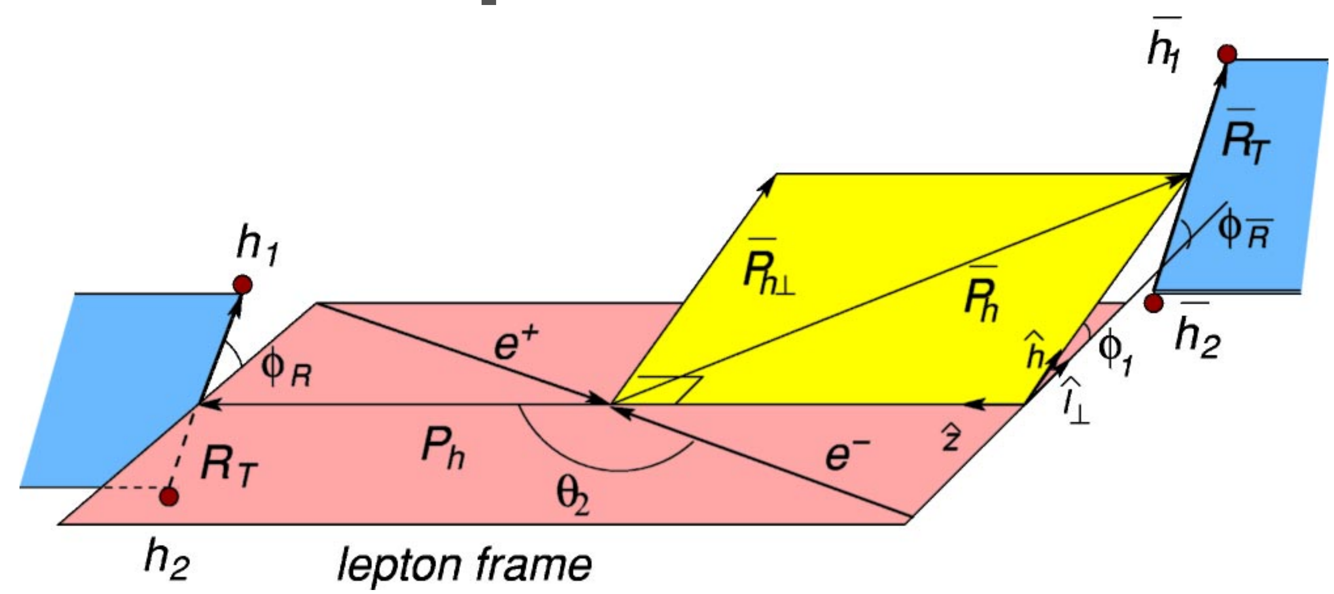
- ***Can access both helicity and transverse pol. dependent DiFFs:***

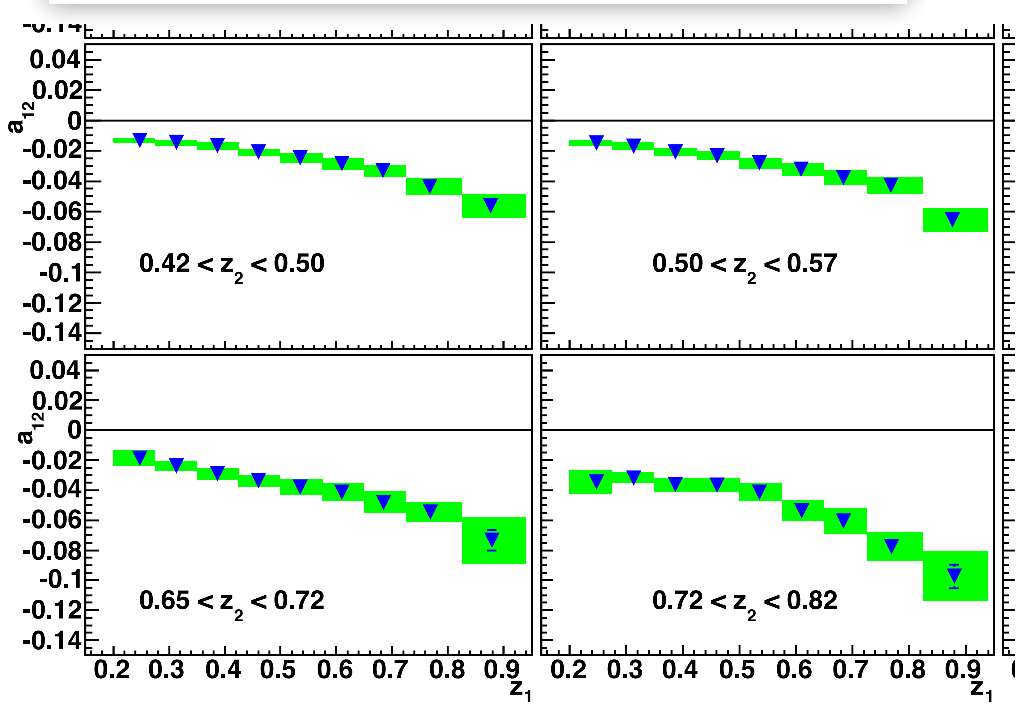


$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

✦ ***BELLE results.***

Phys.Rev.Lett. 107 (2011) 072004



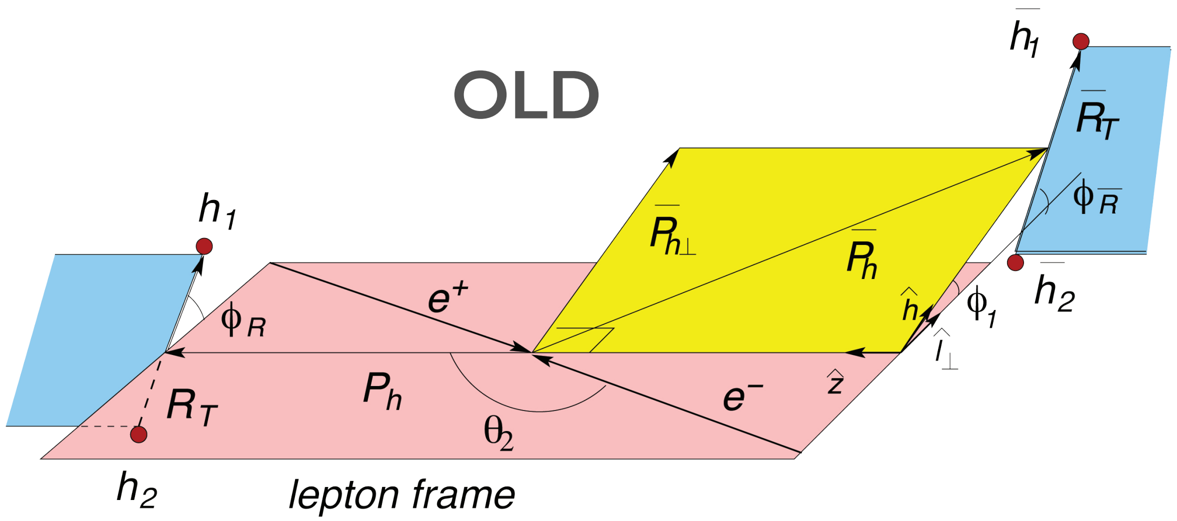
PoS DIS2015 (2015) 216

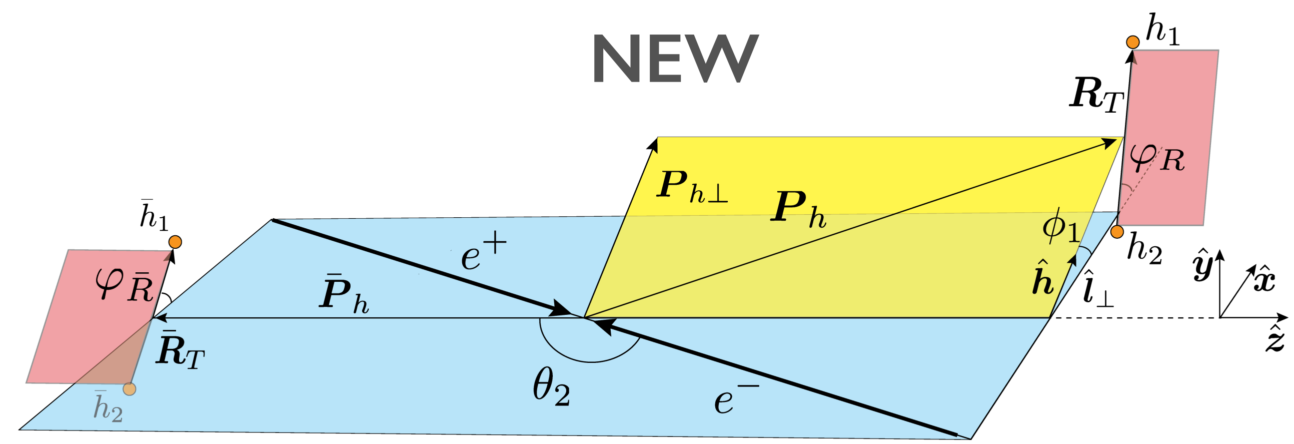
Re-derived e⁺e⁻ Cross Section

H.M. , **Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).**

published today!

- ***An error in kinematics was found:***





- ***The new fully differential cross-section expression:***

$$
\begin{aligned}
\frac{d\sigma\left(e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X\right)}{d^2\boldsymbol{q}_T\,dz\,d\xi\,d\varphi_R\,dM_h^2\,d\bar{z}\,d\bar{\xi}\,d\varphi_{\bar{R}}\,d\bar{M}_h^2\,dy} &= \frac{3\alpha^2}{\pi Q^2} z^2\bar{z}^2 \sum_{a,\bar{a}} e_a^2 \Bigg\{ A(y)\mathcal{F}\left[D_1^a \bar{D}_1^{\bar{a}}\right] \\
&+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar{M}_h}\cos(\varphi_k+\varphi_{\bar{k}})H_1^{\perp a}\bar{H}_1^{\perp\bar{a}}\right] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar{M}_h}\cos(\varphi_R+\varphi_{\bar{R}})H_1^{\sphericalangle a}\bar{H}_1^{\sphericalangle\bar{a}}\right] \\
&+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar{M}_h}\cos(\varphi_k+\varphi_{\bar{R}})H_1^{\perp a}\bar{H}_1^{\sphericalangle\bar{a}}\right] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar{M}_h}\cos(\varphi_R+\varphi_{\bar{k}})H_1^{\sphericalangle a}\bar{H}_1^{\perp\bar{a}}\right] \\
&- A(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|\,|\boldsymbol{k}_T|}{M_h^2}\frac{|\bar{\boldsymbol{R}}_T|\,|\bar{\boldsymbol{k}}_T|}{\bar{M}_h^2}\sin(\varphi_k-\varphi_R)\sin(\varphi_{\bar{k}}-\varphi_{\bar{R}})G_1^{\perp a}\bar{G}_1^{\perp\bar{a}}\right] \Bigg\}.
\end{aligned}
$$

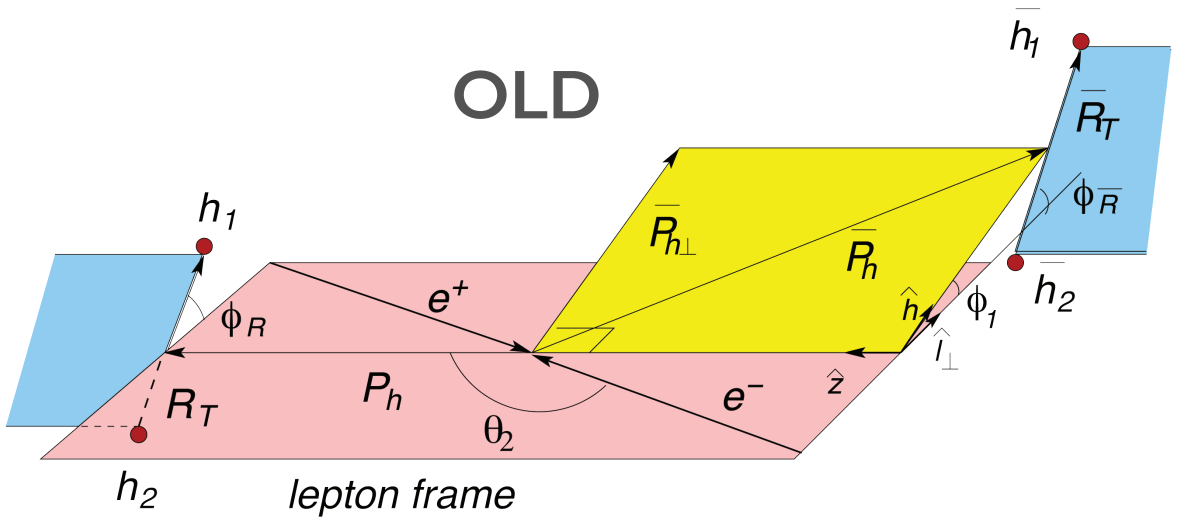
Re-derived e⁺e⁻ Cross Section

H.M., Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).

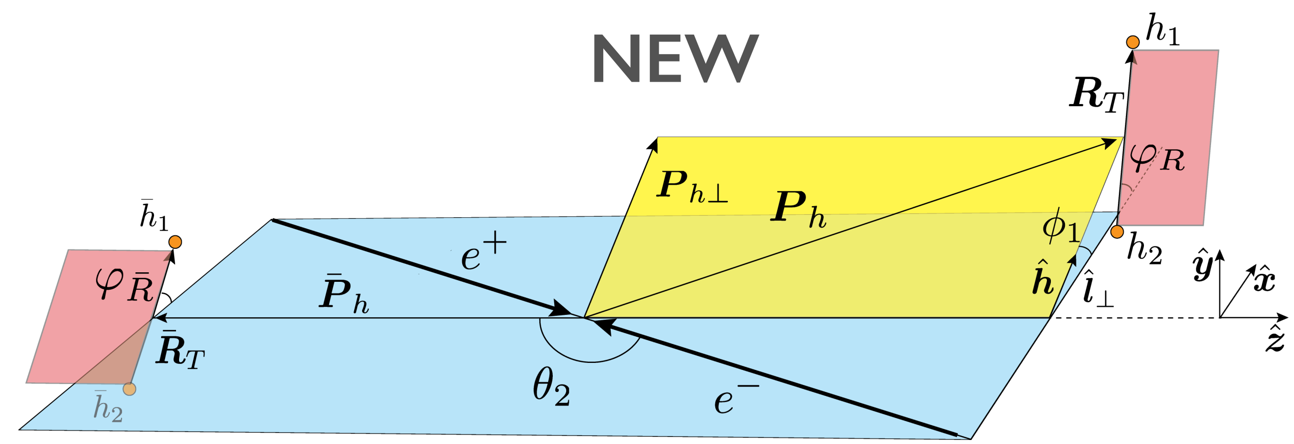
published today!

- *An error in kinematics was found:*

OLD



NEW



- *The new fully differential cross-section expression:*

$$\frac{d\sigma\Big(e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X\Big)}{d^2\boldsymbol{q}_T dz d\xi d\varphi_R dM_h^2 d\bar{z} d\bar{\xi} d\varphi_{\bar{R}} d\bar{M}_h^2 dy} = \frac{3\alpha^2}{\pi Q^2} z^2 \bar{z}^2 \sum_{a,\bar{a}} e_a^2 \Bigg\{ A(y)\mathcal{F}\Big[D_1^a \bar{D}_1^{\bar{a}}\Big]$$

$$\mathcal{F}[wD^a\bar{D}^{\bar{a}}] = \int d^2\boldsymbol{k}_T d^2\bar{\boldsymbol{k}}_T \; \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) w(\boldsymbol{k}_T, \bar{\boldsymbol{k}}_T, \boldsymbol{R}_T, \bar{\boldsymbol{R}}_T) \; D^a \; D^{\bar{a}}.$$

$$- A(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|\,|\boldsymbol{k}_T|}{M_h^2}\frac{|\bar{\boldsymbol{R}}_T|\,|\bar{\boldsymbol{k}}_T|}{\bar{M}_h^2}\sin(\varphi_k - \varphi_R)\sin(\varphi_{\bar{k}} - \varphi_{\bar{R}}) G_1^{\perp a}\bar{G}_1^{\perp\bar{a}}\right]\Bigg\}.$$

IFFs in e⁺e⁻ and SIDIS.

H.M. , Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).

- *The asymmetry now involves **exactly the same** integrated IFF as in SIDIS!*

$$A^{\cos(\varphi_R+\varphi_{\bar R})} = \frac{1}{2}\frac{B(y)}{A(y)}\frac{\sum_{a,\bar a} e_a^2 H_1^{\sphericalangle a}(z,M_h^2)\bar H_1^{\sphericalangle \bar a}(\bar z,\bar M_h^2)}{\sum_{a,\bar a} e_a^2 D_1^{a}(z,M_h^2)\bar D_1^{\bar a}(\bar z,\bar M_h^2)}$$

$$D_1(z,M_h^2) \equiv z^2\int d^2\boldsymbol{k}_T\int d\xi\, D_1^{[0]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

$$H^{\sphericalangle}_{1,e^+e^-}(z,M_h^2) = H_1^{\sphericalangle,[0]} + H_1^{\perp,[1]} \equiv H^{\sphericalangle}_{1,SIDIS}(z,M_h^2)$$

- ***All the previous extractions of the transversity are valid !***

Helicity-dependent DiFF in e^+e^-

H.M. , Kotzinian, Thomas: arXiv:1712.06384.

- *The relevant terms involving* $G_1^\perp$:

$$d\sigma_L \sim \mathcal{F}\Big[\frac{(\boldsymbol{R}_T\times\boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T\times\bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2}G_1^{\perp a}(\boldsymbol{R}_T\cdot\boldsymbol{k}_T)\bar{G}_1^{\perp\bar{a}}(\bar{\boldsymbol{R}}_T\cdot\bar{\boldsymbol{k}}_T)\Big]$$

- ***Note:*** *any azimuthal moment involving only* $\varphi_R,\ \varphi_{\bar{R}}$ *is zero.*

Break-up the convolution: $\int d^2\boldsymbol{q}_T\delta^2(\boldsymbol{k}_T+\bar{\boldsymbol{k}}_T-\boldsymbol{q}_T)$

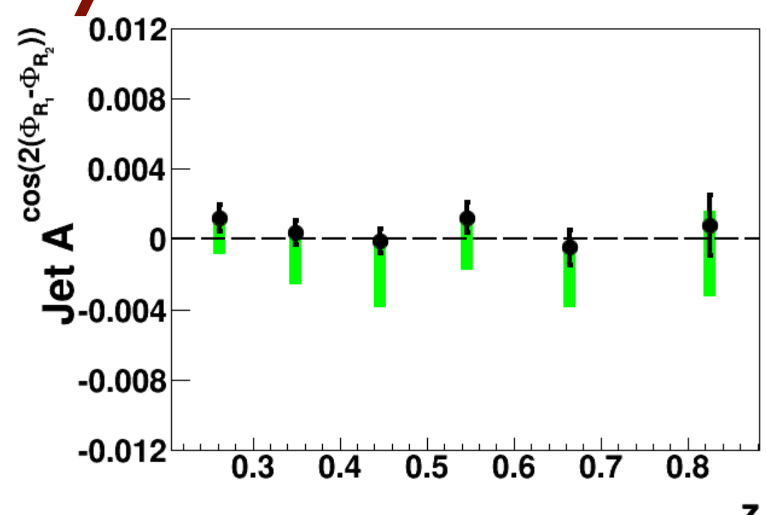
decouple $\boldsymbol{k_T}$ on both sides

Using: $\varphi_k \to \varphi_k' + \varphi_R$, $\int d^2\boldsymbol{k}_T \sin(\varphi_k)\cos(n\varphi_k) = 0$

$$\boxed{\langle f(\varphi_R,\varphi_{\bar{R}})\rangle_L = 0}$$

- ***The old asymmetry by Boer et. al. exactly vanishes!***
- ***Explains the BELLE results.***

$$\boxed{A^{\Rightarrow} = \frac{\langle\cos(2(\varphi_R-\varphi_{\bar{R}}))\rangle}{\langle 1\rangle} = 0!}$$



New way to access $G_1^{\perp}$ DiFF in e$^+$e$^-$

H.M., Kotzinian, Thomas: arXiv:1712.06384.

- *The relevant terms involving* $G_1^{\perp}$ *:*

$$d\sigma_L \sim \mathcal{F}\Big[\frac{(\boldsymbol{R}_T \times \boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T \times \bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2} G_1^{\perp a}(\boldsymbol{R}_T \cdot \boldsymbol{k}_T)\bar{G}_1^{\perp \bar{a}}(\bar{\boldsymbol{R}}_T \cdot \bar{\boldsymbol{k}}_T)\Big]$$

- ***Need a q_T-weighted asymmetry to get non-zero result***

$$\left\langle \frac{q_T^2\left(3\sin(\varphi_q-\varphi_R)\sin(\varphi_q-\varphi_{\bar{R}})+\cos(\varphi_q-\varphi_R)\cos(\varphi_q-\varphi_{\bar{R}})\right)}{M_h\bar{M}_h}\right\rangle$$
$$= \frac{12\alpha^2 A(y)}{\pi Q^2}\sum_{a,\bar{a}} e_a^2\left(G_1^{\perp a,[0]}-G_1^{\perp a,[2]}\right)\left(\bar{G}_1^{\perp \bar{a},[0]}-G_1^{\perp \bar{a},[2]}\right),$$

- ***A new asymmetry to access*** $G_1^{\perp a} \equiv G_1^{\perp a,[0]} - G_1^{\perp a,[2]}$

$$\boxed{A_{e^+e^-}^{\Rightarrow}(z,\bar{z},M_h^2,\bar{M}_h^2) = 4\frac{\sum_{a,\bar{a}} G_1^{\perp a}(z,M_h^2)\; G_1^{\perp \bar{a}}(\bar{z},\bar{M}_h^2)}{\sum_{a,\bar{a}} D_1^{a}(z,M_h^2)\; D_1^{\bar{a}}(\bar{z},\bar{M}_h^2)}}$$

New way to access $G_1^{\perp}$ DiFF in e+e-

H.M. , Kotzinian, Thomas: arXiv:1712.06384.

- *The relevant terms involving $G_1^{\perp}$:*

$$d\sigma_L \sim \mathcal{F}\Big[\frac{(\boldsymbol{R}_T\times\boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T\times\bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2}G_1^{\perp a}(\boldsymbol{R}_T\cdot\boldsymbol{k}_T)\bar{G}_1^{\perp\bar{a}}(\bar{\boldsymbol{R}}_T\cdot\bar{\boldsymbol{k}}_T)\Big]$$

- ***Need a q_T-weighted asymmetry to get non-zero result***

additional $\sin(\varphi_k-\varphi_R)$

$$\left\langle\frac{q_T^2\left(3\sin(\varphi_q-\varphi_R)\sin(\varphi_q-\varphi_{\bar{R}})+\cos(\varphi_q-\varphi_R)\cos(\varphi_q-\varphi_{\bar{R}})\right)}{M_h\bar{M}_h}\right\rangle$$

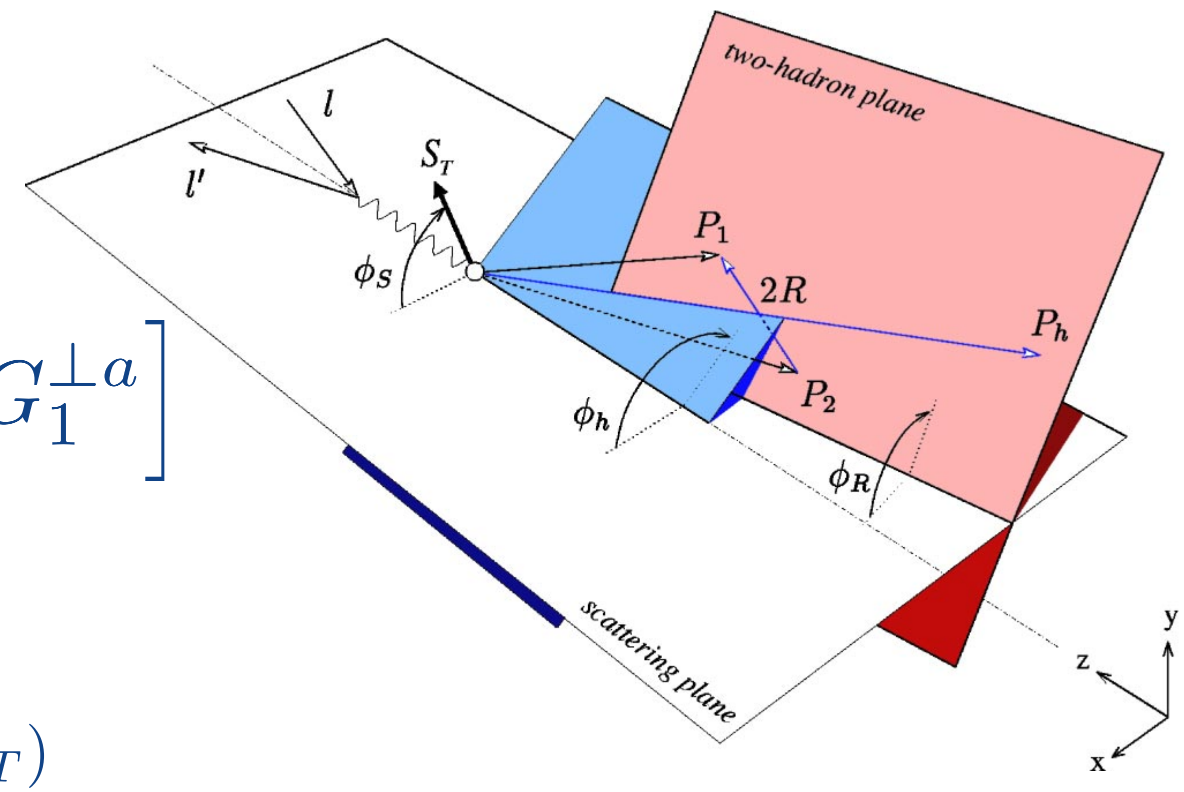
$$=\frac{12\alpha^2A(y)}{\pi Q^2}\sum_{a,\bar{a}}e_a^2\left(G_1^{\perp a,[0]}-G_1^{\perp a,[2]}\right)\left(\bar{G}_1^{\perp\bar{a},[0]}-G_1^{\perp\bar{a},[2]}\right),$$

- ***A new asymmetry to access*** $G_1^{\perp a}\equiv G_1^{\perp a,[0]}-G_1^{\perp a,[2]}$

$$\boxed{A_{e^+e^-}^{\Rightarrow}(z,\bar{z},M_h^2,\bar{M}_h^2)=4\frac{\sum_{a,\bar{a}}G_1^{\perp a}(z,M_h^2)\;G_1^{\perp\bar{a}}(\bar{z},\bar{M}_h^2)}{\sum_{a,\bar{a}}D_1^a(z,M_h^2)\;D_1^{\bar{a}}(\bar{z},\bar{M}_h^2)}}$$

New way to access $G_1^\perp$ DiFF in *SIDIS*

H.M., Kotzinian, Thomas: arXiv:1712.06384.



- ***The relevant terms involving*** $G_1^\perp$ **:**

$$d\sigma_{UL} \sim S_L \mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2}\; g_{1L}^a\; G_1^{\perp a}\Big]$$

$$\mathcal{G}[w f^q D^q] \equiv \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T \delta^2\Big(\boldsymbol{k}_T - \boldsymbol{p}_T + \frac{\boldsymbol{P}_{h\perp}}{z}\Big)$$
$$\times\, w(\boldsymbol{p}_T, \boldsymbol{k}_T, \boldsymbol{R}_T) f^q(x, \boldsymbol{p}_T^2) D^q(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

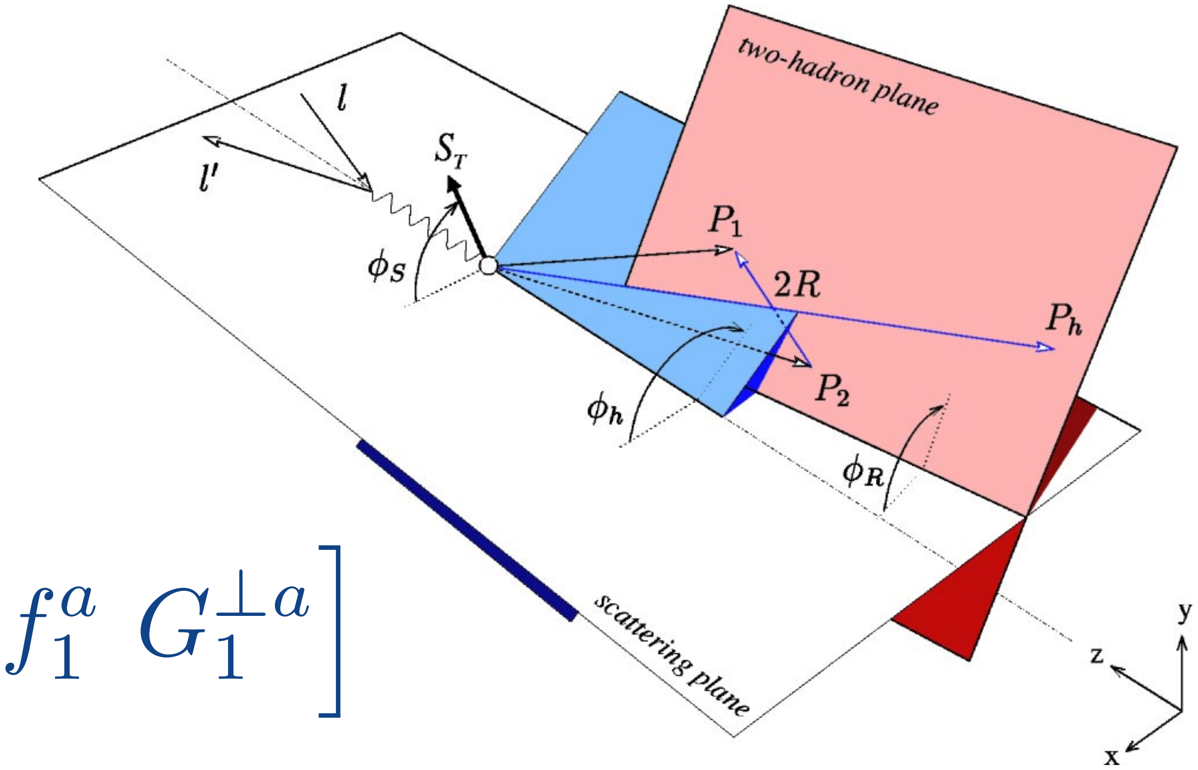
- ***Weighted moment accesses same*** $G_1^\perp$ ***as in*** **e^+e^- .**

$$\Big\langle \frac{P_{h\perp}\sin(\varphi_h - \varphi_R)}{M_h}\Big\rangle_{UL} \sim S_L \sum_a e_a^2\; g_{1L}^a(x)\; z\; G_1^{\perp a}(z, M_h^2)$$

$$\boxed{A^{\rightrightarrows}_{SIDIS}(x, z, M_h^2) = S_L \frac{\sum_a g_{1L}^a(x)\;\; z\; G_1^{\perp a}(z, M_h^2)}{\sum_a f_1^a(x)\;\; D_1^a(z, M_h^2)}.}$$

New way to access $G_1^{\perp}$ DiFF in *SIDIS: II*

- ***The relevant terms involving*** $G_1^{\perp}$ **:**



Consider a polarized beam.

$$d\sigma_{LU} \sim \lambda_e \mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2}\; f_1^a\; G_1^{\perp a}\Big]$$

- ***Weighted moment accesses same*** $G_1^{\perp}$ ***as in*** **e^+e^- .**

$$\left\langle \frac{P_{h\perp}\sin(\varphi_h - \varphi_R)}{M_h} \right\rangle_{LU} \sim \lambda_e \sum_a e_a^2\; f_1^a(x)\; z\; G_1^{\perp a}(z, M_h^2)$$

$$\boxed{A_{SIDIS}^{\hookrightarrow}(x, z, M_h^2) \sim \lambda_e \frac{C'(y)}{A'(y)} \frac{\sum_a f_1^a(x)\; z\; G_1^{\perp a}(z, M_h^2)}{\sum_a f_1^a(x)\; D_1^a(z, M_h^2)}.}$$

CONCLUSIONS I

❖ DiFFs provide information on the polarization of the fragmenting quark.

❖ Two problems appeared recently:

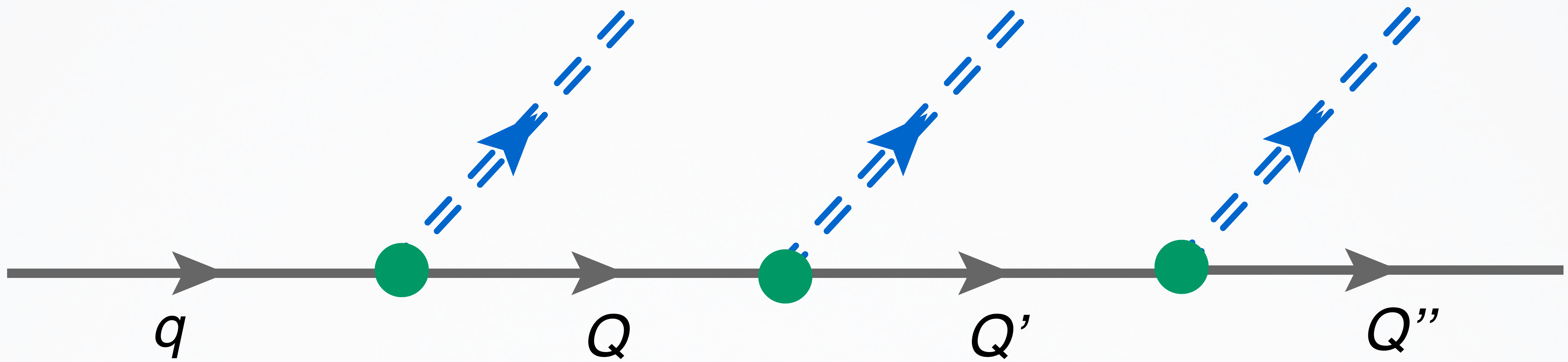
- *Inconsistency* of IFF definitions in *SIDIS* and **e^+e^-** asymmetries.
- *No signal* for the helicity-dependent DiFF from BELLE.

❖ *Re-derived* cross section for **e^+e^-** *resolved both issues*.

❖ *New asymmetries* to measure $G_1^\perp$ in *SIDIS* and **e^+e^-** .

PART II

Dihadron Correlations In Polarized Quark Hadronization:



The Quark-jet Framework

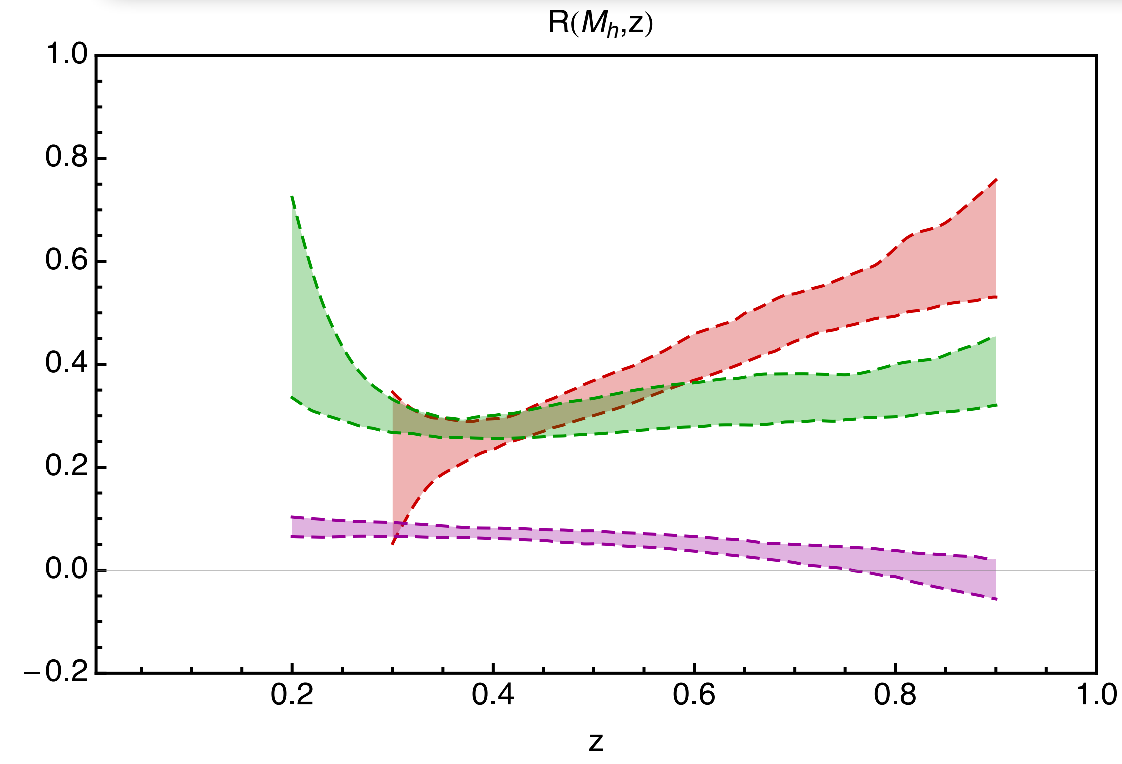
Phys. Rev. D96 074010, (2017); Phys. Rev. D97, 014019 (2018).

Current Challenges

1) Phenomenological Extractions of DiFFs.

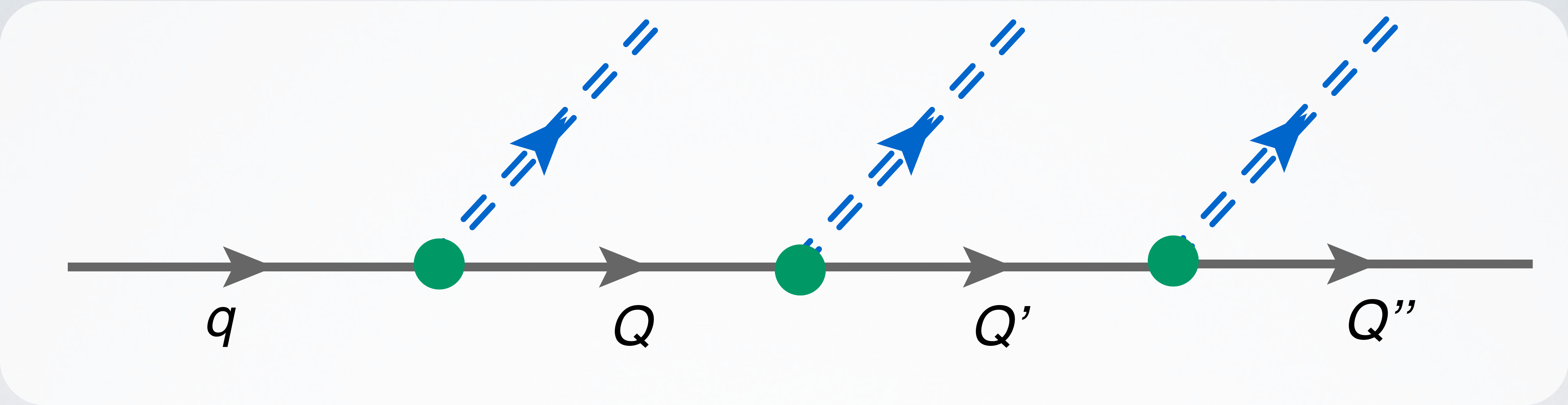
- Unpolarised DiFFs from *PYTHIA*
- Still Large Uncertainties.
- Simplistic Approximations.
- Limited kinematic region.

Radici et al: JHEP JHEP 1505 (2015) 123.



2) Full Event Generators:

- *No* Mainstream MC generator *includes spin* in Full Hadronization *yet*: *PYTHIA, HERWIG, SHERPA…*
- MC generators are needed to support mapping of the 3D structure of nucleon at *JLab12, BELLE II, EIC.*



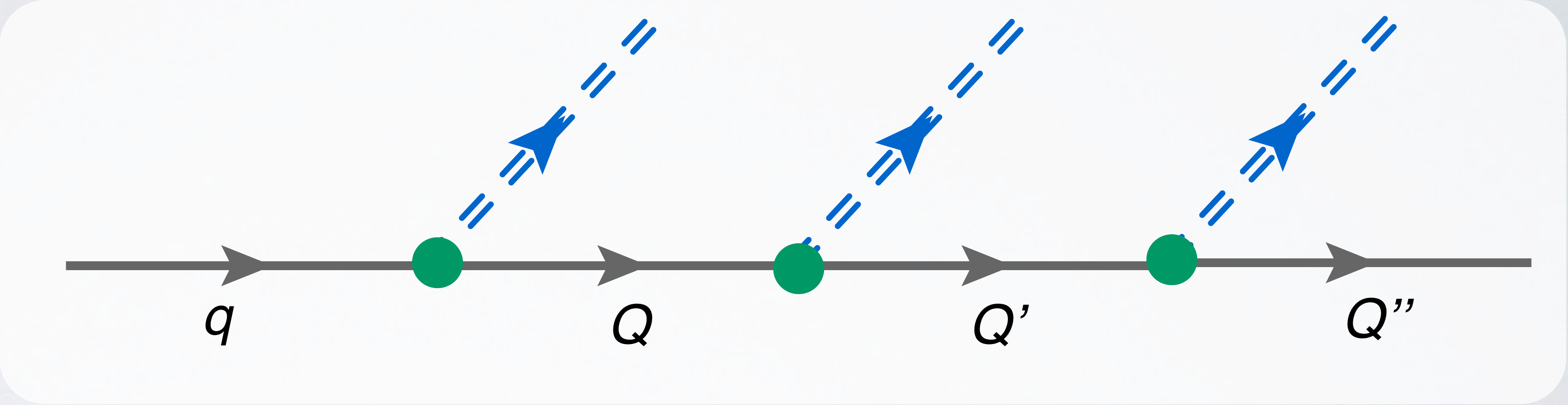
The Quark-jet Framework

THE QUARK JET MODEL

Field, Feynman: Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



$$D_q^h(z) \quad = \hat{d}_q^h(z) \quad + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h(\frac{z}{y})\frac{1}{y}$$

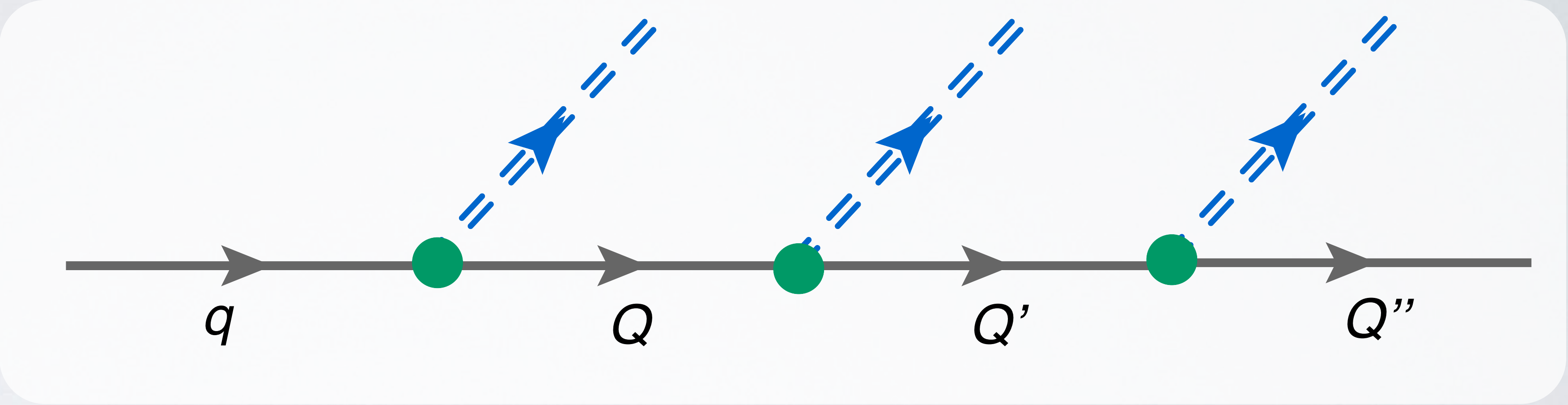
$$\hat{d}_q^h(z) = \hat{d}_q^{Q'}(1-z)|_{h=\bar{Q}'q}$$

THE QUARK JET MODEL

Field, Feynman: Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



Probability of finding hadron h with mom. frac. [z, z+dz] in a jet of quark q

The probability scales with mom. fraction

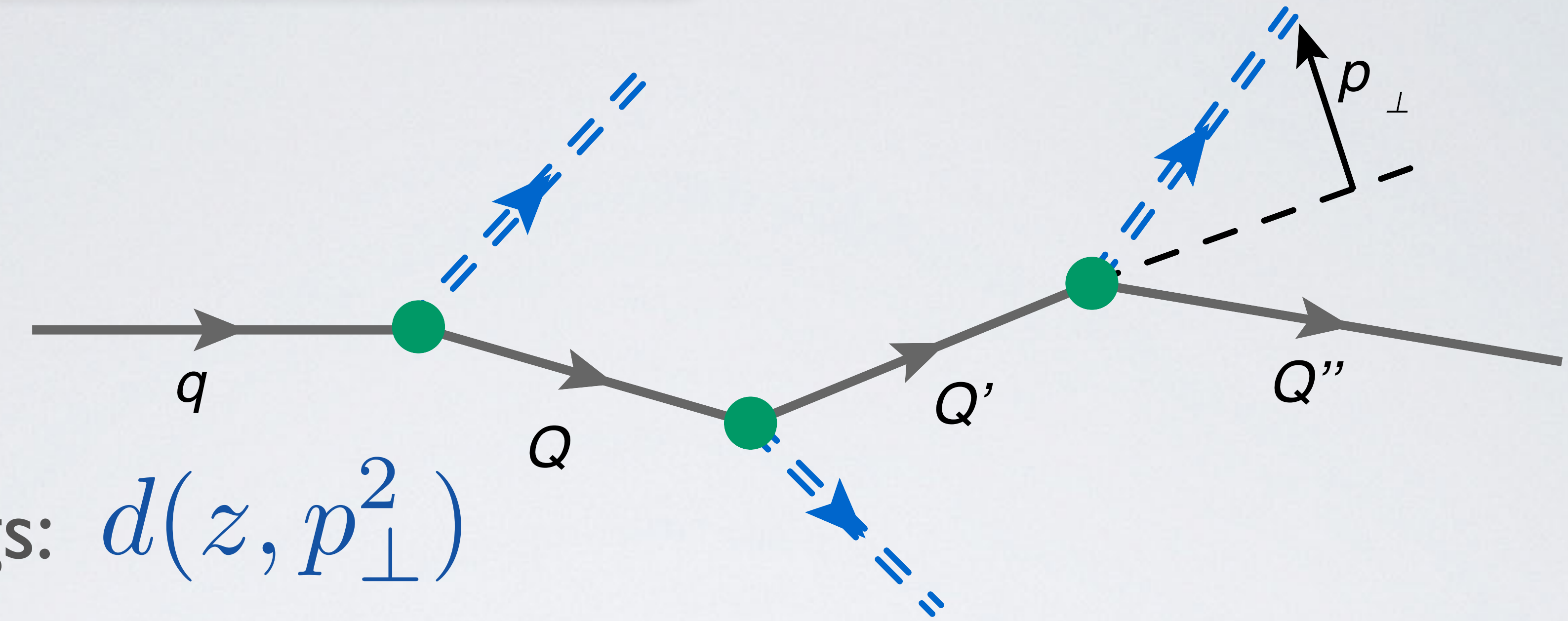
$$D_q^h(z)dz = \hat{d}_q^h(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h\left(\frac{z}{y}\right)\frac{dz}{y}$$

Prob. of emitting at step 1

Prob. of mom. [y, y+dy] is transferred to jet at step 1.

INCLUDING THE *TRANSVERSE MOMENTUM*

H.M., Bentz, Cloet, Thomas, PRD.85:014021, 2012

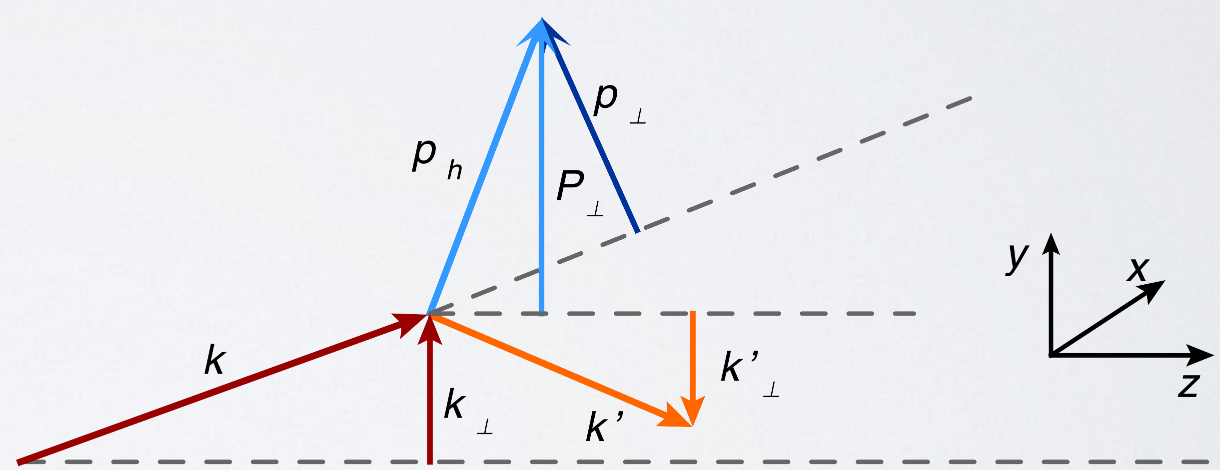


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp$$

$$\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp$$



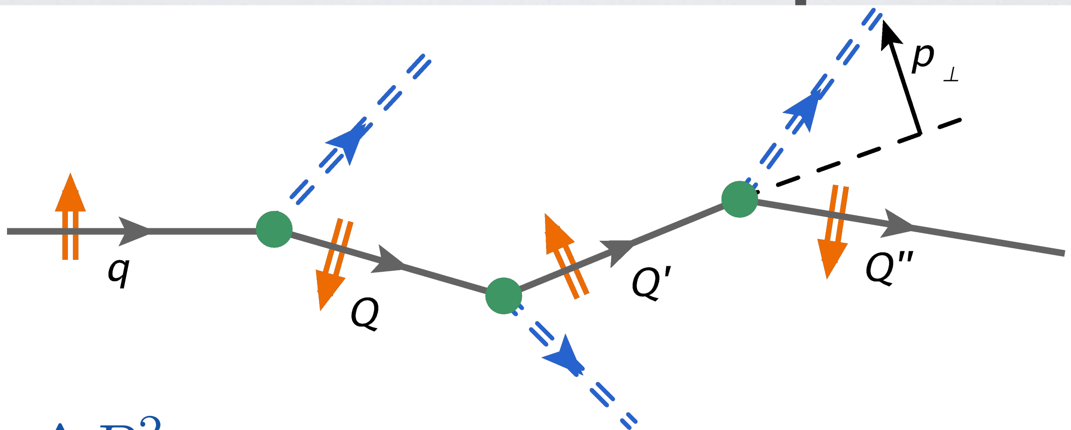
- Calculate the Number Density

$$D_q^h(z, P_\perp^2)\Delta z\ \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

POLARIZATION IN QUARK-JET FRAMEWORK

H.M.,Bentz, Thomas, PRD.86:034025, (2012). H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

- Extend Quark-jet Model to include Spin.

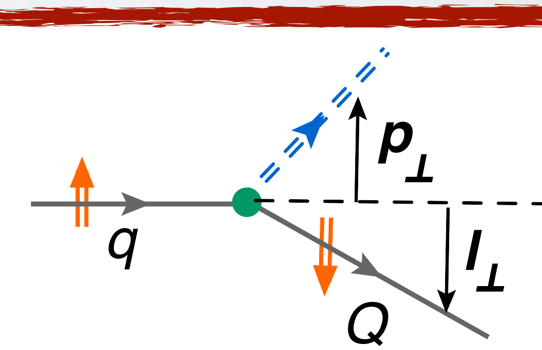


$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\, \Delta z\, \frac{\Delta P_\perp^2}{2}\, \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$

- Input Elementary Collins Function: *Model or Parametrization*

- *Calc. Spin of the remnant quark:* $\mathbf{S}'$

Previously: constant values for spin flip probability: $\mathcal{P}_{SF}$



✦ Use fit form to extract unpol. and Collins FFs from $D_{h/q^\uparrow}$.

$$\boxed{F(c_0, c_1) \equiv c_0 - c_1 \sin(\varphi_C)}$$

$$\boxed{D_{h/q^\uparrow}(z, p_\perp^2, \varphi) = D^{h/q}(z, p_\perp^2) - H^{\perp h/q}(z, p_\perp^2) \frac{p_\perp s_T}{z m_h} \sin(\varphi_C)}$$

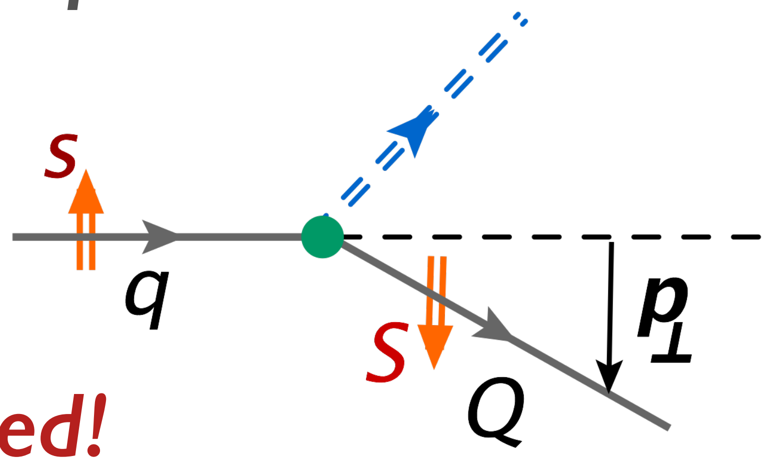
SPIN TRANSFER

Bentz, Kotzinian, H.M, Ninomiya, Thomas, Yazaki: Phys.Rev. D94 034004 (2016).

✦NJL-jet MKIII:

- *The probability for the process* $q \to Q$, *initial spin* $\mathbf{s}$ *to* $\mathbf{S}$

$$F^{q\to Q}(z,\mathbf{p}_\perp;\mathbf{s},\mathbf{S}) = \alpha_\mathbf{s} + \boldsymbol{\beta}_\mathbf{s}\cdot\mathbf{S}$$

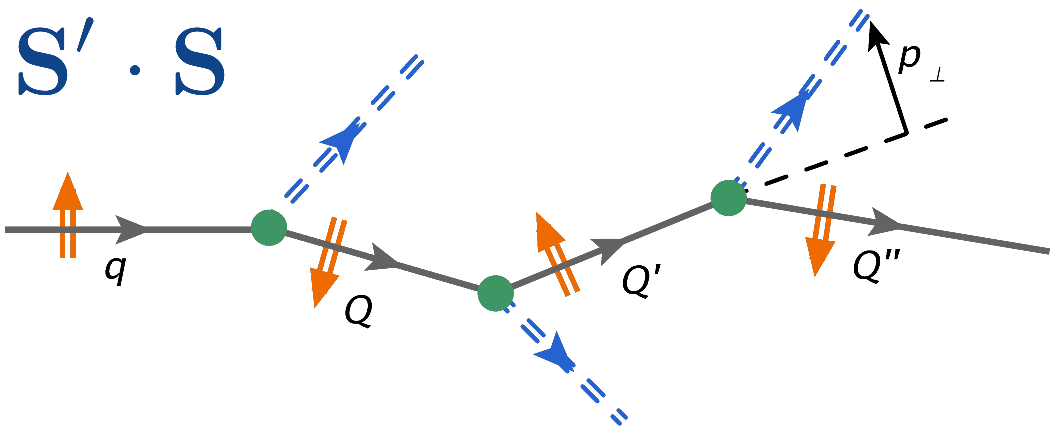


- ***Intermediate*** *quarks in quark-jet are* ***unobserved!***

Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii: **QUANTUM ELECTRODYNAMICS (1982).**

$$F^{q\to Q}(z,\mathbf{p}_\perp;\mathbf{s},\mathbf{S}) \sim \mathrm{Tr}[\rho^{\mathbf{S}'}\rho^{\mathbf{S}}] \sim 1 + \mathbf{S}'\cdot\mathbf{S}$$

$$\mathbf{S}' = \frac{\boldsymbol{\beta}_\mathbf{s}}{\alpha_\mathbf{s}}$$



- *Remnant quark's* $\mathbf{S}'$ ***uniquely determined by*** $z,\mathbf{p}_\perp$ ***and*** $\mathbf{s}$ ***!***

- *Process probability is* ***the same*** *as transition to* ***unpolarized state.***

$$F^{q\to Q}(z,\mathbf{p}_\perp;\mathbf{s},\mathbf{0}) = \alpha_s$$

REMNANT QUARK'S POLARISATION

✦ We can express the spin of the remnant quark $\mathrm{S}' = \dfrac{\beta_{\mathrm{s}}}{\alpha_{\mathrm{s}}}$ in terms of *quark-to-quark TMD FFs*.

$$\alpha_q \equiv D(z, \boldsymbol{p}_\perp^2) + (\boldsymbol{p}_\perp \times \boldsymbol{s}_T) \cdot \hat{\boldsymbol{z}} \frac{1}{z\mathcal{M}} H^\perp(z, \boldsymbol{p}_\perp^2)$$

$$\beta_{q\parallel} \equiv s_L \, G_L(z, \boldsymbol{p}_\perp^2) - (\boldsymbol{p}_\perp \cdot \boldsymbol{s}_T) \frac{1}{z\mathcal{M}} H_L^\perp(z, \boldsymbol{p}_\perp^2)$$

$$\boldsymbol{\beta}_{q\perp} \equiv \boldsymbol{p}'_\perp \frac{1}{z\,\mathcal{M}} D_T^\perp(z, \boldsymbol{p}_\perp^2) - \boldsymbol{p}_\perp \frac{1}{z\mathcal{M}} s_L G_T(z, \boldsymbol{p}_\perp^2) + \boldsymbol{s}_T \, H_T(z, \boldsymbol{p}_\perp^2) + \boldsymbol{p}_\perp (\boldsymbol{p}_\perp \cdot \boldsymbol{s}_T) \frac{1}{z^2\mathcal{M}^2} H_T^\perp(z, \boldsymbol{p}_\perp^2)$$

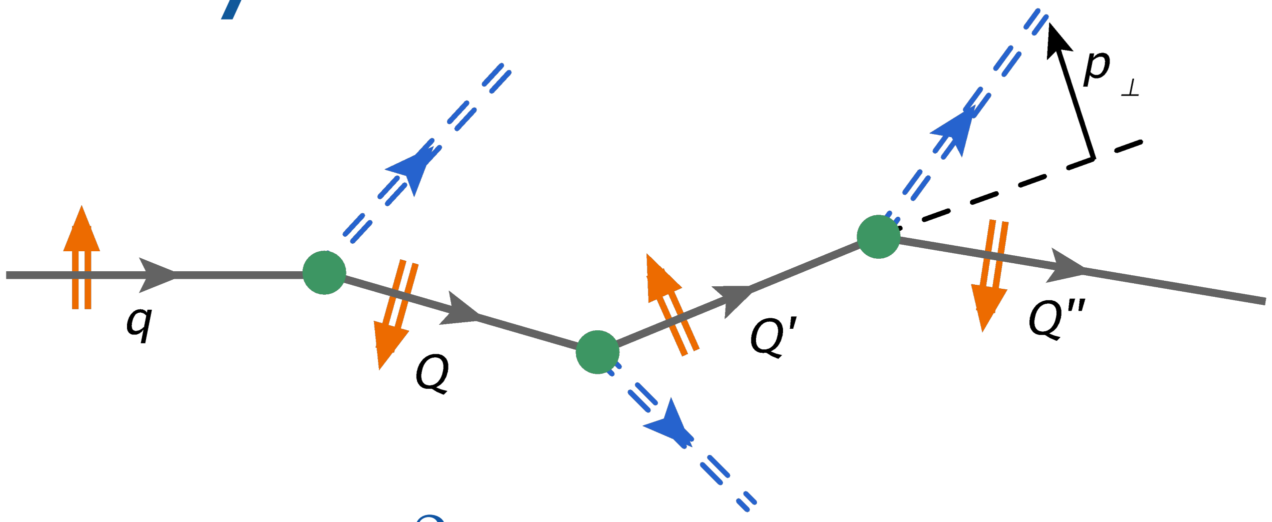
$$F^{q \to Q}(z, \boldsymbol{p}_\perp; \boldsymbol{s}, \boldsymbol{S})$$

Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

MC SIMULATION OF FULL HADRONIZATION

H.M., Kotzinian, Thomas: Phys. Rev. D95 04021, (2017)

✦ We can consider many hadron emissions.



✦ We can sample the $h, z, p_{\perp}^{2}, \varphi_{h}$ using

$$\boxed{f^{q\to h}(z, p_{\perp}^{2}, \varphi_{h}; \mathbf{S}_{T})}$$

✦ Determine the momenta in the initial frame and calculate

$$\Delta N = \langle N_{q}^{h_1 h_2}(z, z+\Delta z, \varphi, \varphi+\Delta\varphi, ...)\rangle$$

✦ *Calculate the remnant quark's spin:* $\mathrm{S}' = \dfrac{\boldsymbol{\beta}_{\mathbf{s}}}{\alpha_{\mathbf{s}}}$

✦ We only need the "elementary" splittings.

$\boxed{f^{q\to h}}$ $\boxed{f^{q\to Q}}$

ELEMENTARY SPLITTINGS

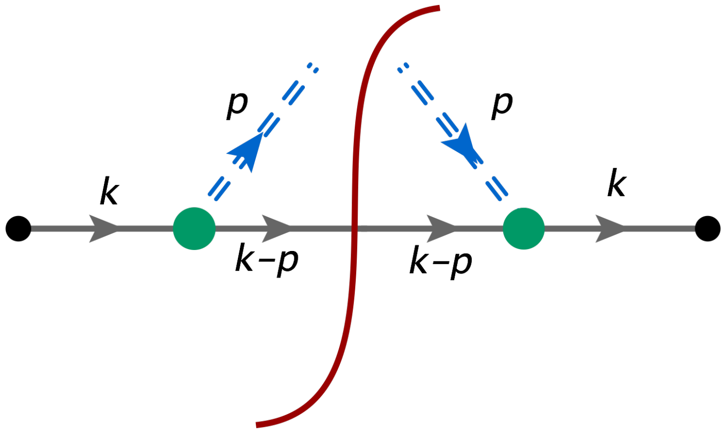
H.M., Thomas, Bentz: PRD. 83:07400; PRD.83:114010, 2011.

- Quark-quark correlator:

$$\Delta_{ij}(z,p_\perp) = \frac{1}{2N_c\, z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, e^{ip\cdot\xi} \times \langle 0|\mathcal{U}_{(\infty,\xi)}\,\psi_i(\xi)|h,X\rangle_{\text{out out}}\langle h,X|\bar{\psi}_j(0)\,\mathcal{U}_{(0,\infty)}|0\rangle\Big|_{\xi^-=0}$$

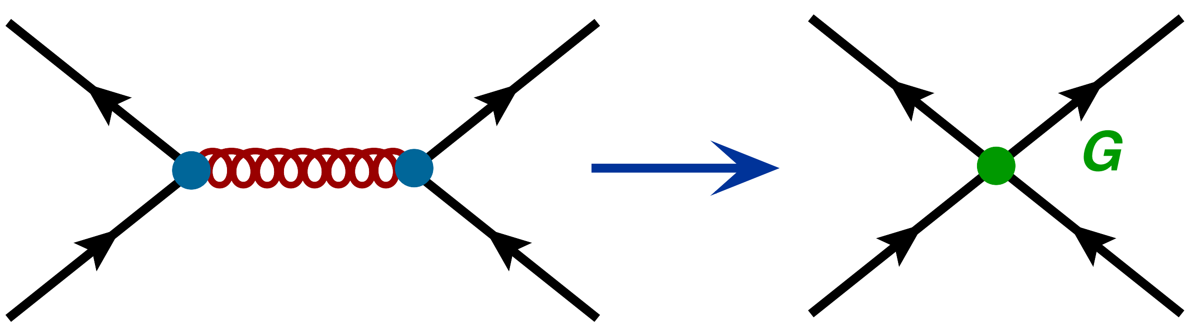
- One-quark truncation of the wavefunction: $q \to Qh$

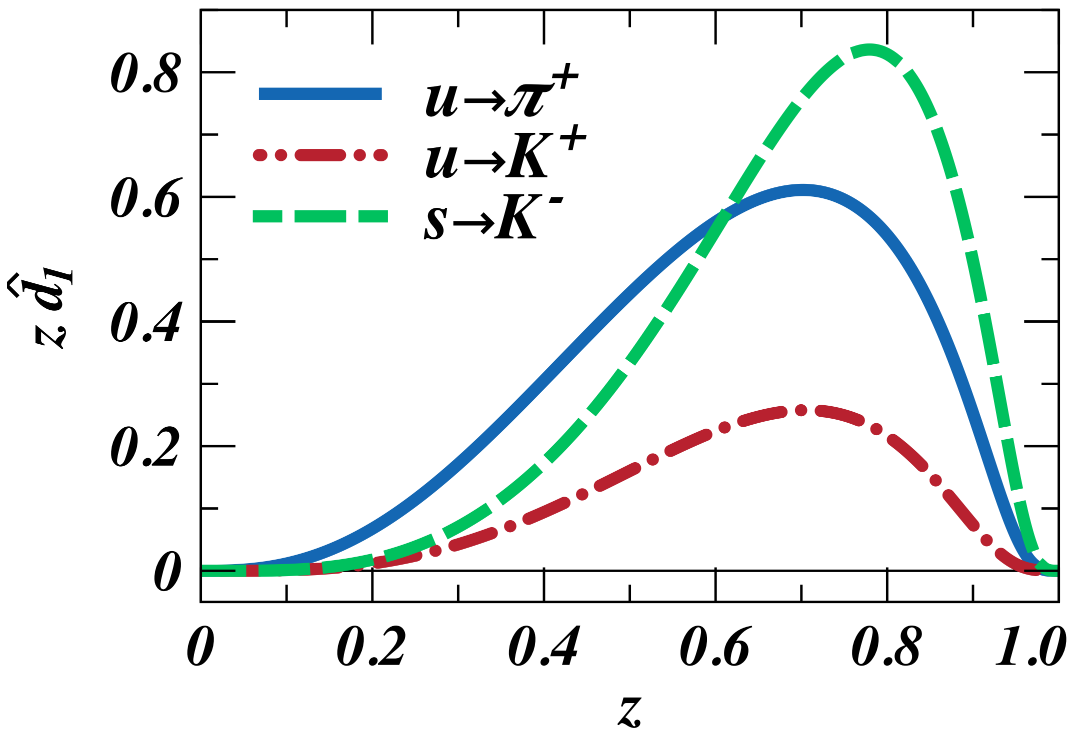
$$d_q^h(z,p_\perp^2) = \frac{1}{2}\mathrm{Tr}[\Delta_0(z,p_\perp^2)\gamma^+]$$



- Use Nambu--Jona-Lasinio (NJL) Effective quark model:

$$\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$$





TWO HADRON CORRELATIONS:
DIHADRON FRAGMENTATION FUNCTIONS

Number Densities

- ***The full number density:***

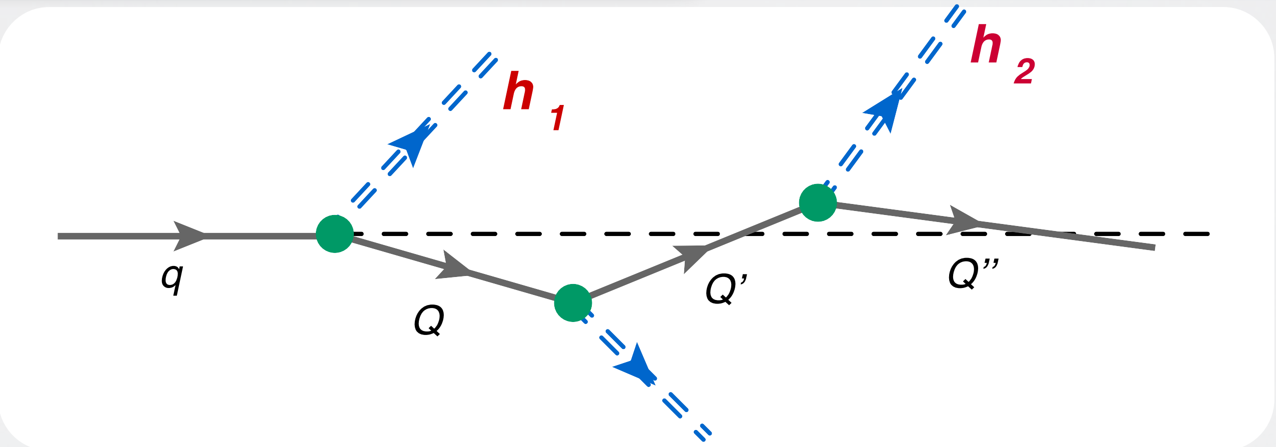
$$
\begin{aligned}
F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;\boldsymbol{s}) =\ & D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) \\
& + s_L\,\frac{(\boldsymbol{R}_T\times\boldsymbol{k}_T)\cdot\hat{\boldsymbol{z}}}{M_h^2}G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) \\
& + \frac{(\boldsymbol{s}_T\times\boldsymbol{R}_T)\cdot\hat{\boldsymbol{z}}}{M_h}H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) \\
& + \frac{(\boldsymbol{s}_T\times\boldsymbol{k}_T)\cdot\hat{\boldsymbol{z}}}{M_h}H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)
\end{aligned}
$$

- ***The differential number of hadron pairs:***

$$
dN_q^{h_1h_2} = F_q^{h_1h_2}(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;\boldsymbol{s})\ dz\ d\xi\ d^2\boldsymbol{k}_T\ d^2\boldsymbol{R}_T
$$

UNPOLARIZED DIHADRON FRAGMENTATIONS

H.M. , Thomas, Bentz: PRD.88:094022, (2013)



- The probability density for observing two hadrons:

$$P_1 = (z_1 k^-, P_1^+, \boldsymbol{P}_{1,\perp}),\ \ P_1^2 = M_{h1}^2$$

$$P_2 = (z_2 k^-, P_2^+, \boldsymbol{P}_{2,\perp}),\ \ P_2^2 = M_{h2}^2$$

- The corresponding number density:

$$\boxed{D_q^{h_1 h_2}(z, M_h^2)\ \Delta z\ \Delta M_h^2 = \langle N_q^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2 + \Delta M_h^2)\rangle}$$

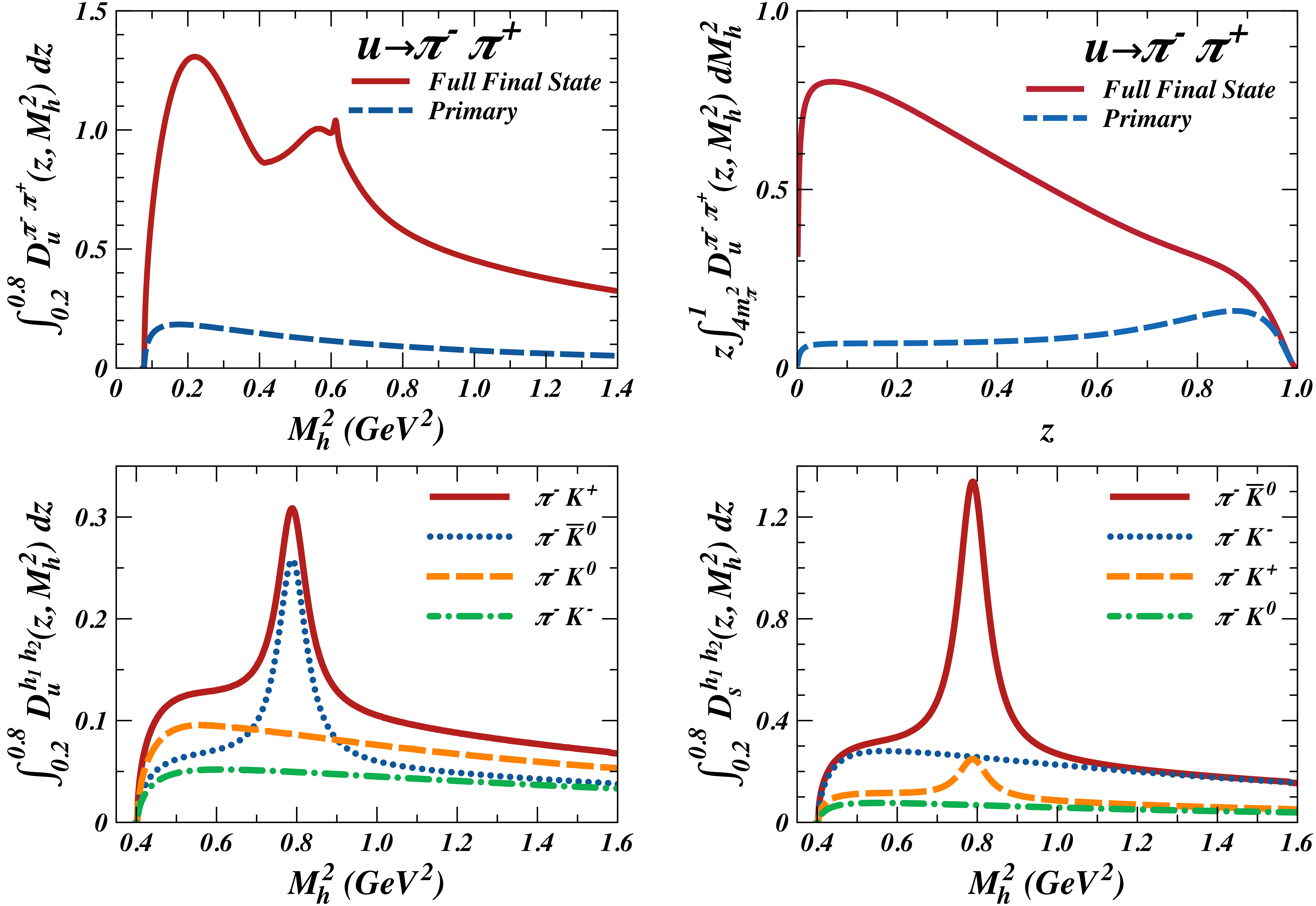
$$z = z_1 + z_2 \qquad M_h^2 = (P_1 + P_2)^2$$

- Kinematic Constraint.

$$\boxed{z_1 z_2 M_h^2 - (z_1 + z_2)(z_2 M_{h1}^2 + z_1 M_{h2}^2) \geq 0}$$

- In MC simulations record all the pairs in every decay chain.

Effect of VMs on Unpol. DiFFs



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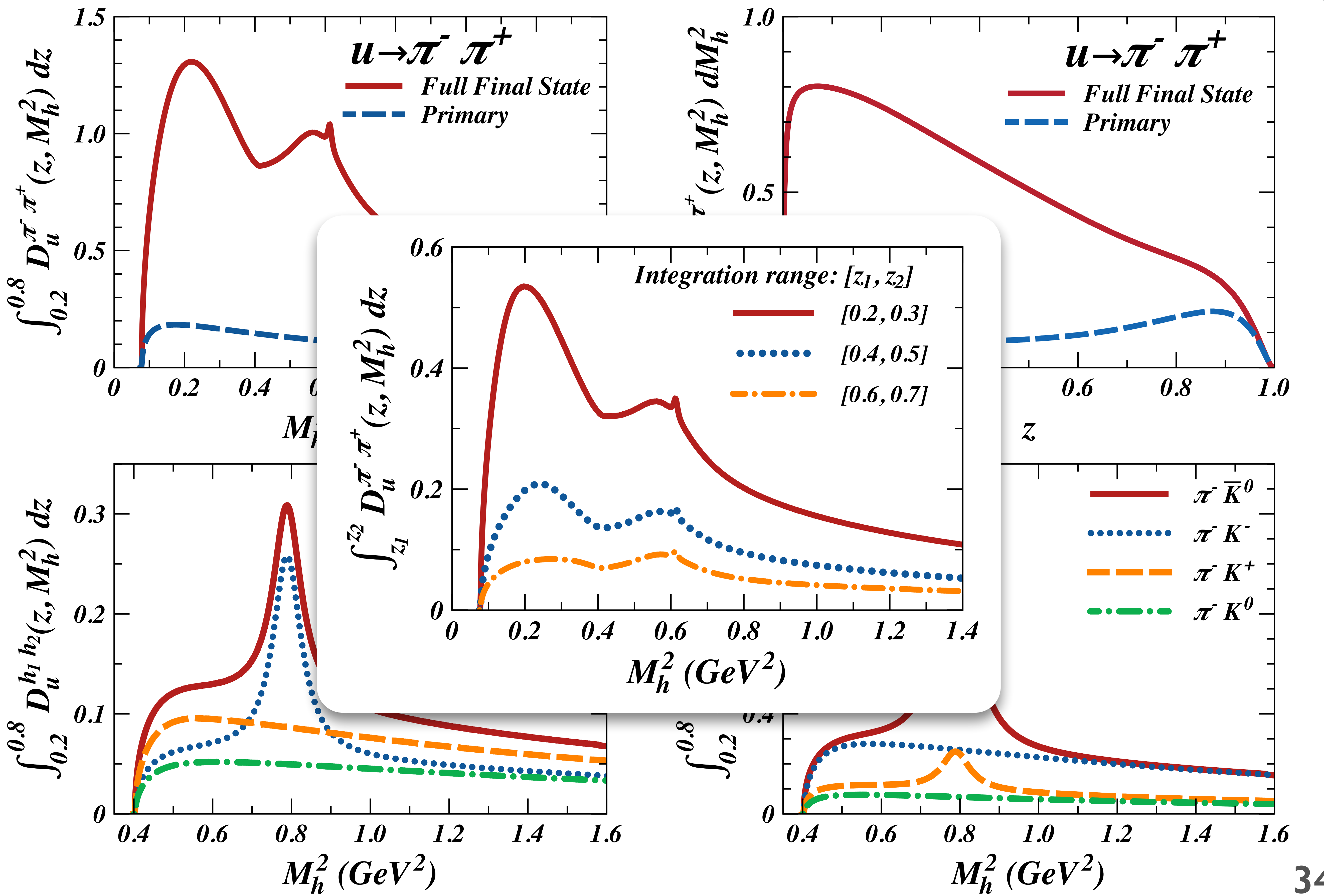

Longitudinal Polarisation in DiHadron FFs

DIFFS FROM THE NUMBER DENSITY

H.M., Kotzinian, Thomas: Phys. Rev. D96 074010, (2017)

- ✦ **Can only calculate number density form MC simulations.**
- ✦ **Extract DiFFs from specific angular modulations.**

$$F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L) = D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK})) - s_L \frac{R_T k_T \sin(\varphi_{RK})}{M_h^2} G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK}))$$

- ✦ **Unpolarized DiFF:** ***straight forward integration*** **of number density.**

$$\boxed{D_1(z,M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \; F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L)}$$

- ✦ **Need** $\cot(\varphi_{RK})$ **to extract** ***helicity dependent DiFF!***

$$\tilde{G}_1^{\perp,[n]}(z,M_h^2) = \int d\xi \int d^2\boldsymbol{k}_T \frac{R_T k_T}{M_h^2} G_1^{\perp,[n]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2)$$

$$\boxed{\tilde{G}_1^{\perp,[n]}(z,M_h^2) = -\frac{1}{s_L}\int d\xi \int d^2\boldsymbol{k}_T \int d\varphi_R \frac{\cos(n\,\varphi_{RK})}{\sin(\varphi_{RK})} F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T)}$$

$$\tilde{G}_1^{\perp} \equiv \tilde{G}_1^{\perp,[1]} = -\frac{1}{s_L}\int d\xi \int d^2\boldsymbol{k}_T \int d\varphi_R \cot(\varphi_{RK}) F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T)$$

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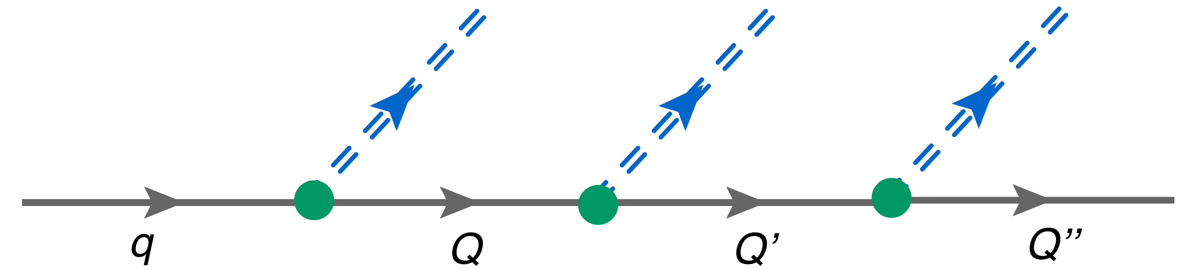
Note: here we use the definition by Boer et. al.

$$\tilde{G}_1^{\perp} \equiv \tilde{G}_1^{\perp,[1]} = -\frac{1}{s_L}\int d\xi \int d^2\boldsymbol{k}_T \int d\varphi_R \cot(\varphi_{RK}) F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T)$$

The Total Number of Pion Pairs

✦ **Validate MC by analytically calculating the total number of pion pairs produced for given N_L.**

$$(\underbrace{\pi^+, \pi^-, \pi^+, \ldots,}_{N_L - n_0} \underbrace{\pi^0, \pi^0, \pi^0}_{n_0}).$$



$$\boxed{\mathcal{N}^{(\pi^+\pi^-)}(N_L) = \sum_{n_0=0}^{n_0=N_L} C_{N_L}^{n_0} \left(\frac{2}{3}\right)^{N_L - n_0} \left(\frac{1}{3}\right)^{n_0} U\left(\frac{N_L - n_0}{2}\right) D\left(\frac{N_L - n_0}{2}\right).}$$

✦ **Extraction from DiFFs.**

$$\mathcal{N}_{MC}^{(\pi^+\pi^-)}(N_L) = \int_0^1 dz\ D_{1,[N_L]}^{u\to\pi^+\pi^-}(z)$$

N_L	$\mathcal{N}^{(\pi^+\pi^-)}$	$\mathcal{N}_N^{(\pi^+\pi^-)}$	$\mathcal{N}_{MC}^{(\pi^+\pi^-)}$	$\mathcal{N}_{MC,z_{min}}^{(\pi^+\pi^-)}$
2	$\frac{4}{9}$	0.44444	0.4444	0.350175
3	$\frac{28}{27}$	1.03704	1.03694	0.683999
4	$\frac{152}{81}$	1.87654	1.87641	0.959588
5	$\frac{712}{243}$	2.93004	2.92992	1.11531
6	$\frac{3068}{729}$	4.2085	4.20882	1.18162
7	$\frac{12484}{2187}$	5.70828	5.70867	1.20282
8	$\frac{48752}{6561}$	7.43057	7.43047	1.20809

✓ **MC simulations and Integral Expressions agree very well!**

✓ **z cuts allow fast convergence with N_L.**

LONGITUDINAL POLARISATION

✦ **DiFF for longitudinally polarized quark:** $s_L \; (\boldsymbol{k}_T \times \boldsymbol{R}_T) \cdot \hat{z}$

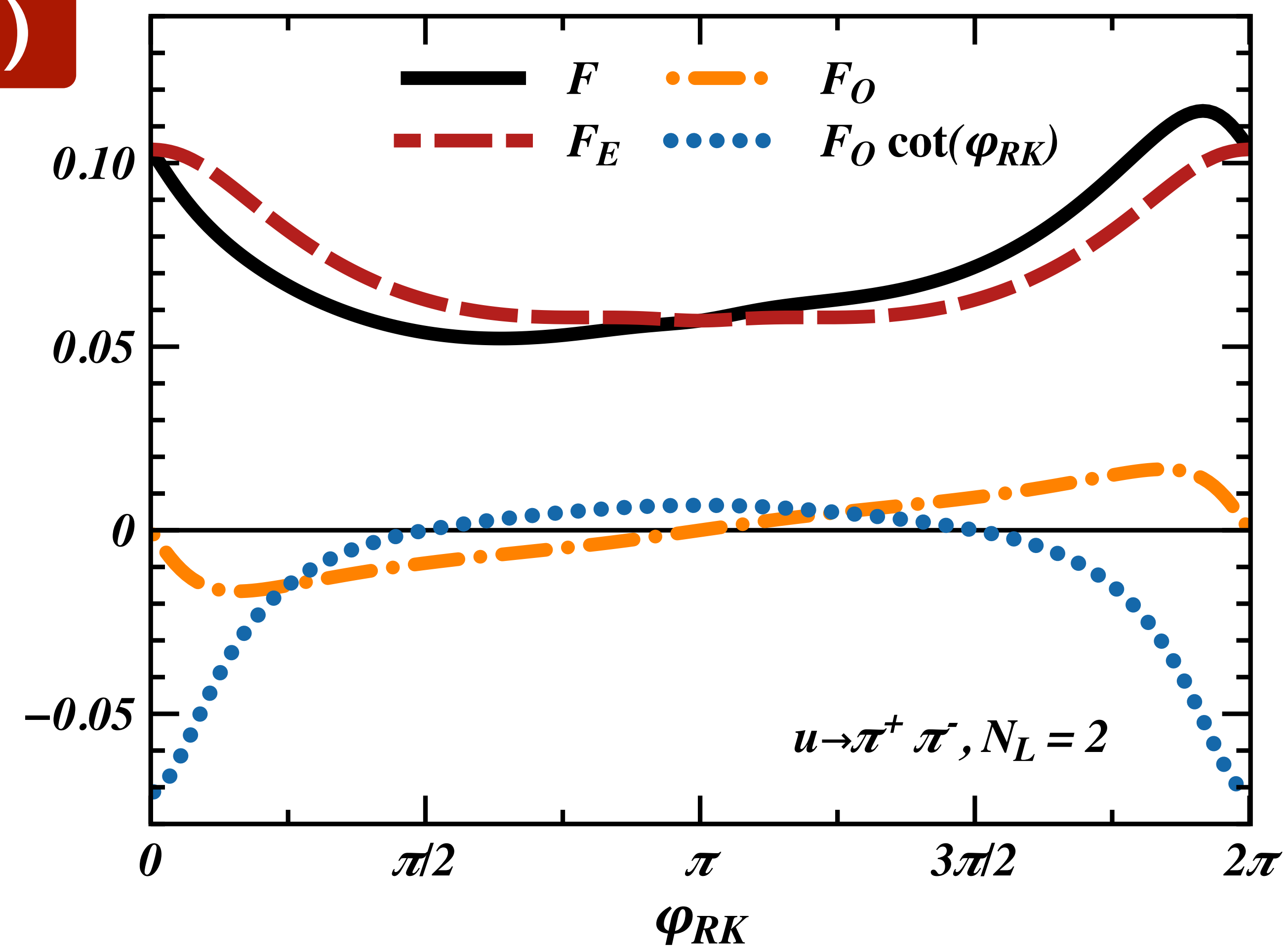
$$\tilde{G}_1^{\perp}(z) = -\frac{1}{s_L} \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \; \cot(\varphi_{RK}) F(z, \xi, \boldsymbol{k}_T, \boldsymbol{R}_T; s_L).$$

✦ **The extraction method works: the angular dependence for N_L=2.**

(given *large enough* statistics!)

$$F_E(\varphi_{RK}) = \frac{F(\varphi_{RK}) + F(2\pi - \varphi_{RK})}{2}$$

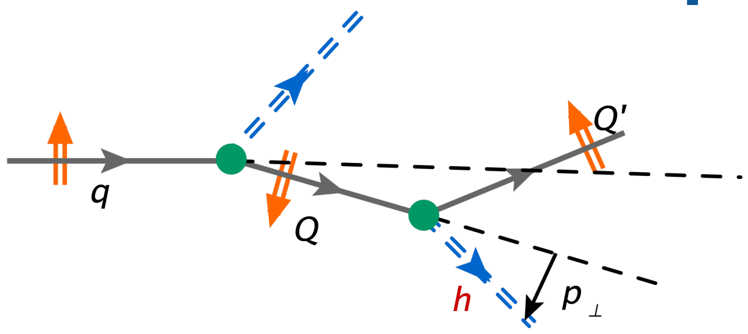
$$F_O(\varphi_{RK}) = \frac{F(\varphi_{RK}) - F(2\pi - \varphi_{RK})}{2}$$



VALIDATION: 2 PRODUCED HADRONS

✦ **Validate MC simulations by comparing to explicit Integral Expressions (IE). Only pions produced in the first two steps!**

$$F^{(2)}_{q\to h_1 h_2} = \sum_{q_1} \hat{f}^{q\to q_1+h_1} \otimes \hat{f}^{q_1\to h_2}.$$

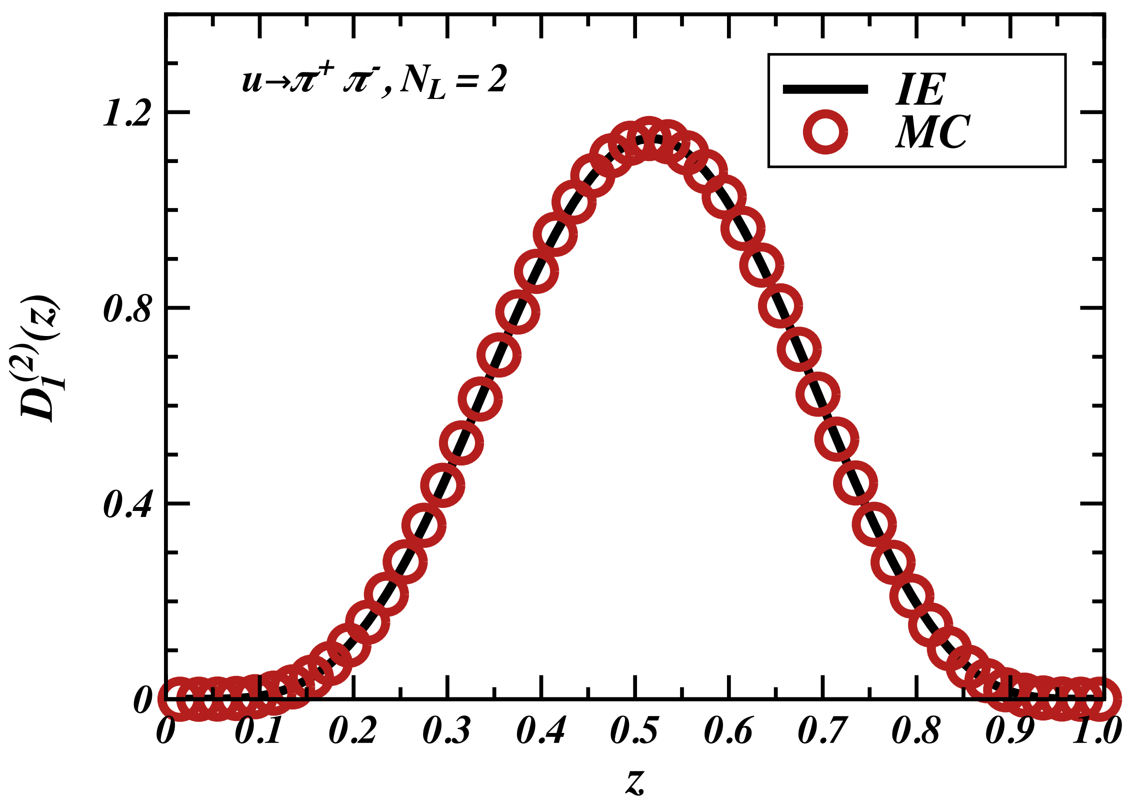


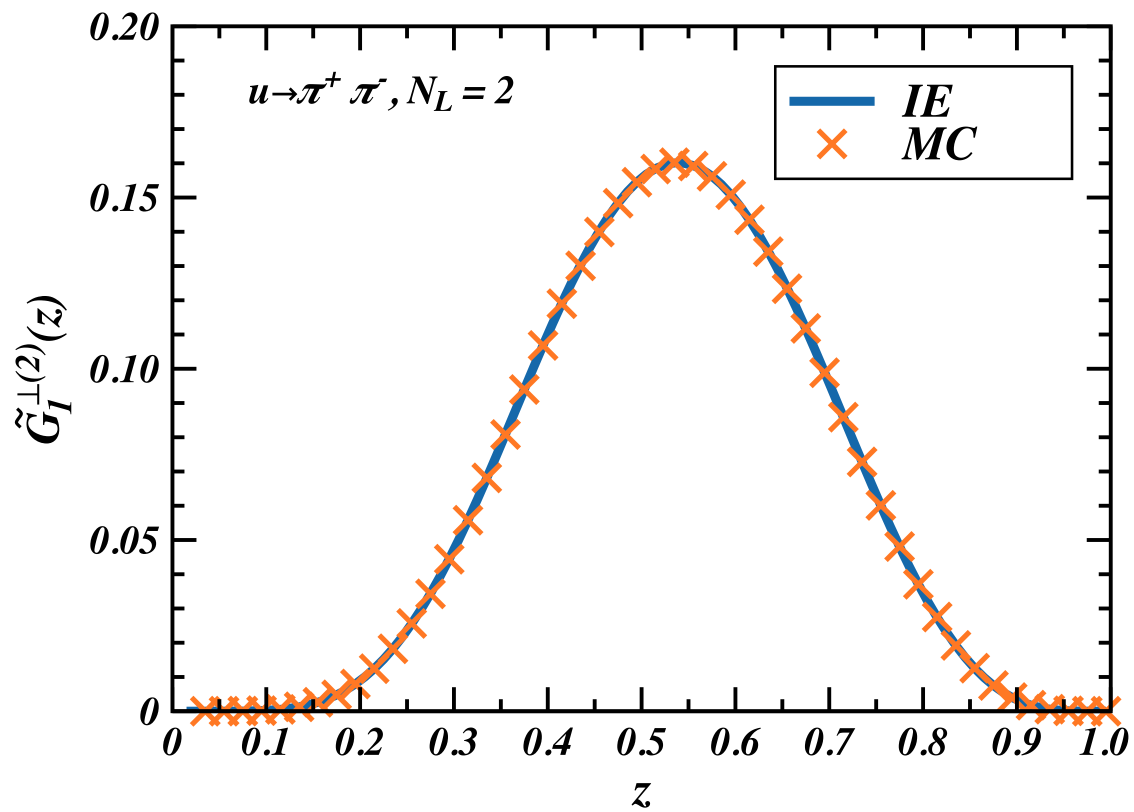
$$D_1^{(2)}(z) = \hat{D}^{q\to q_1} \otimes \hat{D}^{q_1\to h}$$

$$\tilde{G}_1^{\perp(2)} = \hat{G}_T^{q\to q_1} \otimes \hat{H}^{\perp(q_1\to h)}$$

Spin rotation

Collins effect at 2-nd emission

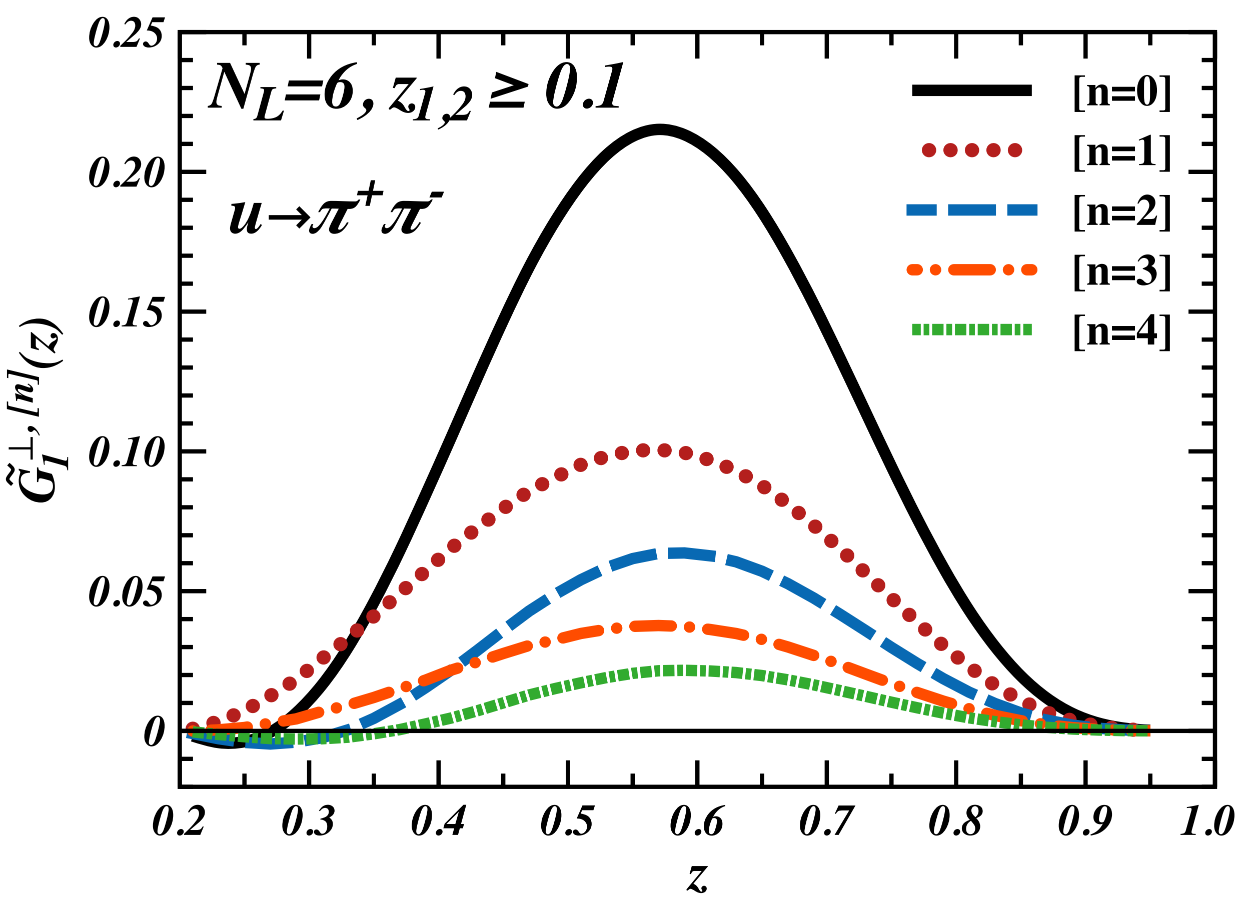


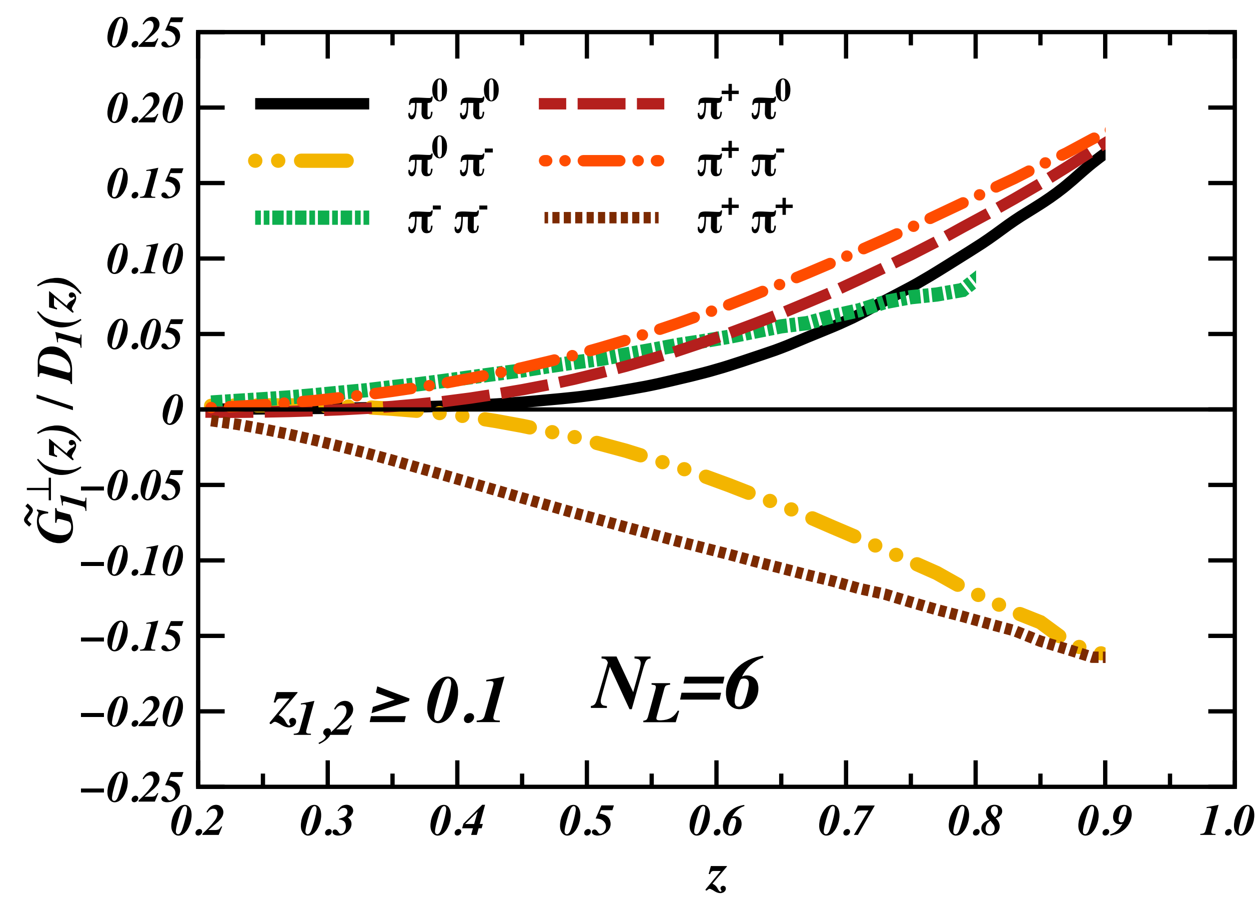


✓ **Collins effect generates helicity dep. two-hadron correlation!**

Results for $G_1^{\perp}$

✦ Results for helicity DiFFs, ***several*** moments, various pairs. Cuts: $z_{1,2} \geq 0.1$





✦ Non-zero signal for various channels, ***sign change for*** $\pi^+\pi^+$ ***pairs*** !

✦ $z_{1,2} \geq 0.1$ ***cut enhances the analysing power at high-z for larger N$_L$*** !

Transverse Polarisation in DiHadron FFs

TRANSVERSELY POL. DIFFS FROM NUMBER DENSITY

H.M., Kotzinian, Thomas, Phys. Rev. D 97, 014019 (2018).

✦ **Slightly more complicated procedure:**

$$\begin{aligned} F(\varphi_R, \varphi_k; \boldsymbol{s}_T) = & D_1(\cos(\varphi_{RK})) \\ & + a_R \sin(\varphi_R - \varphi_s) H_1^{\sphericalangle}(\cos(\varphi_{RK})) \\ & + a_K \sin(\varphi_k - \varphi_s) H_1^{\perp}(\cos(\varphi_{RK})) \end{aligned}$$

✦ **n-th moment of DiFFs:**

$$H_1^{\sphericalangle,[n]} = \frac{2}{s_T} \Big\langle \cos(\varphi_k - \varphi_s) \frac{\cos(n \cdot \varphi_{RK})}{\sin(\varphi_{RK})} F \Big\rangle$$

$$H_1^{\perp,[n]} = -\frac{2}{s_T} \Big\langle \cos(\varphi_R - \varphi_s) \frac{\cos(n \cdot \varphi_{RK})}{\sin(\varphi_{RK})} F \Big\rangle$$

✦ **SIDIS DiFFs:**

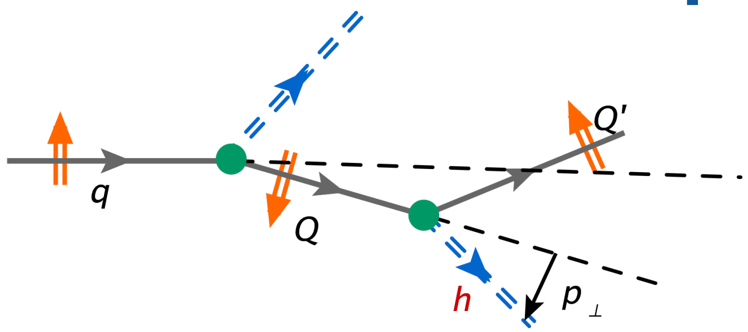
$$H_1^{\sphericalangle,SIDIS}(z) = \frac{2}{s_T} \Big\langle \sin(\varphi_R - \varphi_s) F \Big\rangle$$

$$H_1^{\perp,SIDIS}(z) = \frac{2}{s_T} \Big\langle \sin(\varphi_k - \varphi_s) F \Big\rangle$$

VALIDATION: 2 PRODUCED HADRONS

✦ **Validate MC simulations by comparing to explicit Integral Expressions (IE). Only pions produced in the first two steps!**

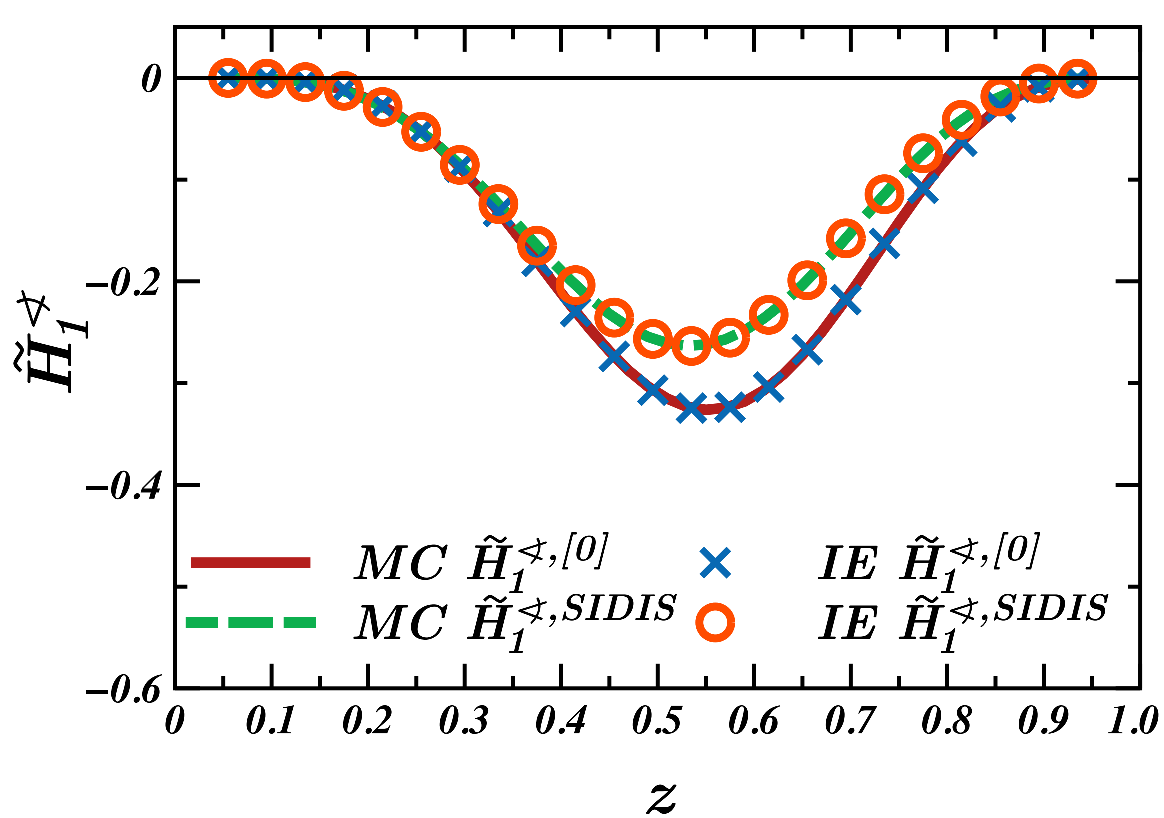
$$F^{(2)}_{q\to h_1 h_2} = \sum_{q_1} \hat{f}^{q\to q_1 + h_1} \otimes \hat{f}^{q_1 \to h_2}.$$

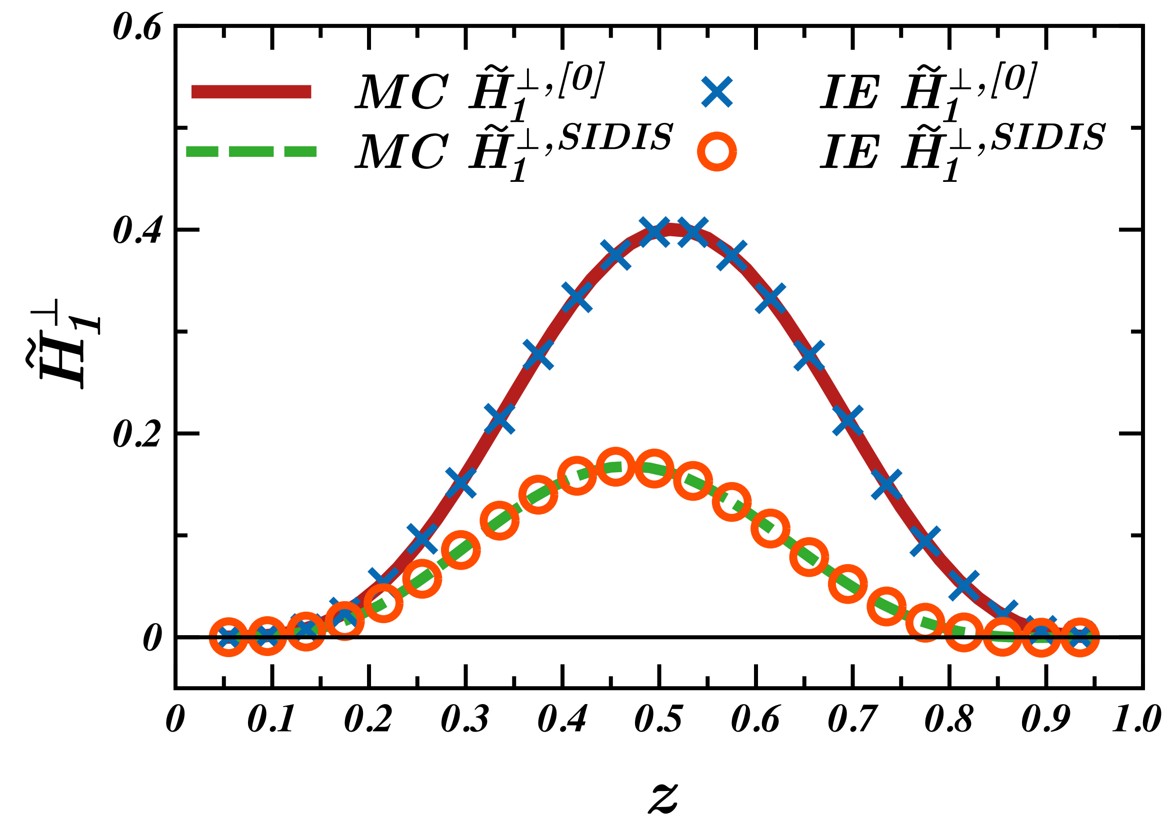


$$H_1^{\sphericalangle(2)} = \hat{H}^{\perp(q\to q_1)} \otimes \hat{D}^{(q_1\to h)} + \hat{H}_T^{(q\to q_1)} \otimes \hat{H}^{\perp(q_1\to h)} + \hat{H}_T^{\perp(q\to q_1)} \otimes \hat{H}^{\perp(q_1\to h)}$$

Recoil TM Modulation

Collins effect at 2-nd emission

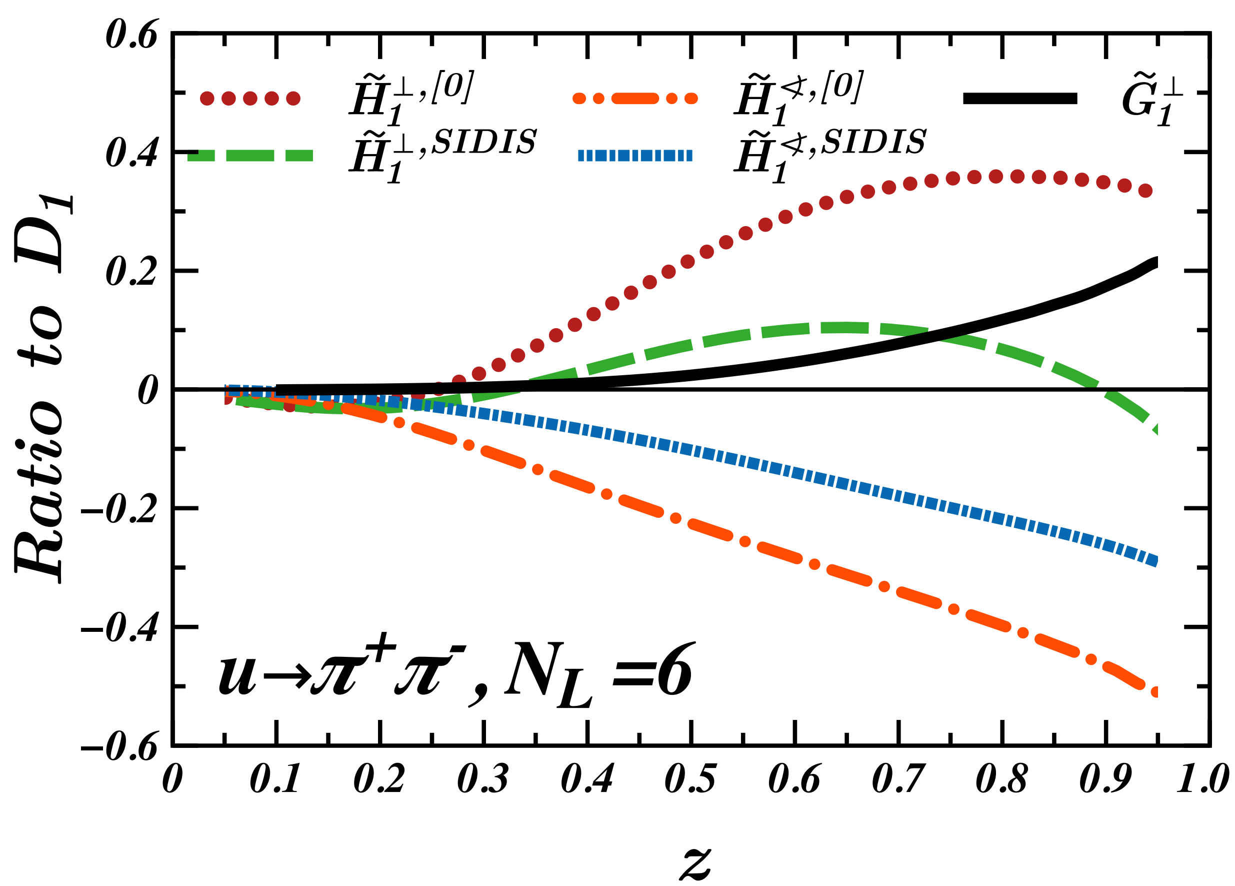


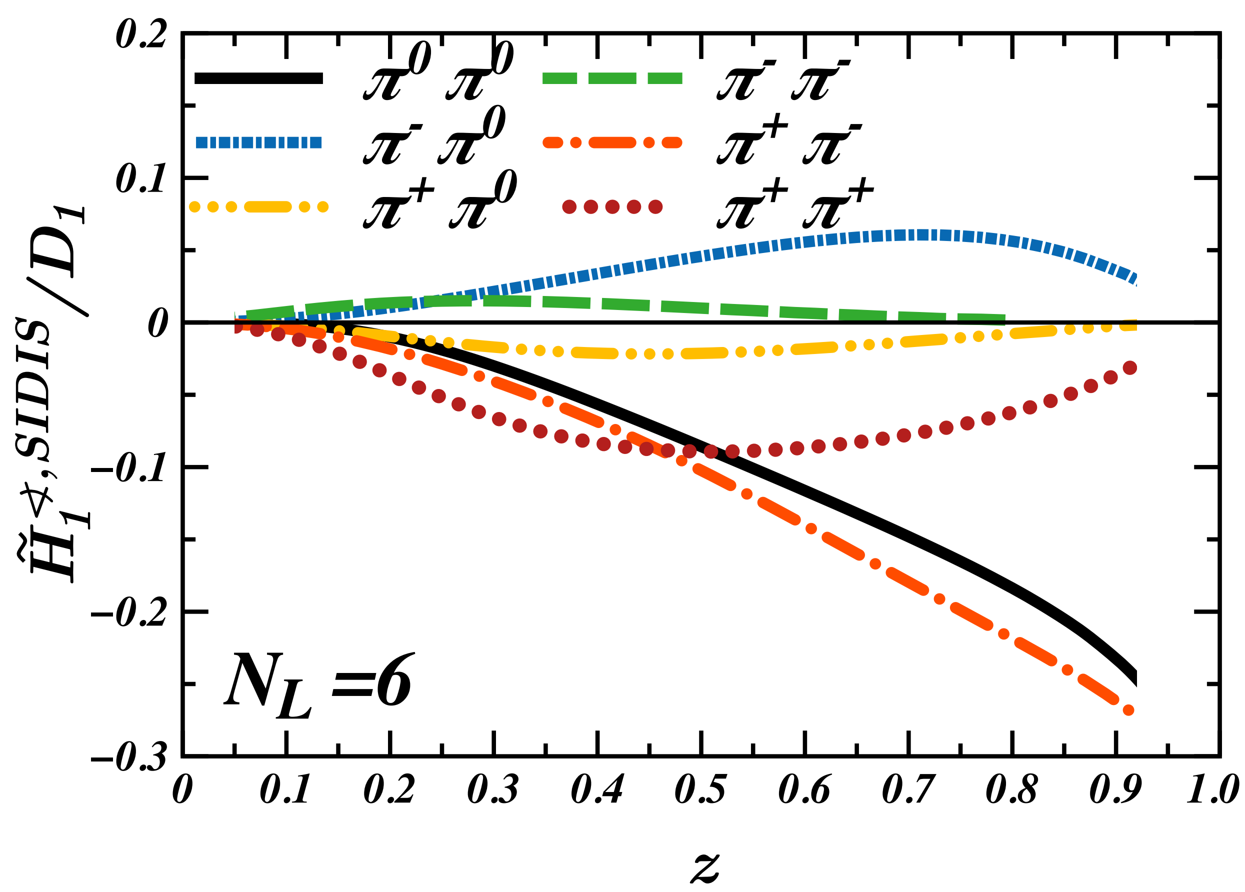


✓ **Collins effect generates S_T dep. DiFF correlations as well !**

Analysing Power for Transverse Spin

✦ **Comparing the analysing powers for all polarized DiFFs.**





✦ **Alternate signs for the two DiFFs.**

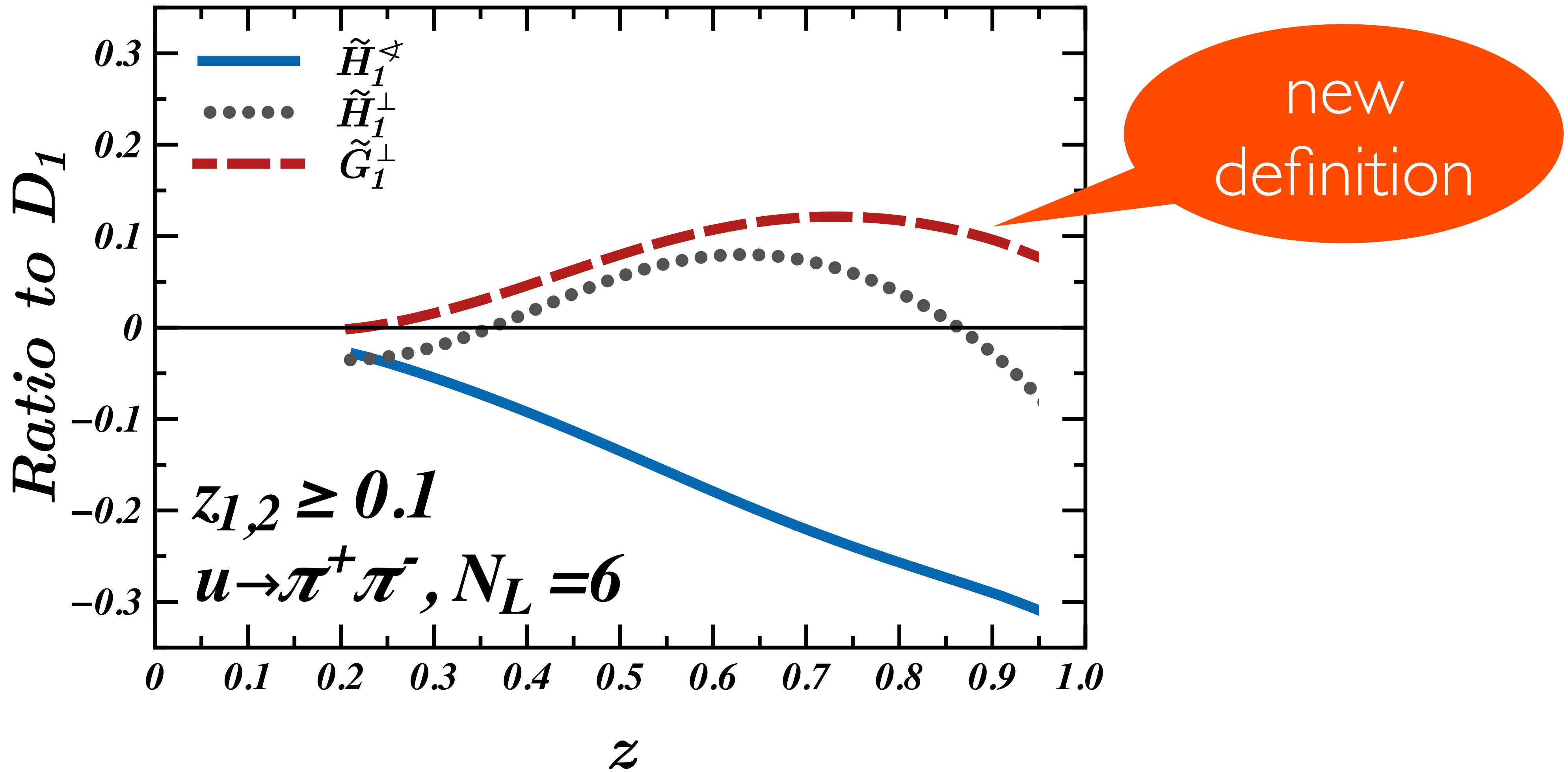
✦ **Significant differences between SIDIS and 0-th moments!**

✦ ***Signals for all possible hadron pairs.***

Feasibility of new measurements of $G_1^{\perp}$

✦ **The analysing powers of DiFFs from quark-jet framework.**

- $G_1^{\perp}$ *naturally smaller than* $H_1^{\sphericalangle}$*, but* ***should be measurable!***



✦ **Reanalyze BELLE and COMPASS data.**

✦ **Measure it at BELLE II and JLab 12GeV.**

CONCLUSIONS II

- ❖ Hadronization Models are needed to calculate polarised TMD FFs and DiFFs, and study various correlations between them.
- ❖ Polarised hadronisation in *MC generators: support for future experiments* to map the 3D structure of nucleon (*COMPASS, JLab12, BELLE II, EIC*).
- ❖ The *quark-jet* framework describes hadronization of a quark with arbitrary polarization via spin density matrix formalism.
- ❖ *All 3* DiHadron spin correlations from single-hadron effects in quark-jet!
- ❖ *Naturally small*, *but measurable* signal for helicity-dependent DiFFs.
- ❖ *Measurements* in $\mathbf{e^+e^-}$ (BELLE) and *SIDIS* (JLab, COMPASS) would test the universality of the helicity-dependent DiFFs.

Thanks!

BACKUP SLIDES

Different Hadronization Mechanisms.

LUND Model

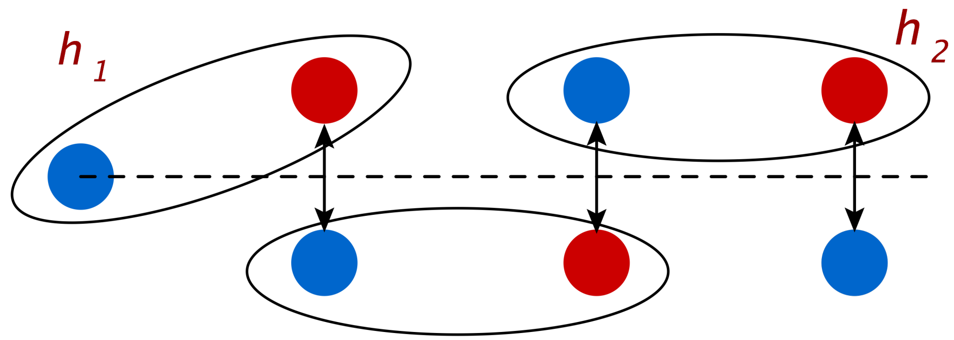
✦ Fragmentation of $q\bar{q}$ pair: break-up of the string.

✦ Independent breaking of the string.

✦ Quark TM indep. of hadron type.

$$u \to u + s\bar{s}, \quad s \to s + s\bar{s}$$

✦ No correlation in TM: h_1 and h_2.



Quark-Jet

✦ Fragmentation of q, similar to QFT definition of FFs.

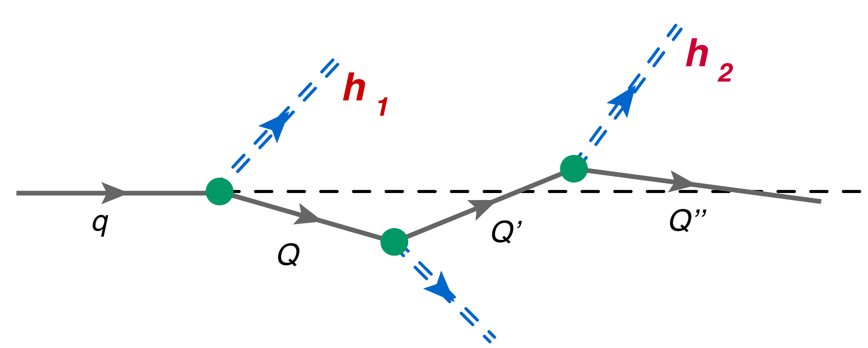
✦ Time-ordered hadron emissions.

✦ $q \to Qh$ depends on **h** (spin, mass).

$$u \to K^+ + s, \quad s \to \phi + s$$

$$u \to K^{*+} + s$$

✦ Recoil TM of h_1 affects h_2



❖ Can we find a signature in polarized FFs? Perhaps Dihadron FFs?

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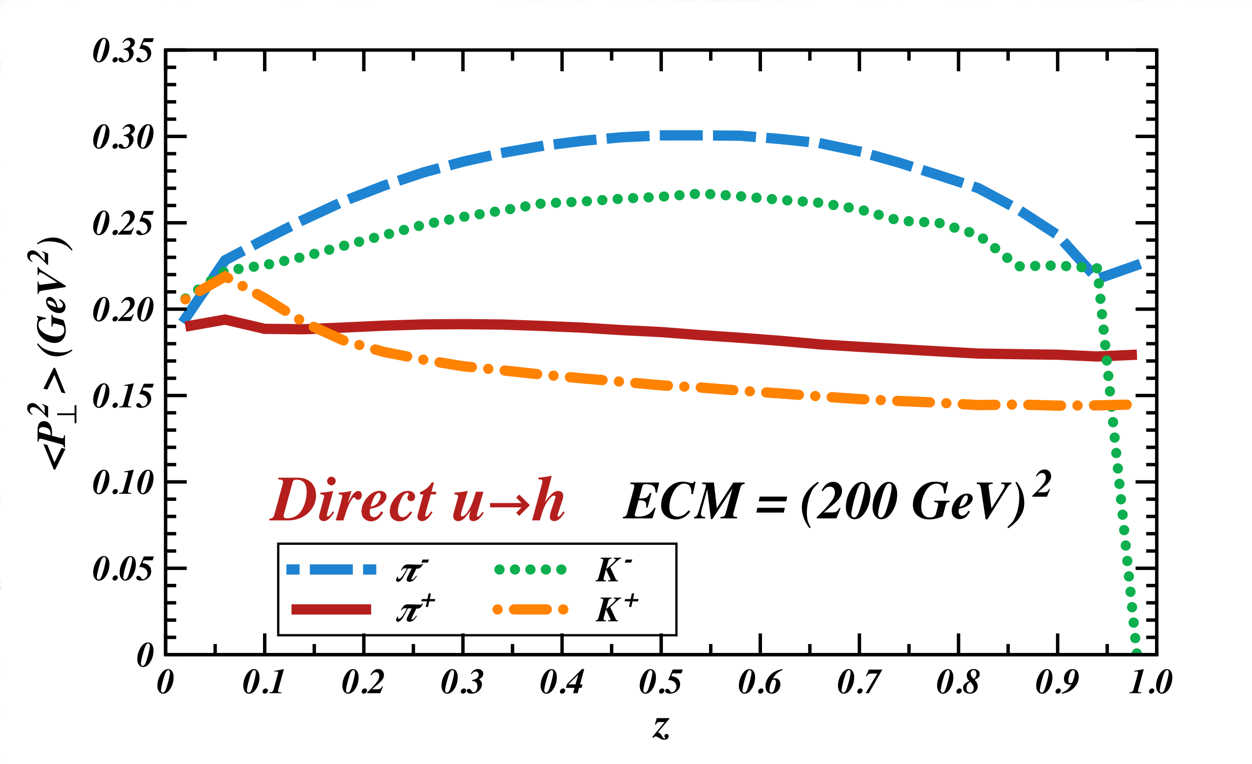

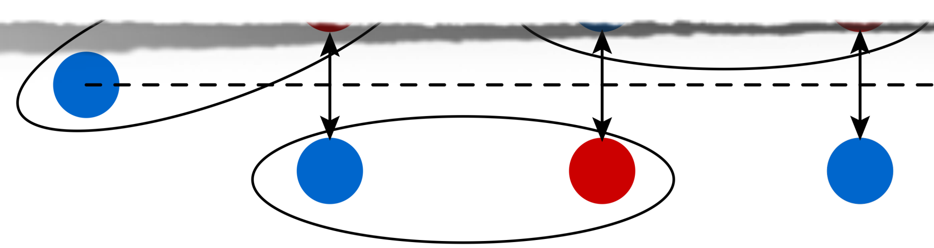

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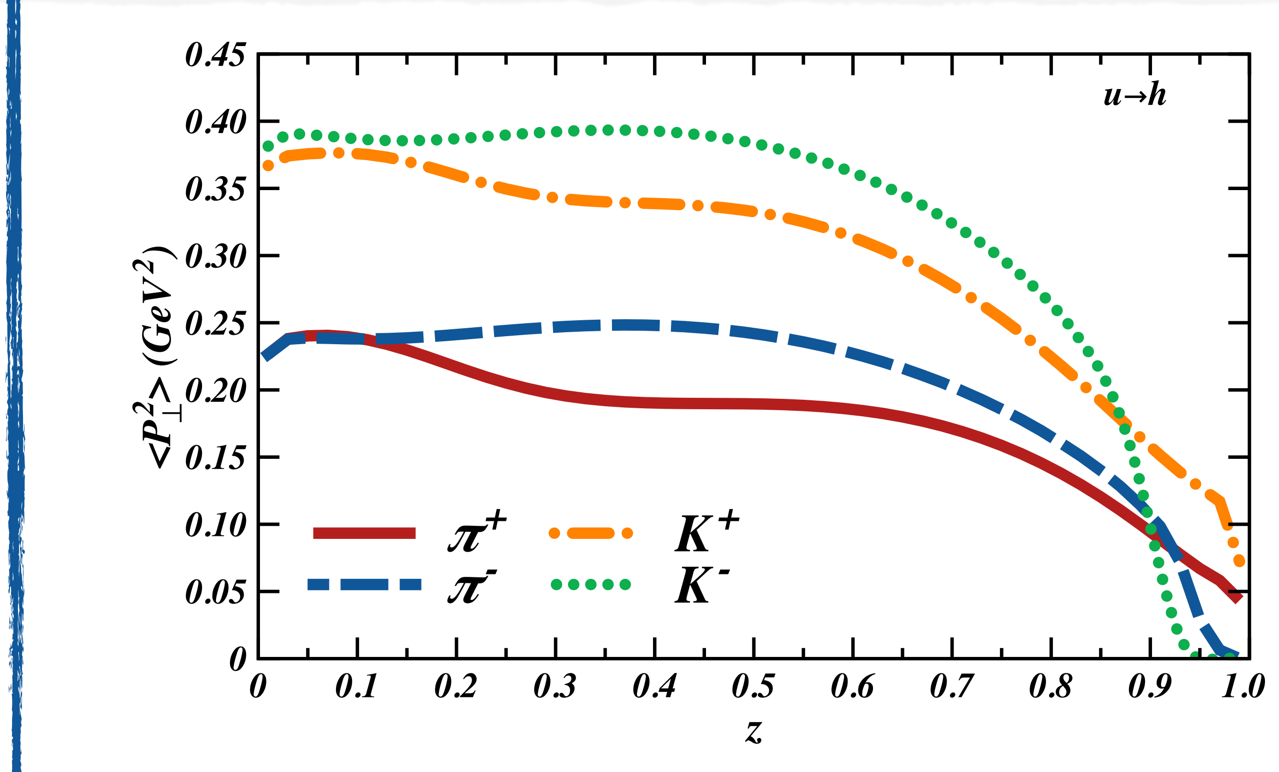

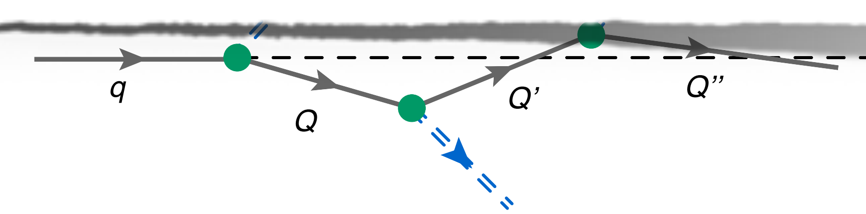

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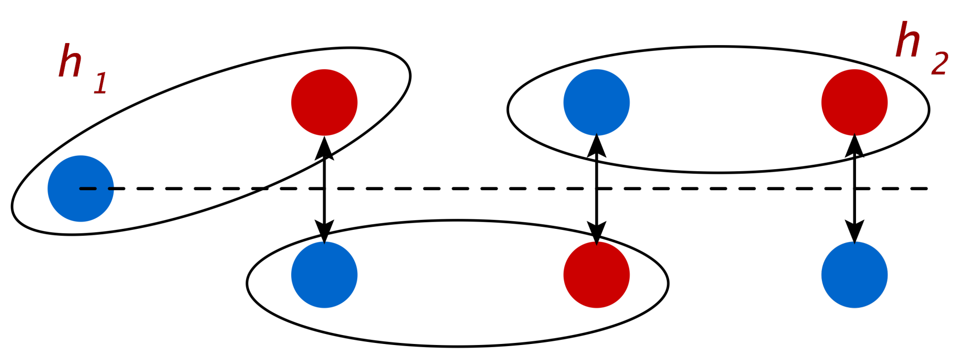

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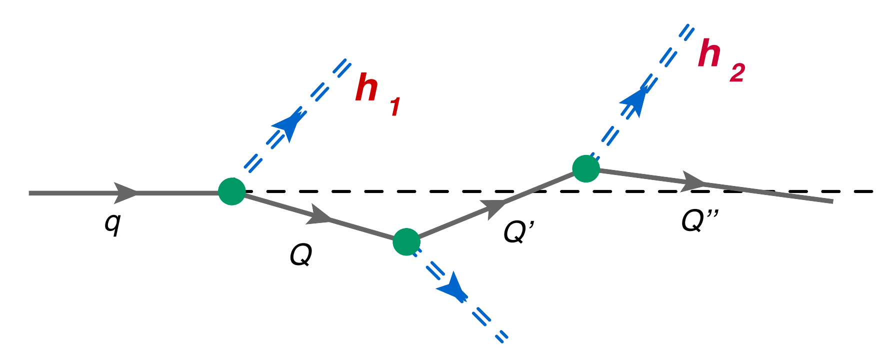
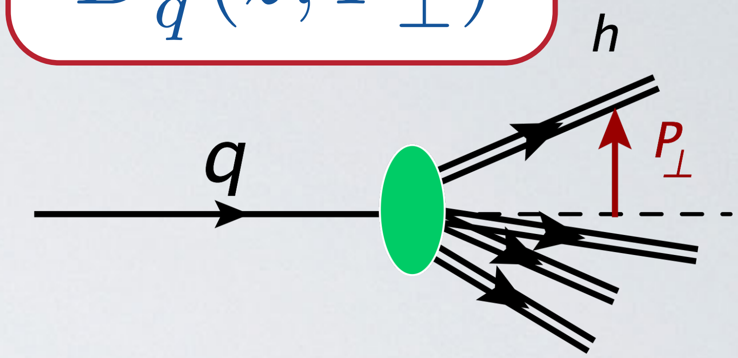


❖ Can we find a signature in polarized FFs? Perhaps Dihadron FFs?

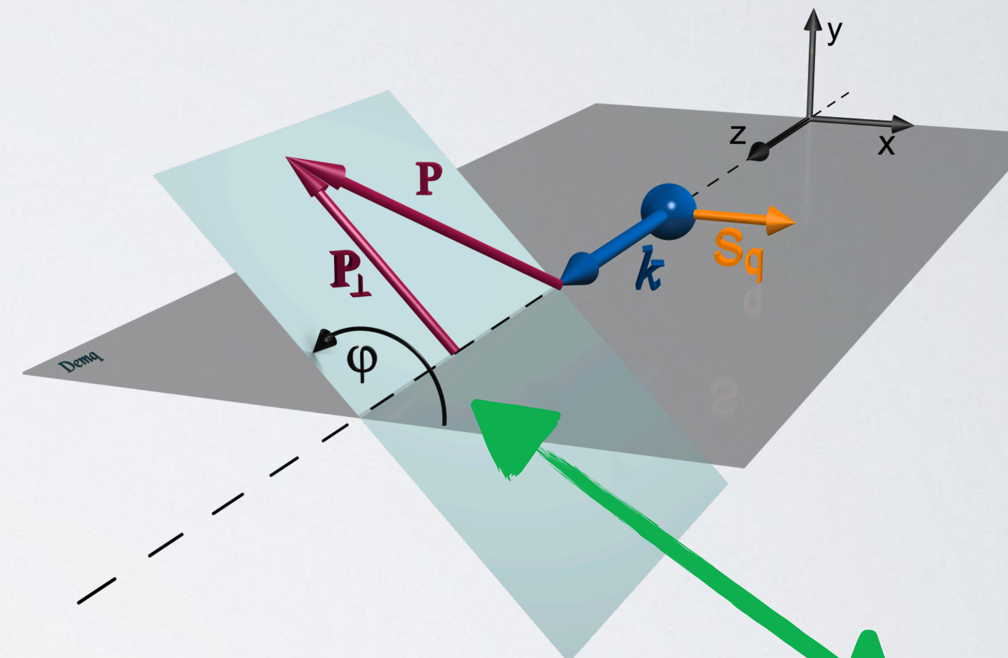
TMD FFs and Collins Fragmentation Function

- **Unpolarized TMD FF:** number density for quark ***q*** to produce unpolarized hadron ***h*** carrying LC fraction **z** and TM $P_\perp$.

$$D_q^h(z, P_\perp^2)$$



- **Collins Effect:** Azimuthal Modulation of Transversely Polarized Quark' FF. Fragmenting quark's transverse spin couples with produced hadron's TM!



$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

Unpolarized

Collins

- ***Collin FF*** is **Chiral-ODD:** Should to be coupled with another chiral-odd PDF/FF in observables.

TMD FFs for Spin-0 and Spin-1/2 Hadrons

❖ ***The transverse momentum (TM) of the hadron can couple with both its own spin and the spin of the quark!***

$$F^{q\to\pi}(z, \boldsymbol{p}_\perp; \boldsymbol{s})$$

π/q	U	L	T
U	D_1		$H_1^\perp$

$$F^{q\to h^\uparrow}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S})$$

h/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

✦ **TMD Polarized Fragmentation Functions at LO.**

- *Only two for unpolarised final state hadrons.*
- *8 for spin 1/2 final state (including quark). Similar to TMD PDFs.*

- ***The quark-quark correlator.***

$$\Delta^{[\Gamma]}(z,\vec{p}_T) \equiv \frac{1}{4}\int \frac{dp^+}{(2\pi)^4} Tr[\Delta\Gamma]|_{p^-=zk^-}$$

$$= \frac{1}{4z}\sum_X \int \frac{d\xi^+ d^2\vec{\xi}_T}{2(2\pi)^3} e^{i(p^-\xi^+/z-\vec{\xi}_T\cdot\vec{p}_T)} \langle 0|\psi(\xi^+,0,\vec{\xi}_T)|p,S_h,X\rangle\langle p,S_h,X|\bar{\psi}(0)\Gamma|0\rangle$$

- ***The definitions of FFs from the quark correlator***

$$\Delta^{[\gamma^+]} = D(z,p_\perp^2) - \frac{1}{M}\epsilon^{ij}k_{Ti}S_{Tj}D_T^\perp(z,p_\perp^2)$$

$$\Delta^{[\gamma^+\gamma_5]} = S_L G_L(z,p_\perp^2) + \frac{\boldsymbol{k}_T\cdot S_T}{M}\, G_T(z,p_\perp^2)$$

$$\Delta^{[i\sigma^{i+}\gamma_5]} = S_T^i H_T(z,p_\perp^2) + \frac{S_L}{M}k_T^i H_L^\perp(z,p_\perp^2) + \frac{k_T^i(\boldsymbol{k}_T\cdot S_T)}{M^2}H_T^\perp(z,p_\perp^2) - \frac{\epsilon^{ij}k_{Tj}}{M}H^\perp(z,p_\perp^2)$$

Positivity and Polarisation of Quark

Bacchetta et al, PRL 85, 712 (2000) .

✦ **The probability density is Positive Definite: constraints on FFs.**

✦ **Leading-order T-Even functions FULLY Saturate these bounds!**

✦ **For non-vanishing $H^{\perp}$ and $D_T^{\perp}$, need to calculate T-Even FFs at next order!**

✦ **Average value of remnant quark's spin.**

$$\langle \boldsymbol{S}_T \rangle_Q = \boldsymbol{s}_T \frac{\int dz \left[h_T^{(q\to Q)}(z) + \frac{1}{2z^2 M_Q^2}\ h_T^{\perp[1](q\to Q)}(z) \right]}{\int dz\ d^{(q\to Q)}(z)}$$

✦ **In spectator model, at leading order:** $h_T(z) = -d(z)$

✦ **Non-zero** $h_T^{\perp}$ **means** $\langle \boldsymbol{S}_T \rangle_Q \neq -\boldsymbol{s}_T$ **(full flip of the spin)!**

SPECTATOR MODELS

E.G. - Bacchetta et al, PLB 659:234, 2008

- ***Use Field-theoretical definition of FFs from a Correlator.***

$$\boxed{\Delta(z, k_T) = \frac{1}{2z}\int dk^+\,\Delta(k, P_h) = \frac{1}{2z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi}\langle 0|\mathcal{U}^{n_+}_{(+\infty,\xi)}\psi(\xi)|h, X\rangle\langle h, X|\bar{\psi}(0)\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\big|_{\xi^-=0}}$$

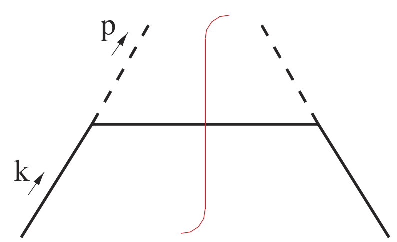
$$D_1(z, z^2\vec{k}_T^2) = \mathrm{Tr}[\Delta(z, \vec{k}_T)\gamma^-].$$

$$\frac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^\perp(z, k_T^2) = \frac{1}{2}\mathrm{Tr}[\Delta(z, k_T) i\sigma^{i-}\gamma_5]$$

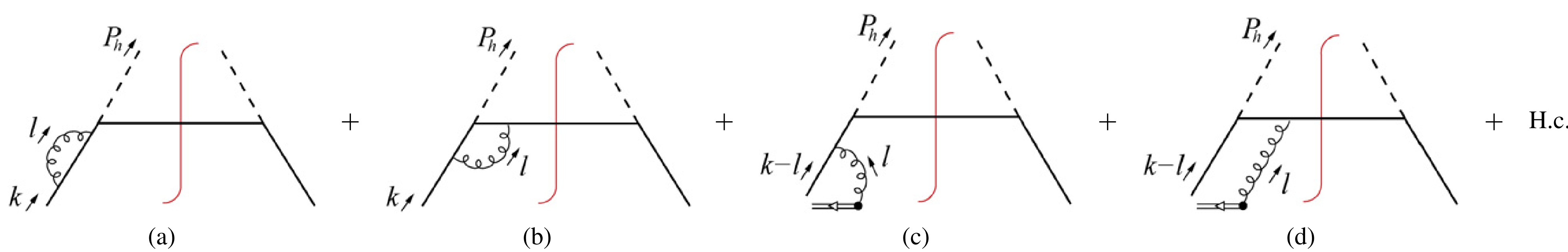
- ***Approximate the remnant X as a "spectator" (quark).***

- ***Calculate the FFs at leading-order in favourite quark model.***

$D_1(z, p_\perp^2)$



$H_1^\perp(z, p_\perp^2)$

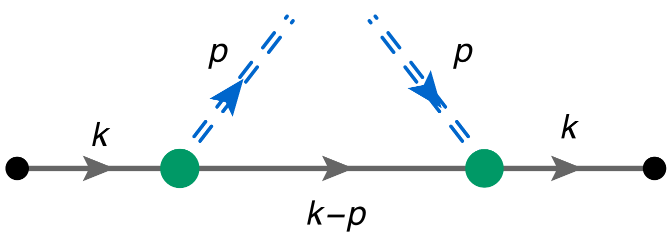


Model Calculations of $q \to Q$ Splittings

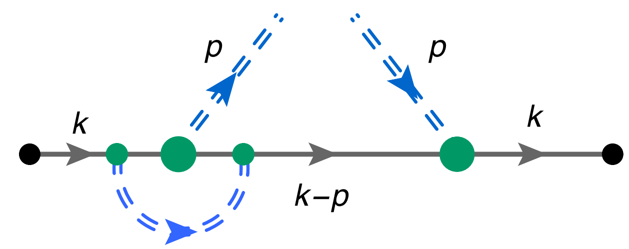
E.G. - Meissner et al, PLB 690, 296 (2010).

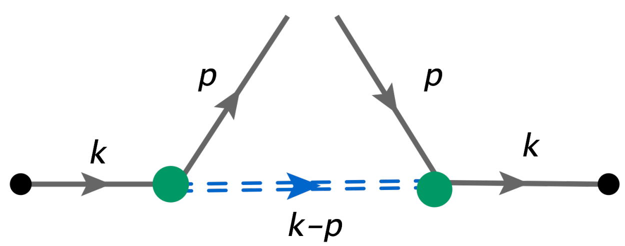
✦We can use the same "spectator" type calculations as for pion.

T-even ***T-odd***

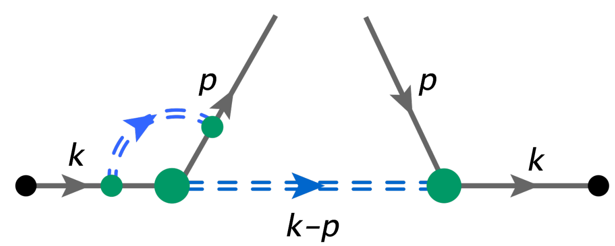


$$q \to h$$





$$q \to Q$$



✦ Positivity Constraints on TMD FFs:

Bacchetta et al, P.R.L. 85, 712 (2000).

$$(H_L^{\perp[1]})^2 + (D_T^{\perp[1]})^2 \le \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \le \frac{p_\perp^2}{4z^2M^2}D^2$$

$$(G_T^{[1]})^2 + (H^{\perp[1]})^2 \le \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \le \frac{p_\perp^2}{4z^2M^2}D^2$$

✦ T-odd parts from previous models ***violate positivity***!

$$(\hat{G}_T^{[1]})^2 = (\hat{H}_L^{\perp[1]})^2 = \frac{p_\perp^2}{4z^2M^2}(\hat{D}+\hat{G}_L)(\hat{D}-\hat{G}_L) \le \frac{p_\perp^2}{4z^2M^2}\hat{D}^2$$

$$\boxed{\hat{H}^\perp(z,p_\perp^2) = 0, \quad \hat{D}_T^\perp(z,p_\perp^2) = 0.}$$

Model Calculations of $q \to Q$ Splittings

✦ **Simple Model that is positive-definite:**

$$\hat{d}(z, p_\perp^2) = 1.1\, \hat{d}_{tree}(z, p_\perp^2),$$

✦ **Use Collins-ansatz for T-odd**

J. C. Collins, NPB 396, 161 (1993)

$$\frac{p_\perp}{zM} \frac{\hat{h}^{\perp(q\to h)}(z, p_\perp^2)}{\hat{d}^{(q\to h)}(z, p_\perp^2)} = 0.4 \frac{2\, p_\perp M_Q}{p_\perp^2 + M_Q^2}$$

$$d_T^\perp = -h^\perp$$

✦ **Ensures the inequalities**

$$(H_L^{\perp[1]})^2 + (D_T^{\perp[1]})^2 \le \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \le \frac{p_\perp^2}{4z^2M^2}D^2$$

$$(G_T^{[1]})^2 + (H^{\perp[1]})^2 \le \frac{p_\perp^2}{4z^2M^2}(D+G_L)(D-G_L) \le \frac{p_\perp^2}{4z^2M^2}D^2$$

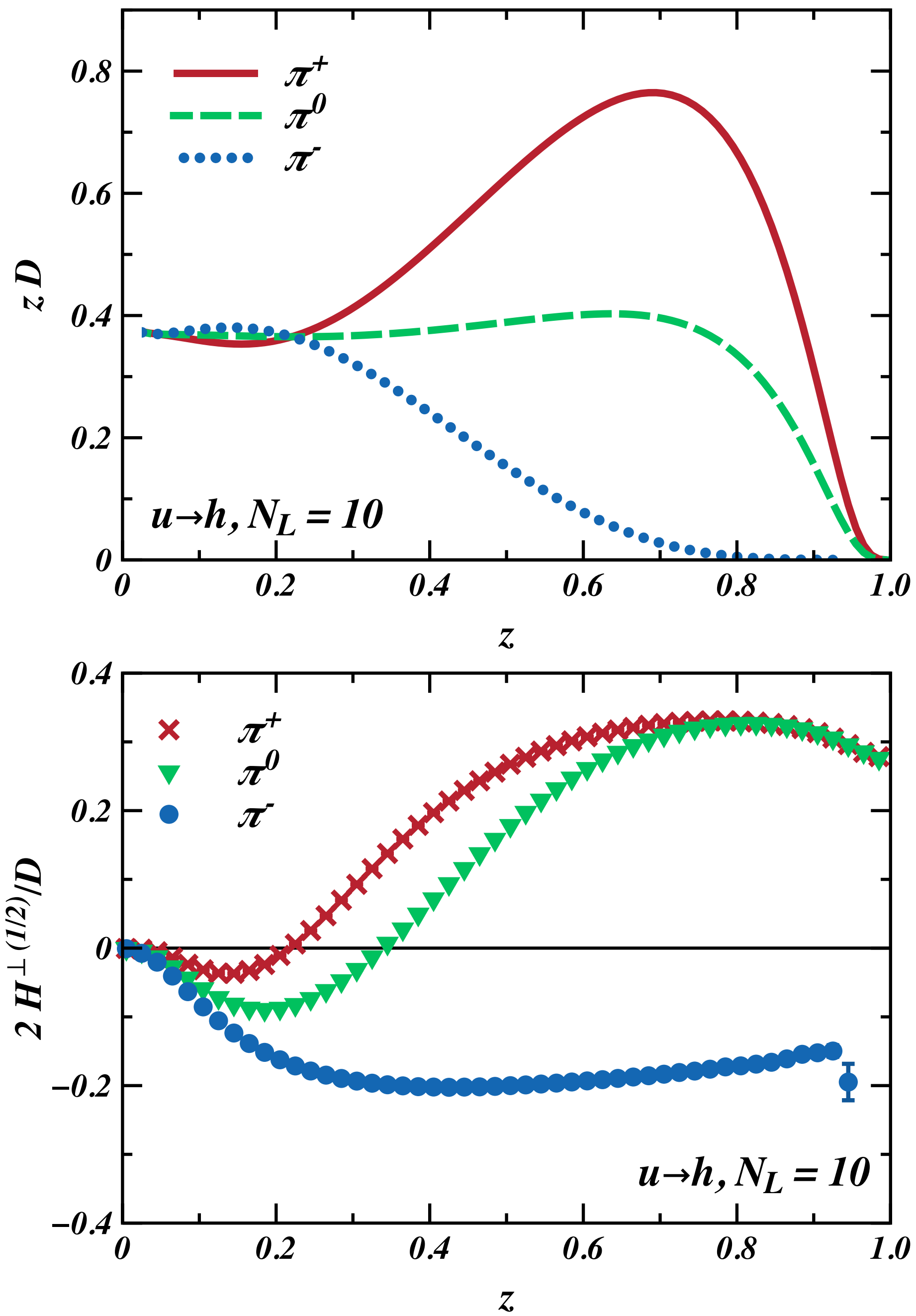
✶ **Also: *Evolution - mimicking ansatz***

$$\boxed{\hat{d}'(z, p_\perp^2) = (1-z)^4 \hat{d}(z, p_\perp^2)}$$

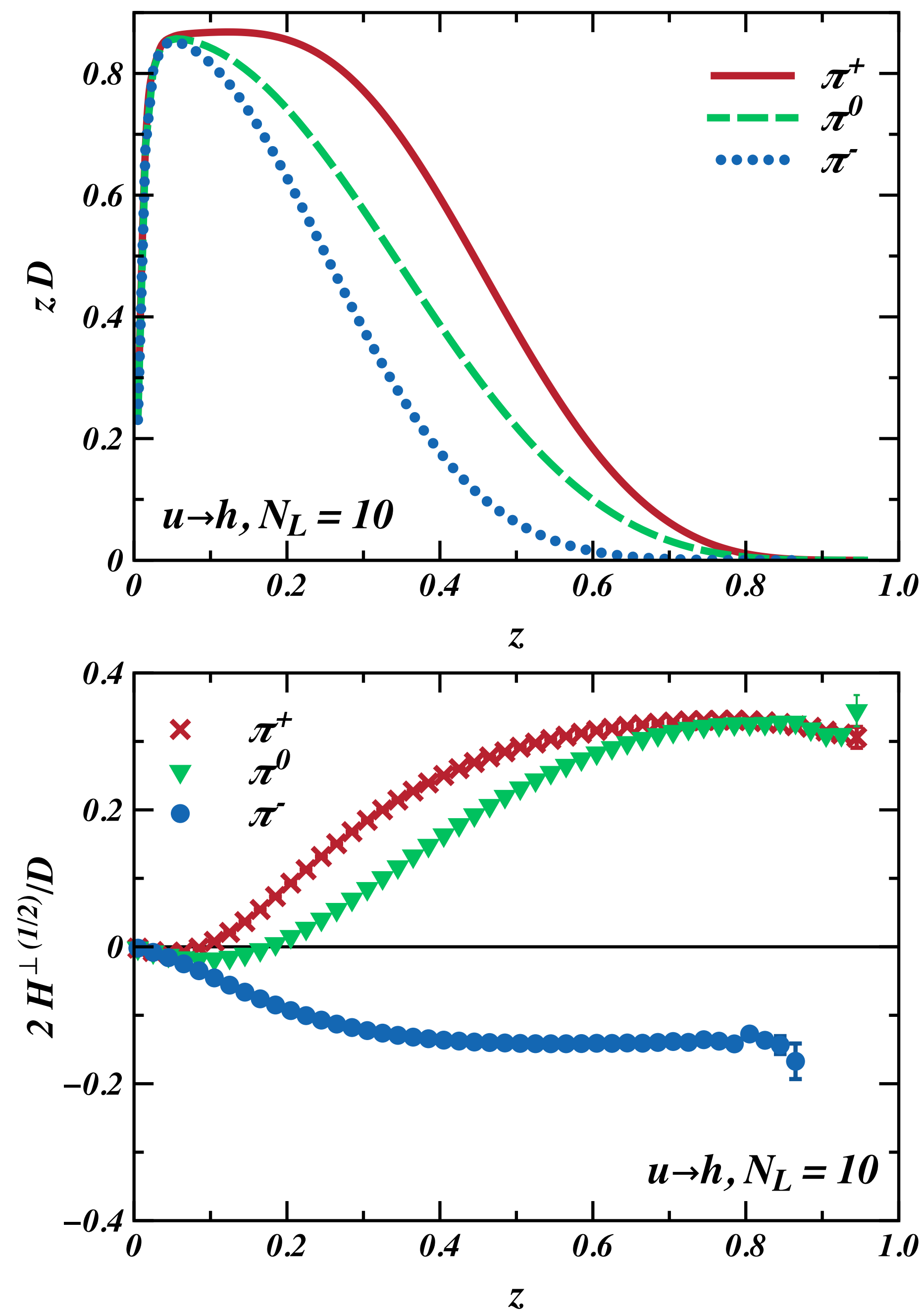
Results for Collins Effect

HM et al, Phys. Rev. D95 04021, (2017)

- *NJL* Model



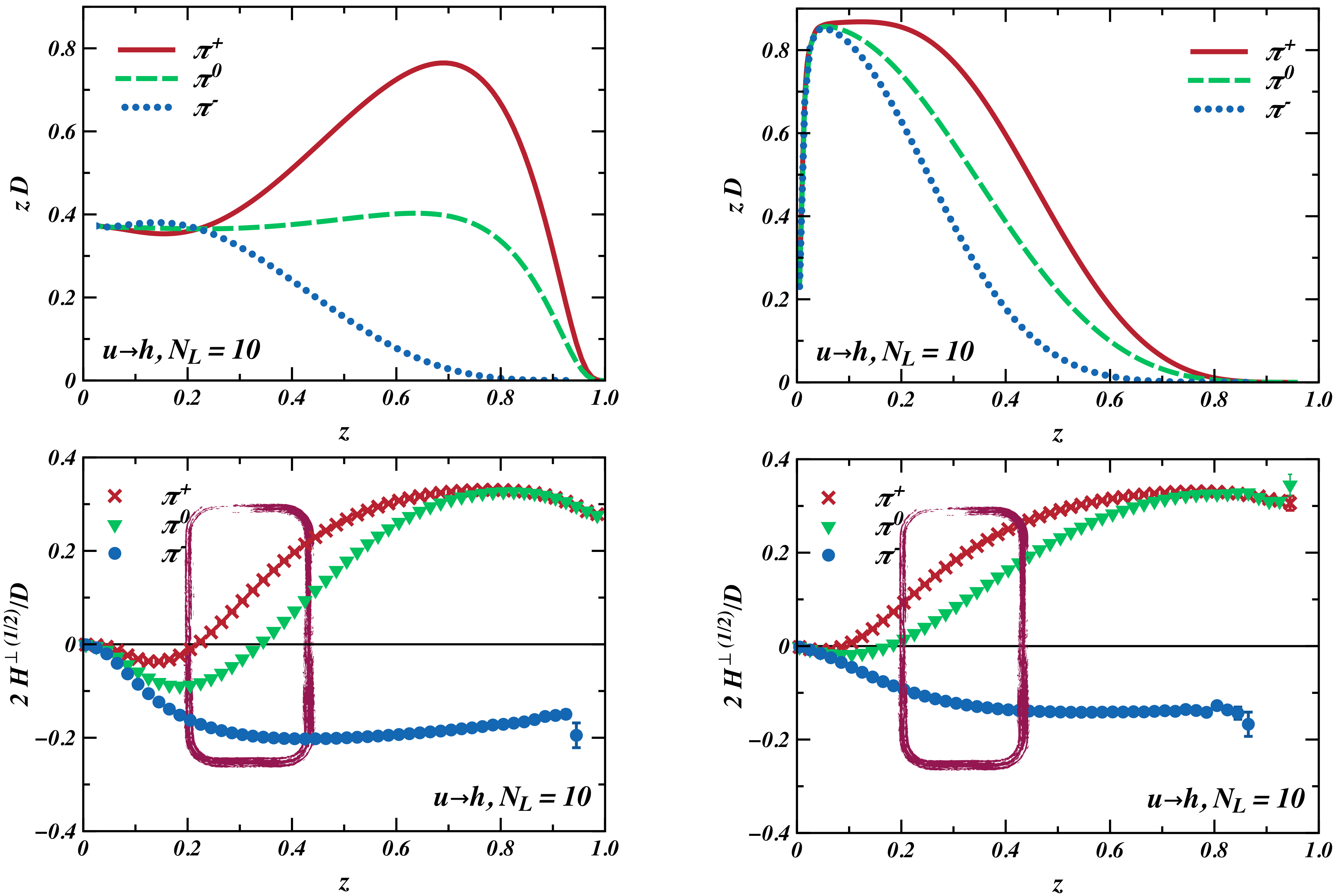
- Evolution-mimicking *Ansatz.*



Results for Collins Effect

HM et al, Phys. Rev. D95 04021, (2017)

- *NJL* Model
- Evolution-mimicking *Ansatz.*

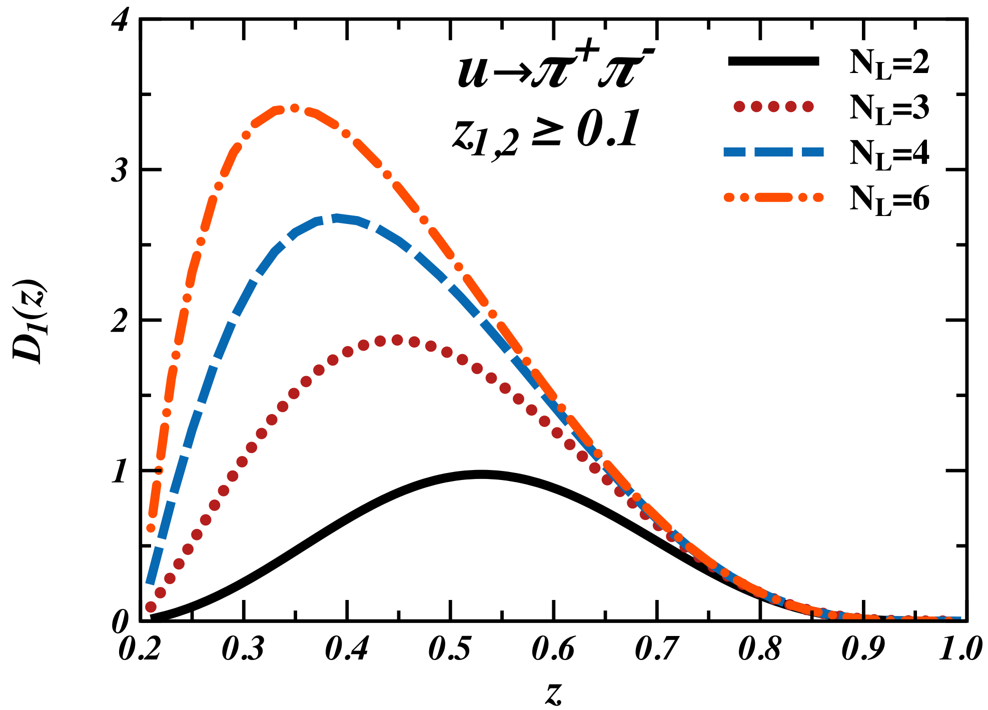


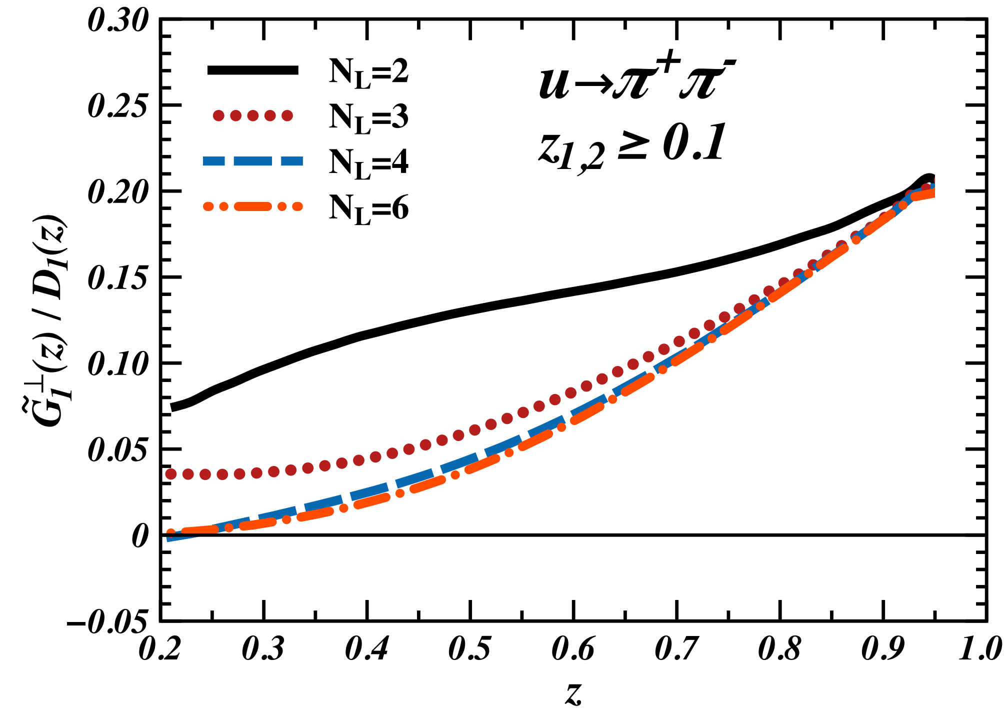
✦ ***Opposite sign and similar size*** in mid-z range for charged pions. (Similar to empirical extractions).

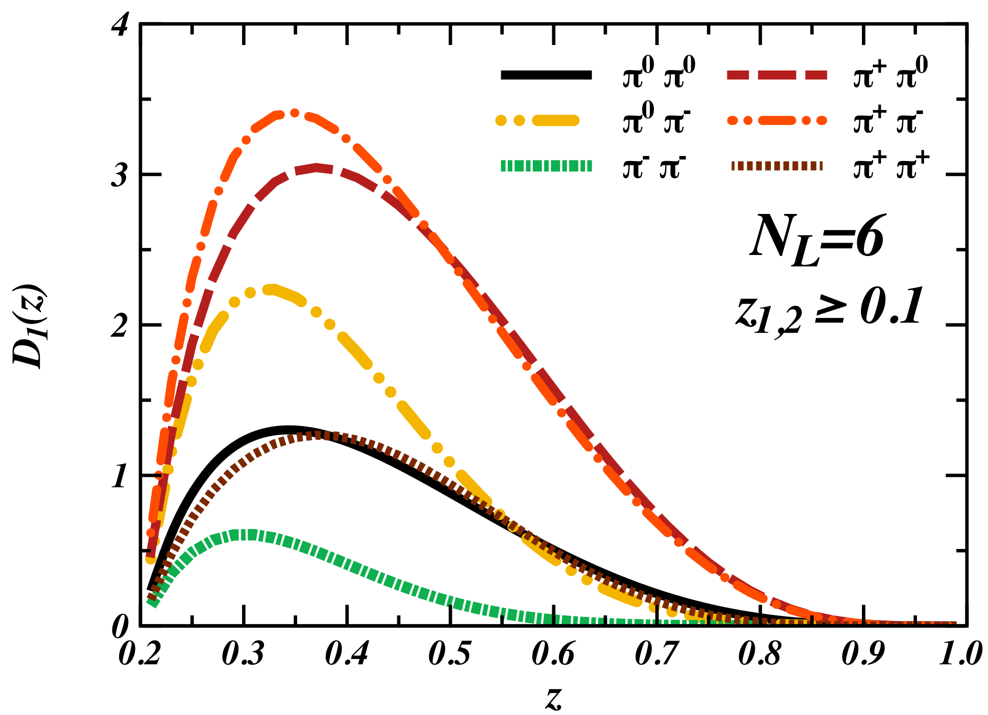
✦ ***Dependence on model inputs:*** can be tuned to data.

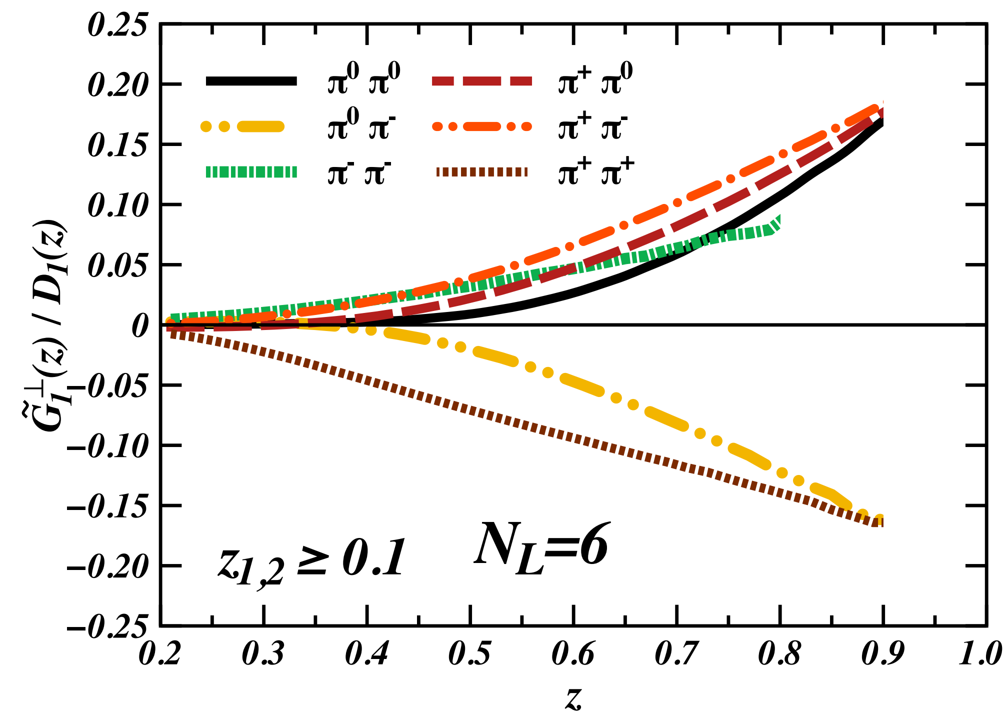
Results for helicity dependant DiFFs

✦ Results for helicity DiFFs, ***N*$_L$** dependence, various pairs. Cuts: $z_{1,2} \geq 0.1$









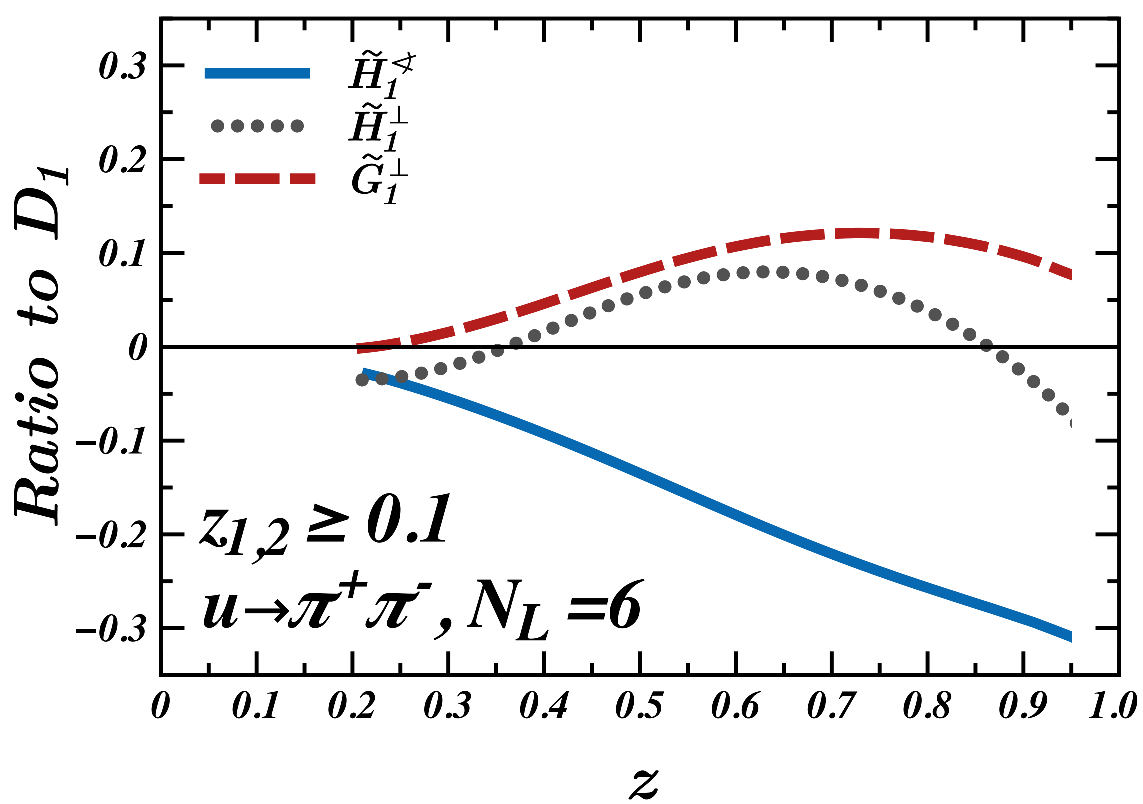
✦ Non-zero signal for various channels, ***sign change for*** $\pi^+\pi^+$ ***pairs*!**

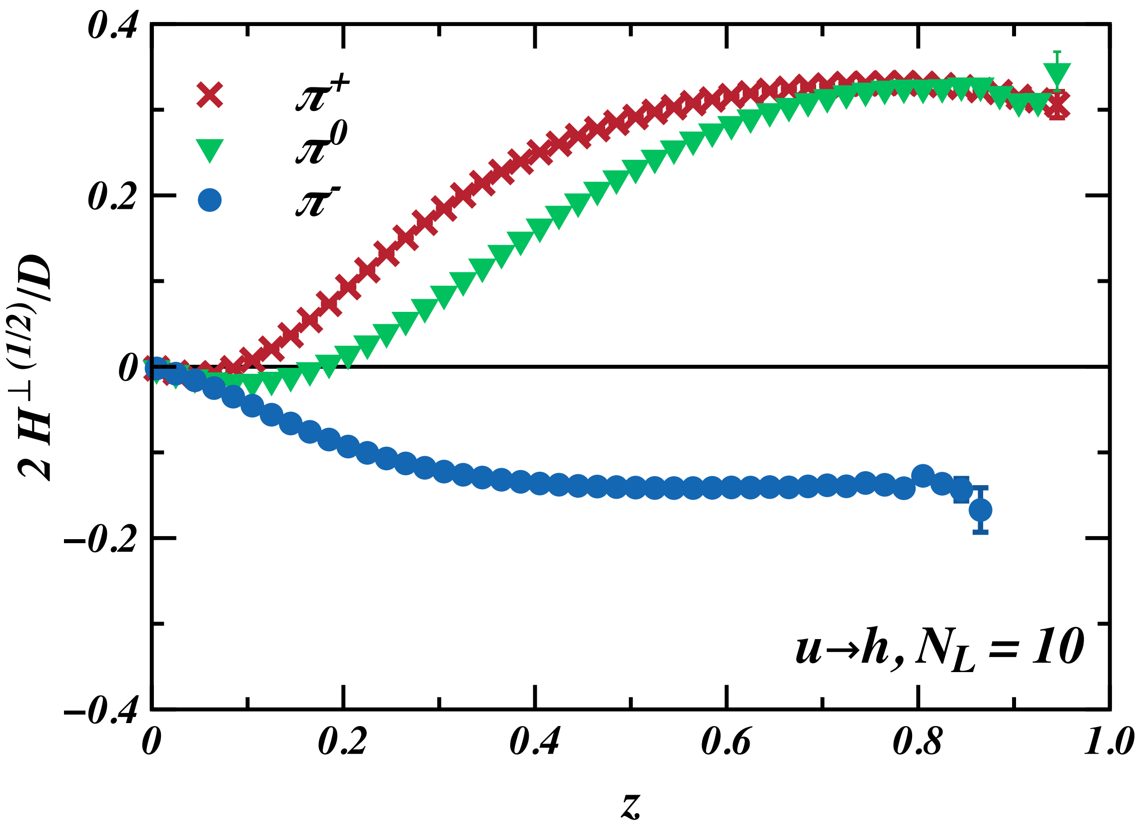
✦ $z_{1,2} \geq 0.1$ ***cut enhances the analysing power at high-z for larger N$_L$*!**

Analysing powers for DiFFs in e⁺e⁻

✦ The analysing powers of DiFFs from quark-jet framework.

- $G_1^{\perp}$ *naturally smaller than* $H_1^{\sphericalangle}$, *but* ***should be measurable!***





INCLUSION OF VECTOR MESONS AND (STRONG) DECAYS

- A naive assumption: VMs should have modest contribution due to relatively small production probability $P(\pi^+)/P(\rho^+) \approx 1.7$
- ***But***: Combinatorial factors enhance VM contribution significantly!
- Let's consider only two hadron emission

Direct: $u \to d + \pi^+ \to u + \pi^- + \pi^+$

VM: $u \to d + \pi^+ \to u + \rho^- + \pi^+$

$$\rho^- \to \pi^- \pi^0$$

...

$$u \to u + \rho^0 \to u + \rho^0 + \rho^0 \to \pi^+ \pi^-$$

$$\rho^0 \to \pi^+ \pi^-$$

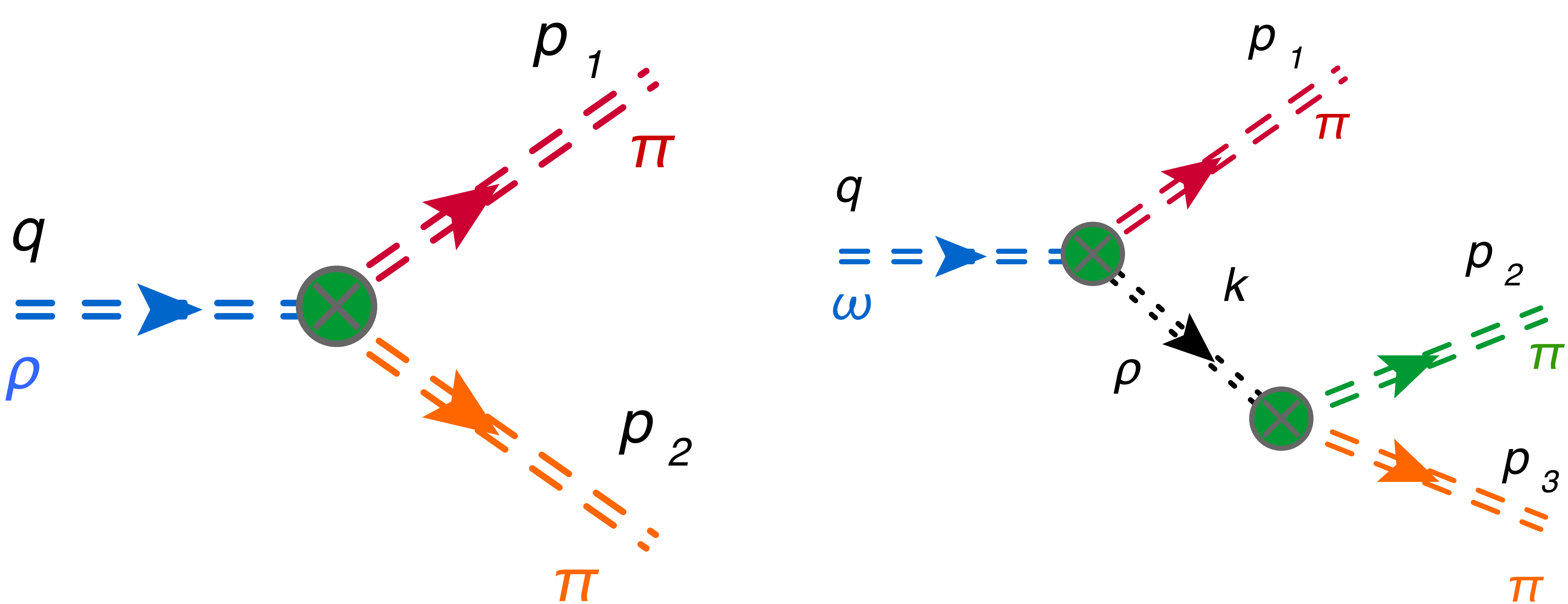
$$P_{Dir}(\pi^+\pi^-)/P_{VM}(\pi^+\pi^-) \approx \frac{1}{4}$$

2- AND 3-BODY DECAYS

The M_h^2 spectrum of pseudoscalars is strongly affected by VM decays.

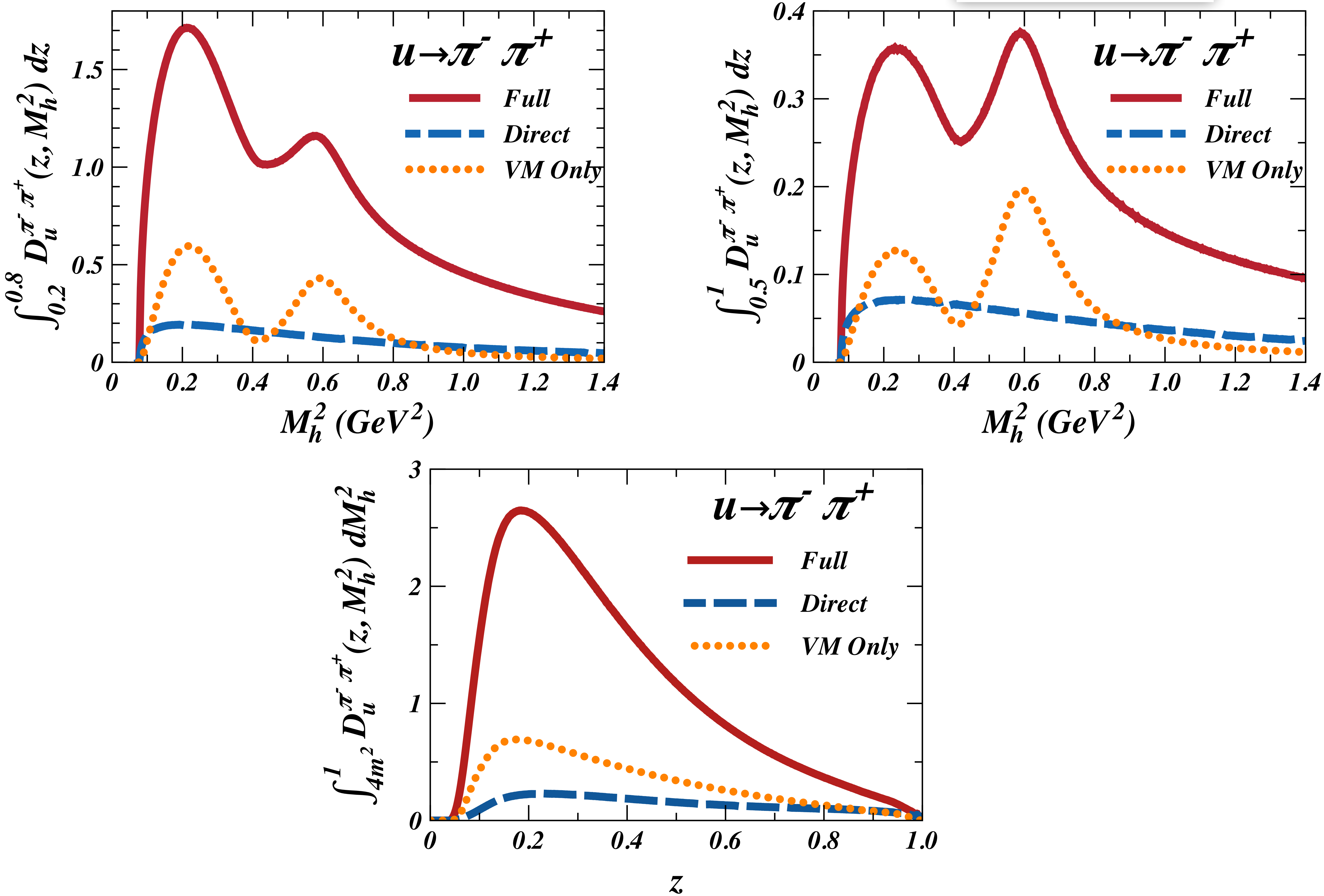
- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ.

Achasov et al. (SND), PRD 68, 052006, (2003).



PYTHIA RESULTS FOR $u \to \pi^- \pi^+$

arXiv:1706.08348

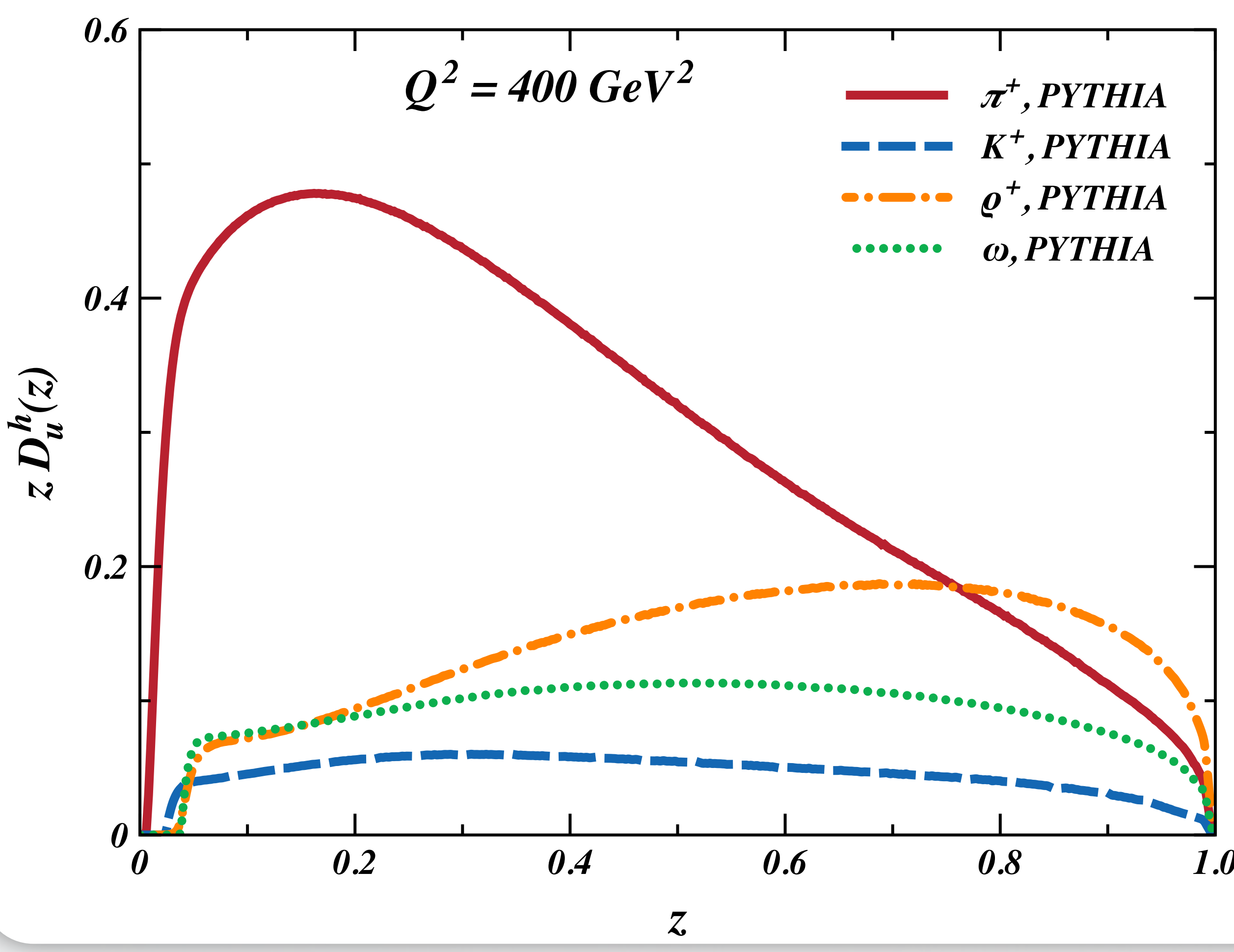


PYTHIA SIMULATIONS

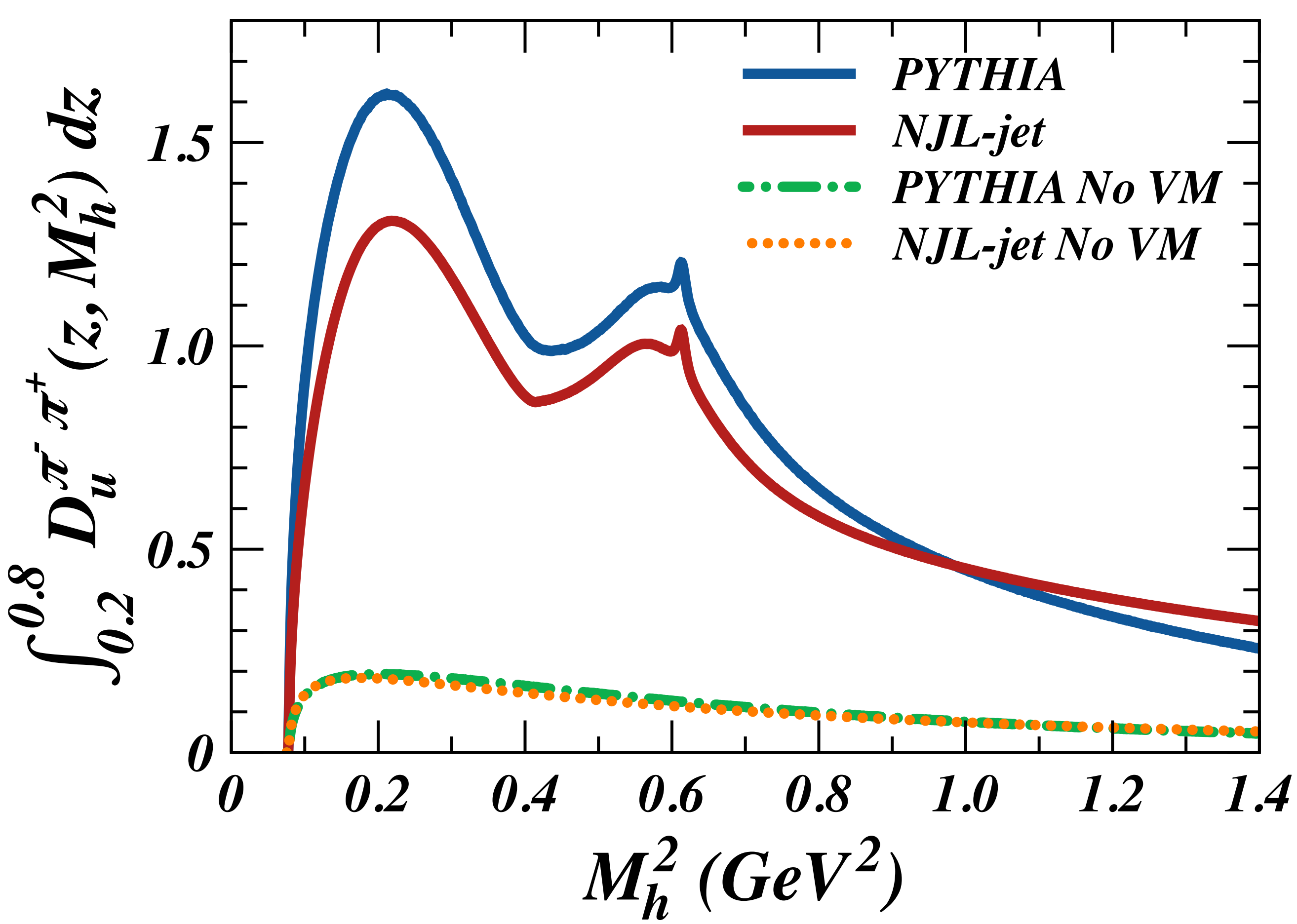
- Setup hard process with back to back $q\ \bar{q}$ along **z** axis.
- ***Only Hadronize.*** Allow the same resonance decays as NJL-jet.
- Assign hadrons with positive p_z to q fragmentation.

$$E_q = 10\ \mathrm{GeV}$$

Single Hadron

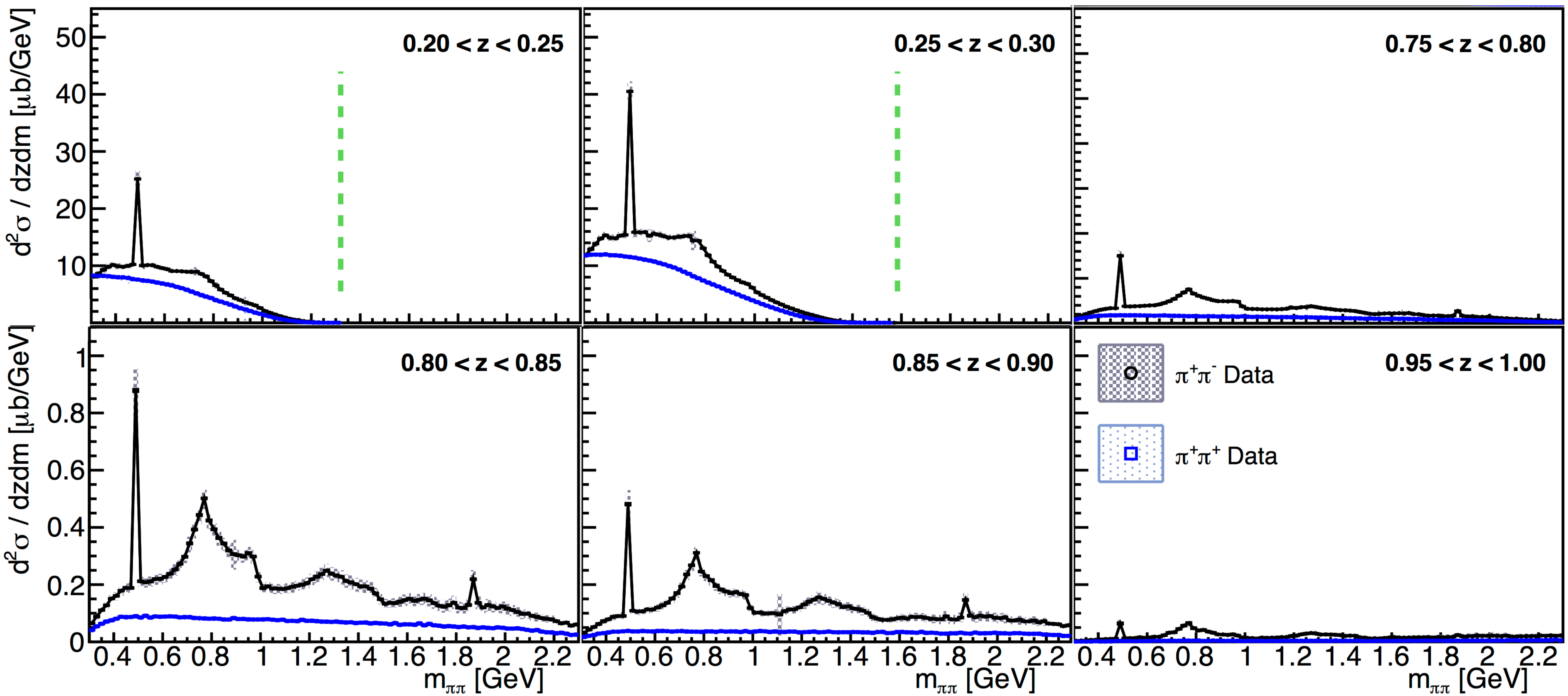


Dihadron



Recent BELLE Results

✦ **Invariant mass dependence of unroll DiFFs:** arXiv:1706.08348



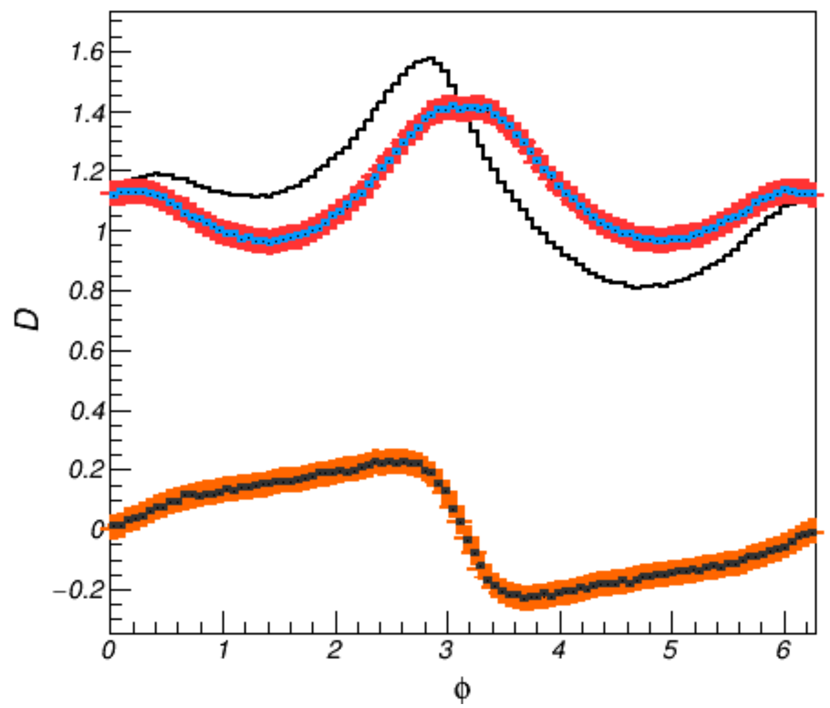
✦ **Note:** $D(z, M_h) dM_h = 2M_h \; D(z, M_h^2) dM_h$

✦ **Large z favours large M_h !**

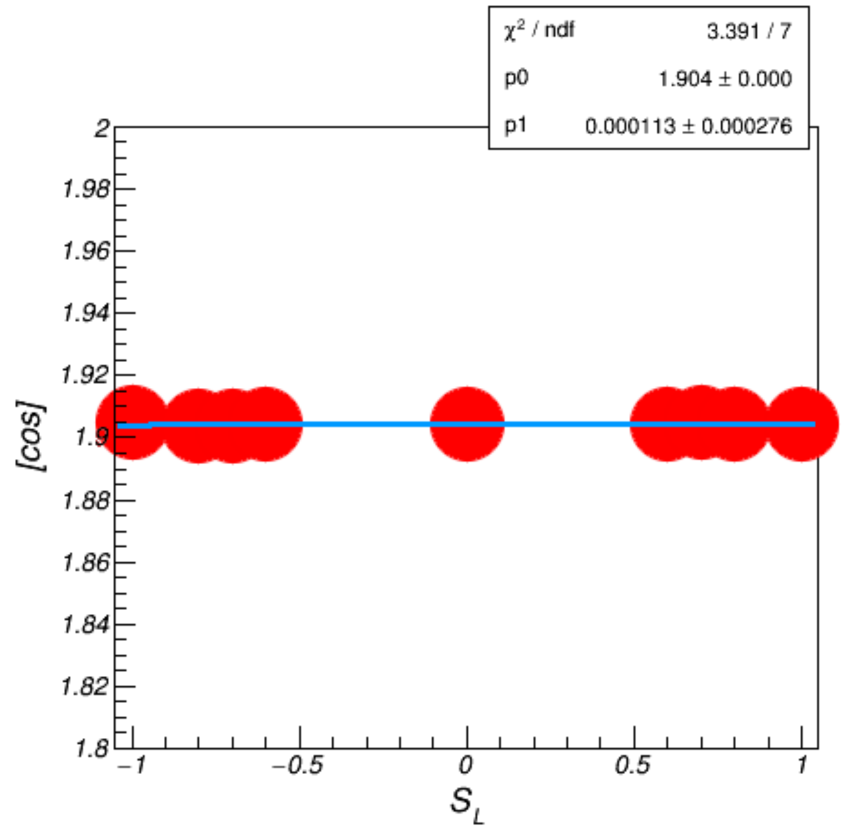
✦ ***Non-resonant channels have no M_h structure, but are amplified!***

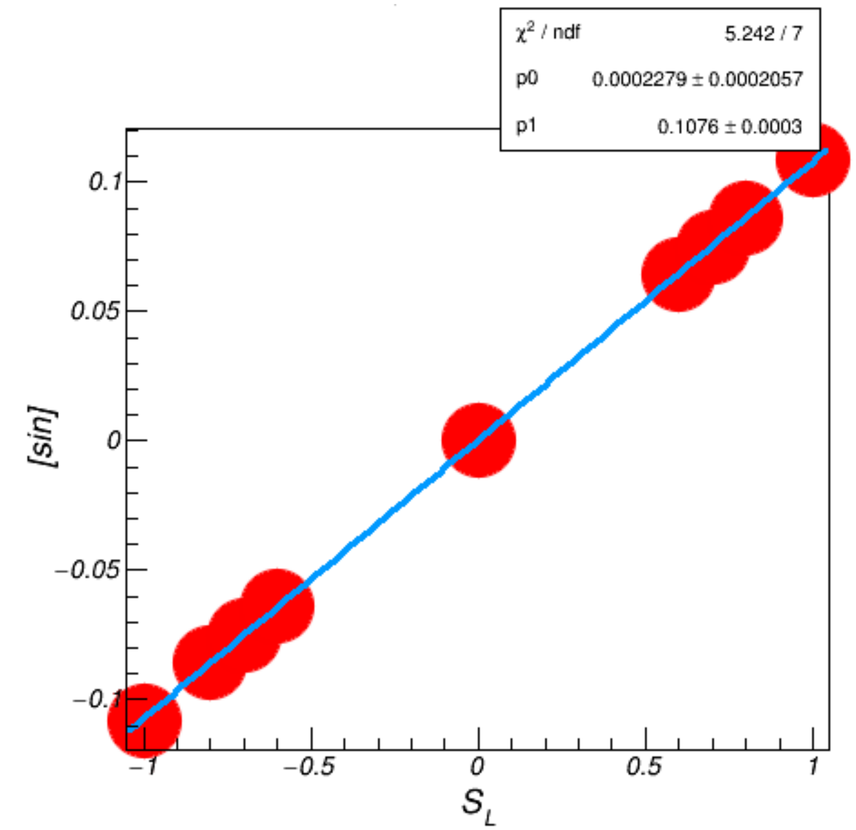
✦ **FF for longitudinally polarized quark:** $(\mathbf{R} \times \mathbf{T}) \cdot \mathbf{s}_L$

$$D_{q\rightarrow}^{h_1 h_2}(\varphi_{R-T}) = D_q^{h_1 h_2}[cos(\varphi_{R-T})] + s_L \sin(\varphi_{R-T})\mathcal{G}[cos(\varphi_{R-T})]$$
$$\varphi_{R-T} \equiv \varphi_R - \varphi_T$$



✦ **Proof of linear dependence on s_L: 9 values of $(s_L, \mathbf{s}_T)$ for $N_L = 6$.**

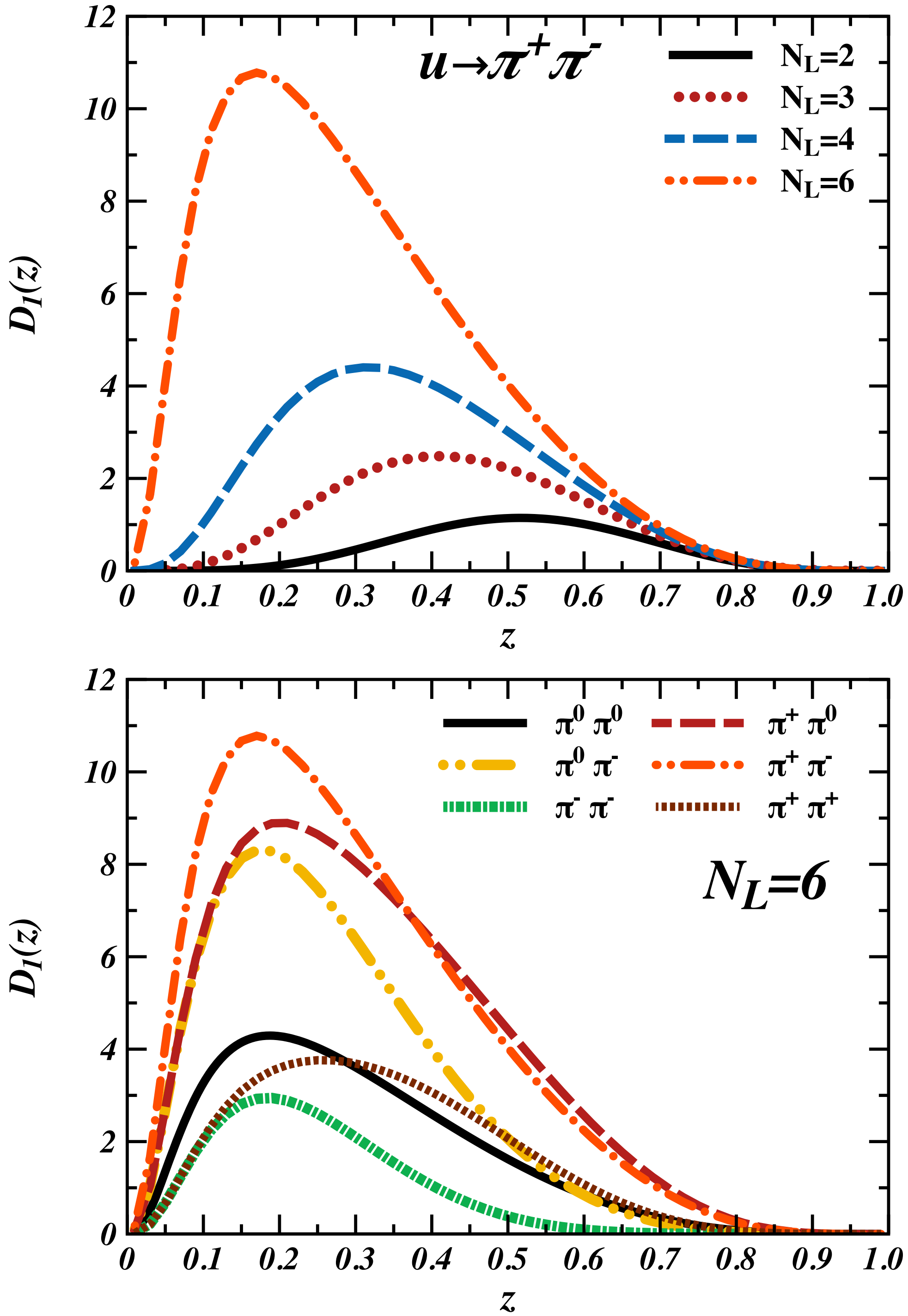




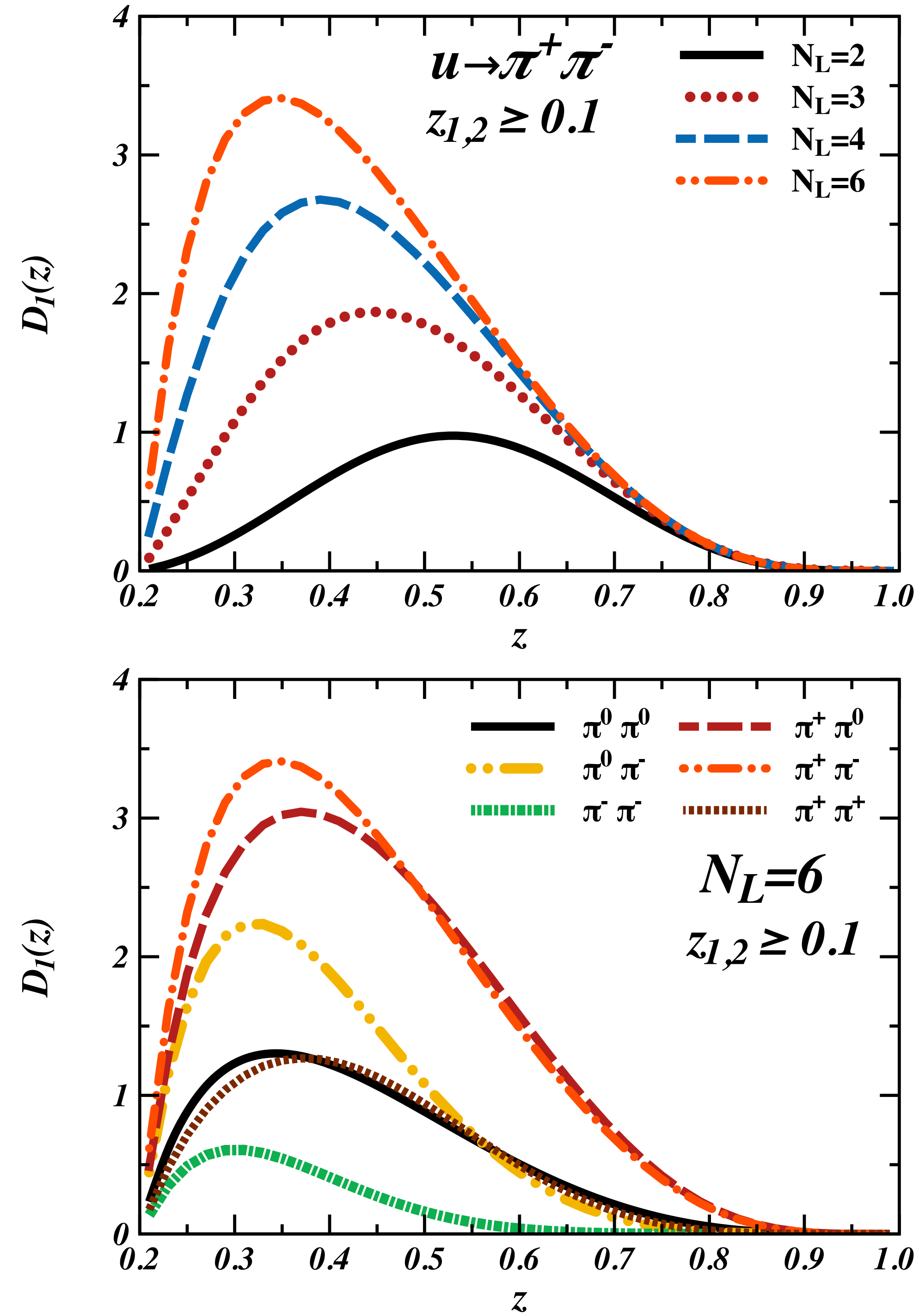
Results for unpolarized DiFF

✦ **Results for unpolarized DiFFs, *N_L* dependence, various pairs:**

- ***No Cuts***



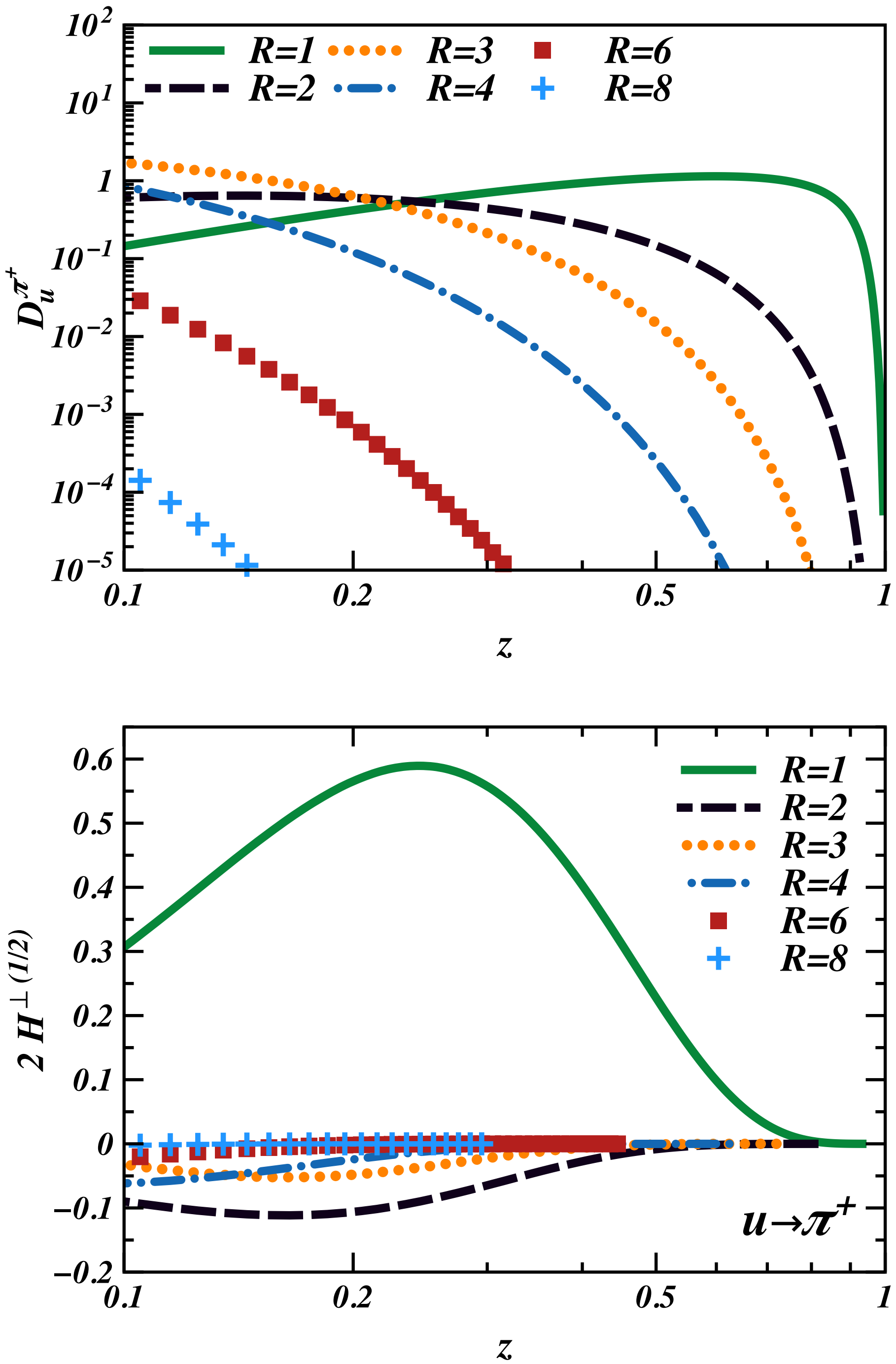
- **z Cuts:** $z_{1,2} \geq 0.1$



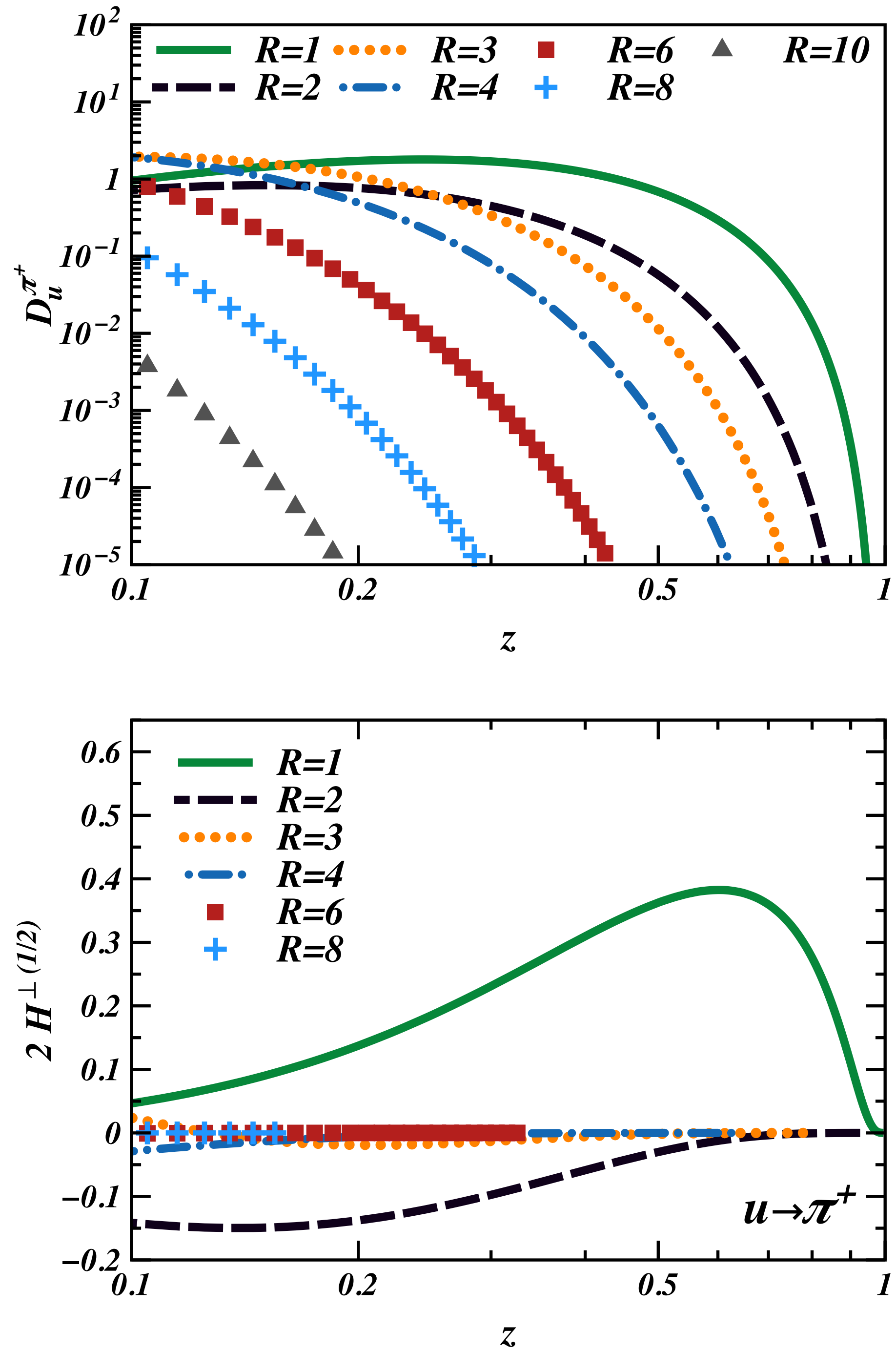
✦ $z_{1,2} \geq 0.1$ ***cut brings in convergence with N_L*** !

✦ FFs vs Rank of produced hadron.

- ***NJL* Model**



- **Evolution-mimicking *Ansatz.***



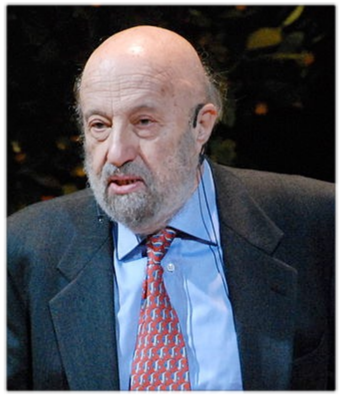
✓ **Hadrons of *Rank > 4* are negligible for FFs at *z > 0.1***

NAMBU--JONA-LASINIO MODEL

Yoichiro Nambu and Giovanni Jona-Lasinio:

"Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I"

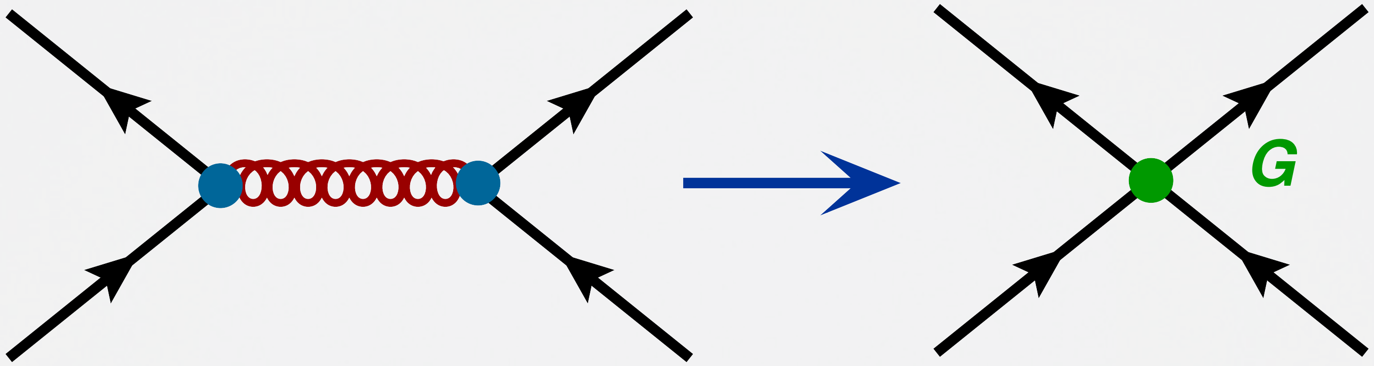
Phys.Rev. 122, 345 (1961)



Effective Quark model of QCD

- Effective Quark Lagrangian

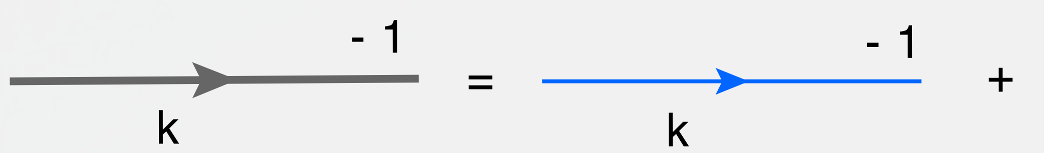
$$\mathcal{L}_{NJL} = \overline{\psi}_q (i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q \Gamma \psi_q)^2$$

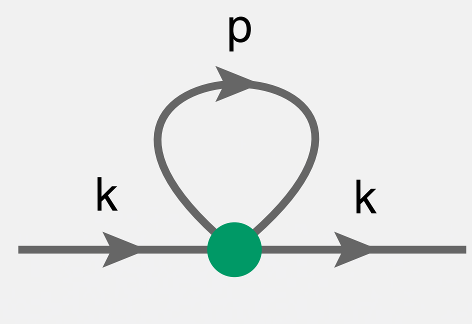


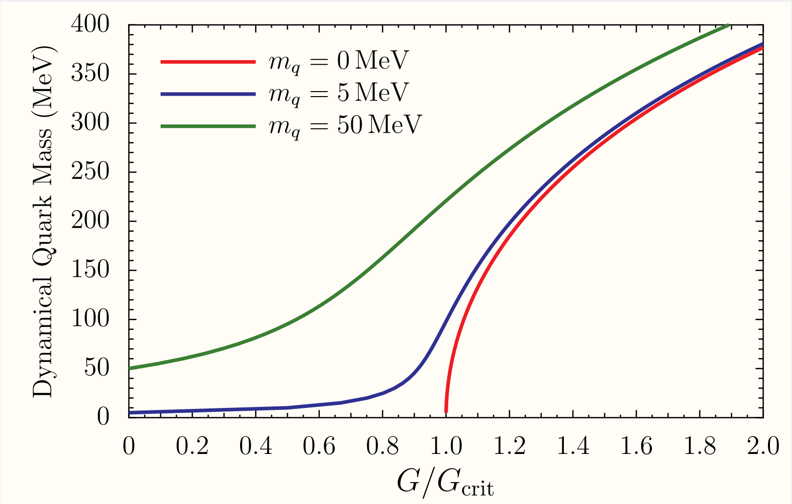
- Low energy chiral effective theory of QCD.
- Covariant, has the same flavor symmetries as QCD.

NAMBU--JONA-LASINIO MODEL

•Dynamically Generated Quark Mass from GAP Eqn.

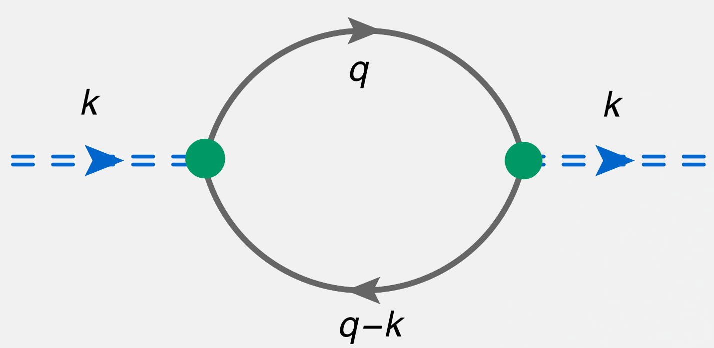


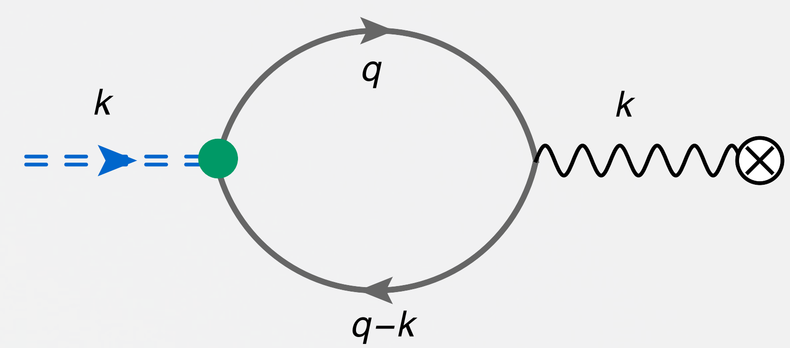




•Pion mass and quark-pion coupling from t-matrix pole.

•Pion decay constant





Fixing Model Parameters

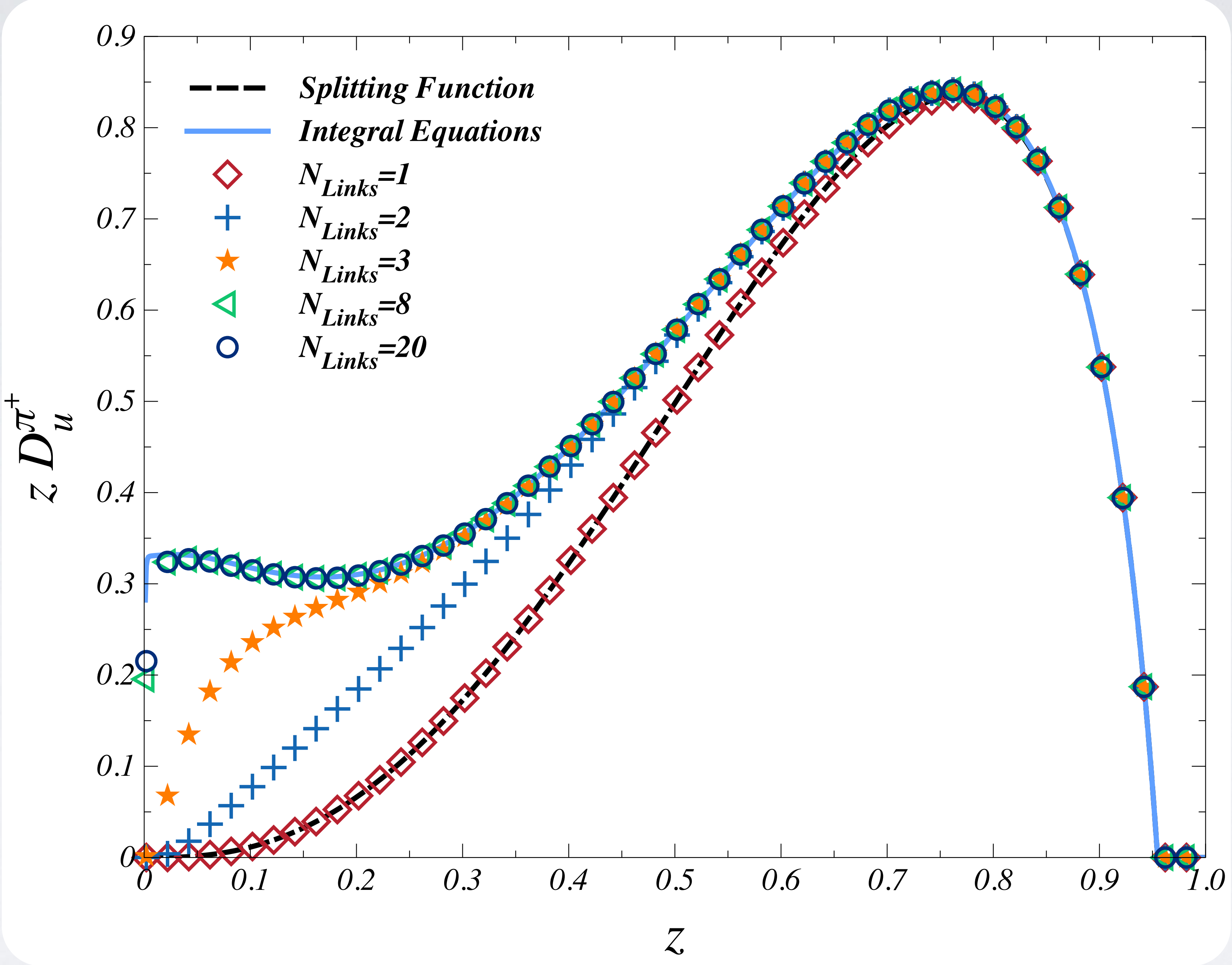
•Use Lepage-Brodsky Invariant Mass cut-off regularisation scheme.

$$M_{12} \leq \Lambda_{12} = \sqrt{\Lambda_3^2 + M_1^2} + \sqrt{\Lambda_3^2 + M_2^2}$$

• Choose a $M_{u(d)}$ and use physical f_π, m_π, m_K to fix model parameters $\Lambda_3,\ G, M_s$ and calculate g_{hqQ}.

DEPENDENCE ON NUMBER OF EMITTED HADRONS

- Restrict the number of emitted hadrons, N_{Links} in MC.

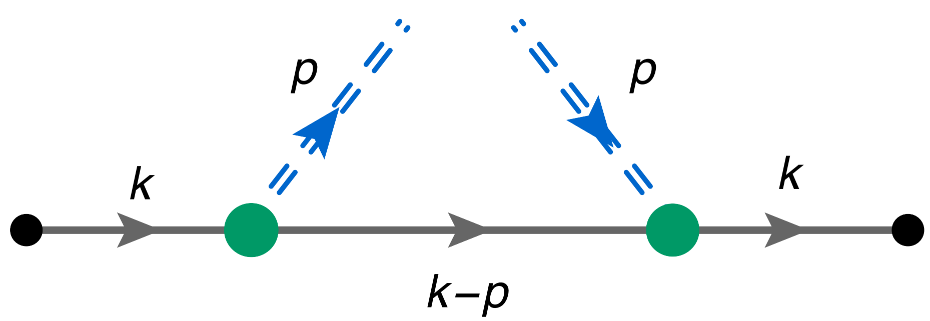


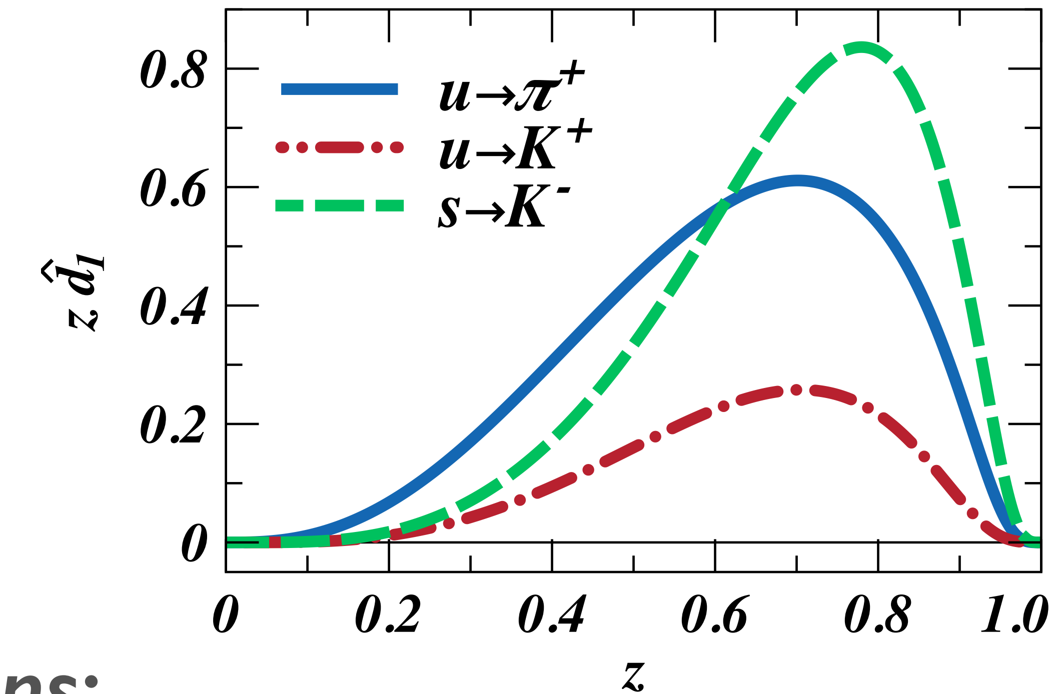
- We reproduce the splitting function and the full solution perfectly.
- The low z region is saturated with just a few emissions.

SOLUTIONS OF THE INTEGRAL EQUATIONS

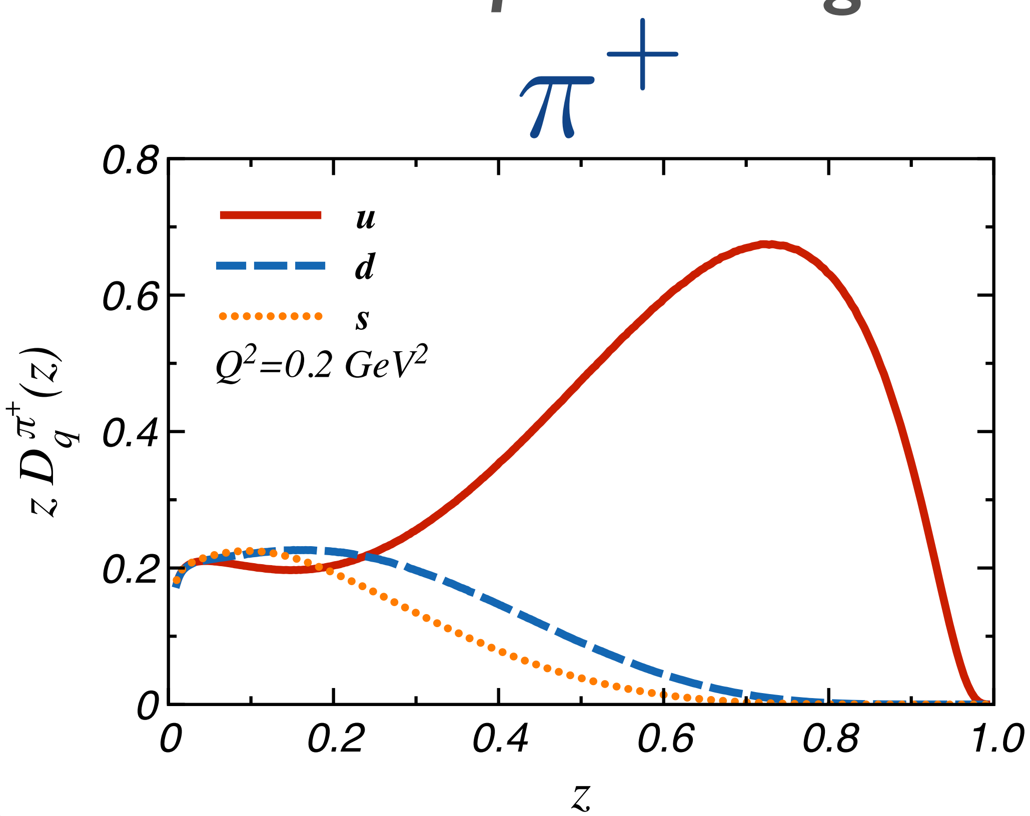
H.M., Thomas, Bentz, PRD. 83:074003, 2011

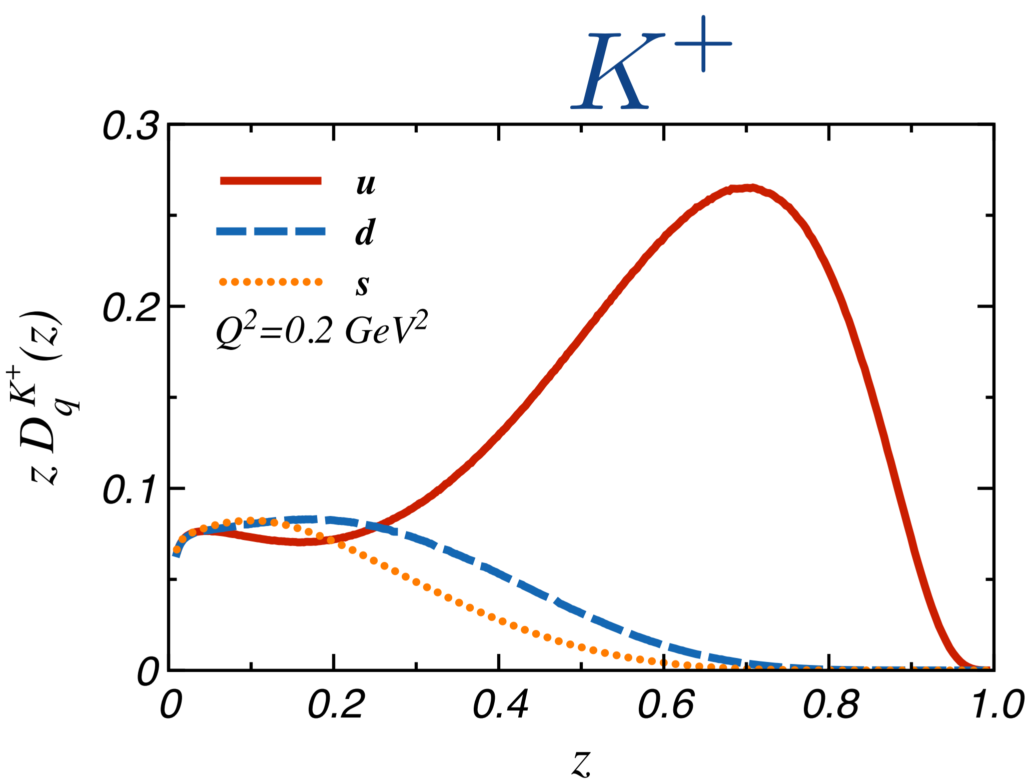
✦ ***Input elementary probabilities from NJL:***





✦ ***Solutions of the integral equations:***





SOLUTIONS OF THE INTEGRAL EQUATIONS

H.M., Thomas, Bentz, PRD. 83:074003, 2011

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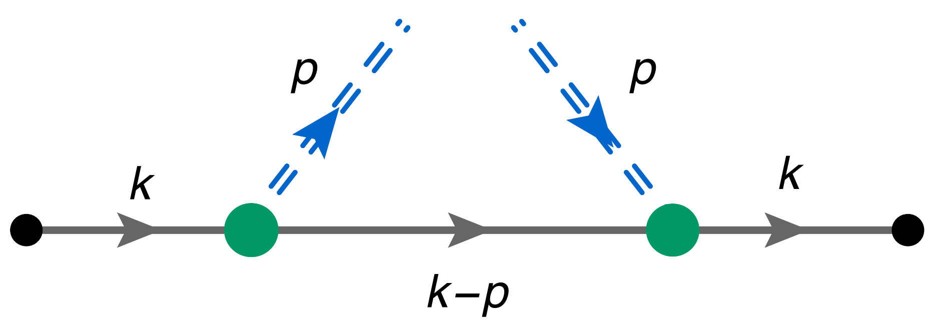
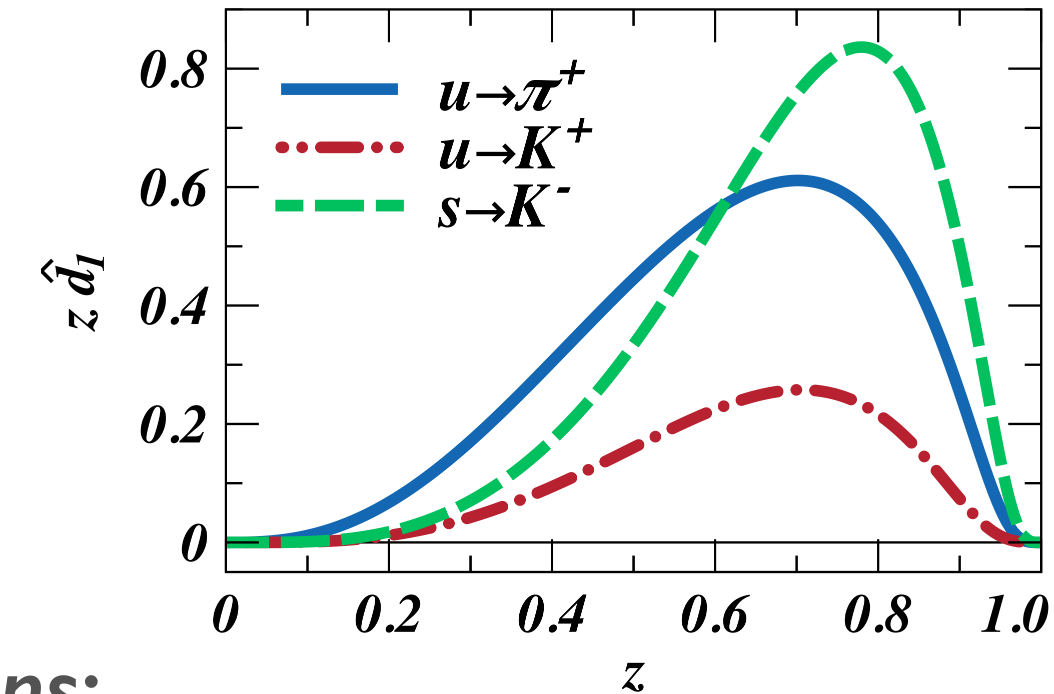
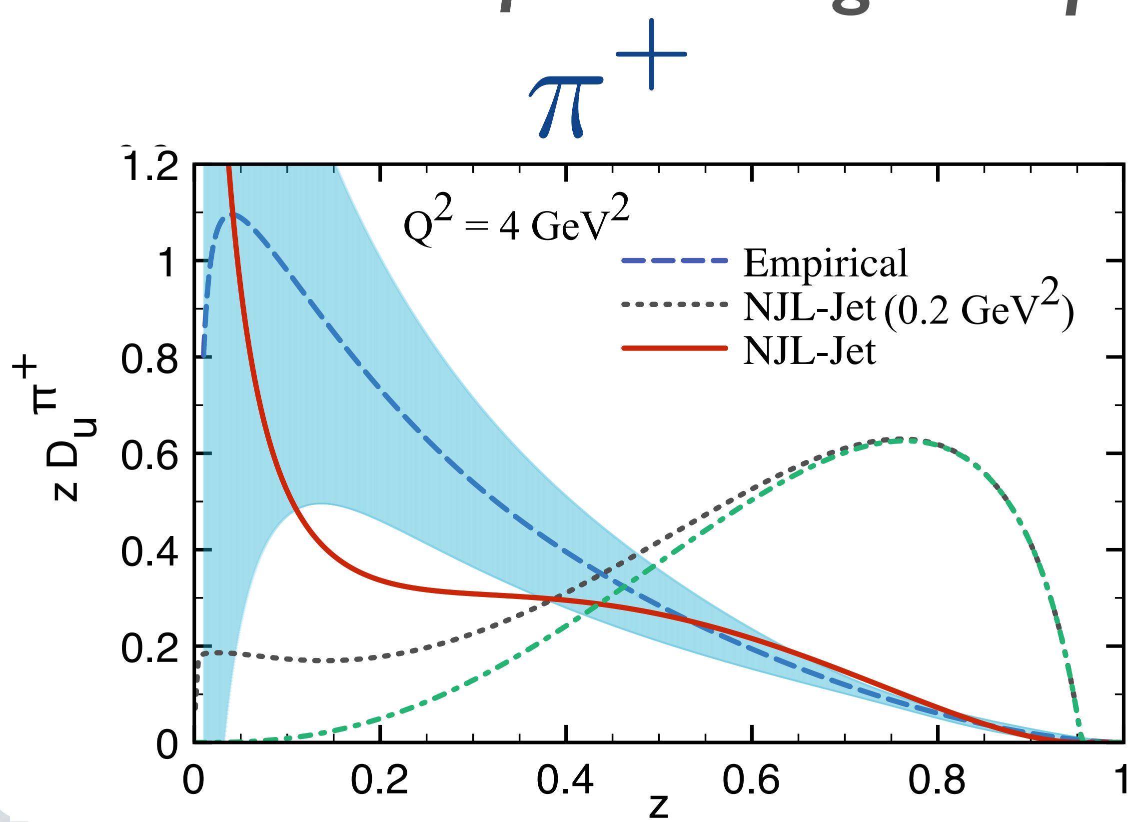
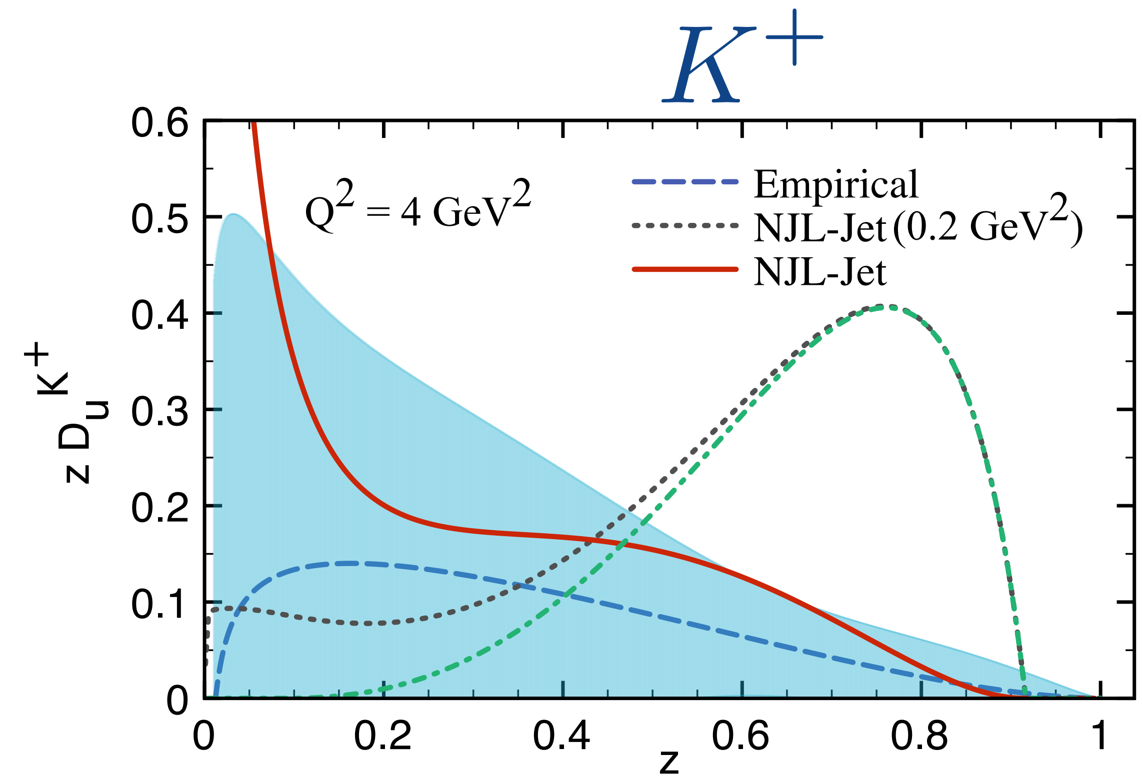




✦ ***Solutions of the integral equations:***

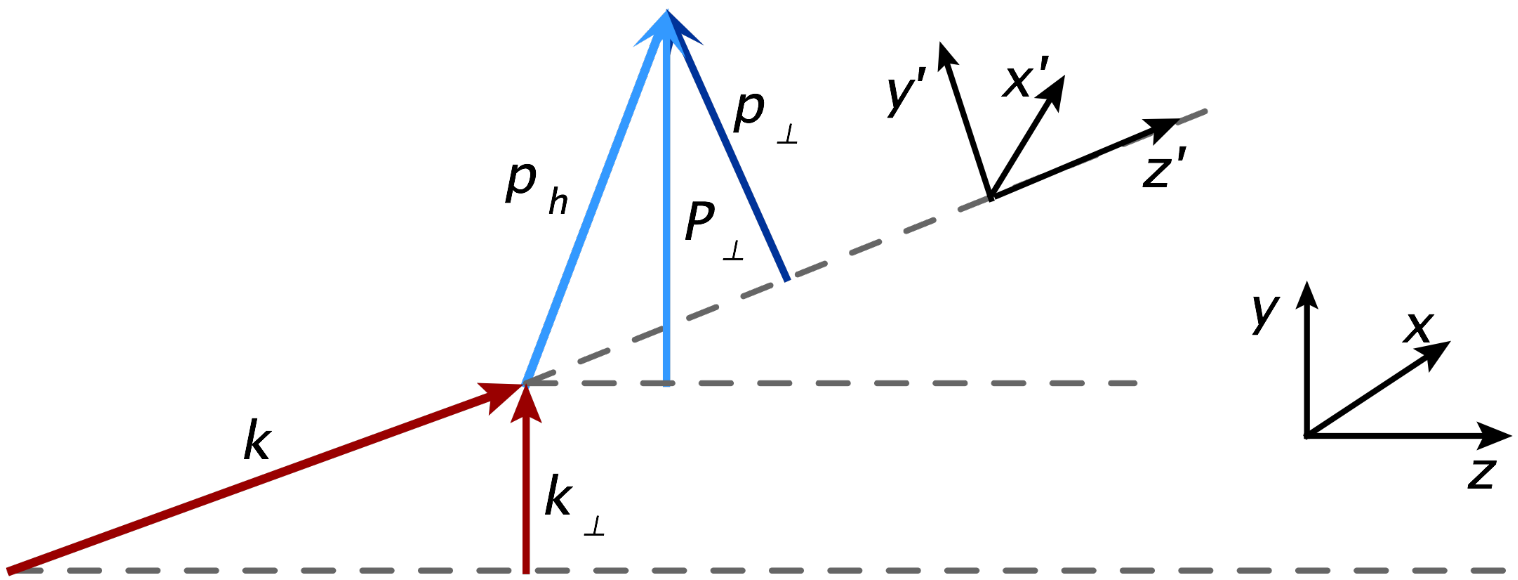




Lorentz Transforms of TM

Diehl: NPB 596, 33 (2001)(2015)

- Boosts from *0* TM frame that preserve "-" *component.*



$$\begin{pmatrix} 1 & \frac{\boldsymbol{k}_\perp^2}{2(k^-)^2} & \frac{k_1}{k^-} & \frac{k_2}{k^-} \\ 0 & 1 & 0 & 0 \\ 0 & \frac{k_1}{k^-} & 1 & 0 \\ 0 & \frac{k_2}{k^-} & 0 & 1 \end{pmatrix}$$

	q	h
$\mathcal{L}'$	$(k'^+, k'^-, \boldsymbol{k}'_\perp = 0)$	$(p^+, p^-, \boldsymbol{p}_\perp)$
$\mathcal{L}$	$(k^+, k^- = k'^-, \boldsymbol{k}_\perp)$	$(P^+, P^- = p^-, \boldsymbol{P}_\perp = \boldsymbol{p}_\perp + z\boldsymbol{k}_\perp)$

$$\boxed{z \equiv \frac{p^-}{k^-} = \frac{p'^-}{k'^-}}$$

$$\boxed{\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp}$$

- In case of two (or more) hadrons: same story!

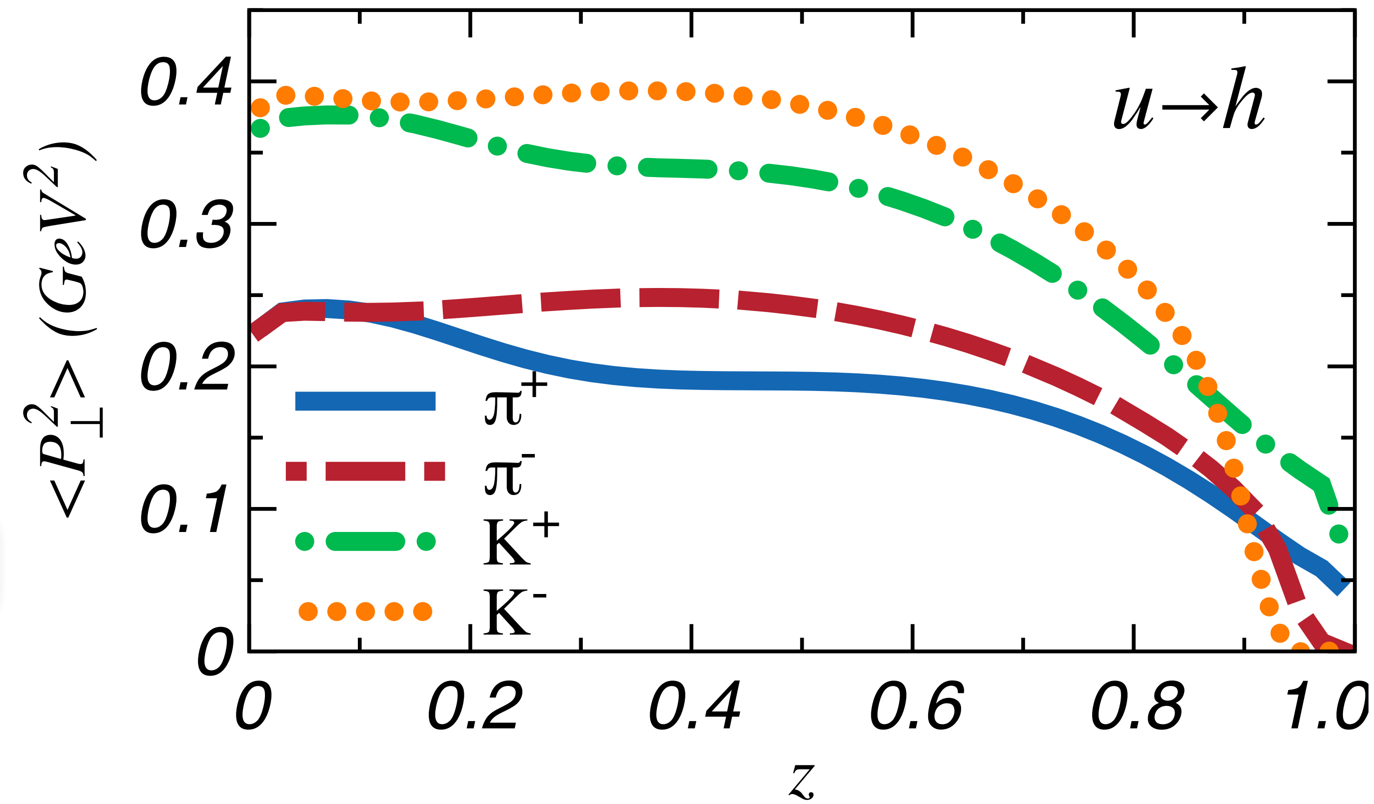
$$\boxed{\boldsymbol{P}_{1\perp} = \boldsymbol{p}_{1\perp} + z_1\boldsymbol{k}_\perp \qquad \boldsymbol{P}_{2\perp} = \boldsymbol{p}_{2\perp} + z_2\boldsymbol{k}_\perp}$$

AVERAGE Transverse Momenta vs z

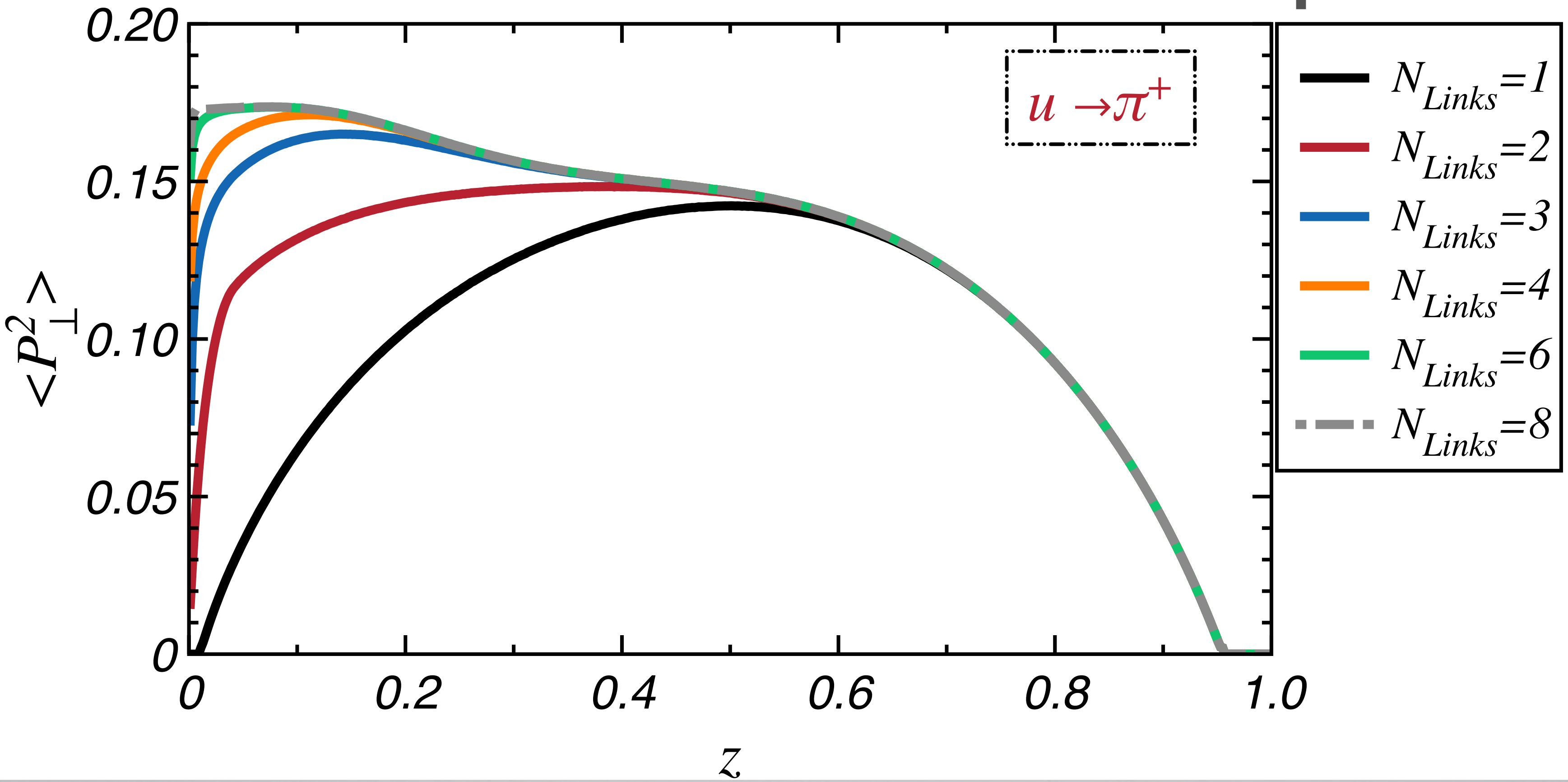
FRAGMENTATION

$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦Indications from HERMES data: A. Signori, et al: JHEP 1311, 194 (2013)



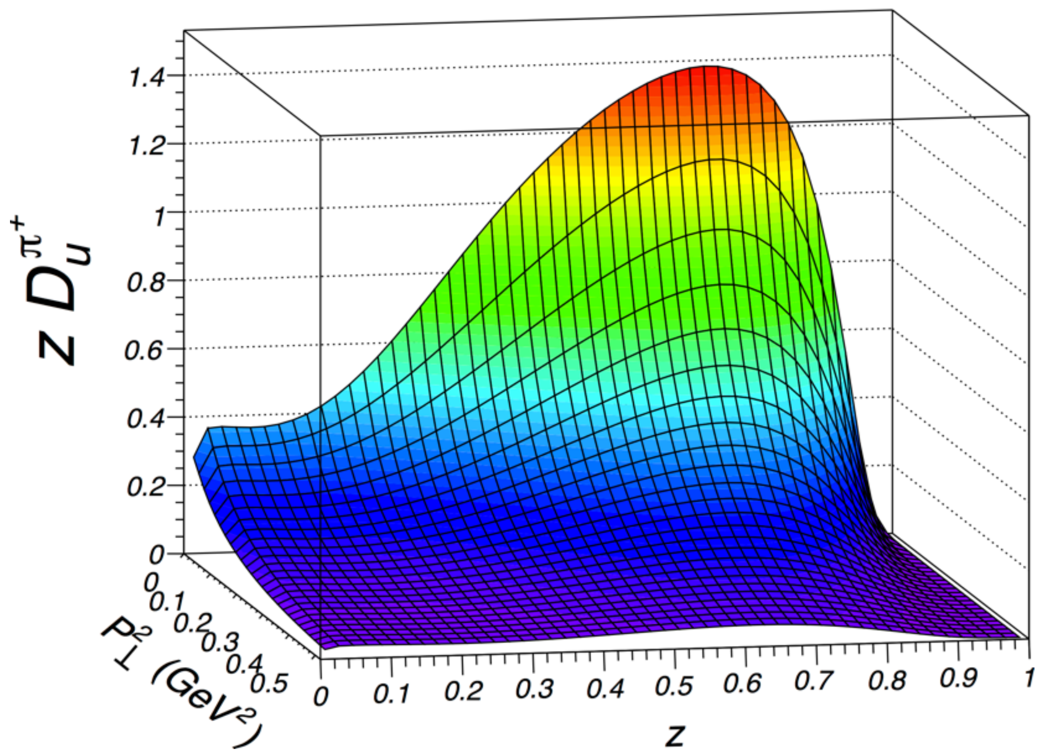
✓Multiple hadron emissions: ***broaden*** the TM dependence at ***low z***!

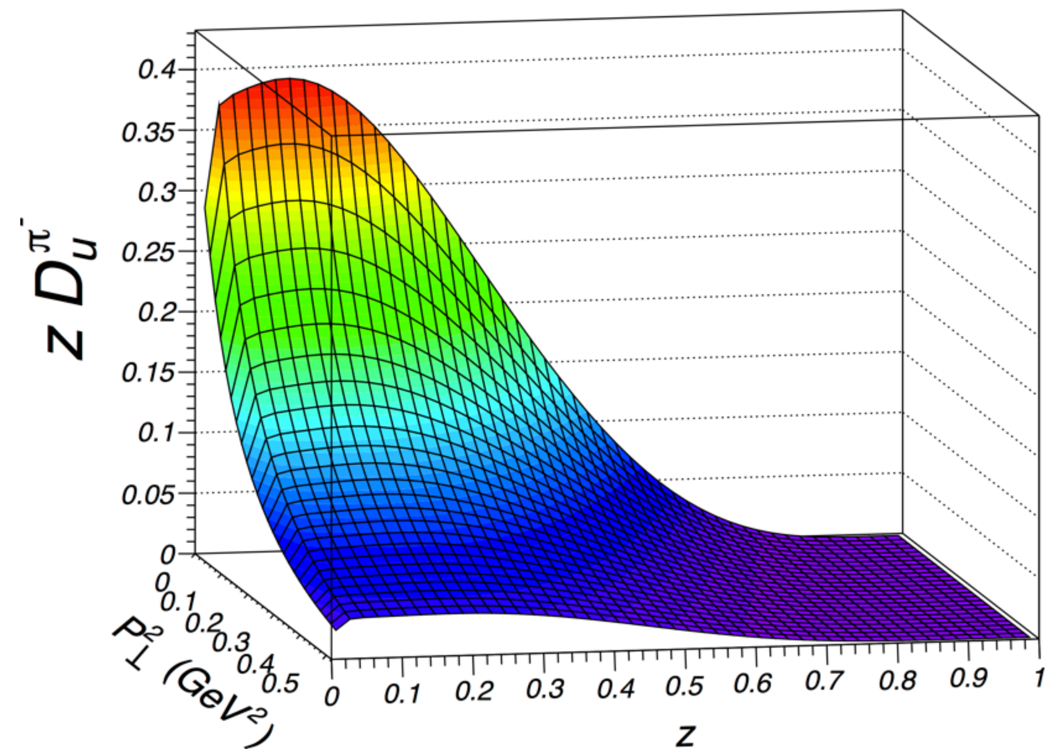


TMD FRAGMENTATION FUNCTIONS

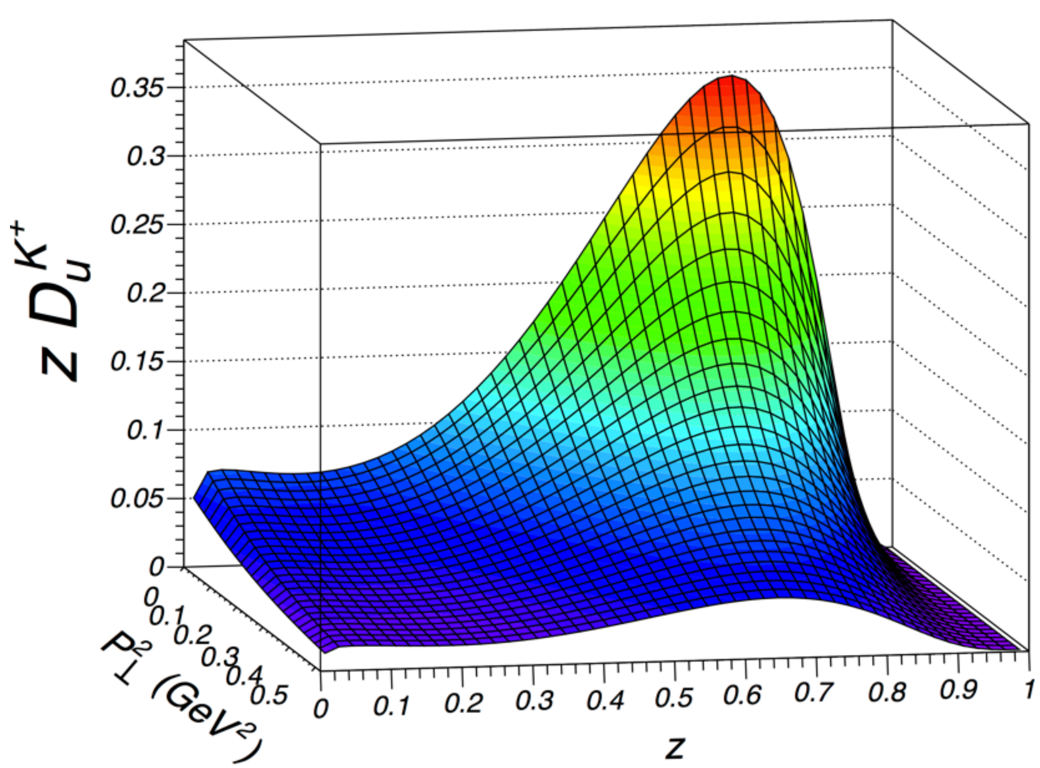
FAVORED

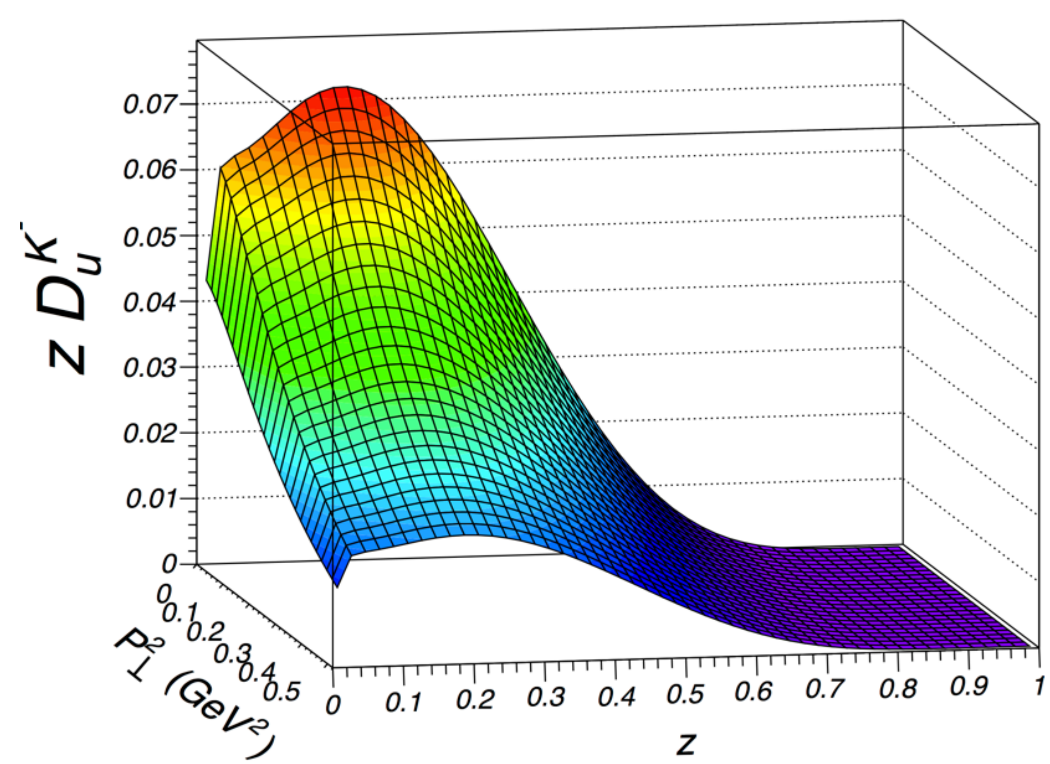
• UNFAVORED





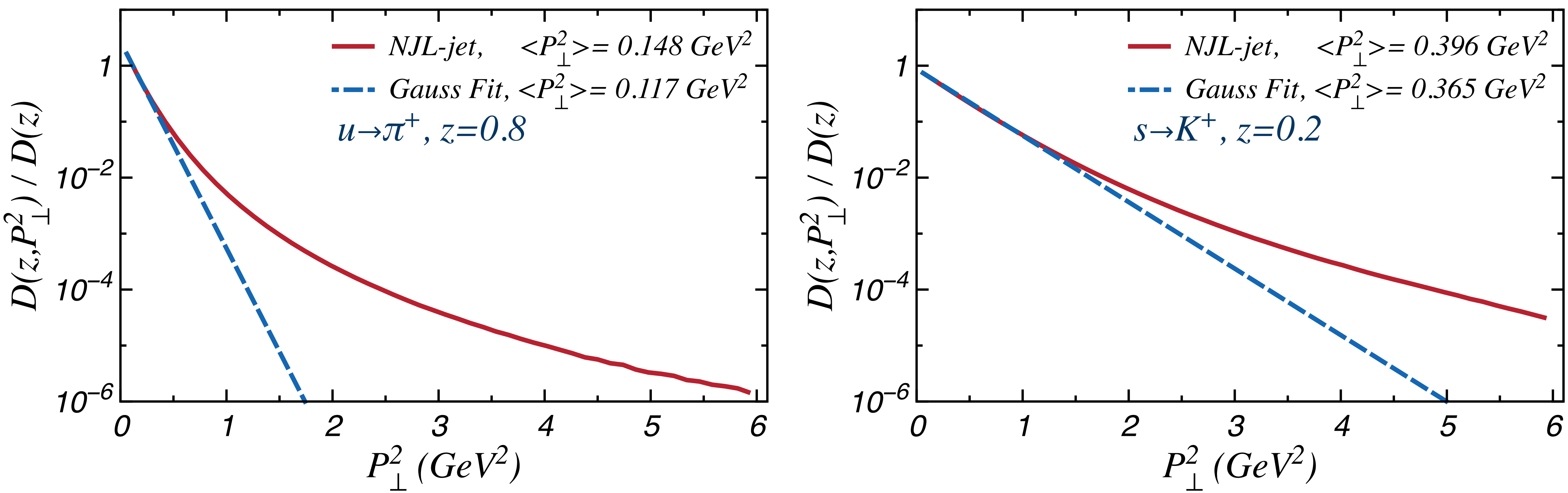
π





K

COMPARISON WITH GAUSSIAN ANSATZ



- Average TM: $\langle P_\perp^2 \rangle \equiv \dfrac{\int d^2\mathbf{P}_\perp \; P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$
- Gaussian ansatz assumes: $D(z, P_\perp^2) = D(z) \dfrac{e^{-P_\perp^2/\langle P_\perp^2 \rangle}}{\pi \langle P_\perp^2 \rangle}$