
Study of K_S^0 pair production
in single-tag two-photon collisions
at Belle



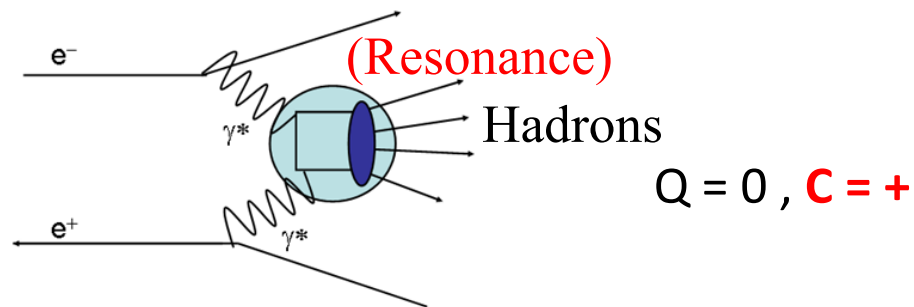
Sadaharu Uehara (KEK)
Belle Collaboration



DIS2018, PI, Kobe
Apr. 16-20, 2018



Two-photon Physics at e^+e^- collider



$\gamma^{(*)} \gamma^{(*)} \rightarrow$ hadron(s) (Exclusive final state):

Useful to Test of QCD

Measurement of resonance production and its properties

Spectroscopy and new-resonance search

Physics motivations of Single-tag measurements, $\gamma^*\gamma$:

- Q^2 dependence of transition form factor (TFF) of resonances
→ Test of QCD, models of meson/exotics, Hadron tomography by GDAs
- Reference of Light-by-Light hadronic contribution for $g-2|_{\mu}$



Measurement of single-tag processes and Form factor

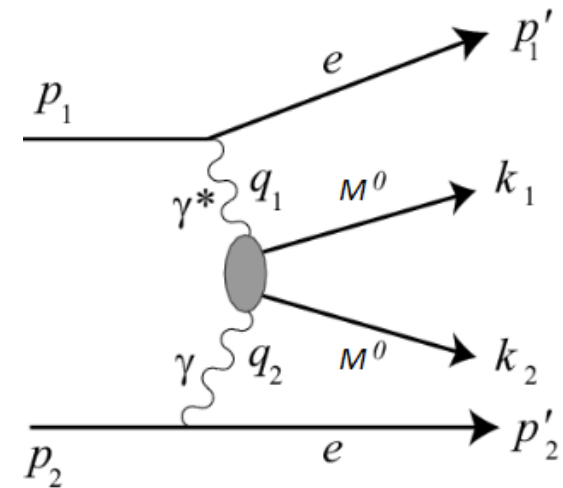
Reaction : $e^+e^- \rightarrow e$ (e) hadrons:

(e) not detected going extremely forward

$\gamma^*\gamma$ cross section $\sigma(W, Q^2)$ is derived using Equivalent Photon Approximation (luminosity function).

W -- $\gamma^*\gamma$ c.m. energy, $Q^2 = -q_1^2$ virtuality

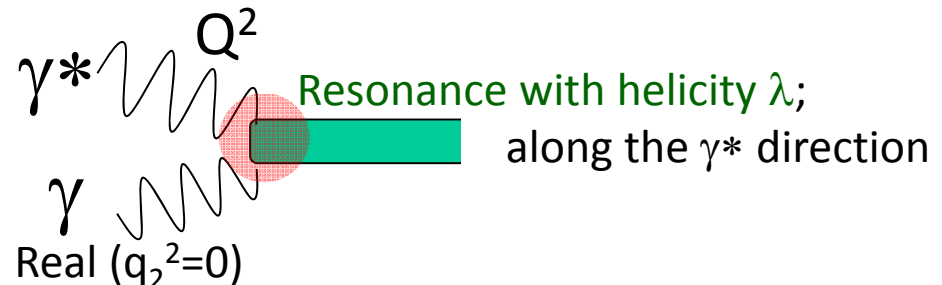
In identical neutral meson pair production (C-even), the Bremsstrahlung diagram (C-odd) is not mixed.



Transition form factor (TFF) of a resonance: $F(Q^2)$

Proportional to the helicity amplitude of the resonance production

$$\sum_{\lambda} |F(Q^2)_{\lambda}|^2 \propto \sigma(\gamma^*\gamma \rightarrow \text{Resonance})$$



KEKB Accelerator and Belle Detector

- Asymmetric $e^- e^+$ collider
8 GeV e^- (HER) x 3.5 GeV e^+ (LER)

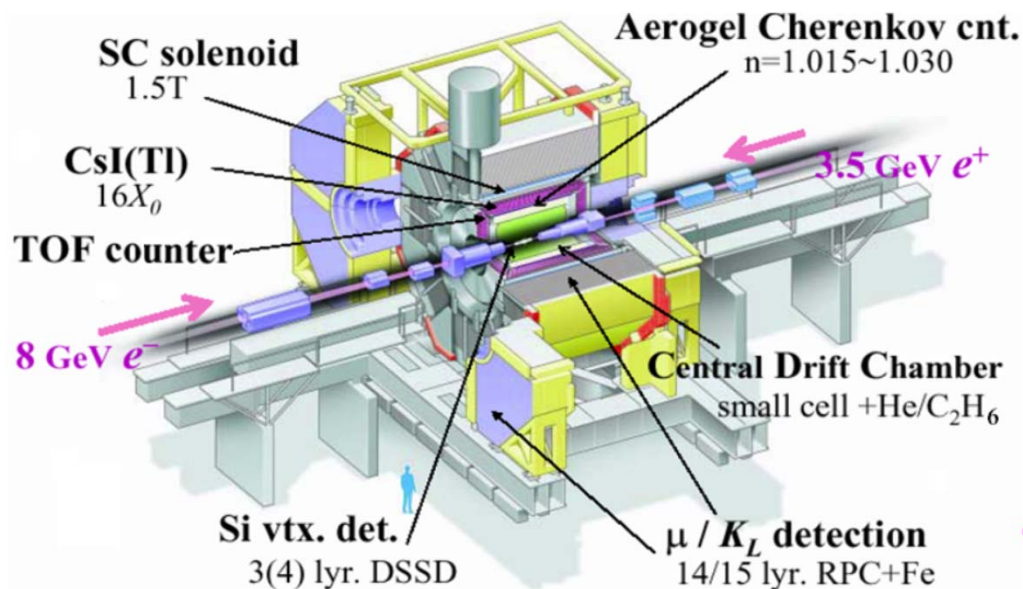
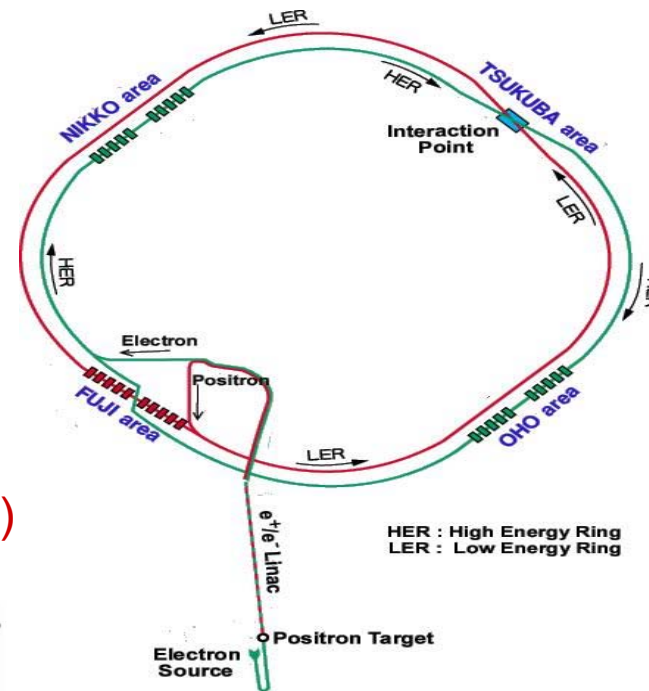
\sqrt{s} = around 10.58 GeV $\Leftrightarrow \Upsilon(4S)$

Beam crossing angle: 22mrad

- World-highest Luminosity

$$L_{\max} = 2.1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$$

$$\int L dt \sim 1040 \text{ fb}^{-1} \text{ (Completed in Jun.2010)}$$



High momentum/energy resolutions

CDC+Solenoid, CsI

Vertex measurement – Si strips

Particle identification

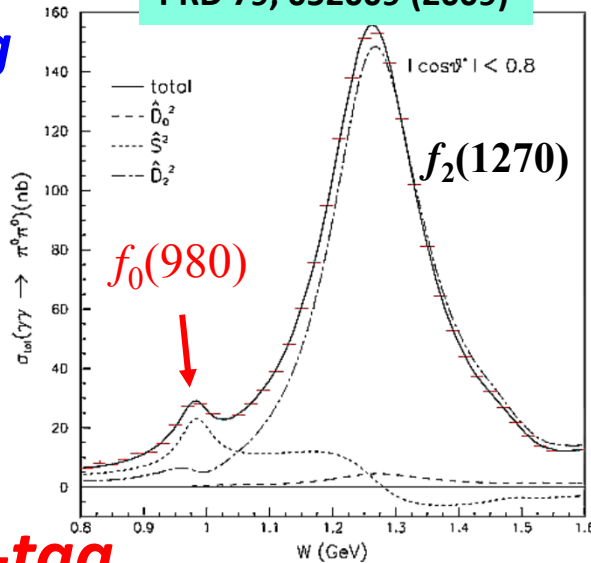
TOF, Aerogel, CDC-dE/dx,

RPC for K_L /muon

$\gamma^*\gamma \rightarrow \pi^0\pi^0 : f_0(980) \text{ and } f_2(1270) \text{ TFF's}$

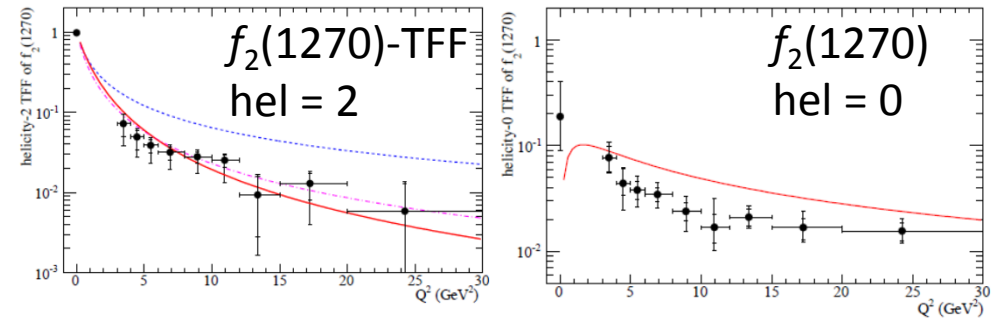
No-tag
($Q^2=0$)

PRD 79, 052009 (2009)

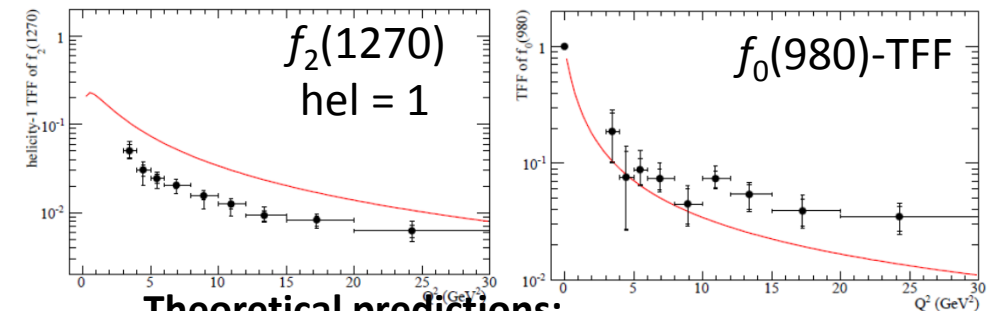
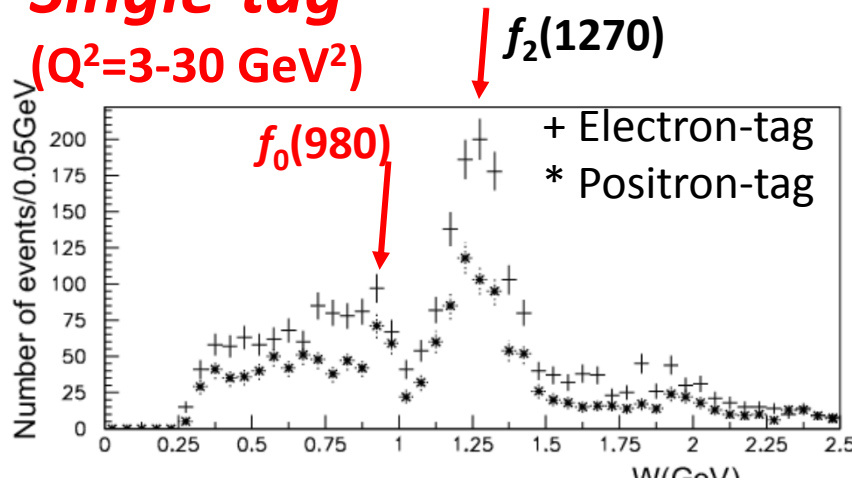


PRD 93, 032003 (2016)

$$|F(Q^2)| = \sqrt{\frac{\sigma_R^1(Q^2)}{\sigma_R^1(0)(1 + \frac{Q^2}{M^2})}}$$



Single-tag
($Q^2=3-30 \text{ GeV}^2$)



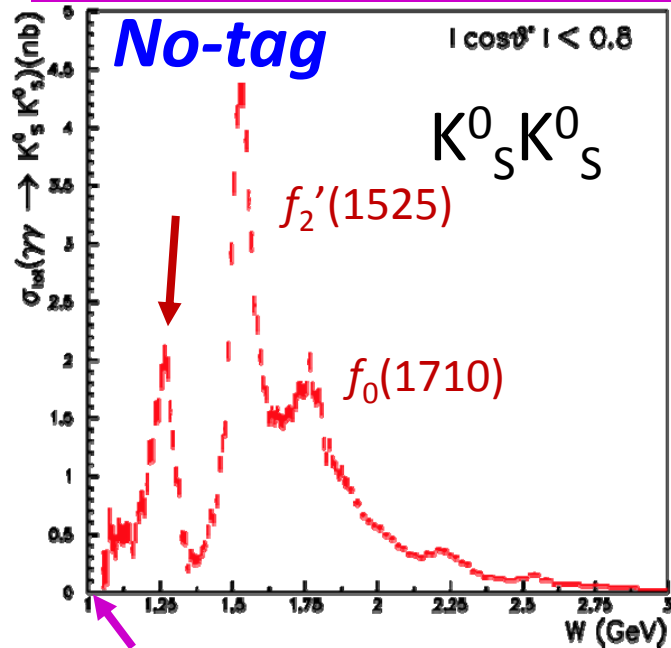
Theoretical predictions:

- Schuler, Berends, van Gulik, a heavy quark approx. NPB 523, 423 (1998) (SBG)
- Pascalutes, Pauk, Vanderhaeghen, saturated sum rule, PRD 85, 116001 (2012), η 's
- - - ibid., axial-vector mesons

The f_0/f_2 ratio larger than in the no-tag case.

Different Q^2 dependences in the helicities. 5

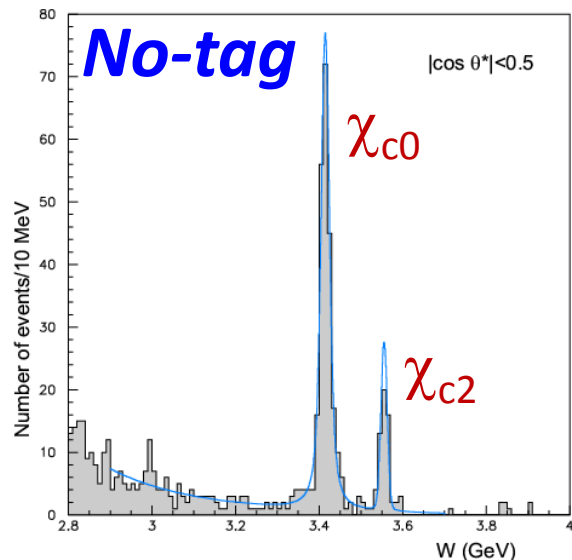
How about in the $K_S^0 K_S^0$ process?



PTEP 2013, 123C01 (2013)

Maximum at the $f_2'(1525)$ peak
 \downarrow $f_2(1270)/a_2(1320)$ destructive interference
 Two-photon coupling of $f_0(1710)$

\nwarrow No data near the $K_S^0 K_S^0$ mass threshold
 lack of trigger efficiency for low- p_t tracks



χ_{cJ} Yield

Interference	$N_{\chi_{c0}}$	$N_{\chi_{c2}}$	$-2 \ln \mathcal{L}/ndf$
not included	$248.3^{+17.9}_{-17.2}$	$53.0^{+8.1}_{-7.4}$	57.34/73
included	266 ± 53	53^{+14}_{-12}	57.22/71

Two-photon decay
 width $\times B(K_S^0 K_S^0)$

Interference	$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c0})$ (eV)	$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c2})$ (eV)
not included	$8.09 \pm 0.58 \pm 0.83$	$0.268^{+0.041}_{-0.037} \pm 0.028$
included	$8.7 \pm 1.7 \pm 0.9$	$0.27^{+0.07}_{-0.06} \pm 0.03$
Belle 2007	$7.00 \pm 0.65 \pm 0.71$	$0.31 \pm 0.05 \pm 0.03$
PDG 2012	7.3 ± 0.5	0.297 ± 0.026

Experimental analysis of Single-tag $K^0_s K^0_s$

Masuda et al. (Belle), PRD 97, 052003 (2018)

$e^+e^- \rightarrow e (e) K^0_s K^0_s, K^0_s \rightarrow \pi^+ \pi^-$ 759 fb^{-1}

Topology: 1 electron(or positron) and 4 charged pions

Event Selection Criteria:

for tracks 5 tracks satisfy $p_t > 0.1 \text{ GeV}/c$, ≥ 2 of them satisfy $p_t > 0.4 \text{ GeV}/c$,
1 of them satisfies **e-identification** and $p > 1.0 \text{ GeV}/c$

for K^0_s Charged π/K separation

Reconstructed $K^0_s K^0_s$ masses (two-dimensional cut) :

$492.6 < \text{ave}[M(K^0_s)_s] < 502.6 \text{ MeV}/c^2$ and $\text{diff}[M(K^0_s)_s] < 10 \text{ MeV}/c^2$

K^0_s decay vertex: $0.3 < v_r < 8 \text{ cm}$

(a finite decay flight length in the $r\phi$ plane)

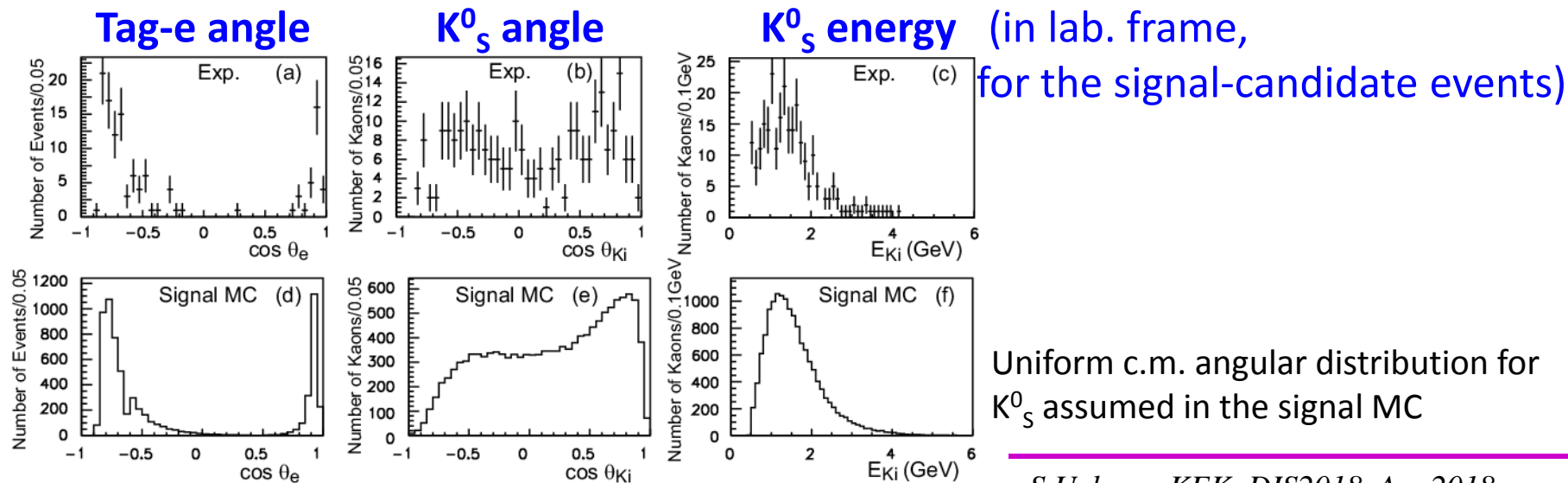
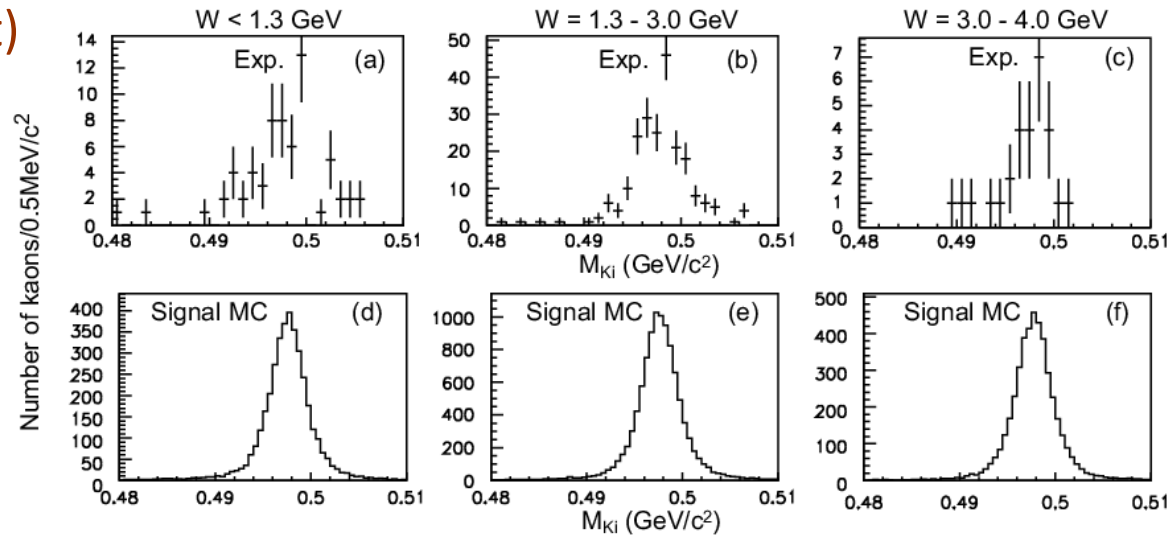
Kinematical cuts (Energy/momentum conservation and transverse-momentum balance)

$$E_{\text{ratio}} = \frac{E_{K^0_s K^0_s}^{\text{measured}}}{E_{K^0_s K^0_s}^{\text{expected}}} \text{ and } |\Sigma p_t^*| \text{ satisfy } \sqrt{\left(\frac{E_{\text{ratio}}-1}{0.04}\right)^2 + \left(\frac{|\Sigma p_t^*|}{0.1 \text{ GeV}/c}\right)^2} \leq 1$$



Reconstructed mass, angles and Energy of the Signal candidates

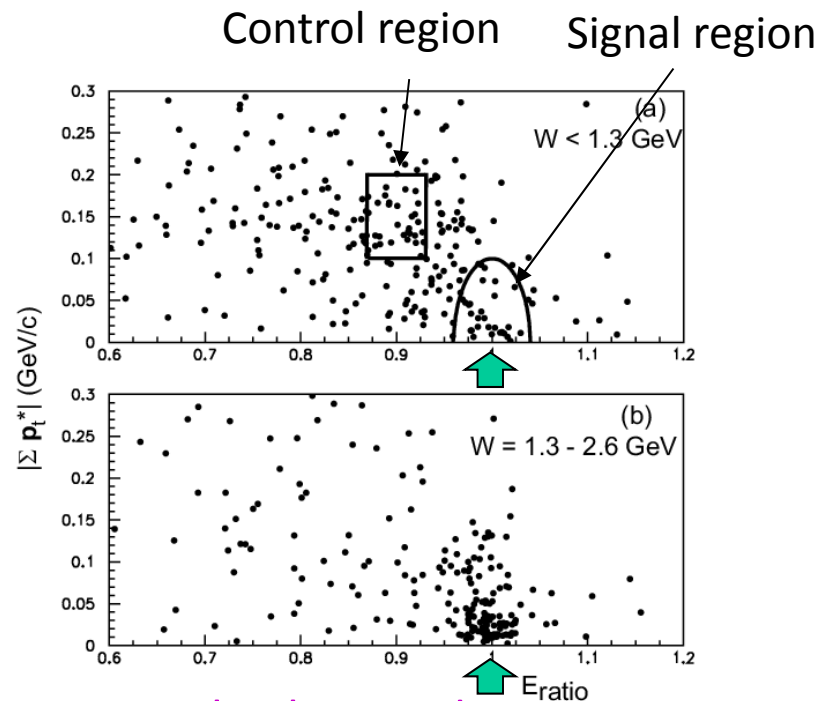
Reconstructed K^0_s mass (with a looser cut)



Uniform c.m. angular distribution for K^0_s assumed in the signal MC

Background processes

Rejection of non-exclusive background,
 $K_S^0 K_S^0 X$ using $|\Sigma p_t^*|$ vs. E_{ratio}



14% background
 only for $W < 1.3 \text{ GeV}$

Systematic uncertainty

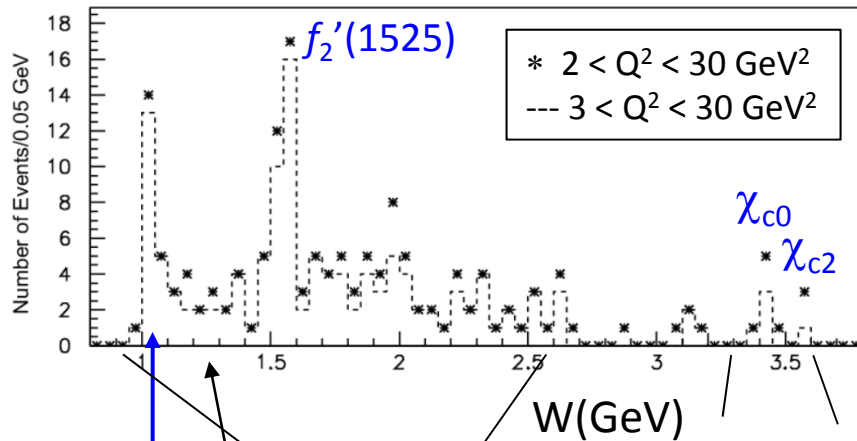
TABLE V: Sources of systematic uncertainties. The values are indicated for specific W ranges. DCS stands for the differential cross section.

Source	Uncertainty (%)
Tracking	2
Electron-ID	1
Pion-ID (for four pions)	2
K_S^0 reconstruction (for two K_S^0 's)	3
Kinematic selection	4
Geometrical acceptance	1
Trigger efficiency	1 - 3
Background effect for the efficiency	2
Angular dependence of DCS	6 - 22
Background subtraction	3 - 7
No unfolding applied	1
Radiative correction	3
Luminosity function	4
Integrated luminosity	1.4
Total	13 - 24

Total: 13% - 24%



W dependence and $\gamma^*\gamma$ cross section at Q^2 bins



* $2 < Q^2 < 30 \text{ GeV}^2$
 --- $3 < Q^2 < 30 \text{ GeV}^2$

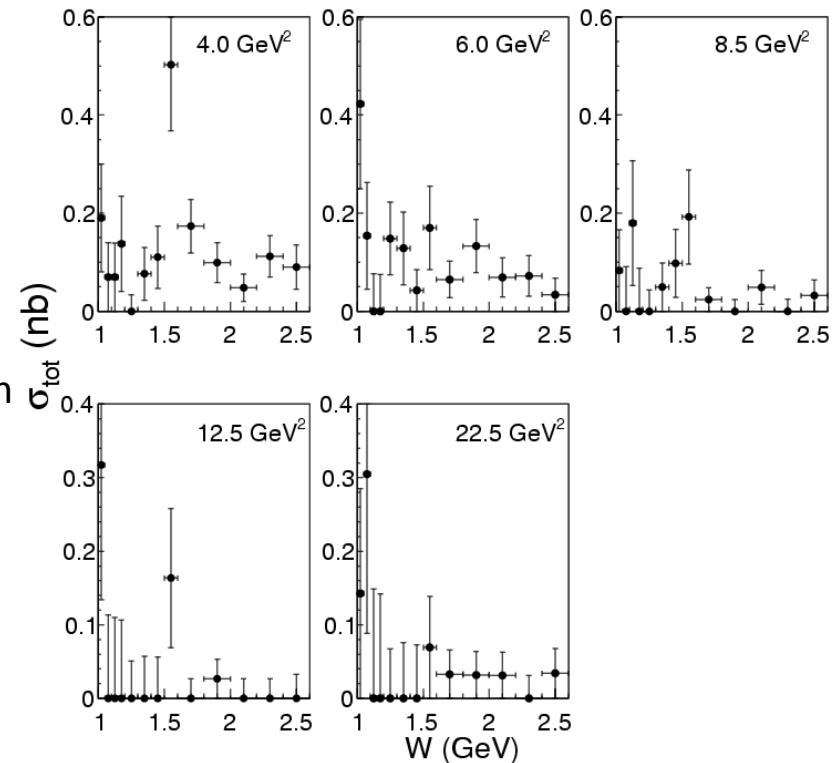
121 events
 $2M(K_S^0) < W < 2.6 \text{ GeV}$
 $(2 < Q^2 < 30 \text{ GeV}^2)$

10 events
 χ_{c1} charmonium region
 $(2 < Q^2 < 30 \text{ GeV}^2)$

No $a_2(1320)/f_2(1270)$ seen

Threshold enhancement
 (including backgrounds)

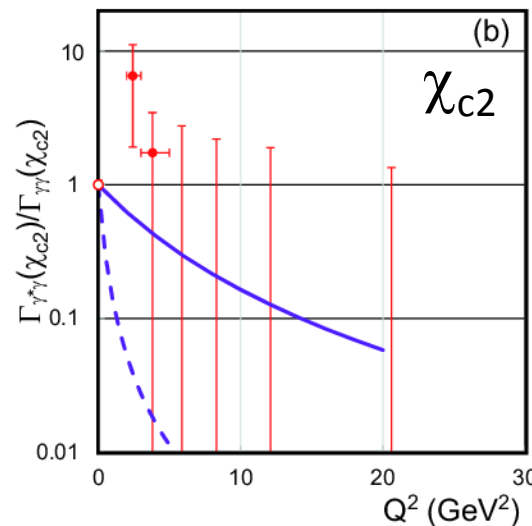
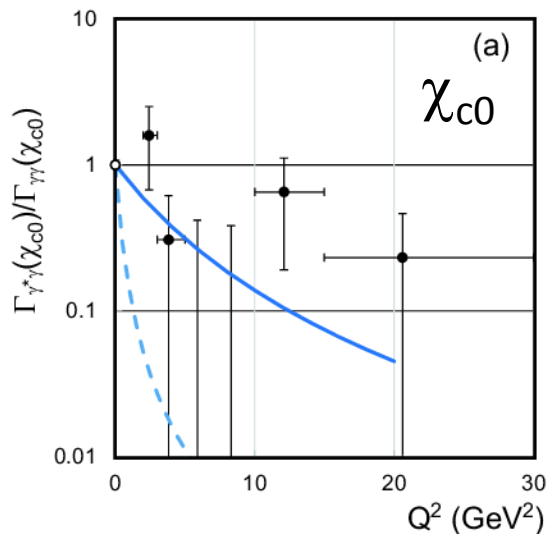
$$\sigma_{\text{tot}}(\gamma^*\gamma \rightarrow K_S^0 K_S^0) = \frac{1}{2} \frac{d^2 L_{\gamma^*\gamma}}{dW dQ^2} \frac{Y(W, Q^2)}{(1 + \delta)\epsilon(W, Q^2)\Delta W \Delta Q^2 \int \mathcal{L} dt B^2}$$



χ_{cJ} charmonia

Assume that in total 7 events (3 events) peaking near the χ_{c0} (χ_{c2}) mass are purely from the charmonium (backgrounds are estimated <1 event in total)

$$\frac{d\sigma_{ee}}{dQ^2} = 4\pi^2 \left(1 + \frac{Q^2}{M_R^2}\right) \frac{(2J+1)}{M_R^2} \frac{2d^2 L_{\gamma^*\gamma}}{dW dQ^2} \Gamma_{\gamma^*\gamma}(Q^2) \mathcal{B}(K_S^0 K_S^0) \quad : \text{Definition of } \Gamma_{\gamma^*\gamma}$$



The first measurement of χ_{cJ} in the single-tag two-photon production

Solid curve: SBG with the charmonium-mass scale ← much favored

Dashed curve: With the ρ -mass scale (VDM like)



Partial Wave Analysis for TFF of $f'_2(1525)$

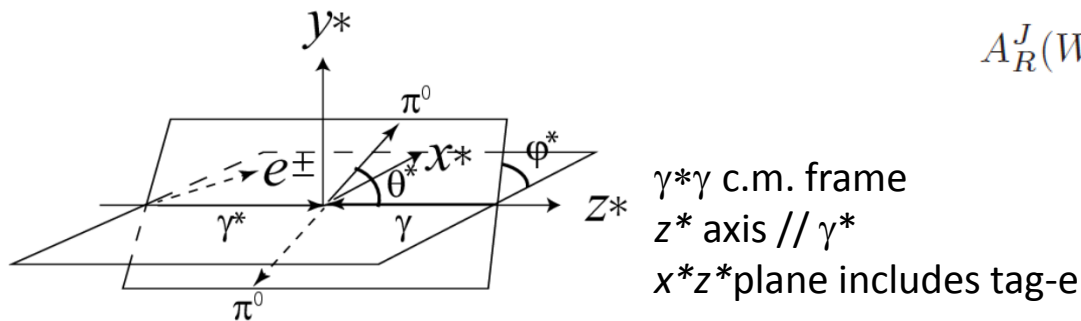
Applied for $W < 1.8$ GeV. We take into account partial waves up to $J=2$. $J=1$ does not couple with $K_S^0 K_S^0$ ($\rightarrow J^P = 0^+$ and 2^+)

PRD 97, 052003 (2018)

$$\frac{d\sigma(\gamma^* \gamma \rightarrow K_S^0 K_S^0)}{d\Omega} = \sum_{n=0}^2 t_n \cos(n\varphi^*),$$

Resonance amplitude for f'_2 , etc.

$$A_R^J(W) = F_R(Q^2) \sqrt{1 + \frac{Q^2}{m_R^2}} \sqrt{\frac{8\pi(2J+1)m_R}{W}} \\ \times \frac{\sqrt{\Gamma_{\text{tot}}(W)\Gamma_{\gamma\gamma}(W)\mathcal{B}(K_S^0 K_S^0)}}{m_R^2 - W^2 - im_R\Gamma_{\text{tot}}(W)}$$



$$t_0 = |SY_0^0 + D_0Y_2^0|^2 + |D_2Y_2^2|^2 + 2\epsilon_0|D_1Y_2^1|^2, \\ t_1 = 2\epsilon_1 \Re [(D_2^*|Y_2^2| - S^*Y_0^0 - D_0^*Y_2^0)D_1|Y_2^1|], \\ t_2 = -2\epsilon_0 \Re [D_2^*|Y_2^2|(SY_0^0 + D_0Y_2^0)].$$

TFF of f'_2 for helicity $i = \lambda$

$$\sqrt{r_{ifp}} F_{f2p} \quad (i = 0, 1, 2) \\ r_{0fp} + r_{1fp} + r_{2fp} = 1$$

S, D_0 , etc. --- Partial-wave amplitudes

ϵ_0, ϵ_1 --- Spin-dependent flux factor ratios for the virtual photon

Y_j^m --- Spherical harmonics



Formalism of PWA and parametrizations

Problems: Low statistics

Only 3 out of S , D_0 , D_1 and D_2 are independent

Non-unique solution (multiple solutions for resonances)

→ Parametrization of the amplitudes with modelled W and Q^2 dependences

$$\begin{aligned}
 S &= A_{BW} e^{i\phi_{BW}} + B_S e^{i\phi_{BS}}, \\
 D_i &= \sqrt{r_{ifa}(Q^2)} (A_{f_2(1270)} - A_{a_2(1320)}) e^{i\phi_{faDi}} \\
 &\quad + \sqrt{r_{ifp}(Q^2)} A_{f'_2(1525)} e^{i\phi_{fpDi}} \\
 &\quad + B_{Di} e^{i\phi_{BDi}},
 \end{aligned}$$

$$\begin{aligned}
 A_{BW}(W) &= \sqrt{\frac{8\pi m_S}{W}} \frac{f_S}{m_S^2 - W^2 - im_S g_S} \\
 &\quad \times \frac{1}{(Q^2/m_0^2 + 1)^{ps}},
 \end{aligned}$$

Nominal fit

$B_S = 0$

$$B_S = \frac{\beta a_S (W_0/W)^{b_S}}{(Q^2/m_0^2 + 1)^{c_S}},$$

$$B_{D0} = \frac{\beta^5 a_{D0} (W_0/W)^{b_{D0}}}{(Q^2/m_0^2 + 1)^{c_{D0}}},$$

$$B_{D1} = \frac{\beta^5 Q^2 a_{D1} (W_0/W)^{b_{D1}}}{(Q^2/m_0^2 + 1)^{c_{D1}}},$$

$$B_{D2} = \frac{\beta^5 a_{D2} (W_0/W)^{b_{D2}}}{(Q^2/m_0^2 + 1)^{c_{D2}}},$$

$\beta = \sqrt{1 - 4m_{K_S^0}^2/W^2}$ is the K_S^0 velocity

$$r_{0fp} : r_{1fp} : r_{2fp} = k_0 Q^2 : k_1 \sqrt{Q^2} : 1$$

-Destructive interference between $f_2(1270)$ and $a_2(1320)$

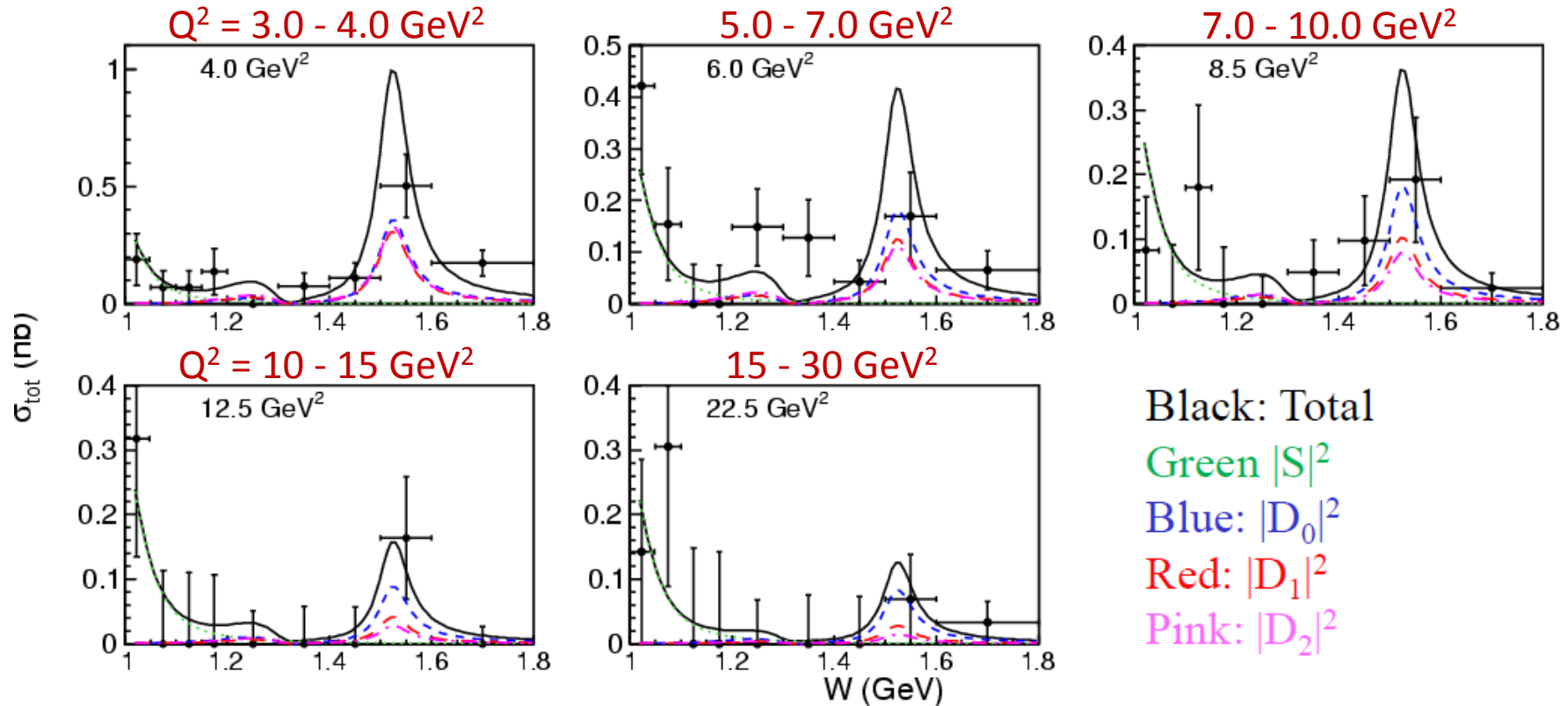
- $r_i(Q^2)$ and TFF for $f_2(1270)$ and $a_2(1320)$ are the same;

use the values obtained in single-tag $\pi^0\pi^0$

Determine each component and the relative phase by a fit



Fit results in W dependence at Q² bins



Show indications of:

- Non-zero D_0 and D_1 components in the f_2' (1525).
- $f_2(1270)/a_2(1320)$ not visible
- An enhancement near the threshold (0.995 GeV).



Angular dependence and the PWA fit

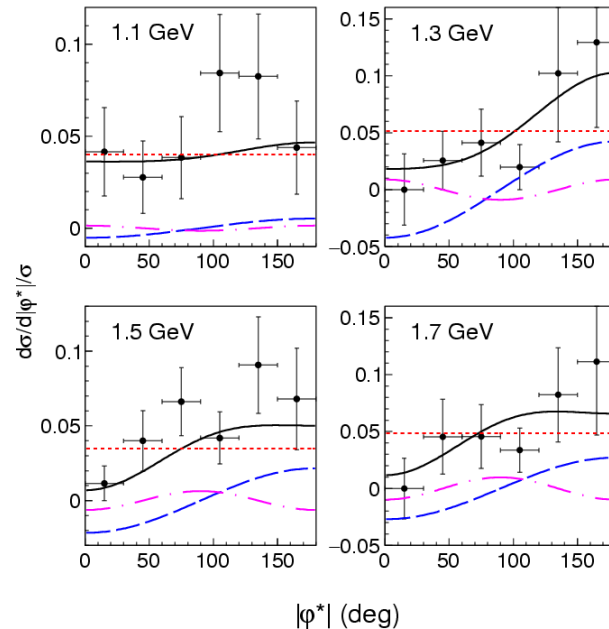
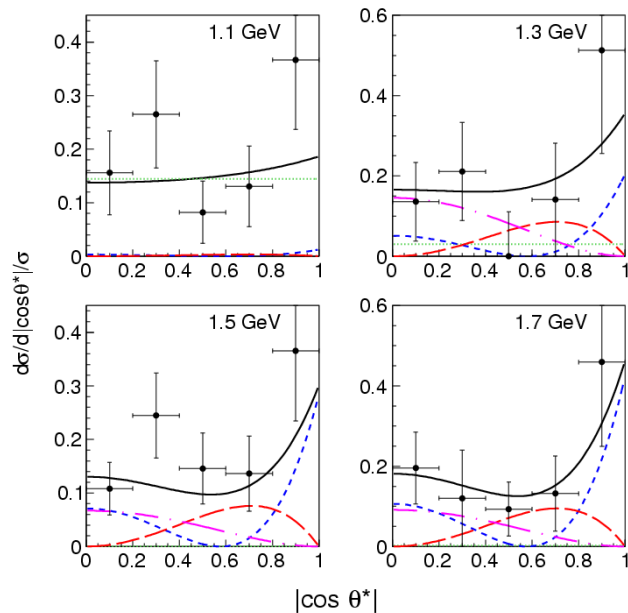
Due to a lack of statistics, we use **Q²-integrated angular differential cross section** derived with the following convention (MC generated isotropically)

$$\frac{d^2\sigma/d|\cos\theta^*|d|\varphi^*|}{N_{\text{EXP}}(|\cos\theta^*|, |\varphi^*|)/N_{\text{MC}}(|\cos\theta^*|, |\varphi^*|)} \propto$$

Q²: integrated over the full range between 3 and 30 GeV²
W: 4 bins

|cos θ*| dependence (|φ*| integrated)

|φ*| dependence (|cos θ*| integrated)



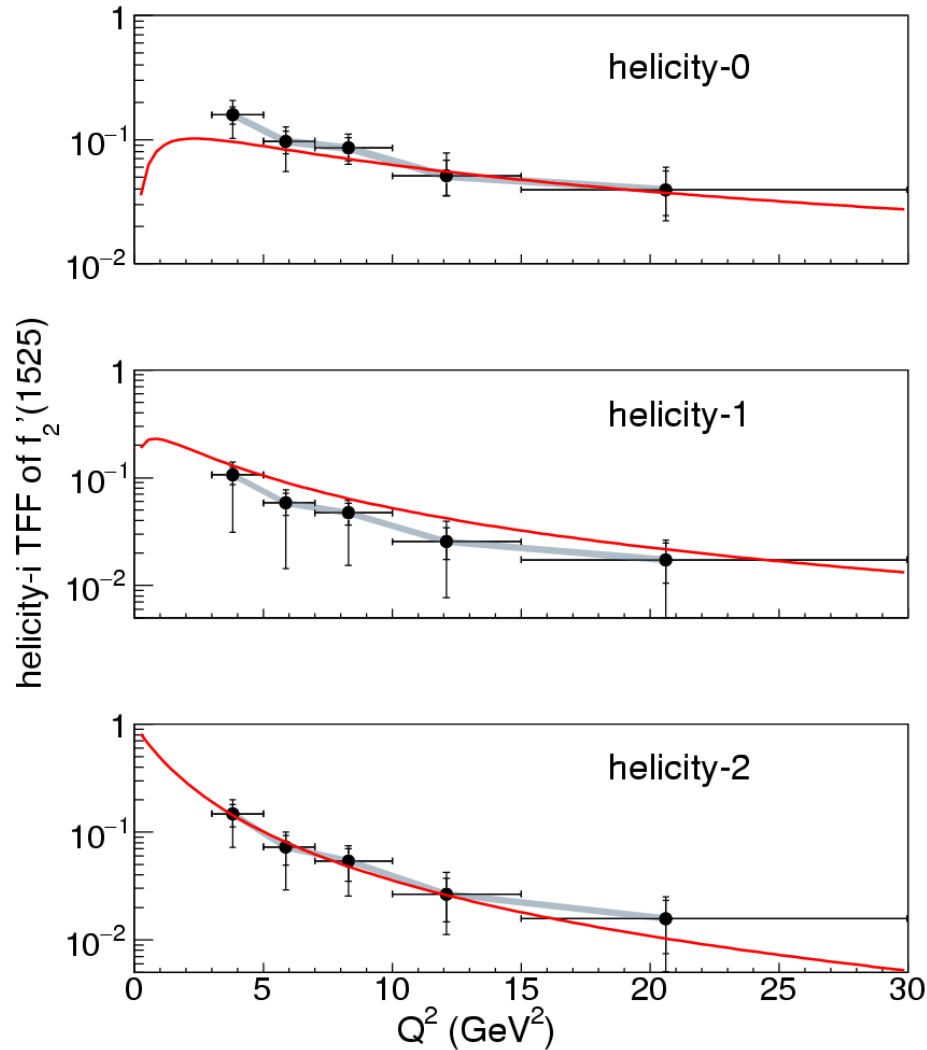
We regard this as the angular dependence at $\langle Q^2 \rangle = 6.5 \text{ GeV}^2$

Fit:
Black: total
Red: t_0
Blue: $t_1 \cos\varphi^*$
Magenta: $t_2 \cos 2\varphi^*$

The fit is applied to the two-dimensional angular-dependence data.
Forward enhancement is from the helicity-0 component.



$f'_2(1525)$ TFF Result



Shorter error bars ; statistical
 Longer error bars ; statistical and systematic
 Shaded areas; overall systematic

— Schuler, Berends, van Glick (SBG)
 Nucl. Phys. B 523, 423, (1998).

helicity-0 and -2 -- agree well with SBG.
 helicity-1 -- slightly smaller, but not inconsistent.

Note: the Q^2 dependence of each helicity fraction is assumed as follows

$$r_{0fp} : r_{1fp} : r_{2fp} = k_0 Q^2 : k_1 \sqrt{Q^2} : 1$$

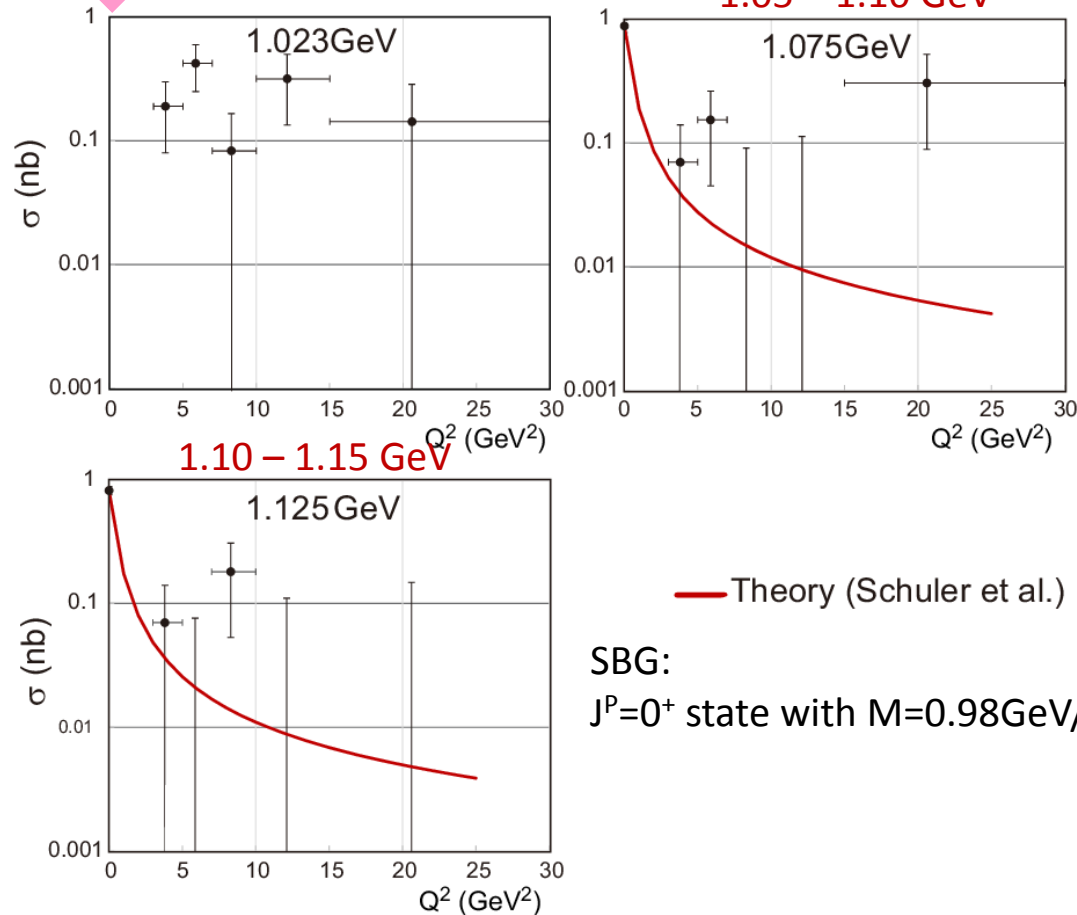
Fractions k_0 and k_1 are floated.



The Threshold Enhancement

No $Q^2=0$ measurement

Mass threshold – 1.05 GeV



The threshold enhancement exists.

- Not inconsistent with SBG.
- The limited statistics currently preclude a conclusive interpretation.



Summary

- Cross section for $\gamma^*\gamma \rightarrow K_S^0 K_S^0$ has been measured for $2M(K_S^0) < W < 2.6 \text{ GeV}$, $3 \text{ GeV}^2 < Q^2 < 30 \text{ GeV}^2$
- Q^2 dependence of $\Gamma_{\gamma^*\gamma}$ of χ_{c0} and χ_{c2} has been measured. Preferable to the charmonium mass scale.
- Q^2 dependence of $f_2'(1525)$ -TFF has been measured.
- Signature of an enhancement near the $K_S^0 K_S^0$ mass threshold is observed.

The measured Q^2 dependences are not inconsistent to theoretical predictions.



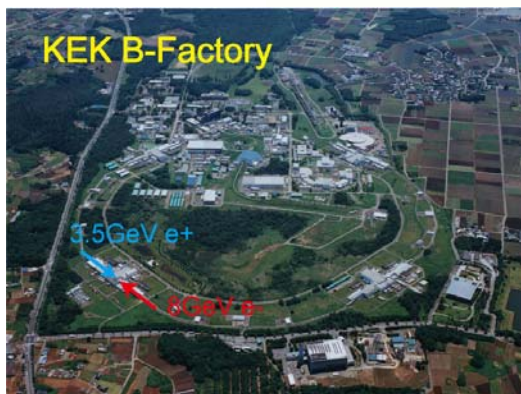
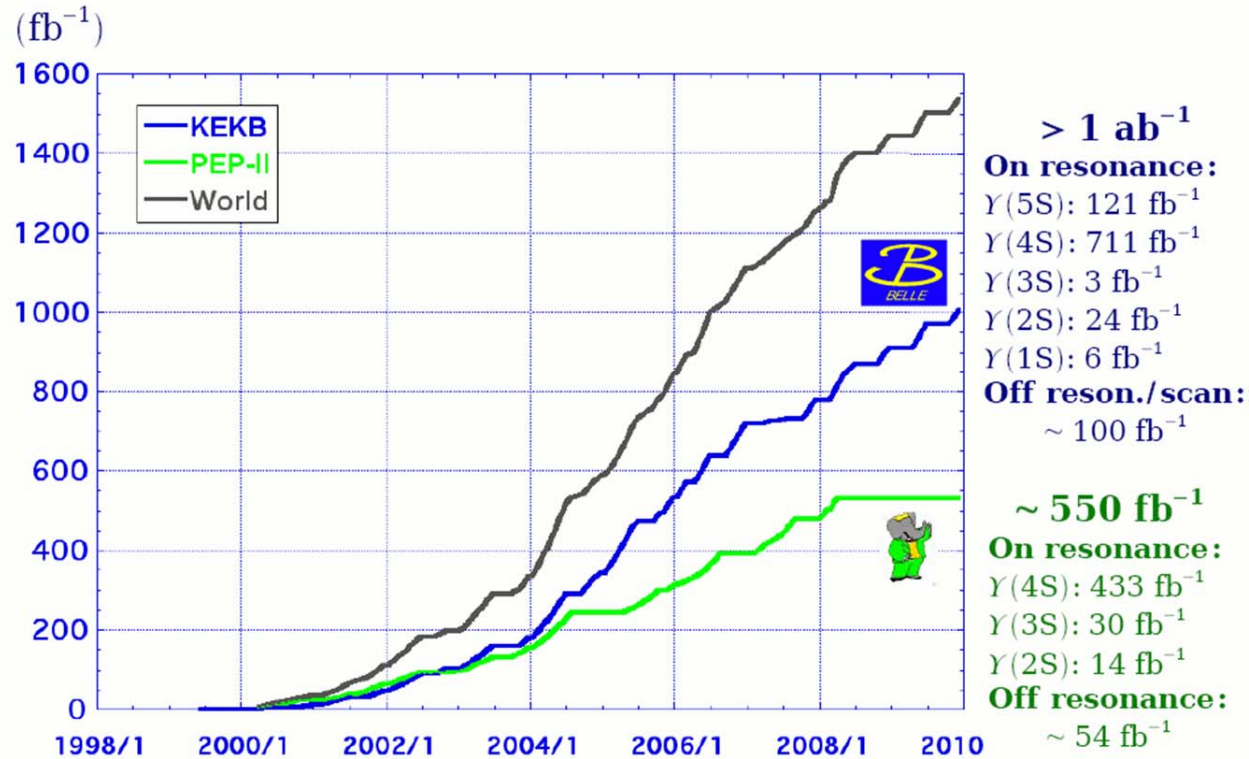


Backup



History of integrated luminosity at Belle

Luminosity at B factories



- 1999 The Belle experiment started
- 2001 CP violation in B mesons was verified and the KEKB accelerator achieved the world's highest luminosity
- 2002 Anomalous CP violation in $b \rightarrow s$ was measured
- 2003 The $B \rightarrow Kll$ decay was discovered
- 2004 The New particle X (3872) was discovered
- 2005 Direct violation of CP in $B \rightarrow K\pi$ was found. The $B \rightarrow \rho\gamma$ decay was discovered
- 2006 $B \rightarrow \tau\nu$ was observed
- 2007 D meson mixing was discovered. A new particle composed of 4 quarks Z (4430) + was discovered
- 2008 Dr. Makoto Kobayashi and Dr. Toshihide Maskawa were awarded the Nobel Prize in Physics
- 2010 The Belle experiment was completed

Formalism of PWA

$$|F(Q^2)| = \sqrt{\frac{\sigma_R^\lambda(Q^2)}{\sigma_R^\lambda(0)(1 + \frac{Q^2}{M^2})}}$$

TFF is defined for each resonance R produced with each helicity λ

To obtain the resonance amplitudes:

Perform PWA, parameterizing W dependence of the resonance and continuum components of each helicity amplitude, e.g.,

$$\frac{d\sigma(\gamma^*\gamma \rightarrow \pi^0\pi^0)}{d\Omega} = \sum_{n=0}^2 t_n \cos(n\varphi^*),$$

$$\begin{aligned} t_0 &= |M_{++}|^2 + |M_{+-}|^2 + 2\epsilon_0 |M_{0+}|^2, \\ t_1 &= 2\epsilon_1 \Re((M_{+-}^* - M_{++}^*)M_{0+}), \\ t_2 &= -2\epsilon_0 \Re(M_{+-}^* M_{++}), \end{aligned}$$

$$M_{++} = S + D_0,$$

$$S = B_S(W) + A_{f_0}(W)$$

$$D_0 = 4\pi [B_{D_0}(W) + A_{f_2}(W)\sqrt{r_{20}}] Y_2^0$$

etc.

Determine each component as well as the relative phase by a fit

++ etc. --- Helicity state of the incident photons

S, D₀ etc. -- Partial-wave amplitude in $\pi^0\pi^0$ scattering

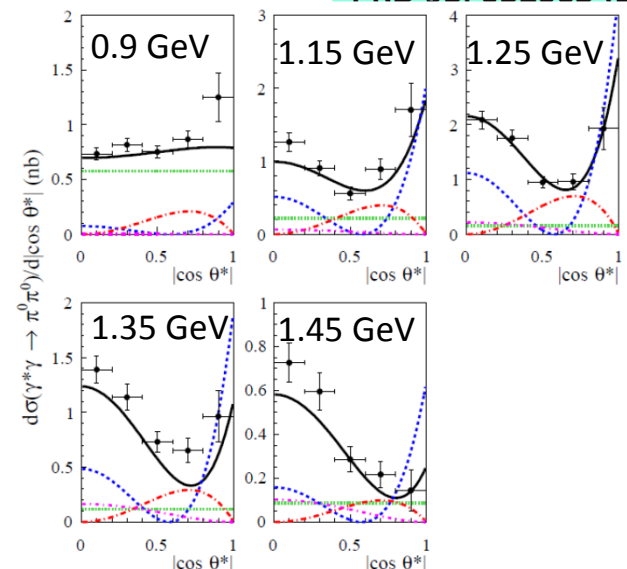
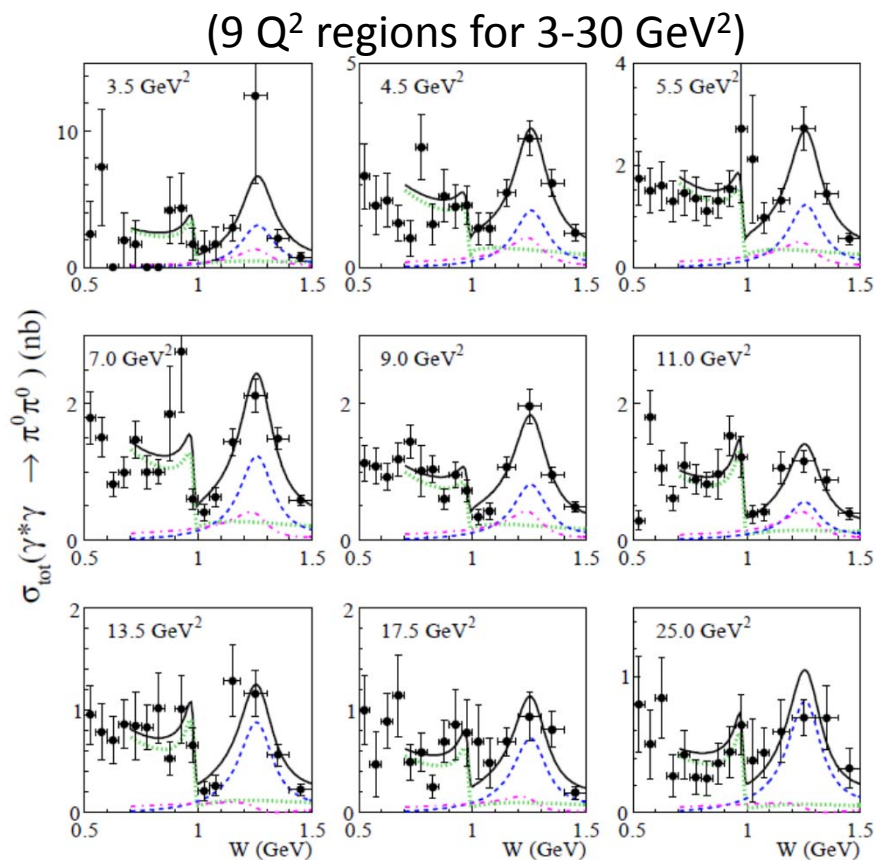
B, A_f -- Background and f-resonance components.

ϵ_0, ϵ_1 --- A spin-dependent flux factor ratio for the virtual-photons



$\gamma^*\gamma \rightarrow \pi^0\pi^0 : f_0(980) \text{ and } f_2(1270) \text{ TFF's}$

PRD 93. 032003 (2016)



lines: solid= total,
dotted= $|S|^2$, dashed= $|D_0|^2$,
and dash-dotted= $|D_2|^2$

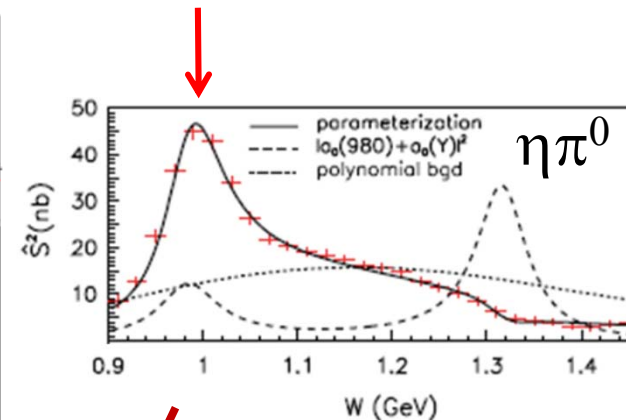
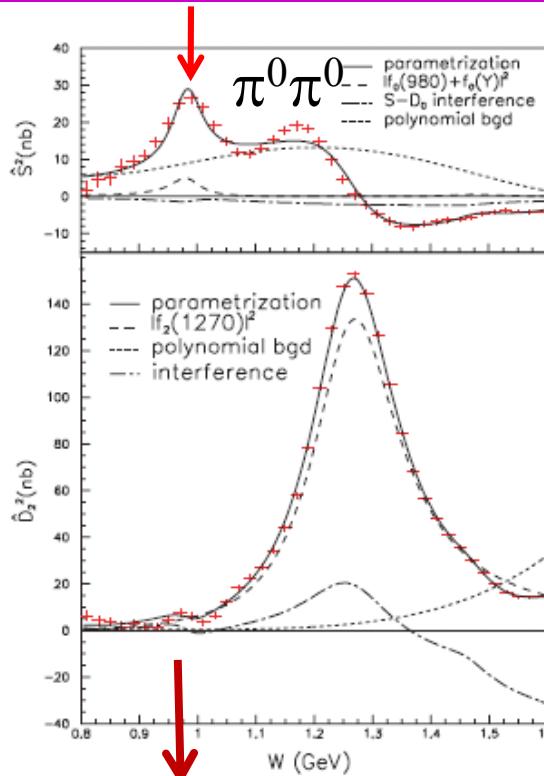
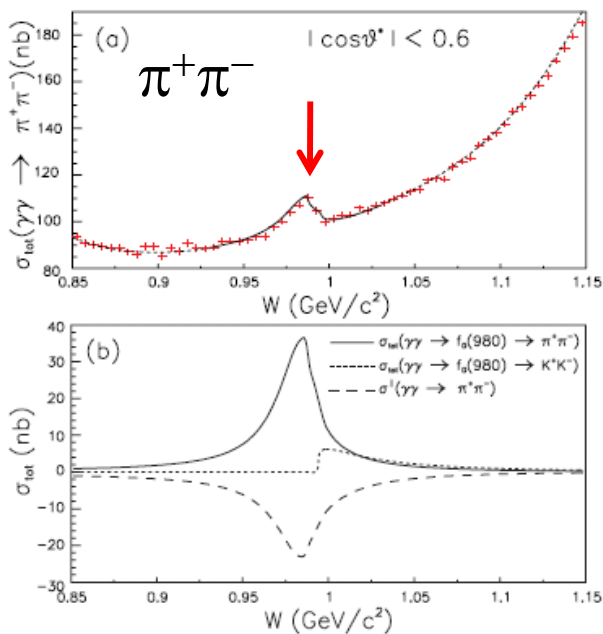
$|\cos \theta^*|$ dependence for
 $Q^2 = 9 \text{ GeV}^2$ and different
W bins

Significant contributions
from hel.=0 and 1 in contrast
to the no-tag ($Q^2=0$) case

The curves are PWA fit constructed by parameterized $f_0(980)$ and $f_2(1270)$ etc. (see the paper)



Two-photon decay width of $f_0(980)$ and $a_0(980)$



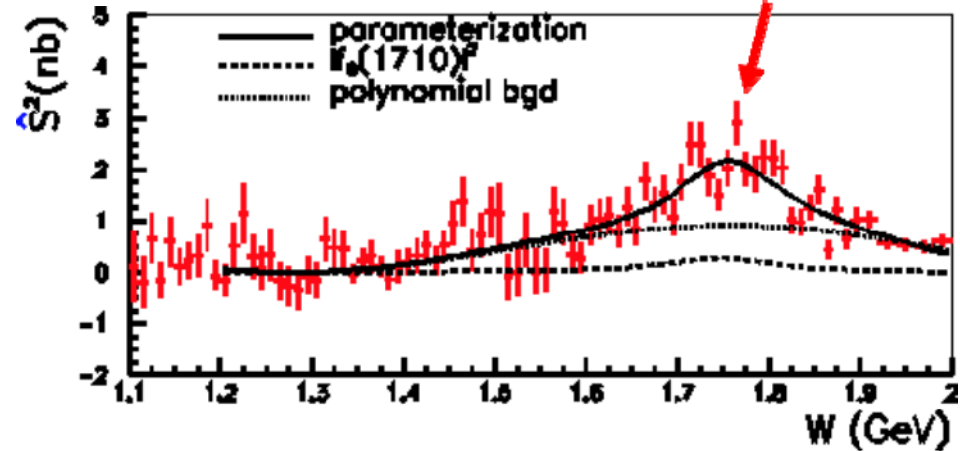
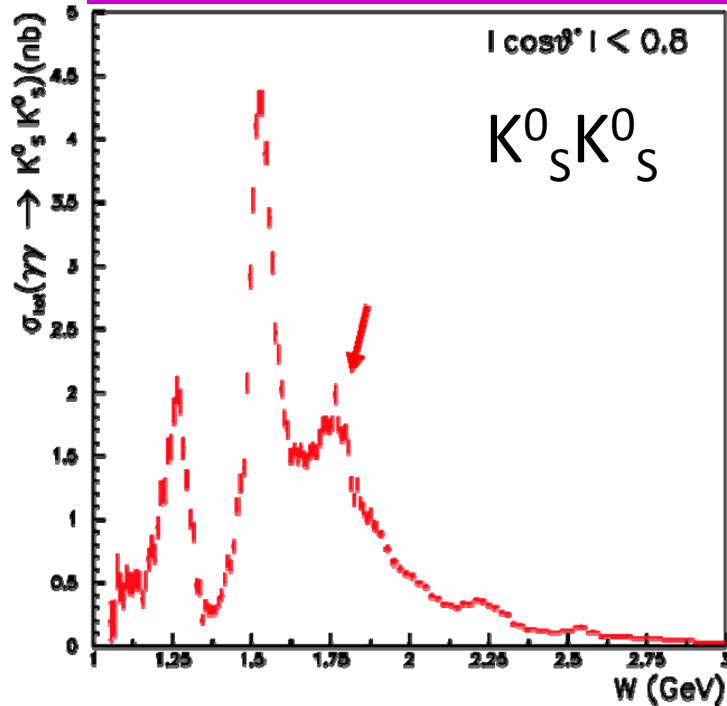
Predictions for $f_0(980)$

Meson	$f_0(980)$	$f_0(980)$	$a_0(980)$
M[MeV/c ²]	985.6 ^{+1.2+1.1} _{-1.5-1.6}	982.2 ± 1.0 ^{+8.1} _{-8.0}	982.3 ^{+0.6+3.1} _{-0.7-4.7}
$\Gamma_{\pi\pi/\text{tot}}$ [MeV]	51.3 ^{+20.9+13.2} _{-17.7-3.8}	66.9 ^{+13.9+8.8} _{-11.8-2.5}	75.6 ± 1.6 ^{+17.4} _{-10.0} (Γ_{tot})
$\Gamma_{\gamma\gamma}$ [eV]	205 ⁺⁹⁵⁺¹⁴⁷ ₋₈₃₋₁₁₇	286 ± 17 ⁺²¹¹ ₋₇₀	128 ⁺³⁺⁵⁰² ₋₂₋₄₃ / $\mathcal{B}_{\pi^0\eta}$

Model	$\Gamma_{\gamma\gamma}$ [eV]
<i>uubar, ddbar</i>	1300 – 1800
<i>ssbar</i>	300 – 500
KKbar molecule	200 – 600
Four-quark	270



$f_0(1710)$ formation in $K_S^0 K_S^0$



Assuming a single resonance,
 $J = 0$ or 2 ? $J = 0$ is much preferred.

Parameter	$f_0(1710)$ fit				$f_2(1710)$ fit	
	fit-H	fit-L	H,L combined	PDG	fit-H	fit-L
$f_J(1710)$						
χ^2/ndf	694.2/585	701.6/585	Two solutions of interference		796.3/585	831.5/585
Mass(f_J) (MeV/ c^2)	1750^{+5+29}_{-6-18}	1749^{+5+31}_{-6-42}	1750^{+6+29}_{-7-18}	1720 ± 6	1750^{+6}_{-7}	1729^{+6}_{-7}
$\Gamma_{\text{tot}}(f_J)$ (MeV)	138^{+12+96}_{-11-50}	145^{+11+31}_{-10-54}	139^{+11+96}_{-12-50}	135 ± 6	132^{+12}_{-11}	150 ± 10
$\Gamma_{\gamma\gamma} \mathcal{B}(K\bar{K})_{f_J}$ (eV)	12^{+3+227}_{-2-8}	21^{+6+38}_{-4-26}	12^{+3+227}_{-2-8}	unknown	$2.1^{+0.5}_{-0.3}$	1.6 ± 0.2

$f_0(1710) \rightarrow K_S^0 K_S^0$ is confirmed in two-photon process.

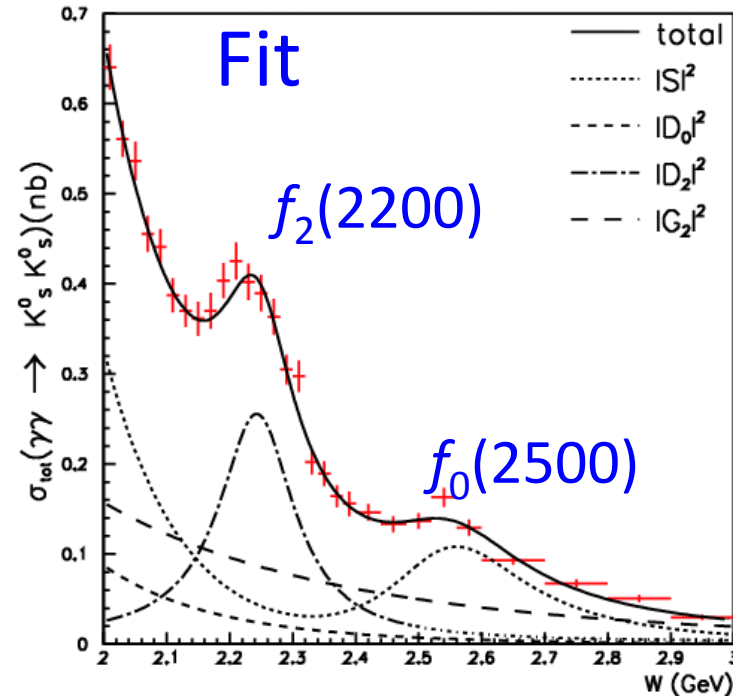
Fit Results for resonances in $K^0_s K^0_s$

$f_2(2200)-f_0(2500)$ is the best solution (in all the J= 0, 2, 4 combinations)

Parameter	$f_2(2200)$	$f_0(2500)$
Mass (MeV/ c^2)	2243^{+7+3}_{-6-29}	$2539 \pm 14^{+38}_{-14}$
Γ_{tot} (MeV)	$145 \pm 12^{+27}_{-34}$	$274^{+77+126}_{-61-163}$
$\Gamma_{\gamma\gamma} \mathcal{B}(K\bar{K})$ (eV)	$3.2^{+0.5+1.3}_{-0.4-2.2}$	40^{+9+17}_{-7-40}

Significances

- 3.4σ for $f_2(2200)$ over $f_0(2200)$
- 4.3σ for $f_0(2500)$ over $f_2(2500)$

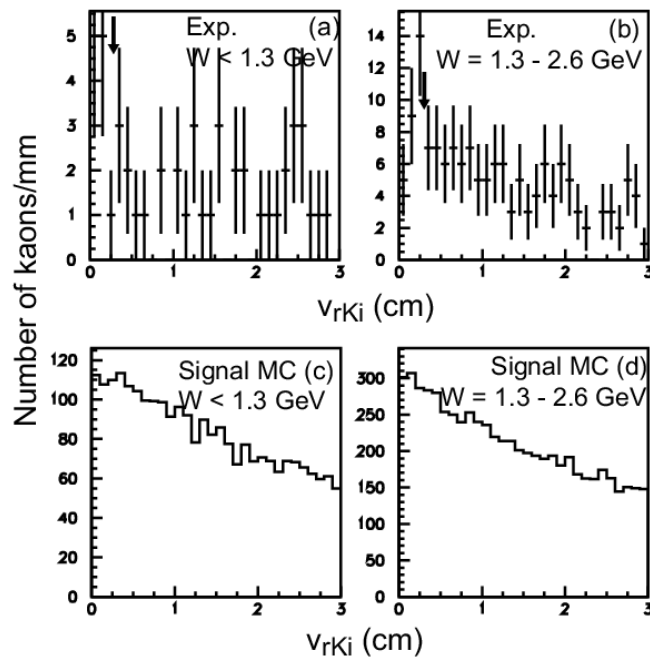


- There can be an only wide state around 2240 MeV.
- Narrow appearances in previous measurements may be due to an interference effect and/or statistical fluctuation.
- A high-mass state at 2.5 GeV may be the heaviest light-quark scalar meson so far found.

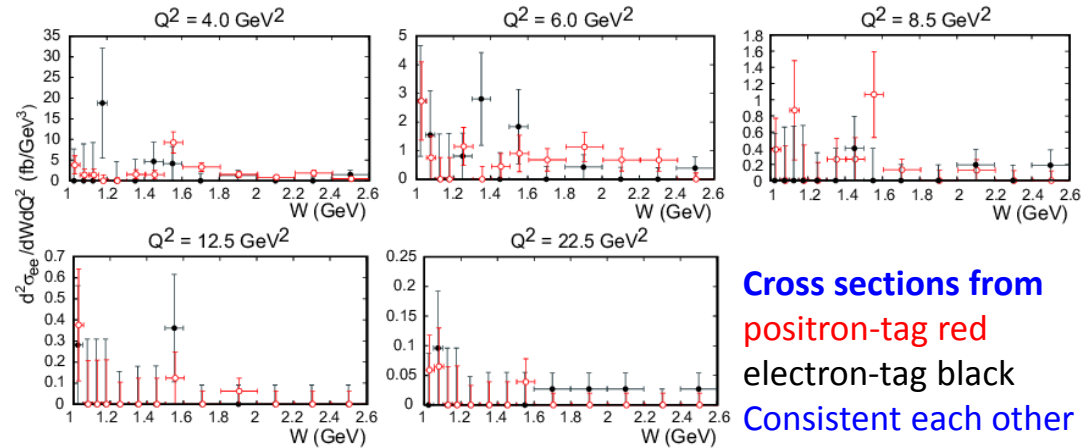
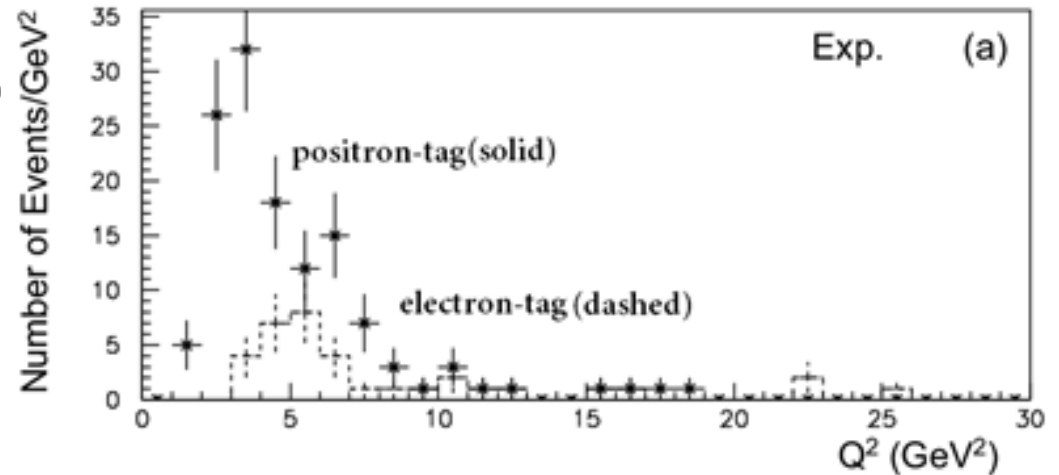


$K^0_S K^0_S$ Experimental data

Searching for non- $K^0_S K^0_S$ background, looking for an enhancement near $v_r=0$ in a loosely selected sample



No enhancement.
 <1 event background
 in the final candidates



Cross sections from
 positron-tag red
 electron-tag black
 Consistent each other



Fit results

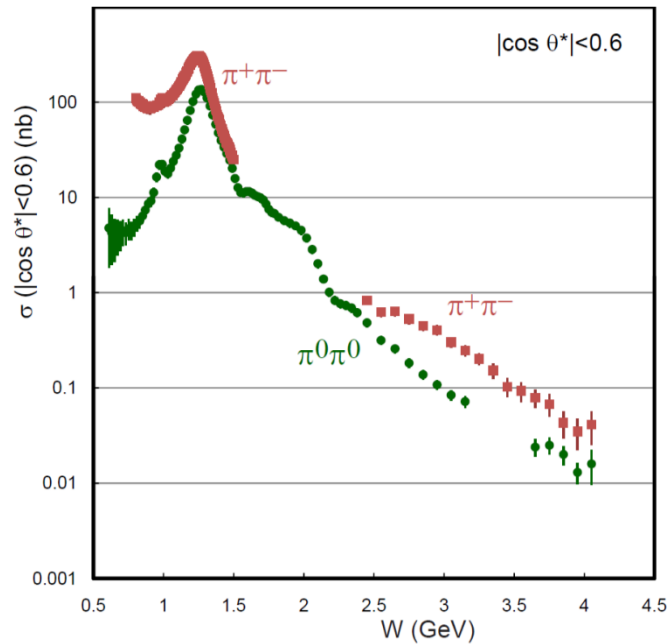
TABLE VI: Fitted parameters of cross sections and the number of solutions obtained under the conditions noted below. In each category, only solutions assuming $k_0 \neq 0 \cap k_1 \neq 0$ are shown. Only the single solution that gives the minimum χ^2 in category 3 is shown, while two viable solutions in categories 1 and 2 are shown.

Parameter	Category 1		Category 2		Category 3
Conditions	$A_{BW} \neq 0 \cap B_S = 0$		$A_{BW} = 0 \cap B_S \neq 0$		$A_{BW} = B_S = 0$
Number of solutions	2		2		3
	Solution 1a	Solution 1b	Solution 2a	Solution 2b	
χ^2_{P}/ndf	152.4/150	159.8/150	154.9/151	156.1/151	293.9/155
k_0 (GeV ⁻²)	$0.30^{+0.31}_{-0.14}$	$0.31^{+0.34}_{-0.15}$	$0.31^{+0.34}_{-0.15}$	$0.29^{+0.31}_{-0.14}$	$0.33^{+0.31}_{-0.14}$
k_1 (GeV ⁻¹)	$0.27^{+0.30}_{-0.14}$	$0.27^{+0.44}_{-0.15}$	$0.29^{+0.33}_{-0.15}$	$0.24^{+0.29}_{-0.13}$	$0.23^{+0.25}_{-0.12}$
$F_{f2p}(0.0); (\times 10^{-2})$	100 ± 7				
$F_{f2p}(4.0); (\times 10^{-2})$	$24.1^{+2.6}_{-2.5}$	$24.4^{+2.7}_{-2.6}$	$24.3^{+2.6}_{-2.5}$	$24.4^{+2.6}_{-2.5}$	$27.1^{+2.7}_{-2.6}$
$F_{f2p}(6.0); (\times 10^{-2})$	$13.4^{+2.6}_{-2.5}$	$13.9^{+2.5}_{-2.4}$	$14.3^{+2.5}_{-2.3}$	$14.4^{+2.5}_{-2.3}$	$15.5^{+2.5}_{-2.4}$
$F_{f2p}(8.5); (\times 10^{-2})$	$11.2^{+2.3}_{-2.2}$	$11.3^{+2.3}_{-2.2}$	$11.5^{+2.3}_{-2.2}$	$11.6^{+2.3}_{-2.1}$	$12.4^{+2.3}_{-2.2}$
$F_{f2p}(12.5); (\times 10^{-2})$	$6.3^{+2.1}_{-1.9}$	$6.3^{+2.1}_{-1.9}$	$6.3^{+2.1}_{-1.9}$	$6.3^{+2.1}_{-1.9}$	$7.0^{+2.1}_{-1.9}$
$F_{f2p}(22.5); (\times 10^{-2})$	$4.6^{+1.9}_{-1.7}$	$4.6^{+1.9}_{-1.7}$	$4.6^{+1.9}_{-1.7}$	$4.7^{+1.9}_{-1.7}$	$5.1^{+2.0}_{-1.8}$
ϕ_{fpD1} (°);	33^{+28}_{-81}	177^{+27}_{-27}	112^{+23}_{-35}	108^{+24}_{-37}	47^{+24}_{-33}
ϕ_{fpD2} (°);	199^{+34}_{-75}	218^{+27}_{-29}	209^{+30}_{-35}	213^{+28}_{-33}	218^{+23}_{-27}
ϕ_{faD1} (°);	137^{+27}_{-34}	328^{+34}_{-39}	18^{+28}_{-30}	340^{+33}_{-33}	234^{+22}_{-24}
ϕ_{faD2} (°);	166^{+30}_{-32}	180^{+29}_{-29}	162^{+29}_{-32}	182^{+27}_{-28}	0 (fixed)
f_S ($\sqrt{\text{nb}}$ GeV ²); ($\times 10^{-2}$)	$1.3^{+1.1}_{-0.6}$	$0.9^{+0.8}_{-0.4}$	0 (fixed)		0 (fixed)
g_S (GeV)	$0.10^{+0.05}_{-0.04}$	$0.06^{+0.05}_{-0.05}$	0 (fixed)		0 (fixed)
p_S	$0.06^{+0.25}_{-0.24}$	$0.01^{+0.26}_{-0.25}$	0 (fixed)		0 (fixed)
ϕ_{BW} (°);	297^{+21}_{-21}	150^{+35}_{-24}	0 (fixed)		0 (fixed)
a_S ($\sqrt{\text{nb}}$); ($\times 10^{-3}$)	0 (fixed)		$4.3^{+12.5}_{-5.9}$	$2.2^{+5.7}_{-3.0}$	0 (fixed)
b_S	0 (fixed)		$19.6^{+4.6}_{-4.1}$	$21.9^{+6.0}_{-4.0}$	0 (fixed)
c_S	0 (fixed)		$0.00^{+0.23}_{-0.06}$	$0.00^{+0.21}_{-0.05}$	0 (fixed)
ϕ_{BS} (°);	0 (fixed)		99^{+19}_{-21}	311^{+20}_{-18}	0 (fixed)



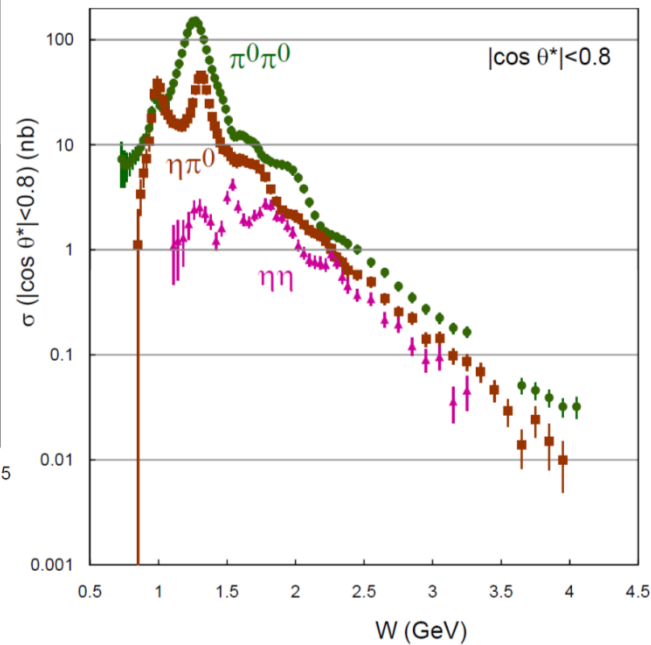
The six processes; in total ~20 peaks

Charged vs Neutral $\pi\pi$

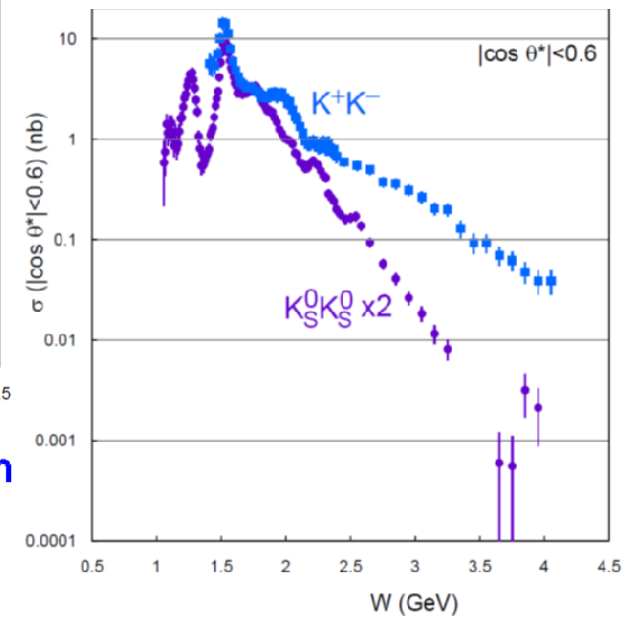


Three neutral-pair processes

$\pi^0\pi^0$, $\eta\pi^0$, $\eta\eta$



Charged vs Neutral $K\bar{K}$



Horizontal axis:

W -- $\gamma\gamma$ c.m. energy = invariant mass of the two-meson system

$W < \sim 2.5 \text{ GeV}$: Dominated by resonances

$W > \sim 2.5 \text{ GeV}$: (Netnegative) Power law works + (χ_c charmonia)

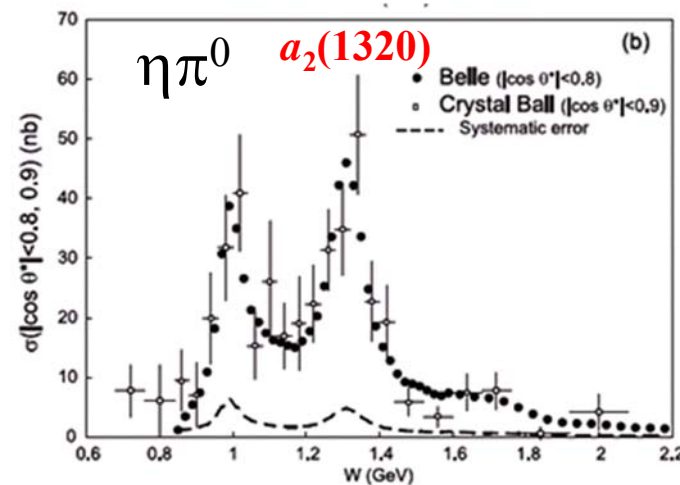
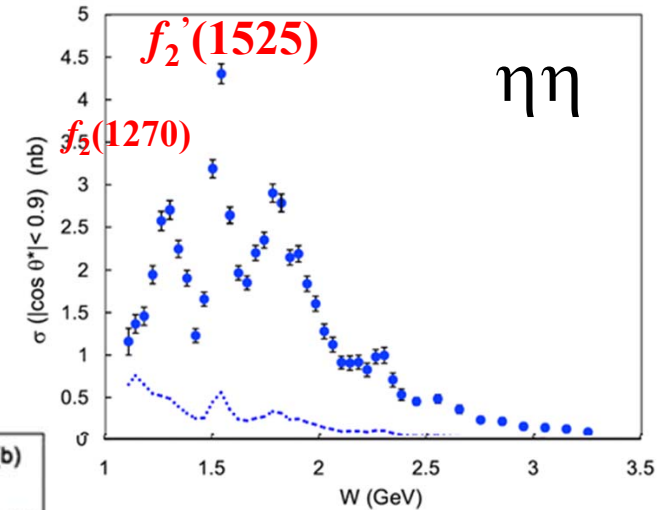
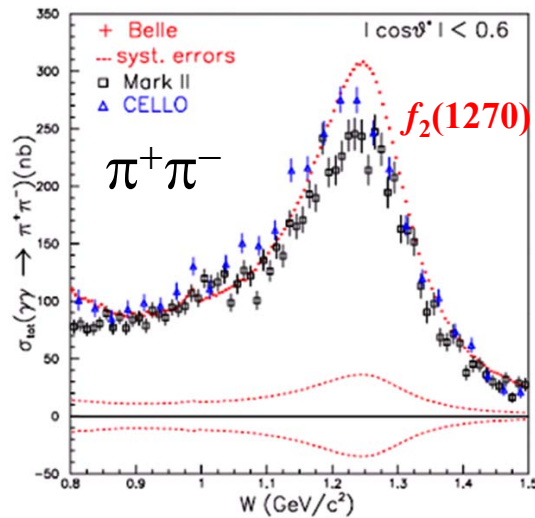


The tensor-meson triplet, $f_2(1270)$, $a_2(1320)$, $f_2'(1525)$

$f_2(1270)$: The largest peak in $\pi^+\pi^-$ and $\pi^0\pi^0$. Also seen in $\eta\eta$

$a_2(1320)$: Large peak in $\eta\pi^0$

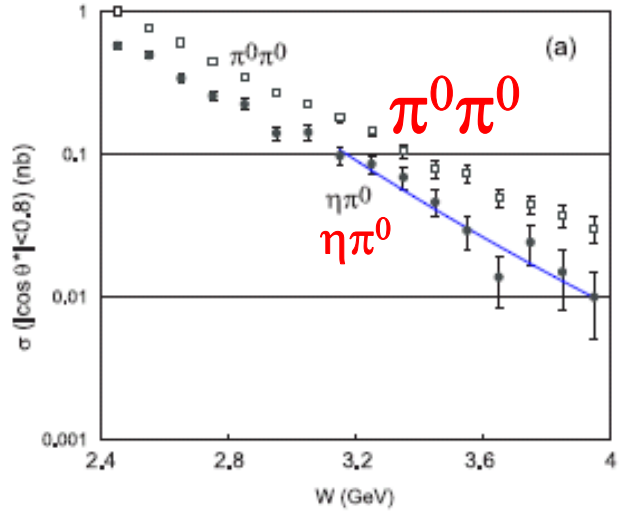
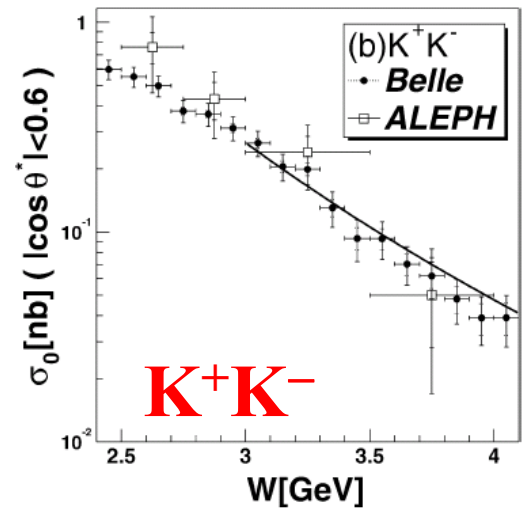
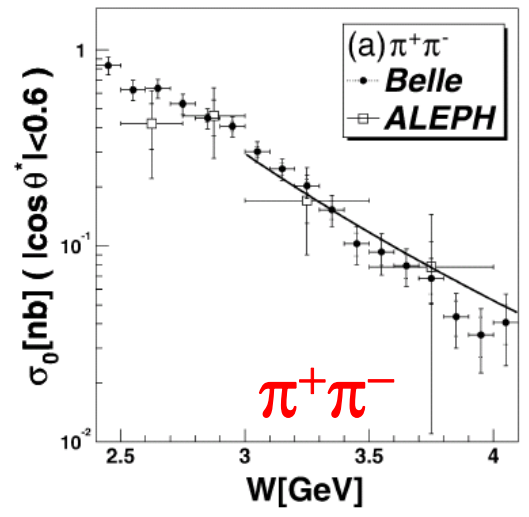
$f_2'(1525)$: Large peak in $\eta\eta$, K^+K^- , and $K_S^0K_S^0$



W-dependences at high energies

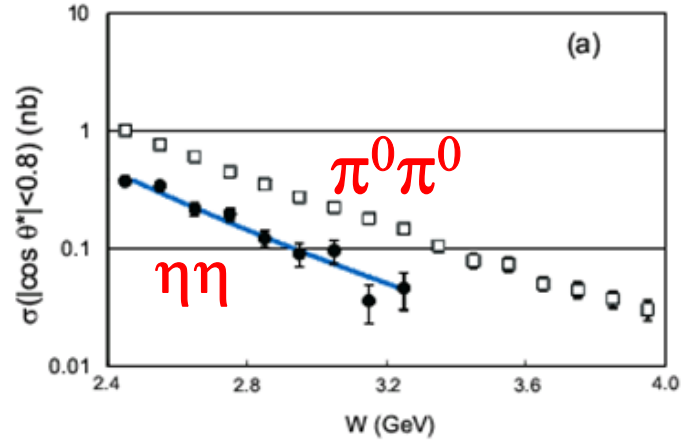
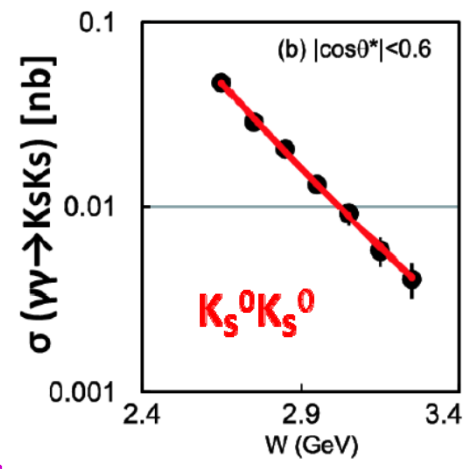
$W \equiv W_{\gamma\gamma} \equiv \sqrt{s_{\gamma\gamma}}$ Collision's c.m. energy

Assume or expect $\sigma(W) \sim W^{-n}$



Fitted and reproduced
Slope parameter **n** different
among the reactions

Charmonium contributions
not included/removed



Cross sections and their ratios

Process	n	$W(\text{GeV})$	$ \cos \theta^* $	BL	BC	DKV
$K_S^0 K_S^0$	$11.0 \pm 0.4 \pm 0.4$	$2.4 - 4.0^\dagger$	< 0.8		10	
$\pi^+ \pi^-$	$7.9 \pm 0.4 \pm 1.5$	$3.0 - 4.1$	< 0.6	6	6	
$K^+ K^-$	$7.3 \pm 0.3 \pm 1.5$	$3.0 - 4.1$	< 0.6	6	6	
$\pi^0 \pi^0$	$8.0 \pm 0.5 \pm 0.4$	$3.1 - 4.1^\dagger$	< 0.8		10	
$\eta \pi^0$	$10.5 \pm 1.2 \pm 0.5$	$3.1 - 4.1$	< 0.8		10	
$\eta \eta$	$7.8 \pm 0.6 \pm 0.4$	$2.4 - 3.3$	< 0.8		10	
Process	σ_0 ratio	$W(\text{GeV})$	$ \cos \theta^* $	BL	BC	DKV
$K^+ K^- / \pi^+ \pi^-$	$0.89 \pm 0.04 \pm 0.15$	$3.0 - 4.1$	< 0.6	2.3	1.06	
$K_S K_S / K^+ K^-$	~ 0.10 to ~ 0.03	$2.4 - 4.0$	< 0.6		0.005	2/25
$\pi^0 \pi^0 / \pi^+ \pi^-$	$0.32 \pm 0.03 \pm 0.06$	$3.1 - 4.1$	< 0.6		0.04-0.07	0.5
$\eta \pi^0 / \pi^0 \pi^0$	$0.48 \pm 0.05 \pm 0.04$	$3.1 - 4.0$	< 0.8			$0.24 R_f (0.46 R_f)^\ddagger$
$\eta \eta / \pi^0 \pi^0$	$0.37 \pm 0.02 \pm 0.03$	$2.4 - 3.3$	< 0.8			$0.36 R_f^2 (0.62 R_f^2)^\ddagger$

† Exclude χ_{cJ} region, 3.3 - 3.6 GeV.

‡ Assuming η is a member of SU(3) octet (superposition of octet and singlet with mixing angle of $\theta_p = -18^\circ$).
 R_f is a ratio of decay constants, $f_\eta^2 / f_{\pi^0}^2$.

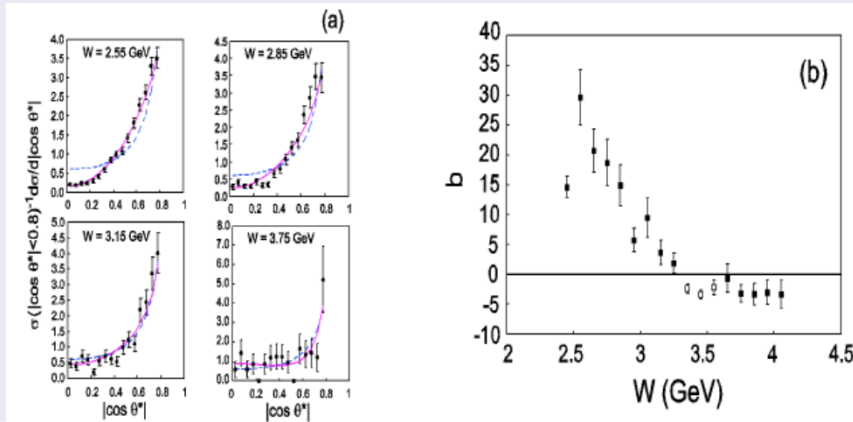
- n ranges 7 to 11. Close or not far from QCD prediction of 6 and 10.
- Cross section ratios tend to be constant above 3 GeV.

Summarized by H.Nakazawa
Hadron2013



Angular dependence

$$\gamma\gamma \rightarrow \pi^0\pi^0$$



$d\sigma/d|\cos\theta^*| \propto \sin^{-4}\theta^*$ is predicted by $q\bar{q}$ -meson model and perturbative QCD

- Fit to $\sin^{-4}\theta^* + b \cos\theta^*$
- b becomes constant above 3.2 GeV.

mode	α in $\sin^{-\alpha}\theta^*$	GeV	$ \cos\theta^* $
$K_S K_S$	3 – 8	2.6 - 3.3	< 0.8
$\pi^+ \pi^-$	Good agreement with 4	3.0 - 4.1	< 0.6
$K^+ K^-$	Good agreement with 4	3.0 - 4.1	< 0.6
$\pi^0 \pi^0$	Better agreement with $\sin^{-4}\theta^* + b \cos\theta^*$ Approaches $\sin^{-4}\theta^*$ above 3.1 GeV	2.4 - 4.1 [†]	< 0.8
$\eta\pi^0$	Good agreement with 4 above 2.7 GeV	3.1 - 4.1	< 0.8
$\eta\eta$	Poor agreement with 4 Close to 6 above 3 GeV	2.4 - 3.3	< 0.9

Summarized by H.Nakazawa
Hadron2013

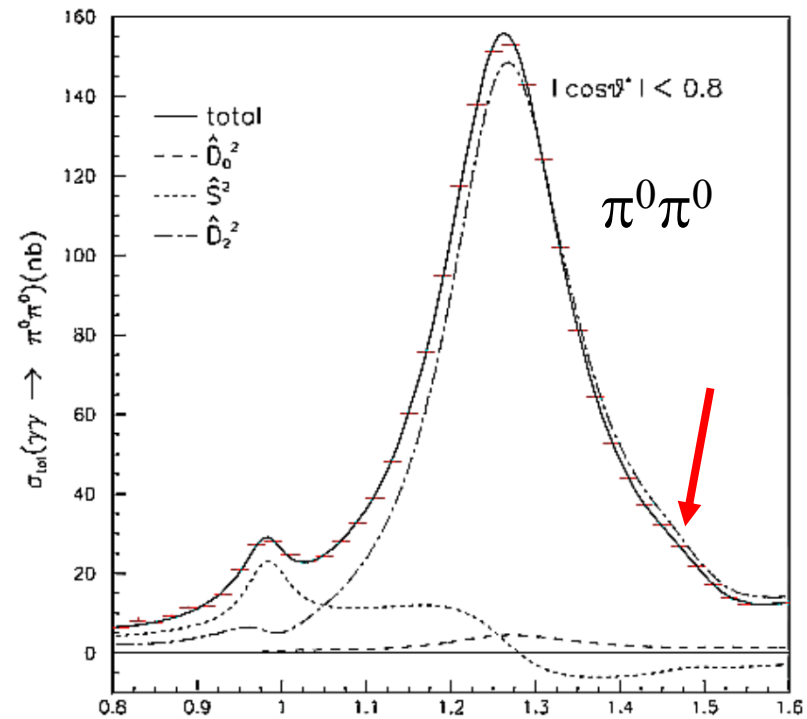
Exclude [†] χ_{ω} region, 3.3 - 3.6 GeV

, DIS2018, Apr.2018

Scalars in the 1.2 – 1.6 GeV region

- Hadron experiments report a wide $f_0(1370)$ and a narrow $f_0(1500)$.
- Some of previous two-photon measurements provide a hint of $f_0(1100-1400) \rightarrow \pi\pi$ under the huge peak of $f_2(1270)$
- Belle's $\pi^0\pi^0$ measurement reports $f_0(1470)$.
May be visible in the line shape.
 → favorable to the narrow $f_0(1500)$,
 but also consistent with $f_0(1370)$.

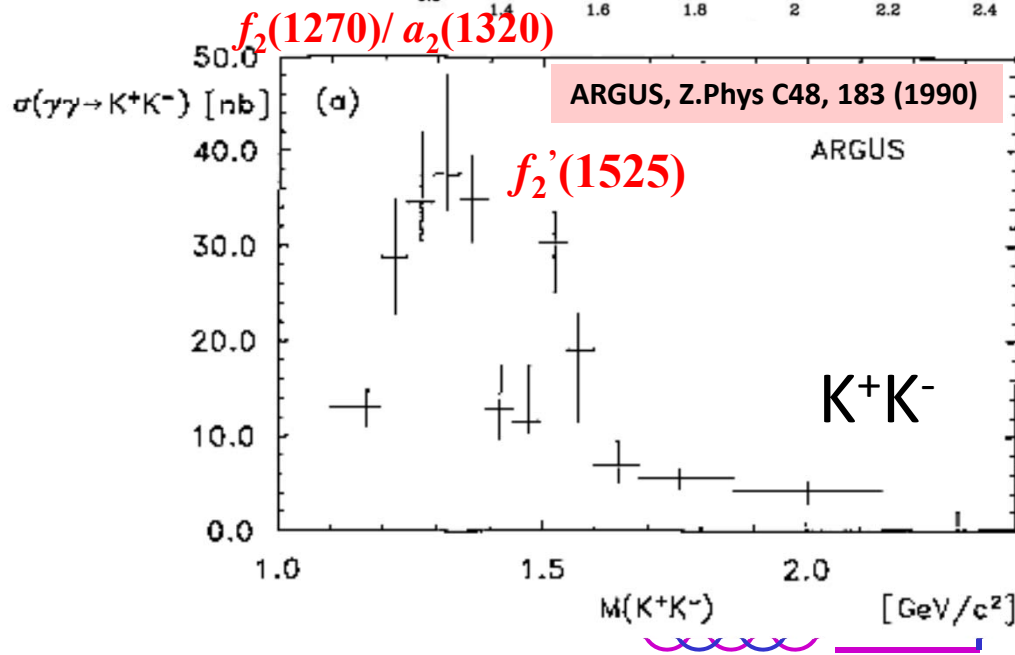
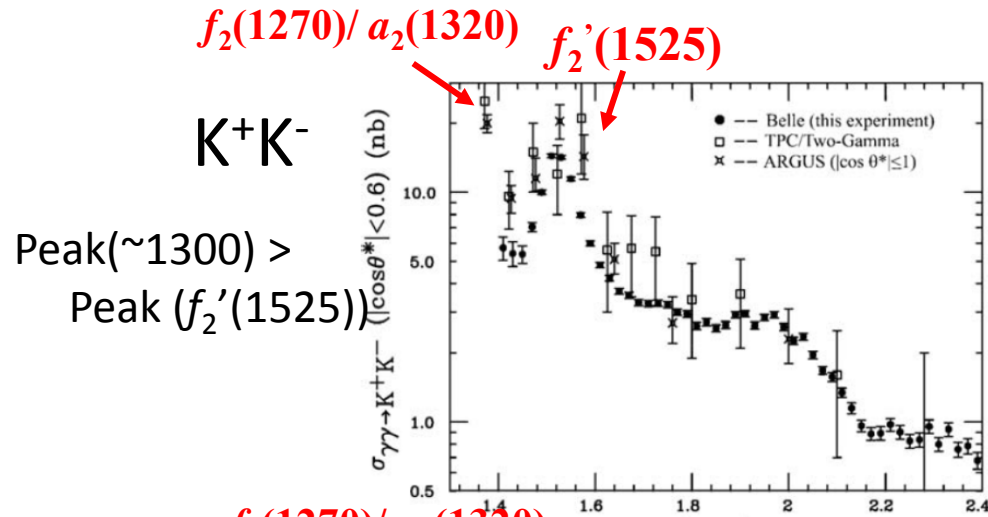
$f_0(1370)$ [l]	$I^G(J^{PC}) = 0^+(0^{++})$		
Mass $m = 1200$ to 1500 MeV Full width $\Gamma = 200$ to 500 MeV			
$f_0(1370)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)	
$\pi\pi$	seen	672	
$f_0(1500)$ [n]	$I^G(J^{PC}) = 0^+(0^{++})$		
Mass $m = 1505 \pm 6$ MeV ($S = 1.3$) Full width $\Gamma = 109 \pm 7$ MeV			
$f_0(1500)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor	p (MeV/c)
$\pi\pi$	$(34.9 \pm 2.3)\%$	1.2	741



Parameter	Belle ($\pi^0\pi^0$)	Crystal Ball	Unit
Mass	1470^{+6+72}_{-7-255}	1250	MeV/c ²
Γ_{tot}	90^{+2+50}_{-1-22}	268 ± 70	MeV
$\Gamma_{\gamma\gamma} \mathcal{B}(\pi^0\pi^0)$	11^{+4+603}_{-2-7}	430 ± 80	eV



$f_2(1270)$ - $a_2(1320)$ interference in $K\bar{K}$



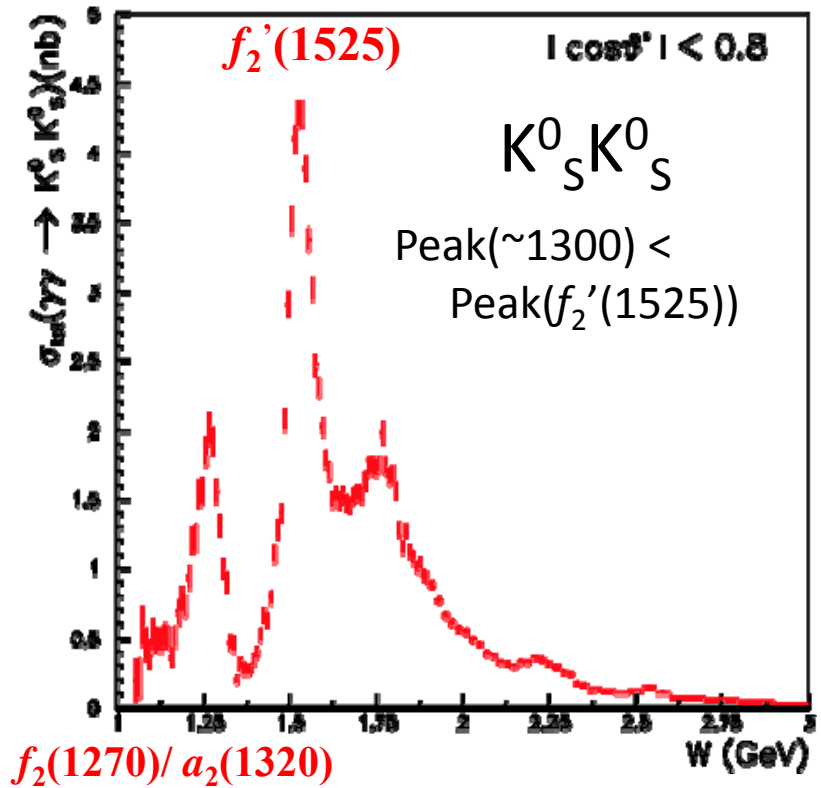
Constructive interference

$f_2(1270)+a_2(1320)$ in K^+K^-

Destructive interference

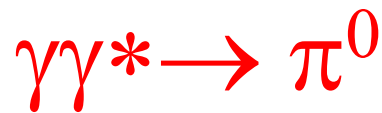
$f_2(1270)-a_2(1320)$ in $K_S^0 K_S^0$

Explained by a phase relation in isospin composition

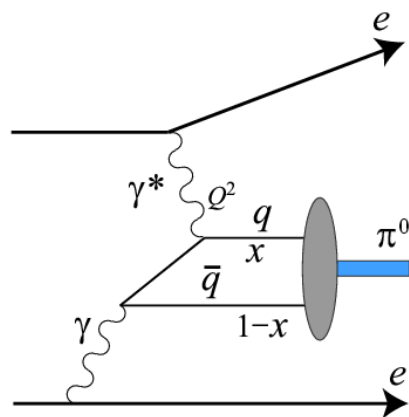


π^0 Transition Form Factor (TFF)

PRD 86, 092007 (2012)



Coupling of neutral pion with two photons
Good test for QCD at high Q^2



Single-tag π^0 production in two-photon process with a large- Q^2 and a small- Q^2 photon

Theoretically calculated from pion distribution amplitude and decay constant

$$F(Q^2) = \frac{\sqrt{2}f_\pi}{3} \int T_H(x, Q^2, \mu) \phi_\pi(x, \mu) dx$$

BaBar has reported a significant deviation from the expectation.

Measurement:

$$|F(Q^2)|^2 = |F(Q^2, 0)|^2 = (d\sigma/dQ^2) / (2A(Q^2)) \quad A(Q^2) \text{ is calculated by QED}$$

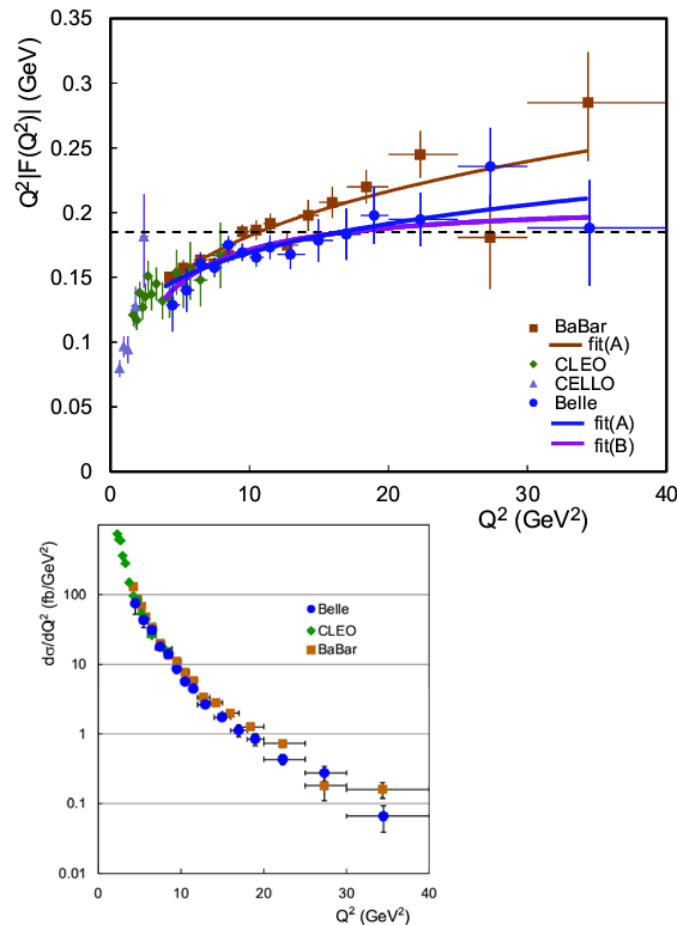
$$|F(0, 0)|^2 = 64\pi\Gamma_{\gamma\gamma} / \{(4\pi\alpha)^2 m_R^3\}$$

Detects e (tag side) and π^0

$Q^2 = 2EE'(1 - \cos \theta)$ from energy and polar angle of the tagged electron



Comparisons with Previous Measurements and Fits



No rapid growth above $Q^2 > 9 \text{ GeV}^2$ is seen in Belle result.

$\sim 2.3\sigma$ difference between Belle and BaBar in $9 - 20 \text{ GeV}^2$

Fit A (suggested by BaBar)

$$Q^2 |F(Q^2)| = A (Q^2/10 \text{ GeV}^2)^\beta$$

BaBar: —

$$A = 0.182 \pm 0.002 (\pm 0.004) \text{ GeV}$$

$$\beta = 0.25 \pm 0.02$$

BaBar, PRD 80, 052002 (2009)

Belle: —

$$A = 0.169 \pm 0.006 \text{ GeV}$$

$$\beta = 0.18 \pm 0.05$$

$$\chi^2/\text{ndf} = 6.90/13 \quad \sim 1.5\sigma \text{ difference from BaBar}$$

Fit B (with an asymptotic parameter)

$$Q^2 |F(Q^2)| = BQ^2/(Q^2+C)$$

Belle: —

$$B = 0.209 \pm 0.016 \text{ GeV}$$

$$C = 2.2 \pm 0.8 \text{ GeV}^2$$

$$\chi^2/\text{ndf} = 7.07/13$$

B is consistent with the QCD value (0.185 GeV)

