Results for Heavy Flavor and Quarkonium production in high multiplicity p+p and p+A collisions in the CGC framework

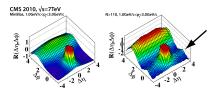
Kazuhiro Watanabe

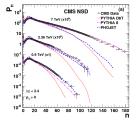
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with Y.-Q. Ma (Peking U), P. Tribedy (BNL), R. Venugopalan (BNL) arXiv:1803.11093







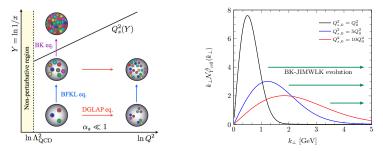
- Discovery of ridge like structure: the starting point.
- p+p vs p+A vs A+A: Initial state (fluctuation) or Final state (hydro) origins?
- Interplay of hard and soft collisions.
- Gluon saturation is a natural way to explain this phenomenon. cf. [Dumitru et al. (2010)]

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Motivation

Gluon saturation describes systematically heavy flavor and onium production in high multiplicity p+p and p+A collisions ?

Gluon saturation and high multiplicity



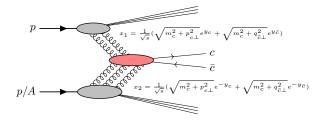
• Gluon recombination at small- $x \rightarrow$ Glauon Saturation

$$Q_{s,A}^2 \propto \# \frac{\alpha_s}{S_{A\perp}} x G_A \sim \# A^{1/3} \left(\frac{1}{x}\right)^\lambda \sim \# A^{1/3} Q_{s,p}^2$$

- $\checkmark k_{\perp} > Q_s$: Gluons are hard, pQCD is applicable.
- $\checkmark k_{\perp} < Q_s$: Gluons are soft and high occupied in "Saturation region".
- High multiplicity events: Spacial and momentum structure of rare parton configurations are important. \implies Such rare parton configurations can be controlled by $Q_s(x)$. cf. [Dusling, Venugopalan (2012)]

Large Q_s (hadron fluctuation) \leftrightarrow Soft gluon modes are enhanced \leftrightarrow High multiplicity

- \checkmark Heavy flavor and Onium are complementary observables to light hadron, dijet production. \rightarrow Event engineering studies.
- \checkmark Provides unique playground to study gluon saturation or test the CGC framework. Remind $c\bar{c}$ is largely produced via initial gluon fusion at collider energies.
- \checkmark Saturation scales are semihard at the energy frontier: m_c vs $Q_{sp}(x_1)$ vs $Q_{sA}(x_2)$
- \checkmark The CGC can be helpful in understanding of heavy flavor and onium production mechanisms at low- p_{\perp} .



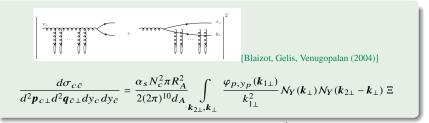
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The Color-Glass-Condensate (CGC) framework

• projectile moving $\rightarrow x^+ = +\infty$, target moving $\rightarrow x^- = +\infty$.

• Solving classical Yang-Mills eq. :

$$[D^{\mu}, F_{\mu\nu}] = J^{\nu} \Longrightarrow |\mathcal{T}\rangle = \sum_{i}^{\infty} |\underbrace{gg \cdots gg}_{i}\rangle$$



- Unintegrated gluon distribution function: $\varphi_{p,y_p}(\mathbf{k}_{\perp}) = \pi R_p^2 \frac{N_c k_{\perp}^2}{4\alpha_s} N_{y_p}^A(\mathbf{k}_{\perp})$
- Dipole amplitude: $N_{y_P(Y)}(\boldsymbol{k}_{\perp}) = \int d^2 \boldsymbol{r}_{\perp} e^{-i\boldsymbol{k}_{\perp}\cdot\boldsymbol{r}_{\perp}} \frac{1}{N_c} \left\langle \operatorname{Tr} \left[V_F(\boldsymbol{r}_{\perp}) V_F^{\dagger}(\boldsymbol{0}_{\perp}) \right] \right\rangle_{y_P(Y)}$
- The rcBK equation describes *x*-evolution of the dipole amplitude. $(N_Y(k_{\perp}) = F \cdot T \cdot of D_Y(r_{\perp}))$ [Balitsky (2006)]

$$-\frac{dD_{Y,\boldsymbol{r}_{\perp}}}{dY} = \int d^2 \boldsymbol{r}_{1\perp} \mathcal{K}(\boldsymbol{r}_{\perp}, \boldsymbol{r}_{1\perp}) \Big[D_{Y,\boldsymbol{r}_{\perp}} - D_{Y,\boldsymbol{r}_{1\perp}} D_{Y,\boldsymbol{r}_{2\perp}} \Big]$$

Minimum Bias events

- D meson production
- J/ψ production
- $\psi(2S)$ production
- Y production

High multiplicity events

- D meson production vs N_{ch}
- J/ψ meson production vs N_{ch}

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A quick guide to numerical calculations

- m = 1.3 GeV for D meson and J/ψ . But m = 1.5 GeV $\approx m_{J/\psi}/2$ is used only when NRQCD factorization is employed.
- α_s is fixed.
- Initial saturation scale of proton: $Q_{sp,0}^2 = Q_0^2 = 0.168$ GeV at x = 0.01. \leftarrow HERA-DIS global data fitting. [AAMQS (2010)]

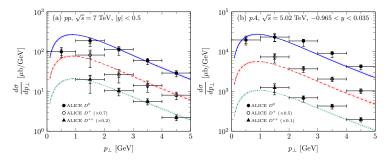
$$D_{Y=Y_0,\boldsymbol{r}_{\perp}} = \exp\left[-\frac{\left(r_{\perp}^2 Q_{sp,0}^2\right)^{\gamma}}{4}\ln\left(\frac{1}{r_{\perp}\Lambda} + e\right)\right],$$

- Initial saturation scale of nucleus: $Q_{sA,0}^2 = 2Q_0^2$. \leftarrow A fit to the available NMC (New Muon Collab.) data (fixed target eA) on the nuclear structure functions $F_{2,A}(x, Q^2)$ at $x \sim 0.01$. [Dusling, Gelis, Lappi, Venugopalan (2009)]
- φ at large-x: Matching between φ and collinear PDF xG with CTEQ6M set at x = 0.01.

$$\mathcal{N}_Y^A(\boldsymbol{k}_{\perp}) \stackrel{x > x_0}{=} \frac{x G_{\text{CTEQ}}(x, Q_0^2)}{x G^{\text{Dipole}}(x_0, Q_0^2)} \mathcal{N}_{Y_0}^A(\boldsymbol{k}_{\perp})$$

with the identity $xG^{\text{Dipole}}(x, Q_0^2) = \frac{1}{4\pi^3} \int_0^{Q_0^2} dk_{\perp}^2 \varphi_Y(k_{\perp})$. Switch from φ to xG at $x > x_0$.

• $R_p \sim 0.5$ fm, $Q_0 \sim 5$ GeV are obtained from the matching. R_A is chosen to reproduce $R_{pA} = \frac{d\sigma_{pA}}{Ad\sigma_{pp}} = 1$ when $p_{\perp} \rightarrow \infty$. [Fujii, KW (2013)][Ma, Tribedy, Venugopalan, KW (2018)]



$$\frac{d\sigma_D}{d^2 \boldsymbol{p}_{D\perp} dy} = \int_{z_{min}}^1 dz \frac{D_{c \to D}(z)}{z^2} \int dy_{\bar{c}} \int_{\boldsymbol{q}_{\bar{c}\perp}} \frac{d\sigma_{c\bar{c}}}{d^2 \boldsymbol{p}_{c\perp} d^2 \boldsymbol{q}_{\bar{c}\perp} dy dy_{\bar{c}}}$$

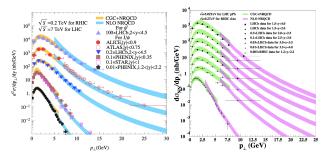
• $D_{c \to D}(z)$: BCFY Fragmentation Function [Braaten, Cheung, Fleming, Yuan(1994)]

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J/ψ production in the CGC + NRQCD

[Ma, Venugopalan (2014)][Ma, Venugopalan, Zhang (2015)]



• The CGC cross sections at short distance are matched to NRQCD LDMEs.

$$\frac{d\sigma^{\psi}}{dydp_{\perp}^{2}} = \sum_{\kappa} \underbrace{\frac{d\hat{\sigma}_{c\bar{c}}^{\kappa}}{dydp_{\perp}^{2}}}_{CGC} \times \underbrace{\langle O_{\kappa}^{\psi} \rangle}_{LDMEs} \qquad (\kappa = {}^{2S+1}L_{J}^{[c]})$$

• The LDMEs are extracted from high p_{\perp} data fitting at Tevatron. [Chao et al. (2012)]

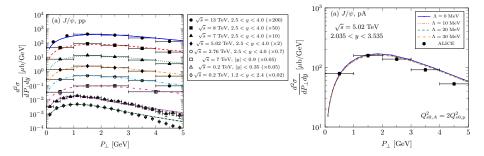
$$\begin{split} \langle O^{J/\psi} [{}^1S_0^{[8]}] \rangle &= 0.089 \pm 0.0098 \text{GeV}^3, \\ \langle O^{J/\psi} [{}^3S_1^{[8]}] \rangle &= 0.0030 \pm 0.0012 \text{GeV}^3, \\ \langle O^{J/\psi} [{}^3P_0^{[8]}] \rangle &= 0.0056 \pm 0.0021 \text{GeV}^3 \end{split}$$

• The contribution of CS channel is relatively small. (10% in pp, 15% - 20% in pA at small- p_{\perp})

J/ψ production in the CGC + Improved CEM

- Color Evaporation Model (CEM) is consistent with NRQCD in the sense that color octet $c\bar{c}$ is mainly considered.
- Improved CEM (ICEM) can reproduce different p_{\perp} distributions of J/ψ and $\psi(2S)$ correctly. [Ma, Vogt (2016)]

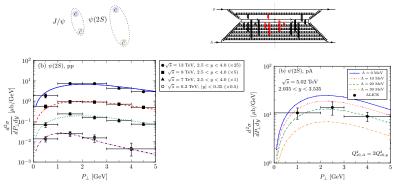
$$\frac{d\sigma_{\psi}}{d^2 p_{\perp} dy} = F_{c\bar{c} \to \psi} \int_{m_{\psi}}^{2m_D} dM \left(\frac{M}{m_{\psi}}\right)^2 \frac{d\sigma_{c\bar{c}}}{dM d^2 p_{\perp}' dy} \bigg|_{p_{\perp}' = \frac{M}{m_{\psi}} p_{\perp}}$$



[Ma, Venugopalan, Zhang, KW (2017)]

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• Soft color exchanges between partonic comovers and the $c \bar{c}$ can affect greatly $\psi(2S)$ production. [Ma, Venugopalan, Zhang, KW (2017)]



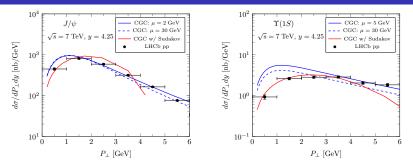
• In p+p collisions, $F_{q\bar{q}\rightarrow\psi}$ is fitted and should include the effect of soft color exchanges at final stage.

• Important assumption : the role of soft color exchanges should be enhanced in p+A collisions. $\rightarrow \Lambda$ is responsible for the nuclear enhancement effect.

$$\frac{d\sigma_{\psi}}{d^2 p_{\perp} dy} = F_{c\bar{c} \to \psi} \int_{m_{\psi}}^{2m_D - \Lambda} dM \left(\frac{M}{m_{\psi}}\right)^2 \frac{d\sigma_{c\bar{c}}}{dM d^2 p'_{\perp} dy} \bigg|_{p'_{\perp} = \frac{M}{m_{\psi}} p_{\perp}}$$

where Λ denotes the average momentum kick given by additional nuclear parton comovers. $\exists \exists \forall \land \land \land$

$\Upsilon(1S)$ production



• Large Mass scale $\rightarrow \# \alpha_s \ln^2 \frac{M^2}{p_{\perp}^2} \sim O(1)$. (Sudakov double logs)

• The soft gluon shower effect and the saturation effect can be described straightforwardly in the hybrid framework: [K.W., Xiao (2015)]

$$d\sigma_{\text{resum}}^{c\bar{c}} \propto \int \frac{d^2 u_{\perp} d^2 v_{\perp}}{(2\pi)^4} e^{-ip_{\text{rel}} \cdot u_{\perp}} e^{ip_{\perp} \cdot v_{\perp}} x_1 G\left(x_1, \frac{c_0}{v_{\perp}}\right) D_Y(x_{\perp}) D_Y(y_{\perp}) e^{-S_{\text{Sud}}(M, v_{\perp})} \hat{H}_{\text{LO}},$$

with $x_{\perp} = v_{\perp} + (1 - z)u_{\perp}$ and $y_{\perp} = v_{\perp} - zu_{\perp}$.

- S_{Sud} must be "Universal". cf. [Sun, Yuan, Yuan (2012)]
- Sudakov effect is dominant for low- p_{\perp} Y production in p+p collisions. cf. [Qiu, K.W. (2017)]
- In p+A collisions, Saturation effect can be comparable to Sudakov effect.

Minimum Bias events

- D meson production
- J/ψ production
- $\psi(2S)$ production
- Y production

High multiplicity events

- D meson production vs N_{ch}
- J/ψ meson production vs N_{ch}

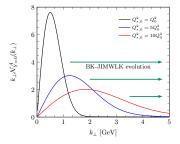
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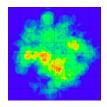
Caveats

• Matching between φ_p and xG is no more justified. In lieu, the adopt the simple extrapolation ansatz for φ : [Gelis, Stasto, Venugopalan (2006)]

$$\varphi_{p,y_p}(\pmb{k}_\perp) = \varphi_{p,y_0}(\pmb{k}_\perp) \left(\frac{1-x}{1-x_0}\right)^4 \left(\frac{x_0}{x}\right)^{0.15}.$$

- Hadronization of charm to D meson, J/ψ production models are the same as used in MB events.
- All the input parameters are taken to be the same, except for Q_s .





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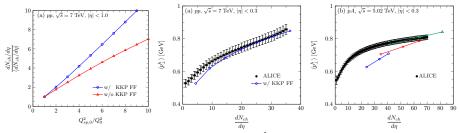
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Charged hadron multiplicity in the CGC framework

• $p + p/A \rightarrow g(\rightarrow h) + X$: [Kovchegov, Tuchin (2001)]

$$\frac{d\sigma_g}{d^2 \boldsymbol{p}_{g\perp} dy} = \frac{\alpha_s \hat{\boldsymbol{K}}_b}{(2\pi)^3 \pi^3 C_F} \frac{1}{p_{g\perp}^2} \int d^2 \boldsymbol{k}_\perp \varphi_{p,y_p}(\boldsymbol{k}_\perp) \varphi_{A,Y}(\boldsymbol{p}_{g\perp} - \boldsymbol{k}_\perp)$$
$$\frac{dN_{ch}}{d\eta} = \frac{\hat{\boldsymbol{K}}_{ch}}{\sigma_{\text{inel}}} \int d^2 \boldsymbol{p}_\perp \int_{z_{\min}}^1 dz \frac{D_h(z)}{z^2} J_{y \to \eta} \frac{d\sigma_g}{d^2 \boldsymbol{p}_{g\perp} dy}$$

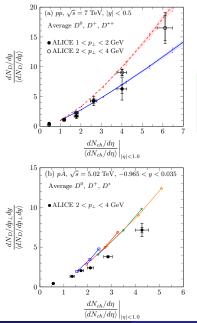
where
$$J_{y \to \eta} = p_{g\perp} \cosh \eta / \sqrt{p_{g\perp}^2 \cosh^2 \eta} + m_h^2$$
 and $p_{\perp} \equiv z p_{g\perp}$.



It is clear that the relative N_{ch} grows almost linearly as $Q_{sp,0}^2$ increases when the KKP FF is used.

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D meson production vs N_{ch}



← p+p collisions

• Normalized in MB p+p collisions.

$$\begin{array}{l} \times \ Q_{sp,proj}^2 > Q_0^2 \ \text{and} \ Q_{sp,targ}^2 = Q_0^2. \\ \times \ Q_{sp,targ}^2 > Q_0^2 \ \text{and} \ Q_{sp,proj}^2 = Q_0^2. \\ \checkmark \ Q_{sp_1}^2 = Q_{sp_2}^2 > Q_0^2. \end{array}$$

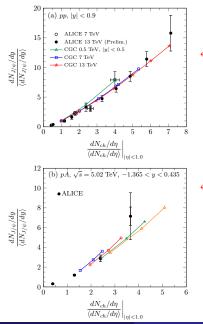
← p+A collisions

- Normalized in MB p+A collisions.
- Different colors: Different $Q_{sA,0}^2 \Rightarrow$ Fluctuation effect of proton is important to achieve high N_{ch} when $Q_{sA,0}^2$ is being fixed.

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• The same qualitative trend in p+p and p+A collisions.

J/ψ meson production vs N_{ch} at mid rapidity



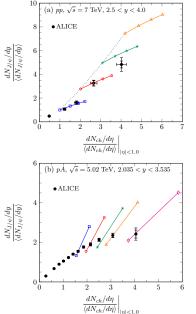
- ← p+p collisions
 - The CGC+ICEM framework is used.
 - The ratios are \sqrt{s} -independent! In the CGC, events at different energies with the same Q_s are identical.

- ← p+A collisions
- Different colors: Different $Q_{sA,0}^2$

The similar trends are seen for *D* meson and J/ψ production. \rightarrow Hadronization dynamics is irrelevant, rather saturation effect at short distance plays a key role in describing data.

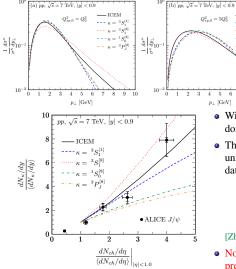
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J/ψ meson production vs N_{ch} at forward rapidity



- ← p+p collisions
- Different colors: Different $Q_{sp1,0}^2$ and $Q_{sp2,0}^2 \ge Q_{sp1,0}^2$.
- In contrast to mid rapidity, the symmetrical treatment; $Q_{sp_1,0}^2 = Q_{sp_2,0}^2$ overshoots the data slightly in p + p collisions (Dashed line). Data point at $dN_{ch}/\langle dN_{ch} \rangle \sim 4$ seems to favor the asymmetrical treatment; $Q_{sp_1,0}^2 < Q_{sp_2,0}^2$.

- ← p+A collisions
- Different colors: Different $Q_{sA,0}^2$.
- Lower points: $Q_{sp,0}^2 = Q_0^2$, Upper points: $Q_{sp,0}^2 = 2Q_0^2$.



- The normalized $c\bar{c}$ differential cross section for the ${}^{3}S_{1}^{[8]}$ channel is close to that of the ICEM over the entire p_{\perp} range at large $Q_{sp,0}^{2}$.
- With increasing event activity, the ${}^{3}S_{1}^{[8]}$ state dominates J/ψ production.
- This is remarkably consistent with the universality requirement from BELLE e^+e^- data:

$$\langle O^{J/\psi} [{}^{1}S_{0}^{[8]}] \rangle + 4.0 \langle O^{J/\psi} [{}^{3}P_{0}^{[8]}] \rangle / m^{2}$$

< $2.0 \pm 0.6 \times 10^{-2} \text{GeV}^{3}$

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[Zhang, Ma, Wang, Chao (2009)]

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• Note: the LDMEs used in the CGC+NRQCD previously violates this upper bound.

- Event engineered Heavy flavor and Onium production in high multiplicity p+p and p+A collisions provide unique opportunity to study gluon saturation phenomenon.
- Excellent agreement is found between the CGC computations and the LHC heavy flavor data on D and J/ψ production in high multiplicity events in p+p and p+A collisions.
- Data comparisons using the CGC+NRQCD framework potentially can distinguish between intermediate states with differing quantum numbers that contribute to the hadronization of J/ψ mesons.

Thank you!



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[Kang, Ma, Venugopalan (2013)]

$$\frac{d\sigma_{c\bar{c},CS}^{\kappa}}{d^{2}\boldsymbol{p}_{\perp}dy} = \frac{\alpha_{s}\pi R_{A}^{2}}{(2\pi)^{9}d_{A}} \int_{\boldsymbol{k}_{\perp\perp},\boldsymbol{k}_{\perp},\boldsymbol{k}_{\perp}'} \frac{\varphi_{p,y_{p}}(\boldsymbol{k}_{\perp\perp})}{k_{\perp\perp}^{2}} N_{Y}(\boldsymbol{k}_{\perp}) N_{Y}(\boldsymbol{k}_{\perp}) N_{Y}(\boldsymbol{k}_{\perp\perp} - \boldsymbol{k}_{\perp} - \boldsymbol{k}_{\perp}') \mathcal{G}_{1}^{\kappa}$$

$$\frac{d\sigma_{c\bar{c},CO}^{\kappa}}{d^{2}\boldsymbol{p}_{\perp}dy} = \frac{\alpha_{s}\pi R_{A}^{2}}{(2\pi)^{7}d_{A}} \int_{\boldsymbol{k}_{\perp\perp},\boldsymbol{k}_{\perp}} \frac{\varphi_{p,y_{p}}(\boldsymbol{k}_{\perp\perp})}{k_{\perp\perp}^{2}} N_{Y}(\boldsymbol{k}_{\perp}) N_{Y}(\boldsymbol{k}_{\perp} - \boldsymbol{k}_{\perp}) \Gamma_{8}^{\kappa}$$

BCFY FF

We follow [Cacciari et al. (2012)]:

$$\begin{split} D_{c \to D^{0}}(z;r) &= 0.168 D_{\text{BCFY}}^{(P)}(z;r) + 0.39 \tilde{D}_{\text{BCFY}}^{(V)}(z;r), \\ D_{c \to D^{+}}(z;r) &= 0.162 D_{\text{BCFY}}^{(P)}(z;r) + 0.07153 \tilde{D}_{\text{BCFY}}^{(V)}(z;r), \\ D_{c \to D^{*}}(z;r) &= 0.233 D_{\text{BCFY}}^{(V)}(z;r), \end{split}$$

where the original BCFY FFs are given by

$$\begin{split} D_{\text{BCFY}}^{(P)}(z;r) = & N \frac{rz(1-z)^2}{[1-(1-r)z]^6} \Big[6 - 18(1-2r)z + (21 - 74r + 68r^2)z^2 - 2(1-r)(6 - 19r + 18r^2)z^3 \\ &\quad + 3(1-r)^2(1-2r + 2r^2)z^4 \Big], \\ D_{\text{BCFY}}^{(V)}(z;r) = & 3N \frac{rz(1-z)^2}{[1-(1-r)z]^6} \Big[2 - 2(3 - 2r)z + 3(3 - 2r + 4r^2)z^2 - 2(1-r)(4 - r + 2r^2)z^3 \\ &\quad + (1-r)^2(3 - 2r + 2r^2)z^4 \Big]. \end{split}$$

N is determined analytically from $\int_0^1 dz D_{\text{BCFY}}^{(P,V)}(z;r) = 1$.

$$\tilde{D}_{\rm BCFY}^{(V)}(z;r) = \theta \left(\frac{m_D}{m_D^*} - z\right) D_{\rm BCFY}^{(V)}\left(\frac{m_D^*}{m_D}z;r\right) \frac{m_D^*}{m_D}.$$

We fix $m_D = (m_{D^0} + m_{D^{\pm}})/2 = 1.867$ GeV and $m_{D^*} = (m_{D^{*0}} + m_{D^{*\pm}})/2 = 2.009$ GeV. *r* is a single nonperturbative parameter.