TMD splitting functions in kT factorization

[EPJC 78 (2018) 174, 1711.04587]
[JHEP 01 (2016) 181, 1511.08439]
[PRD 94, 114013 (2016), 1607.01507]

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Introduction

- Parton distribution functions (PDFs) together with parton level matrix elements allow for a very accurate description of 'hard' events in hadron-hadron and hadron-electron collisions.

- The bulk of such analysis is carried out within the framework of collinear factorization.

- However, there exist classes of multi-scale processes where the use of more general schemes is of advantage
  - E.g. high-energy or low x limit of hard processes $s \gg M^2 \gg \Lambda^2_{\mathrm{QCD}}$ where $x = M^2/s$.
  - In such a scenario it is necessary to resum terms enhanced by logarithms $\ln 1/x$ to all orders in the α_s, which is achieved by BFKL evolution equation.
  - The resulting formalism called high energy (or k_T) factorization provides a factorization of such cross-sections into a TMD coefficient or 'impact factor' and an 'unintegrated' gluon density.

Introduction

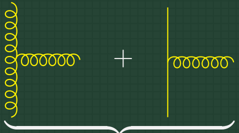
Limitations of high-energy factorization framework:

- valid only in low $x \lesssim 10^{-2}$ region
  - problems for observables involving fragmentation functions which involve integrals over the full x range of initial state PDFs
  - limited to exclusive observables which allow to fix x of both gluons
- limited to gluon-to-gluon splittings in the low x evolution, with quarks being absent.
  - omits a resummation of collinear logarithms associated with quark splittings which can provide sizable contributions at intermediate and large x
  - For hard processes initiated by quarks the appropriate unintegrated parton density functions are needed

What can we do?

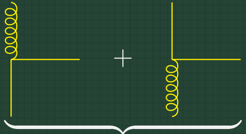
- Partial solution: use CCFM evolution equation instead of BFKL

✓ based on QCD coherence
$\rightarrow$ includes also resummation
of soft logarithms



contains $\frac{1}{z}$, $\frac{1}{1-z}$

✗ still limited to low x
✗ evolution equation for gluons only
✗ missing collinear logarithms



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What do we want?

- Resum low x logarithms.
- Smooth continuation to the large x region.
- Reproduce the correct collinear limit (DGLAP).
- Ultimately: a coupled system of evolution equations for unintegrated PDFs
  - need: k_T-dependent splitting functions

We will try to achieve this goal by extending Curci-Furmanski-Petronzio (CFP) and Catani-Hautmann (CH) formalisms.

Rest of the talk

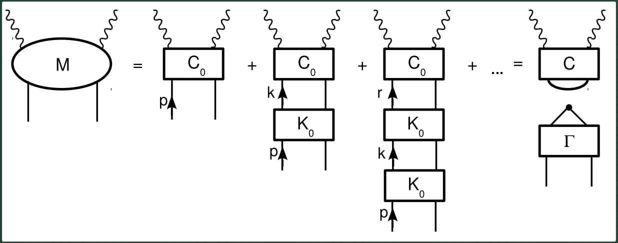
1. Basics of Curci-Furmanski-Petronzio method of splitting function calculation in collinear factorization.

2. Generalization to the high-energy case and kernel calculation.

3. Results: new TMD splitting functions

Curci-Furmanski-Petronzio (CFP) methodology [NPB175 (1980) 2792]

- Factorization based on generalized ladder expansion (in terms of 2PI kernels) [Ellis et al. NPB152 (1979), 285]



- Axial gauge instrumental
  - integration over outgoing legs leads to collinear singularities
  - incoming propagators amputated
- 2PI kernels connected only by convolution in x
  - this is achieved by introducing appropriate projector operators.

Curci-Furmanski-Petronzio (CFP) methodology [NPB175 (1980) 2792]

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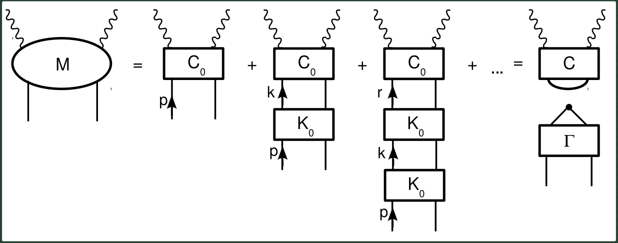


$$M = C^{(0)}\,\mathcal{G}^{(0)} \quad \text{with} \quad \mathcal{G}^{(0)} = \left(1 + K^{(0)} + K^{(0)}\,K^{(0)} + \ldots\right) = \frac{1}{1-K^{(0)}}$$

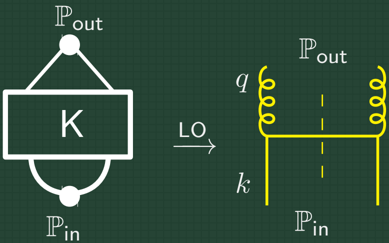
introduce projectros: $K^{(0)} = (1-\mathbb{P})\;K^{(0)} + \mathbb{P}\,K^{(0)}$

$$M = \underbrace{\left(C_0\,\frac{1}{1-(1-\mathbb{P})K^{(0)}}\right)}_{C}\underbrace{\left(\frac{1}{1-\mathbb{P}K}\right)}_{\Gamma} \quad \text{with} \quad K = K^{(0)}\left(\frac{1}{1-(1-\mathbb{P})K^{(0)}}\right)$$

Curci-Furmanski-Petronzio (CFP) methodology

[Nucl. Phys. B175 (1980) 2792]

- CFP projector operators: $\mathbb{P} = \mathbb{P}^{\epsilon} \otimes \mathbb{P}^{s} = \mathbb{P}^{\epsilon} \otimes \mathbb{P}^{s}_{\text{in}} \otimes \mathbb{P}^{s}_{\text{out}}$



$$\begin{cases} \mathbb{P}^{\mu\nu}_{g,\,\text{out}} = -g^{\mu\nu} \\ \mathbb{P}_{q,\,\text{out}}(q) = \frac{\not{n}}{2q\cdot n} \end{cases}$$

$$\begin{cases} \mathbb{P}^{\mu\nu}_{g,\,\text{in}}(k) = \frac{1}{m-2}\left(-g^{\mu\nu} + \frac{k^{\mu}n^{\nu}+n^{\mu}k^{\nu}}{k\cdot n}\right) \\ \mathbb{P}_{q,\,\text{in}} = \frac{\not{k}}{2} \\ \text{incoming legs put on-shell } k^2 = 0 \end{cases}$$

- performs integration over $dq^2 d^{m-2}\mathbf{q}$
- takes pole part

Rest of the talk

1. Basics of Curci-Furmanski-Petronzio method of splitting function calculation in collinear factorization.

2. Generalization to the high-energy case and kernel calculation.

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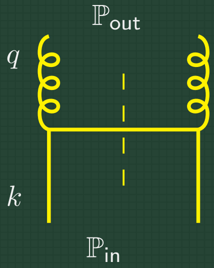
Generalization to high energy kinematics

- High energy kinematics



$$k^\mu = yp^\mu + k_\perp^\mu$$

$$q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2 + \mathbf{q}^2}{2xp \cdot n} n^\mu$$

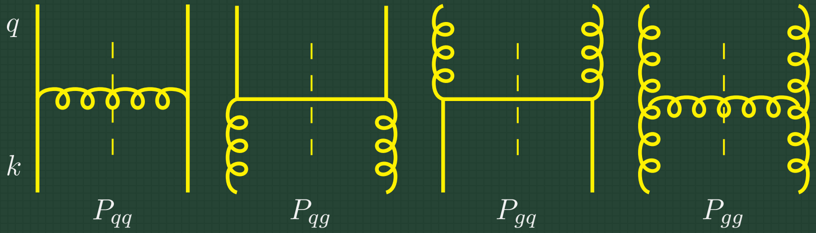
We will define/constrain splitting functions by requiring:

- gauge invariance/current conservation of vertices
- correct collinear limit
- correct high energy limit

Generalization to high energy kinematics

- Partly obtained by Catani and Hautmann for the case of P_{qg}
  [Catani, Hautmann NPB427 (1994) 475524, hep-ph/9405388]

- We want to extend it to general case including all splittings
  - P_{gq} and P_{qq} case done in [JHEP 01 (2016) 181, 1511.08439]
  - P_{gg} done in [EPJC 78 (2018) 174, 1711.04587]



To achieve this goal we need to provide:

- Appropriate projector operators
- Generalize QCD vertices

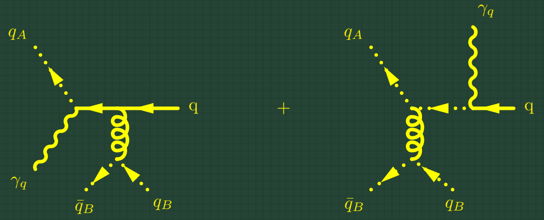
Generalization of QCD vertices

We need to ensure gauge invariance of vertices in the presence of off-shell k, q momenta.

- We use spinor helicity formalism to construct appropriate gauge invariant amplitudes and extracted vertices from them [Kutak, van Hameren, Serino, JHEP 02, 009 (2017)]
  - Off-shell particles are introduced as pairs of auxiliary on-shell particles $\rightarrow$ increased no. of Feynman diagrams.
  - Sum relevant diagrams to obtain gauge invariant amplitudes.
  - "Deconstruct" amplitude by removing polarisation vectors ($\epsilon^{\mu}(p)$ for an on-shell particle vs. p^{μ} for an off-shell particle)

- Alternatively (but with some ambiguity for P_{gg}) this can be obtained using the reggeized quark formalism (Lipatov high-energy action) [Lipatov, Vyazovsky, NPB 597 (2001) 399]

Generalization of QCD vertices: $\Gamma^{\mu}_{g^*q^*q}$ from $\mathcal{A}(1^*, \bar{q}^{*-}, q^+)$



$$\mathcal{A}(q,k,p') = \frac{\langle p|\gamma^\mu|p]}{\sqrt{2}} \frac{d_{\mu\nu}(q)}{q^2} \langle p| \frac{\not{\epsilon}_{p+}}{\sqrt{2}} \frac{\not{k}}{k^2} \frac{\gamma^\nu}{\sqrt{2}} - \frac{\gamma^\nu}{\sqrt{2}} \frac{\not{p}}{2p\cdot p'} \frac{\not{\epsilon}_{p+}}{\sqrt{2}} |n]$$

$$= n^\mu \frac{d_{\mu\nu}(q)}{q^2} [n| \left\{ \gamma^\nu - \frac{p^\nu}{p\cdot p'} \not{k} \right\} \frac{\not{k}}{k^2} |p]$$

$$\equiv n^\mu \frac{d_{\mu\nu}(q)}{q^2} [n| \Gamma^\nu_{g^*q^*q}(q,k,p') |p]$$

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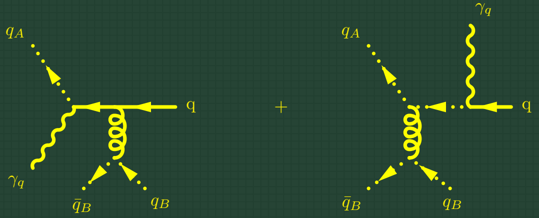
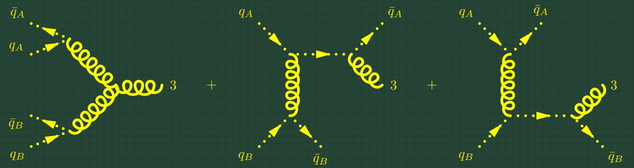
Generalization of QCD vertices: $\Gamma^{\mu}_{g^* q^* q}$ from $\mathcal{A}(1^*, \bar{q}^{*-}, q^+)$



$$
\begin{aligned}
\mathcal{A}(q,k,p') &= \frac{\langle p|\gamma^{\mu}|p]}{\sqrt{2}}\,\frac{d_{\mu\nu}(q)}{q^2}\,\langle p|\frac{\not{\epsilon}_{p+}}{\sqrt{2}}\frac{\not{k}}{k^2}\frac{\gamma^{\nu}}{\sqrt{2}} - \frac{\gamma^{\nu}}{\sqrt{2}}\frac{\not{p}}{2p\cdot p'}\frac{\not{\epsilon}_{p+}}{\sqrt{2}}|n] \\
&= \frac{n^{\mu}}{q^2}\,[n|\left\{ d_{\mu\nu}(q)\left(\gamma^{\nu} - \frac{p^{\nu}}{p\cdot p'}\not{k}\right)\right\}\frac{\not{k}}{k^2}|p] \\
&\equiv \frac{n^{\mu}}{q^2}\,[n|\,\Gamma^{\nu}_{g^* q^* q}(q,k,p')\,|p]
\end{aligned}
$$

[EPJC 78 (2018) 174, 1711.04587]

Generalization of QCD vertices: $\Gamma^{\mu}_{g^*g^*g}$ from $\mathcal{A}(1^*, 2^*, 3)$



$$\mathcal{A}(q,k,p') = (\sqrt{2})\, \frac{p_{\mu_1}\, n_{\mu_2}\, \epsilon_{\mu_3}(p')}{q^2\, k^2} \Big\{ \mathcal{V}^{\lambda\kappa\mu_3}(q,k,p')\, d^{\mu_1}{}_{\lambda}(q)\, d^{\mu_2}{}_{\kappa}(k) + d^{\mu_1\mu_2}(k)\, \frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\, \frac{k^2 p^{\mu_3}}{p\cdot p'} \Big\}$$

$$\equiv (\sqrt{2})\, \frac{p_{\mu_1}\, n_{\mu_2}\, \epsilon_{\mu_3}(p')}{q^2\, k^2}\, \Gamma^{\mu_1\mu_2\mu_3}(q,k,p')$$

- $d_{\mu\nu}(q) = -g_{\mu\nu} + \frac{n^\mu q^\nu + n^\nu q^\mu}{q^2}$ not invertable in light-cone gauge $(n^2 = 0) \Rightarrow$ it has to be kept everywhere!

Generalization of QCD vertices

The full set of gauge invariant off-shell vertices are:

$$\Gamma^{\mu}_{q^*g^*q}(q,k,p') = igt^a\, d^{\mu}{}_{\nu}(k) \left(\gamma^{\nu} - \frac{n^{\nu}}{k\cdot n}\not{q}\right)$$

$$\Gamma^{\mu}_{g^*q^*q}(q,k,p') = igt^a\, d^{\mu}{}_{\nu}(q) \left(\gamma^{\nu} - \frac{p^{\nu}}{p\cdot q}\not{k}\right)$$

$$\Gamma^{\mu}_{q^*q^*g}(q,k,p') = igt^a \left(\gamma^{\mu} - \frac{p^{\mu}}{p\cdot p'}\not{k} + \frac{n^{\mu}}{n\cdot p'}\not{q}\right)$$

$$\Gamma^{\mu_1\mu_2\mu_3}_{g^*g^*g}(q,k,p') = i\,g\,f^{abc}\left\{\mathcal{V}^{\lambda\kappa\mu_3}(q,k,p')\, d^{\mu_1}{}_{\lambda}(q)\, d^{\mu_2}{}_{\kappa}(k) + d^{\mu_1\mu_2}(k)\,\frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\,\frac{k^2 p^{\mu_3}}{p\cdot p'}\right\}$$

Generalization of projector operators 1

Since the incoming momentum is no longer collinear the corresponding projector operators need to be modified.

- Gluon case [Catani, Hautmann NPB427 (1994) 475524]:

$$\mathbb{P}_{g,\,\text{in}}^{\mu\nu} = \frac{k_\perp^\mu k_\perp^\nu}{\mathbf{k}^2}$$

- Quark case [JHEP 01 (2016) 181]:

$$\mathbb{P}_{q,\,\text{in}} = \frac{y\,\not{p}}{2}$$

✓ Both operators reduce to the CFP projectors in the collinear limit

$$< \frac{k_\perp^\mu k_\perp^\nu}{\mathbf{k}^2} >_\phi \overset{k_\perp \to 0}{=} \frac{1}{m-2}\left(-g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{k \cdot n}\right)$$

$$k \overset{k_\perp \to 0}{=} yp$$

Generalization of projector operators 2

- The form of the CH projectors were derived based on heavy quark production in which case numerators of the gluon propagators factorize [Catani, Ciafaloni, NPB 366 (1991) 135-188]

$$\mathcal{M}^{g^* g^* \to q\bar{q}}(k_1, k_2; p_3, p_4) = \frac{2\, x_1\, x_2\, p_1^{\mu_1}\, p_2^{\mu_2}}{\sqrt{k_{1\,\perp}^2\, k_{2\,\perp}^2}}\, d_{\mu_1\nu_1}(k_1)\, d_{\mu_2\nu_2}(k_2)\, \hat{\mathcal{M}}^{g^* g^* \to q\bar{q}}_{\mu_1,\mu_2}(k_1, k_2; p_3, p_4$$

- Since the numerators of gluon propagators do not factorize in case of $\Gamma^{\mu_1\mu_2\mu_3}_{g^* g^* g}$ we need further modifications.
- The form of the projectors is determined by
  - condition: $\mathbb{P}^2 = \mathbb{P}$
  - and a proper collinear limit

Final set of projectors:

$$\mathbb{P}^{\mu\nu}_{g,\,\text{in}} = -y^2\, \frac{p^\mu p^\nu}{k_\perp^2} \qquad \mathbb{P}^{\mu\nu}_{g,\,\text{out}} = -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n} - k^2\, \frac{n_\mu n_\nu}{(k\cdot n)^2}$$

$$\mathbb{P}_{q,\,\text{in}} = \frac{y\, \not{p}}{2} \qquad \mathbb{P}_{q,\,\text{out}} = \frac{\not{n}}{2\, n\cdot l}$$

Rest of the talk

1. Basics of Curci-Furmanski-Petronzio method of splitting function calculation in collinear factorization.
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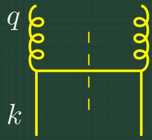
TMD splitting function definition

- Angular-dependent splitting function

$$\hat{K}_{ij}\left(z,\frac{\mathbf{k}^2}{\mu^2},\epsilon\right)=z\int\frac{d^{2+2\epsilon}\mathbf{q}}{2(2\pi)^{4+2\epsilon}}\underbrace{\int dq^2\,\mathbb{P}_{j,\,\mathrm{in}}\otimes\hat{K}^{(0)}_{ij}(q,k)\otimes\mathbb{P}_{i,\,\mathrm{out}}}_{\tilde{P}^{(0)}_{ij}(z,\mathbf{k},\tilde{\mathbf{q}},\epsilon)}\,\Theta(\mu_F^2+q^2)$$

$$\hat{K}^{(0)}_{ij}(q,k)=$$



$$q^\mu=xp^\mu+q_\perp^\mu+\frac{q^2+\mathbf{q}^2}{2xp\cdot n}n^\mu$$
$$k^\mu=yp^\mu+k_\perp^\mu$$
$$\tilde{\mathbf{q}}=\mathbf{q}-z\mathbf{k},\qquad z=x/y$$

- Angular-average splitting function

$$\hat{K}_{ij}\left(z,\frac{\mathbf{k}^2}{\mu_F^2},\epsilon\right)=\frac{\alpha_s}{2\pi}\,z\int_0^{(1-z)(\mu_F^2-z\mathbf{k}^2)}\frac{d\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2}\left(\frac{\tilde{\mathbf{q}}^2}{\mu^2}\right)^{\epsilon}\frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)}\bar{P}^{(0)}_{ij}\left(z,\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2},\epsilon\right)$$

Results for splitting functions

With the new projection operators we reproduce our earlier results [JHEP 01 (2016) 181, 1511.08439]

$$\tilde{P}_{qg}^{(0)} = T_R \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\,\mathbf{k}^2} \right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \right]$$

$$\tilde{P}_{gq}^{(0)} = C_F \left[\frac{2\tilde{\mathbf{q}}^2}{z|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} - \frac{\tilde{\mathbf{q}}^2(\tilde{\mathbf{q}}^2(2-z) + \mathbf{k}^2 z(1-z^2))}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)^2} + \frac{\epsilon z \tilde{\mathbf{q}}^2(\tilde{\mathbf{q}}^2 + (1-z)^2\mathbf{k}^2)}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)^2} \right]$$

$$\tilde{P}_{qq}^{(0)} = C_F \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2} \right) \left[\frac{\tilde{\mathbf{q}}^2 + (1-z^2)\mathbf{k}^2}{(1-z)|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} + \frac{z^2\tilde{\mathbf{q}}^2 - z(1-z)(1-3z+z^2)\mathbf{k}^2 + (1-z)^2\epsilon(\tilde{\mathbf{q}}^2 + z^2\mathbf{k}^2)}{(1-z)(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)} \right]$$

✓ In the collinear limit, $\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \to 0$, standard DGLAP results are reproduced.

Results for splitting functions

The new result is [EPJC 78 (2018) 174, 1711.04587]

$$
\begin{aligned}
\tilde{P}_{gg}^{(0)}(z,\tilde{\mathbf{q}},\mathbf{k}) = C_A \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2}\right)^2 \frac{\tilde{\mathbf{q}}^2}{\left(\tilde{\mathbf{q}}-(1-z)\mathbf{k}\right)^2} \\
\times \Bigg[-\frac{4z^2-4z+2}{z(1-z)} - z(1-z)(4z^4-12z^3+9z^2+1)\frac{\mathbf{k}^4}{\tilde{\mathbf{q}}^4} \\
-4z(1-z)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}^2}{\mathbf{k}^2\tilde{\mathbf{q}}^2} + 2(4z^3-6z^2+6z-3)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2} \\
-4z(1-z)^2(3-5z)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^4} - (4z^4-8z^3+5z^2-3z-2)\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \\
+8z(1-z)^2\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}^3}{\mathbf{k}^2\tilde{\mathbf{q}}^4} - 2z^2(1-z)(3-4z)(3-2z)\frac{\mathbf{k}^2\,\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^4}\Bigg] \\
-\epsilon\, C_A\, z(1-z)\frac{\tilde{\mathbf{q}}^2}{\mathbf{k}^2}\left(\frac{(2z-1)\mathbf{k}^2+2\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2}\right)^2
\end{aligned}
$$

Results for splitting functions

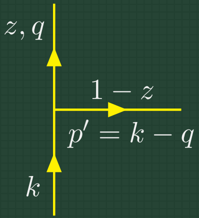
or in an angular integrated form (with $\epsilon = 0$)

$$\bar{P}_{gg}^{(0)} = \frac{1}{\pi}\int_0^\pi d\phi\, \sin^{2\epsilon}\phi\, \tilde{P}_{gg}^{(0)}$$

$$= C_A \frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2}\left[\frac{(2-z)\tilde{\mathbf{q}}^2 + (z^3 - 4z^2 + 3z)\mathbf{k}^2}{z(1-z)\,|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|}\right.$$

$$\left. + \frac{(2z^3 - 4z^2 + 6z - 3)\tilde{\mathbf{q}}^2 + z(4z^4 - 12z^3 + 9z^2 + z - 2)\mathbf{k}^2}{(1-z)(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)}\right]$$

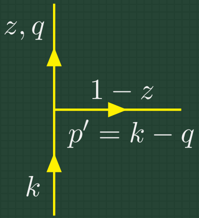
Kinematic limits of $\tilde{P}_{gg}^{(0)}$



- Collinear (DGLAP) limit

$$\lim_{\mathbf{k}^2 \to 0} \bar{P}_{gg}^{(0)} = 2\, C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z\,(1-z) \right]$$

Kinematic limits of $\tilde{P}_{gg}^{(0)}$



- Collinear (DGLAP) limit

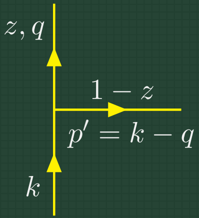
$$\lim_{\mathbf{k}^2 \to 0} \bar{P}_{gg}^{(0)} = 2\, C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z\,(1-z) \right]$$

- High-energy limit $(z \to 0)$

$$\lim_{z \to 0} \hat{K}_{gg}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \epsilon, \alpha_s\right) = \frac{\alpha_s C_A}{\pi (e^{\gamma_E} \mu^2)^{\epsilon}} \int \frac{d^{2+2\epsilon}\tilde{\mathbf{p}}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - (\mathbf{k} - \tilde{\mathbf{p}})^2\right) \frac{1}{\tilde{\mathbf{p}}^2}$$

$$= \int \frac{d^{2+2\epsilon}\mathbf{q}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - \mathbf{q}^2\right) \frac{\alpha_s C_A}{\pi (e^{\gamma_E} \mu^2)^{\epsilon}} \frac{1}{(\mathbf{q} - \mathbf{k})^2},$$

Kinematic limits of $\tilde{P}_{gg}^{(0)}$



- **Collinear** (DGLAP) limit

$$\lim_{\mathbf{k}^2 \to 0} \bar{P}_{gg}^{(0)} = 2\, C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z\,(1-z) \right]$$

- **High-energy** limit $(z \to 0)$

$$\lim_{z \to 0} \hat{K}_{gg}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \epsilon, \alpha_s \right) = \frac{\alpha_s C_A}{\pi (e^{\gamma_E} \mu^2)^{\epsilon}} \int \frac{d^{2+2\epsilon}\tilde{\mathbf{p}}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - (\mathbf{k} - \tilde{\mathbf{p}})^2\right) \frac{1}{\tilde{\mathbf{p}}^2}$$
$$= \int \frac{d^{2+2\epsilon}\mathbf{q}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - \mathbf{q}^2\right) \frac{\alpha_s C_A}{\pi (e^{\gamma_E} \mu^2)^{\epsilon}} \frac{1}{(\mathbf{q} - \mathbf{k})^2},$$

- **Soft** (CCFM) limit ($\tilde{\mathbf{p}} = \frac{\mathbf{k}-\mathbf{q}}{1-z}$): $\tilde{\mathbf{p}}^2 \to 0$

$$\hat{K}_{gg}\left(z, \frac{\mathbf{k}^2}{\mu^2}, 0, \alpha_s \right) = z \int_0 \frac{d\tilde{\mathbf{p}}^2}{\tilde{\mathbf{p}}^2} \frac{\alpha_s C_a}{\pi} \left[\frac{1}{z} + \frac{1}{1-z} + \mathcal{O}\left(\frac{\tilde{\mathbf{p}}^2}{\mathbf{k}^2} \right) \right]$$

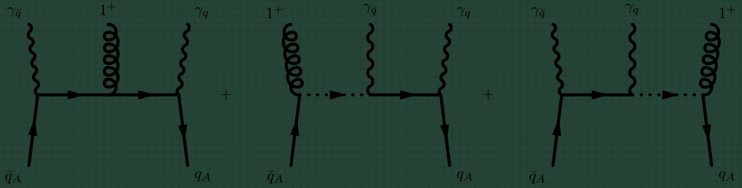
we obtain real/unresummed CFFM kernel “for free”

Summary and Outlook

- We successfully extended method of Curci, Furmanski and Petronzio to the TMD case using gauge invariant vertices.
  - The essential subtleties which prevent the Catani-Hautmann generalisation from being directly extended to the P_{gg} case were uncovered and worked out.
- With the new projectors we have reproduced our earlier results for real emission $k_\perp$-dependent P_{qq}, P_{gq} and P_{qg} splitting functions confirming our formalism.
- We used the formalism to calculate P_{gg} TMD splitting function which feature correct
  - collinear limit (DGLAP kernels)
  - high-energy limit (BFKL kernel)
  - soft limit (CCFM kernel)
- The next step is to calculate virtual corrections.
- In a longer perspective construct a complete set of evolution equations.

BACKUP SLIDES

$\Gamma^{\mu}_{q^*,q^*,g}$ from $\mathcal{A}(g^+, \bar{q}^{*+}, q^{*-})$



$$\mathcal{A}(1^+, \bar{q}^{*+}, q^{*-}) \to \langle \bar{q}| \frac{\not{k}_{\bar{q}}}{k_{\bar{q}}^2} \left\{ \gamma^{\mu} + \frac{p_{\bar{q}}^{\mu}}{p_{\bar{q}} \cdot k_q} \not{k}_{\bar{q}} + \frac{p_q^{\mu}}{p_q \cdot k_{\bar{q}}} \not{k}_q \right\} \frac{\not{k}_q}{k_q^2} |q]$$