Inclusive photon production in DIS off nuclei at small x

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Ian Balitsky, Yacine Mehtar-Tani, Al Mueller, Anna Stasto and Andrey Tarasov, for useful discussions on NLO computation
Inclusive photon production in e+A DIS at LO, LLx and all-twist contributions

Interesting limits (k_T & collinear factorization, di-jets)

Fermion and Gluon shockwave propagators in the “wrong” light cone gauge – connection to Lipatov’s Reggeon Field Theory

The structure of higher order computations: the NLO inclusive photon impact factor
The CGC EFT

EFT in the Regge limit of $Q^2 >> \Lambda_{QCD}^2$ to study matter at high parton densities

arXiv:1708.01527
DIS inclusive photon production at LO

Right moving nucleus with momentum $P_N^+$ is Lorentz contracted in $x^-$ direction

Glue fields satisfy Yang-Mills eqns.

\[
[D_\mu, F^{\mu\nu}](x) = g\delta^{\nu^+}\delta(x^-)\rho_A(x_\perp)
\]

$A^{-\alpha} = 0, F_{\alpha\beta} = 0$ with $A^{+,\alpha}, A^{i,\alpha}$ static (independent of $x^+$)

Suppressed at small $x$
DIS inclusive photon production at LO

\[ e(\tilde{l}) + A(P) \to e(\tilde{l}') + Q(k) + \bar{Q}(p) + \gamma(k_\gamma) + X \]

\[
\frac{d\sigma}{dx dQ^2} = \frac{2\pi y^2}{64\pi^3 Q^2} \frac{d^3k}{(2\pi)^3 2E_k} \frac{d^3p}{(2\pi)^3 2E_p} \frac{d^3k_{\gamma}}{(2\pi)^3 2E_{k_{\gamma}}} \frac{1}{2q^-} \left( \frac{1}{2} \sum_{\text{spins,} \lambda} \left\langle |\tilde{M}|^2 \right\rangle_{Y_A} \right) (2\pi) \delta(P^- - q^-) 
\]

\[
\frac{1}{2} \sum_{\text{spins,} \lambda} \left\langle |\tilde{M}|^2 \right\rangle_{Y_A} = L_{\mu\nu} X^{\mu\nu}
\]

L_{\mu\nu} is well known—the lepton tensor X^{\mu\nu} - the hadron tensor for inclusive photon production is what we compute.

How do we compute these 10 diagrams?
**Shockwave fermion propagator**


Solve Dirac equation in LC gauge background field ($A^-=0$, $A^+=0$, $F_{ij}=0$) on either side of source – match solutions at $x^-=0$

**LC propagator can be expressed as**

$$S_{LC}(x, y) = G(x^-, x_\perp)S_{Lorenz} G^\dagger(y^-, y_\perp)$$

where $G(x^-, x_\perp) = \theta(-x^-) + \theta(x^-)\tilde{U}(x_\perp)$

$$\tilde{U}(x_\perp) = \mathcal{P}_- \exp \left[ -ig \int_{-\infty}^{\infty} dz^- \frac{\rho_A^a(z^-, x_\perp)t^a}{\nabla^2_{\perp}} \right]$$

The mom. space Lorenz gauge propagator has the simple form

$$S_{Lorenz}(q, p) = (2\pi)^4 \delta^{(4)}(q - p) + S_0(q)T(q, p)S_0(p)$$

**Effective vertex**

$$T(q, p) = 2\pi \delta(p^- - q^-) \text{sign}(p^-)\gamma^- \int_{x_\perp} e^{i(q_\perp - p_\perp) \cdot x_\perp} \left[ \tilde{U}\text{sign}(p^-) - 1 \right]$$
DIS inclusive cross-section at LO

Roy, RV; arXiv:1802.09550

\[
\frac{d\sigma}{dx dQ^2 d^2k_{\perp} d\eta_{k_{\perp}}} = \frac{\alpha^2 q^4 y^2 N_c}{512\pi^5 Q^2} \frac{1}{2q} \int_0^{+\infty} \frac{dk^-}{k^-} \int_0^{+\infty} \frac{dp^-}{p^-} \int_{k_{\perp},p_{\perp}} L^{\mu\nu} \tilde{X}_{\mu\nu} (2\pi) \delta(P^- - q^-)
\]

\[
L^{\mu\nu} = \frac{2e^2}{Q^4} \left[ (\tilde{l}^\mu \tilde{l}^\nu + \tilde{l}^{\prime \mu} \tilde{l}^{\prime \nu}) - \frac{Q^2}{2} g^{\mu\nu} \right]
\]

\[
\tilde{X}_{\mu\nu} = \int_{x_{\perp}, y_{\perp} \neq x_{\perp}^\prime, y_{\perp}^\prime} e^{-i(P_{\perp} - l_{\perp}) \cdot x_{\perp} + i(l_{\perp} - P_{\perp}) \cdot y_{\perp} + i(l_{\perp}^\prime - P_{\perp}^\prime) \cdot x_{\perp}^\prime + i(l_{\perp}^\prime - P_{\perp}^\prime) \cdot y_{\perp}^\prime} \tau_{\mu\nu}^{q\bar{q}, q\bar{q}} (1_{\perp}, 1_{\perp}^\prime | P_{\perp}) \Xi(x_{\perp}, y_{\perp}; x_{\perp}^\prime, y_{\perp}^\prime)
\]

All the nonperturbative info about strongly correlated gluons is in

\[
\Xi(x_{\perp}, y_{\perp}; x_{\perp}^\prime, y_{\perp}^\prime) = 1 - D(x_{\perp}, y_{\perp}) - D(y_{\perp}, x_{\perp}^\prime) + Q(x_{\perp}, y_{\perp}; x_{\perp}^\prime, y_{\perp}^\prime)
\]

**Dipoles:**

\[
D(x_{\perp}, y_{\perp}) = \frac{1}{N_c} \langle \text{Tr} \left( \tilde{U}(x_{\perp}) \tilde{U}^\dagger(y_{\perp}) \right) \rangle_{Y_A}
\]

**Quadrupoles:**

\[
Q(x_{\perp}, y_{\perp}) = \frac{1}{N_c} \langle \text{Tr} \left( \tilde{U}(y_{\perp}) \tilde{U}^\dagger(x_{\perp}^\prime) \tilde{U}(x_{\perp}) \tilde{U}^\dagger(y_{\perp}) \right) \rangle_{Y_A}
\]

Building blocks of high energy QCD
The leading LLx one loop real and virtual contributions below the quark loop ($Y < Y_A$) in rapidity can be absorbed in the JIMWLK evolution of $W_{Y A}[\rho_A]$

– thereby also giving the LLx evolution equations for Dipole and Quadrupole Wilson line correlators
Some interesting limits

When $k_\gamma \to 0$, the amplitude satisfies the Low-Burnett-Kroll theorem:

$$\mathcal{M}_\mu(q, k, p, k_\gamma) \to -(e q_f) e_\alpha^*(k_\gamma, \lambda) \left( \frac{p_\alpha}{p \cdot k_\gamma} - \frac{k_\alpha}{k \cdot k_\gamma} \right) \mathcal{M}_\mu^{NR}(q, k, p)$$

Recover the result of Dominguez, Marquet, Xiao and Yuan for di-jet production
-as they show, it is sensitive to the gluon Weizsäcker-Williams distribution for large pair momenta
Some interesting limits

At high $P_T$, recover leading twist $k_T$ and collinear factorization expressions

\[ \tilde{X}^{LT}_{\mu\nu} = \frac{2\alpha_S}{N_C} \frac{\phi_Y A(P_\perp)}{P_\perp^2} \Theta_{\mu\nu}(P_\perp) \]

\[ \tilde{X}^{\text{coll.}}_{\mu\nu} = \frac{2\alpha_S \pi^2}{N_C} (2\pi)^2 \delta^{(2)}(p_\perp + k_\perp + k_{\gamma\perp}) x_A G_A(x_A, Q^2) \lim_{P_\perp \to 0} \Theta_{\mu\nu}(P_\perp) \frac{P_\perp^2}{P_\perp} \]

Inclusive photon production is directly sensitive to the nuclear gluon distribution - result at small $x$ agrees exactly with Aurenche et al (Z. Phys. C24, 309 (1984))
Structure of higher order computations: Shockwave propagators

Convenient to work in the “wrong” light cone gauge $A^- = 0$ for this problem (Gauge links in pdf definitions are unity in the right LC gauge $A^+ = 0$)

Just as for the Lorenz gauge quark propagator, the gluon propagator has an identical simple form where $G_0$ is the free propagator in $A^- = 0$ gauge

$$G^{\mu\nu;ab}(p, p') = (2\pi)^4 \delta^{(4)}(p - p') G_0^{\mu\nu;ab} + G_0^{\mu\rho;ac} T_{\rho\sigma;cd} G_0^{\sigma\nu;db}(p')$$

Note the slightly different form of the vertices—they will be useful in higher order computations

This structure of vertices is identical to the quark-quark-reggeon and gluon-gluon-reggeon in Lipatov’s Reggeon EFT

Bondarenko, Lipatov, Pozdnyakov, Prygarin, arXiv:1708.05183

Hentschinski, arXiv:1802.06755

McLerran, RV; hep-ph/9402335
Ayala, Jalilian-Marian, McLerran, RV; hep-ph/9508302
Balitsky, Belitsky, hep-ph/0110158
DIS inclusive photon production at NLO

NLO contributions of this sort – give rise to NLLx JIMWLK evolution
--significant progress in this direction

Balitsky, Chirilli, arXiv:1309.7644
Grabovsky, arXiv:1307.5414
Caron-Huot, arXiv:1309.6521
Kovner, Lublinsky, Mulian, arXiv:1310.0378
Lublinsky, Mulian, arXiv:1610.03453

Our method of computation, entirely in momentum space,
is different from most of these approaches – so would be interesting to compare.
This is work in progress with Roy.
The NLO inclusive photon impact factor

Several computations exist for fully inclusive DIS – subtleties in choice of scheme, etc.

Balitsky, Chirilli, arXiv:1009.4729
Beuf, arXiv:1606.00777, 1708.06557
Hanninen, Lappi, Paatelainen, 1711.08207
also, Boussarie, Grabovsky, Szymanowski, Wallon, 1606.00419

Real contributions at NLO:

\[ \mathcal{M}_{\mu\alpha;b}^{\text{real}} = 2\pi(\epsilon q_f)^2 g \delta(q^- - P_{tot}^-) \ d\Pi_R \ u(k) \left( T_R^{(1)}_{\mu\alpha;b} \left( (\tilde{U}(x_\perp) t^a \tilde{U}^\dagger(y_\perp))_{ij} U_{ba}(z_\perp) - (t_b)_{ij} \right) \right. \]

\[ + \left. T_R^{(2)}_{\mu\alpha;b} \left( (t_b \tilde{U}(x_\perp) \tilde{U}^\dagger(y_\perp))_{ij} - (t_b)_{ij} \right) + T_R^{(3)}_{\mu\alpha;b} \left( (\tilde{U}(x_\perp) \tilde{U}^\dagger(y_\perp) t_b)_{ij} - (t_b)_{ij} \right) \right) v(p) \]

\[ D\Pi_R \text{ represents phase space integrals} \]

Twenty such diagrams in the amplitude

\[ T_R^{(1)} \]

\[ T_R^{(2)} \]

\[ T_R^{(3)} \]

Roy, RV, in preparation
The NLO inclusive photon impact factor

Virtual contributions at NLO:

\[
M^{\text{vertex}}_{\mu \alpha} = 2\pi(cqfg)^2 \delta(q^- - P^-) \, d\Pi_V \, \bar{u}(k) \left( T_V^{(1)}_{\mu \alpha} \left( (t^a \bar{U}(x_\perp)t^b \bar{U}^\dagger(y_\perp))_{ij} \, U_{ab}(z_\perp) - C_F \delta_{ij} \right) \right.
\]
\[
+ T_V^{(2)}_{\mu \alpha} \left( (\bar{U}(x_\perp)t^a \bar{U}^\dagger(y_\perp)t^b)_{ij} \, U_{ba}(z_\perp) - C_F \delta_{ij} \right) + T_V^{(3)}_{\mu \alpha} \left( C_F (\bar{U}(x_\perp)\bar{U}^\dagger(y_\perp) - 1)_{ij} \right)
\]
\[
+ T_V^{(4)}_{\mu \alpha} \left( (t^a \bar{U}(x_\perp)\bar{U}^\dagger(y_\perp)t^b)_{ij} - C_F \delta_{ij} \right) \bigg) v(p),
\]

Overall, 24 such diagrams in the amplitude

Roy, RV, in preparation
The NLO inclusive photon impact factor

Inclusive photon cross-section: \( \text{NLO-real} \times \text{NLO*real} + \text{LO} \times \text{NLO virtual} \)

<table>
<thead>
<tr>
<th>Wilson line factor</th>
<th>Real emission</th>
<th>Virtual: Vertex</th>
<th>Virtual: Self-energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{N_c^2}{2} \left( 1 - D_{x'<em>y}D</em>{y} - D_{y'<em>x}D</em>{x} + \frac{1}{2} \Xi(x_{y \perp}, y_{\perp}; y'<em>{\perp}, x'</em>{\perp}) \right) )</td>
<td>( T_R^{(1)}T_R^{(1)} )</td>
<td>( T_{LO}T_V^{(3)} + c.c )</td>
<td>( T_{LO}T_S^{(3)} + c.c )</td>
</tr>
<tr>
<td>( \frac{N_c}{2} \Xi(x_{\perp}, y_{\perp}; y'<em>{\perp}, x'</em>{\perp}) )</td>
<td>( T_R^{(2)}T_R^{(2)} + T_R^{(3)}T_R^{(3)} )</td>
<td>( T_{LO}T_V^{(4)} + c.c )</td>
<td>( T_{LO}T_S^{(4)} + c.c )</td>
</tr>
<tr>
<td>( \frac{N_c}{2} \left( 1 - D_{x'y'}(1 - D_{y'x'}) \right) - \frac{1}{2} \Xi(x_{\perp}, y_{\perp}; y'<em>{\perp}, x'</em>{\perp}) )</td>
<td>( T_R^{(2)}T_R^{(2)} + T_R^{(3)}T_R^{(3)} + c.c )</td>
<td>( T_{LO}T_V^{(4)} + c.c )</td>
<td>( T_{LO}T_S^{(4)} + c.c )</td>
</tr>
</tbody>
</table>

For each Wilson line structure, collinear divergences cancel between real and interference contributions.

Rapidity and UV divergent pieces: these can be absorbed, in a subtraction scheme, into the NLLx JIMWLK expressions.

Roy, RV, in preparation
Summary and Outlook

- Inclusive photon production in inclusive DIS is a clean probe of the dynamics of strongly correlated gluons at high energies.

- A rich structure in terms of 2-point and 4-point Wilson line correlators is seen. Well-known results for inclusive di-jet production (and extraction of the Weizsäcker-Williams gluon distribution) are recovered in the soft photon limit. In the leading twist limit, the cross-section is directly proportional to the nuclear gluon distribution.

- The structure of dressed quark and gluon propagators in the “wrong” light cone gauge $A^- = 0$ is very simple and facilitates higher order computations in momentum space (using otherwise standard techniques in pQCD).

- The computation of the NLO inclusive photon impact factor is nearing completion and will be presented shortly.