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FORWARD DI-JETS IN $p+A$ COLLISIONS IN THE ITMD FRAMEWORK

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OUTLINE

- I. What is saturation?
 - I. Gluon branching
 - II. Parton distribution functions
 - III. Where to look for saturation effects?
 - IV. Saturation inside the nucleus
- II. Di-jet events at forward rapidity
 - I. Use of nuclear effects
 - II. Cross section calculation
 - III. Nuclear modification factor
- III. Conclusions

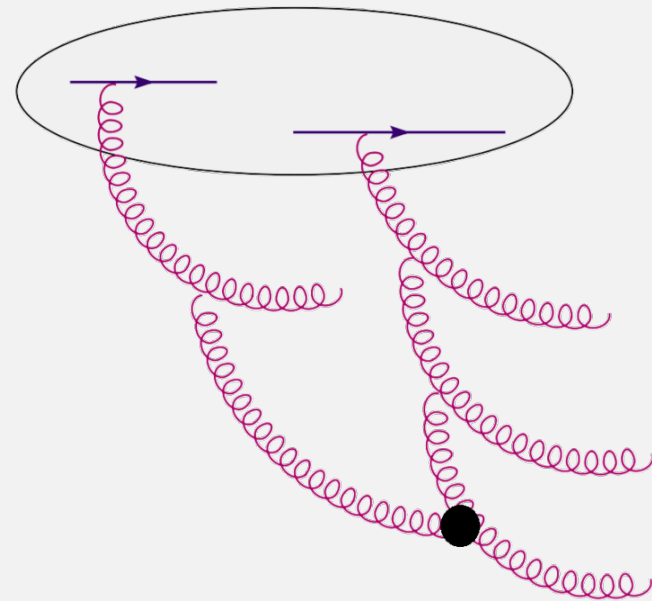
WHAT IS SATURATION?

GLUON BRANCHING

Gluon branching – The number of gluons grows as x decreases.

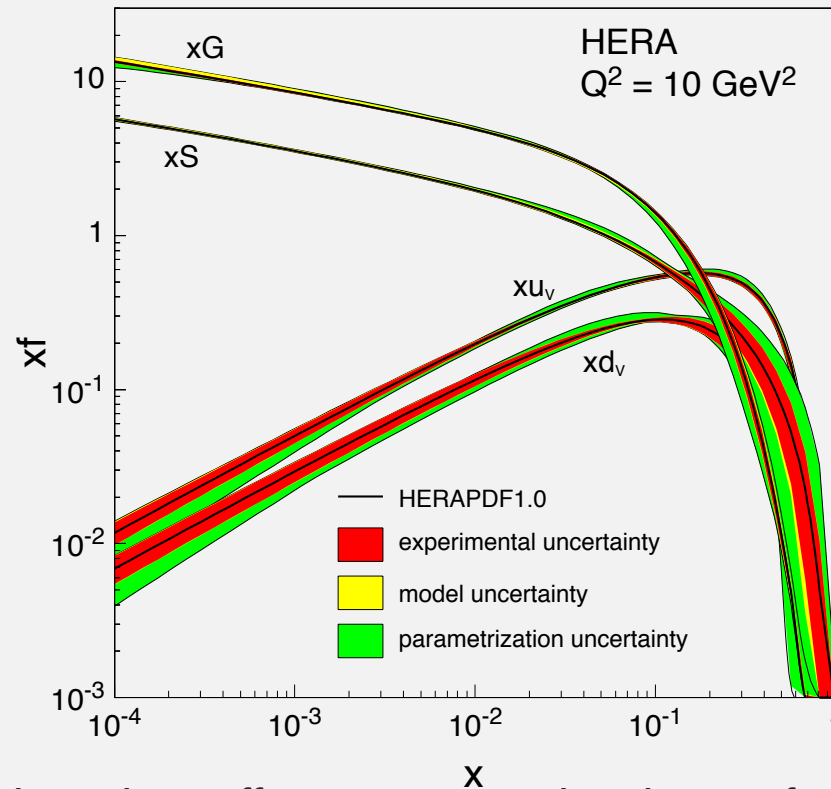
- The more gluons there are in a proton, the more they recombine.

Gluon recombination – High density of gluons can lead to overlapping of their wave functions and two gluons can merge into one.



PARTON DISTRIBUTION FUNCTIONS

- xS – Sea quark distribution
- xG – Gluon distribution
- xu_v – Valence u-quark distribution
- xd_v – Valence d-quark distribution



We can see the contribution of gluon branching effects in parton distribution functions.

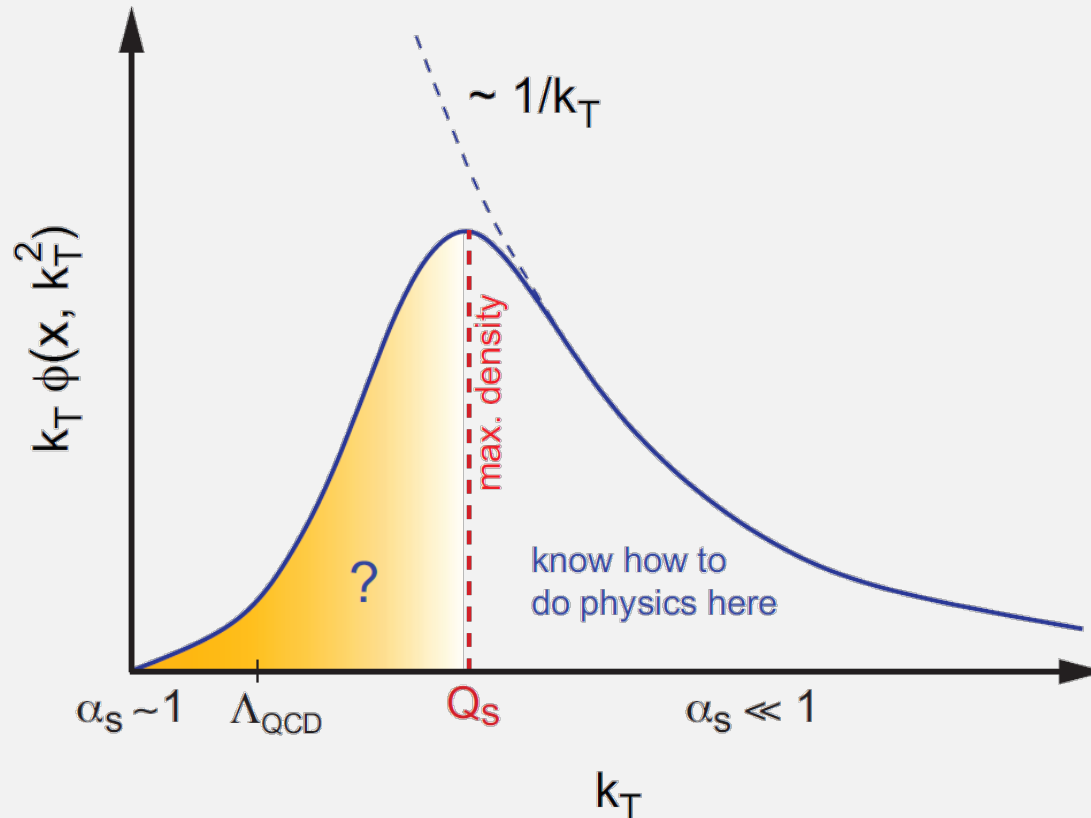
Do the gluon recombination effects also contribute?

WHERE TO LOOK FOR SATURATION EFFECTS?

- To observe saturation effects, we need to reach low values of x .
- With a fixed collision energy, lower- x corresponds to lower values of k_t , the typical transverse momentum of the process.
- Low k_t jets can be however difficult to measure.

$$x \sim \frac{k_t}{\sqrt{s}}$$

WHERE TO LOOK FOR SATURATION EFFECTS?

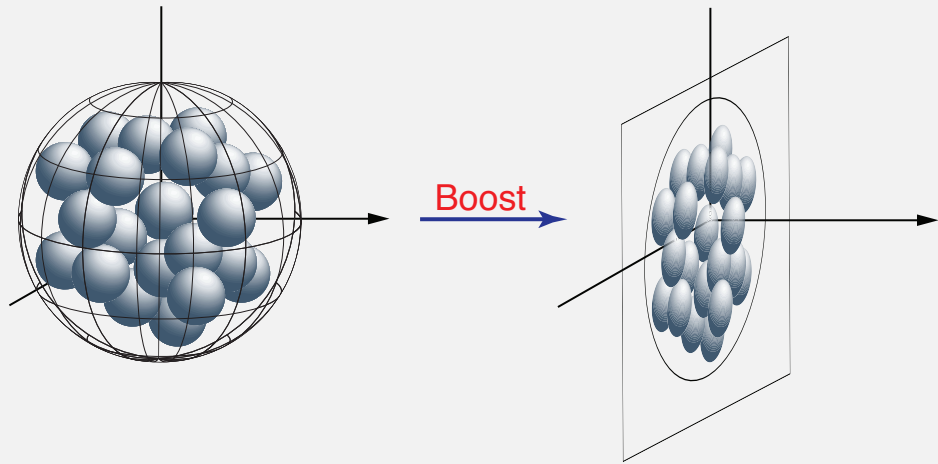


Saturation effects suppress distribution functions at low values of k_t . Maximum density is at Q_s .

SATURATION INSIDE THE NUCLEUS

If we look at the influence of nuclear effects on saturation, we find out that

$$Q_{sA}^2 \sim A^{1/3} Q_{sp}^2$$



Compared to the proton case, the saturation scale inside a nucleus is larger, because of Lorentz contraction.

The goal is to use forward dijets in pA collisions to look for saturation effects in lead.

WHAT PROCESSES TO FOCUS ON?

STUDIED PROCESSES

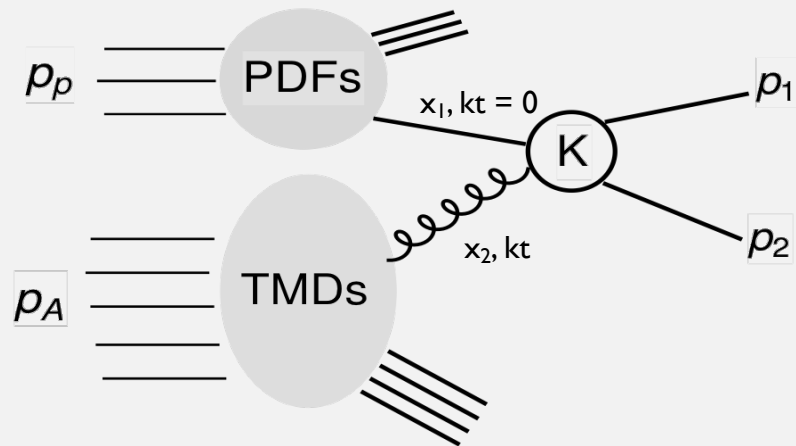
For our computation, we focused on back-to-back jets in the forward region of rapidity.

Why?

STUDIED PROCESSES

We want to reach low values of k_t .

Although $p_{1t}, p_{2t} \gg Q_s$

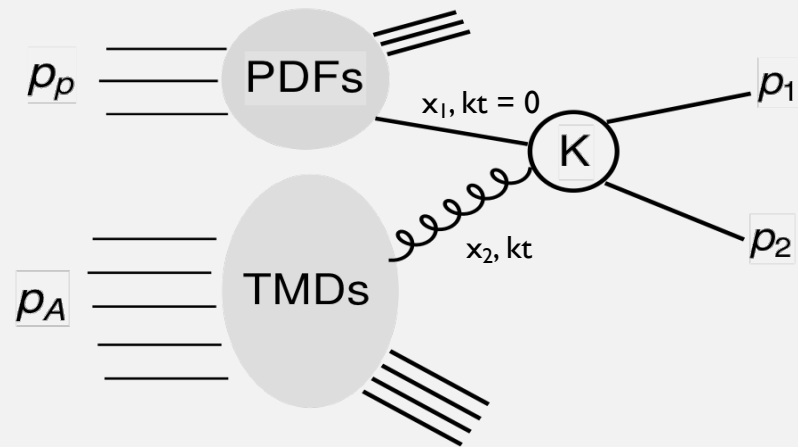


If we focus on back-to-back jets in the transverse momentum plane, we can get

$$k_t = |\vec{p}_{1t} + \vec{p}_{2t}| \text{ where } k_t \sim Q_s$$

STUDIED PROCESSES

Furthermore, it is necessary to reach
the region where x_1 is large and $x_2 \ll 1$.



$x_2 \ll 1$ is necessary to detect saturation effects; large x_1 is required, because in this region of x , we can use parton distribution functions that are known with great precision from previous experiments.

STUDIED PROCESSES

For x_1 and x_2 holds:

$$x_1 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}) , \quad x_2 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2})$$

Therefore for $y_1, y_2 \gg 1$ we obtain $x_1 \sim 1$ and $x_2 \ll 1$.

That is why we shall focus on studying back-to-back jets in the forward region in rapidity to detect saturation effects.

HOW DO WE DETERMINE WHETHER
SATURATION EFFECTS ARE PRESENT?

USE OF NUCLEAR EFFECTS

We use the fact that in nuclei, the saturation scale reaches higher values than in protons.

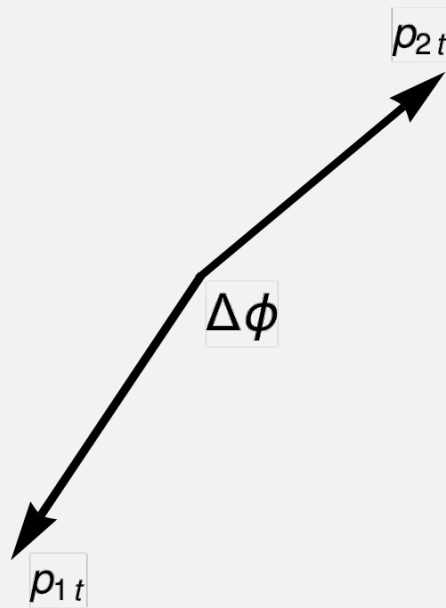
$$Q_s^2 \sim A^{1/3}$$

For the detection of saturation effects we use:

$$R_{pPb} = \frac{\frac{d\sigma^{p+Pb}}{d\mathcal{O}}}{A \frac{d\sigma^{p+p}}{d\mathcal{O}}}$$

We look for suppression of this ratio with respect to unity.

USE OF NUCLEAR EFFECTS

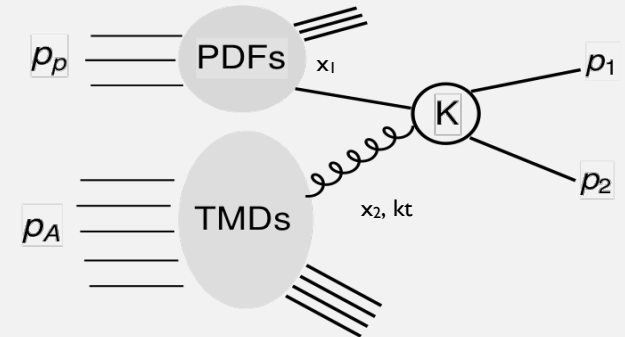


When we take a look at the nuclear modification factor with respect to the angle between the two jets, we can see the non-linear effects as a suppression at about $\sim 180^\circ$ when $p_{1t} \sim p_{2t}$.

$$R_{pPb} = \frac{\frac{d\sigma^{p+Pb}}{d\mathcal{O}}}{A \frac{d\sigma^{p+p}}{d\mathcal{O}}}$$

HOW DO WE PREDICT THESE
CROSS SECTIONS?

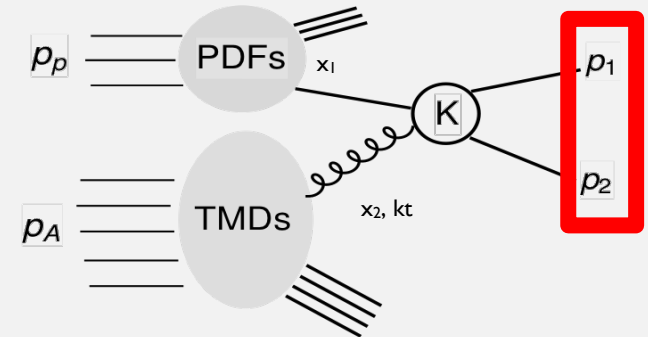
CROSS SECTION CALCULATION



The cross section is calculated as:

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

CROSS SECTION CALCULATION

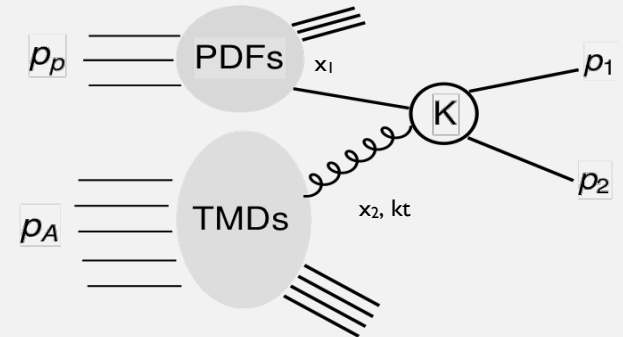


$$d^2 P_t d^2 k_t dy_1 dy_2 = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

- P_t - Single jet transverse momentum, k_t - jet pair transverse momentum,
 y_1, y_2 - jet rapidities $\in [3.5, 4.5]$, p_{t1}, p_{t2} - jet momenta $\in [20, 250]$ GeV/c

$$k_t = \sqrt{p_{t1}^2 + p_{t2}^2 + 2p_{t1}p_{t2}\cos(\Delta\varphi)} \quad \vec{P}_t = \frac{p_2^+ \vec{p}_1 - p_1^+ \vec{p}_2}{p_1^+ + p_2^+} \quad p_i^+ = p_{ti} e^{y_i} / \sqrt{2}$$

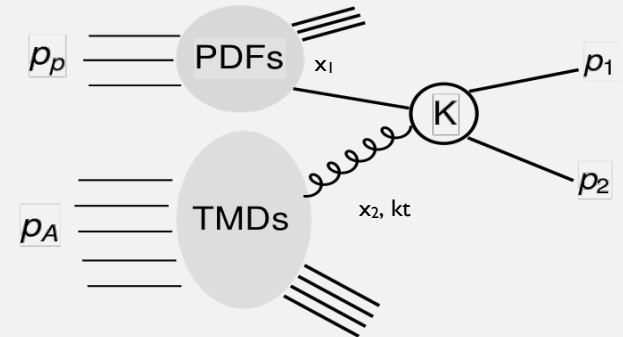
CROSS SECTION CALCULATION



$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

- Running coupling – cancels in the ratio, so we fix it in our computation.

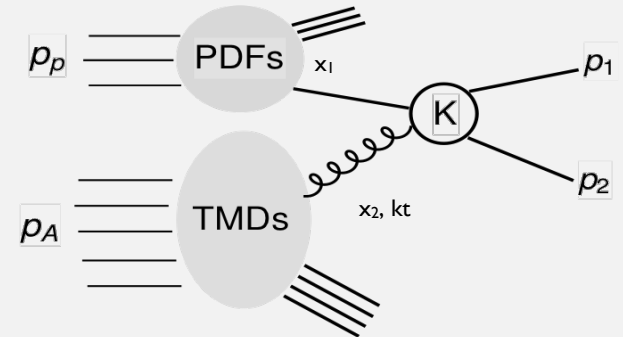
CROSS SECTION CALCULATION



$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{x_1 x_2 s^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

- x_1 corresponds to the projectile particle, x_2 to the target particle and s is the energy of the collision.

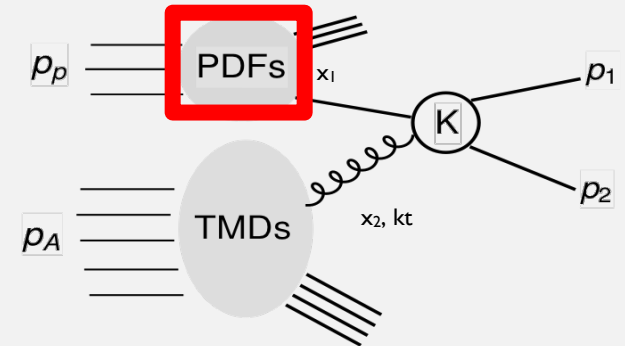
CROSS SECTION CALCULATION



$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

- The factor of $1/2$ is included in processes with indistinguishable outgoing particles.

CROSS SECTION CALCULATION



I) Projectile gluon distribution

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

Is obtained from data from previous experiments.

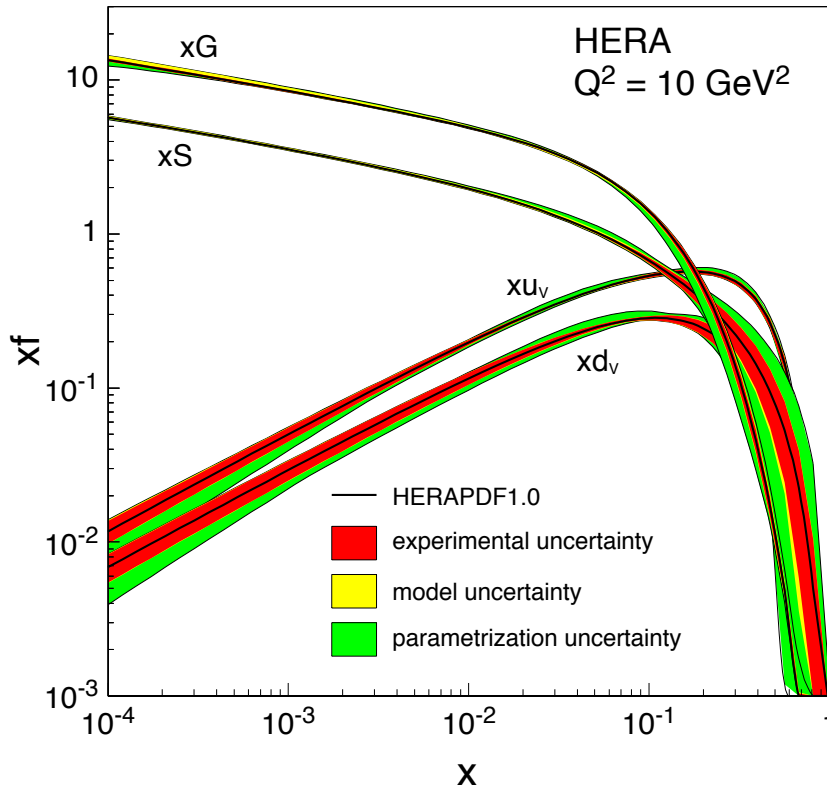
CROSS SECTION CALCULATION



Projectile gluon distrib

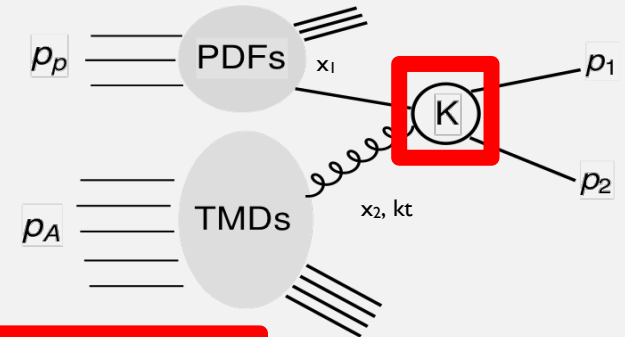
$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha}{(x_1 x_2)}$$

Is obtained from data from



$k_t)$

CROSS SECTION CALCULATION



2) Matrix elements

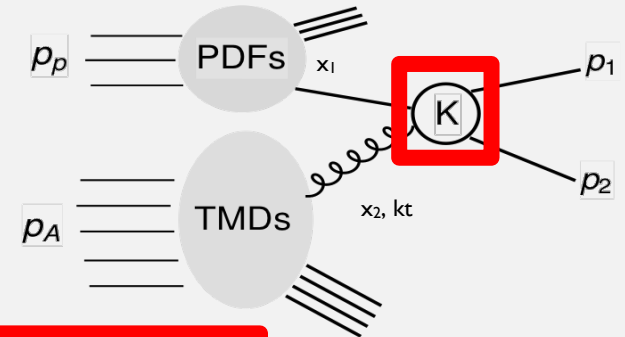
$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 \boxed{K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t)} \delta_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

2-to-2 matrix elements with non-zero gluon k_t .

CROSS SECTION CALCULATION



Matrix elements



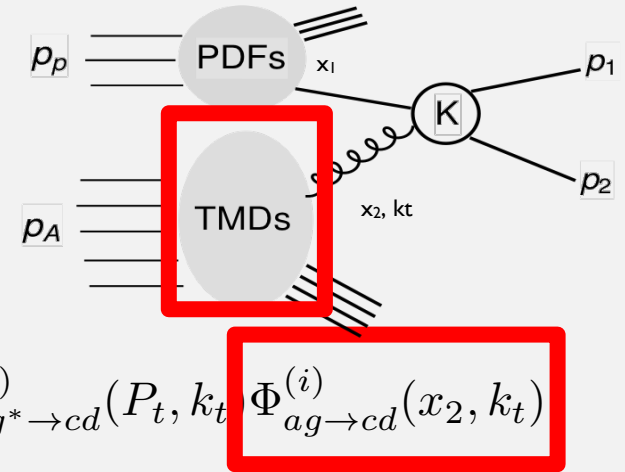
$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 \boxed{K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t)} \tilde{\Phi}_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

Matrix elements have been computed in:

F. Dominguez, C. Marquet, B. -W. Xiao and F. Yuan, Phys. Rev. D 83 (2011) 105005 – with zero gluon k_t .

A. van Hameren, P. Kotko, K. Kutak, C. Marquet, E. Petreska and S. Sapeta, arXiv:1607.03121 – with non-zero gluon k_t .

CROSS SECTION CALCULATION



3) Transverse momentum distributions

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

Five TMDs are needed to describe this process. These are now process dependent and not universal, unlike the usual PDFs.

For the computation, we need to undergo several substeps.

CROSS SECTION CALCULATION

a) Scattering amplitude

- The scattering amplitude corresponds to the cross section of the interaction of a color dipole with a nucleus. It is a solution of the Balitsky-Kovchegov equation.
- In our approach, we focus on rcBK with factorized b-dependence. (The integral over impact parameter is represented with a multiplicative factor.)

$$N_F(x, \mathbf{r}) = 1 - S_F(x, \mathbf{r})$$

CROSS SECTION CALCULATION

a) Scattering amplitude

$$\frac{\partial N(r, Y)}{\partial \ln Y} = \int d\vec{r}_1 K(\vec{r}, \vec{r}_1, \vec{r}_2) (N(\vec{r}_1, Y) + N(\vec{r}_2, Y) - N(\vec{r}, Y) - N(\vec{r}_1, Y)N(\vec{r}_2, Y))$$

with kernel $K(\vec{r}, \vec{r}_1, \vec{r}_2) = \frac{\alpha_s(r^2)N_c}{2\pi} \left(\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right)$ where $\vec{r}_2 = \vec{r} - \vec{r}_1$

The initial condition at $x_0 = 0.01$ is $N^{MV}(r) = 1 - \exp\left(-\frac{(r^2 Q_{s0}^2)}{4} \ln\left(\frac{1}{r^2 \Lambda_{QCD}^2} + e\right)\right)$

We used $\Lambda_{QCD} = 0.241 \text{ GeV}$, $Q_{s0}^2 = 0.6 \text{ GeV}^2$ for lead and $Q_{s0}^2 = 0.2 \text{ GeV}^2$ for protons.

CROSS SECTION CALCULATION



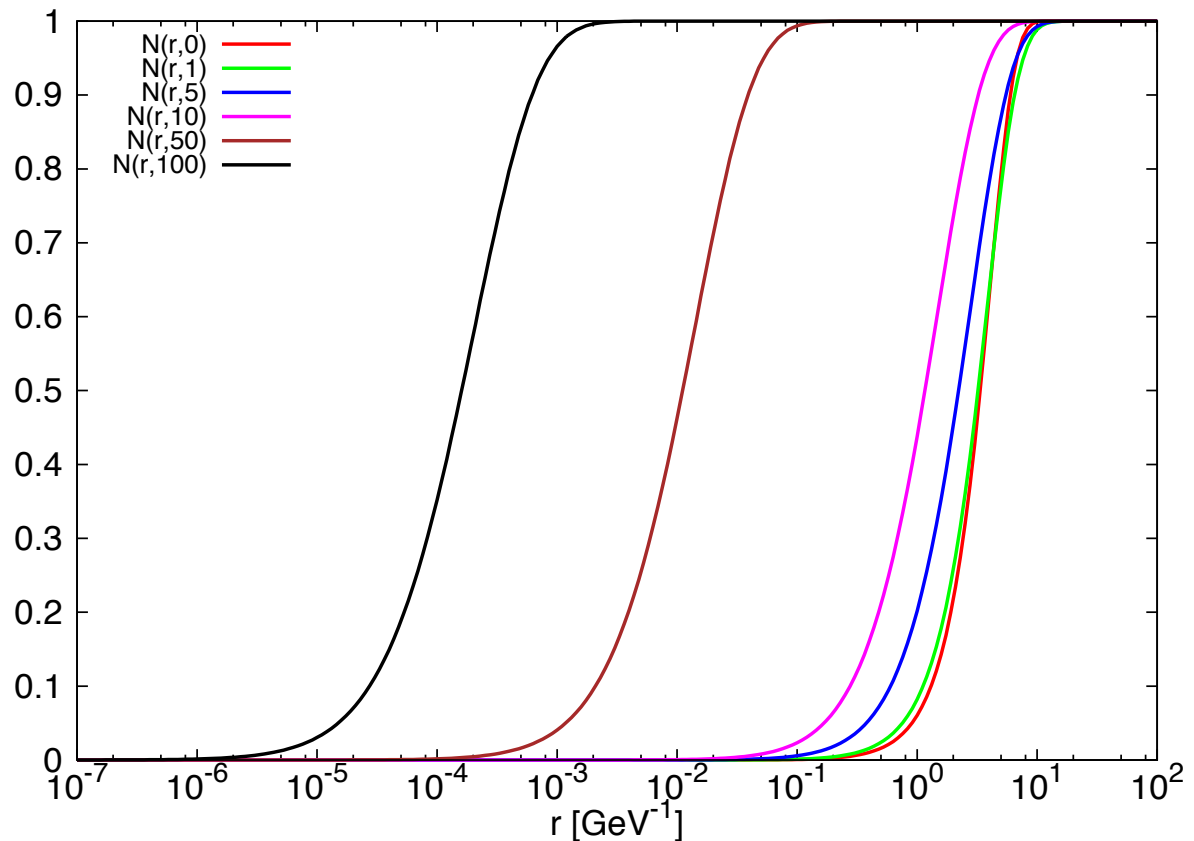
Scattering amplitude

$$\frac{\partial N(r, Y)}{\partial \ln Y} = \int d\vec{r}_1 K$$

with kernel $K(\vec{r}, \vec{r}_1, \vec{r}_2) = \frac{\alpha_s}{N(r, Y)}$

Initial condition at $x_0 = 0.0$

We used $\Lambda_{QCD} = 0.241 \text{ GeV}$



Y. V. Kovchegov, Phys. Rev. D 60, 034008 (1999)
J. L. Albacete et al, Eur.Phys.J. C71 (2011) 1705

CROSS SECTION CALCULATION

b) Fourier transform

- We transform the scattering amplitude from the coordinate space to the momentum space

with the Fourier transform:

$$F(x_2, k_t) = \int \frac{d^2 \mathbf{r}}{(2\pi)^2} e^{-ik_t \cdot \mathbf{r}} S_F(x_2, \mathbf{r})$$

c) Compute the dipole gluon distribution and Weizsacker-Williams gluon distribution.

$$x_2 G^{(2)}(x_2, k_t) = \frac{N_c k_t^2 S_\perp}{2\pi^2 \alpha_s} F(x_2, k_t) \qquad S_A(x, \mathbf{r}) = [S_F(x, \mathbf{r})]^2$$

$$x_2 G^{(1)}(x_2, k_t) = \frac{C_F}{2\alpha_s \pi^4} \int d^2 b \int \frac{d^2 \mathbf{r}}{\mathbf{r}^2} e^{-ik_t \cdot \mathbf{r}} [1 - S_A(x_2, \mathbf{r})]$$

CROSS SECTION CALCULATION

Fourier tra

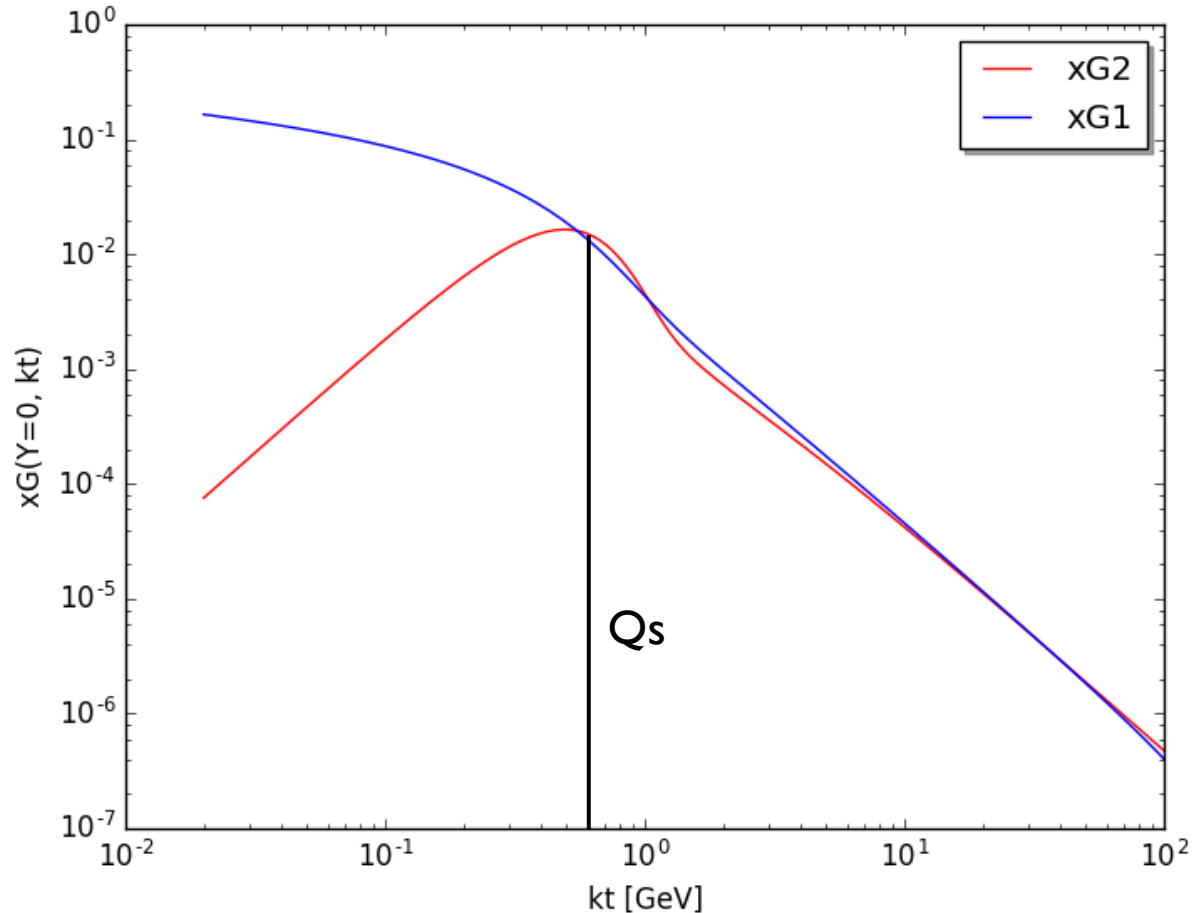
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Compute t

$$x_2 G^{(2)}(x_2$$

$$x_2 G^{(1)}(x_2$$



space

on.

$$c)]^2$$

CROSS SECTION CALCULATION

- d) Compute the transverse momentum distributions as a convolution of the Fourier transform and the gluon distributions.

$$\mathcal{F}_{qg}^{(1)}(x_2, k_t) = x_2 G^{(2)}(x_2, q_t),$$

$$\mathcal{F}_{qg}^{(2)}(x_2, k_t) = \int d^2 q_t x_2 G^{(1)}(x_2, q_t) F(x_2, k_t - q_t),$$

$$\mathcal{F}_{gg}^{(1)}(x_2, k_t) = \int d^2 q_t x_2 G^{(2)}(x_2, q_t) F(x_2, k_t - q_t),$$

$$\mathcal{F}_{gg}^{(2)}(x_2, k_t) = - \int d^2 q_t \frac{(k_t - q_t) \cdot q_t}{q_t^2} x_2 G^{(2)}(x_2, q_t) F(x_2, k_t - q_t),$$

$$\mathcal{F}_{gg}^{(6)}(x_2, k_t) = \int d^2 q_t d^2 q'_t x_2 G^{(1)}(x_2, q_t) F(x_2, q'_t) F(x_2, k_t - q_t - q'_t)$$

CROSS SECTION CALCULATION



Compute t

transform a

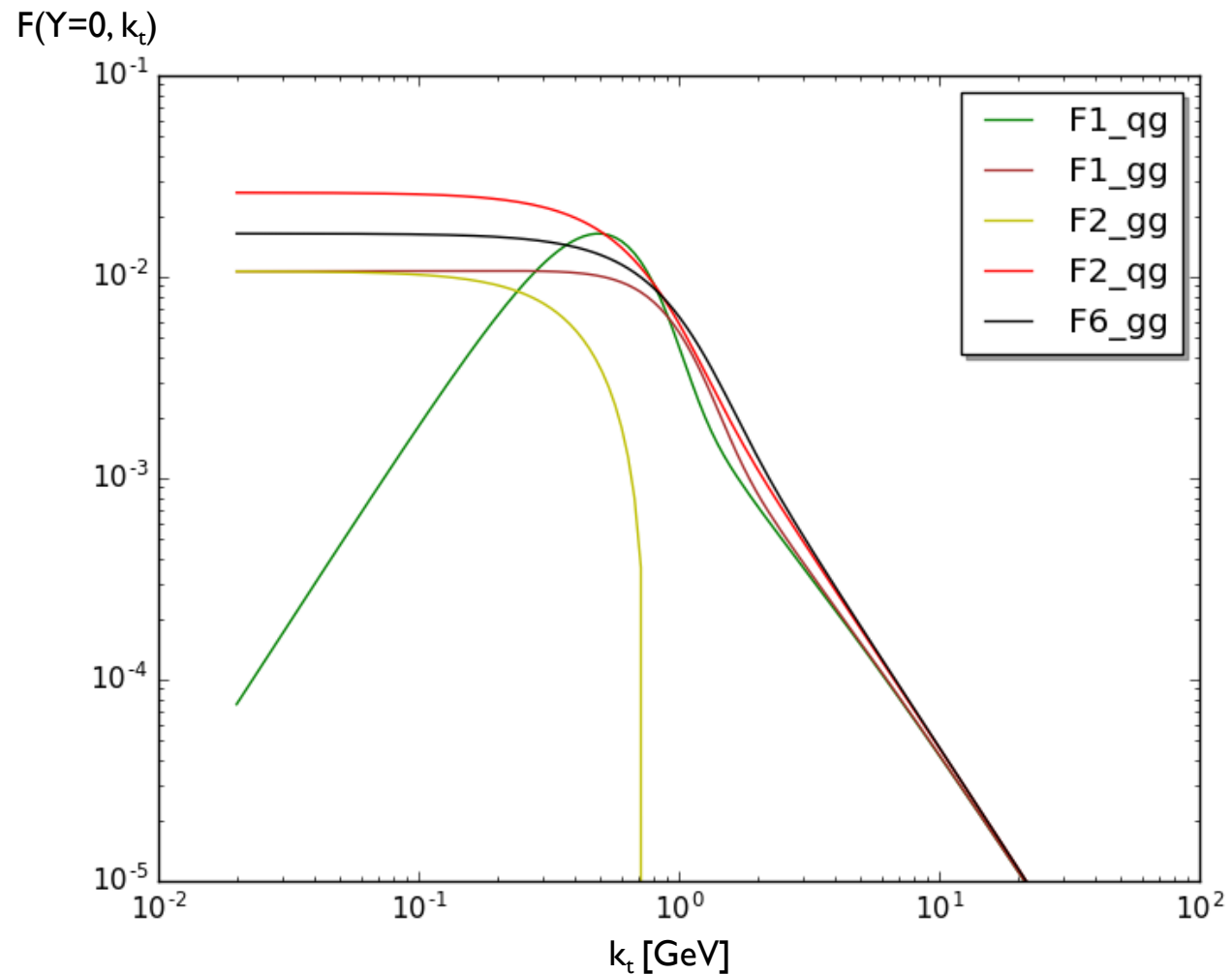
$$\mathcal{F}_{qg}^{(1)}(x_2, k_t)$$

$$\mathcal{F}_{qg}^{(2)}(x_2, k_t)$$

$$\mathcal{F}_{gg}^{(1)}(x_2, k_t)$$

$$\mathcal{F}_{gg}^{(2)}(x_2, k_t)$$

$$\mathcal{F}_{gg}^{(6)}(x_2, k_t)$$



er

CROSS SECTION CALCULATION

e) Redefine the transverse momentum distributions in the large N_c limit as:

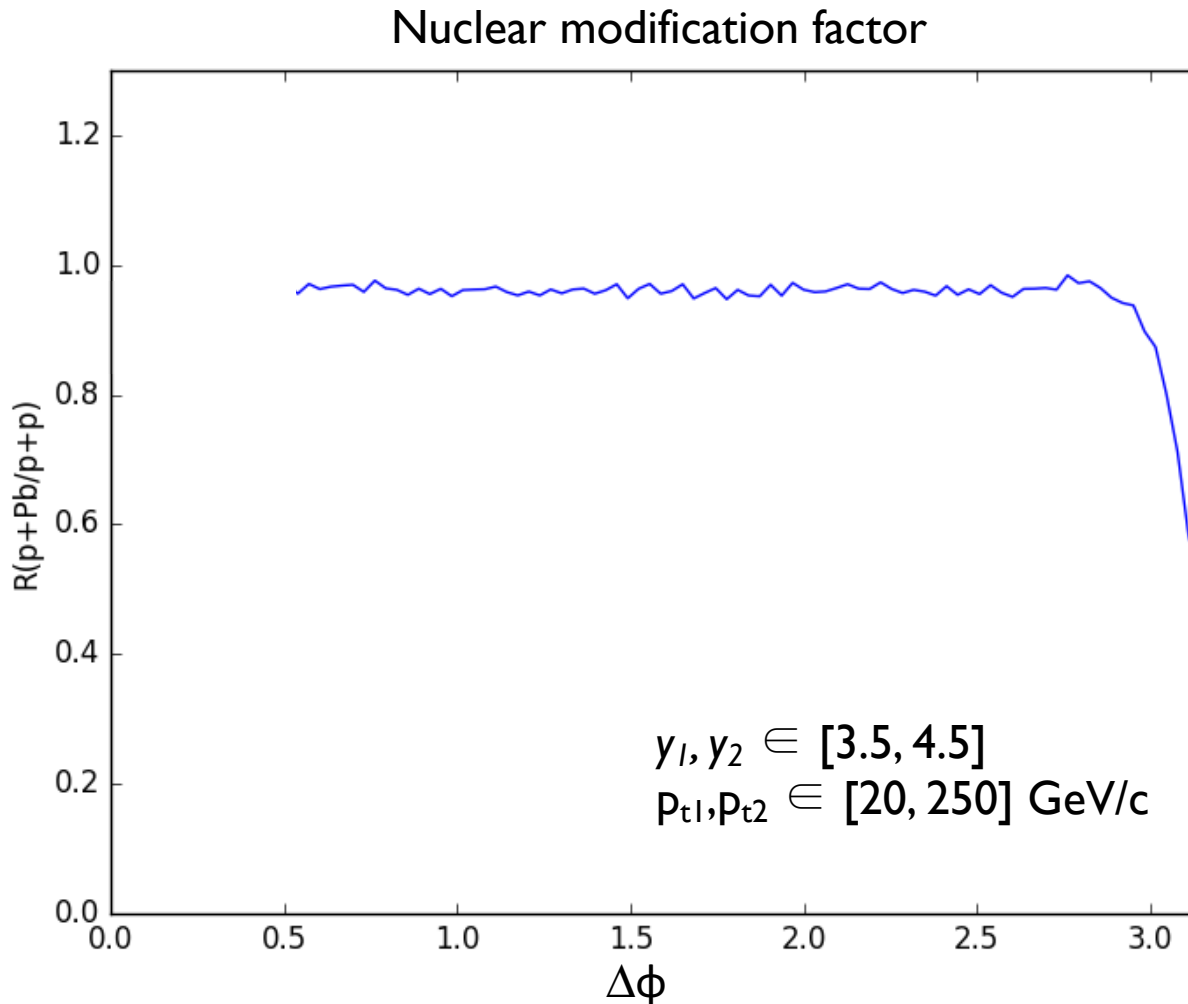
$$\begin{aligned}
 \Phi_{qg \rightarrow qg}^{(1)} &= \mathcal{F}_{qg}^{(1)} & , & & \Phi_{qg \rightarrow qg}^{(2)} &\approx \mathcal{F}_{qg}^{(2)} \\
 \Phi_{gg \rightarrow q\bar{q}}^{(1)} &\approx \mathcal{F}_{gg}^{(1)} & , & & \Phi_{gg \rightarrow q\bar{q}}^{(2)} &\approx -N_c^2 \mathcal{F}_{gg}^{(2)} \\
 \Phi_{gg \rightarrow gg}^{(1)} &\approx \frac{1}{2} \left(\mathcal{F}_{gg}^{(1)} + \mathcal{F}_{gg}^{(6)} \right) & , & & \Phi_{gg \rightarrow gg}^{(2)} &\approx \mathcal{F}_{gg}^{(2)} + \mathcal{F}_{gg}^{(6)}
 \end{aligned}$$

CROSS SECTION CALCULATION

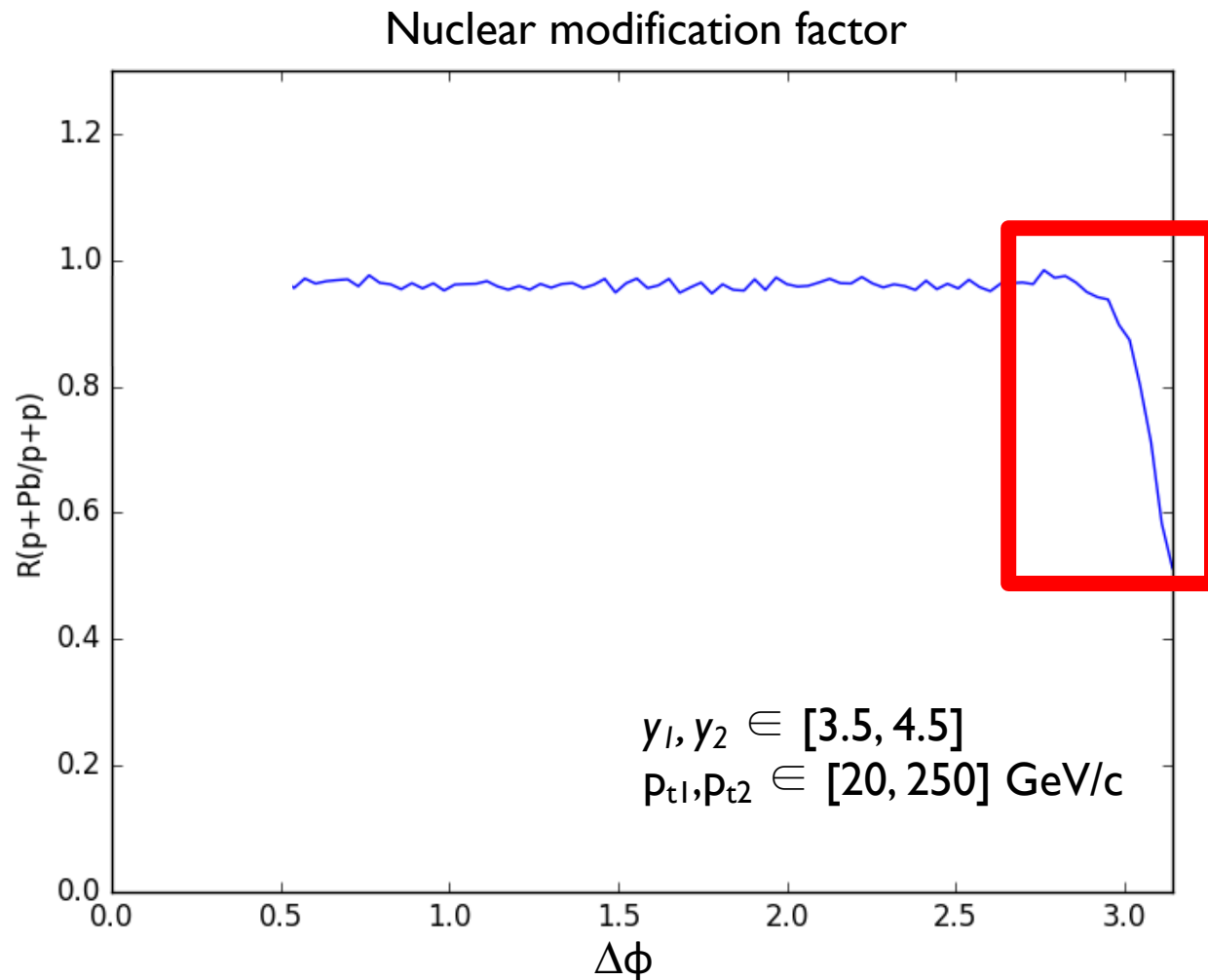
$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1 + \delta_{cd}} \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)}(P_t, k_t) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t)$$

Now we can compute the cross section and from that the nuclear modification factor.

NUCLEAR MODIFICATION FACTOR



NUCLEAR MODIFICATION FACTOR



CONCLUSIONS

- The existence of saturation effects can be studied with the use of the nuclear modification factor in p-Pb collisions.
- Observation of the saturation effects is caused by the fact that the transverse momentum of the outgoing back-to-back jet pair is similar to the nuclear saturation scale.
- These studies are all impact parameter independent. Future incorporation of non-trivial impact parameter dependence in the scattering amplitude computation is highly desired, because it can have a major influence on the studied phenomena.
- Measurements at such forward rapidity would be desired. Is it feasible at the LHC?

THANK YOU FOR YOUR ATTENTION

MATRIX ELEMENTS

i	1	2
$K_{gg^* \rightarrow gg}^{(i)}$	$2 \frac{(\bar{s}^4 + \bar{t}^4 + \bar{u}^4) (\bar{u}\hat{u} + \bar{t}\hat{t})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$	$-\frac{(\bar{s}^4 + \bar{t}^4 + \bar{u}^4) (\bar{u}\hat{u} + \bar{t}\hat{t} - \bar{s}\hat{s})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$
$K_{gg^* \rightarrow q\bar{q}}^{(i)}$	$\frac{1}{2N_c} \frac{(\bar{t}^2 + \bar{u}^2) (\bar{u}\hat{u} + \bar{t}\hat{t})}{\bar{s}\hat{s}\bar{t}\hat{t}\hat{u}}$	$\frac{1}{2N_c^3} \frac{(\bar{t}^2 + \bar{u}^2) (\bar{u}\hat{u} + \bar{t}\hat{t} - \bar{s}\hat{s})}{\bar{s}\hat{s}\bar{t}\hat{t}\hat{u}}$
$K_{qg^* \rightarrow qg}^{(i)}$	$-\frac{\bar{u} (\bar{s}^2 + \bar{u}^2)}{2\bar{t}\hat{t}\hat{s}}$	$-\frac{\bar{s} (\bar{s}^2 + \bar{u}^2)}{2\bar{t}\hat{t}\hat{u}}$