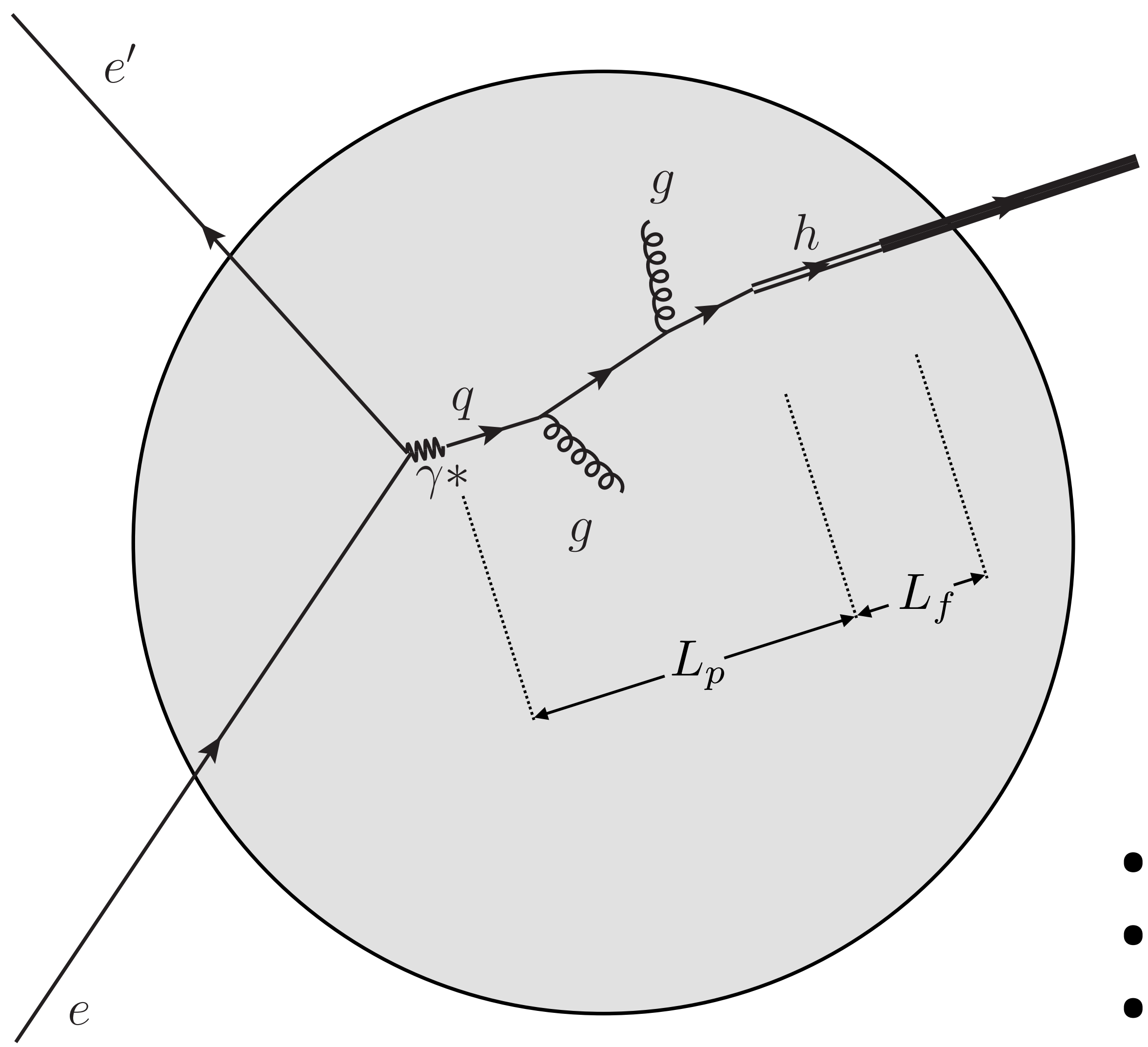
# A Space–time Analysis of Semi–inclusive Deep Inelastic Scattering on Nuclei

William Brooks, Jorge López
Universidad Técnica Federico Santa María, Valparaíso, Chile

**DIS 2018 – Kobe, Japan**
**April 2018**

# Space-time perspective of DIS on nuclei



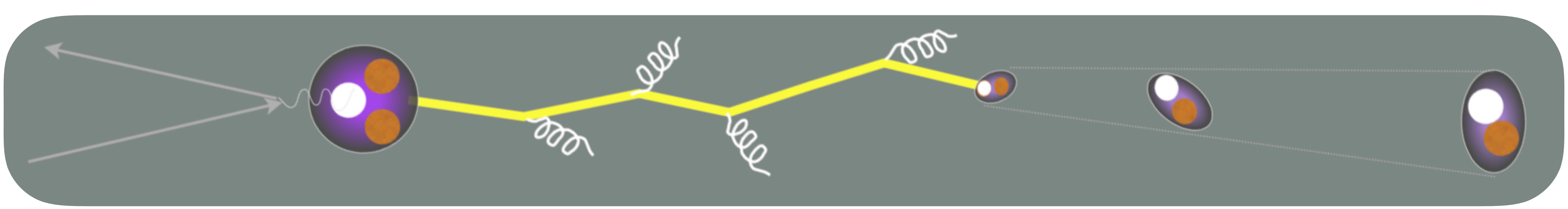

**Production length ($L_p$):** distance required for a colored system to evolve into a color singlet system.

***"color lifetime"***

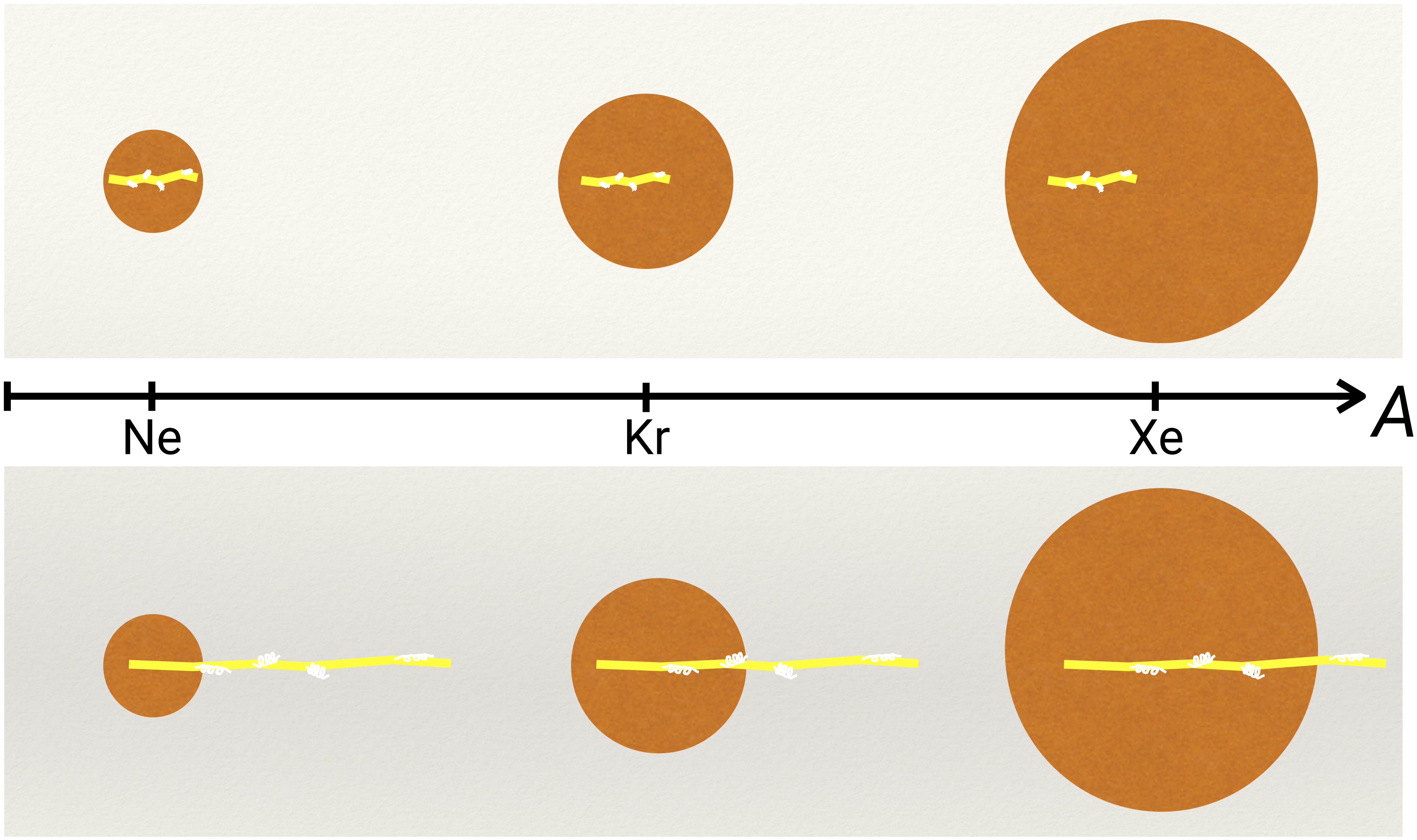
**Formation length ($L_f$):** distance required for a *pre-h*adron to get fully formed.

**FUNDAMENTAL QCD PROCESSES:**

- Partonic elastic scattering in medium
- Gluon bremsstrahlung in vacuum and in medium
- Color neutralization
- Hadron formation



# Methodology




By comparing **$p_T$ broadening** and **hadron attenuation** in nuclei of different sizes, one can measure the length of the process of color propagation at the femtometer scale

# HERMES data – Observables

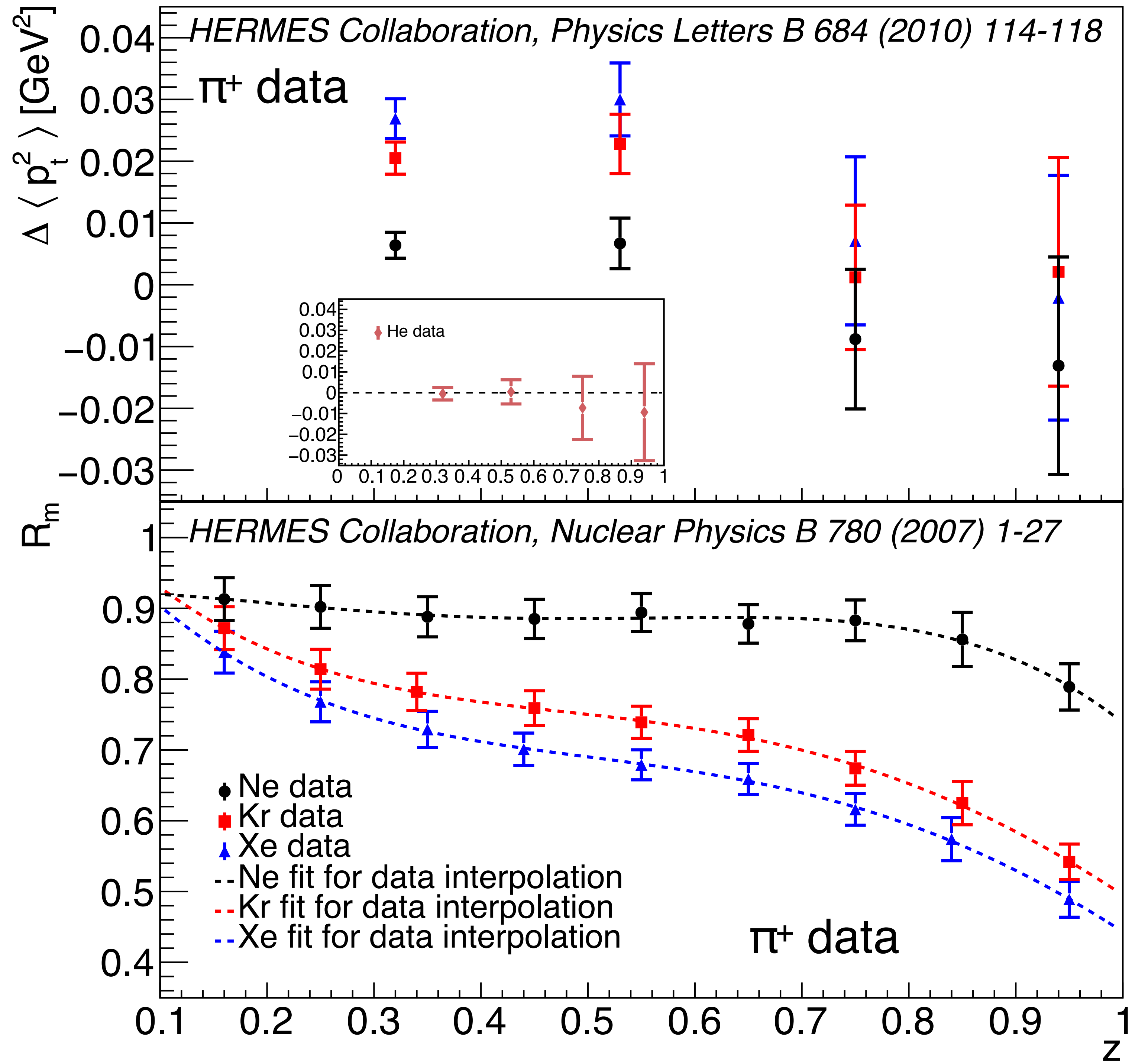
**Transverse momentum broadening**

$$\Delta\langle p_{\mathrm{T}}^2\rangle = \langle p_{\mathrm{T}}^2\rangle_A - \langle p_{\mathrm{T}}^2\rangle_p$$

**Multiplicity ratio or hadron attenuation**

$$R_{\mathrm{M}} = \frac{N_h(Q^2,\nu,z,p_{\mathrm{T}})/N_e(Q^2,\nu)|_A}{N_h(Q^2,\nu,z,p_{\mathrm{T}})/N_e(Q^2,\nu)|_p}$$

**Simultaneous description of both observables**



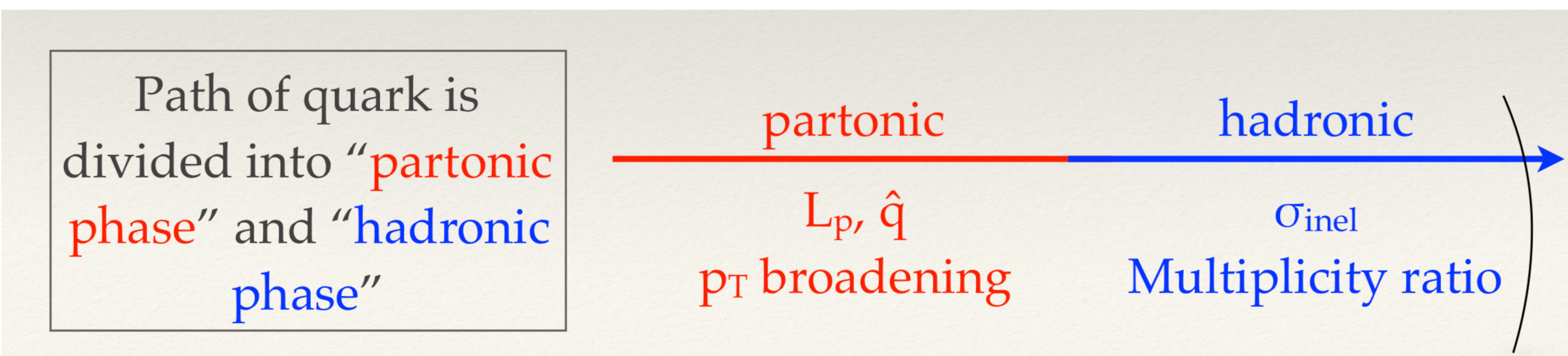

http://www-hermes.desy.de/notes/pub/publications.html

# Geometric model

- Propagating quark causes $p_T$ broadening of final hadron
- Propagating pre-hadron "disappears" when it undergoes an inelastic interaction with cross section σ.
- Implemented as Monte Carlo calculation.

$$\Delta\langle p_T^2\rangle = \left\langle q_0 \int_{z_0}^{z_0+L_p} \rho(x_0, y_0, z)\mathrm{d}z \right\rangle_{\mathrm{MC}}$$

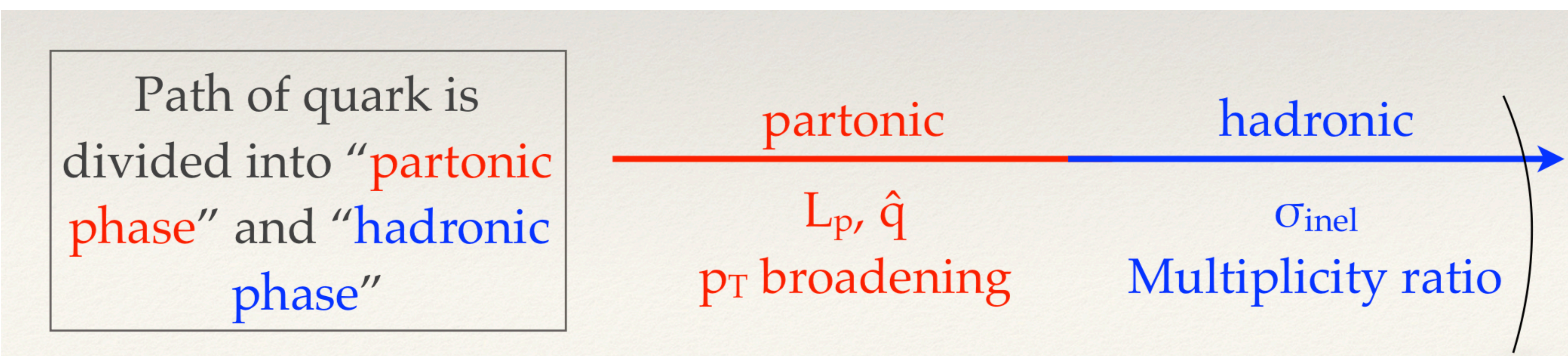
$$R_{\mathrm{M}} = \left\langle \exp\left( -\sigma_{\mathrm{hn}} \int_{z_0+L_p}^{r_A} \rho(x_0, y_0, z)\mathrm{d}z \right) \right\rangle_{\mathrm{MC}}$$



# Geometric model

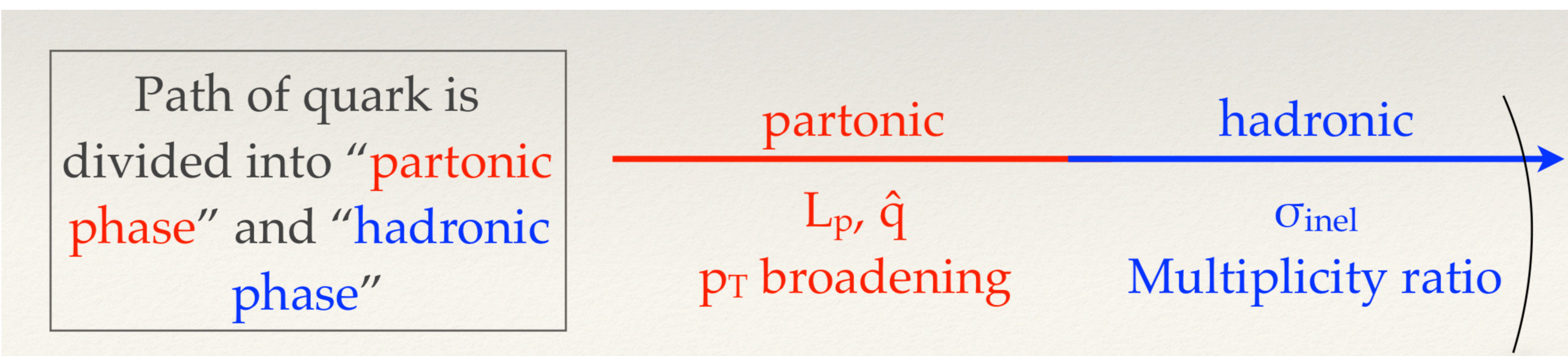
- Baseline model implemented with 3 parameters:
  - $q_0$: sets the scale of $p_T$ broadening
  - Production length $L_p$: distance over which $p_T$ broadening and energy loss occur. Assumed exponential form.
  - Cross section for pre-hadron to interact with nucleus.
- Interaction point $(x_0, y_0, z_0)$ thrown uniformly in sphere, weighted by $\rho(x_0, y_0, z_0)$.
- **No dynamical information is assumed; it emerges from fit.**

$$\chi^2 = \left(\frac{\text{data} - \text{model}}{\text{uncertainties}}\right)^2_{p_T\text{-broadening}} + \left(\frac{\text{data} - \text{model}}{\text{uncertainties}}\right)^2_{\text{multiplicity}}$$

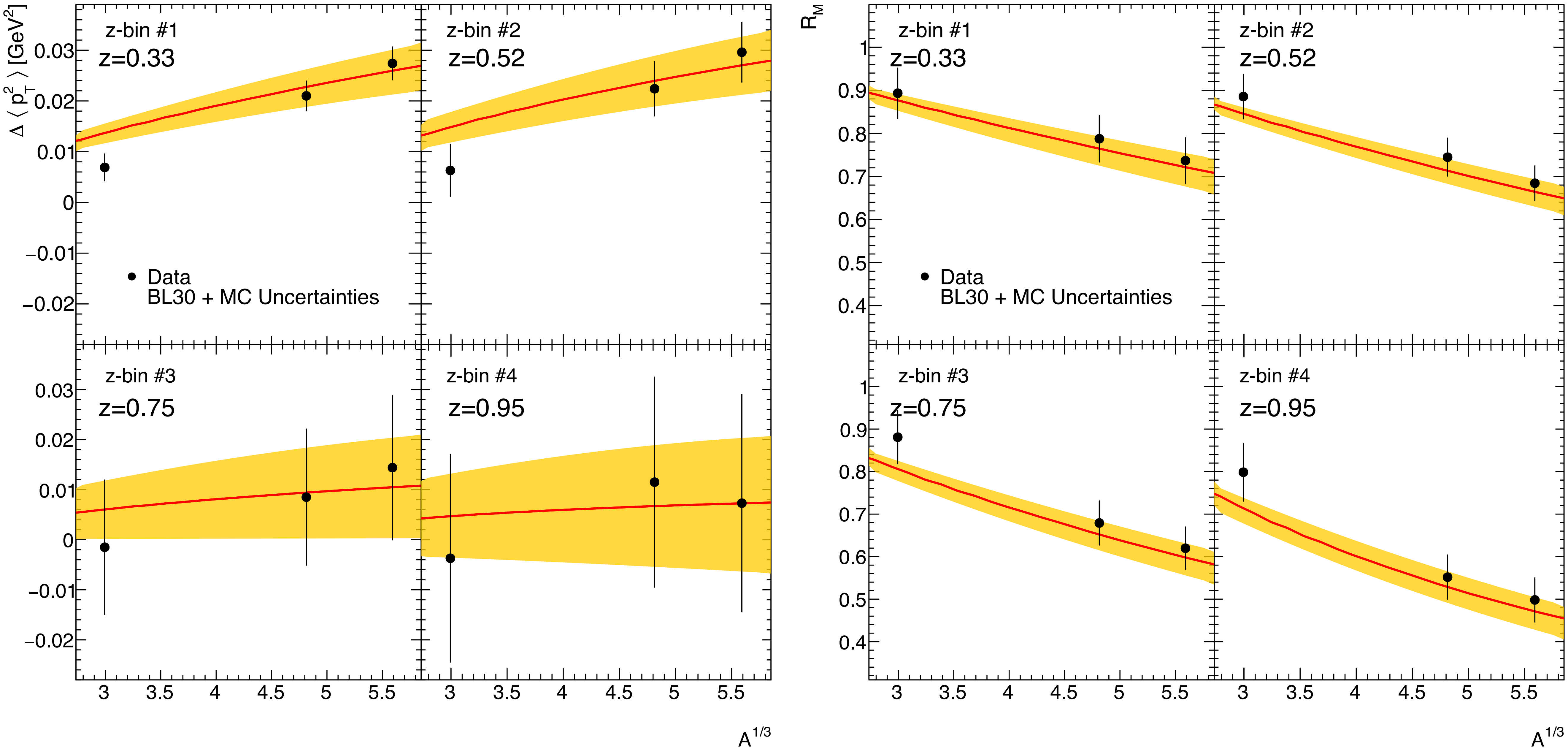


# Model variants

| Variant | Number of free parameters | |
|---|---|---|
| BLE | 4 | Free parameters are: $q_0$, $L_p$, $\sigma$, and energy loss |
| BL30 | 2 | Fixed cross-section @ 30 mb, no energy loss |
| BLE30 | 3 | Fixed cross-section @ 30 mb |

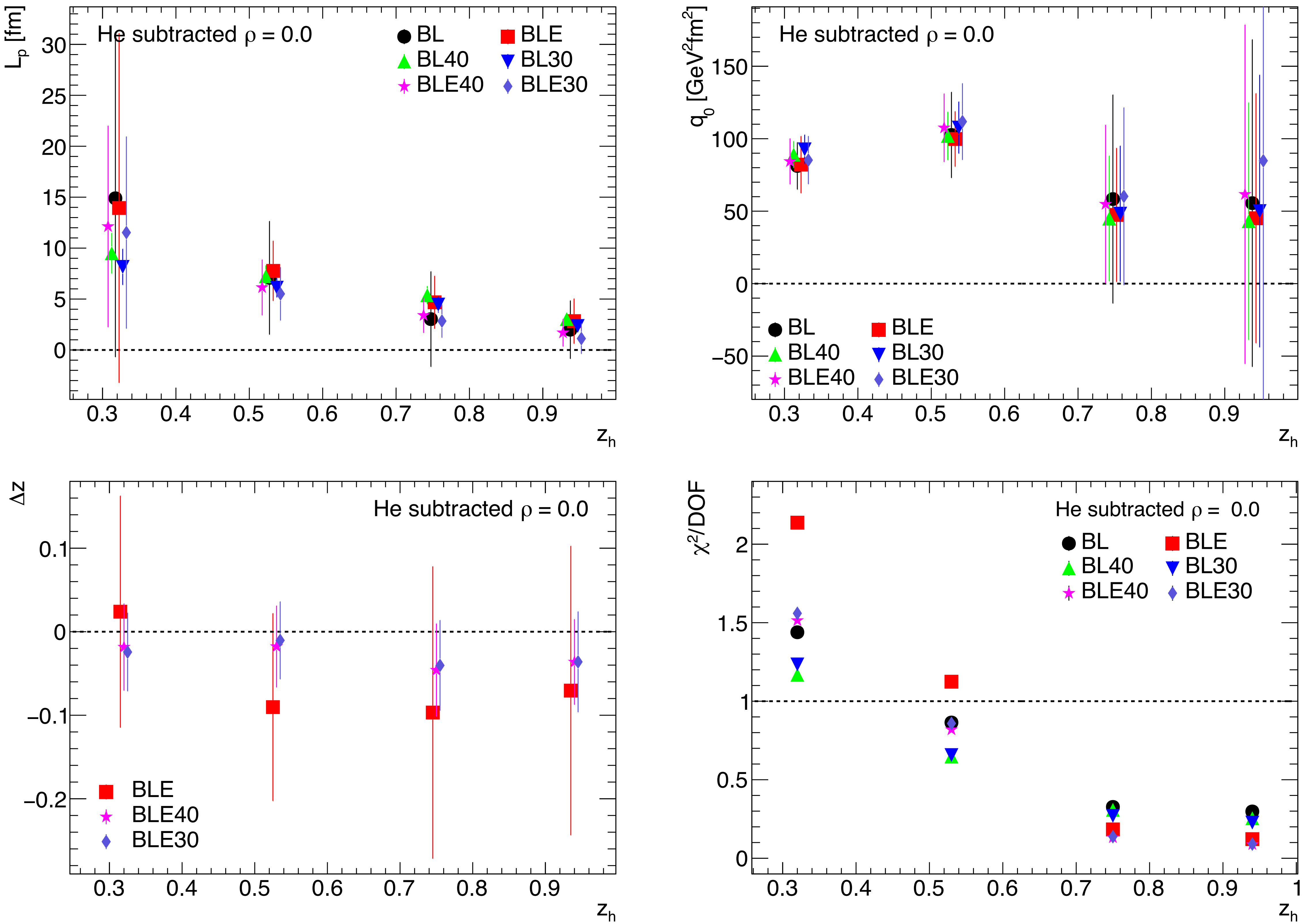


# Results: model vs nucleus size ~ $A^{1/3}$




Result for model variant BL30, baseline model at fixed pre-hadron cross-section 30 mb.

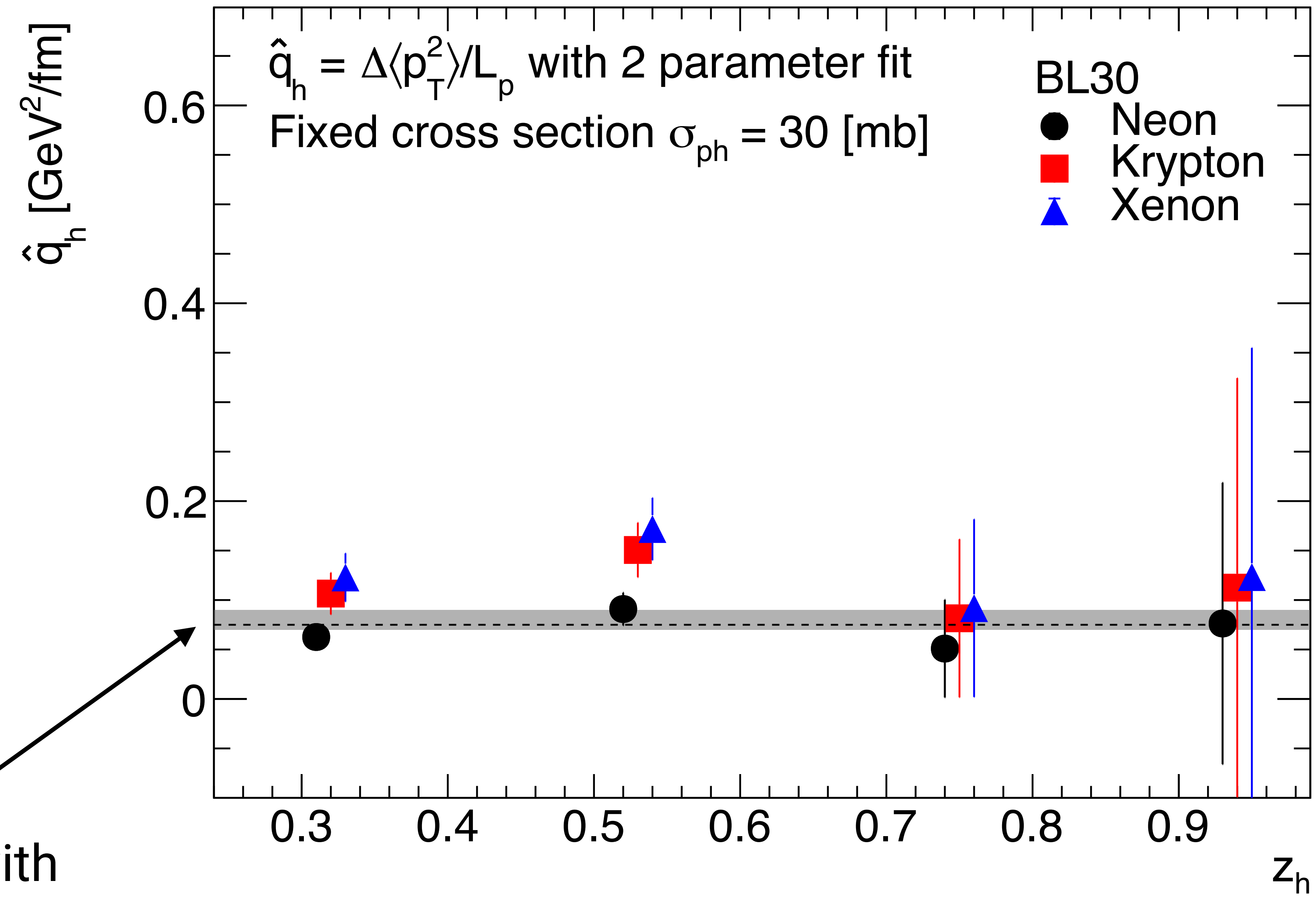
# Results: model parameters vs $z_h$



# Transport coefficient

In a simple picture:

$$\hat{q} = \frac{\Delta\langle p_{\mathrm{T}}^2\rangle}{L_{\mathrm{p}}}$$




Compatible with calculations for p+A physics at the LHC:

$$\hat{q} = 0.075^{+0.015}_{-0.005}\ \mathrm{GeV}^2/\mathrm{fm}$$

https://arxiv.org/pdf/1212.0434.pdf
https://arxiv.org/pdf/1707.09973.pdf

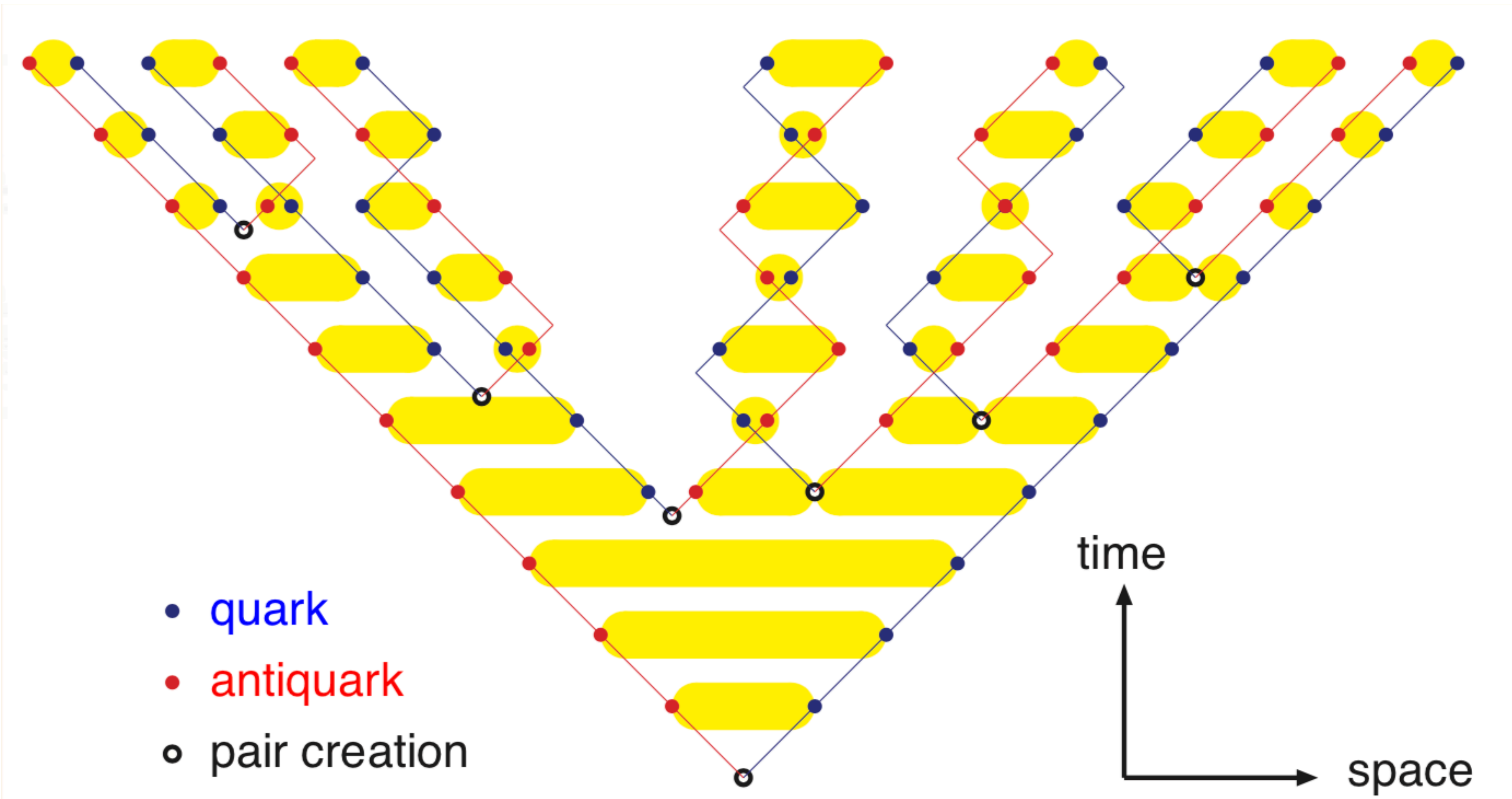
Average over $z_h$:

$$\hat{q}(\mathrm{Ne}) = 0.072 \pm 0.006\ \mathrm{GeV}^2/\mathrm{fm}$$

$$\hat{q}(\mathrm{Kr}) = 0.120 \pm 0.010\ \mathrm{GeV}^2/\mathrm{fm}$$

$$\hat{q}(\mathrm{Xe}) = 0.137 \pm 0.011\ \mathrm{GeV}^2/\mathrm{fm}$$

# Lund String Model




Remarkably successful model, foundational tool in HEP

- Alternative physical picture to pQCD: emission of many gluons in vacuum, string as an average; quantitative
- Successful, but few connections to fundamental QCD
- We can *compare* some of our results to the Lund String Model, and other results to pQCD

# Production length $L_p$

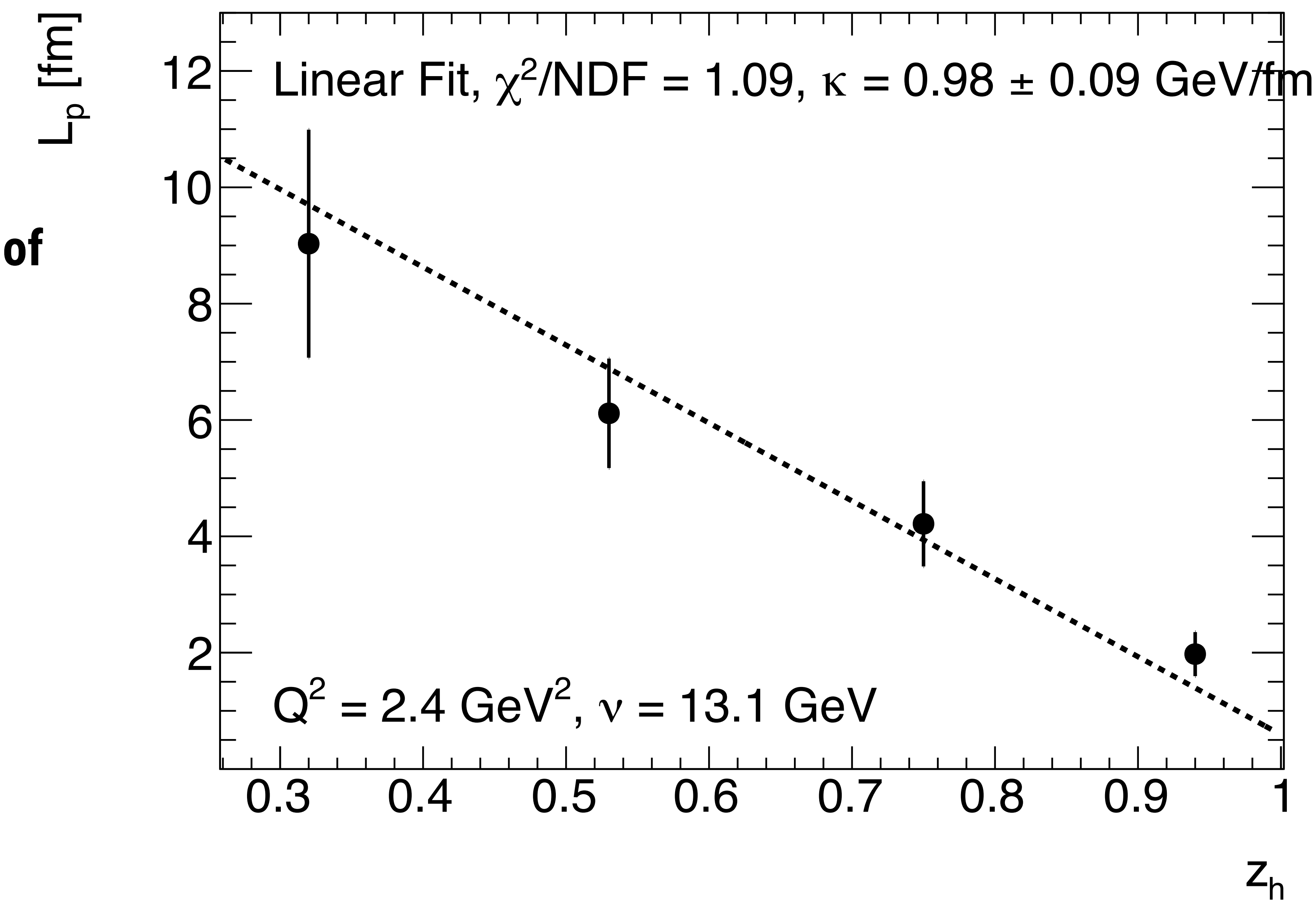
**From the Lund String Model, for the struck quark (in a simple approximation):**

$$2\kappa L_p = M_p + \nu(1 + \sqrt{1 + Q^2/\nu^2}) - 2\nu z$$

**Predicted value**
**κ ~ 1 GeV/fm**

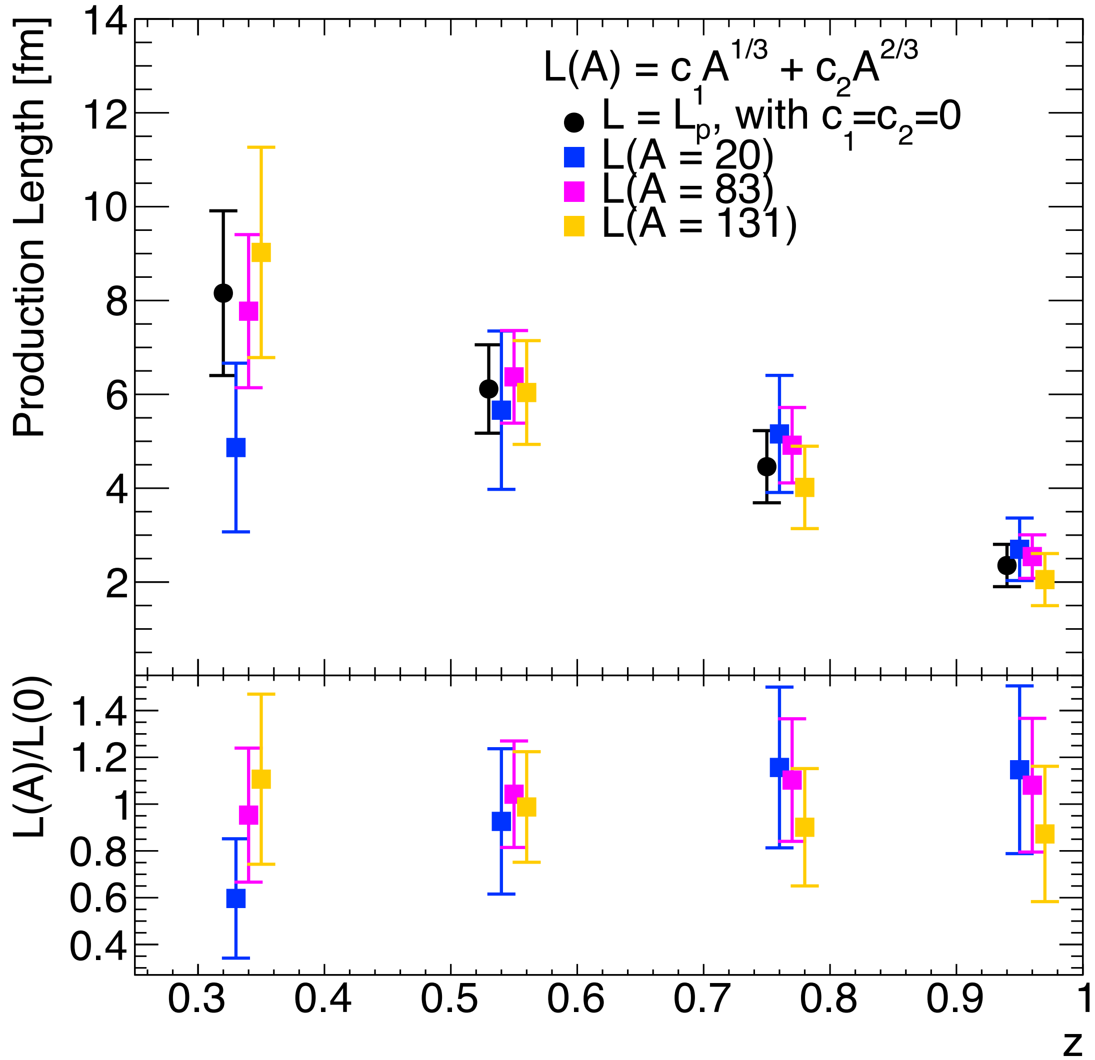
**We recover the known value of the string constant!**

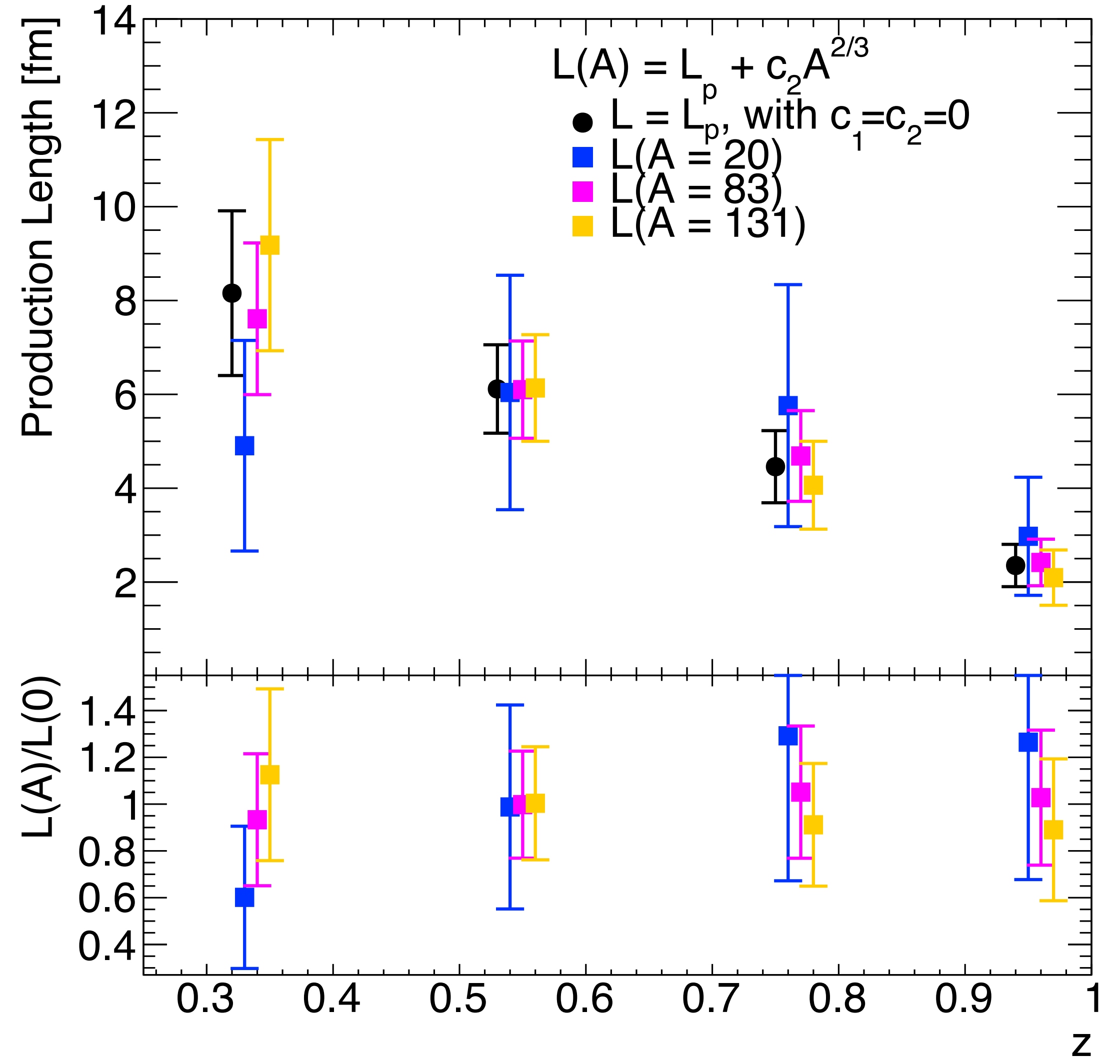
**Strong validation of our model**



# Nuclear size dependent $L_p$

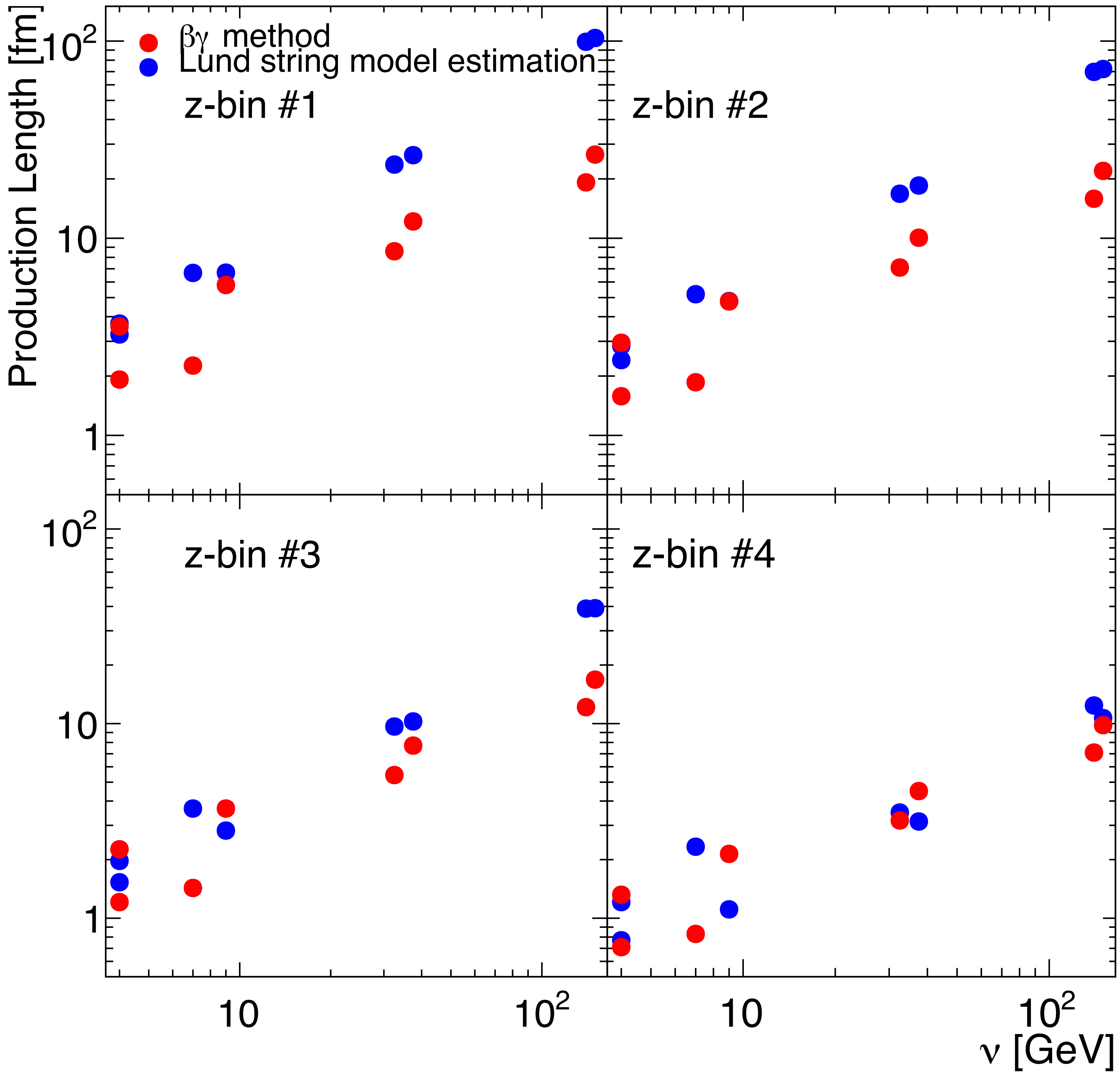
Other parametrization for the production length can be assumed: $\boldsymbol{L_p = L_0 + c_1 A^{1/3} + c_2 A^{2/3}}$




No nuclear size dependence for $L_p$ is observed. Future data could provide more insights into this.

# Estimations for future experiments




Space-time analysis and Lund string model provide close estimates for $L_p$ for future experiments with assumptions of $Q^2$, $\nu$.

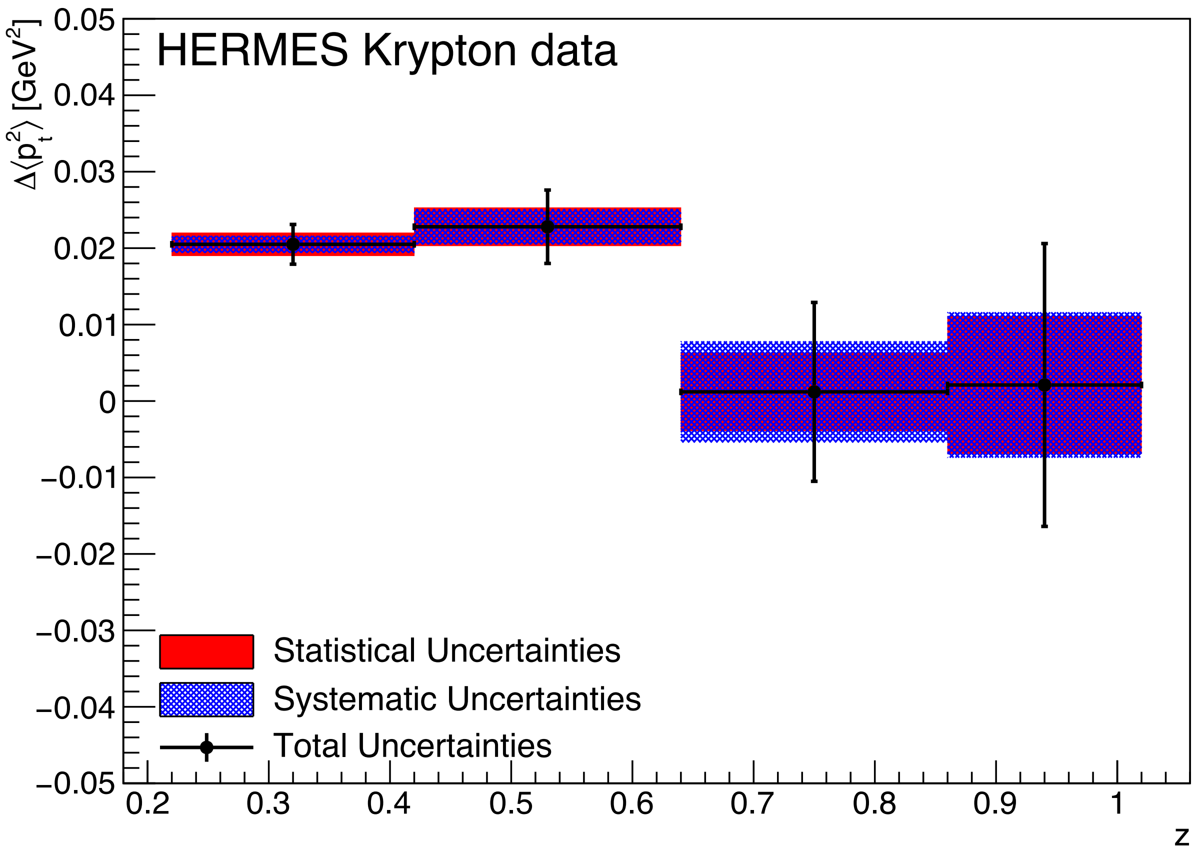
# Summary

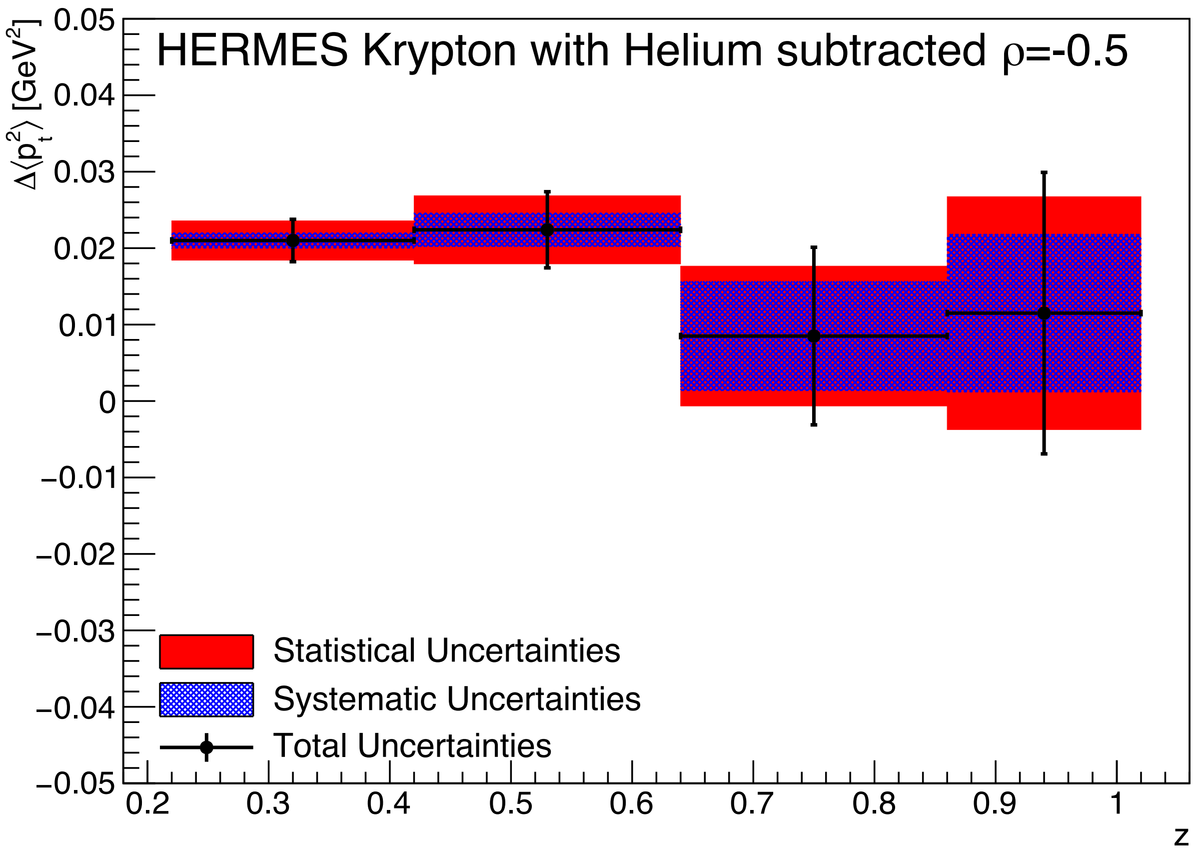
- We extract the characteristic production time of $\pi^+$ HERMES data using a geometric model.
- The model describes transverse momentum broadening and multiplicity ratios simultaneously.
- No dynamical information is assumed; it emerges from fit.
- Transport coefficient is compatible with some theoretical predictions.
- We recover the known value of the string constant completely independently, strong support of our model.
- Our approach estimates production length for future experiments using simple kinematical assumptions.
- This is the first measurement of the color lifetime.

**Research funded by CONICYT, FONDECYT ANILLO ACT1413 & DGIIP-UTFSM**

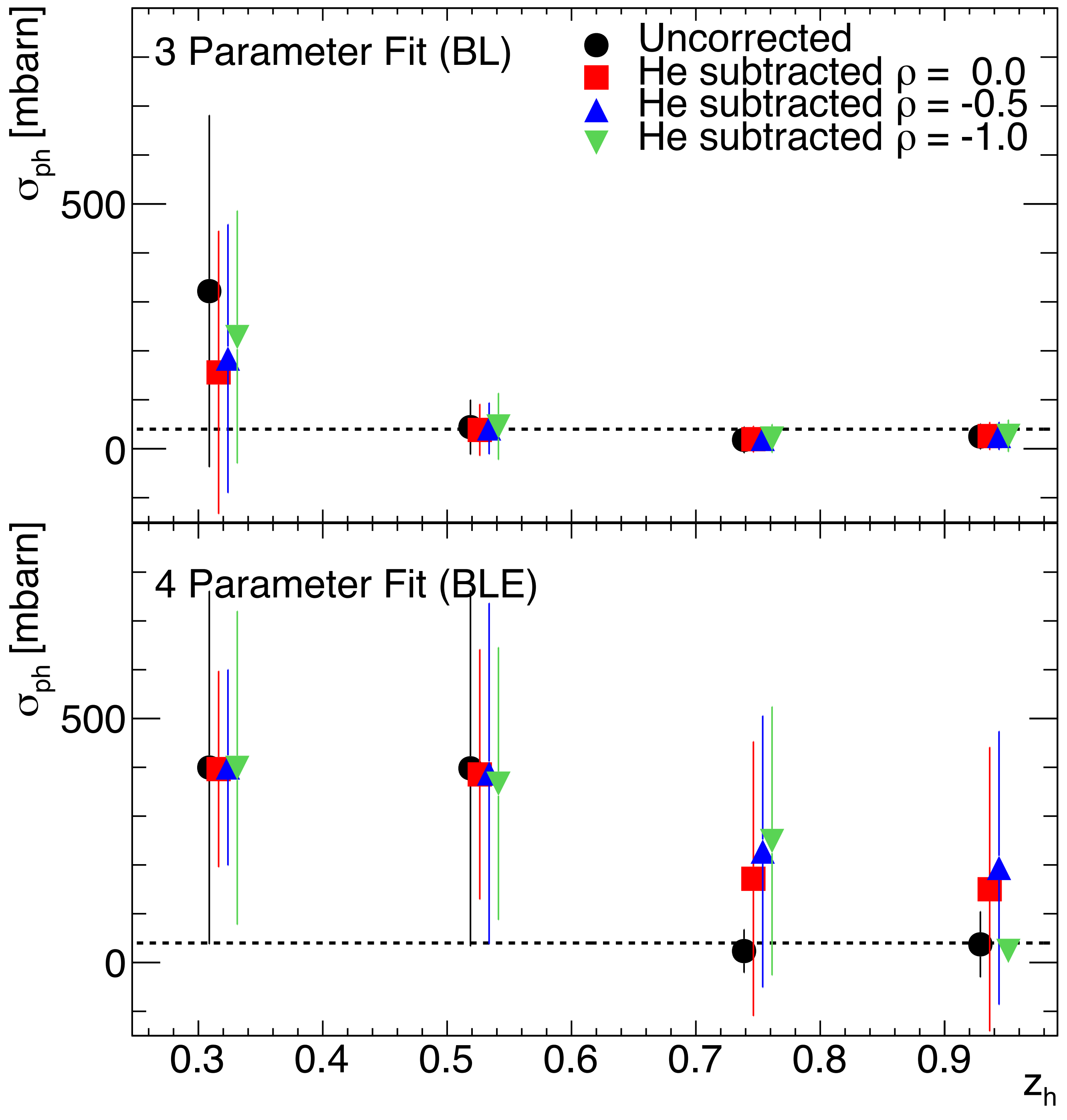
# Additional Slides

# Treatment of the data

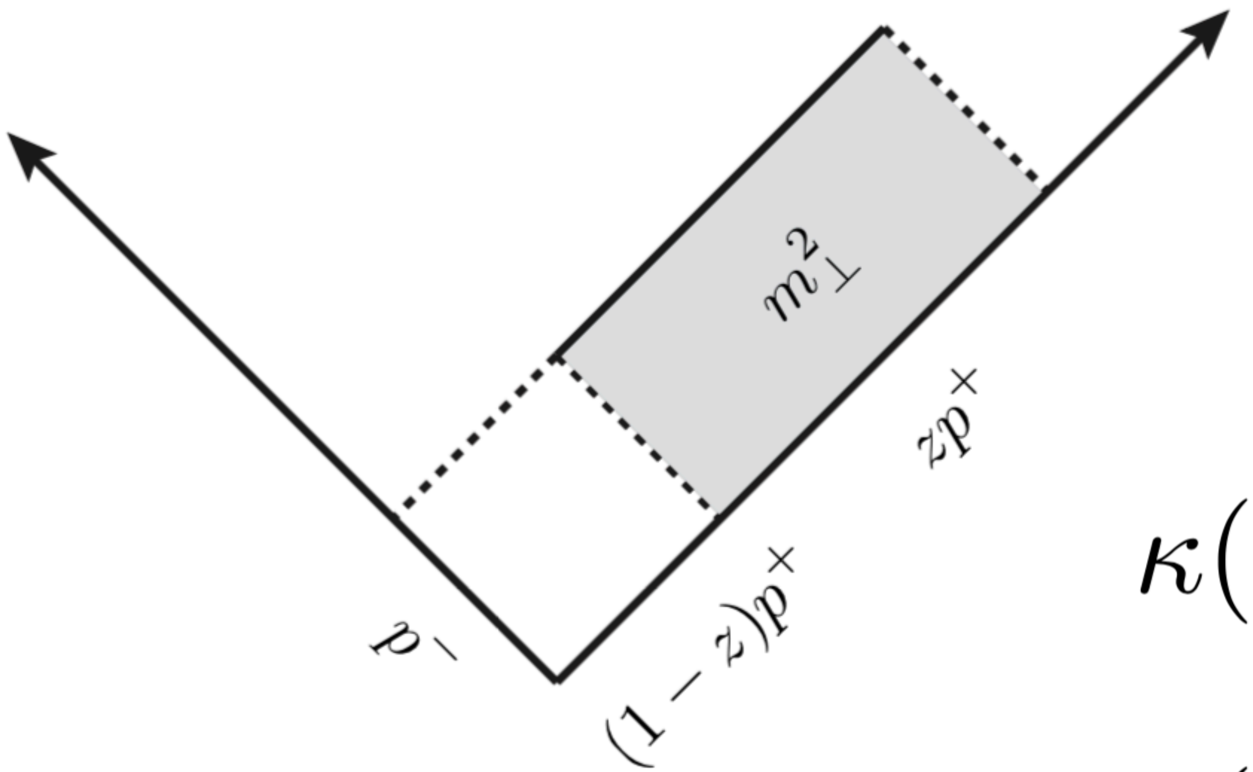


# Pre-hadron cross-section determination



# Production length $L_p$




$$\kappa(t+l) = (1-z)p^+$$
$$\kappa(t-l) = \frac{m_\perp^2}{zp^+}$$

$$2\kappa l = M + \nu\left(1+\sqrt{1+Q^2/\nu^2}\right) - 2\nu z$$

# Estimations for future experiments

## Space-time characteristics of the struck quark

Assume: Single-photon exchange, no quark-pair production

"JLab" example: $Q^2 = 3$ GeV$^2$, $\nu = 3$ GeV. ($x_{Bj}$~0.5)

Struck quark absorbs virtual photon energy $\nu$ and momentum $p_{\gamma^*} = |\vec{p}_{\gamma^*}| = \sqrt{(\nu^2 - Q^2)}$.

- Neglect any initial momentum/mass of quark
- Immediately after the interaction, quark mass $m_q = Q = \sqrt{(Q^2)}$.
- Gamma factor is therefore $\gamma = \nu/Q$, beta is $\beta = p_{\gamma^*}/\nu$.

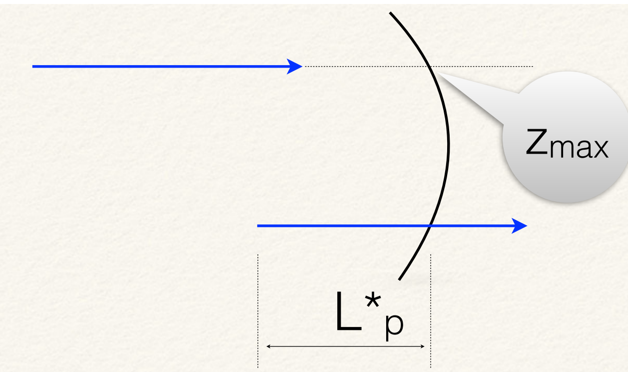
JLab example: $\gamma = 1.73$, $\beta = 0.82$

Rigorous? $\gamma$, $\beta$ allow:

1. extrapolations to EIC kinematics,
2. test of time dilation in CLAS fits, and
3. direct comparison between JLab and HERMES fits

# Geometric model

B-L Geometric model description III




$$\langle \Delta p_T^2 \rangle = \langle\ \hat{q}_0 \int_{z=z_0}^{z=z_0+L_p^*} \rho(x_0, y_0, z)\ dz\ \rangle_{x_0,y_0,z_0,L_p}$$

$L_p$ is distributed as exponential

$x_0$,$y_0$,$z_0$ thrown uniformly in sphere, weighted by ρ(x,y,z)

$L^*_p = L_p$ except where truncated by integration sphere

$$\langle R_M \rangle = \langle\ exp(-\sigma \int_{z=z_0+L_p}^{z=z_{max}} \rho(x, y, z)\ dx\ dy\ dz\ )\ \rangle_{x_0,y_0,z_0,L_p}$$

The above are computed sequentially (same $x_0$, $y_0$, $z_0$, $L_p$)

Data in ($x$,$Q^2$,$z$) bin: fitted to model, 3 parameters: $\hat{q}_0$,$\langle L_p \rangle$,σ

*<u>No dynamical information is assumed; it emerges from fit</u>*

Systematic errors: 3% for multiplicity ratio, 4% for $p_T$ broadening

## *Comment on the B-L model*

I believe that studies of this kind can be carried out at the same level of validity as the estimation of centrality in heavy ion collisions.

This model has the same foundation as the well-known “Glauber Model” used to estimate centrality in heavy ion collisions: the spatial mass distribution of protons and neutrons in the nucleus.

Conclusion: good evidence for the following functional form. The vacuum term $L_{po}$ is determined, but with large uncertainties. There are hints that may help us to understand color propagation mechanisms at lower and higher $z_h$. The JLab data should allow a more precise study.

$$L_p(A) = L_{po} + c_2 A^{2/3}$$

## Aims

### Quark-Hadron Transition

Discover new fundamental features of hadronization

- Characteristic time distributions
- Mechanisms of color neutralization

### Quark-Nucleus Interaction

Understand how color interacts within nuclei

- Partonic interactions with medium (“tomography”)
  - energy loss in-medium: $\hat{e}$
  - transverse momentum broadening: $\hat{q}$

*Method: struck quark from DIS probes nuclei of different sizes*

# Comparison of Color Propagation in Three Processes