

Longitudinal-transverse double-spin asymmetry with a $\cos \phi_s$ modulation in SIDIS

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OUTLINE



1. Formalism

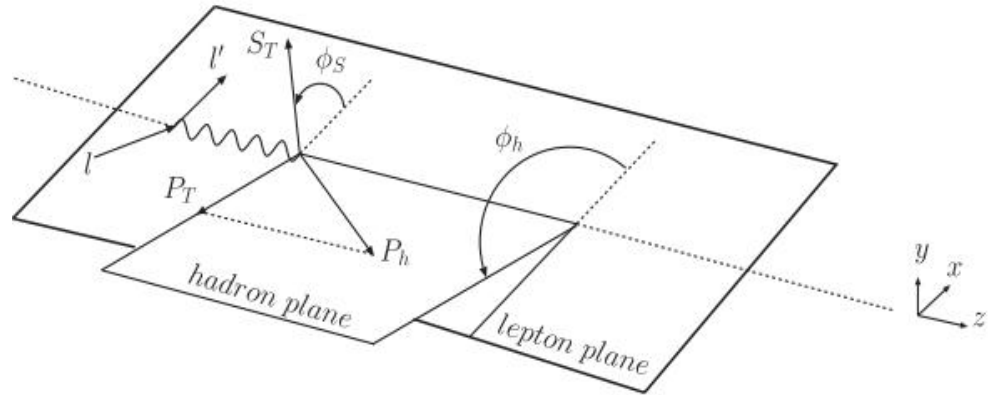
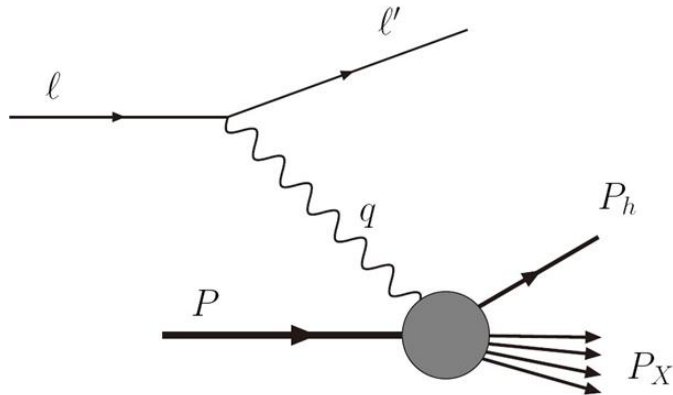
2. Calculation on the structure functions

3. Numerical Estimate at CLAS12 and EIC

4. Summary

- ◆ Semi-inclusive DIS by longitudinal polarized lepton beam off a transversely polarized nucleon target

$$e^{\rightarrow}(l) + p^{\uparrow}(P) \rightarrow e(l') + h(P_h) + X$$



- ◆ The invariants defined as

$$x = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2Mx}{Q}$$
$$Q^2 = -q^2, \quad s = (P + \ell)^2, \quad W^2 = (P + q)^2,$$

- ◆ The ratio of the longitudinal and transverse photon flux ([Bacchetta et al., JHEP0702, 093 \(2007\)](#))

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}.$$

- ◆ The general form for the differential cross section with a transversely polarized nucleon target ([Bacchetta et al., JHEP0702, 093 \(2007\)](#))

$$\begin{aligned}
 & \frac{d^6\sigma}{dx dy dz d\phi_h d\phi_S dP_{hT}^2} \\
 &= \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \\
 & \times |S_T| \lambda_e \{ \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S}(x, z, P_{hT}) \\
 & + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)}(x, z, P_{hT}) \\
 & + \text{leading twist terms} \}, \tag{3}
 \end{aligned}$$

$$A_{LT}^{\cos\phi_S} \equiv \frac{F_{LT}^{\cos\phi_S}}{F_{UU}}$$

$$A_{LT}^{\cos(2\phi_h - \phi_S)} \equiv \frac{F_{LT}^{\cos(2\phi_h - \phi_S)}}{F_{UU}}$$

At twist-3 level

Two double spin asymmetries

- ◆ Sizable SSAs or DSAs cannot be explained by pQCD (Ahmed & Gehrmann, PLB465, 297 (1999)).
- ◆ In the (assumed) TMD factorization (Bacchetta, Mulders, Pijlman PLB595, 309 (2004); Bacchetta et al., JHEP0702, 093 (2007))
- ◆ With the notation (Bacchetta et al., JHEP0702, 093 (2007))

$$\begin{aligned} \mathcal{C}[\omega f D] &= x \sum_q e_q^2 \int d^2 \mathbf{p}_T d^2 \mathbf{k}_T \delta^{(2)}(\mathbf{k}_T - \mathbf{p}_T - \mathbf{P}_{hT}/z) \\ &\quad \times \omega(\mathbf{p}_T, \mathbf{k}_T) f^q(x, \mathbf{k}_T^2) D^q(z, \mathbf{p}_T^2), \end{aligned}$$

- ◆ The unpolarized structure function F_{UU} can in the be given as

$$F_{UU} = \mathcal{C}[f_1 D_1],$$

- ◆ Focus on the $\cos\phi_s$ azimuthal asymmetry, the structure function can be given as

$$F_{LT}^{\cos\phi_s}(x, z, P_{hT}) = \frac{2M}{Q} \mathcal{C} \left\{ - \left(x g_T D_1 + \frac{M_h}{M} h_1 \frac{\tilde{E}}{z} \right) + \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x e_T H_1^\perp - \frac{M_h}{M} g_{1T} \frac{\tilde{D}^\perp}{z} \right) + \left(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{G}^\perp}{z} \right) \right] \right\},$$

- ◆ Considering the transverse momentum of the outgoing pion is integrated to ignore the contributions from the Collins function and the twist-3 TMD fragmentation functions \tilde{D}^\perp , \tilde{G}^\perp

- ◆ After the integral $\int d^2\mathbf{P}_{hT}$ is performed, the fourfold differential cross section has the form

$$\frac{d^4\sigma}{dx dy dz d\phi_S} = \frac{2\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \times \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S}(x, z).$$

- ◆ The collinear counter part of the structure function can be written as

$$\begin{aligned} F_{LT}^{\cos\phi_S}(x, z) &= \int d^2\mathbf{P}_{hT} F_{LT}^{\cos\phi_S}(x, z, P_{hT}) \\ &= -x \sum_q e_q^2 \frac{2M}{Q} \left(x g_T^q(x) D_1^q(z) \right. \\ &\quad \left. + \frac{M_h}{M} h_1^q(x) \frac{\tilde{E}^q(z)}{z} \right). \end{aligned}$$

- ◆ The longitudinal-transver spin asymmetry may be defined as

$$A_{LT} \sim \frac{\sigma(+\lambda_e, \mathbf{S}_T) - \sigma(-\lambda_e, \mathbf{S}_T)}{\sigma(+\lambda_e, \mathbf{S}_T) + \sigma(-\lambda_e, \mathbf{S}_T)},$$

- ◆ Thus, the x-dependent or z-dependent $\cos\phi_s$ asymmetry can be written as

$$\begin{aligned} A_{LT}^{\cos\phi_s}(x) &= \frac{\int dy \int dz \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} (1 + \frac{r^2}{2x}) \sqrt{2\varepsilon(1-\varepsilon)} F_{LT}^{\cos\phi_s}(x, z)}{\int dy \int dz \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} (1 + \frac{r^2}{2x}) F_{UU}(x, z)}, \\ A_{LT}^{\cos\phi_s}(z) &= \frac{\int dx \int dy \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} (1 + \frac{r^2}{2x}) \sqrt{2\varepsilon(1-\varepsilon)} F_{LT}^{\cos\phi_s}(x, z)}{\int dx \int dy \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} (1 + \frac{r^2}{2x}) F_{UU}(x, z)}. \end{aligned}$$

- ◆ From the collinear counter part of the structure function

$$\begin{aligned} F_{LT}^{\cos \phi_S}(x, z) &= \int d^2 \mathbf{P}_{hT} F_{LT}^{\cos \phi_S}(x, z, P_{hT}) \\ &= -x \sum_q e_q^2 \frac{2M}{Q} \left(x g_T^q(x) D_1^q(z) \right. \\ &\quad \left. + \frac{M_h}{M} h_1^q(x) \frac{\tilde{E}^q(z)}{z} \right). \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \sum_q e_q^2 g_T^q(x) &= g_1(x) + g_2(x), \\ g_1(x) &= \frac{1}{2} \sum_q e_q^2 g_1^q(x), \\ g_2(x) &= g_2^{\text{WW}}(x) + g_2^{\text{tw}-3}(x). \end{aligned}$$

- ◆ Here we use the WW ([Wandzura-Wilczek](#)) approximation

$$g_2^{\text{WW}} \approx g_2^{\text{WW}}(x) = -g_1(x) + \int_x^1 dy \frac{g_1(y)}{y},$$

STRUCTURE FUNCTION



◆ For the genuine twist-3 contribution $g_{2,p}^{\text{twist-3}}$, we use the result from V.M. Braun, T. Lautenschlager, A.N. Manashov and B. Pirnay, Phys. Rev. D 83, 094023(2011) at the reference scale $Q^2 = 1\text{GeV}^2$ for proton and neutron targets

$$g_{2,p}^{\text{tw-3}}(x) = 0.0436772 \left(\ln x + \bar{x} + \frac{1}{2} \bar{x}^2 \right) + \bar{x}^3 (1.57357 - 5.94918\bar{x} + 6.74412\bar{x}^2 - 2.19114\bar{x}^3), \quad (17)$$

$$g_{2,n}^{\text{tw-3}}(x) = 0.0655158 \left(\ln x + \bar{x} + \frac{1}{2} \bar{x}^2 \right) + \bar{x}^3 (0.130996 - 1.12101\bar{x} + 2.31342\bar{x}^2 - 1.20598\bar{x}^3), \quad (18)$$

- ◆ In the valence region, applying the isospin symmetry

$$g_{2,p}^{\text{tw}-3}(x) = \frac{4}{9}g_2^{\text{tw}-3,u}(x) + \frac{1}{9}g_2^{\text{tw}-3,d}(x),$$

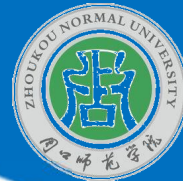
$$g_{2,n}^{\text{tw}-3}(x) = \frac{1}{9}g_2^{\text{tw}-3,u}(x) + \frac{4}{9}g_2^{\text{tw}-3,d}(x),$$

we obtain the expression for g_T^q

$$g_T^u(x) = \int_x^1 dy \frac{g_1^u(y)}{y} + \frac{6}{5}(4g_{2,p}^{\text{tw}-3}(x) - g_{2,n}^{\text{tw}-3}(x)),$$

$$g_T^d(x) = \int_x^1 dy \frac{g_1^d(y)}{y} + \frac{6}{5}(4g_{2,n}^{\text{tw}-3}(x) - g_{2,p}^{\text{tw}-3}(x)).$$

STRUCTURE FUNCTION



- ◆ To estimate the asymmetry, we also need to know the chiral-odd fragmentation function $\tilde{E}^q(z)$ (there is still no theoretical or experimental information on it.)

$$\begin{aligned} F_{LT}^{\cos \phi_S}(x, z) &= \int d^2 \mathbf{P}_{hT} F_{LT}^{\cos \phi_S}(x, z, \mathbf{P}_{hT}) \\ &= -x \sum_q e_q^2 \frac{2M}{Q} \left(x g_T^q(x) D_1^q(z) \right. \\ &\quad \left. + \frac{M_h}{M} h_1^q(x) \frac{\tilde{E}^q(z)}{z} \right). \end{aligned}$$

- ◆ For a rough estimate, we assume

$$\tilde{E}^q(z) = \frac{m'_q}{M_h} \frac{z^2}{1-z} D_1^q(z).$$

the motion relation

$$\frac{E^q(z)}{z} = \frac{\tilde{E}^q(z)}{z} + \frac{m_q}{M_h} D_1^q(z),$$

chiral quark model result

$$E^q(z) = \frac{m'_q}{M_h} \frac{z}{1-z} D_1^q(z),$$

STRUCTURE FUNCTION



- ◆ For the transversity $h_1^q(x)$, we adopt the standard parametrization
M. Anselmino, M. Boglione *et al.* Phys. Rev. D 87 094019(2013)

$$\begin{aligned} F_{LT}^{\cos\phi_S}(x, z) &= \int d^2\mathbf{P}_{hT} F_{LT}^{\cos\phi_S}(x, z, \mathbf{P}_{hT}) \\ &= -x \sum_q e_q^2 \frac{2M}{Q} \left(x g_T^q(x) D_1^q(z) \right. \\ &\quad \left. + \frac{M_h}{M} h_1^q(x) \frac{\tilde{E}^q(z)}{z} \right). \end{aligned}$$

$$h_1^q(x) = \frac{1}{2} \mathcal{N}_q^T(x) [f_1^q(x) + g_1^q(x)],$$

with

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta}.$$

- ◆ For the distribution $f_1^q(x)$, we adopt GRV98 LO parametrization
- ◆ For the distribution $g_1^q(x)$, we adopt GRV2000 LO parametrization
- ◆ For the fragmentation function $D_1^q(z)$, we adopt LO set of the DSS parametrization

NUMERICAL ESTIMATE

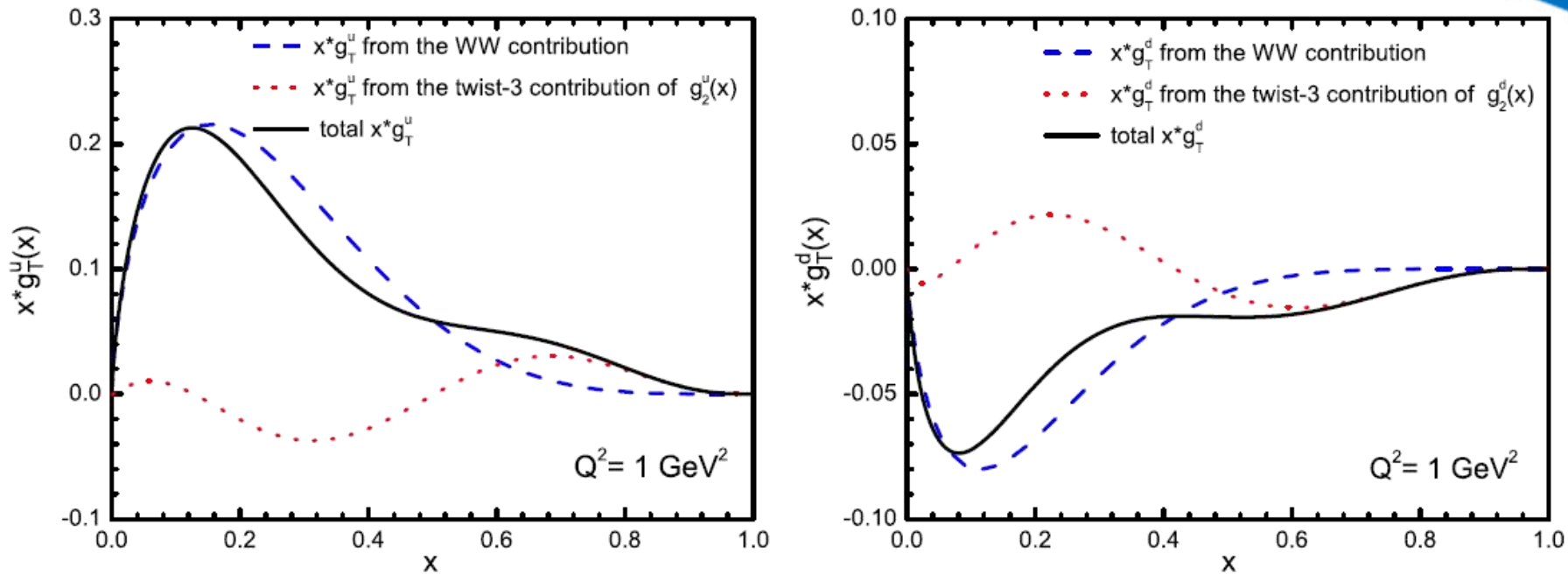


FIG. 2. The twist-3 distribution function $xg_T^q(x)$ for up and down quarks vs x at $Q^2 = 1 \times \text{GeV}^2$.



- ◆ At CLAS12, the kinematics are as follows

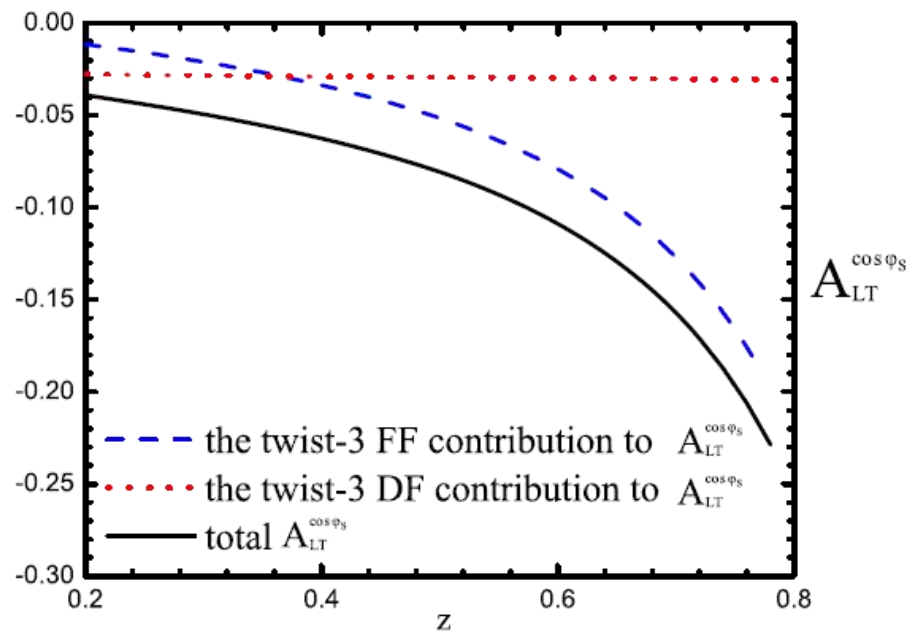
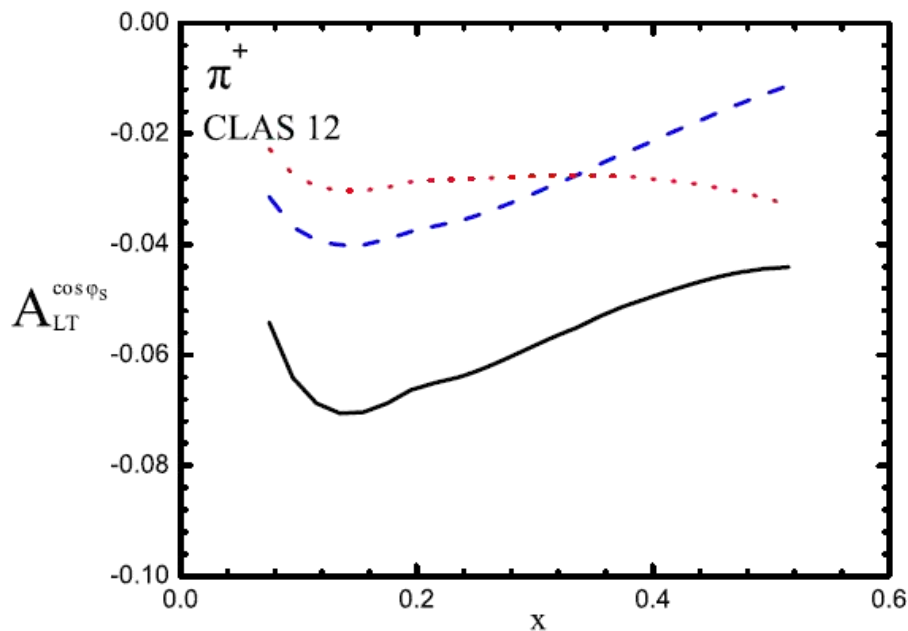
$$0.072 < x < 0.532, \quad 0.2 < z < 0.8, \quad E_e = 11 \text{ GeV}, \\ W^2 > 4 \text{ GeV}^2, \quad 1 < Q^2 < 6.3 \text{ GeV}^2,$$

- ◆ The invariant mass of the photon-nucleon system is

$$W^2 = (P + q)^2 \approx \frac{1-x}{x} Q^2.$$

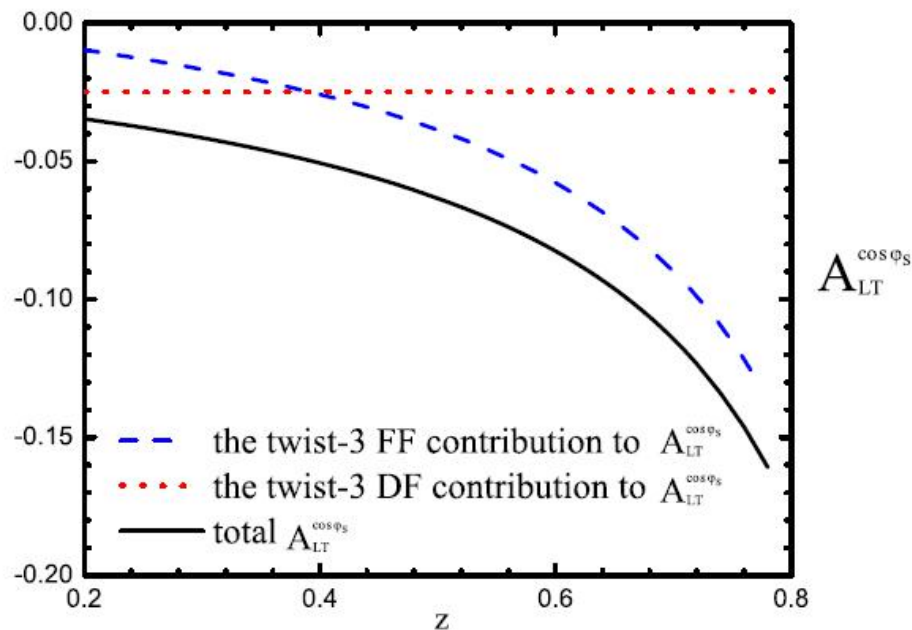
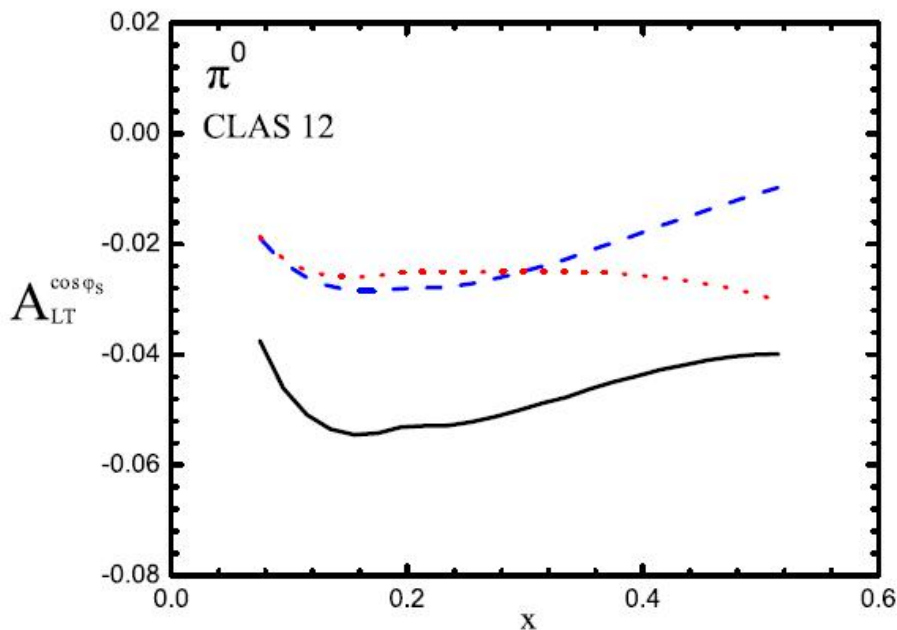
NUMERICAL ESTIMATE at CLAS12

- ◆ The x- and z- dependent double-spin asymmetry $A_{LT}^{\cos\phi_S}$ of π^+ production in SIDIS at CLAS12.



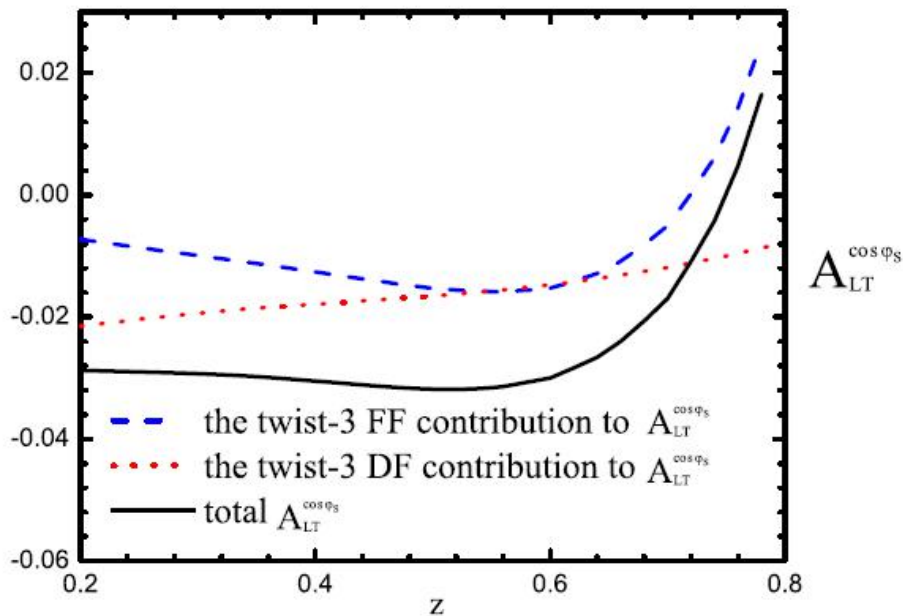
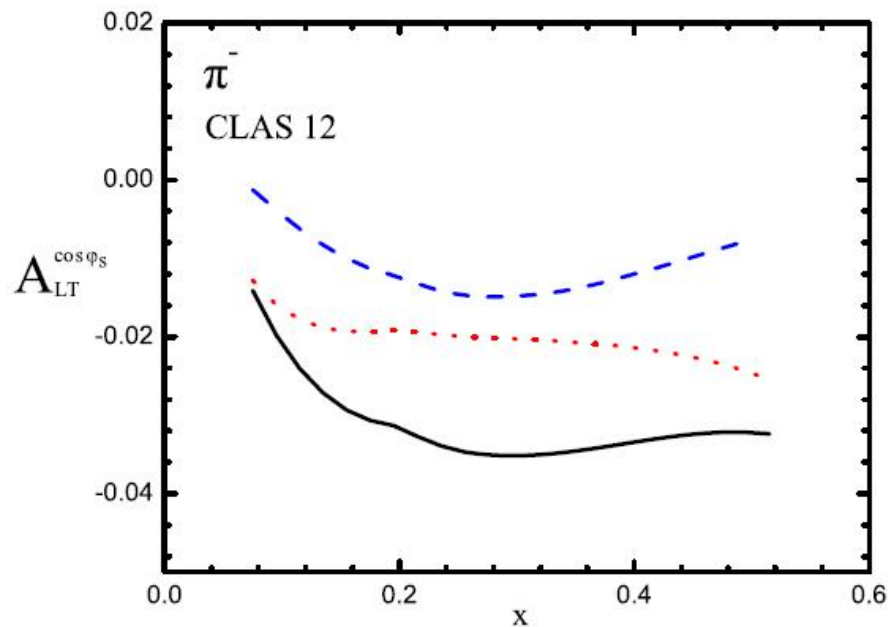
NUMERICAL ESTIMATE at CLAS12

- ◆ The x- and z- dependent double-spin asymmetry $A_{LT}^{\cos\phi_S}$ of π^0 production in SIDIS at CLAS12.



NUMERICAL ESTIMATE at CLAS12

- ◆ The x- and z- dependent double-spin asymmetry $A_{LT}^{\cos\phi_S}$ of π^- production in SIDIS at CLAS12.



NUMERICAL ESTIMATE at EIC



- ◆ At EIC, the kinematical cuts are as follows

$$Q^2 > 1 \text{ GeV}^2, \quad 0.001 < x < 0.4, \quad 0.01 < y < 0.95,$$

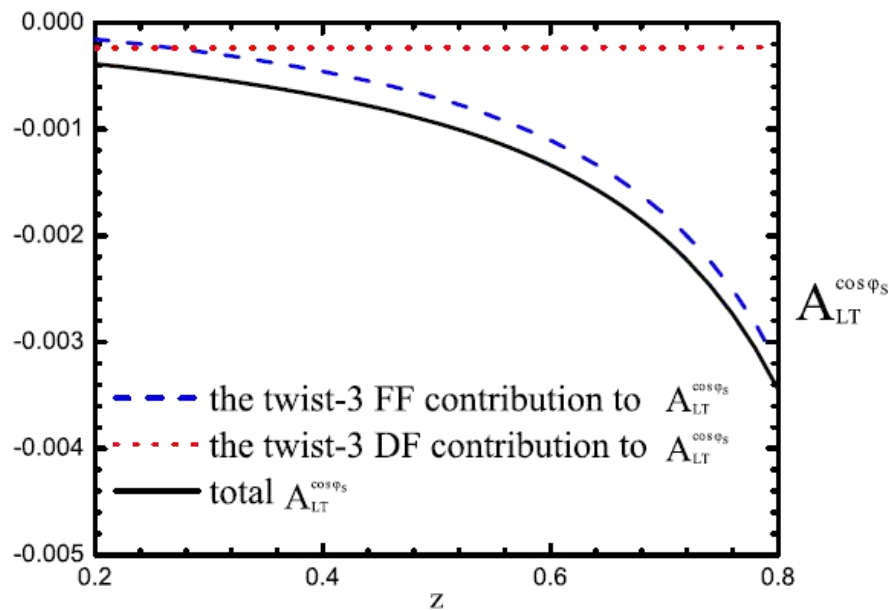
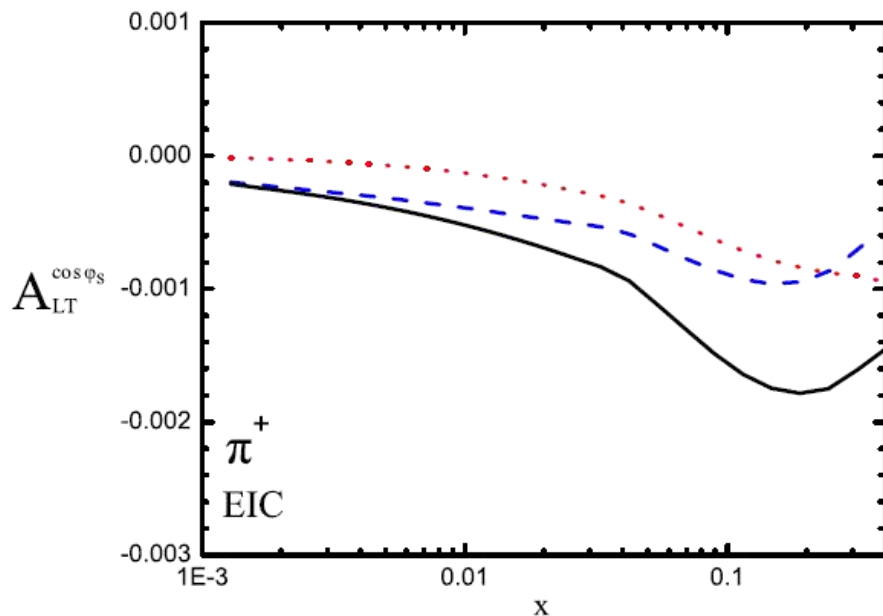
$$0.2 < z < 0.8, \quad \sqrt{s} = 45 \text{ GeV}, \quad W > 5 \text{ GeV}.$$

- ◆ At the twist-3 level, the effect will be suppressed by $1/Q$, since the averaged Q value at EIC is much higher than that at CLAS12

NUMERICAL ESTIMATE at EIC



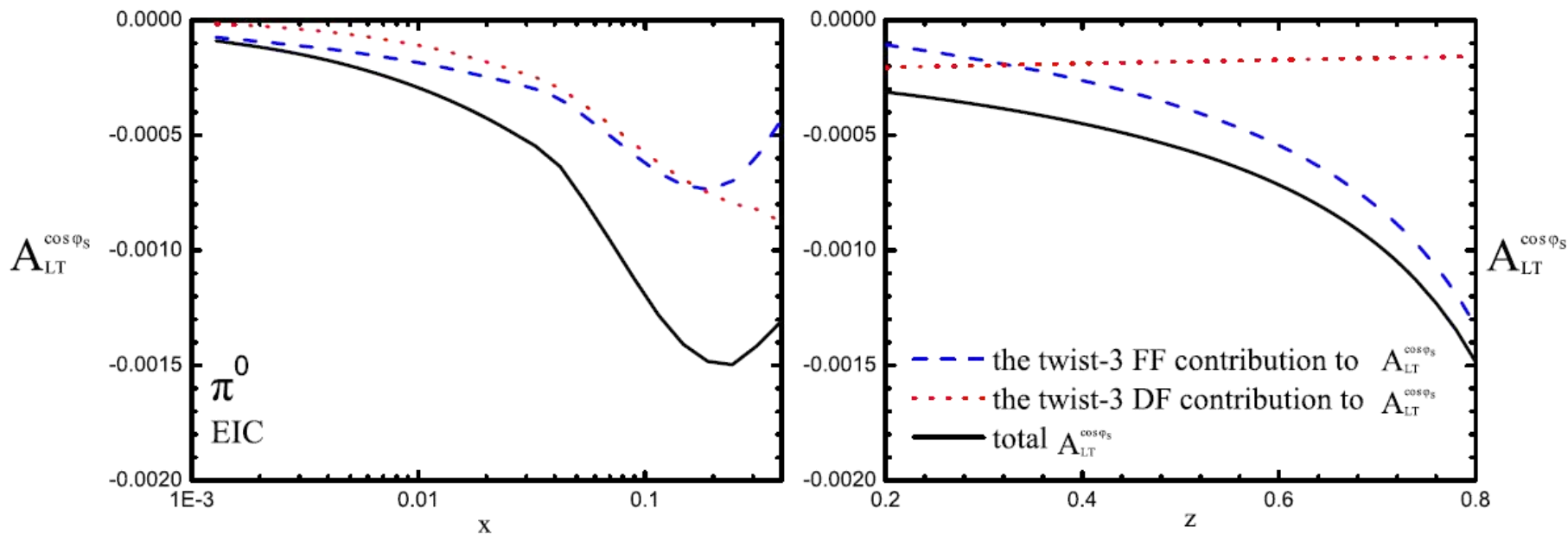
- ◆ The x- and z- dependent double-spin asymmetry $A_{LT}^{\cos\phi_S}$ of π^+ production in SIDIS at EIC.



NUMERICAL ESTIMATE at EIC



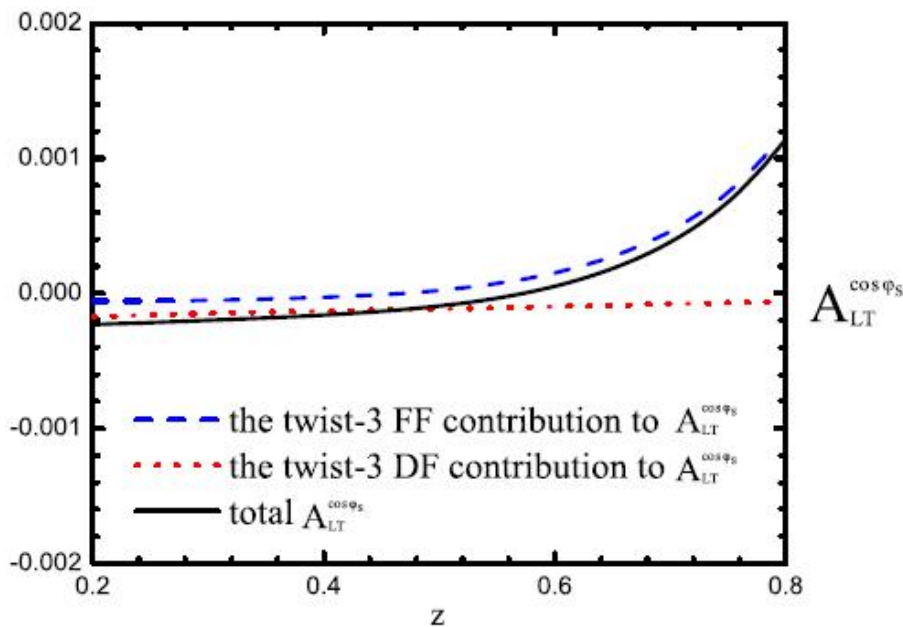
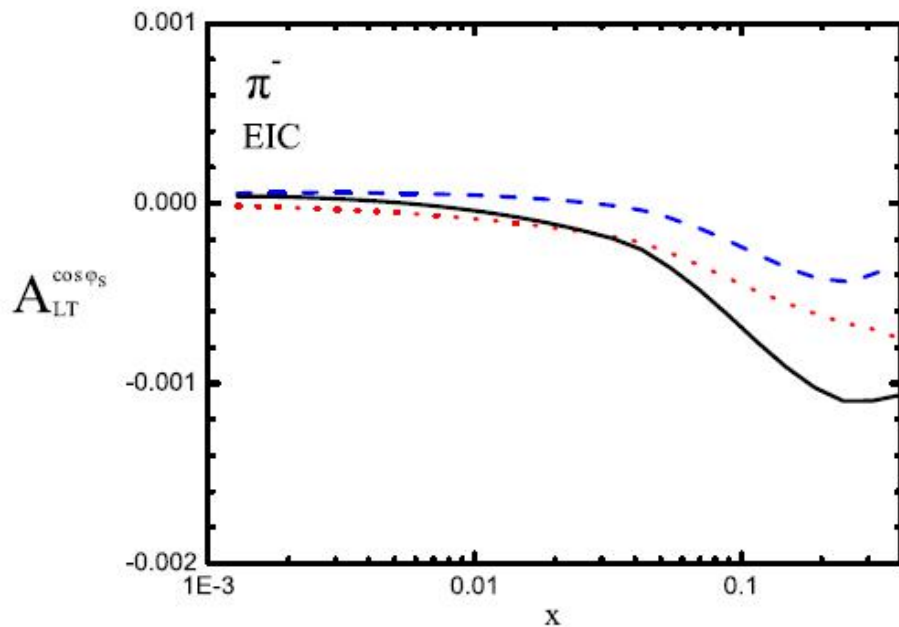
- ◆ The x- and z- dependent double-spin asymmetry $A_{LT}^{\cos\phi_S}$ of π^0 production in SIDIS at EIC.



NUMERICAL ESTIMATE at EIC



- ◆ The x- and z- dependent double-spin asymmetry $A_{LT}^{\cos\phi_S}$ of π^- production in SIDIS at EIC.



The transverse double-spin asymmetries in semi-inclusive DIS at subleading with the TMDs

- ◆ The structure function at twist-3 level

$$F_{LT}^{\cos\phi_S}(x, z, P_{hT}) = \frac{2M}{Q} C \left\{ - \left(x g_T D_1 + \frac{M_h}{M} h_1 \frac{\tilde{E}}{z} \right) + \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x e_T H_1^\perp - \frac{M_h}{M} g_{1T} \frac{\tilde{D}^\perp}{z} \right) + \left(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{G}^\perp}{z} \right) \right] \right\},$$

- ◆ Use the WW (Wandzura-Wilczek) approximation to ignore the contributions from the twist-3 TMD fragmentation functions

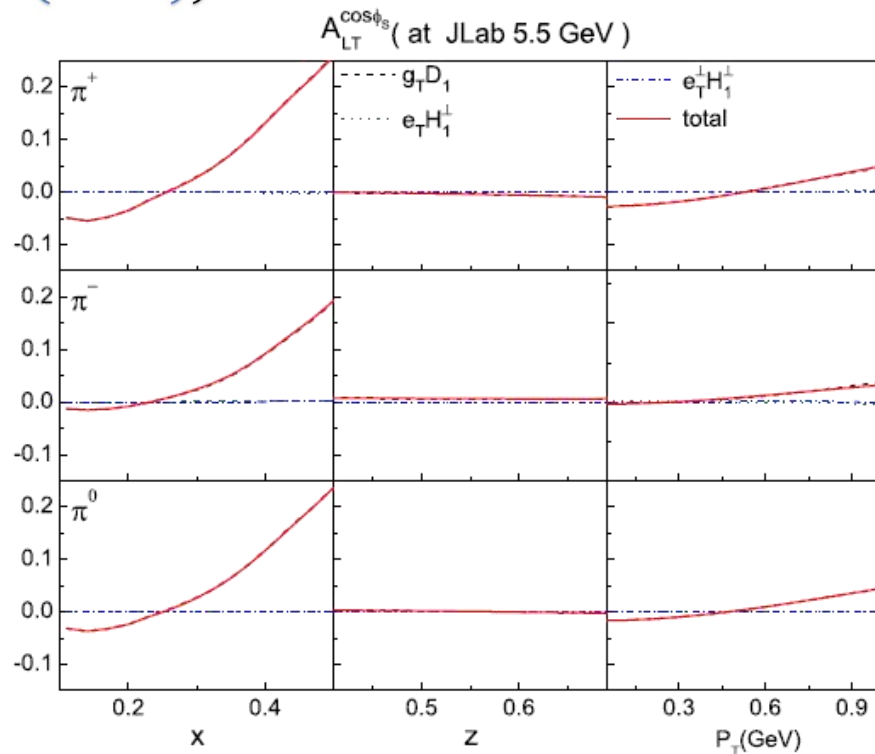
$$F_{LT}^{\cos\phi_S} \approx \frac{2M}{Q} C \left\{ -x g_T D_1 - \frac{\mathbf{p}_T \cdot \mathbf{k}_T}{2zMM_h} (x e_T H_1^\perp + x e_T^\perp H_1^\perp) \right\}$$

Based on Phys. Rev. D 91, 034029 (2015)

Predictions at JLab



- ◆ Kinematics at JLab 5.5 GeV (H. Avakian, Nuovo Cimento Soc. Ital. Fis. C 36, 73 (2013)):

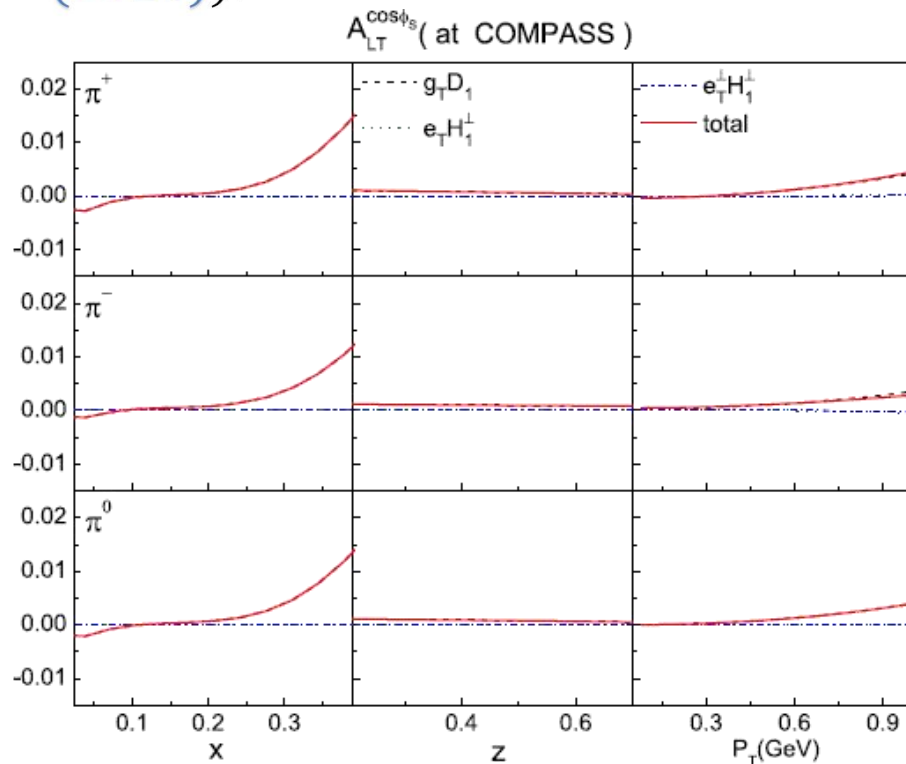


$$0.1 < x < 0.6, \quad 0.4 < z < 0.7, \quad Q^2 > 1 \text{ GeV}^2, \\ P_T > 0.05 \text{ GeV}, \quad W^2 > 4 \text{ GeV}^2.$$

Predictions at COMPASS



- ◆ Kinematics at COMPASS 160GeV (M.G. Alekseev et al., Phys. Lett. B 692, 240 (2010)):



$$0.004 < x < 0.7, \quad 0.1 < y < 0.9, \quad z > 0.2,$$

$$P_T > 0.1 \text{ GeV}, \quad Q^2 > 1 \text{ GeV}^2,$$

$$W > 5 \text{ GeV}, \quad E_h > 1.5 \text{ GeV}.$$

SUMMARY



- ◆ We estimate the double-spin asymmetry $A_{LT}^{\cos\phi_S}$ of charged and neutral pion production in SIDIS at CLAS12 and a future EIC.
- ◆ We consider the particular case where the transverse momentum of the final-state hadron is integrated out.
- ◆ We focus on the contributions from the twist-3 distribution function term $g_T(x)D_1(z)$ and the twist-3 fragmentation function term $h_1(x)\tilde{E}(z)$.
- ◆ The asymmetry of pion at CLAS12 is sizable, and $\tilde{E}(z)$ can play an important role in the large- z region.



THANK YOU!