Generalized distribution amplitudes and gravitational form factors for pion

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Outline

Generalized distribution amplitude (GDA) of pion

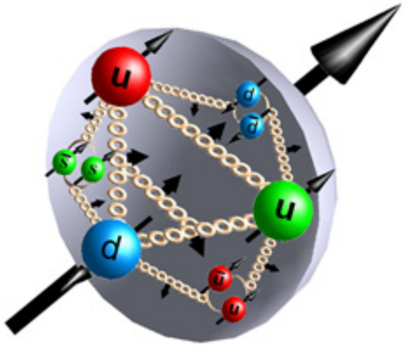
- Motivation
- GDA in two-photon process
- GDA analysis for Belle data

Structure of hadrons: 3D structure

Spin puzzle of proton

$$\Delta u^{+}+\Delta d^{+}+\Delta s^{+}\approx 0.3$$

$$\Delta g+\Delta L\neq 0$$



Generalized Parton Distributions (GPDs) provide information on ΔL to solve the proton puzzle!

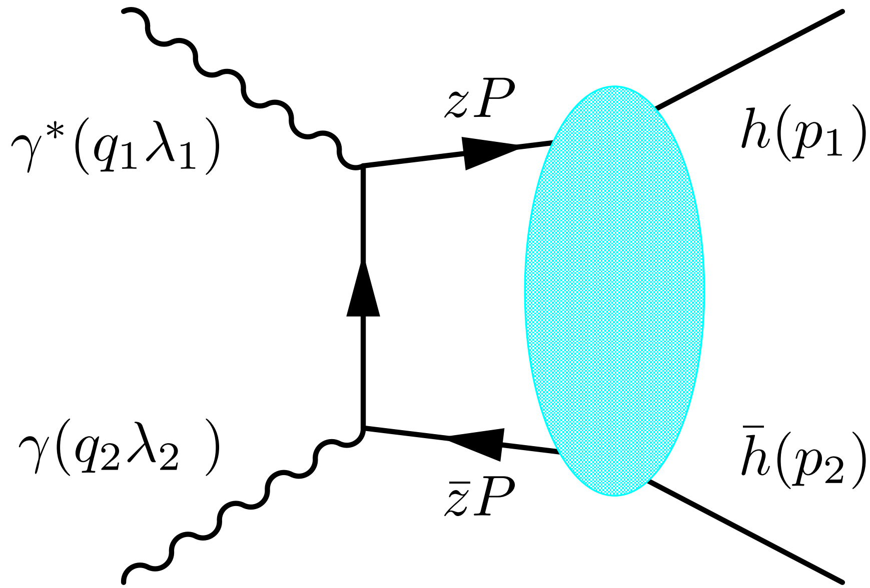
Generalized Distribution Amplitudes (GDAs) <--> s-t crossing of GPDs

Pion GDAs are investigated.

GDA carry many important physical quantities of the hadron, such as distribution amplitudes (DAs) and timelike form factors.

Generalized distribution amplitude for pion

In the process $\gamma\gamma^*\to$h bar{h}, an hard part describing the process $\gamma\gamma^*\to$q bar{q} with produced collinear and on-shell quark, and a soft part describing the production of the hadron h pair from a q bar{q}.This soft part is called Generalized Distribution Amplitude (GDA).



The process $\gamma^*\gamma\to$h bar{h}

GDA is an important quantity of hadron, it is defined as

$$\Phi^q\left(z,\xi,W^2\right)=\int\frac{dx^-}{2\pi}e^{-izP^+x}\left\langle h(p)\bar{h}(p')\,|\,\bar{q}(x^-)\gamma^+q(0)\,|\,0\right\rangle$$

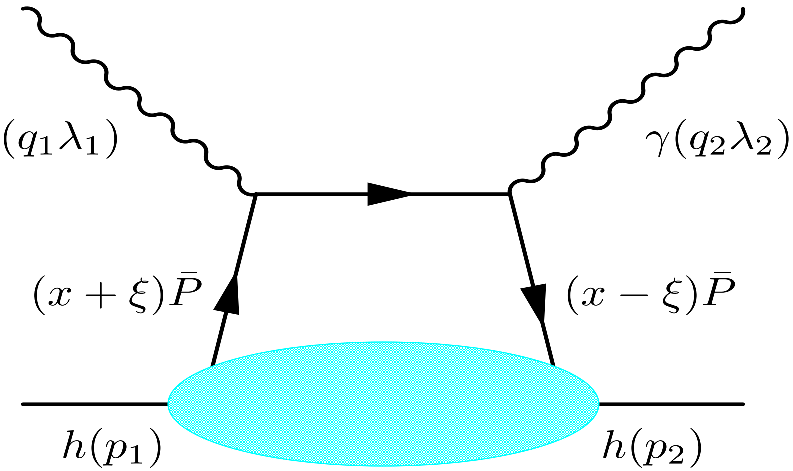
$$z=\frac{k^+}{P^+},\ \xi=\frac{p^+}{P^+},\ \mathrm{s=W}^2=(p+p')^2=P^2$$

M. Diehl, Phys. Rep. 388 (2003) 41.
M. Diehl and P. Kroll, EPJC 73 (2013) 2397 .

GDA is closely related to generalized parton distribution (GPD) by the s-t crossing, so GDA could provide another way to obtain GPD information.

$$\Phi^q\left(z,\xi,W^2\right) \leftrightarrow H^q\left(x=\frac{1-2z}{1-2\xi}, \varsigma=\frac{1}{1-2\xi}, t=W^2 \right)$$

GDA GPD



$\gamma^* h \to \gamma\ h$

GPD can be used to study the proton spin puzzle!

$$\int \frac{dx^-}{2\pi} e^{-iz(\bar{P}^+ x)} \left\langle h(p_2) | \bar{q}(x^-)\gamma^+ q(0) | h(p_1) \right\rangle$$

$$= \frac{1}{2\bar{P}^+}\left[H^q(x,\xi,t)\bar{u}(p_2)\gamma^+ u(p_1) + E^q(x,\xi,t)\bar{u}(p_2)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p_1) \right]$$

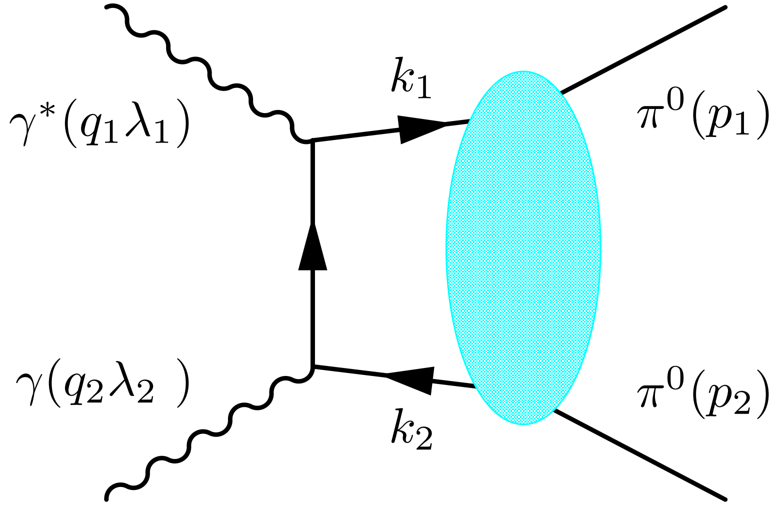
$$\bar{P}=(p_1+p_2)/2,\ \Delta=p_2-p_1,\ x=\frac{-q_1^2}{2p_1 q_1},\ \xi=\frac{\Delta^+}{p_1^+ + p_2^+}$$

M. Diehl, Phys. Rep. 388 (2003), 41.
H. Kawamura and S. Kumano, PRD 89 (2014), 054007.

The cross section of process $\gamma^*\gamma \to \pi^0\pi^0$

$$d\sigma = \frac{1}{4}\frac{1}{4\sqrt{(q_1q_2)^2 - q_1^2q_2^2}}\sum_{\lambda_1\lambda_2}|-iT_{\mu\nu}\varepsilon^{\mu}(q_1)\varepsilon^{\nu}(q_2)|^2 d\Phi_2$$

$$d\sigma = \frac{\pi\alpha^2\sqrt{1-\frac{4m^2}{s}}}{4(Q^2+s)}|A_{++}|^2 \sin\theta d\theta$$



$A_{\lambda 1\lambda 2}$ is the helicity amplitude, and there are 3 independent helicity amplitudes, they are A_{++}, A_{0+} and A_{+-}. The leading-twist amplitude A_{++} has a close relation with the generalized distribution amplitude (GDA) $\Phi^q(z, \xi, W^2)$.

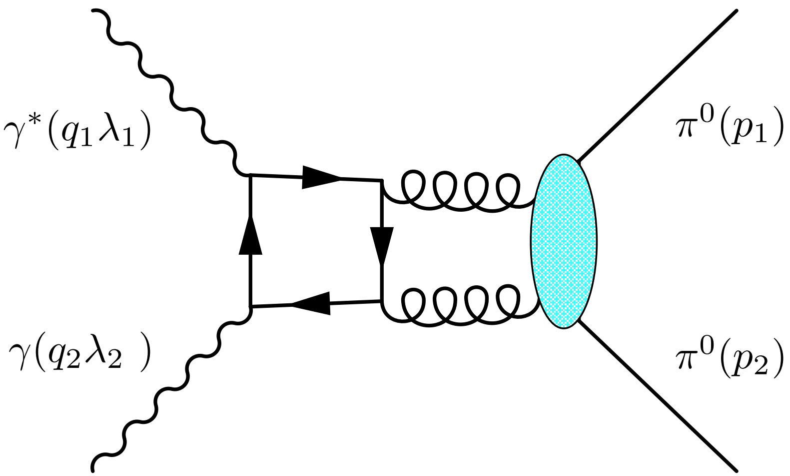
$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\varepsilon^{\mu}(\lambda_1)\varepsilon^{\nu}(\lambda_2)/e^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{2z-1}{z(1-z)}\Phi^q\left(z,\xi,W^2\right)$$

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.

Higher twist and higher order contributions

Higher-twist contribution A_{0+} requires a helicity flip along the fermion line, and it decreases as 1/Q. Higher-order contribution A_{+-} contributes with the GDA of gluon, since A_{+-} indicates the angular momentum $L_z = 2$. Therefore, A_{+-} is suppressed by running coupling constant α_s.



Gluon GDA

M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.
N. Kivel, L. Mankiewicz and M.V. Polyakov PLB 467 (1999) 263.

GDA expression

At very high energy Q^2, we can have the asymptotic form of the GDA

$$\sum_q \Phi_q^+\left(z,\xi,W^2\right) = 18 n_f z(1-z)(2z-1)[B_{10}(W) + B_{12}(W) P_2(2\xi-1)]$$

$$= 18 n_f z(1-z)(2z-1)[\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)]$$

The GDAs are related to the energy-momentum form factor in the timelike region.

$$\int dz(2z-1)\Phi_q^+\left(z,\xi,W^2\right) = \frac{2}{(P^+)^2}\left\langle \pi^+(p_1)\pi^-(p_2)\middle| T_q^{++}(0) \middle| 0\right\rangle$$

where the energy-momentum form factor for quarks is defined as

$$\left\langle \pi^0(p_1)\pi^0(p_2)\middle| T^{\mu\nu}(0) \middle| 0\right\rangle = \frac{1}{2}\left[\left(s g^{\mu\nu} - P^\mu P^\nu\right)\Theta_1 + \Delta^\mu \Delta^\nu \Theta_2\right]$$

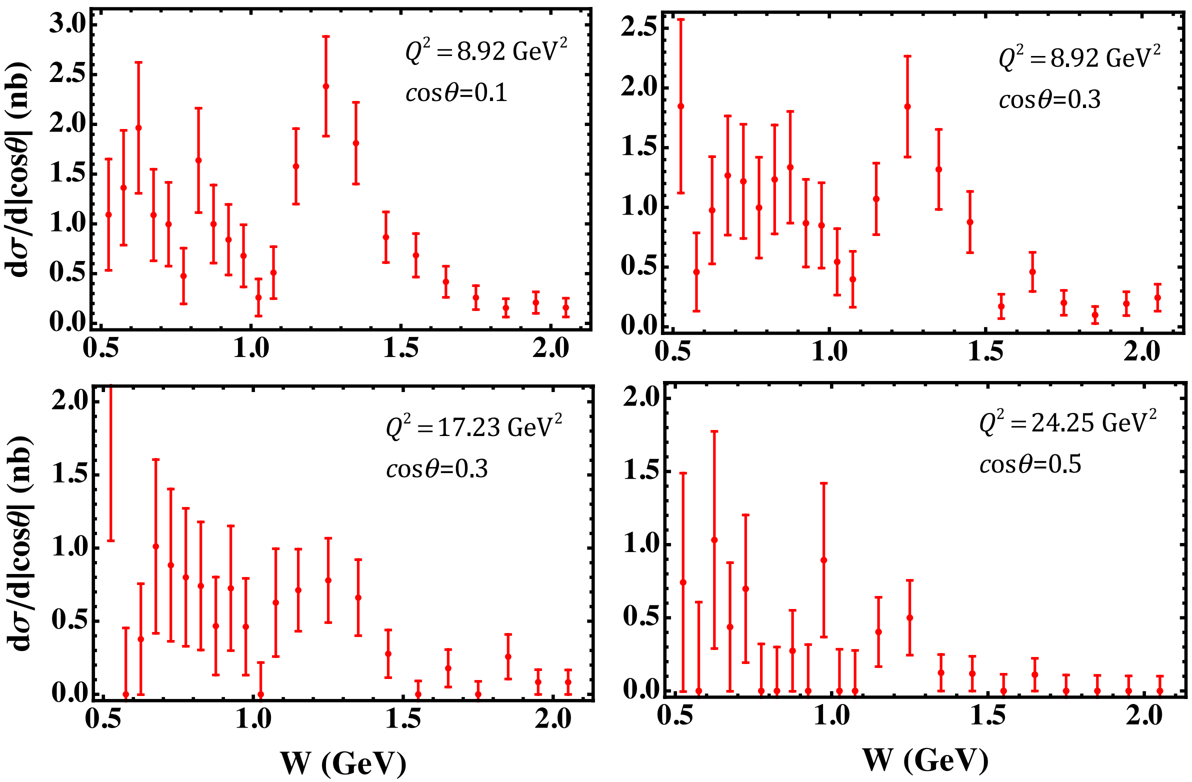
$$P = p_1 + p_2 \,,\ \Delta = p_1 - p_2$$

By using this sum rule we can obtain $B_{12}(0) = \frac{5R_\pi}{9}$

where R_π is the momentum fraction carried by quarks in the pion.

M. V. Polyakov, NPB **555** (1999) 231.
M. V. Polyakov and C. Weiss PRD 60 (1999) 114017.

In 2016, the Belle Collaboration released the measurements of differential cross section for $\boldsymbol{\gamma}^{*}\boldsymbol{\gamma}\to\pi^0\pi^0$. The GDAs can be obtained by analyzing the Belle data.



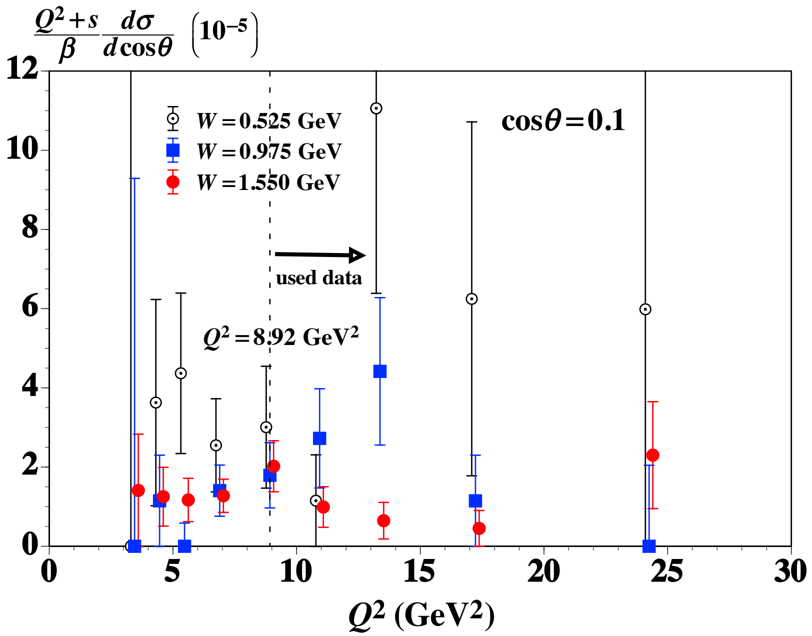
Differential cross section for $\gamma^{*}\gamma\to\pi^0\pi^0$

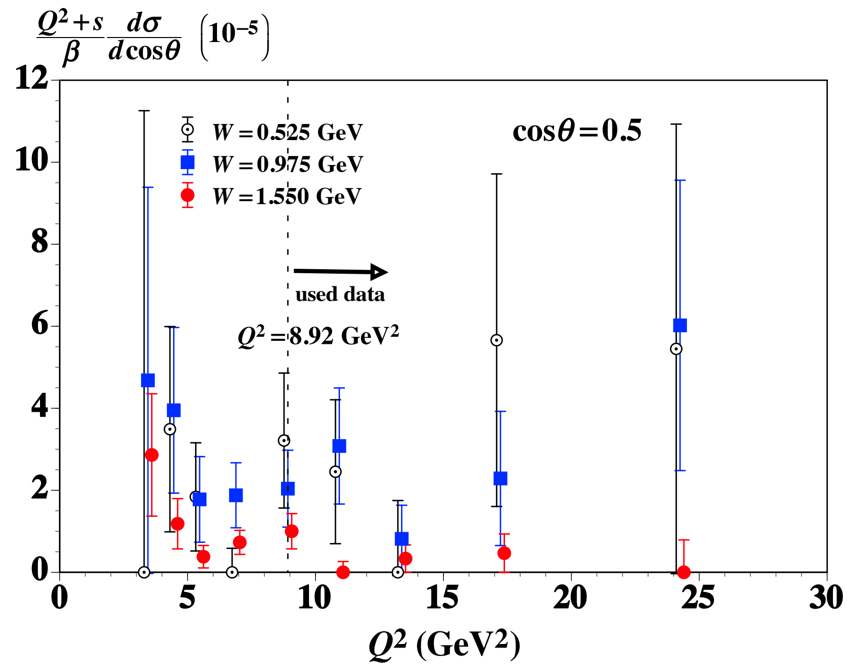
In these figures, the resonance $f_2(1270)$ is clearly seen around W = 1.25 GeV, however, other resonances are not clearly seen due to the large errors.

M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.

Scale violation of GDA based on Belle data

$$\frac{(Q^2+s)d\sigma}{\beta d|\cos\theta|} \propto \left| \Phi^{\pi^0\pi^0}(z,\ \cos\theta,\ W,\ Q) \right|^2$$





The scale dependence of the Belle data. We have red color for W = 0. 525 GeV, blue color for W = 0. 975 GeV, and green color for W = 1. 55 GeV.

The scaling violation of the GDAs is not so obvious in the Belle data on account of the large errors, so that the Q^2-independent GDAs could be used in analyzing the Belle data.

Q^2-independent (asymptotic form) GDAs

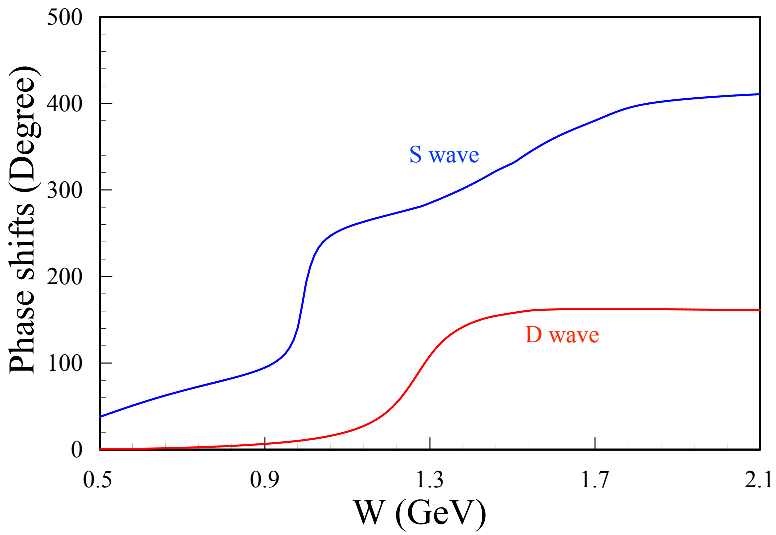
$$\sum_q \Phi_q^+\left(z,\xi,W^2\right) = 18 n_f z(1-z)(2z-1)[B_{10}(W)+B_{12}(W)P_2(2\xi-1)]$$

$$= 18 n_f z(1-z)(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W) = \bar{B}_{10}(W)e^{i\delta_0}, \tilde{B}_{12}(W) = \bar{B}_{12}(W)e^{i\delta_2}$$

In the above equation δ_0 and δ_2 and are the $\pi\pi$ elastic scattering phase shifts in the isospin=0 channel (see the figure). Above the KK threshold, the additional phase is introduced for S-wave

The S wave and D-wave $\pi\pi$ scattering phase shifts.



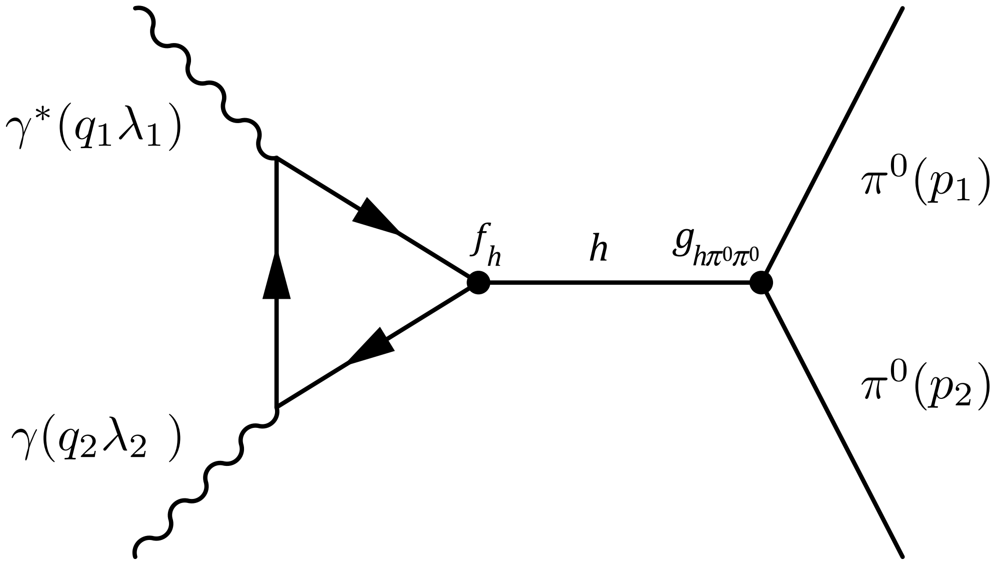
M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.
P. Bydzovsky, R. Kamiski and V. Nazari, PRD 90 (2014) , 116005; PRD 94 (2016), 116013.

Resonance effects

In the process $\gamma^*\gamma\to\pi^0\pi^0$, the $\pi^0\pi^0$ can be produced through intermediate meson state h. The q bar{q}→h amplitude should be proportional to the decay constant f_h or the distribution amplitude (DA), and the $h\to\pi^0\pi^0$ amplitude can be expressed by the coupling constant $g_{h\pi\pi}$. These resonance contributions read

$$\bar{B}_{12}(W)=\beta^2\frac{10g_{f_2\pi\pi}f_{f_2}M_{f_2}^2}{9\sqrt{2}\sqrt{(M_{f_2}^2-W^2)^2-\Gamma_{f_2}^2M_{f_2}^2}}$$

$$\bar{B}_{10}(W)=\frac{5g_{f_0\pi\pi}f_{f_0}}{3\sqrt{2}\sqrt{(M_{f_0}^2-W^2)^2-\Gamma_{f_0}^2M_{f_0}^2}}$$



The resonance effects play an important role in the resonance regions.

We adopt a simple expression of GDA to analyze Belle data, here resonance effects of $f_0(500)$ and $f_2(1270)$ are introduced.

$$\Phi_q^+\left(z,\xi,W^2\right)=N_h z^\alpha(1-z)^\alpha(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

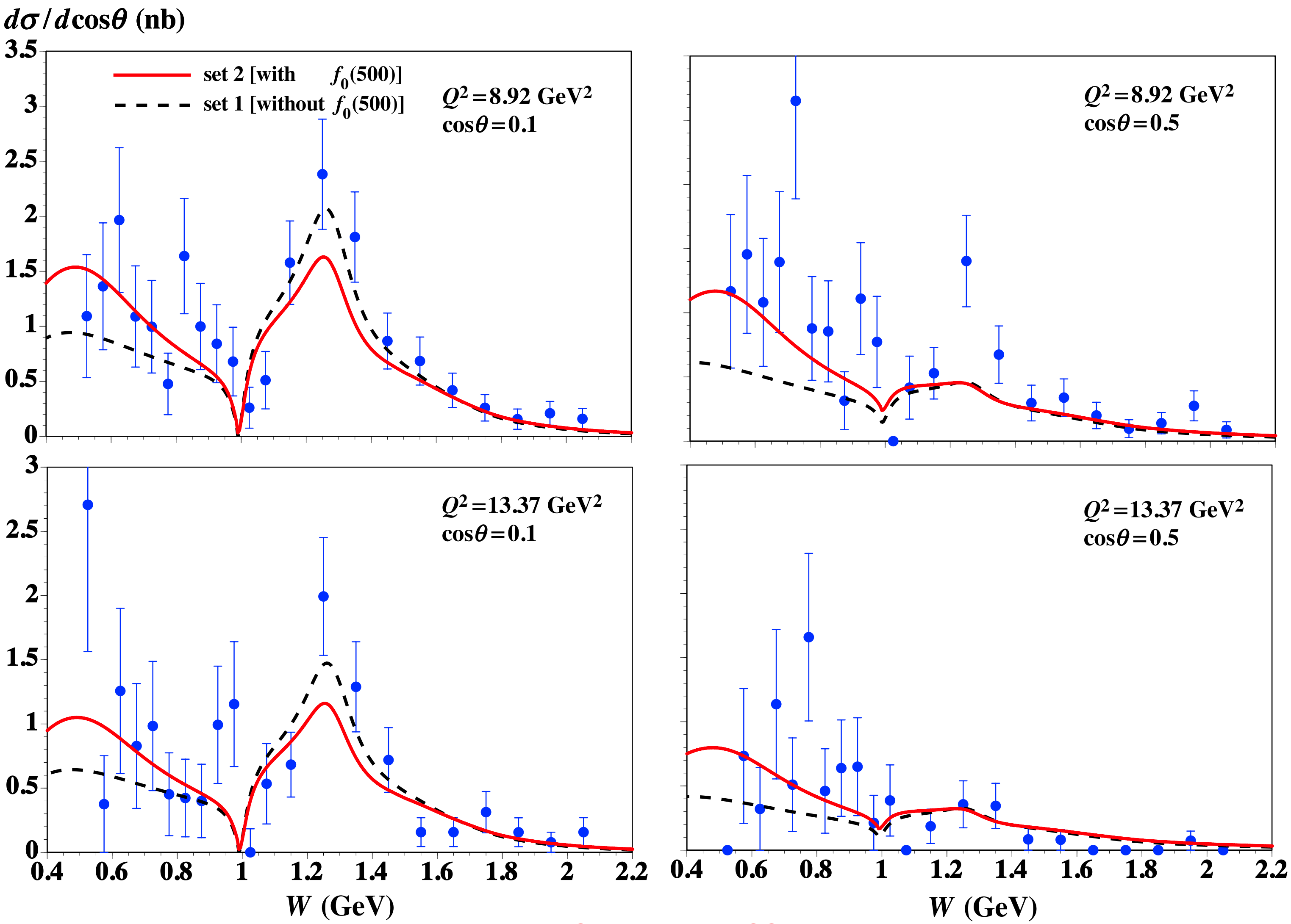
$$\tilde{B}_{10}(W)=[\frac{-3+\beta^2}{2}\frac{5R_\pi}{9}F_h(W^2)+\frac{5g_{f_0\pi\pi}f_{f_0}}{3\sqrt{2}\sqrt{(M_{f_0}^2-W^2)^2-\Gamma_{f_0}^2M_{f_0}^2}}]e^{i\delta_0}$$

$$\tilde{B}_{12}(W)=[\beta^2\frac{5R_\pi}{9}F_h(W^2)+\beta^2\frac{10g_{f_2\pi\pi}f_{f_2}M_{f_2}^2}{9\sqrt{2}\sqrt{(M_{f_2}^2-W^2)^2-\Gamma_{f_2}^2M_{f_2}^2}}]e^{i\delta_2}$$

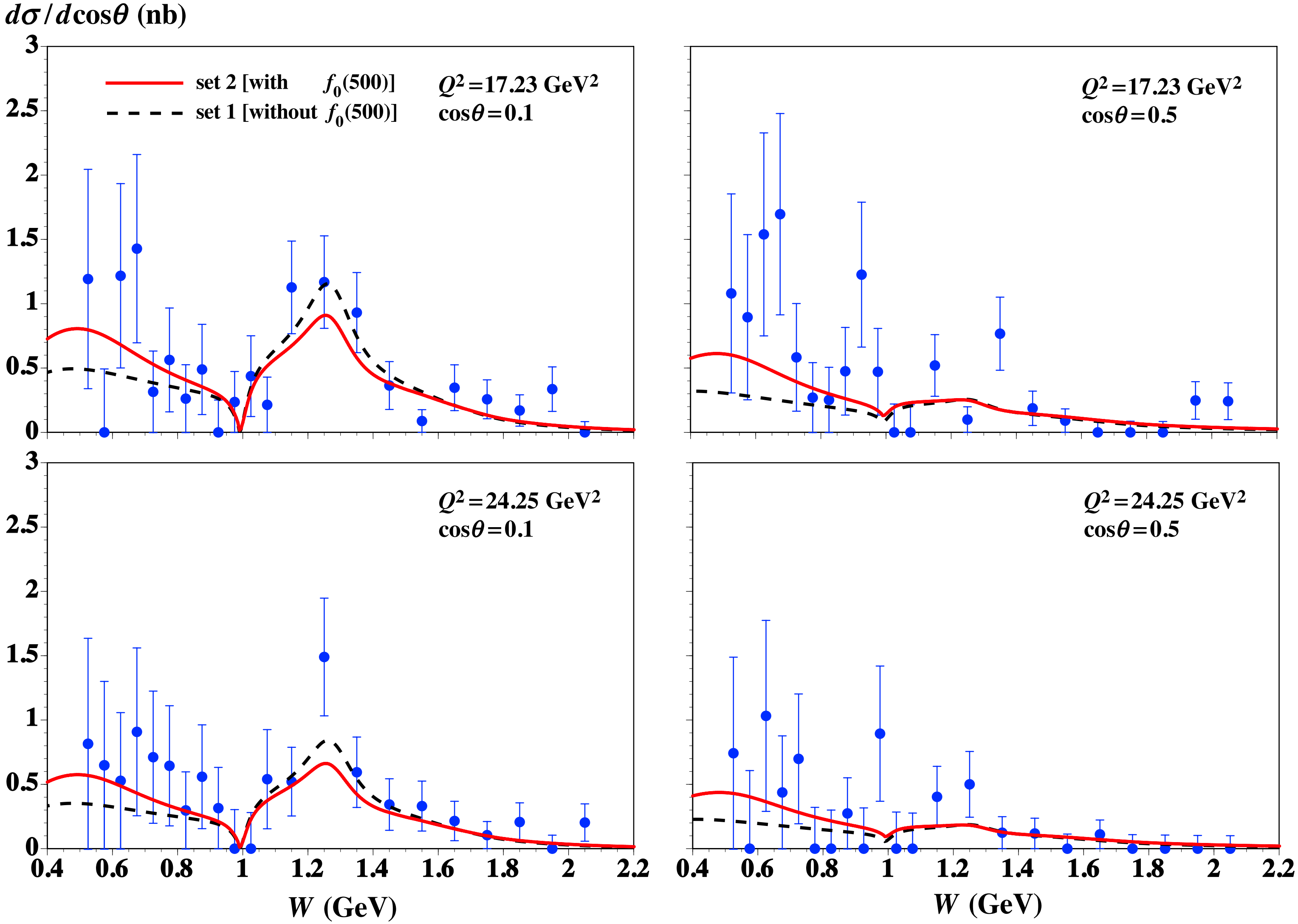
$$F_h(W^2)=\frac{1}{\left[1+\frac{W^2-4m_\pi^2}{\Lambda^2}\right]^{n-1}}$$

The function $F_h(W^2)$ is the form factor of the quark part of the energy-momentum tensor, and the parameter Λ is the momentum cutoff in the form factor. The parameter n is predicted as $n = 2$ at very high energy, because we have $d\sigma/d|\cos\theta|/\sim 1/W^6$ by the counting rule. In the asymptotic limit, $\boldsymbol{\alpha} = 1$.

By analyzing the Belle data, the values of parameters are obtained.



The W dependence of the differential cross section (in units of nb), and in comparison with Belle data.



The W dependence of the differential cross section (in units of nb), and in comparison with Belle data.

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

By considering the following sum rule, we can also obtain the energy-momentum form factors for pion.

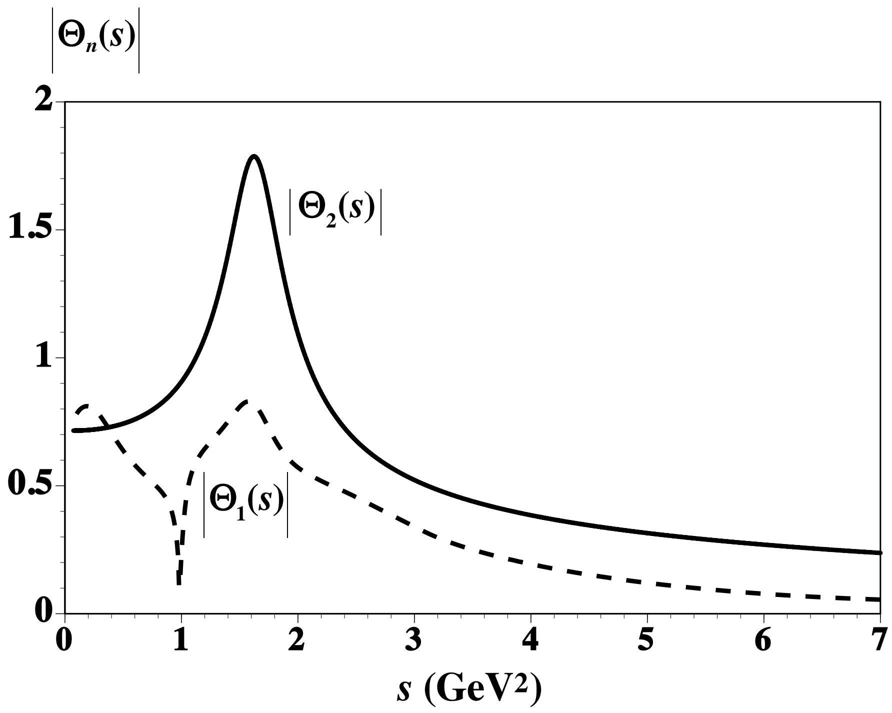
$$\int dz(2z-1)\Phi_q^+\left(z,\xi,W^2\right)=\frac{2}{(P^+)^2}\left\langle \pi^0(p_1)\pi^0(p_2)\right| T_q^{++}(0)\left|0\right\rangle$$

$$\left\langle \pi^0(p_1)\pi^0(p_2)\right| T^{\mu\nu}(0)\left|0\right\rangle=\frac{1}{2}\left[\left(sg^{\mu\nu}-P^\mu P^\nu\right)\Theta_1+\Delta^\mu\Delta^\nu\Theta_2\right]$$

$$\Theta_1=\frac{3}{5}(\tilde{B}_{12}-2\tilde{B}_{10}),\ \Theta_2=\frac{9}{5\beta^2}\tilde{B}_{12}$$

M. V. Polyakov, NPB **555** (1999) 231.
M. V. Polyakov and C. Weiss PRD 60 (1999) 114017.

$\Theta_1 \rightarrow$ Mechanical (pressure and shear force)
$\Theta_2 \rightarrow$ Mass



The timelike form factors Θ_1 and Θ_2

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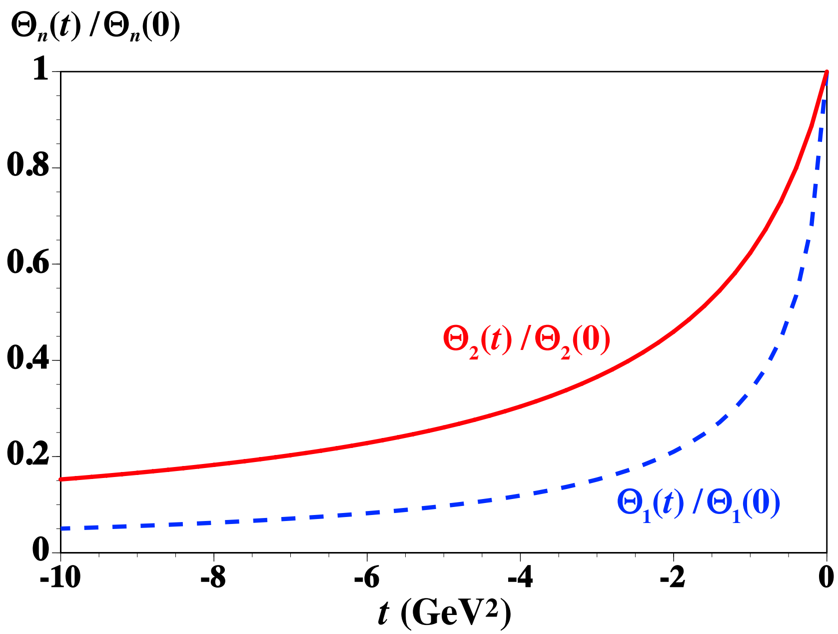
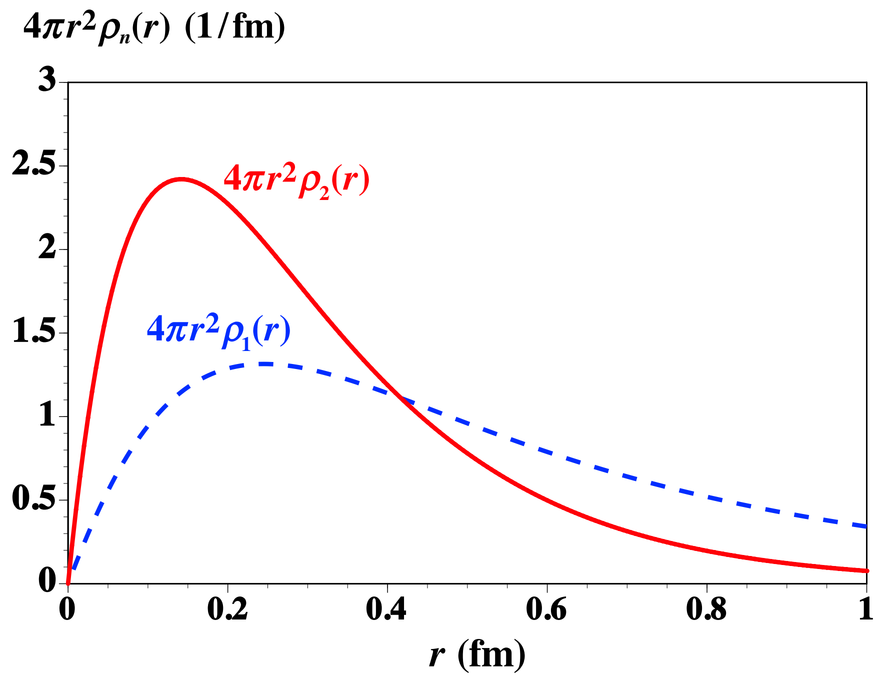
Timelike form factor → Spacelike form factor (pion radius) : dispersion relation

$$F(t)=\int_{4m^2}^{\infty}\frac{ds}{\pi}\frac{\mathrm{Im}(F(s))}{s-t-i\varepsilon}$$





The spacelike form factors Θ_1 and Θ_2

Fourier Transform of Θ_1 and Θ_2

Radius can be obtained by the following equation

$$<r^2>=6\int_{4m^2}^{\infty}\frac{\mathrm{Im}(F(s))}{s^2}$$

$\sqrt{\langle r^2\rangle}=$ 0.69 fm for Θ_2 Mass radius

$\sqrt{\langle r^2\rangle}=$ 1.45 fm for Θ_1 Mechanical radius(pressure and shear force)

In our analysis we introduce the additional phase for S-wave above the KK threshold. However, the additional phase could be add to D-wave phase above the threshold, in this case we have

Mass radius: 0.56-0.69 fm, Mechanical radius: 1.45-1.56 fm

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

Summary

- By analyzing the Belle data the pion GDAs are obtained, and the obtained GDAs can also give a good description of experimental data.

- The energy-momentum form factors for pion are calculated from the GDA of pion.

- This is the first finding on gravitational radii of hadrons from actual experimental measurements: we obtain the mass radius (0.56-0.69fm) and the mechanical radius (1.45-1.56fm).

Thank you very much