The complete twist-4 contributions to Semi-Inclusive DIS $e^- + N \rightarrow e^- + q(jet) + X$

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References: SY. Wei, Y.K. Song, K.B. Chen, & ZTL, PRD95, 074017 (2017);
K.B. Chen, S.Y. Wei & ZTL, Fron. Phys. 10, 101204 (2015);
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Introduction

Three-dimensional structure of the nucleon

↓

Study of azimuthal asymmetry → Cahn effect → higher twists

Study of single-spin asymmetry → Sivers effect → gauge link

Questions: Where does the gauge link come from? How do we deal with higher twist contributions systematically?

Look at the simplest case, the inclusive DIS, first!
Collinear approximation in inclusive DIS without QCD

Parton model without QCD:

\[ W_{\mu\nu}(q,p,S) = \left| \langle p | \bar{\psi}(0) \gamma_\lambda \gamma_\mu \gamma_\nu (k+q) \psi(z) | p, S \rangle \right|^2 \]

\[ W_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \hat{H}_{\mu\nu}(k,q) \hat{\phi}(k,p,S) \right] \]

the calculable hard part \( \hat{H}_{\mu\nu}(k,q) = \gamma_\mu(k+q)\gamma_\nu(2\pi)\delta_+(k+q)^2 \)

the quark-quark correlator \( \hat{\phi}(k,p,S) = \int d^4z e^{ikz} \langle p,S | \bar{\psi}(0) \psi(z) | p, S \rangle \)

Collinear approximation: \( p \approx p^+ \bar{n}, \quad k \approx xp \) (neglecting M/Q, i.e. taking \( M/Q \sim 0 \))

\[ W_{\mu\nu}(q,p) \approx \left[ (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) + \frac{1}{2xq \cdot p} (q + 2xp)_\mu (q + 2xp)_\nu \right] f_q(x) \]

operator expression of the number density: \( f_q(x) = \int \frac{dz^-}{2\pi} e^{-ixp^+z^-} \langle p | \bar{\psi}(0) \gamma^+ \psi(z) | p \rangle \)

no (local) gauge invariance!
Collinear expansion in DIS with "QCD multiple gluon scattering"

To get the gauge invariance, we need to take the "multiple gluon scattering" into account.

Collinear approximation:

- Approximating the hard part at \( k = x p \): \( \hat{H}^{(0)}_{\mu\nu}(k,q) \approx \hat{H}^{(0)}_{\mu\nu}(x), \quad \hat{H}^{(1)}_{\mu\nu}(k_1,k_2,q) \approx \hat{H}^{(1)}_{\mu\nu}(x_1,x_2) \)

- Keeping only the longitudinal component of the gluon field: \( A_\rho(y) \approx n \cdot A(y) \frac{p_\rho}{n \cdot p} \)

- Using Ward identities, e.g., \( p_\rho \hat{H}^{(1L)}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon} \) all \( \hat{H}^{(j)}_{\mu\nu}(x_i) \)'s reduce to \( \hat{H}^{(0)}_{\mu\nu}(x) \)

- Adding all the terms together
**Inclusive DIS: LO pQCD, leading twist**

\[
W_{\mu\nu}(q,p,S) \approx \tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \hat{\Phi}^{(0)}(k;p,S)\hat{H}^{(0)}_{\mu\nu}(x) \right]
\]

\[
\hat{\Phi}^{(0)}(k;p,S) = \int d^4ze^{ikz} \langle p,S | \bar{\psi}(0) \mathcal{A}(0,z)\psi(z) | p,S \rangle
\]

The gauge invariant un-integrated quark-quark correlator: contain QCD interaction!

\[
\mathcal{A}(0,z) = \mathcal{A}^\dagger(-\infty,0)\mathcal{A}(-\infty,z),
\]

\[
\mathcal{A}(-\infty,z) = Pe^{-ig\int^{z^\perp}_{-\infty} dy^- A^+(z^+, y^- , z^\perp)} = 1 + ig\int_{-\infty}^{z^\perp} dy^- A^+(z^+, y^- , z^\perp) + \frac{1}{2}(ig)^2 \int_{-\infty}^{z^\perp} dy^- \int_{-\infty}^{y^-} dy'^- A^+(z^+, y'^- , z^\perp) A^+(z^+, y'^- , z^\perp) + \ldots
\]

**Graphically:**

**collinear approximation**

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DIS2018
Collinear expansion:

Expanding the hard part at \( k = xp \):

\[
\hat{H}_{\mu\nu}^{(0)}(k,q) = \hat{H}_{\mu\nu}^{(0)}(x) + \frac{\partial \hat{H}_{\mu\nu}^{(0)}(x)}{\partial k^\rho} \omega^\rho_{\ \rho'} k_{\rho'} + \ldots
\]

\[
\hat{H}_{\mu\nu}^{(1)^\rho}(k_1, k_2, q) = \hat{H}_{\mu\nu}^{(1)^\rho}(x_1, x_2) + \frac{\partial \hat{H}_{\mu\nu}^{(1)^\rho}(x_1, x_2)}{\partial k^\sigma_{\ 1}} \omega_{\sigma'_{\ 1}} k_{\sigma'} + \ldots
\]

Decomposition of the gluon field:

\[
A_{\rho}(y) = n \cdot A(y) \frac{p^\rho_{\ n}}{n \cdot p} + \omega^\rho_{\ \rho'} A_{\rho'}(y)
\]

Using the Ward identities such as,

\[
\frac{\partial \hat{H}_{\mu\nu}^{(0)}(x)}{\partial k^\rho} = - \hat{H}_{\mu\nu}^{(1)^\rho}(x,x)
\]

\[
p^\rho_{\ n} \hat{H}_{\mu\nu}^{(1,1)^\rho}(x_1, x_2) = \frac{\hat{H}_{\mu\nu}^{(0)}(x_1)}{x_2 - x_1 - i\epsilon}
\]

to replace the derivatives etc.

Adding all terms with the same hard part together:

\[
\hat{H}_{\mu\nu}^{(0)}(x) \equiv \hat{H}_{\mu\nu}^{(0)}(k = xp, q)
\]

\[
\frac{\partial \hat{H}_{\mu\nu}^{(0)}(x)}{\partial k^\rho} \equiv \frac{\partial \hat{H}_{\mu\nu}^{(0)}(k, q)}{\partial k^\rho} \bigg|_{k = xp}
\]

\[
x = k^+ / p^+
\]

\[
\omega^\rho_{\ \rho'} \equiv g_{\rho\rho'} - n_{\rho} n_{\rho'}
\]

\[
\omega^\rho_{\ \rho'} k_{\rho'} = (k - xp)_\rho
\]

\[
k^\pm = \frac{1}{\sqrt{2}} (k_0 \pm k_3)
\]

\[
n = (0,1,0_\perp)
\]

\[
\bar{n} = (1,0,0_\perp)
\]
Inclusive DIS: LO pQCD, leading & higher twists

\[ W_{\mu\nu}(q, p, S) = \tilde{W}_{\mu\nu}^{(0)}(q, p, S) + \tilde{W}_{\mu\nu}^{(1)}(q, p, S) + \tilde{W}_{\mu\nu}^{(2)}(q, p, S) + \ldots \]

\[ \tilde{W}_{\mu\nu}^{(0)}(q, p, S) = \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \hat{\Phi}^{(0)}(k, p, S) \hat{H}_{\mu\nu}^{(0)}(x) \right] \]

\[ \hat{\Phi}^{(0)}(k, p, S) = \int d^4z e^{ikz} \langle p, S \mid \bar{\psi}(0) \mathcal{A}(0, z) \psi(z) \mid p, S \rangle \]

\[ \text{depends on } x \text{ only!} \]

\[ \tilde{W}_{\mu\nu}^{(1)}(q, p, S) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \text{Tr} \left[ \hat{\Phi}^{(1)}_\rho(k_1, k_2, p, S) \hat{H}_{\mu\nu}^{(1)}(x_1, x_2) \omega_\rho \rho' \right] \]

\[ \hat{\Phi}^{(1)}_\rho(k_1, k_2, p, S) = \int d^4z d^4y e^{ik_1y+ik_2(z-y)} \langle p, S \mid \bar{\psi}(0) \mathcal{A}(0, y) D_\rho(y) \mathcal{A}(y, z) \psi(z) \mid p, S \rangle \]

\[ D_\rho(y) = -i\partial_\rho + gA_\rho(y) \]

gauge invariant quark-gluon-quark correlator

twist-2, 3 and 4 contributions

twist-3, 4 and 5 contributions

\[ \text{A consistent framework for inclusive DIS } e^- N \rightarrow e^- X \text{ including leading & higher twists} \]

collinear expansion

DIS2018

2018年4月，神戸
Simplified expressions for hadronic tensors

The “collinearly expanded hard parts” take very simple forms such as:

\[ \hat{H}^{(0)}_{\mu\nu}(x) = \hat{h}^{(0)}_{\mu\nu}\delta(x - x_B), \quad \hat{h}^{(0)}_{\mu\nu} = \gamma_{\mu} \bar{\nu}_{\nu} \]

\[ \hat{H}^{(1, L)}_{\mu\nu}(x_1, x_2) \omega^{\rho^\prime}_{\rho} = \frac{\pi}{2q \cdot p} \hat{h}^{(1)}_{\mu\nu} \omega^{\rho^\prime}_{\rho} \delta(x_1 - x_B), \quad \hat{h}^{(1)}_{\mu\nu} = \gamma_{\mu} \bar{\nu}_{\nu} \gamma_{\rho^\prime} \gamma_{\rho} \]

\( \hat{W}^{(0)}_{\mu\nu}(q, p, S) = \text{Tr} \left[ \hat{\Phi}^{(0)}(x_B; p, S) \hat{h}^{(0)}_{\mu\nu} \right] \)

\( \hat{\Phi}^{(0)}(x; p, S) \equiv \int \frac{d^4k}{(2\pi)^4} \delta(x - \frac{k^+}{p^+}) \hat{\Phi}^{(0)}(k; p, S) = \int \frac{p^+ dz^-}{2\pi} e^{ixp^+z^-} \langle p, S | \bar{\psi}(0) \mathcal{L}(0, z^-) \psi(z^-) | p, S \rangle \)

\( \text{one-dimensional gauge invariant quark-quark correlator} \)

\( \hat{W}^{(1)}_{\mu\nu}(q, p, S) = \frac{\pi}{2q \cdot p} \text{Tr} \left[ \hat{\phi}^{(1)}_{\rho}(x_B; p, S) \hat{h}^{(1)}_{\mu\nu} \omega^{\rho^\prime}_{\rho} \right] \)

\( \hat{\phi}^{(1)}_{\rho}(x; p, S) \equiv \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \delta(x - \frac{k_1^+}{p^+}) \hat{\Phi}^{(1)}_{\rho}(k_1, k_2; p, S) = \int \frac{p^+ dz^-}{2\pi} e^{ixp^+z^-} \langle p, S | \bar{\psi}(0) D_{\rho}(0) \mathcal{L}(0, z^-) \psi(z^-) | p, S \rangle \)

\( \text{the involved one-dimensional gauge invariant quark-gluon-quark correlator} \)

**ONE dimensional, depend only on ONE parton momentum!**
PDFs defined via quark-quark correlator

- Expand the quark-quark correlator in terms of the $\Gamma$-matrices:

\[
\hat{\Phi}^{(0)}(x;p,S) = \frac{1}{2} \left[ \Phi^{(0)}(x;p,S) + i\gamma_5 \tilde{\Phi}^{(0)}(x;p,S) + \gamma^\alpha \Phi_\alpha^{(0)}(x;p,S) + \gamma_5 \gamma^\alpha \tilde{\Phi}_\alpha^{(0)}(x;p,S) + i\gamma_5 \sigma^{\alpha\beta} \Phi_{\alpha\beta}^{(0)}(x;p,S) \right]
\]

(scalar) (pseudo-scalar) (vector) (axial vector) (tensor)

- Make Lorentz decompositions

\[
\Phi^{(0)}(x;p,S) = Me(x)
\]
\[
\tilde{\Phi}^{(0)}(x;p,S) = \lambda M e_L(x)
\]
\[
\Phi_\alpha^{(0)}(x;p,S) = p^+ \bar{n}_\alpha f_1(x) + M \varepsilon_{\perp\alpha\rho} S^\rho_T f_T(x) + \frac{M^2}{p^+} n_\alpha f_3(x)
\]
\[
\tilde{\Phi}_\alpha^{(0)}(x;p,S) = \lambda p^+ \bar{n}_\alpha g_{1L}(x) + M S_{T\alpha} g_T(x) + \lambda \frac{M^2}{p^+} n_\alpha g_{3L}(x)
\]
\[
\Phi_{\rho\alpha}^{(0)}(x;p,S) = p^+ \bar{\eta}_{[\rho} S_{T\alpha]} h_{1T}(x) - M \varepsilon_{T\rho\alpha} h_T(x) + \lambda M \bar{n}_{[\rho} n_{\alpha]} h_L(x) + \frac{M^2}{p^+} n_{[\rho} S_{T\alpha]} h_{3T}(x)
\]

the scalar functions are the one-dimensional PDFs, e.g.,

\[
f_1(x) = \frac{1}{p^+} n^\alpha \Phi_\alpha^{(0)}(x;p,S) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p,S \mid \bar{\psi}(0) \mathcal{L}(0,z^-) \gamma^+ \psi(z^-) \rangle \mid p,S \rangle
\]
Conclusion: Collinear expansion provides a systematical way of dealing with leading and higher twist contributions including gauge invariance.

Advantages:
- Gauge link obtained automatically
- Leading and higher twist systematically
- Extremely simplified expressions

Question: Can we extend collinear expansion to semi-inclusive DIS?
Collinear expansion in high energy reactions

Successfully to all processes where only ONE hadron is explicitly involved.

Inclusive DIS  \( e^- N \rightarrow e^- X \)

Yes!

where collinear expansion was first formulated.


Semi-Inclusive DIS  \( e + N \rightarrow e + q(jet) + X \)

Yes!

ZTL & X.N. Wang, PRD (2007);

Inclusive  \( e^- + e^+ \rightarrow h + X \)

Yes!

S.Y. Wei, Y.K Song, ZTL, PRD (2014);

Semi-Inclusive  \( e^- + e^+ \rightarrow h + \bar{q}(jet) + X \)

Yes!

Semi-Inclusive DIS $e^- + N \rightarrow e^- + q(jet) + X$

$W^{(si)}_{\mu\nu}(q, p, S, k') = \tilde{W}^{(0,si)}_{\mu\nu}(q, p, S, k') + \tilde{W}^{(1,si)}_{\mu\nu}(q, p, S, k') + \tilde{W}^{(2,si)}_{\mu\nu}(q, p, S, k') + ...$

\[
\tilde{W}^{(0,si)}_{\mu\nu}(q, p, S, k') = \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\Phi^{(0)}(k, p, S)\hat{H}^{(0)}_{\mu\nu}(x)] (2E_k)(2\pi)^3 \delta^3(\tilde{k}' - \tilde{k} - \tilde{q})
\]

\[
\hat{\Phi}^{(0)}(k, p, S) = \int d^4ze^{ikz} \langle p, S | \bar{\psi}(0) \mathcal{L}(0, z) \psi(z) | p, S \rangle
\]

depends on $x$ only!

\[
\tilde{W}^{(1,si)}_{\mu\nu}(q, p, S, k') = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \text{Tr}[\Phi^{(1)}(k_1, k_2, p, S) \hat{H}^{(1,\rho)}_{\mu\nu}(x_1, x_2) \omega_{\rho}^{\rho'}] (2E_k)(2\pi)^3 \delta^3(\tilde{k}' - \tilde{k}_c - \tilde{q})
\]

\[
\hat{\Phi}^{(1)}(k_1, k_2, p, S) = \int d^4zd^4ye^{ik_1y + ik_2(z-y)} \langle p, S | \bar{\psi}(0) \mathcal{L}(0, y) D_{\rho}(y) \mathcal{L}(y, z) \psi(z) | p, S \rangle
\]

twist-2, 3 and 4 contributions

twist-3, 4 and 5 contributions

A consistent framework for $e^- N \rightarrow e^- + q(jet) + X$ at LO pQCD including higher twists

Semi-Inclusive DIS \( e^- + N \rightarrow e^- + q(\text{jet}) + X \)

Simplified expressions for hadronic tensors

The “collinearly expanded hard parts” take the simple forms such as:

\[
\hat{H}^{(0)}_{\mu\nu}(x) = \hat{h}^{(0)}_{\mu\nu}(x - x_B), \quad \hat{h}^{(0)}_{\mu\nu} = \gamma_\mu \gamma_\nu
\]

\[
\hat{H}^{(1, L)\rho}_{\mu\nu}(x_1, x_2) \omega^\rho_{\rho'} = \frac{\pi}{2q \cdot p} \hat{h}^{(1)}_{\mu\nu} \omega^\rho_{\rho'}(x_1 - x_B), \quad \text{where} \quad \hat{h}^{(1)}_{\mu\nu} = \gamma_\mu \gamma^\rho \gamma_\nu, \quad \text{depends only on } x_1!
\]

The involved three-dimensional gauge invariant quark-quark correlator

\[
\hat{W}^{(0, si)}_{\mu\nu}(q, p, S; k_\perp) = \text{Tr} \left[ \hat{\Phi}^{(0)}(x_B, k_\perp; p, S) \hat{h}^{(0)}_{\mu\nu} \right]
\]

\[
\hat{\Phi}^{(0)}(x, k_\perp; p, S) = \int \frac{d^4k_1}{(2\pi)^4} \delta(x - x_B) \delta^2(k_\perp - k_\perp) \hat{\Phi}^{(0)}(k; p, S) = \int \frac{p^+ dz^-}{2\pi} d^2z_\perp \exp^{z_\perp - i k_\perp \cdot z_\perp} \langle N | \bar{\psi}(0) \mathcal{L}(0, z) \psi(z) | N \rangle
\]

The involved three-dimensional gauge invariant quark-gluon-quark correlator

THREE dimensional, depend only on ONE parton momentum!
TMD PDFs defined via **quark-j-gluon-quark correlator**

Relationships obtained from the QCD equation of motion \( \gamma \cdot D(z)\psi(z) = 0 \)

E.g.:

**Twist-3:**

\[
f^K_{ds}(x,k_\perp) - g^K_{ds}(x,k_\perp) = -x \left[ f^K_S(x,k_\perp) - ig^K_S(x,k_\perp) \right]
\]

\( K = \text{null}, \quad S = T; \quad K = \perp, \quad S = \text{null, L, or T} \)

**Twist-4:**

\[
x^2 f_3(x,k_\perp) = xf_{-3d}(x,k_\perp) = -f^M_{3dd}(x,k_\perp)
\]

\[
x^2 f^\perp_{3T}(x,k_\perp) = xf^\perp_{-3dT}(x,k_\perp) = -f^M_{-3ddT}(x,k_\perp)
\]

\[
x^2 g_{3L}(x,k_\perp) = xf_{-3dL}(x,k_\perp) = -f^M_{-3ddL}(x,k_\perp)
\]

\[
x^2 g^\perp_{3T}(x,k_\perp) = xf^\perp_{-3dT}(x,k_\perp) = -f^M_{-3ddT}(x,k_\perp)
\]

See e.g., Y.K. Song, J.H. Gao, ZTL & X.N. Wang, PRD (2014); SY. Wei, Y.K. Song, K.B. Chen, & ZTL, PRD95, 074017 (2017); .......
General kinematic analysis for $e^{-} + N \rightarrow e^{-} + h + X$

The cross section in the $\gamma^*p$ c.m. frame (only parity conserved part)

$$\frac{d\sigma}{dx dy d\phi_s d^2p_{hT}} = \frac{\alpha^2}{xyQ^2} \mathcal{K}(\mathcal{F}_{UU} + \lambda_T \mathcal{F}_{LU} + \lambda_N \mathcal{F}_{UL} + \lambda_T \lambda_N \mathcal{F}_{LL} + |\tilde{S}_T| \mathcal{F}_{UT} + \lambda_T |\tilde{S}_T| \mathcal{F}_{LT})$$

- $\mathcal{F}_{UU} = F_{UU,T} + \varepsilon F_{UU,L} + \cos\phi \sqrt{2\varepsilon(1+\varepsilon)} F_{UU}^{\cos\phi} + \cos 2\phi \varepsilon F_{UU}^{\cos 2\phi}$
- $\mathcal{F}_{LU} = \sin\phi \sqrt{2\varepsilon(1-\varepsilon)} F_{LU}^{\sin\phi}$
- $\mathcal{F}_{UL} = \sin\phi \sqrt{2\varepsilon(1+\varepsilon)} F_{UL}^{\sin\phi} + \sin 2\phi \varepsilon F_{UL}^{\sin 2\phi}$
- $\mathcal{F}_{LL} = \sqrt{1-\varepsilon^2} F_{LL} + \cos\phi \sqrt{2\varepsilon(1-\varepsilon)} F_{LL}^{\cos\phi}$
- $\mathcal{F}_{UT} = \sin\phi_s \sqrt{2\varepsilon(1+\varepsilon)} F_{UT}^{\sin\phi_s} + \sin(\phi - \phi_s)(F_{UT,T}^{\sin(\phi-\phi_s)} + \varepsilon F_{UT,L}^{\sin(\phi-\phi_s)})$
  + $\sin(\phi + \phi_s)\varepsilon F_{UT}^{\sin(\phi+\phi_s)} + \sin(2\phi - \phi_s)\sqrt{2\varepsilon(1+\varepsilon)} F_{UT}^{\sin(2\phi-\phi_s)} + \sin(3\phi - \phi_s)\varepsilon F_{UT}^{\sin(3\phi-\phi_s)}$
- $\mathcal{F}_{LT} = \cos\phi_s \sqrt{2\varepsilon(1-\varepsilon)} F_{LT}^{\cos\phi_s} + \cos(\phi - \phi_s)\sqrt{1-\varepsilon^2} F_{LT}^{\cos(\phi-\phi_s)} + \cos(2\phi - \phi_s)\sqrt{2\varepsilon(1-\varepsilon)} F_{LT}^{\cos(2\phi-\phi_s)}$

Totally 18 structure functions: $F_{jx}^{yy}(x, \xi, p_{hT}^2, Q)$
General kinematic analysis for \( e^- + N \rightarrow e^- + q(jet) + X \)

The cross section in the \( \gamma^*p \) c.m. frame (only parity conserved part)

\[
\frac{d\sigma}{dx dy d\phi_S d^2 k_T} = \frac{\alpha^2}{xyQ^2} \mathcal{K}(\mathcal{V}_{UU} + \lambda_t \mathcal{V}_{LU} + \lambda_N \mathcal{V}_{UL} + \lambda_t \lambda_N \mathcal{V}_{LL} + |\vec{S}_T| \mathcal{V}_{UT} + \lambda_t |\vec{S}_T| \mathcal{V}_{LT})
\]

\[
\mathcal{V}_{UU} = W_{UU,t} + \varepsilon W_{UU,L} + \cos \phi \sqrt{2\varepsilon(1+\varepsilon)} W_{UU}^{\cos \phi} + \cos 2\varepsilon W_{UU}^{\cos 2\phi}
\]

\[
\mathcal{V}_{LU} = \sin \phi \sqrt{2\varepsilon(1-\varepsilon)} W_{LU}^{\sin \phi}
\]

\[
\mathcal{V}_{UL} = \sin \phi \sqrt{2\varepsilon(1+\varepsilon)} W_{UL}^{\sin \phi} + \sin 2\varepsilon W_{UL}^{\sin 2\phi}
\]

\[
\mathcal{V}_{LL} = \sqrt{1-\varepsilon^2} W_{LL} + \cos \phi \sqrt{2\varepsilon(1-\varepsilon)} W_{LL}^{\cos \phi}
\]

\[
\mathcal{V}_{UT} = \sin \phi_S \sqrt{2\varepsilon(1+\varepsilon)} W_{UT}^{\sin \phi_S} + \sin(\phi - \phi_S)(W_{UT,T}^{\sin(\phi-\phi_S)} + \varepsilon W_{UT,L}^{\sin(\phi-\phi_S)})
\]

\[
+ \sin(\phi + \phi_S) W_{UT}^{\sin(\phi+\phi_S)} + \sin(2\phi - \phi_S) \sqrt{2\varepsilon(1+\varepsilon)} W_{UT}^{\sin(2\phi-\phi_S)} + \sin(3\phi - \phi_S) \varepsilon W_{UT}^{\sin(3\phi-\phi_S)}
\]

\[
\mathcal{V}_{LT} = \cos \phi_S \sqrt{2\varepsilon(1-\varepsilon)} W_{LT}^{\cos \phi_S} + \cos(\phi - \phi_S) \sqrt{1-\varepsilon^2} W_{LT}^{\cos(\phi-\phi_S)} + \cos(2\phi - \phi_S) \sqrt{2\varepsilon(1-\varepsilon)} W_{LT}^{\cos(2\phi-\phi_S)}
\]

Totally 18 structure functions: \( W^{yy}_{jxx} = W^{yy}_{jxx}(x, k_T^2, Q) \)
Semi-Inclusive DIS: \( e^- + N \rightarrow e^- + q(jet) + X \)

Complete results for structure functions up to twist-4

\[
W_{U,U,T} = x f_1 + 4 x^2 \kappa_M^2 f_{+3dd}, \quad W_{U,U,L} = 8 x^3 \kappa_M^2 f_3
\]

\[
W_{U,U}^{\cos2\phi} = -2 x^2 \kappa_M^2 \bar{k}_\perp f_{-3d}\]

\[
W_{U,U}^{\sin2\phi} = 2 x^2 \kappa_M^2 \bar{k}_\perp f_{+3dL}\]

\[
W_{L,L} = x g_{1L} + 4 x^2 \kappa_M^2 f_{+3ddL}\]

\[
W_{U,L}^{\sin(\phi-\phi_S)} = \bar{k}_\perp (x f_{1T} + 4 x^2 \kappa_M^2 f_{+3ddT}), \quad W_{U,L}^{\sin(\phi+\phi_S)} = -2 x^2 \kappa_M^2 \bar{k}_\perp f_{-3d}\]

\[
W_{U,U}^{\sin(3\phi-\phi_S)} = -x^2 \kappa_M^2 \bar{k}_\perp^3 (f_{+3dT} - f_{-3dT})\]

\[
W_{L,L}^{\cos(\phi-\phi_S)} = \bar{k}_\perp (x g_{1T} + 4 x^2 \kappa_M^2 f_{+3ddT})\]

\[
W_{U}^{\sin2\phi} = -2 x^2 \kappa_M^2 \bar{k}_\perp f_{1T}\]

\[
W_{L}^{\sin(2\phi-\phi_S)} = -x^2 \kappa_M^2 \bar{k}_\perp^2 f_{L}\]

\[
W_{L}^{\cos(2\phi-\phi_S)} = -x^2 \kappa_M^2 \bar{k}_\perp^2 g_{L}\]

\[
\kappa_M \equiv \frac{M}{Q}, \quad \bar{k}_\perp \equiv \frac{|\bar{k}_\perp|}{M}\]

\[
W_{U,U}^{\cos\phi} = \bar{k}_\perp f_{L}\]

\[
W_{L,U}^{\sin\phi_S} = -2 x^2 \kappa_M^2 \bar{k}_\perp g_{L}\]

\[
W_{L,T}^{\cos\phi_S} = -x^2 \kappa_M^2 \bar{k}_\perp g_{T}\]

(1) twist 2 and 4 \( \leftrightarrow \) even number of \( \phi \) and \( \phi_S \)

(2) Wherever there is twist-2 contribution, there is a twist-4 addendum to it.

Semi-Inclusive DIS: \( e^- + N \rightarrow e^- + q(jet) + X \)

Azimuthal asymmetries up to twist-4, e.g.,

At leading twist (twist-2)

\[
\langle \sin(\phi - \phi_S) \rangle_{UT} |_{\text{twist2}} = k_\perp \frac{xf_{1T}^\perp}{2 f_1} \left(1 - \alpha_{UT}^2 \right)
\]

Sivers asymmetry \[\rightarrow\] Sivers function \( f_{1T}^\perp \)

\[
\langle \cos(\phi - \phi_S) \rangle_{LT} |_{\text{twist2}} = k_\perp \frac{C(y) g_{1T}^\perp}{A(y) 2 f_1} \left(1 - \alpha_{LT}^2 \right)
\]

\[\rightarrow\] tran-helicity \( g_{1T}^\perp \)

Including twist-4 contributions

\[
\langle \sin(\phi - \phi_S) \rangle_{UT} = k_\perp \frac{xf_{1T}^\perp}{2 f_1} (1 - \alpha_{UT}^2 \kappa_M^2)
\]

\( \alpha_{UT} = 16x^2 \frac{1 - y}{1 + (1 - y)^2} \left( \frac{f_3}{f_1} - \frac{f_{3T}^\perp}{f_{1T}^\perp} \right) + 4x \left( \frac{f_{+3dd}}{f_1} - \frac{f_{+3dd}^\perp}{f_{1T}^\perp} \right) \)

\[
\langle \cos(\phi - \phi_S) \rangle_{LT} = k_\perp \frac{C(y) g_{1T}^\perp}{A(y) 2 f_1} (1 - \alpha_{LT}^2 \kappa_M^2)
\]

\( \alpha_{LT} = 16x^2 \frac{1 - y}{1 + (1 - y)^2} \frac{f_3}{f_1} + 4x \left( \frac{f_{+3dd}}{f_1} \frac{f_{+3dd}^\perp}{g_{1T}^\perp} \right) \)

The Cahn’s effect:

\[
\langle \cos \phi \rangle_{UU} = -k_\perp \kappa_M \frac{B(y) xf_{1T}^\perp}{A(y) f_1}
\]

\[
\langle \cos 2\phi \rangle_{UU} = -k_\perp^2 \kappa_M^2 E(y) \frac{f_{-3d}^\perp}{A(y) 2 f_1}
\]
Semi-Inclusive DIS: \( e^- + N \rightarrow e^- + q(\text{jet}) + X \)


Relationships between twist-4 PDFs and their twist-2 counterparts at \( g=0 \) (Fermion gas model)

\[
\begin{align*}
xf_{3d} &= \frac{k^2}{2M^2} xf_{3d}^\perp = x^2 f_3 = -\frac{k^2}{2M^2} xf_1 \\
xg_{3dL} &= i \frac{k^2}{2M^2} xg_{3dL}^\perp = -x^2 g_{3L} = \frac{k^2}{2M^2} xg_{1L} \\
xf_{3dT}^\perp &= -\frac{k^2}{2M^2} xf_{3dT}^\perp = x^2 f_{3T}^\perp = -\frac{k^2}{2M^2} f_{1T} \\
xg_{3dT}^{13} &= -i \frac{k^2}{2M^2} xg_{3dT}^{14} = -x^2 g_{3T}^\perp = \frac{k^2}{2M^2} g_{1T}^\perp
\end{align*}
\]

\[
\alpha_{UT} \approx \frac{k^2}{M^2} \left[ \frac{\partial \ln f_1}{\partial \ln x} - \frac{\partial \ln f_{1T}^\perp}{\partial \ln x} \right]
\]

\[
\alpha_{LT} \approx -\frac{k^2}{M^2} \left[ \frac{8(1-y)}{1+(1-y)^2} \frac{\partial \ln f_1}{\partial \ln x} - \frac{\partial \ln g_{1T}^\perp}{\partial \ln x} \right]
\]

\[
\langle \cos \phi \rangle_{UU} \approx -k^2 \frac{B(y)}{A(y)} \quad \langle \cos 2\phi \rangle_{UU} \approx k^2 \frac{E(y)}{A(y)}
\]
Collinear expansion is crucial to deal with leading & higher twists systematically and is shown to be applicable to SIDIS $e + N \rightarrow e + q(jet) + X$

Complete twist-4 results for the semi-inclusive DIS $e + N \rightarrow e + q(jet) + X$ have been obtained.

We also propose to use relationships between twist-4 PDFs and their twist-2 counterparts obtained in the “Fermion gas model” to make a rough estimation of twist-4 contributions. The results suggest that twist-4 contributions might be very significant to the well-known twist-2 asymmetries such as the Sivers asymmetry thus have large impact on the study of TMDs.

Thank you for your attention!
TMD PDFs defined via quark-quark correlator $\hat{\Phi}^{(0)}(x,k_\perp; p, S)$

<table>
<thead>
<tr>
<th>quark polarization</th>
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</thead>
<tbody>
<tr>
<td>nucleon polarization</td>
<td>$f_1(x,k_\perp)$ number density</td>
<td>$g_{1\perp}(x,k_\perp)$ helicity distribution</td>
<td>$h_1^T(x,k_\perp)$ Boer-Mulders function</td>
</tr>
<tr>
<td>Sivers function</td>
<td>Worm-gear/longi-transversity</td>
<td></td>
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</tr>
</tbody>
</table>

**Twist-2 TMD PDFs**

**Twist-3 TMD PDFs**

| nucleon polarization | $e(x,k_\perp), f_1^+, f_T(x,k_\perp)$ number density | $g_1^+(x,k_\perp)$ helicity distribution | $h(x,k_\perp)$ Boer-Mulders function |
| Sivers function | Worm gear/longi-transversity |

| nucleon polarization | $e_+^T(x,k_\perp), f_+^T(x,k_\perp), f_T^+(x,k_\perp)$ number density | $g_+^T(x,k_\perp), g_1^+(x,k_\perp)$ helicity distribution | $h_+^T(x,k_\perp)$ Boer-Mulders function |
| Sivers function | Worm gear/longi-transversity |

**Twist-4 TMD PDFs**

| nucleon polarization | $f_3(x,k_\perp)$ number density | $g_{2\perp}(x,k_\perp)$ helicity distribution | $h_2^T(x,k_\perp)$ Boer-Mulders function |
| Sivers function | Worm-gear/longi-transversity |

| nucleon polarization | $f_3^+(x,k_\perp)$ Sivers function | $g_{3\perp}^+(x,k_\perp)$ helicity distribution | $h_3^T(x,k_\perp)$ Boer-Mulders function |
| Sivers function | Worm-gear/longi-transversity |

**Twist-3 TMD PDFs**

| nucleon polarization | $e(x,k_\perp), f_1^+, f_T(x,k_\perp)$ number density | $g_1^+(x,k_\perp)$ helicity distribution | $h(x,k_\perp)$ Boer-Mulders function |
| Sivers function | Worm gear/longi-transversity |

| nucleon polarization | $e_+^T(x,k_\perp), f_+^T(x,k_\perp), f_T^+(x,k_\perp)$ number density | $g_+^T(x,k_\perp), g_1^+(x,k_\perp)$ helicity distribution | $h_+^T(x,k_\perp)$ Boer-Mulders function |
| Sivers function | Worm gear/longi-transversity |

**Twist-4 TMD PDFs**

| nucleon polarization | $f_3(x,k_\perp)$ number density | $g_{2\perp}(x,k_\perp)$ helicity distribution | $h_2^T(x,k_\perp)$ Boer-Mulders function |
| Sivers function | Worm-gear/longi-transversity |

| nucleon polarization | $f_3^+(x,k_\perp)$ Sivers function | $g_{3\perp}^+(x,k_\perp)$ helicity distribution | $h_3^T(x,k_\perp)$ Boer-Mulders function |
| Sivers function | Worm-gear/longi-transversity |
The Lorentz decomposition of the quark-gluon-quark correlator:

\[ \hat{\phi}_\rho^{(1)} (x, k_\perp ; p, S) = \frac{1}{2} \left[ \phi_\rho^{(1)} + i \gamma_5 \tilde{\phi}_\rho^{(1)} + \gamma^\alpha \phi_\rho^{(1)} + \gamma_5 \gamma^\alpha \tilde{\phi}_\rho^{(1)} + i \gamma_5 \sigma^{\alpha\beta} \phi_{\rho\alpha\beta} \right] \]

E.g.:

\[ \phi_{\rho\alpha}^{(1)} (x, k_\perp ; p, S) = p^+ n_\alpha \left( k_{\perp\rho} f_{d\perp} - M \bar{S}_{T\rho} f_{dT} - \lambda \bar{k}_{\perp\rho} f_{dL} - \frac{k_{\perp(\rho} k_{\perp\beta)} S^\beta_{T} f_{dT}^{\perp} \right) \]

a subscript “d” to denote that they are from D-type quark-gluon-quark correlator

\[ + M^2 g_{\perp\rho\alpha} \left( f_{3d} - \frac{\epsilon_{\perp}^{KS}}{M} f_{3dT}^{\perp} \right) + k_{\perp(\rho} k_{\perp\alpha)} \left( f_{3d}^{\perp} + \frac{\epsilon_{\perp}^{KS}}{M} f_{3dT}^{\perp 2} \right) \]

\[ + i M^2 \epsilon_{\perp\rho\alpha} \left( \lambda \bar{f}_{3dL} - \frac{k_{\perp} \cdot S_T}{M} f_{3dT}^{\perp 3} \right) + \frac{1}{2} k_{\perp(\rho} \tilde{k}_{\perp\alpha)} \left( \lambda \bar{f}_{3dL}^{\perp} + \frac{k_{\perp} \cdot S_T}{M} f_{3dT}^{\perp 4} \right) + ... \]

\[ \downarrow \quad \text{twist-3} \]

\[ \downarrow \quad \text{twist-4} \]
Introduction: Azimuthal asymmetries and higher twist contributions

Azimuthal asymmetry and higher twist

$e^- + N \rightarrow e^- + q(\text{jet}) + X$

1977, Georgi & Politzer: gluon radiation $\longrightarrow$ azimuthal asymmetry $\longrightarrow$ “Clean test to pQCD”

1978, Cahn: generalize parton model to include an intrinsic $\vec{k}_\perp$:

$$\langle \cos \varphi \rangle = -\frac{|k_\perp|}{Q} \frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2} \quad (\text{twist 3})$$

$$\langle \cos 2\varphi \rangle = \frac{|k_\perp|^2}{Q^2} \frac{2(1-y)}{1+(1-y)^2} \quad (\text{twist 4})$$

higher twist, nevertheless significant!

$$|k_\perp| \sim 0.3 - 0.7\,GeV \quad |k_\perp|/Q \sim 0.1$$

Lesson: do not forget higher twists!
Introduction: Azimuthal asymmetries and gauge link

Sivers function and gauge link

1991, Sivers: introduced the asymmetric quark distribution in polarized nucleon (Sivers function)

\[ f_q(x, k_\perp; S_\perp) = f_q(x, k_\perp) + (\hat{k}_\perp \times \hat{p}) \cdot \vec{S}_T \cdot f_{1T}(x, k_\perp) \]

1993, Boros, Liang & Meng:
proposed an intuitive picture:
quark orbital angular momentum + “surface effect”

1993, Collins: P&T invariance \( \iff f_{1T}(x, k_\perp) = 0 \)

2002, Brodsky, Hwang, Schmidt:

2002, Collins: “final state interaction” = “gauge link”.

Lesson: do not forget the gauge link!
General kinematic analysis for $e^- + N \rightarrow e^- + q(\text{jet}) + X$

The azimuthal asymmetries in terms of structure functions

1. \[ \langle \cos \phi \rangle_{UU} = \sqrt{2\varepsilon(1 + \varepsilon)} W_{UU}^{\cos \phi} / W_{UU} \]
   \[ \langle \cos 2\phi \rangle_{UU} = \varepsilon W_{UU}^{\cos 2\phi} / W_{UU} \]

2. \[ \langle \sin \phi \rangle_{LU} = \sqrt{2\varepsilon(1 - \varepsilon)} W_{LU}^{\sin \phi} / W_{UU} \]

3. \[ \langle \sin \phi \rangle_{UL} = \sqrt{2\varepsilon(1 + \varepsilon)} W_{UL}^{\sin \phi} / W_{UU} \]
   \[ \langle \sin 2\phi \rangle_{UL} = \varepsilon W_{UL}^{\sin 2\phi} / W_{UU} \]

4. \[ \langle \cos \phi \rangle_{LL} = \sqrt{2\varepsilon(1 + \varepsilon)} [W_{UU}^{\cos \phi} + \lambda_I \lambda_N W_{LL}^{\cos \phi} / (2 - y)] / (W_{UU} + \lambda_I \lambda_N \sqrt{1 - \varepsilon^2} W_{LL}) \]

5. \[ \langle \sin \phi_S \rangle_{UT} = \sqrt{2\varepsilon(1 + \varepsilon)} W_{UT}^{\sin \phi_S} / W_{UU} \]
   \[ \langle \sin(\phi + \phi_S) \rangle_{UT} = \varepsilon W_{UT}^{\sin(\phi + \phi_S)} / W_{UU} \]

6. \[ \langle \sin(\phi - \phi_S) \rangle_{UT} = W_{UT}^{\sin(\phi - \phi_S)} / W_{UU} \]
   \[ \langle \sin(2\phi - \phi_S) \rangle_{UT} = \sqrt{2\varepsilon(1 + \varepsilon)} W_{UT}^{\sin(2\phi - \phi_S)} / W_{UU} \]

7. \[ \langle \sin(3\phi - \phi_S) \rangle_{UT} = \varepsilon W_{UT}^{\sin(3\phi - \phi_S)} / W_{UU} \]

8. \[ \langle \cos \phi_S \rangle_{LT} = \sqrt{2\varepsilon(1 - \varepsilon)} W_{LT}^{\cos \phi_S} / W_{UU} \]
   \[ \langle \cos(\phi - \phi_S) \rangle_{LT} = \sqrt{1 - \varepsilon^2} W_{LT}^{\cos(\phi - \phi_S)} / W_{UU} \]

\[ \langle \cos(2\phi - \phi_S) \rangle_{LT} = \sqrt{2\varepsilon(1 - \varepsilon)} W_{LT}^{\cos(2\phi - \phi_S)} / W_{UU} \]

9. \[ \langle \sin(3\phi - \phi_S) \rangle_{UT} = \varepsilon W_{UT}^{\sin(3\phi - \phi_S)} / W_{UU} \]

Totally 15 azimuthal asymmetries in different polarized cases.
Inclusive deep inelastic scattering (DIS) \( e^- + N \rightarrow e^- + X \)

The differential cross section

\[
d\sigma = \frac{\alpha^2_{em}}{sQ^4} L^{\mu\nu}(l, \lambda, l', \lambda') W_{\mu\nu}(q, p, S) \frac{d^3l'}{2E'}
\]

The hadronic tensor:

\[
W_{\mu\nu}(q, p, S) = \sum_x \langle p, S | J_\mu(0) | X\rangle \langle X | J_\nu(0) | p, S \rangle (2\pi)^4 \delta^4(p + q - p_X)
\]

\[
W_{\mu\nu}(q, p, S) = \left| \frac{2}{m^2} \right|
\]

\[
= \frac{m \times m^*}{m^2}
\]
General kinematic analysis for $e^- N \rightarrow e^- hX$

$$d\sigma = \frac{\alpha^2}{s Q^4} L^{\mu\nu}(l, \lambda_l, l') W^{(si)}_{\mu\nu}(q, p, S, p_h) \frac{d^3l' d^3p_h}{(2\pi)^3 2E_l 2E_h}$$

The reference frame

- c.m. frame of $\gamma^* N$
- $p$ in z-direction
- lepton-hadron plane = oxz plane

$N: \quad p = (E, 0, 0, |\vec{p}|)$

$e^-: \quad l = E_l (1, \sin \theta, 0, \cos \theta)$

$\gamma^*: \quad q = (q_0, 0, 0, -|\vec{q}|)$

$h: \quad p_h = (E_h, |\vec{p}_{hT}| \cos \phi, |\vec{p}_{hT}| \cos \phi, p_{hz})$

In dependent variables used:

$$s = q^2 = Q^2 \quad y = l \cdot p / q \cdot p$$
$$x = 2q \cdot p / Q^2 \quad p_{hT}^2 \equiv -|\vec{p}_{hT}|^2, \quad \phi_h$$
$$\xi = 2q \cdot p_h / Q^2$$
The cross section:

\[ d\sigma^{(si)} = \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l,\lambda_l,l',\lambda_{l'}) W^{(si)}_{\mu\nu}(q,p,S,k') \frac{d^3l'}{2E'} \frac{d^3k'}{2E_k} \]

\[ W^{(si)}_{\mu\nu}(q,p,S,k') = W^{(0,si)}_{\mu\nu}(q,p,S,k') + W^{(1,si)}_{\mu\nu}(q,p,S,k') + W^{(2,si)}_{\mu\nu}(q,p,S,k') + \ldots \]

To compare

\[ W^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\hat{\phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(k,q)] \]

\[ W^{(1)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \text{Tr}[\hat{\phi}^{(1)}_{\rho}(k_1,k_2,p,S)\hat{H}^{(1,c,si)}_{\mu\nu}\rho(k_1,k_2,q,k')] \]
Semi-Inclusive DIS $e^- + N \rightarrow e^- + q\,(\text{jet}) + X$

The hard parts

\[ \hat{H}^{(0,\text{si})}_{\mu\nu}(k, q, k') = \gamma_\mu (k + q) \gamma_\nu (2\pi)^4 \delta^4 (k' - k - q) \]

\[ \hat{H}^{(1,L;\text{si})}_{\mu\nu}(k_1, k_2, q, k') = \gamma_\mu \frac{(k_1 + q) \gamma^\rho (k_2 + q)}{(k_2 + q)^2 - i\epsilon} \gamma_\nu (2\pi)^4 \delta^4 (k' - k_1 - q) \]

To compare

\[ \hat{H}^{(0)}_{\mu\nu}(k, q) = \gamma_\mu (k + q) \gamma_\nu (2\pi) \delta_+ ((k + q)^2) \]

\[ \hat{H}^{(1,L)}_{\mu\nu}(k_1, k_2, q) = \gamma_\mu \frac{(k_1 + q) \gamma^\rho (k_2 + q)}{(k_2 + q)^2 - i\epsilon} \gamma_\nu (2\pi) \delta_+ ((k_1 + q)^2) \]

Ward identities:

\[ \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} = -\hat{H}^{(1)\rho}_{\mu\nu}(x, x) \]

\[ p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1, x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\epsilon} \]

\[ \frac{\partial \hat{H}^{(0,\text{si})}_{\mu\nu}(x, x')}{\partial k^\rho} \neq -\hat{H}^{(1,\text{si})\rho}_{\mu\nu}(x, x; x') \]

\[ p_\rho \hat{H}^{(1,L,\text{si})\rho}_{\mu\nu}(x_1, x_2; x') \neq \frac{\hat{H}^{(0,\text{si})}_{\mu\nu}(x_1, x')}{x_2 - x_1 - i\epsilon} \]
Semi-Inclusive DIS \( e^- + N \rightarrow e^- + q(\text{jet}) + X \)

Using the identity:

\[
(2\pi)^4 \delta^4(k' - k - q) = (2\pi)^4 \delta^4(k + q) (2\pi)^3 (2E_k') \delta^3(\vec{k}' - \vec{k} - \vec{q})
\]

We obtain:

\[
\hat{H}_{\mu\nu}^{(0,si)}(k,q,k') = \hat{H}_{\mu\nu}^{(0)}(k,q)(2E_k')(2\pi)^3 \delta^3(\vec{k}' - \vec{k} - \vec{q})
\]

\[
\hat{H}_{\mu\nu}^{(1,\rho,c,si)}(k_1,k_2,q,k') = \hat{H}_{\mu\nu}^{(1,\rho,c)}(k_1,k_2,q)(2E_k')(2\pi)^3 \delta^3(\vec{k}' - \vec{k}_c - \vec{q})
\]

\[
W^{(0,si)}_{\mu\nu}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\hat{\phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(k,q)] (2E_k')(2\pi)^3 \delta^3(\vec{k}' - \vec{k} - \vec{q})
\]

\[
W^{(1,si)}_{\mu\nu}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \text{Tr}[\hat{\phi}^{(1)}_{\rho}(k_1,k_2,p,S)\hat{H}_{\mu\nu}^{(1,c)}(k_1,k_2,q)] (2E_k')(2\pi)^3 \delta^3(\vec{k}' - \vec{k}_c - \vec{q})
\]

The same Taylor expansion of \( \hat{H}_{\mu\nu}^{(j)}(k_1,\ldots;q) \) as that for inclusive DIS
Inclusive DIS: LO pQCD, leading & higher twists

Simplified expressions for hadronic tensors

The “collinearly expanded hard parts” take the simple forms such as:

$$\hat{H}^{(0)}_{\mu\nu}(x) = \hat{h}^{(0)}_{\mu\nu} \delta(x - x_B), \quad \hat{h}^{(0)}_{\mu\nu} = \gamma_\mu n \gamma_\nu$$

$$\hat{H}^{(1,L)}_{\mu\nu}(x_1, x_2) \omega_\rho \omega_\rho' = \frac{\pi}{2 q \cdot p} \hat{h}^{(1)}_{\mu\nu} \omega_\rho \omega_\rho' \delta(x_1 - x_B), \quad \hat{h}^{(1)}_{\mu\nu} = \gamma_\mu n \gamma_\nu$$

each depends only on one longitudinal variable $x$!

Contributes at twist-2, 3 and 4

$$\tilde{W}^{(0)}_{\mu\nu}(q, p, S) = \int dx \text{Tr} \left[ \hat{\Phi}^{(0)}(x; p, S) h^{(0)}_{\mu\nu} \right] \delta(x - x_B) = \text{Tr} \left[ \hat{\Phi}^{(0)}(x_B; p, S) h^{(0)}_{\mu\nu} \right]$$

$$\hat{\Phi}^{(0)}(x; p, S) = \int \frac{d^4 k}{(2\pi)^4} \delta(x - \frac{k^+}{p^+}) \hat{\Phi}^{(0)}(k; p, S) = \int \frac{p^+ dz^-}{2\pi} e^{i p^+ z^-} \langle p, S | \bar{\psi}(0) \mathcal{L}(0, z^-) \psi(z^-) | p, S \rangle$$

Contributes at twist-3, 4 and 5

$$\tilde{W}^{(1)}_{\mu\nu}(q, p, S) = \frac{\pi}{2 q \cdot p} \int dx \text{Tr} \left[ \hat{\phi}^{(1)}_\rho(x; p, S) h^{(1)}_{\mu\nu} \omega_\rho \omega_\rho' \right] \delta(x - x_B) = \frac{\pi}{2 q \cdot p} \text{Tr} \left[ \hat{\phi}^{(1)}_\rho(x_B; p, S) h^{(1)}_{\mu\nu} \omega_\rho \omega_\rho' \right]$$

$$\hat{\phi}^{(1)}_\rho(x; p, S) = \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \delta(x - \frac{k_1^+}{p^+}) \hat{\phi}^{(1)}_\rho(k_1, k_2; p, S) = \int \frac{p^+ dz^-}{2\pi} e^{i p^+ z^-} \langle p, S | \bar{\psi}(0) D_\rho(0) \mathcal{L}(0, z^-) \psi(z^-) | p, S \rangle$$

the involved one-dimensional gauge invariant quark-gluon-quark correlator

**ONE dimensional, depend only on ONE parton momentum!**