



# The complete twist-4 contributions to Semi-Inclusive DIS $e^- + N \rightarrow e^- + q(jet) + X$

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References: SY. Wei, Y.K. Song, K.B. Chen, & ZTL, PRD95, 074017 (2017);  
K.B. Chen, S.Y. Wei & ZTL, Fron. Phys. 10, 101204 (2015);  
ZTL & X.N. Wang, PRD 75, 094002 (2007).

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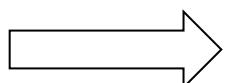
## Three-dimensional structure of the nucleon



Study of azimuthal asymmetry  $\longrightarrow$  Cahn effect  $\longrightarrow$  higher twists

Study of single-spin asymmetry  $\longrightarrow$  Sivers effect  $\longrightarrow$  gauge link

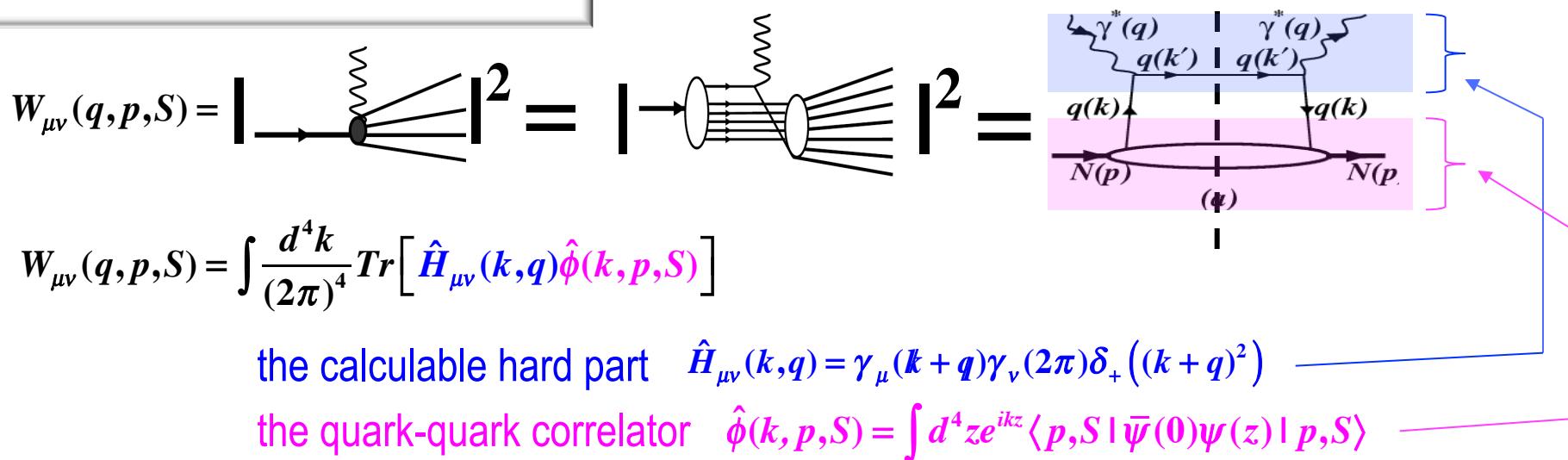
Questions: Where does the gauge link come from?  
How do we deal with higher twist contributions systematically?



Look at the simplest case, the inclusive DIS, first!

# Collinear approximation in inclusive DIS without QCD

Parton model without QCD:



Collinear approximation:  $p \approx p^+ \bar{n}$ ,  $k \approx xp$  (neglecting M/Q, i.e. taking  $M/Q \sim 0$ )

$$\rightarrow W_{\mu\nu}(q, p) \approx \left[ (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) + \frac{1}{2xp} (q + 2xp)_\mu (q + 2xp)_\nu \right] f_q(x)$$

operator expression of the number density :  $f_q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+ z^-} \langle p | \bar{\psi}(0) \frac{\gamma^+}{2} \psi(z) | p \rangle$

no (local) gauge invariance!

# Collinear expansion in DIS with “QCD multiple gluon scattering”

To get the gauge invariance, we need to take the “multiple gluon scattering” into account

$$W_{\mu\nu}(q, p, S) = \begin{array}{c} \text{Diagram 1: } q(k) \xrightarrow{\gamma^*(q)} q(k') \xrightarrow{\gamma^*(q)} q(k) \\ \text{Diagram 2: } q(k_1) \xrightarrow{\gamma^*(q)} q(k_2) \\ \text{Diagram 3: } q(k_1) \xrightarrow{\gamma^*(q)} q(k_3) \xrightarrow{\gamma^*(q)} q(k_4) \xrightarrow{\gamma^*(q)} q(k_2) \end{array} + \dots$$

$$W_{\mu\nu}(q, p, S) = W_{\mu\nu}^{(0)}(q, p, S) + W_{\mu\nu}^{(1)}(q, p, S) + W_{\mu\nu}^{(2)}(q, p, S) + \dots$$

$$W_{\mu\nu}^{(0)}(q, p, S) = \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \hat{H}_{\mu\nu}^{(0)}(k, q) \hat{\phi}^{(0)}(k, p, S) \right]$$

$$W_{\mu\nu}^{(1)}(q, p, S) = \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \text{Tr} \left[ \hat{H}_{\mu\nu}^{(1)\rho}(k_1, k_2, q) \hat{\phi}_\rho^{(1)}(k_1, k_2, p, S) \right]$$

no gauge invariance!

## Collinear approximation:

- ★ Approximating the hard part at  $k = xp$ :  $\hat{H}_{\mu\nu}^{(0)}(k, q) \approx \hat{H}_{\mu\nu}^{(0)}(x)$ ,  $\hat{H}_{\mu\nu}^{(1)\rho}(k_1, k_2, q) \approx \hat{H}_{\mu\nu}^{(1)\rho}(x_1, x_2)$
- ★ Keeping only the longitudinal component of the gluon field:  $A_\rho(y) \approx n \cdot A(y) \frac{p_\rho}{n \cdot p}$
- ★ Using Ward identities, e.g.,  $p_\rho \hat{H}_{\mu\nu}^{(1,L)\rho}(x_1, x_2) = \frac{\hat{H}_{\mu\nu}^{(0)}(x_1)}{x_2 - x_1 - i\varepsilon}$  all  $\hat{H}_{\mu\nu}^{(j)}(x_i)$ 's reduce to  $\hat{H}_{\mu\nu}^{(0)}(x)$
- ★ Adding all the terms together  $\longrightarrow$

# Inclusive DIS: LO pQCD, leading twist



$$\rightarrow W_{\mu\nu}(q, p, S) \approx \tilde{W}_{\mu\nu}^{(0)}(q, p, S) = \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \hat{\Phi}^{(0)}(k; p, S) \hat{H}_{\mu\nu}^{(0)}(x) \right]$$

LO & leading twist

$$\hat{\Phi}^{(0)}(k; p, S) = \int d^4 z e^{ikz} \langle p, S | \bar{\psi}(0) \mathcal{L}(0, z) \psi(z) | p, S \rangle$$

The gauge invariant un-integrated quark-quark correlator: contain QCD interaction!

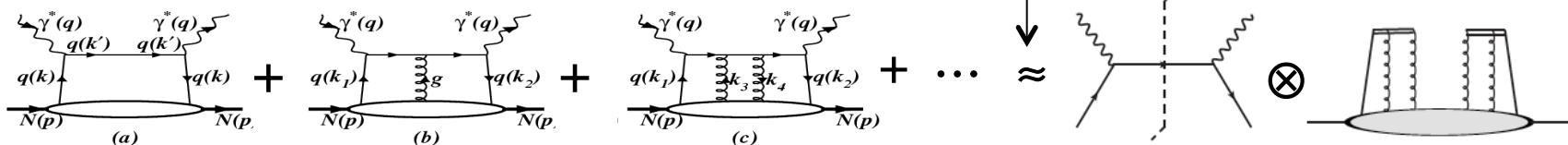
$$\mathcal{L}(0, z) = \mathcal{L}^\dagger(-\infty, 0) \mathcal{L}(-\infty, z),$$

$$\begin{aligned} \mathcal{L}(-\infty, z) &= Pe^{-ig \int\limits_{-\infty}^{z^-} dy^- A^+(z^+, y^-, \vec{z}_\perp)} \\ &= 1 + ig \int\limits_{-\infty}^{z^-} dy^- A^+(z^+, y^-, \vec{z}_\perp) + \frac{1}{2} (ig)^2 \int\limits_{-\infty}^{z^-} dy^- \int\limits_{-\infty}^{y^-} dy'^- A^+(z^+, y^-, \vec{z}_\perp) A^+(z^+, y'^-, \vec{z}_\perp) + \dots \end{aligned}$$

gauge link

Graphically:

collinear approximation





# Inclusive DIS: LO pQCD, leading & higher twists

## Collinear expansion:

- ★ Expanding the hard part at  $k = xp$ :

$$\hat{H}_{\mu\nu}^{(0)}(k, q) = \hat{H}_{\mu\nu}^{(0)}(x) + \frac{\partial \hat{H}_{\mu\nu}^{(0)}(x)}{\partial k^\rho} \omega_\rho^{\rho'} k_{\rho'} + \dots$$

$$\hat{H}_{\mu\nu}^{(1)\rho}(k_1, k_2, q) = \hat{H}_{\mu\nu}^{(1)\rho}(x_1, x_2) + \frac{\partial \hat{H}_{\mu\nu}^{(1)\rho}(x_1, x_2)}{\partial k_1^\sigma} \omega_\sigma^{\sigma'} k_{1\sigma'} + \dots$$

- ★ Decomposition of the gluon field:

$$A_\rho(y) = \mathbf{n} \cdot A(y) \frac{\mathbf{p}_\rho}{\mathbf{n} \cdot \mathbf{p}} + \omega_\rho^{\rho'} A_{\rho'}(y)$$

- ★ Using the Ward identities such as,

$$\frac{\partial \hat{H}_{\mu\nu}^{(0)}(x)}{\partial k^\rho} = -\hat{H}_{\mu\nu}^{(1)\rho}(x, x) \quad p_\rho \hat{H}_{\mu\nu}^{(1,L)\rho}(x_1, x_2) = \frac{\hat{H}_{\mu\nu}^{(0)}(x_1)}{x_2 - x_1 - i\epsilon}$$

to replace the derivatives etc.

- ★ Adding all terms with the same hard part together  $\longrightarrow$

Ellis, Furmanski, Petronzio (1982, 1983)  
Qiu, Sterman (1990, 1991)

$$\hat{H}_{\mu\nu}^{(0)}(x) \equiv \hat{H}_{\mu\nu}^{(0)}(k = xp, q)$$

$$\frac{\partial \hat{H}_{\mu\nu}^{(0)}(x)}{\partial k^\rho} \equiv \left. \frac{\partial \hat{H}_{\mu\nu}^{(0)}(k, q)}{\partial k^\rho} \right|_{k=xp}$$

$$x = k^+ / p^+$$

$$\omega_\rho^{\rho'} \equiv g_\rho^{\rho'} - \bar{n}_\rho n^{\rho'}$$

$$\omega_\rho^{\rho'} k_{\rho'} = (k - xp)_\rho$$

$$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$$

$$\mathbf{n} = (0, 1, \vec{0}_\perp)$$

$$\bar{\mathbf{n}} = (1, 0, \vec{0}_\perp)$$

# Inclusive DIS: LO pQCD, leading & higher twists



$$W_{\mu\nu}(q, p, S) = \tilde{W}_{\mu\nu}^{(0)}(q, p, S) + \tilde{W}_{\mu\nu}^{(1)}(q, p, S) + \tilde{W}_{\mu\nu}^{(2)}(q, p, S) + \dots$$

$$\tilde{W}_{\mu\nu}^{(0)}(q, p, S) = \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \hat{\Phi}^{(0)}(k, p, S) \hat{H}_{\mu\nu}^{(0)}(x) \right] \quad \text{twist-2, 3 and 4 contributions}$$

depends on  $x$  only!

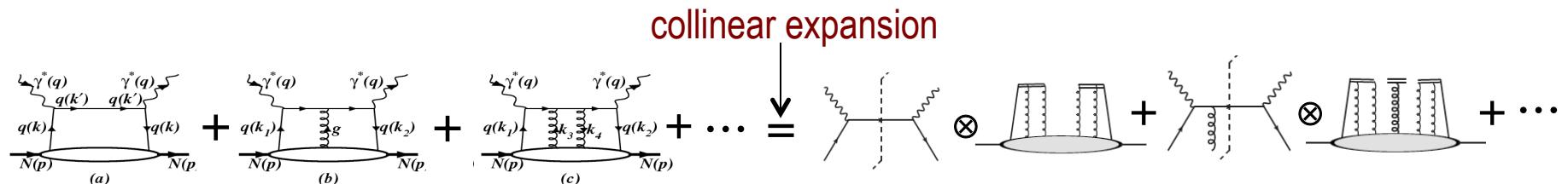
$$\hat{\Phi}^{(0)}(k, p, S) = \int d^4 z e^{ikz} \langle p, S | \bar{\psi}(0) \cancel{\mathcal{L}}(0, z) \psi(z) | p, S \rangle \quad \text{gauge invariant quark-quark correlator}$$

$$\tilde{W}_{\mu\nu}^{(1)}(q, p, S) = \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \text{Tr} \left[ \hat{\Phi}_{\rho'}^{(1)}(k_1, k_2, p, S) \hat{H}_{\mu\nu}^{(1)\rho}(x_1, x_2) \omega_{\rho'}^{\rho'} \right] \quad \text{twist-3, 4 and 5 contributions}$$

$$\hat{\Phi}_{\rho'}^{(1)}(k_1, k_2, p, S) = \int d^4 z d^4 y e^{ik_1 y + ik_2(z-y)} \langle p, S | \bar{\psi}(0) \cancel{\mathcal{L}}(0, y) D_{\rho}(y) \cancel{\mathcal{L}}(y, z) \psi(z) | p, S \rangle$$

$$D_{\rho}(y) = -i\partial_{\rho} + gA_{\rho}(y) \quad \text{gauge invariant quark-gluon-quark correlator}$$

→ A consistent framework for inclusive DIS  $e^- N \rightarrow e^- X$  including leading & higher twists





# Inclusive DIS: LO pQCD, leading & higher twists

## Simplified expressions for hadronic tensors

The “collinearly expanded hard parts” take very simple forms such as:

$$\hat{H}_{\mu\nu}^{(0)}(x) = \hat{h}_{\mu\nu}^{(0)} \delta(x - x_B), \quad \hat{h}_{\mu\nu}^{(0)} = \gamma_\mu \not{n} \gamma_\nu$$

$$\hat{H}_{\mu\nu}^{(1,L)\rho}(x_1, x_2) \omega_\rho^{\rho'} = \frac{\pi}{2q \cdot p} \hat{h}_{\mu\nu}^{(1)\rho} \omega_\rho^{\rho'} \delta(x_1 - x_B), \quad \hat{h}_{\mu\nu}^{(1)\rho} = \gamma_\mu \not{n} \gamma^\rho \not{n} \gamma_\nu$$

each depends only  
on one longitudinal  
variable  $x$ !

$$\tilde{W}_{\mu\nu}^{(0)}(q, p, S) = \text{Tr} \left[ \hat{\Phi}^{(0)}(x_B; p, S) \hat{h}_{\mu\nu}^{(0)} \right] \quad \text{contributes at twist-2, 3 and 4}$$

$$\hat{\Phi}^{(0)}(x; p, S) \equiv \int \frac{d^4 k}{(2\pi)^4} \delta(x - \frac{k^+}{p^+}) \hat{\Phi}^{(0)}(k; p, S) = \int \frac{p^+ dz^-}{2\pi} e^{ixp^+ z^-} \langle p, S | \bar{\psi}(0) \cancel{L}(0, z^-) \psi(z^-) | p, S \rangle$$

one-dimensional gauge invariant quark-quark correlator

$$\tilde{W}_{\mu\nu}^{(1)}(q, p, S) = \frac{\pi}{2q \cdot p} \text{Tr} \left[ \hat{\phi}_\rho^{(1)}(x_B; p, S) \hat{h}_{\mu\nu}^{(1)\rho} \omega_\rho^{\rho'} \right] \quad \text{contributes at twist-3, 4 and 5}$$

$$\hat{\phi}_\rho^{(1)}(x; p, S) \equiv \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \delta(x - \frac{k_1^+}{p^+}) \hat{\Phi}_\rho^{(1)}(k_1, k_2; p, S) = \int \frac{p^+ dz^-}{2\pi} e^{ixp^+ z^-} \langle p, S | \bar{\psi}(0) D_\rho(0) \cancel{L}(0, z^-) \psi(z^-) | p, S \rangle$$

the involved one-dimensional gauge invariant quark-gluon-quark correlator

ONE dimensional, depend only on ONE parton momentum!

# PDFs defined via quark-quark correlator



- Expand the quark-quark correlator in terms of the  $\Gamma$ -matrices:

- Make Lorentz decompositions

$$\Phi^{(0)}(x;p,\textcolor{violet}{S}) = Me(x)$$

$$\tilde{\Phi}^{(0)}(x; p, \textcolor{violet}{S}) = \lambda M e_{\textcolor{violet}{L}}(x)$$

$$\Phi_\alpha^{(0)}(x; p, S) = p^+ \bar{n}_\alpha f_1(x) + M \varepsilon_{\perp \alpha \rho} S_T^\rho f_T(x) + \frac{M^2}{p^+} n_\alpha f_3(x)$$

$$\tilde{\Phi}_\alpha^{(0)}(x; p, S) = \lambda p^+ \bar{n}_\alpha g_{1L}(x) + M S_{T\alpha} g_T(x) + \lambda \frac{M^2}{p^+} n_\alpha g_{3L}(x)$$

$$\Phi_{\rho\alpha}^{(0)}(x;p,\textcolor{violet}{S}) = p^+ \bar{n}_{[\rho} \textcolor{violet}{S}_{\textcolor{violet}{T}\alpha]} \textcolor{blue}{h}_{1\textcolor{violet}{T}}(x) - M \mathcal{E}_{T\rho\alpha} h_{\textcolor{violet}{T}}(x) + \lambda M \bar{n}_{[\rho} n_{\alpha]} h_{\textcolor{violet}{L}}(x) + \frac{M^2}{p^+} n_{[\rho} S_{\textcolor{violet}{T}\alpha]} h_{3\textcolor{violet}{T}}(x)$$

the scalar functions are the one-dimensional PDFs, e.g.,

$$f_1(x) = \frac{1}{p^+} n^\alpha \Phi_\alpha^{(0)}(x; p, S) = \int \frac{dz^-}{2\pi} e^{ixp^+ z^-} \langle p, S | \bar{\psi}(0) \cancel{\mathcal{L}}(\mathbf{0}, z^-) \frac{\gamma^+}{2} \psi(z^-) | p, S \rangle$$



Conclusion: Collinear expansion provides a systematical way of dealing with leading and higher twist contributions including gauge invariance.

advantages

- Gauge link obtained automatically
- Leading and higher twist systematically
- Extremely simplified expressions

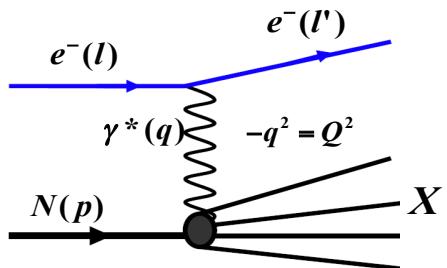
Question: Can we extend collinear expansion to semi-inclusive DIS?

# Collinear expansion in high energy reactions



Successfully to all processes where only ONE hadron is explicitly involved.

Inclusive DIS  $e^- N \rightarrow e^- X$



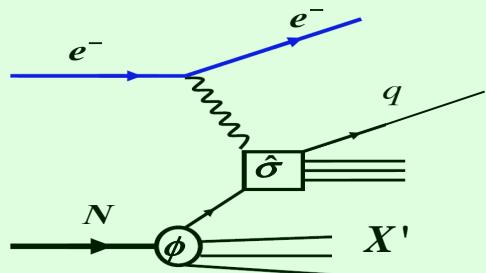
Yes!

where collinear expansion was first formulated.

R. K. Ellis, W. Furmanski and R. Petronzio,  
Nucl. Phys. B207,1 (1982); B212, 29 (1983).

Semi-Inclusive DIS

$e + N \rightarrow e + q(jet) + X'$

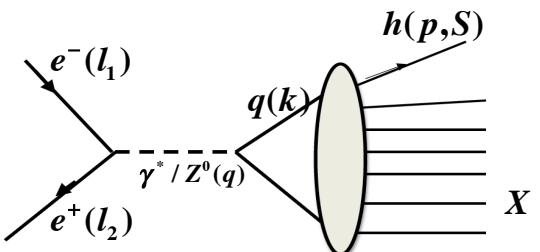


Yes!

ZTL & X.N. Wang,  
PRD (2007);

Inclusive

$e^- + e^+ \rightarrow h + X$

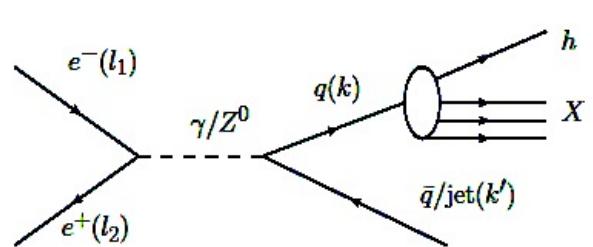


Yes!

S.Y. Wei, Y.K Song, ZTL,  
PRD (2014);

Semi-Inclusive

$e^- + e^+ \rightarrow h + \bar{q}(jet) + X$



Yes!

S.Y. Wei, K.B. Chen, Y.K Song,  
ZTL, PRD (2015).

# Semi-Inclusive DIS $e^- + N \rightarrow e^- + q(jet) + X$



$$W_{\mu\nu}^{(si)}(q, p, S, k') = \tilde{W}_{\mu\nu}^{(0,si)}(q, p, S, k') + \tilde{W}_{\mu\nu}^{(1,si)}(q, p, S, k') + \tilde{W}_{\mu\nu}^{(2,si)}(q, p, S, k') + \dots$$

twist-2, 3 and 4 contributions

$$\tilde{W}_{\mu\nu}^{(0,si)}(q, p, S, k') = \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[\hat{\Phi}^{(0)}(k, p, S) \hat{H}_{\mu\nu}^{(0)}(x)] (2E_{k'}) (2\pi)^3 \delta^3(\vec{k}' - \vec{k} - \vec{q})$$

$$\hat{\Phi}^{(0)}(k, p, S) = \int d^4 z e^{ikz} \langle p, S | \bar{\psi}(0) \cancel{\mathcal{L}}(0, z) \psi(z) | p, S \rangle$$

depends on x only!

twist-3, 4 and 5 contributions

$$\tilde{W}_{\mu\nu}^{(1,si)}(q, p, S, k') = \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \sum_{c=L,R} \text{Tr}[\hat{\Phi}_\rho^{(1)}(k_1, k_2, p, S) \hat{H}_{\mu\nu}^{(1,c)\rho}(x_1, x_2) \omega_\rho^\rho] (2E_{k'}) (2\pi)^3 \delta^3(\vec{k}' - \vec{k}_c - \vec{q})$$

$$\hat{\Phi}_\rho^{(1)}(k_1, k_2, p, S) = \int d^4 z d^4 y e^{ik_1 y + ik_2(z-y)} \langle p, S | \bar{\psi}(0) \cancel{\mathcal{L}}(0, y) D_\rho(y) \cancel{\mathcal{L}}(y, z) \psi(z) | p, S \rangle$$

→ A consistent framework for  $e^- N \rightarrow e^- + q(jet) + X$  at LO pQCD including higher twists

ZTL & X.N. Wang, PRD (2007); Y.K. Song, J.H. Gao, ZTL & X.N. Wang, PRD (2011) & PRD (2014).

# Semi-Inclusive DIS $e^- + N \rightarrow e^- + q(jet) + X$



## Simplified expressions for hadronic tensors

The “collinearly expanded hard parts” take the simple forms such as:

$$\hat{H}_{\mu\nu}^{(0)}(x) = \hat{h}_{\mu\nu}^{(0)}\delta(x - x_B), \quad \hat{h}_{\mu\nu}^{(0)} = \gamma_\mu \not{\epsilon} \gamma_\nu$$

$$\hat{H}_{\mu\nu}^{(1,L)\rho}(x_1, x_2) \omega_\rho^{\rho'} = \frac{\pi}{2q \cdot p} \hat{h}_{\mu\nu}^{(1)\rho} \omega_\rho^{\rho'} \delta(x_1 - x_B), \quad \text{where } \hat{h}_{\mu\nu}^{(1)\rho} = \gamma_\mu \not{\epsilon} \gamma^\rho \not{\epsilon} \gamma_\nu, \text{ depends only on } x_1 !$$

$$\tilde{W}_{\mu\nu}^{(0,si)}(q, p, S; \mathbf{k}_\perp) = \text{Tr} \left[ \hat{\Phi}^{(0)}(x_B, \mathbf{k}_\perp; p, S) \hat{h}_{\mu\nu}^{(0)} \right] \quad \text{twist-2, 3 and 4}$$

$$\hat{\Phi}^{(0)}(x, k'_\perp; p, S) = \int \frac{d^4 k}{(2\pi)^4} \delta(x - \frac{k^+}{p^+}) \delta^2(k_\perp - k'_\perp) \hat{\Phi}^{(0)}(k; p, S) = \int \frac{p^+ dz^-}{2\pi} d^2 z_\perp e^{ixp^+ z^- - ik_\perp \cdot z_\perp} \langle N | \bar{\psi}(0) \mathcal{L}(\mathbf{0}, z) \psi(z) | N \rangle$$

three-dimensional gauge invariant quark-quark correlator

$$\tilde{W}_{\mu\nu}^{(1,si)}(q, p, S; \mathbf{k}_\perp) = \frac{\pi}{2q \cdot p} \text{Tr} \left[ \hat{\phi}_\rho^{(1)}(x_B, \mathbf{k}_\perp; p, S) \hat{h}_{\mu\nu}^{(1)\rho} \omega_\rho^{\rho'} \right] \quad \text{twist-3, 4 and 5}$$

$$\begin{aligned} \hat{\phi}_\rho^{(1)}(x, k_\perp; p, S) &= \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \delta(x - \frac{k_1^+}{p^+}) \delta^2(k_{1\perp} - k_\perp) \hat{\Phi}_\rho^{(1)}(k_1, k_2; p, S) \\ &= \int \frac{p^+ dz^-}{2\pi} d^2 z_\perp e^{ixp^+ z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle N | \bar{\psi}(0) D_\rho(0) \mathcal{L}(\mathbf{0}, z) \psi(z) | N \rangle \end{aligned}$$

the involved three-dimensional gauge invariant quark-gluon-quark correlator

**THREE dimensional, depend only on ONE parton momentum!**



# TMD PDFs defined via quark-j-gluon-quark correlator

Relationships obtained from the QCD equation of motion  $\gamma \cdot D(z)\psi(z) = 0$

E.g.:

Twist-3:

$$f_{dS}^K(x, k_\perp) - g_{dS}^K(x, k_\perp) = -x [f_S^K(x, k_\perp) - ig_S^K(x, k_\perp)]$$

$$K = \text{null}, \quad S = T; \quad K = \perp, \quad S = \text{null}, L, \text{ or } T$$

the subscript d or dd is introduced to denote TMDs defined via D-type quark-gluon-quark or quark-2-gluon-quark correlator

Twist-4:

$$x^2 f_3(x, k_\perp) = x f_{-3d}(x, k_\perp) = -f_{-3dd}^M(x, k_\perp)$$

$$x^2 f_{3T}^\perp(x, k_\perp) = x f_{-3dT}^\perp(x, k_\perp) = -f_{-3ddT}^{M\perp}(x, k_\perp)$$

$$x^2 g_{3L}(x, k_\perp) = x f_{-3dL}(x, k_\perp) = -f_{-3ddL}^M(x, k_\perp)$$

$$x^2 g_{3T}^\perp(x, k_\perp) = x f_{-3dT}^{\perp 3}(x, k_\perp) = -f_{-3ddT}^{M\perp 3}(x, k_\perp)$$

See e.g., Y.K. Song, J.H. Gao, ZTL & X.N. Wang, PRD (2014);  
SY. Wei, Y.K. Song, K.B. Chen, & ZTL, PRD95, 074017 (2017); .....

# General kinematic analysis for $e^- + N \rightarrow e^- + h + X$

The cross section in the  $\gamma^* p$  c.m. frame (only parity conserved part)

$$\frac{d\sigma}{dxdydp_s d^2p_{hT}} = \frac{\alpha^2}{xyQ^2} \mathcal{K}(\mathcal{F}_{UU} + \lambda_U \mathcal{F}_{LU} + \lambda_N \mathcal{F}_{UL} + \lambda_U \lambda_N \mathcal{F}_{LL} + |\vec{S}_T| \mathcal{F}_{UT} + \lambda_U |\vec{S}_T| \mathcal{F}_{LT})$$

- $\bullet$   $\mathcal{F}_{UU} = F_{UU,T} + \epsilon F_{UU,L} + \cos \phi \sqrt{2\epsilon(1+\epsilon)} F_{UU}^{\cos \phi} + \cos 2\phi \epsilon F_{UU}^{\cos 2\phi}$

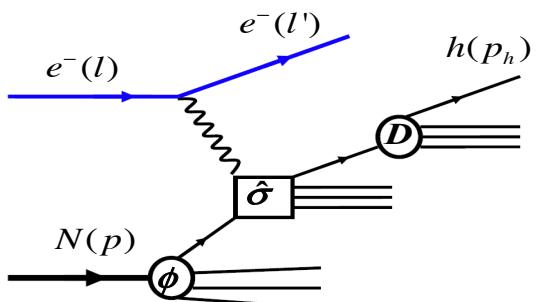
- $\bullet$   $\mathcal{F}_{LU} = \sin \phi \sqrt{2\epsilon(1-\epsilon)} F_{LU}^{\sin \phi}$

- $\bullet$   $\mathcal{F}_{UL} = \sin \phi \sqrt{2\epsilon(1+\epsilon)} F_{UL}^{\sin \phi} + \sin 2\phi \epsilon F_{UL}^{\sin 2\phi}$

- $\bullet$   $\mathcal{F}_{LL} = \sqrt{1-\epsilon^2} F_{LL} + \cos \phi \sqrt{2\epsilon(1-\epsilon)} F_{LL}^{\cos \phi}$

- $\bullet$  
$$\begin{aligned} \mathcal{F}_{UT} &= \sin \phi_s \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin \phi_s} + \sin(\phi - \phi_s)(F_{UT,T}^{\sin(\phi-\phi_s)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_s)}) \\ &\quad + \sin(\phi + \phi_s) \epsilon F_{UT}^{\sin(\phi+\phi_s)} + \sin(2\phi - \phi_s) \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin(2\phi-\phi_s)} + \sin(3\phi - \phi_s) \epsilon F_{UT}^{\sin(3\phi-\phi_s)} \end{aligned}$$

- $\bullet$   $\mathcal{F}_{LT} = \cos \phi_s \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos \phi_s} + \cos(\phi - \phi_s) \sqrt{1-\epsilon^2} F_{LT}^{\cos(\phi-\phi_s)} + \cos(2\phi - \phi_s) \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos(2\phi-\phi_s)}$



nucleon  
 electron

Totally 18 structure functions:  $F_{jxx}^{yy} = F_{jxx}^{yy}(x, \xi, p_{hT}^2, Q)$

# General kinematic analysis for $e^- + N \rightarrow e^- + q(jet) + X$



The cross section in the  $\gamma^* p$  c.m. frame (only parity conserved part)

$$\frac{d\sigma}{dxdy d\phi_s d^2 k_T} = \frac{\alpha^2}{xyQ^2} \mathcal{K}(\mathcal{W}_{UU} + \lambda_U \mathcal{W}_{LU} + \lambda_U \mathcal{W}_{UL} + \lambda_U \lambda_L \mathcal{W}_{LL} + |\vec{S}_T| \mathcal{W}_{UT} + \lambda_U |\vec{S}_T| \mathcal{W}_{LT})$$

- $\mathcal{W}_{UU} = W_{UU,T} + \varepsilon W_{UU,L} + \cos \phi \sqrt{2\varepsilon(1+\varepsilon)} W_{UU}^{\cos \phi} + \cos 2\phi \varepsilon W_{UU}^{\cos 2\phi}$

- $\mathcal{W}_{LU} = \sin \phi \sqrt{2\varepsilon(1-\varepsilon)} W_{LU}^{\sin \phi}$

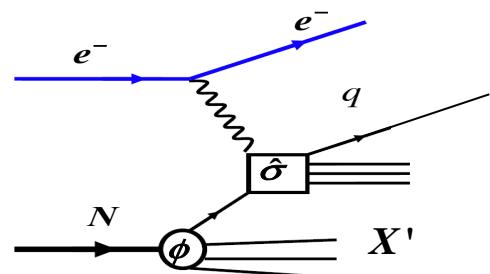
- $\mathcal{W}_{UL} = \sin \phi \sqrt{2\varepsilon(1+\varepsilon)} W_{UL}^{\sin \phi} + \sin 2\phi \varepsilon W_{UL}^{\sin 2\phi}$

- $\mathcal{W}_{LL} = \sqrt{1-\varepsilon^2} W_{LL} + \cos \phi \sqrt{2\varepsilon(1-\varepsilon)} W_{LL}^{\cos \phi}$

- $\mathcal{W}_{UT} = \sin \phi_s \sqrt{2\varepsilon(1+\varepsilon)} W_{UT}^{\sin \phi_s} + \sin(\phi - \phi_s)(W_{UT,T}^{\sin(\phi-\phi_s)} + \varepsilon W_{UT,L}^{\sin(\phi-\phi_s)}) + \sin(\phi + \phi_s) \varepsilon W_{UT}^{\sin(\phi+\phi_s)} + \sin(2\phi - \phi_s) \sqrt{2\varepsilon(1+\varepsilon)} W_{UT}^{\sin(2\phi-\phi_s)} + \sin(3\phi - \phi_s) \varepsilon W_{UT}^{\sin(3\phi-\phi_s)}$

- $\mathcal{W}_{LT} = \cos \phi_s \sqrt{2\varepsilon(1-\varepsilon)} W_{LT}^{\cos \phi_s} + \cos(\phi - \phi_s) \sqrt{1-\varepsilon^2} W_{LT}^{\cos(\phi-\phi_s)} + \cos(2\phi - \phi_s) \sqrt{2\varepsilon(1-\varepsilon)} W_{LT}^{\cos(2\phi-\phi_s)}$

nucleon  
 electron



Totally 18 structure functions:  $W_{jxx}^{yy} = W_{jxx}^{yy}(x, k_\perp^2, Q)$



# Semi-Inclusive DIS: $e^- + N \rightarrow e^- + q(jet) + X$

Complete results for structure functions up to twist-4

$$\kappa_M \equiv \frac{M}{Q}, \quad \bar{k}_\perp \equiv \frac{|\vec{k}_\perp|}{M}$$

$$W_{UU,T} = xf_1 + 4x^2 \kappa_M^2 f_{+3dd}, \quad W_{UU,L} = 8x^3 \kappa_M^2 f_3$$

$$W_{UU}^{\cos 2\phi} = -2x^2 \kappa_M^2 \bar{k}_\perp^2 f_{-3d}^\perp$$

$$W_{UL}^{\sin 2\phi} = 2x^2 \kappa_M^2 \bar{k}_\perp^2 f_{+3dL}^\perp$$

$$W_{LL} = xg_{1L} + 4x^2 \kappa_M^2 f_{+3ddL}$$

$$W_{UT,T}^{\sin(\phi-\phi_s)} = \bar{k}_\perp (xf_{1T}^\perp + 4x^2 \kappa_M^2 f_{+3ddT}), \quad W_{UT,L}^{\sin(\phi-\phi_s)} = 8x^3 \kappa_M^2 \bar{k}_\perp f_{3T}^\perp$$

$$W_{UT}^{\sin(\phi+\phi_s)} = -x^2 \kappa_M^2 \bar{k}_\perp^3 (f_{+3dT}^{\perp 4} + f_{-3dT}^{\perp 2})$$

$$W_{UT}^{\sin(3\phi-\phi_s)} = -x^2 \kappa_M^2 \bar{k}_\perp^3 (f_{+3dT}^{\perp 4} - f_{-3dT}^{\perp 2})$$

$$W_{LT}^{\cos(\phi-\phi_s)} = \bar{k}_\perp (xg_{1T}^\perp + 4x^2 \kappa_M^2 f_{+3ddT}^{\perp 3})$$

$$W_{UU}^{\cos \phi} = -2x^2 \kappa_M \bar{k}_\perp f^\perp$$

$$W_{UL}^{\sin \phi} = -2x^2 \kappa_M \bar{k}_\perp f_L^\perp$$

$$W_{LU}^{\sin \phi} = 2x^2 \kappa_M \bar{k}_\perp g^\perp$$

$$W_{LL}^{\cos \phi} = -2x^2 \kappa_M \bar{k}_\perp g_L^\perp$$

$$W_{UT}^{\sin \phi_s} = -2x^2 \kappa_M f_T$$

$$W_{UT}^{\sin(2\phi-\phi_s)} = -x^2 \kappa_M \bar{k}_\perp^2 f_T^\perp$$

$$W_{LT}^{\cos \phi_s} = -2x^2 \kappa_M g_T$$

$$W_{LT}^{\cos(2\phi-\phi_s)} = -x^2 \kappa_M \bar{k}_\perp^2 g_T^\perp$$

(1) twist 2 and 4  $\iff$  even number  
of  $\phi$  and  $\phi_s$

twist-3  $\iff$  odd number  
of  $\phi$  and  $\phi_s$

(2) Wherever there is twist-2 contribution, there is a twist-4 addendum to it.

S.Y. Wei, Y.K. Song, K.B. Chen, & ZTL, PRD95, 074017 (2017).



# Semi-Inclusive DIS: $e^- + N \rightarrow e^- + q(jet) + X$

Azimuthal asymmetries up to twist-4, e.g.,

At leading twist (twist-2)

$$\langle \sin(\phi - \phi_s) \rangle_{UT} |_{twist\,2} = \bar{k}_\perp \frac{xf_1^\perp}{2f_1} \quad \text{Sivers asymmetry} \quad \longrightarrow \quad \text{Sivers function } f_1^\perp$$

$$\langle \cos(\phi - \phi_s) \rangle_{LT} |_{twist\,2} = \bar{k}_\perp \frac{C(y)}{A(y)} \frac{g_1^\perp}{2f_1} \quad \longrightarrow \quad \text{tran-helicity } g_1^\perp$$

Including twist-4 contributions

$$\langle \sin(\phi - \phi_s) \rangle_{UT} = \bar{k}_\perp \frac{xf_1^\perp}{2f_1} (1 - \alpha_{UT} \kappa_M^2) \quad \alpha_{UT} = 16x^2 \frac{1-y}{1+(1-y)^2} \left( \frac{f_3}{f_1} - \frac{f_3^\perp}{f_1^\perp} \right) + 4x \left( \frac{f_{+3dd}}{f_1} - \frac{f_{+3dd}^\perp}{f_1^\perp} \right)$$

$$\langle \cos(\phi - \phi_s) \rangle_{LT} = \bar{k}_\perp \frac{C(y)}{A(y)} \frac{g_1^\perp}{2f_1} (1 - \alpha_{LT} \kappa_M^2) \quad \alpha_{LT} = 16x^2 \frac{1-y}{1+(1-y)^2} \frac{f_3}{f_1} + 4x \left( \frac{f_{+3dd}}{f_1} \frac{f_{+3dd}^{\perp 3}}{g_1^\perp} \right)$$

The Cahn's effect:

$$\langle \cos \phi \rangle_{UU} = -\bar{k}_\perp \kappa_M \frac{B(y)}{A(y)} \frac{xf_1^\perp}{f_1} \quad \langle \cos 2\phi \rangle_{UU} = -\bar{k}_\perp^2 \kappa_M^2 \frac{E(y)}{A(y)} \frac{f_{-3d}^\perp}{2f_1}$$

Relationships in “Fermion gas model” as a rough estimation of twist-4.

Relationships between twist-4 PDFs and their twist-2 counterparts at g=0 (Fermion gas model)

$$x f_{3d} = \frac{k_\perp^2}{2M^2} x f_{3d}^\perp = x^2 f_3 = -\frac{k_\perp^2}{2M^2} x f_1$$

$$x g_{3dL} = i \frac{k_\perp^2}{2M^2} x g_{3dL}^\perp = -x^2 g_{3L} = \frac{k_\perp^2}{2M^2} x g_{1L}$$

$$x f_{3dT}^\perp = -\frac{k_\perp^2}{2M^2} x f_{3dT}^{\perp 2} = x^2 f_{3T}^\perp = -\frac{k_\perp^2}{2M^2} f_{1T}^\perp$$

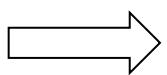
$$x g_{3dT}^{\perp 3} = -i \frac{k_\perp^2}{2M^2} x g_{3dT}^{\perp 4} = -x^2 g_{3T}^\perp = \frac{k_\perp^2}{2M^2} g_{1T}^\perp$$

$$2 \operatorname{Re} f_{3dd} = 2 \operatorname{Re} \frac{k_\perp^2}{2M^2} f_{3dd}^\perp = \frac{k_\perp^2}{2M^2} \frac{\partial}{\partial x} f_1$$

$$2 \operatorname{Re} g_{3ddL} = 2 \operatorname{Re} \frac{k_\perp^2}{2M^2} g_{3ddL}^\perp = -\frac{k_\perp^2}{2M^2} \frac{\partial}{\partial x} g_{1L}$$

$$2 \operatorname{Re} f_{3ddT}^\perp = -2 \operatorname{Re} \frac{k_\perp^2}{2M^2} f_{3ddT}^{\perp 2} = \frac{k_\perp^2}{2M^2} \frac{\partial}{\partial x} f_{1T}^\perp$$

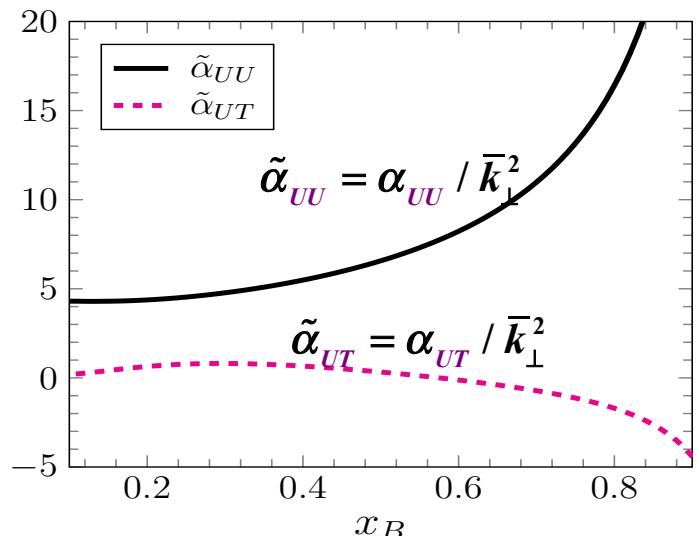
$$2 \operatorname{Re} g_{3ddT}^{\perp 3} = -2 \operatorname{Re} \frac{k_\perp^2}{2M^2} g_{3ddT}^{\perp 4} = -\frac{k_\perp^2}{2M^2} \frac{\partial}{\partial x} g_{1T}^\perp$$



$$\alpha_{UU} \approx \frac{k_\perp^2}{M^2} \left[ \frac{\partial \ln f_1}{\partial \ln x} - \frac{\partial \ln f_{1T}^\perp}{\partial \ln x} \right]$$

$$\alpha_{UT} \approx -\frac{k_\perp^2}{M^2} \left[ \frac{8(1-y)}{1+(1-y)^2} - \frac{\partial \ln f_1}{\partial \ln x} - \frac{\partial \ln g_{1T}^\perp}{\partial \ln x} \right]$$

$$\langle \cos \phi \rangle_{UU} \approx -\bar{k}_\perp \kappa_M \frac{B(y)}{A(y)} \quad \langle \cos 2\phi \rangle_{UU} \approx \bar{k}_\perp^2 \kappa_M^2 \frac{E(y)}{A(y)}$$





# Summary & Outlook

- Collinear expansion is crucial to deal with leading & higher twists systematically and is shown to be applicable to SIDIS  $e + N \rightarrow e + q(jet) + X$
- Complete twist-4 results for the semi-inclusive DIS  $e + N \rightarrow e + q(jet) + X$  have been obtained.

(1) twist 2 and 4  $\longleftrightarrow$  even number of  $\phi$  and  $\phi_s$ ;      twist-3  $\longleftrightarrow$  odd number of  $\phi$  and  $\phi_s$

(2) Wherever there is twist-2 contribution, there is a twist-4 addendum to it.

- We also propose to use relationships between twist-4 PDFs and their twist-2 counterparts obtained in the “Fermion gas model” to make a rough estimation of twist-4 contributions. The results suggest that twist-4 contributions might be very significant to the well-known twist-2 asymmetries such as the Sivers asymmetry thus have large impact on the study of TMDs.

**Thank you for your attention!**

# TMD PDFs defined via quark-quark correlator $\hat{\Phi}^{(0)}(x, k_\perp; p, S)$



## Twist-2 TMD PDFs

quark polarization →		
U	L	T
U	$f_1(x, k_\perp)$ number density	
L		$g_{1L}(x, k_\perp)$ helicity distribution
T	$f_{1T}^\perp(x, k_\perp)$ Sivers function	$g_{1T}^\perp(x, k_\perp)$ Worm-gear/trans-helicity
nucleon polarization ↑		
		$h_{1L}^\perp(x, k_\perp)$ Boer-Mulders function
		$h_{1L}^\perp(x, k_\perp)$ Worm-gear/longi-transversity
		$h_{1T}^\perp(x, k_\perp)$ transversity distribution
		$h_{1T}^\perp(x, k_\perp)$ pretzelosity

## Twist-3 TMD PDFs

nucleon polarization ↑			
U	L	T	
U	$e(x, k_\perp), f^\perp(x, k_\perp)$ number density	$g^\perp(x, k_\perp)$	$h(x, k_\perp)$ Boer-Mulders function
L	$g^\perp(x, k_\perp)$	$e_L(x, k_\perp), g_L^\perp(x, k_\perp)$ helicity distribution	$h_L(x, k_\perp)$ Worm gear/ longi-transversity
T	$e_T^\perp(x, k_\perp),$ Sivers function	$e_T^\perp(x, k_\perp),$ $g_T(x, k_\perp), g_T^\perp(x, k_\perp)$ Worm gear/ trans-helicity	$h_T^\perp(x, k_\perp)$ transversity distribution
		$h_T^\perp(x, k_\perp)$ pretzelosity	

## Twist-4 TMD PDFs

nucleon polarization ↑		
U	L	T
U	$f_3(x, k_\perp)$ number density	
L		$g_{3L}(x, k_\perp)$ helicity distribution
T	$f_{3T}^\perp(x, k_\perp)$ Sivers function	$g_{3T}^\perp(x, k_\perp)$ Worm-gear/trans-helicity
		$h_3^\perp(x, k_\perp)$ Boer-Mulders function
		$h_{3L}^\perp(x, k_\perp)$ Worm-gear/longi-transversity
		$h_{3T}^\perp(x, k_\perp)$ transversity distribution
		$h_{3T}^\perp(x, k_\perp)$ pretzelosity



# TMD PDFs defined via quark-gluon-quark correlator

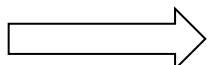
The Lorentz decomposition of the **quark-gluon-quark** correlator:

$$\hat{\phi}_{\rho}^{(1)}(x, k_{\perp}; p, S) = \frac{1}{2} \left[ \varphi_{\rho}^{(1)} + i\gamma_5 \tilde{\varphi}_{\rho}^{(1)} + \gamma^{\alpha} \varphi_{\rho\alpha}^{(1)} + \gamma_5 \gamma^{\alpha} \tilde{\varphi}_{\rho\alpha}^{(1)} + i\gamma_5 \sigma^{\alpha\beta} \varphi_{\rho\alpha\beta}^{(1)} \right]$$

E.g.:

a subscript “d” to denote that they are from D-type quark-gluon-quark correlator

$$\begin{aligned} \varphi_{\rho\alpha}^{(1)}(x, k_{\perp}; p, S) &= p^+ \bar{n}_{\alpha} \left( k_{\perp\rho} f_d^{\perp} - M \tilde{S}_{T\rho} f_{dT} - \lambda \tilde{k}_{\perp\rho} f_{dL}^{\perp} - \frac{k_{\perp(\rho} k_{\perp\beta)}}{M} \tilde{S}_{T}^{\beta} f_{dL}^{\perp} \right) && \text{twist-3} \\ &+ M^2 g_{\perp\rho\alpha} \left( f_{3d} - \frac{\epsilon_{\perp}^{kS}}{M} f_{3dL}^{\perp} \right) + k_{\perp(\rho} k_{\perp\alpha)} \left( f_{3d}^{\perp} + \frac{\epsilon_{\perp}^{kS}}{M} f_{3dL}^{\perp 2} \right) && \text{twist-4} \\ &+ iM^2 \epsilon_{\perp\rho\alpha} \left( \lambda f_{3dL} - \frac{k_{\perp} \cdot S_T}{M} f_{3dL}^{\perp 3} \right) + \frac{1}{2} k_{\perp(\rho} \tilde{k}_{\perp\alpha)} \left( \lambda f_{3dL}^{\perp} + \frac{k_{\perp} \cdot S_T}{M} f_{3dL}^{\perp 4} \right) + \dots \end{aligned}$$

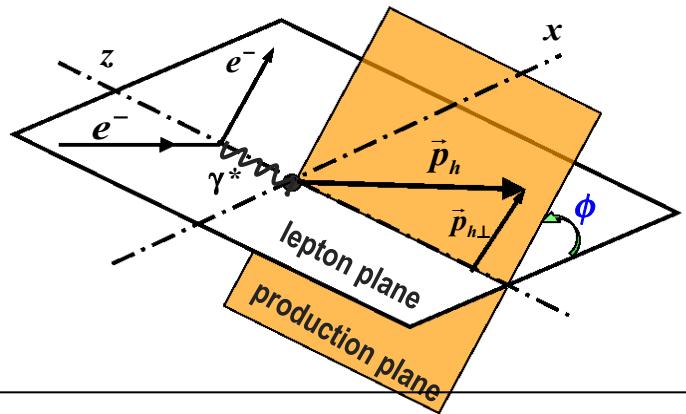
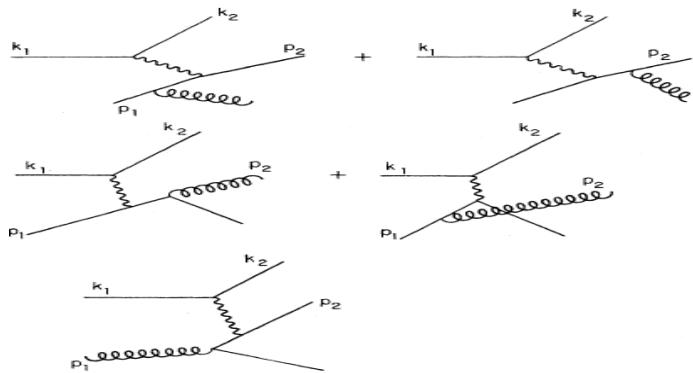


# Introduction: Azimuthal asymmetries and higher twist contributions

## Azimuthal asymmetry and higher twist

$$e^- + N \rightarrow e^- + q(\text{jet}) + X$$

1977, Georgi & Politzer: gluon radiation  $\longrightarrow$  azimuthal asymmetry  $\Longleftrightarrow$  “Clean test to pQCD”



1978, Cahn: generalize parton model to include an intrinsic  $\vec{k}_\perp$ :

$$\langle \cos \phi \rangle = -\frac{|\vec{k}_\perp|}{Q} \frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2} \quad (\text{twist 3}) \quad \langle \cos 2\phi \rangle = \frac{|\vec{k}_\perp|^2}{Q^2} \frac{2(1-y)}{1+(1-y)^2} \quad (\text{twist 4})$$

“Cahn effect”

higher twist, nevertheless significant !

$$|\vec{k}_\perp| \sim 0.3 - 0.7 \text{ GeV} \quad |\vec{k}_\perp|/Q \sim 0.1$$

**Lesson: do not forget higher twists!**

# Introduction: Azimuthal asymmetries and gauge link

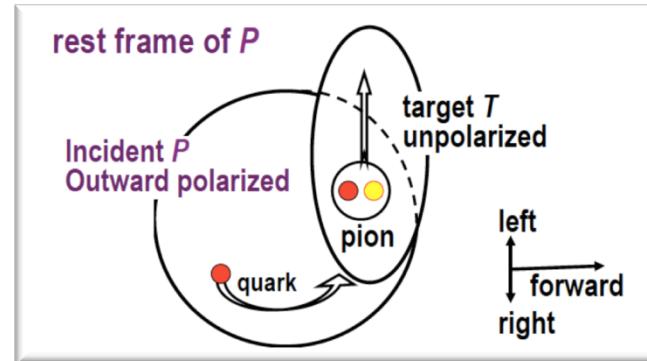
## Sivers function and gauge link

1991, Sivers: introduced the asymmetric quark distribution in polarized nucleon (Sivers function)

$$f_q(x, k_\perp; S_\perp) = f_q(x, k_\perp) + (\hat{k}_\perp \times \hat{p}) \cdot \vec{S}_T f_{1T}^\perp(x, k_\perp)$$

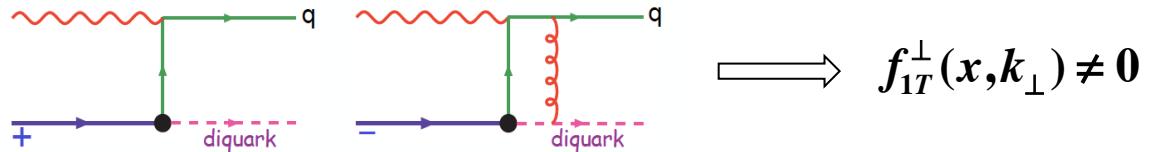
1993, Boros, Liang & Meng:

proposed an intuitive picture:  
quark orbital angular momentum+ “surface effect”



1993, Collins: P&T invariance  $\implies f_{1T}^\perp(x, k_\perp) = 0$

2002, Brodsky, Hwang, Schmidt:



2002, Collins: “final state interaction” = “gauge link”.

**Lesson: do not forget the gauge link!**



## The azimuthal asymmetries in terms of structure functions

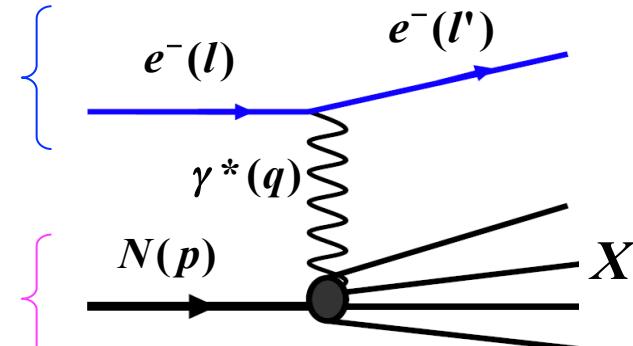
- $\langle \cos \phi \rangle_{UU} = \sqrt{2\epsilon(1+\epsilon)} W_{UU}^{\cos \phi} / W_{UU}$        $\langle \cos 2\phi \rangle_{UU} = \epsilon W_{UU}^{\cos 2\phi} / W_{UU}$
  
- $\langle \sin \phi \rangle_{LU} = \sqrt{2\epsilon(1-\epsilon)} W_{LU}^{\sin \phi} / W_{UU}$
  
- $\langle \sin \phi \rangle_{UL} = \sqrt{2\epsilon(1+\epsilon)} W_{UL}^{\sin \phi} / W_{UU}$        $\langle \sin 2\phi \rangle_{UL} = \epsilon W_{UL}^{\sin 2\phi} / W_{UU}$
  
- $\langle \cos \phi \rangle_{LL} = \sqrt{2\epsilon(1+\epsilon)} [W_{UU}^{\cos \phi} + \lambda_l \lambda_N y W_{LL}^{\cos \phi} / (2-y)] / (W_{UU} + \lambda_l \lambda_N \sqrt{1-\epsilon^2} W_{LL})$
  
- $\langle \sin \phi_s \rangle_{UT} = \sqrt{2\epsilon(1+\epsilon)} W_{UT}^{\sin \phi_s} / W_{UU}$        $\langle \sin(\phi + \phi_s) \rangle_{UT} = \epsilon W_{UT}^{\sin(\phi + \phi_s)} / W_{UU}$   
 $\langle \sin(\phi - \phi_s) \rangle_{UT} = W_{UT}^{\sin(\phi - \phi_s)} / W_{UU}$        $\langle \sin(2\phi - \phi_s) \rangle_{UT} = \sqrt{2\epsilon(1+\epsilon)} W_{UT}^{\sin(2\phi - \phi_s)} / W_{UU}$   
 $\langle \sin(3\phi - \phi_s) \rangle_{UT} = \epsilon W_{UT}^{\sin(3\phi - \phi_s)} / W_{UU}$
  
- $\langle \cos \phi_s \rangle_{LT} = \sqrt{2\epsilon(1-\epsilon)} W_{LT}^{\cos \phi_s} / W_{UU}$        $\langle \cos(\phi - \phi_s) \rangle_{LT} = \sqrt{1-\epsilon^2} W_{LT}^{\cos(\phi - \phi_s)} / W_{UU}$   
 $\langle \cos(2\phi - \phi_s) \rangle_{LT} = \sqrt{2\epsilon(1-\epsilon)} W_{LT}^{\cos(2\phi - \phi_s)} / W_{UU}$

Totally 15 azimuthal asymmetries in different polarized cases.

## The differential cross section

$$d\sigma = \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l, \lambda_l, l', \lambda_{l'}) W_{\mu\nu}(q, p, S) \frac{d^3 l'}{2E'}$$

leptonic tensor      hadronic tensor



The hadronic tensor:  $W_{\mu\nu}(q, p, S) = \sum_X \langle p, S | J_\mu(0) | X \rangle \langle X | J_\nu(0) | p, S \rangle (2\pi)^4 \delta^4(p + q - p_X)$

$$W_{\mu\nu}(q, p, S) = \left| \begin{array}{c} \text{Feynman diagram} \\ \text{with } m^2 \end{array} \right|^2 = \left| \begin{array}{c} \text{Feynman diagram} \\ \text{with } m \times m^* \end{array} \right|^2$$

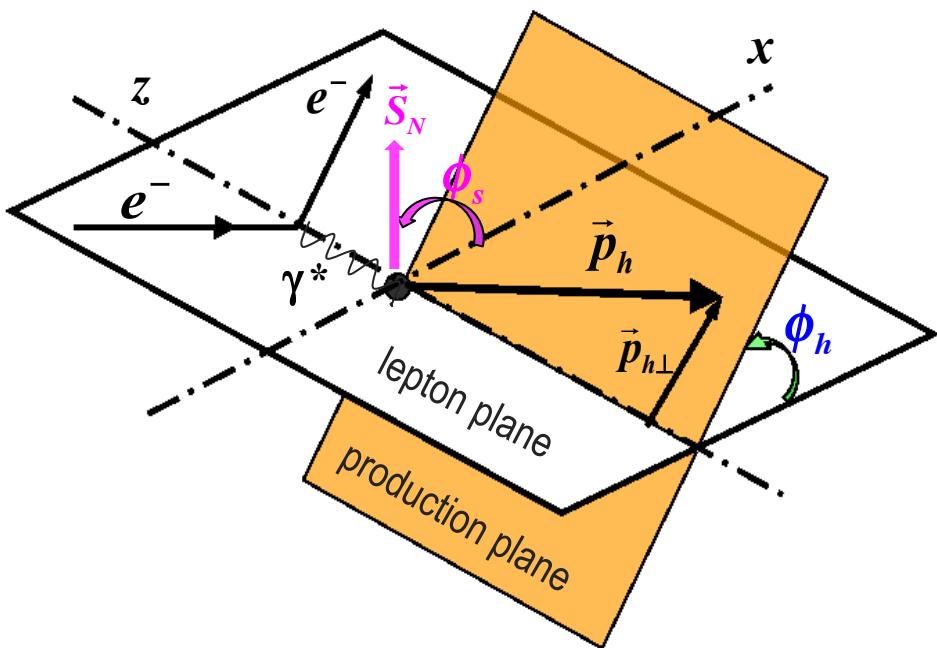
# General kinematic analysis for $e^- N \rightarrow e^- hX$



$$d\sigma = \frac{\alpha^2}{sQ^4} L^{\mu\nu}(l, \lambda_l, l') W_{\mu\nu}^{(si)}(q, p, S, p_h) \frac{d^3 l' d^3 p_h}{(2\pi)^3 2E_l 2E_h}$$

The reference frame

- c.m. frame of  $\gamma^* N$
- $p$  in z-direction
- lepton-hadron plane = oxz plane



$$\textcolor{blue}{N}: \quad p = (E, 0, 0, |\vec{p}|)$$

$$\textcolor{blue}{e^-}: \quad l = E_l(1, \sin\theta, 0, \cos\theta)$$

$$\gamma^*: \quad q = (q_0, 0, 0, -|\vec{q}|)$$

$$\textcolor{blue}{h}: \quad p_h = (E_h, |\vec{p}_{hT}| \cos\phi, |\vec{p}_{hT}| \sin\phi, p_{hz})$$

In dependent variables used:

$$s = q^2 = Q^2 \quad y = l \cdot p / q \cdot p$$

$$x = 2q \cdot p / Q^2 \quad p_{hT}^2 \equiv -|\vec{p}_{hT}|^2, \quad \phi_h$$

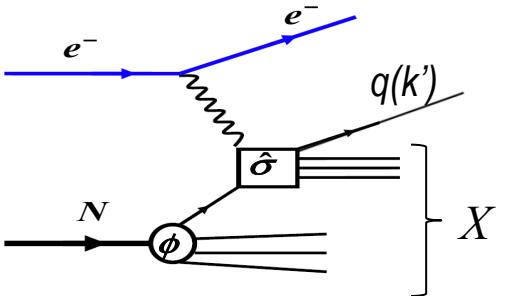
$$\xi = 2q \cdot p_h / Q^2$$

# Semi-Inclusive DIS $e^- + N \rightarrow e^- + q(jet) + X$



The cross section:

$$d\sigma^{(si)} = \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l, \lambda_l, l', \lambda_{l'}) W_{\mu\nu}^{(si)}(q, p, S, \mathbf{k}') \frac{d^3 l'}{2E'} \frac{d^3 k'}{2E_k}$$



$$W_{\mu\nu}^{(si)}(q, p, S, \mathbf{k}') = \sum_{n=0}^{\infty} W_{\mu\nu}^{(n, si)}(q, p, S, \mathbf{k}') = W_{\mu\nu}^{(0, si)}(q, p, S, \mathbf{k}') + W_{\mu\nu}^{(1, si)}(q, p, S, \mathbf{k}') + W_{\mu\nu}^{(2, si)}(q, p, S, \mathbf{k}') + \dots$$

$$W_{\mu\nu}^{(0, si)}(q, p, S, \mathbf{k}') = \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[\hat{\phi}^{(0)}(k, p, S) \hat{H}_{\mu\nu}^{(0, si)}(k, q, k')]$$

$$W_{\mu\nu}^{(1, si)}(q, p, S, \mathbf{k}') = \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \sum_{c=L,R} \text{Tr}[\hat{\phi}_\rho^{(1)}(k_1, k_2, p, S) \hat{H}_{\mu\nu}^{(1, c; si)\rho}(k_1, k_2, q, k')]$$

To compare  $W_{\mu\nu}^{(0)}(q, p, S) = \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[\hat{\phi}^{(0)}(k, p, S) \hat{H}_{\mu\nu}^{(0)}(k, q)]$

$$W_{\mu\nu}^{(1)}(q, p, S) = \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \sum_{c=L,R} \text{Tr}[\hat{\phi}_\rho^{(1)}(k_1, k_2, p, S) \hat{H}_{\mu\nu}^{(1, c)\rho}(k_1, k_2, q)]$$

# Semi-Inclusive DIS $e^- + N \rightarrow e^- + q(jet) + X$



The hard parts

$$\hat{H}_{\mu\nu}^{(0,si)}(k, q, k') = \gamma_\mu (\not{k} + \not{q}) \gamma_\nu (2\pi)^4 \delta^4(k' - k - q)$$

$$\hat{H}_{\mu\nu}^{(1,L;si)}(k_1, k_2, q, k') = \gamma_\mu \frac{(\not{k}_1 + \not{q}) \gamma^\rho (\not{k}_2 + \not{q})}{(k_2 + q)^2 - i\varepsilon} \gamma_\nu (2\pi)^4 \delta^4(k' - k_1 - q)$$

To compare

$$\hat{H}_{\mu\nu}^{(0)}(k, q) = \gamma_\mu (\not{k} + \not{q}) \gamma_\nu (2\pi) \delta_+((k + q)^2)$$

$$\hat{H}_{\mu\nu}^{(1,L)}(k_1, k_2, q) = \gamma_\mu \frac{(\not{k}_1 + \not{q}) \gamma^\rho (\not{k}_2 + \not{q})}{(k_2 + q)^2 - i\varepsilon} \gamma_\nu (2\pi) \delta_+((k_1 + q)^2)$$

Ward identities:

$$\frac{\partial \hat{H}_{\mu\nu}^{(0)}(x)}{\partial k^\rho} = -\hat{H}_{\mu\nu}^{(1)\rho}(x, x)$$

$$p_\rho \hat{H}_{\mu\nu}^{(1,L)\rho}(x_1, x_2) = \frac{\hat{H}_{\mu\nu}^{(0)}(x_1)}{x_2 - x_1 - i\varepsilon}$$

$$\frac{\partial \hat{H}_{\mu\nu}^{(0,si)}(x, x')}{\partial k^\rho} \neq -\hat{H}_{\mu\nu}^{(1,si)\rho}(x, x; x')$$

$$p_\rho \hat{H}_{\mu\nu}^{(1,L,si)\rho}(x_1, x_2; x') \neq \frac{\hat{H}_{\mu\nu}^{(0,si)}(x_1, x')}{x_2 - x_1 - i\varepsilon}$$

# Semi-Inclusive DIS $e^- + N \rightarrow e^- + q(jet) + X$



Using the identity:  $(2\pi)^4 \delta^4(k' - k - q) = (2\pi) \delta_+((k + q)^2) (2\pi)^3 (2E_{k'}) \delta^3(\vec{k}' - \vec{k} - \vec{q})$

We obtain:  $\hat{H}_{\mu\nu}^{(0,si)}(k, q, k') = \hat{H}_{\mu\nu}^{(0)}(k, q) (2E_{k'}) (2\pi)^3 \delta^3(\vec{k}' - \vec{k} - \vec{q})$

$$\hat{H}_{\mu\nu}^{(1,\rho,c,si)}(k_1, k_2, q, k') = \hat{H}_{\mu\nu}^{(1,\rho,c)}(k_1, k_2, q) (2E_{k'}) (2\pi)^3 \delta^3(\vec{k}' - \vec{k}_c - \vec{q})$$

→

$$W_{\mu\nu}^{(0,si)}(q, p, S, k') = \underbrace{\int \frac{d^4 k}{(2\pi)^4} \text{Tr}[\hat{\phi}^{(0)}(k, p, S) \hat{H}_{\mu\nu}^{(0)}(k, q)]}_{W_{\mu\nu}^{(0)}(q, p, S)} (2E_{k'}) (2\pi)^3 \delta^3(\vec{k}' - \vec{k} - \vec{q})$$

← common factor

$$W_{\mu\nu}^{(1,si)}(q, p, S, k') = \underbrace{\int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \sum_{c=L,R} \text{Tr}[\hat{\phi}_\rho^{(1)}(k_1, k_2, p, S) \hat{H}_{\mu\nu}^{(1,c)\rho}(k_1, k_2, q)]}_{W_{\mu\nu}^{(1)}(q, p, S)} (2E_{k'}) (2\pi)^3 \delta^3(\vec{k}' - \vec{k}_c - \vec{q})$$

The same Taylor expansion of  $\hat{H}_{\mu\nu}^{(j)}(k_1, \dots; q)$  as that for inclusive DIS →



# Inclusive DIS: LO pQCD, leading & higher twists

## Simplified expressions for hadronic tensors

The “collinearly expanded hard parts” take the simple forms such as:

$$\hat{H}_{\mu\nu}^{(0)}(x) = \hat{h}_{\mu\nu}^{(0)} \delta(x - x_B), \quad \hat{h}_{\mu\nu}^{(0)} = \gamma_\mu \not{\epsilon} \gamma_\nu$$

$$\hat{H}_{\mu\nu}^{(1,L)\rho}(x_1, x_2) \omega_\rho^{\rho'} = \frac{\pi}{2q \cdot p} \hat{h}_{\mu\nu}^{(1)\rho} \omega_\rho^{\rho'} \delta(x_1 - x_B), \quad \hat{h}_{\mu\nu}^{(1)\rho} = \gamma_\mu \not{\epsilon} \gamma^\rho \not{\epsilon} \gamma_\nu$$

each depends only  
on one longitudinal  
variable  $x$  !

$$\tilde{W}_{\mu\nu}^{(0)}(q, p, S) = \int dx \text{Tr} \left[ \hat{\Phi}^{(0)}(x; p, S) \hat{h}_{\mu\nu}^{(0)} \right] \delta(x - x_B) = \text{Tr} \left[ \hat{\Phi}^{(0)}(x_B; p, S) \hat{h}_{\mu\nu}^{(0)} \right]$$

contributes at twist-2, 3 and 4

$$\hat{\Phi}^{(0)}(x; p, S) \equiv \int \frac{d^4 k}{(2\pi)^4} \delta(x - \frac{k^+}{p^+}) \hat{\Phi}^{(0)}(k; p, S) = \int \frac{p^+ dz^-}{2\pi} e^{ixp^+ z^-} \langle p, S | \bar{\psi}(0) \not{\epsilon}(0, z^-) \psi(z^-) | p, S \rangle$$

one-dimensional gauge invariant quark-quark correlator

$$\tilde{W}_{\mu\nu}^{(1)}(q, p, S) = \frac{\pi}{2q \cdot p} \int dx \text{Tr} \left[ \hat{\phi}_\rho^{(1)}(x; p, S) \hat{h}_{\mu\nu}^{(1)\rho} \omega_\rho^{\rho'} \right] \delta(x - x_B) = \frac{\pi}{2q \cdot p} \text{Tr} \left[ \hat{\phi}_\rho^{(1)}(x_B; p, S) \hat{h}_{\mu\nu}^{(1)\rho} \omega_\rho^{\rho'} \right]$$

contributes at twist-3, 4 and 5

$$\hat{\phi}_\rho^{(1)}(x; p, S) \equiv \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \delta(x - \frac{k_1^+}{p^+}) \hat{\Phi}_\rho^{(1)}(k_1, k_2; p, S) = \int \frac{p^+ dz^-}{2\pi} e^{ixp^+ z^-} \langle p, S | \bar{\psi}(0) D_\rho(0) \not{\epsilon}(0, z^-) \psi(z^-) | p, S \rangle$$

the involved one-dimensional gauge invariant quark-gluon-quark correlator

ONE dimensional, depend only on ONE parton momentum!