# Fragmentation Functions

## measurements at Belle

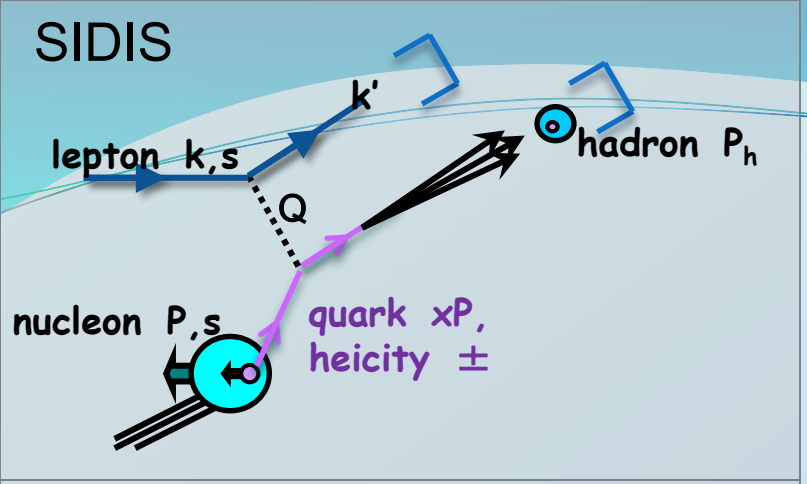
**DIS 2018, Kobe,**
**April 15-20, 2018**

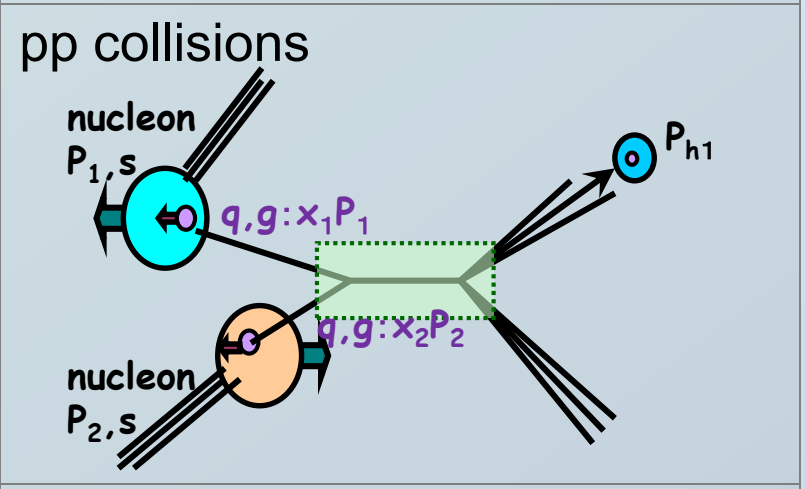
**Ralf Seidl (RIKEN)**

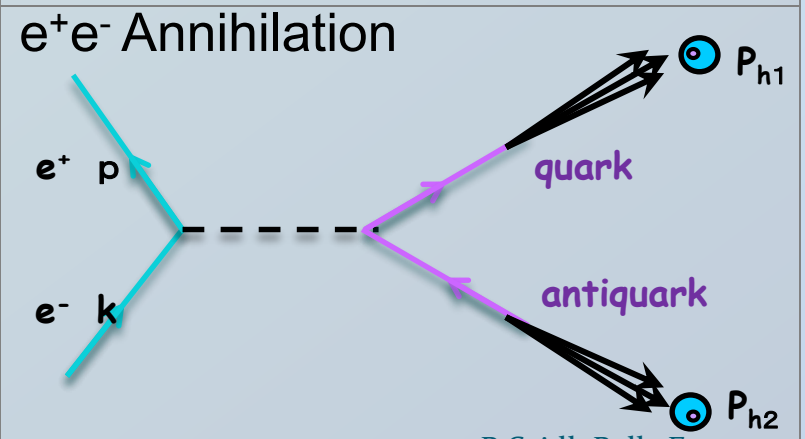
# Outline

- Single hadron fragmentation
  - Hyperon and charmed Baryon fragmentation
  - Λ polarizing fragmentation
- Di-hadron fragmentation
  - Unpolarized mass, z dependence
- Other ongoing measurments (kt deptendence)

# Access to FFs




- SIDIS:

$$\sigma^h(x, z, Q^2, P_{h\perp}) \propto \sum_q e_q^2 q(x, p_t, Q^2) D_{1,q}^h(z, k_t, Q^2)$$

  - Relies on unpol PDFs
  - Parton momentum known at LO
  - Flavor structure directly accessible
  - Transverse momenta convoluted between FF and PDF
- pp:

$$\sigma^h(P_T) \propto \int_{x_1,x_2,z} \sum_{a,a' \in q,g} f_a(x_1) \otimes f_{a'}(x_2) \otimes \sigma_{aa'} \otimes D_{1,q}^h(z)$$

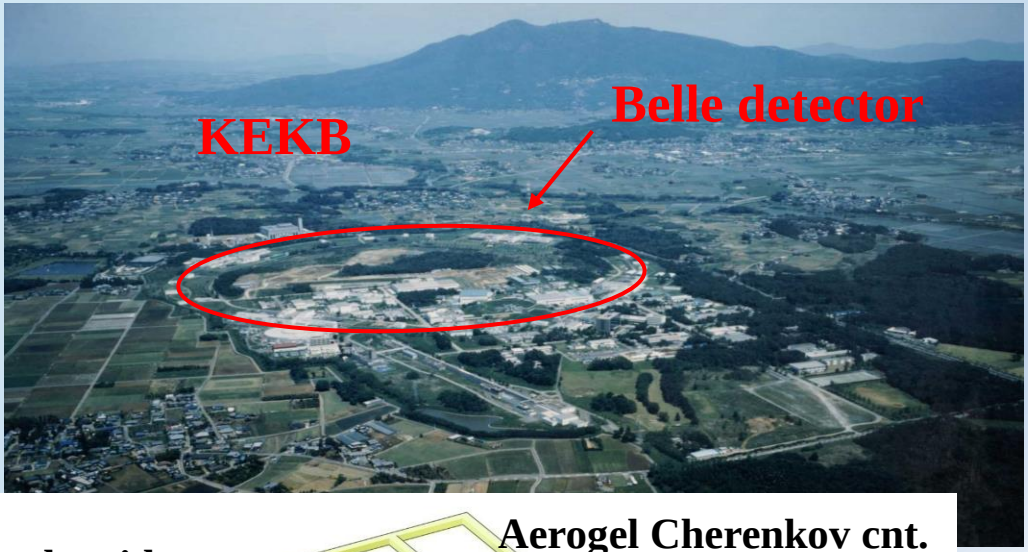
  - Relies on unpol PDFs
  - leading access to gluon FF
  - Parton momenta not directly known
- e+e-:

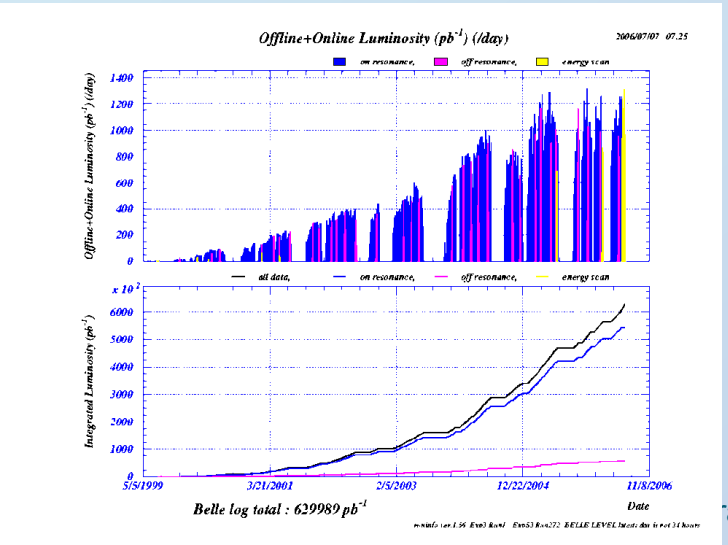
$$\sigma^h(z, Q^2, k_t) \propto \sum_q e_q^2 \left( D_{1,q}^h(z, k_t, Q^2) + D_{1,\bar{q}}^h(z, k_t, Q^2) \right)$$

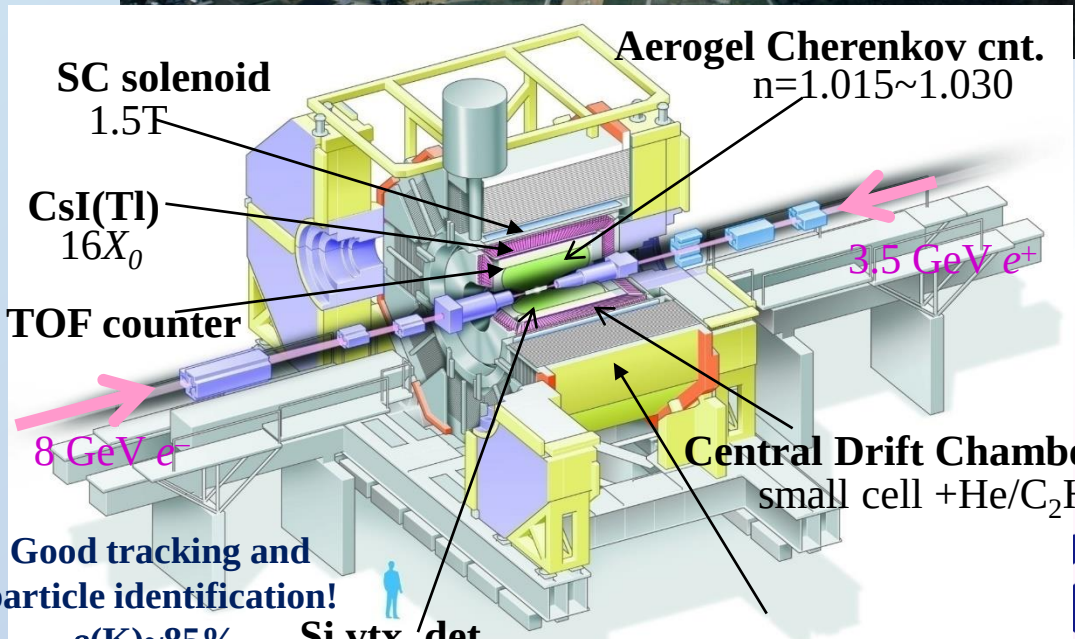
  - No PDFs necessary
  - Clean initial state, parton momentum known at LO
  - Flavor structure not directly accessible

# Belle Detector and KEKB

- Asymmetric collider
- 8GeV $e^-$ + 3.5GeV $e^+$
- $\sqrt{s}$ = 10.58GeV ($\Upsilon(4S)$)
- $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$
- Continuum production: 10.52 GeV
- $e^+e^- \rightarrow q\bar{q}$ (u,d,s,c)
- Integrated Luminosity: >1000 $fb^{-1}$
- >70$fb^{-1}$ => continuum



# Single hadron fragmentation

$$D_{1,q}^{h}(z, Q^2)$$

$$D_{1,q}^{h}(z, k_T, Q^2)$$

$$H_{1,q}^{\perp h}(z, k_T, Q^2)$$

$$D_{1,q}^{\perp h}(z, k_T, Q^2)$$

$$H_{1,q}^{h}(z, Q^2)$$

$$\mathcal{G}_{q}^{h}(z, z_h, \omega_J R, j_\perp, Q^2)$$

In $e^+e^-$ annihilation:

$$Q = \sqrt{s}$$

$$z = \frac{2E_h}{Q} \approx \frac{E_h}{E_q}$$



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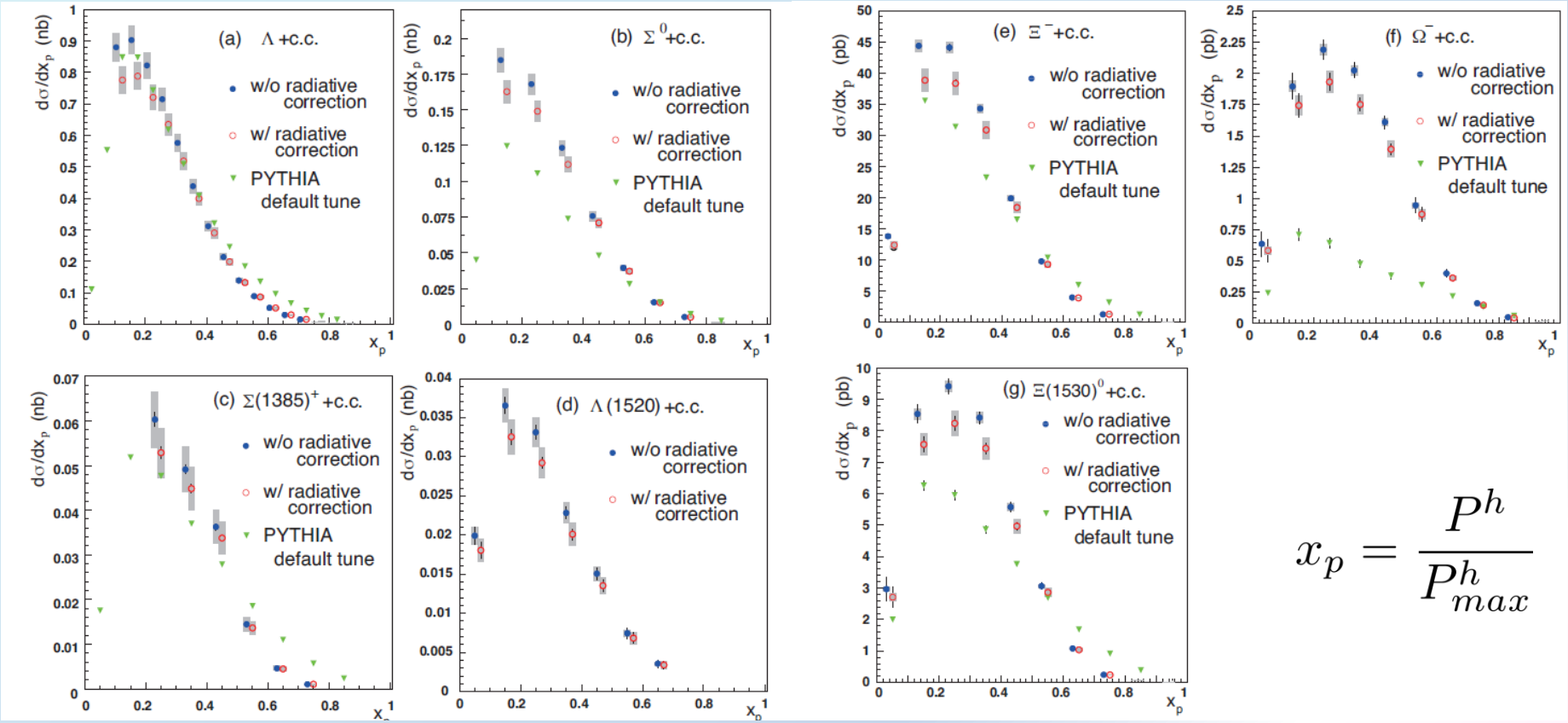
# Hyperon Fragmentation

Belle: Niiyama et. al. PRD 97 (2018), 072005




$$x_p = \frac{P^h}{P^h_{max}}$$

- Hyperons similar to light hadron fragmentation → peaking at low z ($x_p$)
- Baryon production not too well described by Pythia 6 default settings



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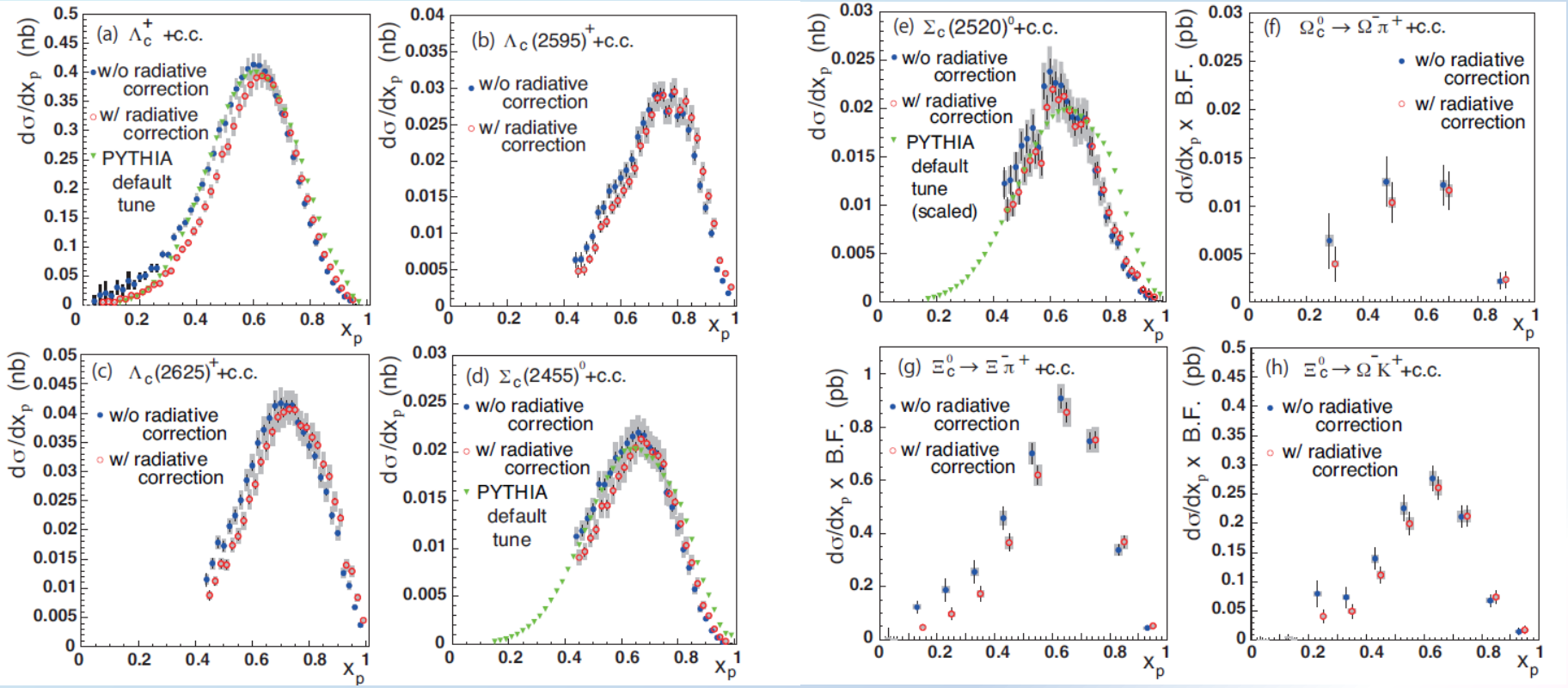
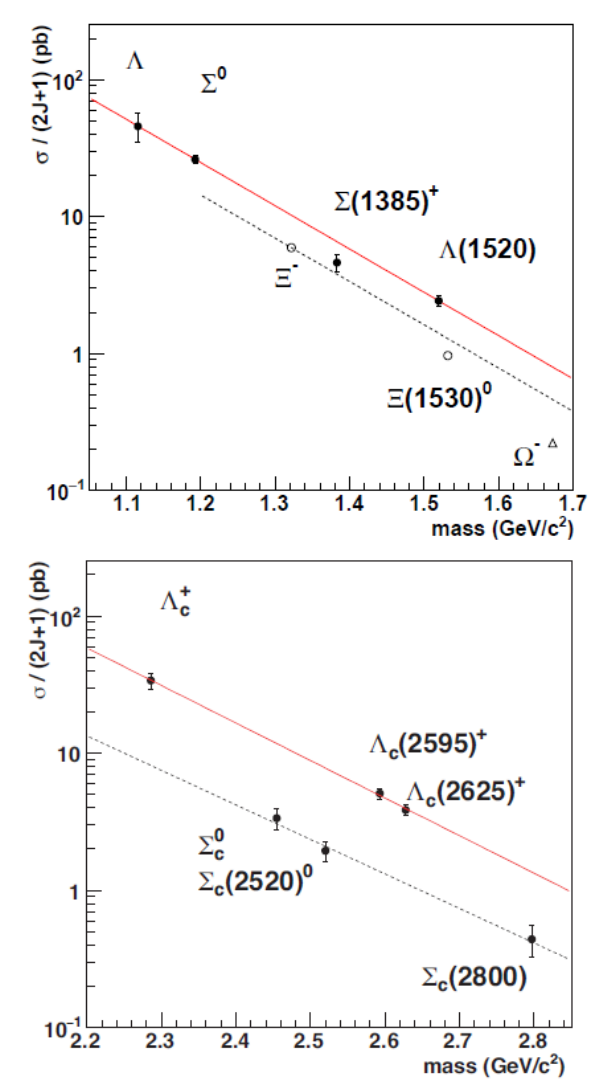
# Charmed baryon Fragmentation

Belle: Niiyama et. al. PRD 97 (2018), 072005




- Charmed baryons carry large fraction of parton momentum, similar to charmed mesons
- Charmed fragmentation reasonably described in Pythia for main states

# Baryon production rates




- First feed-down corrected production rates extracted
- No Λ(1520) enhancement seen
- Strangeness suppression seen for hyperons:

$$\frac{\sigma(S=-1)}{(2J+1)} > \frac{\sigma(S=-2,-3)}{(2J+1)}$$

- Difference in slopes for $\Lambda_c$ and $\Sigma_c$ in support of diquark production picture (spin 1 diquarks suppressed)

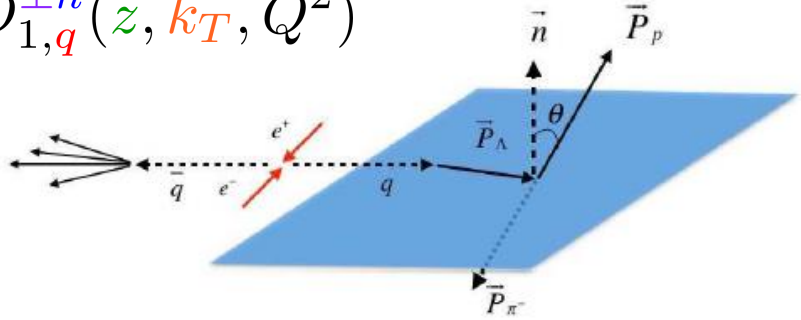
# Single Λ polarization measurements

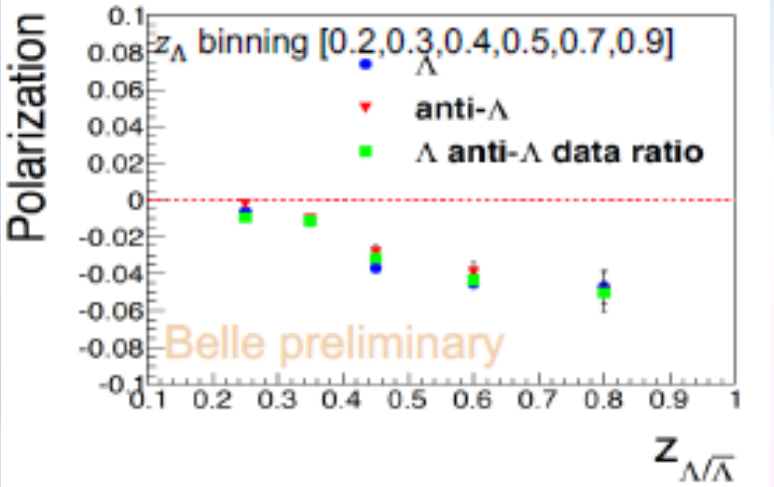
- Related to open question about Λ polarization in hadron collisions from 40 years ago!
- Fragmentation counterpart to the Sivers Function:

  unpolarized parton fragments into transversely polarized baryon with transverse momentum wrt to parton direction
- Reconstruct Λ, its transverse momentum and polarization

YingHui Guan (Indiana/KEK): arXiv:1611.06648

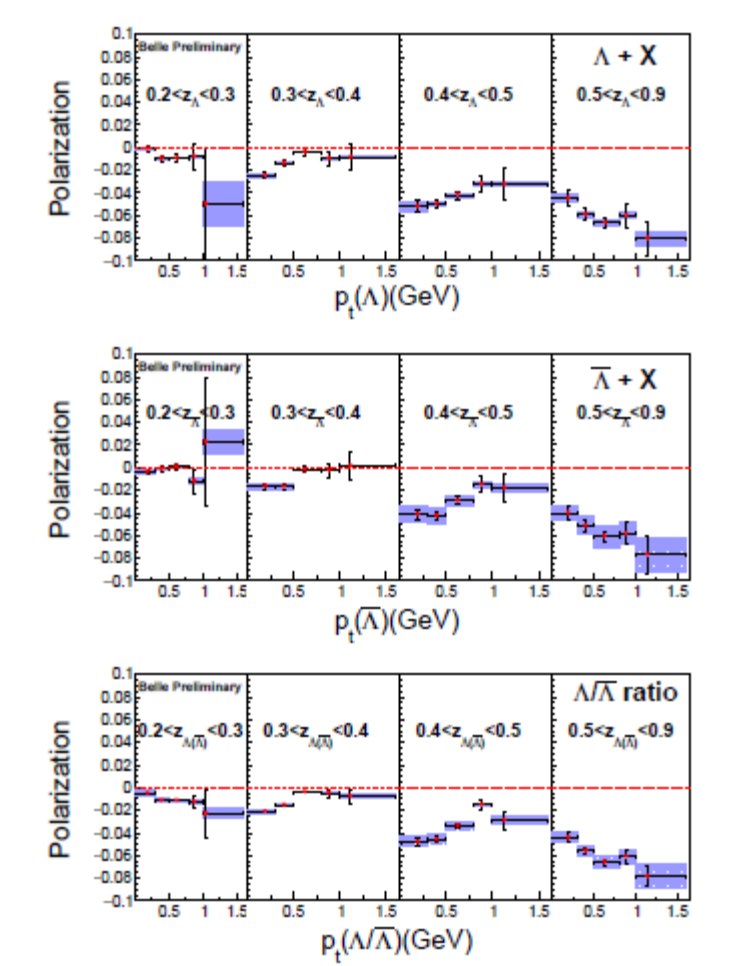
$D_{1,q}^{\perp h}(z, k_T, Q^2)$



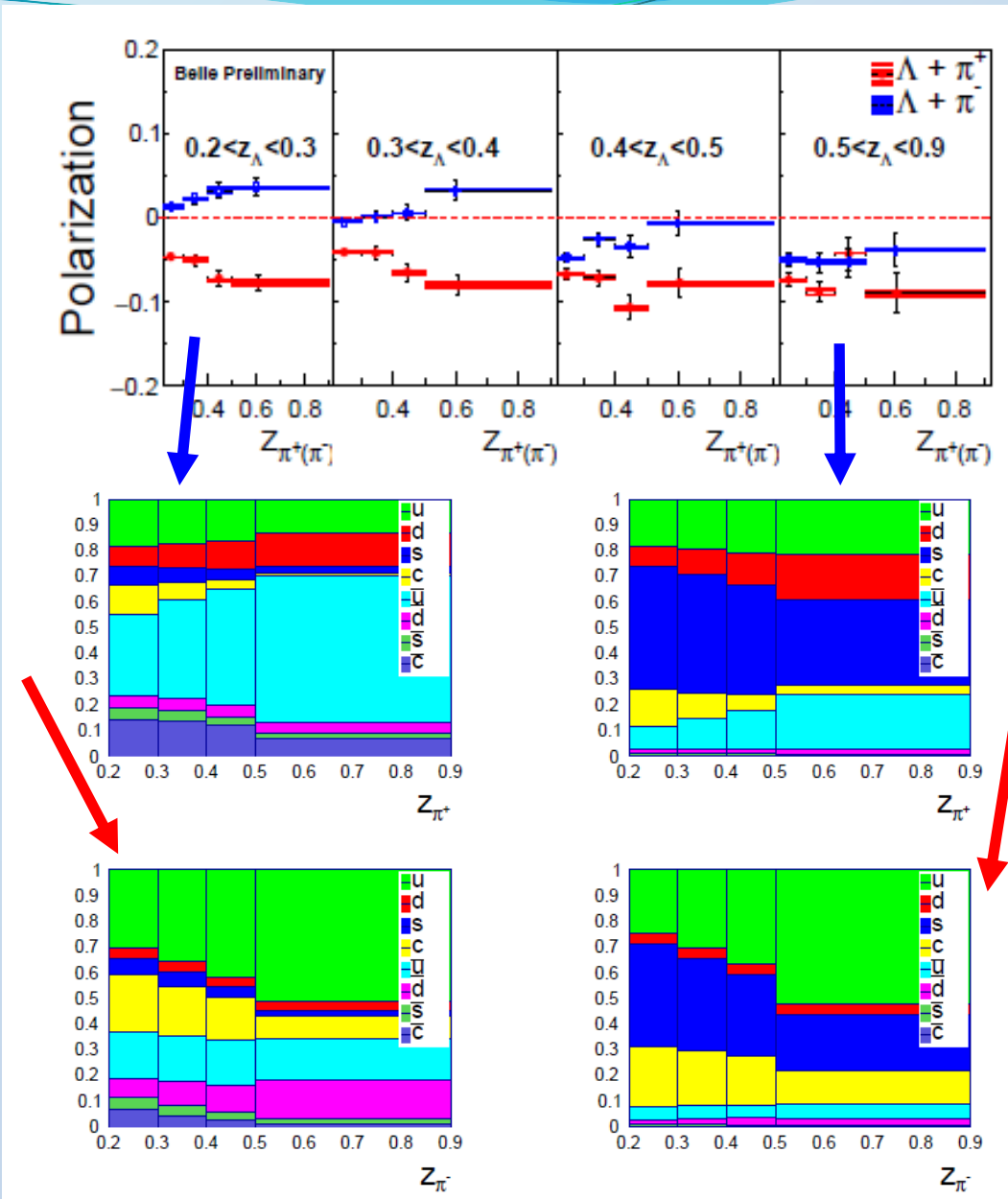
# Transverse momentum dependence

- Different behavior for low and high-z :
- At low z small
- At intermediate z falling Polarization with kt
- At high z increasing polarization with kt



# Opposite hemisphere pion correlation

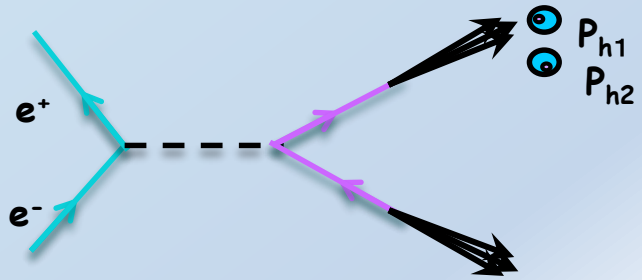
- Interesting $z_\pi$ and $z_\Lambda$ dependence :
- At low $z_\Lambda$ light quark fragmentation dominant, some charm in $\pi^-$ → different signs
- At high $z_\Lambda$ strange + charm fragmentation more relevant → same signs



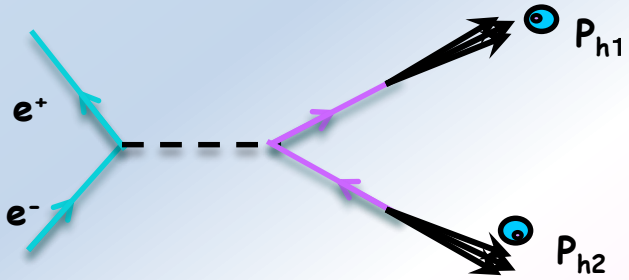
# Di-hadron fragmentation functions

$$D_{1,q}^{h_1 h_2}(z, m, Q^2)$$

$$H_{1,q}^{h_1, h_2, \sphericalangle}(z, Q^2, M_h)$$




$$D_{1,q}^{h}(z_1, Q^2) D_{1,q}^{h}(z_2, Q^2)$$



# Di-hadrons

In $e^+e^-$ annihilation:

$$Q = \sqrt{s}$$

$$z = \frac{2E_h}{Q} \approx \frac{E_h}{E_q}$$

- Single inclusive hadron multiplicities (e+e-→hX) sum over all available flavors and quarks and antiquarks:

$$d\sigma(e^+e^- \to hX)/dz \propto \sum_q e_q^2 (D^h_{1,q}(z,Q^2) + D^h_{1,\overline{q}}(z,Q^2))$$

- Especially distinction between favored (ie u→$\pi^+$) and disfavored ($\bar{u}$ →$\pi^+$) fragmentation would be important
- Idea: Use di-hadron fragmentation, preferably from opposite hemispheres and access favored and disfavored combinations:

$$u\overline{u} \to \pi^+\pi^- X \quad \propto \quad D^{\pi^+}_{u,fav}(z_1,Q^2) \cdot D^{\pi^-}_{\overline{u},fav}(z_2,Q^2) + D^{\pi^+}_{\overline{u},dis}(z_1,Q^2) \cdot D^{\pi^-}_{u,dis}(z_2,Q^2)$$

$$u\overline{u} \to \pi^+\pi^+ X \quad \propto \quad D^{\pi^+}_{u,fav}(z_1,Q^2) \cdot D^{\pi^+}_{\overline{u},dis}(z_2,Q^2) + D^{\pi^+}_{\overline{u},dis}(z_1,Q^2) \cdot D^{\pi^+}_{u,fav}(z_2,Q^2)$$

- Also: unpol baseline for interference fragmentation



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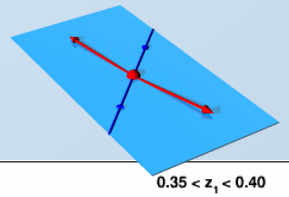
# Ratios to opposite charge pion pairs

$$R \approx \frac{D_{dis}(z_1)D_{fav}(z_2) + D_{fav}(z_1)D_{dis}(z_2)}{D_{fav}(z_1)D_{fav}(z_2) + D_{dis}(z_1)D_{dis}(z_2)}$$

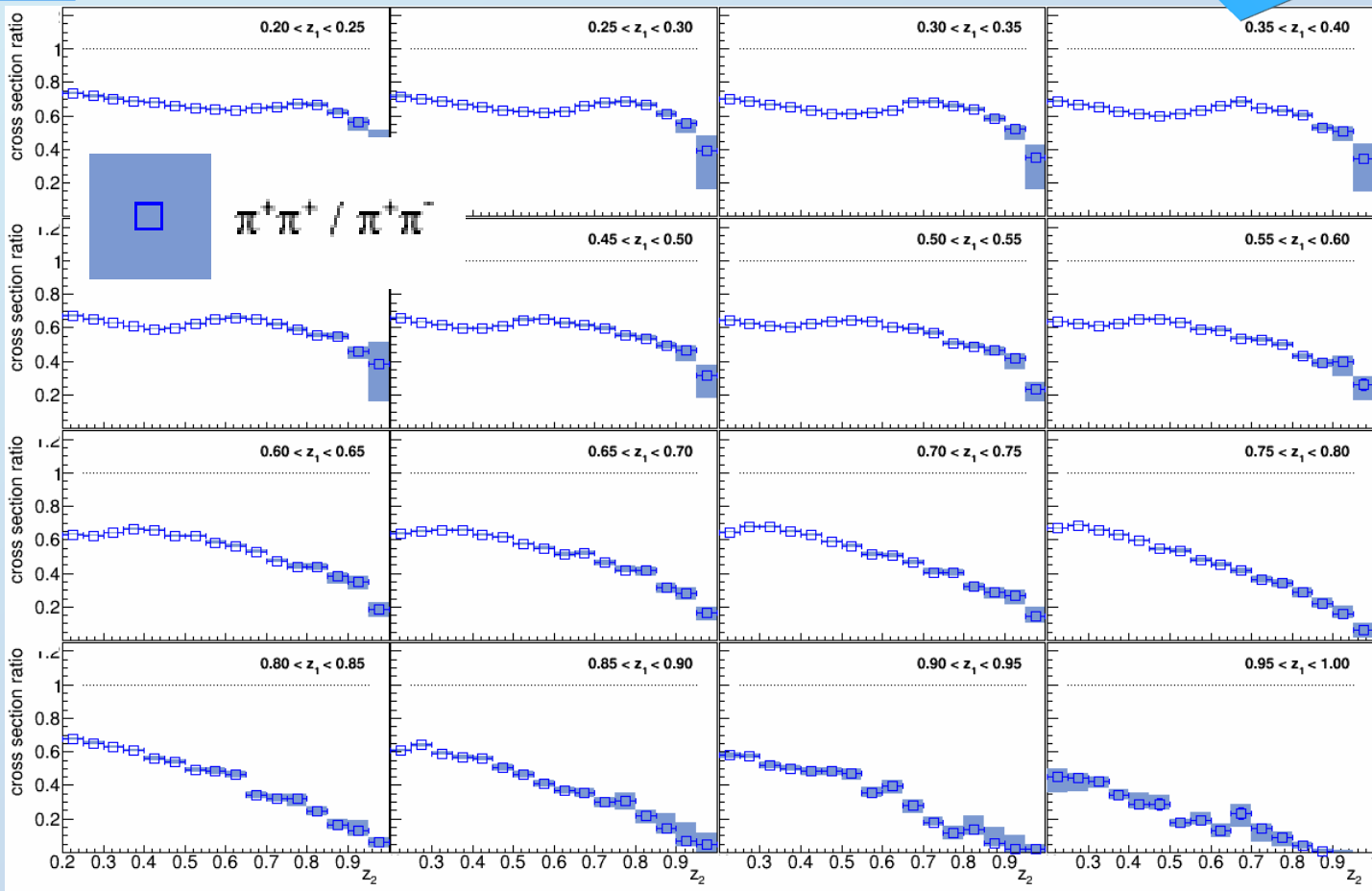
PRD92 (2015) 092007



$\pi^+\pi^+$ comparable to $\pi^+\pi^-$ at low z, decreasing towards high z:

→ Favored and disfavored fragmentation similar at low z

→ Disfavored much smaller at high z



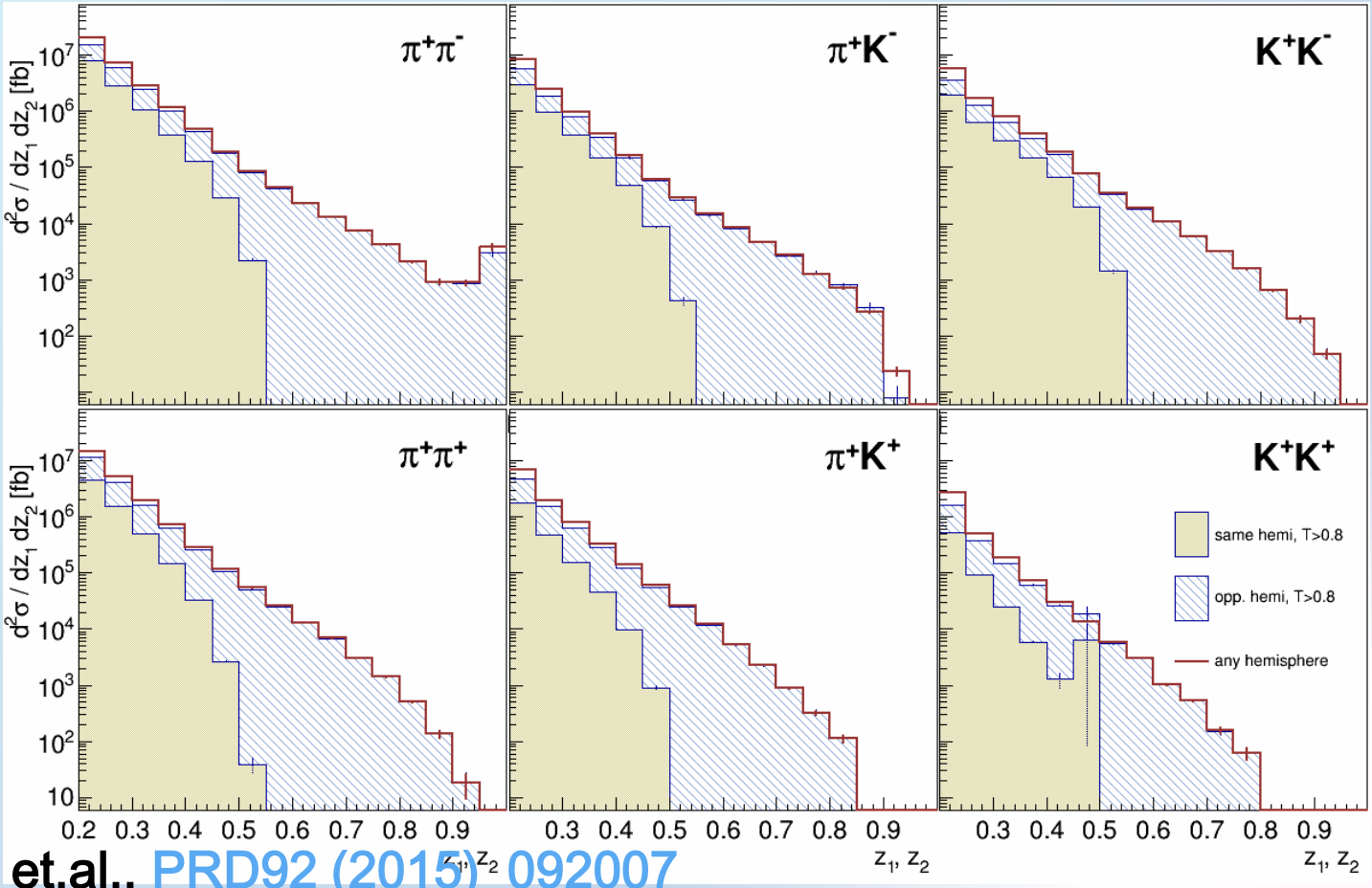
# Hemisphere composition

Same hemisphere contribution drops rapidly: Consistent with LO assumption of

Same hemisphere: single quark → di-hadron FF: ($z_1$+$z_2$ <1)

Opposite hemisphere: single quark→single hadron FF

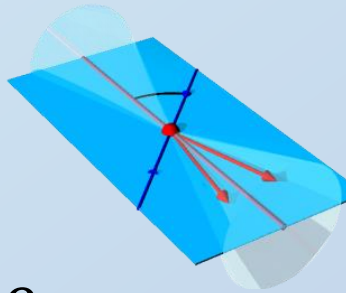
Diagonal $z_1$,$z_2$ bins




**Belle: RS et.al., PRD92 (2015) 092007**

Systematic uncertainties not displayed

# Explicit di-hadron mass dependence



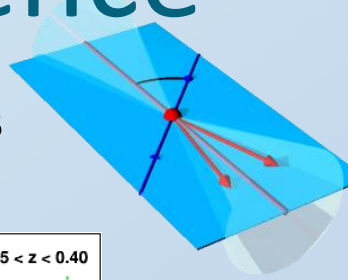
- IFF related asymmetries extracted by Belle in 2011 (PRL107:072004(2011))
- SIDIS (JHEP 0806 (2008),PLB713 (2012)) and RHIC (PRL 115 (2015) 242501) IFF asymmetries published
- Global fits currently missing unpolarized di-hadron FF baseline

➔Belle to the rescue

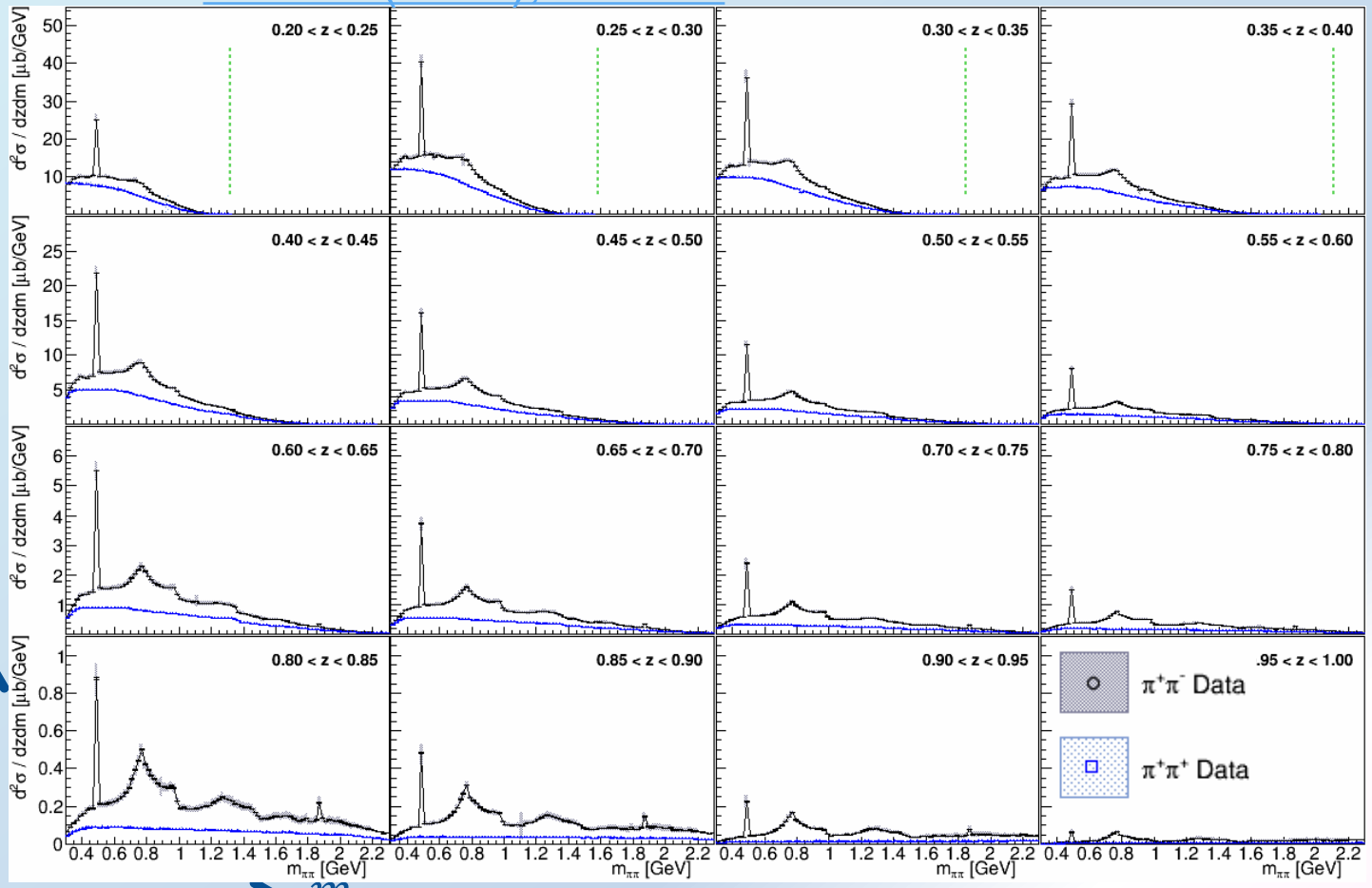
- Use same hemisphere di-hadrons for this analysis
- 16 z bins between 0.2 – 1
- 100 mass bins between 0.3 – 2.3 GeV
- Data analysis and correction steps same as previous di-hadron analysis, except for ISR treatment

# Di-hadron mass dependence

Similar analysis in same hemisphere and mass – combined z binning. Important input for IFF based transversity global analysis



Belle: RS et.al. PRD96 (2017), 032005

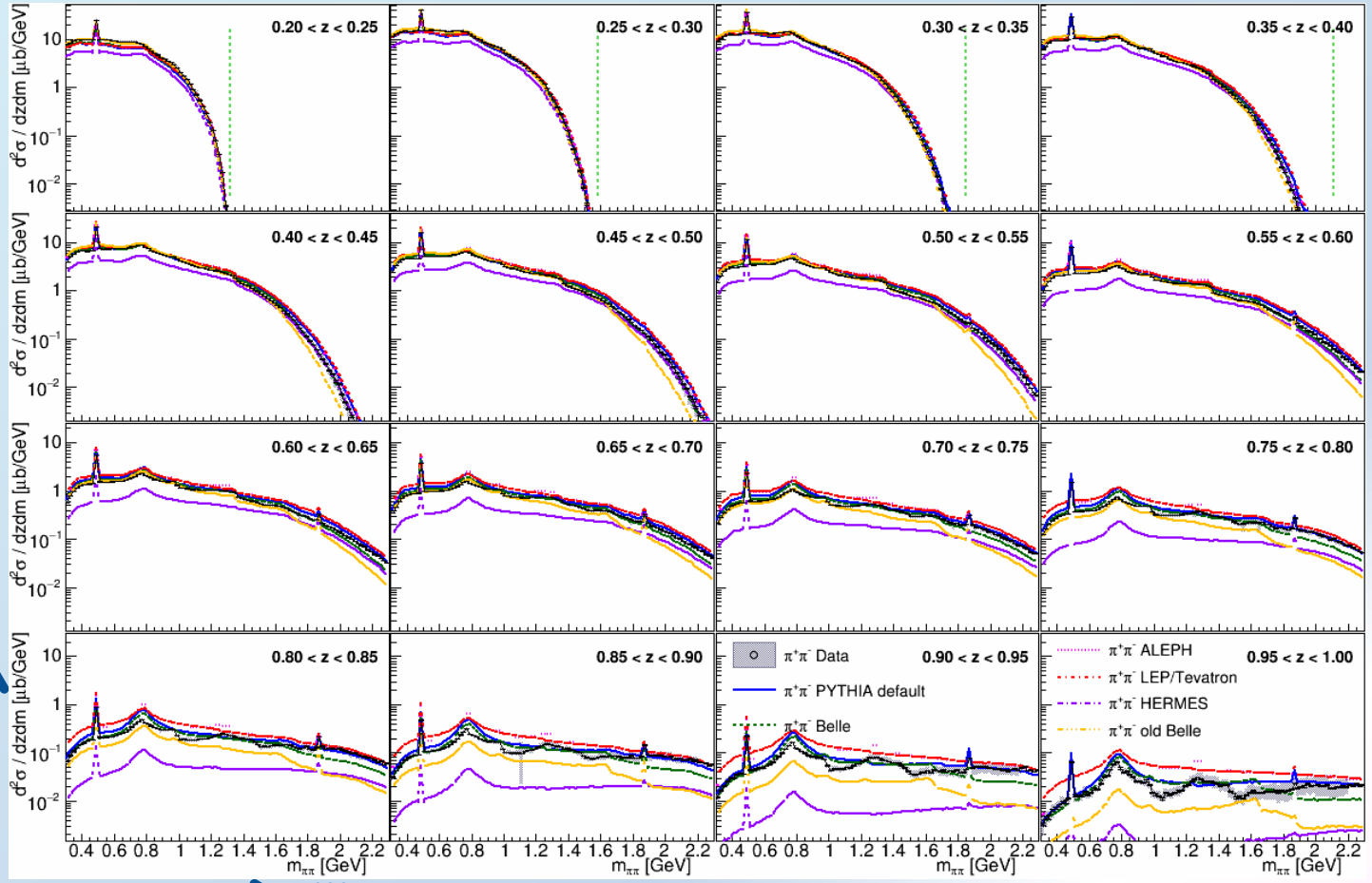



$\frac{d^2\sigma}{\mathrm{dzd}m_{\pi\pi}}$

$m_{\pi\pi}$

# Mass dependence comparisons to Pythia tunes

Magnitude and z dependence reasonable in Pythia 6.4 default,
Intermediate mass structure better described by LEP tunes (higher spin mesons)

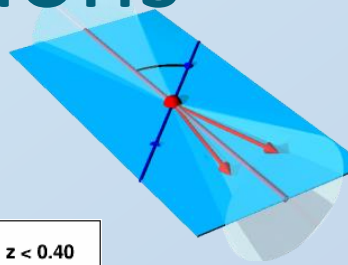



$\frac{d^2\sigma}{\mathrm{dzd}m_{\pi\pi}}$

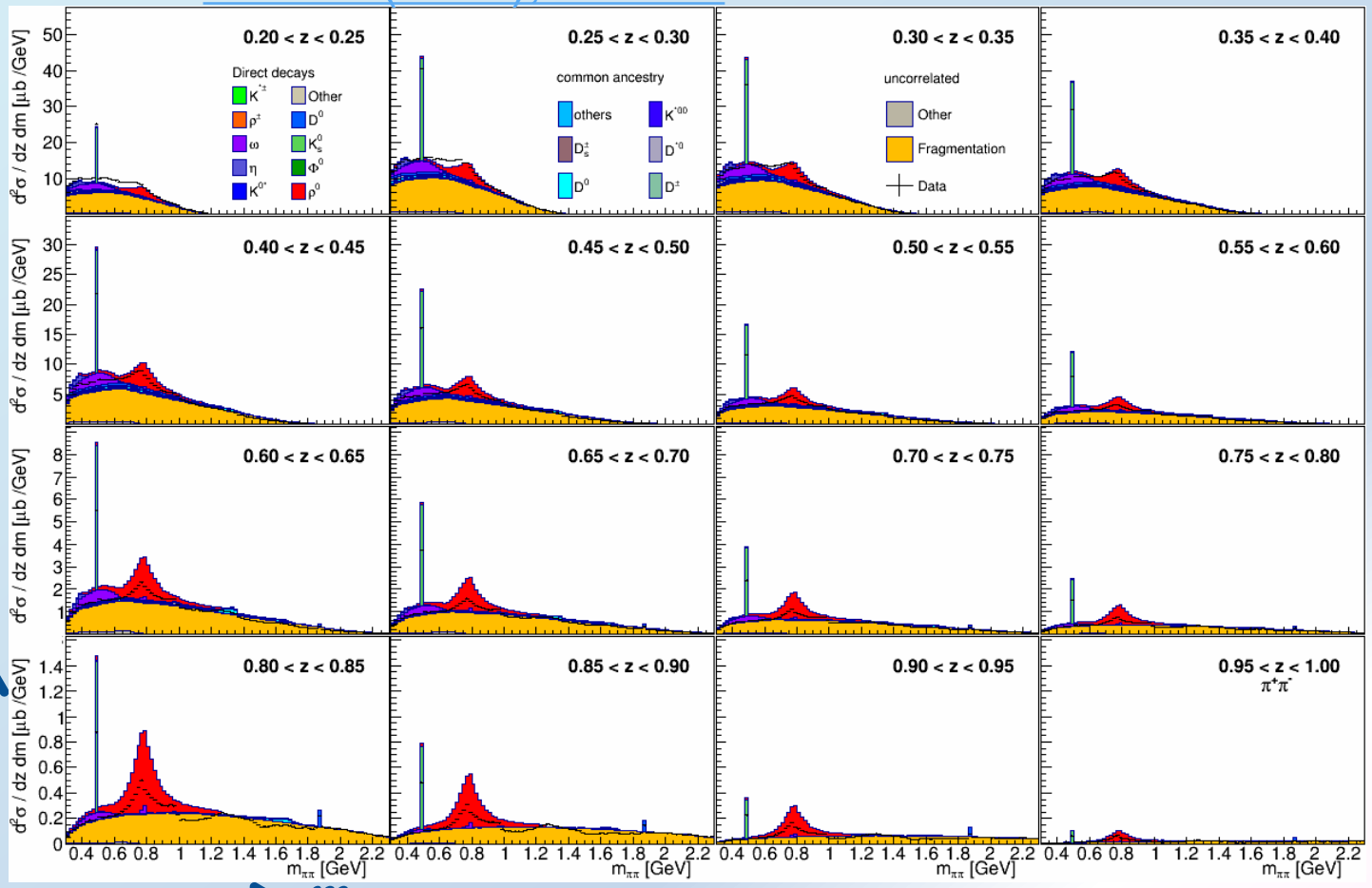
$m_{\pi\pi}$

# Di-pion individual contributions

Contributions from various resonances and direct fragmentation



Belle: RS et.al. PRD96 (2017), 032005




$$\frac{d^2\sigma}{\mathrm{dz}\mathrm{d}m_{\pi\pi}}$$

$m_{\pi\pi}$

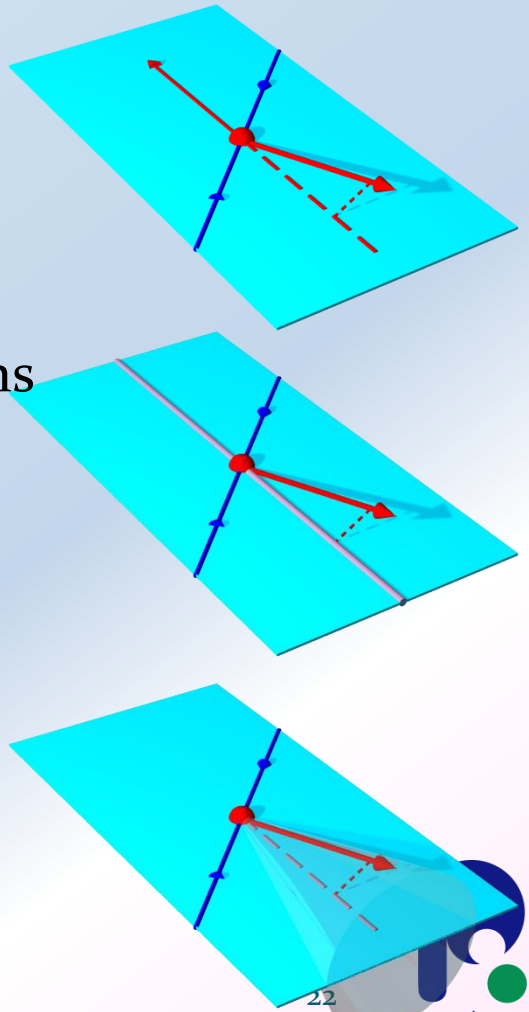
# Transverse momentum dependence

Aka un-integrated PDFs and FFs

$$D_{1,q}^{h}(z, Q^2, k_t)$$

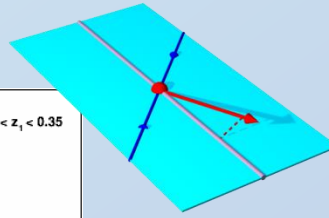
# $K_T$ Dependence of FFs in e+e-

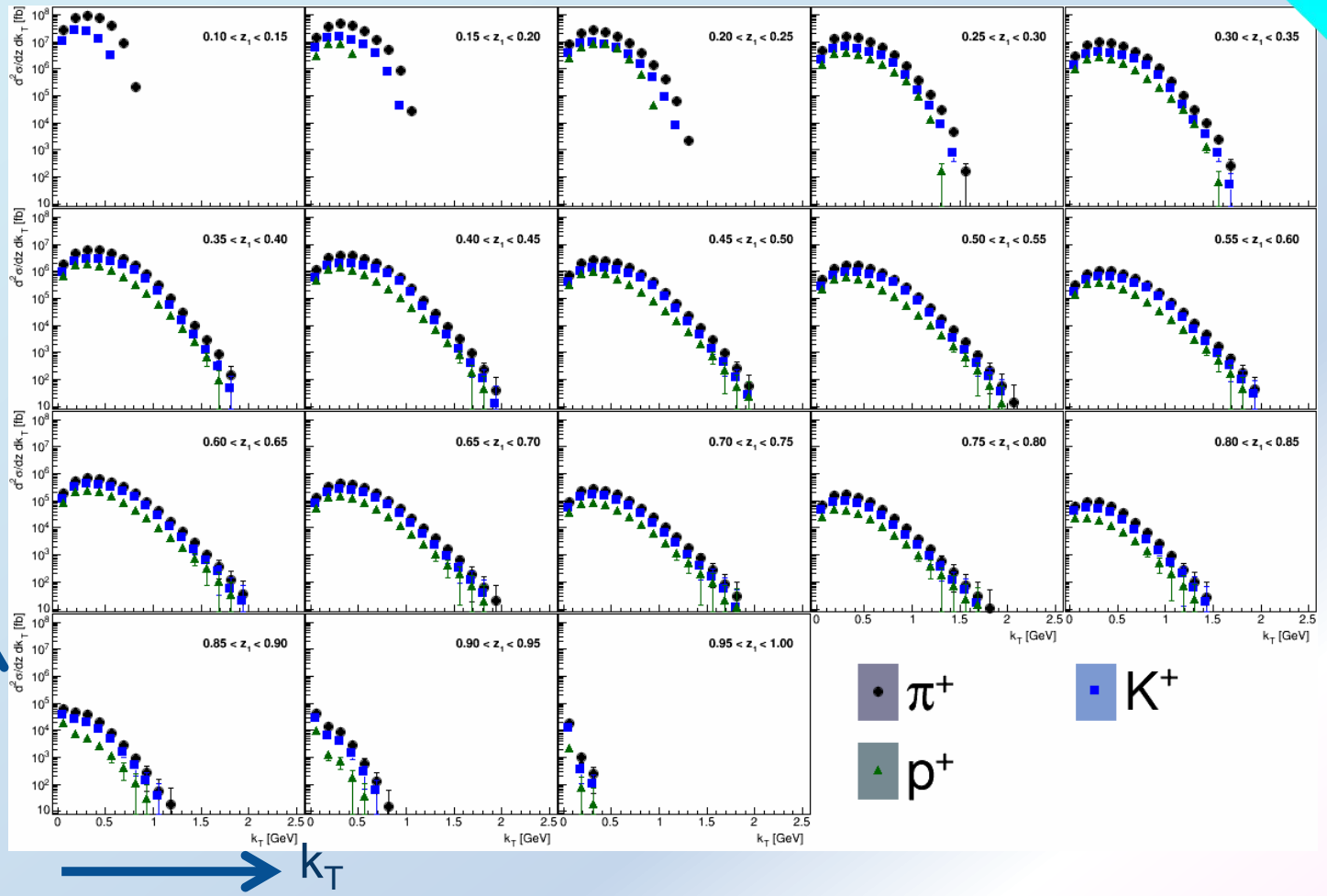
- Gain also sensitivity into transverse momentum generated in fragmentation
- Two ways to obtain transverse momentum dependence
  - Traditional 2-hadron FF
    - → use transverse momentum between two hadrons (in opposite hemispheres)
    - → Usual convolution of two transverse momenta
  - Single-hadron FF wrt to Thrust or jet axis
    - → No convolution
    - → Need correction for $q\bar{q}$ axis



# MC sample for various hadrons

MC simulation





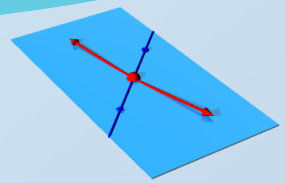

$\frac{d^2\sigma}{dzdk_T}$

$k_T$

# Summary and outlook

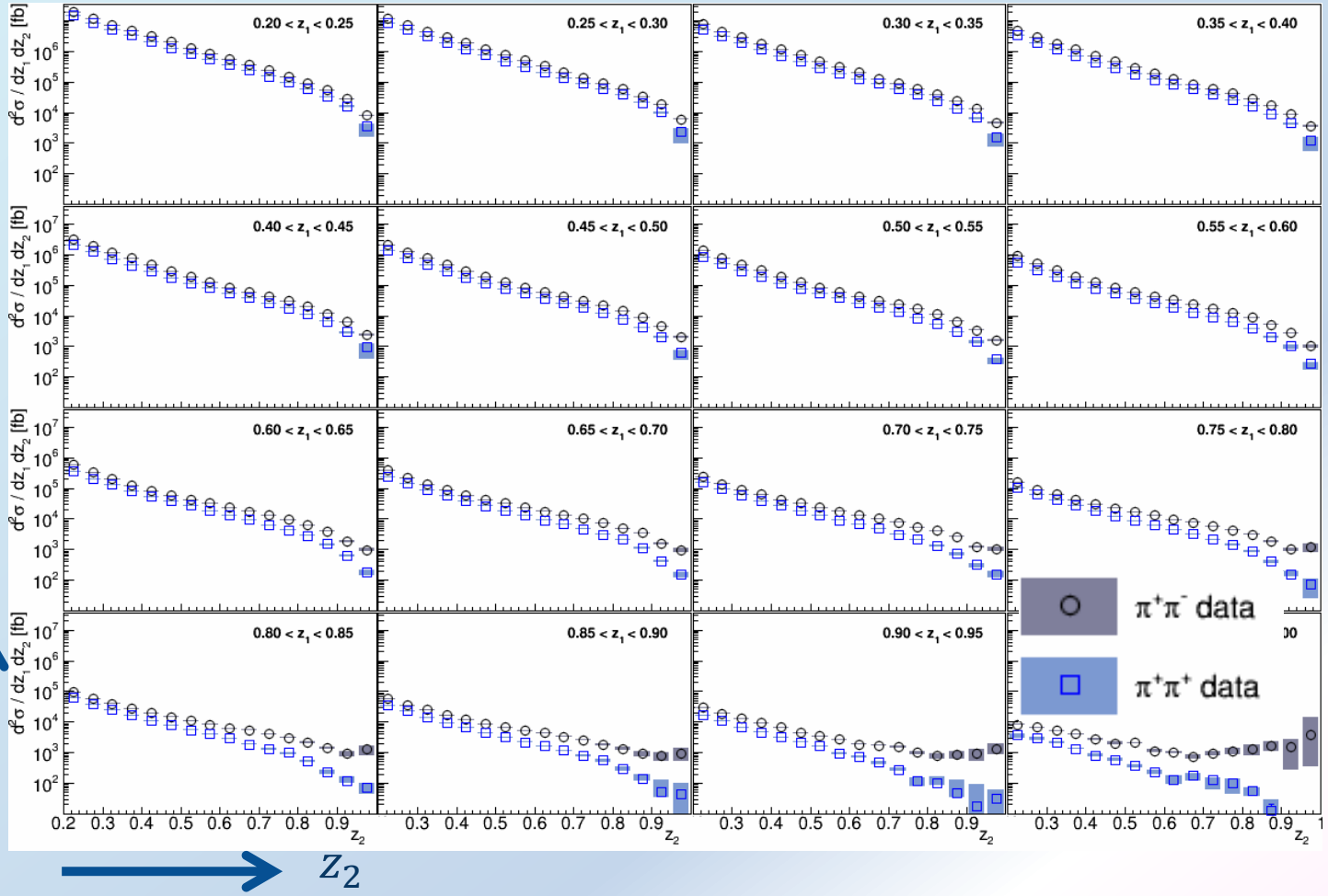
- Hyperon and charmed baryon fragmentation measurements just published, support for diquark picture in charm FF
- Nonzero Lambda polarization measured, interesting flavor dependence
- Di-hadron fragmentation functions measured, important input for di-hadron related Transversity/Tensor charge extractions
- Transverse momentum dependent fragmentation analysis ongoing
- Other results being finalized as well ($\eta$,$\pi^0$ Collins)

# Full results for pion pairs



PRD92 (2015) 092007

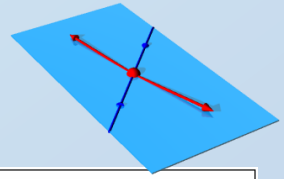
Pion pair example in any topology combination shown here




$\frac{d^2\sigma}{dz_1 dz_2}$

$z_2$

# Results for diagonal $z_1$ $z_2$ bins



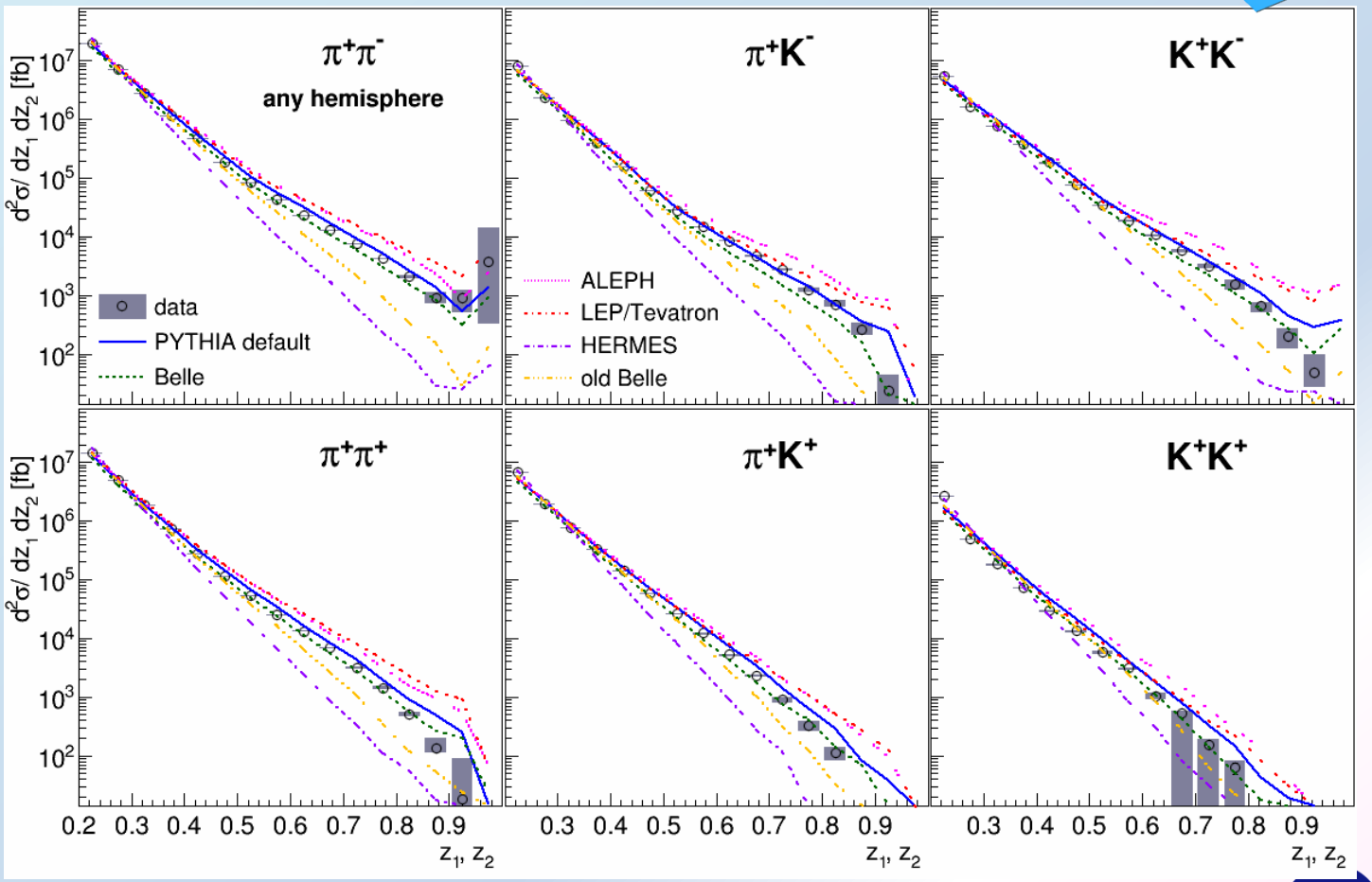
Low z dominates integral:
→Well defined, all tunes agree

High z not well measured, especially at Belle energies:
→large spread in tunes

Default Pythia settings and current Belle setting with good agreement

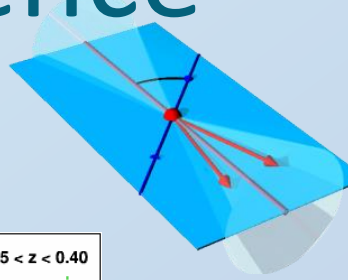
PRD92 (2015) 092007

Diagonal $z_1$,$z_2$ bins

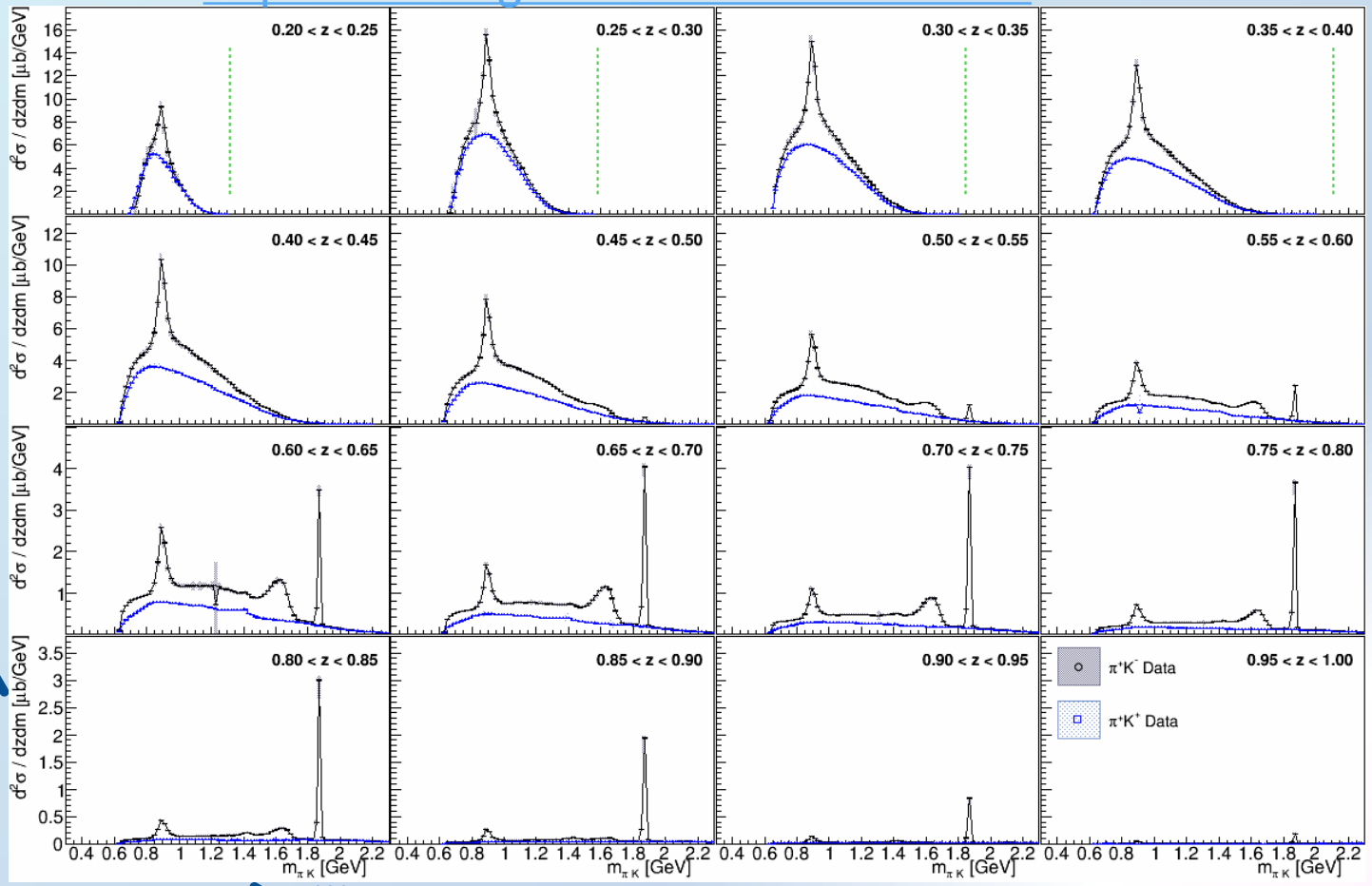


# Di-hadron mass dependence

Pion – kaon pairs



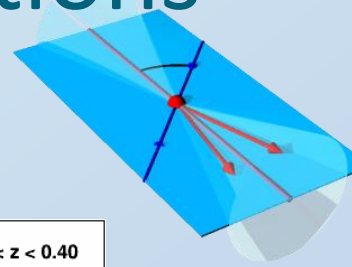
Belle: RS et.al. http://arxiv.org/abs/arXiv:1706.08348




$\frac{d^2\sigma}{\mathrm{dz}\mathrm{d}m_{\pi K}}$

$m_{\pi K}$

# Pion-kaon individual contributions



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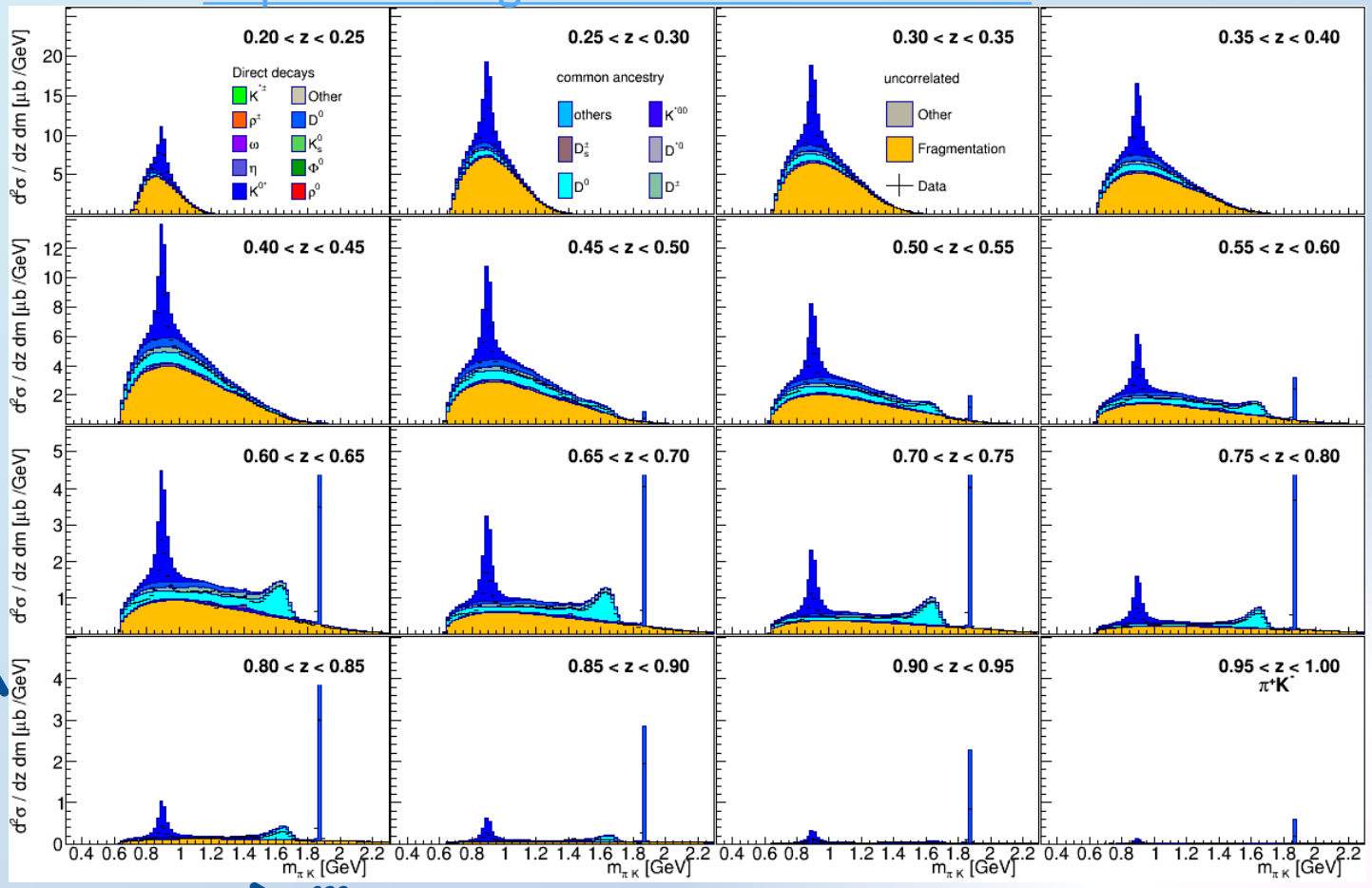
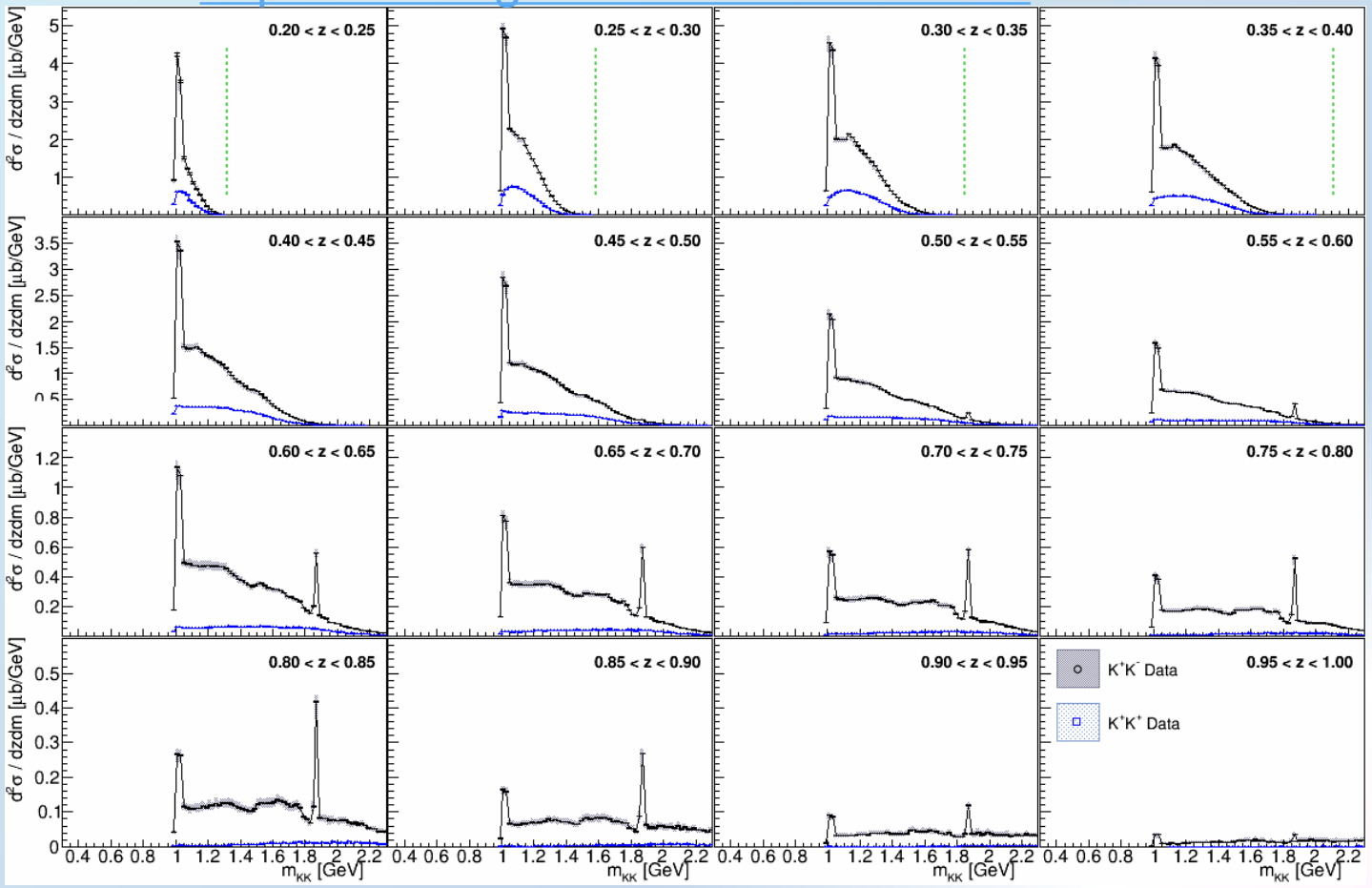



$\frac{d^2\sigma}{\mathrm{dz}\mathrm{d}m_{\pi K}}$

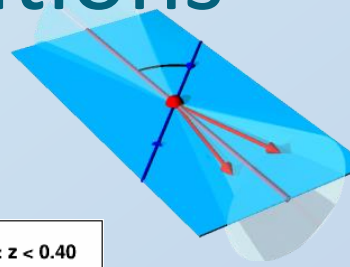
$m_{\pi K}$

# Kaon pairs

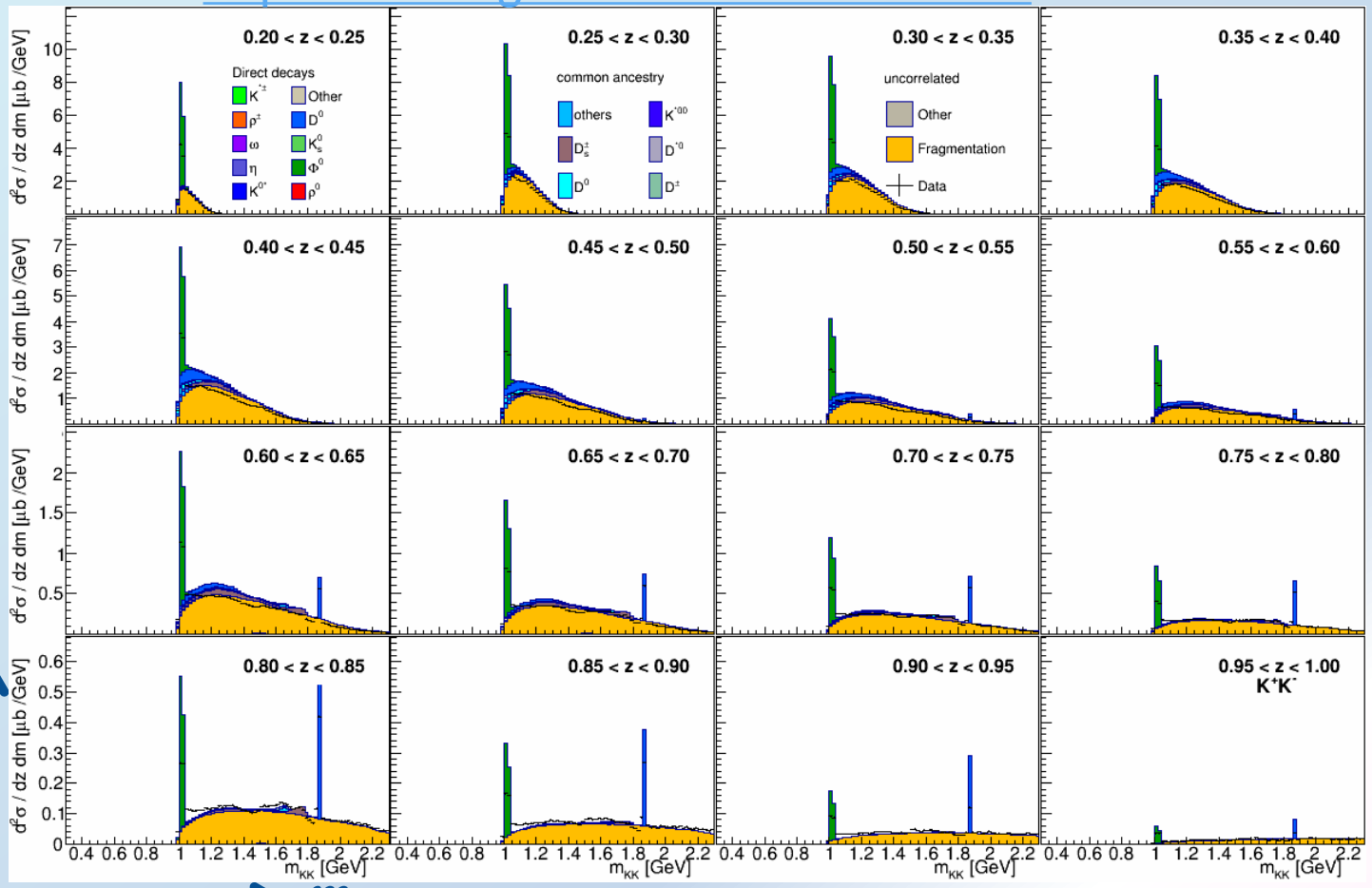
Belle: RS et.al. http://arxiv.org/abs/arXiv:1706.08348



# Kaon-kaon individual contributions



Belle: RS et.al. http://arxiv.org/abs/arXiv:1706.08348




$$\frac{d^2\sigma}{\mathrm{dz}\mathrm{d}m_{\pi K}}$$

$m_{\pi K}$

# Differences in Pythia/JetSet settings

| Par | 0 | 1 | 9 | 10 | 11 | 12 | 13 | udscatlas | udschermes |
|---|---|---|---|---|---|---|---|---|---|
| | Pythia def. | belle | Atlas | Aleph | LEP/tev. | Hermes | gen Belle | | |
| PARJ(1) | 0.1 | | | 0.106 | 0.073 | 0.029 | | | 0.029 |
| PARJ(2) | 0.3 | | | 0.285 | 0.2 | 0.283 | | | 0.283 |
| PARJ(3) | 0.4 | | | 0.71 | 0.94 | 1.2 | | | 1.2 |
| PARJ(4) | 0.05 | | | 0.05 | 0.032 | | | | |
| PARJ(11) | 0.5 | | | 0.55 | 0.31 | | | | |
| PARJ(12) | 0.6 | | | 0.47 | 0.4 | | | | |
| PARJ(13) | 0.75 | | | 0.65 | 0.54 | | | | |
| PARJ(14) | 0.0 | 0.0 | 0.0 | 0.02 | 0.0 | 0.0 | 0.05 | 0.0 | 0.0 |
| PARJ(15) | 0.0 | 0.0 | 0.0 | 0.04 | 0.0 | 0.0 | 0.05 | 0.0 | 0.0 |
| PARJ(16) | 0.0 | | 0.0 | 0.02 | 0.0 | 0.0 | 0.05 | 0.0 | 0.0 |
| PARJ(17) | 0.0 | 0.0 | 0.0 | 0.2 | 0.0 | 0.0 | 0.05 | 0.0 | 0.0 |
| PARJ(19) | 1 | | | 0.57 | | | | | |
| PARJ(21) | 0.36 | | | 0.37 | 0.325 | 0.400 | 0.28 | 0.28 | 0.400 |
| PARJ(25) | 1 | | | | 0.63 | | 0.27 | 0.27 | |
| PARJ(26) | 0.4 | | | 0.27 | 0.12 | | 0 | 0 | |
| PARJ(33) | 0.8 | | 0.8 | 0.8 | 0.8 | 0.3 | | 0.8 | 0.8 |
| PARJ(41) | 0.3 | | | 0.4 | 0.5 | 1.94 | 0.32 | 0.32 | 1.94 |
| PARJ(42) | 0.58 | | | 0.796 | 0.6 | 0.544 | 0.62 | 0.62 | 0.544 |
| PARJ(45) | 0.5 | | | | | 1.05 | | | 1.05 |
| PARJ(46) | 1. | | | | | | 1.0 | 1.0 | |
| PARJ(47) | 1. | | | | 0.67 | | | | |
| PARJ(54) | -0.050 | -0.040 | -0.050 | -0.04 | -0.050 | -0.050 | | -0.050 | -0.050 |
| PARJ(55) | -0.005 | -0.004 | -0.005 | -0.0035 | -0.005 | -0.005 | | -0.005 | -0.005 |
| PARJ(81) | 0.29 | | | 0.292 | 0.29 | | 0.38 | 0.38 | |
| PARJ(82) | 1.0 | | | 1.57 | 1.65 | | 0.5 | 0.5 | |
| MSTJ(11) | 4 | | | 3 | 5 | | 4 | 4 | |
| MSTJ(12) | 2 | | | 3 | | 1 | | | 1 |
| MSTJ(26) | 2 | 0 | 2 | 2 | 2 | 2 | 0 | 2 | 2 |
| MSTJ(45) | 5 | | | | | 4 | | | 4 |
| MSTJ(47) | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |

VM suppression

$P_x$,$P_y$ Gauss width

Lund params

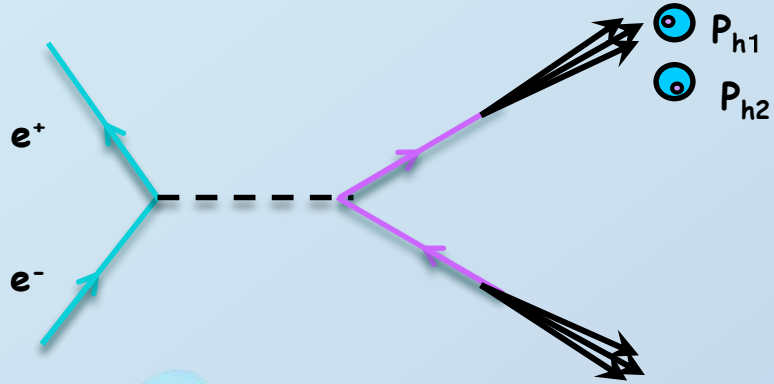
$\Lambda_{QCD}$ and E cutoff

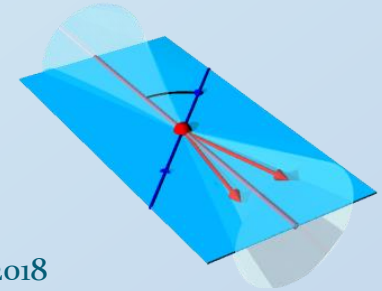
# Pythia/Jetset parameters

| | | |
|---|---|---|
| PARJ(1) | : | Diquark suppression relative to quark antiquark production |
| PARJ(2) | : | Strangeness suppression relative to u or d pair production |
| PARJ(3) | : | Extra suppression of strange diqurks relative to strange quark production |
| PARJ(4) | : | Axial ($ud_1$) vs scalar ($ud_0$) diquark suppression |
| PARJ(11) | : | Light meson with spin 1 probability |
| PARJ(12) | : | Strange meson with spin 1 probability |
| PARJ(13) | : | Charm meson with spin 1 probability |
| PARJ(14) | : | Spin 0 meson with L = 1 and J = 1 probability |
| PARJ(15) | : | Spin 1 meson with L = 1 and J = 0 probability |
| PARJ(16) | : | Spin 1 meson with L = 1 and J = 1 probability |
| PARJ(17) | : | Spin 1 meson with L = 1 and J = 2 probability |
| PARJ(19) | : | Extra baryon suppression relative to regular diquark suppression ( if MSTJ(12) = 3) |
| PARJ(21) | : | Gaussian Width of $p_x$ and $p_y$ for primary hadrons |
| PARJ(25) | : | $\eta$ production suppression factor |
| PARJ(26) | : | $\eta'$ production suppression factor |
| PARJ(33) | : | Energy cutoff of fragmentation process |
| PARJ(41) | : | Lund a parameter: $(1-z)^a$ |
| PARJ(42) | : | Lund b parameter: $exp(-bm_\perp^2/z)$ |
| PARJ(45) | : | addition to a parameter for diquarks |
| PARJ(46) | : | modification of Lund fragmentation for heavy quarks with Bowler, charm, bottom |
| PARJ(47) | : | modification of Lund fragmentation for heavy quarks with Bowler, bottom |
| PARJ(54) | : | charm fragmentation functional form and value if MSTJ(11) = 2 or 3 |
| PARJ(55) | : | bottom fragmentation functional form and value if MSTJ(11) = 2 or 3 |
| PARJ(81) | : | $\Lambda_{QCD}$ for parton showers |
| PARJ(82) | : | invariant mass cut-off for parton showers |

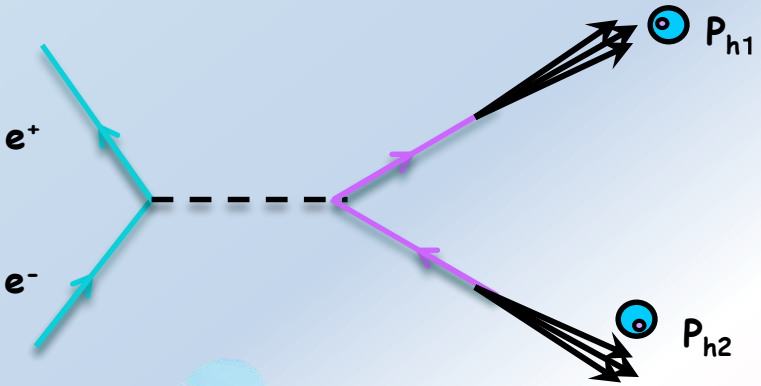
# Di-hadron fragmentation functions

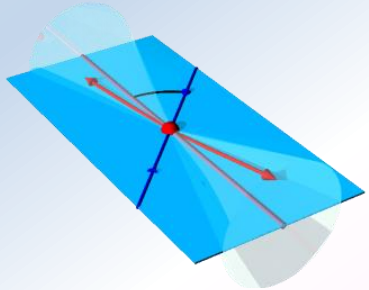
$$D_{1,q}^{h_1 h_2}(z, m, Q^2)$$



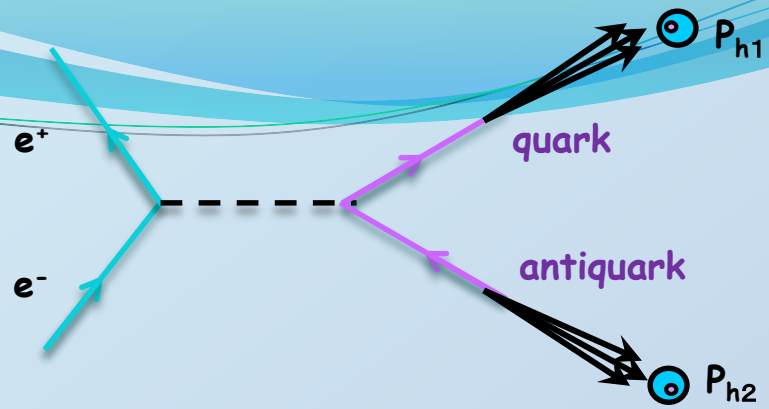



$$D_{1,q}^{h}(z_1, Q^2) D_{1,q}^{h}(z_2, Q^2)$$

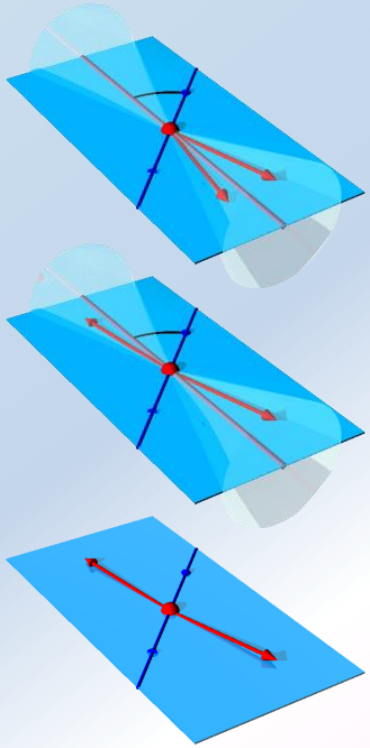


# Setup




- Generally look at 4 x 4 hadron combinations ( π, K, +,-)
  - Keep separate until end: only 6 independent yields
- 3 hemisphere combinations:
  - same hemisphere (thrust >0.8)
  - opposite hemisphere ( thrust >0.8 )
  - any combination ( no thrust selection)
- 16 x 16 $z_1$ $z_2$ binning between 0.2 - 1

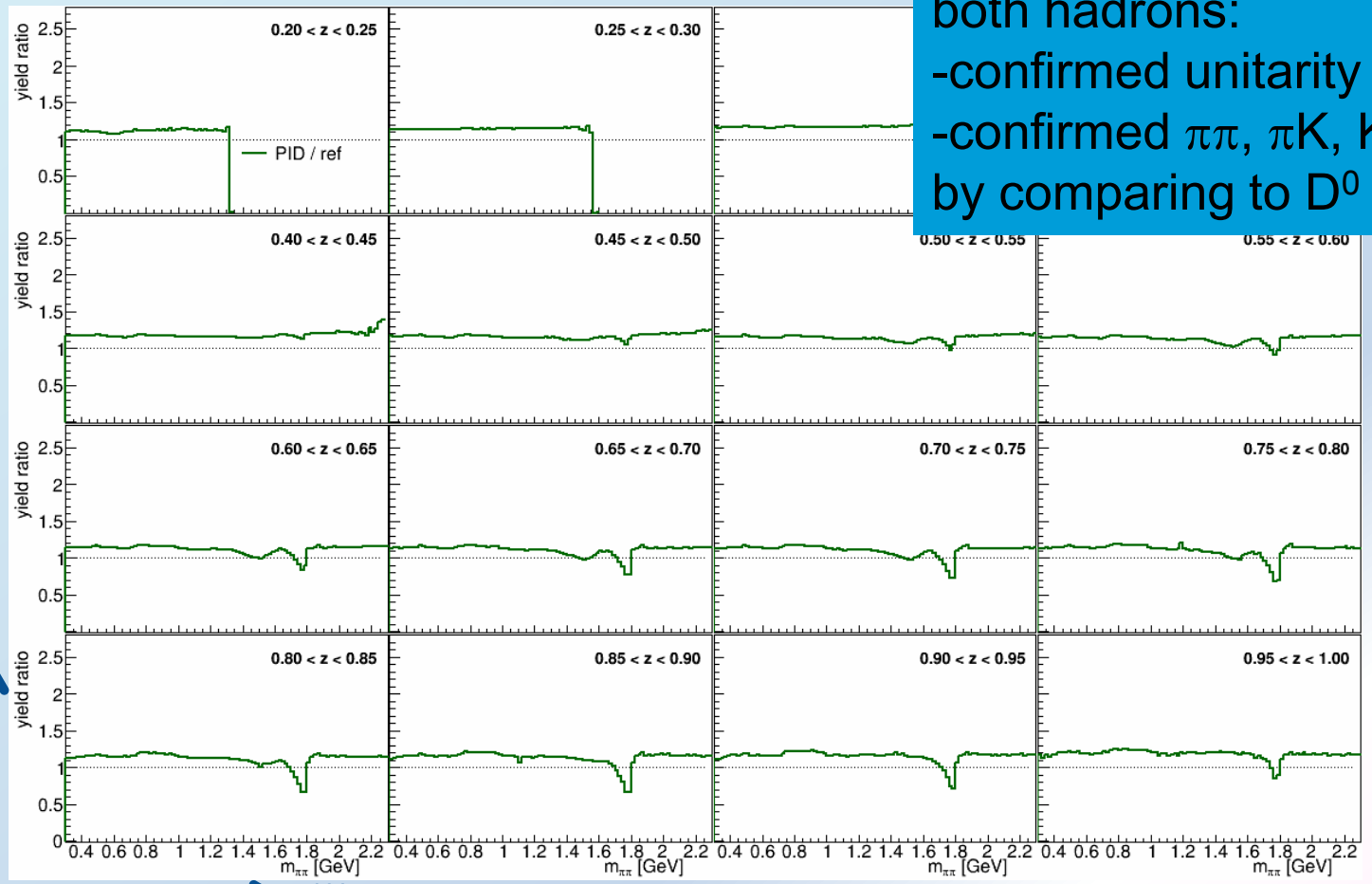


# Correction chain

| Correction | Method | Systematics |
|---|---|---|
| PID mis-id | PID matrices (5x5 for cos $\theta_{lab}$ and $p_{lab}$ ) | MC sampling of inverted matric element uncertainties, variation of PID correction method |
| Momentum smearing | MC based smearing matrices (1600x1600), SVD unfold | SVD unfolding vs analytically inverted matrix, reorganized binning, MC statistics |
| Non-qqbar BG removal | eeuu, eess, eecc, tau MC subtraction | Variation of size, MC statistics |
| Acceptance I (cut efficiency) | In barrel reconstucted vs udsc generated in barrel | MC statistics |
| Acceptance II | udsc Gen MC barrel to $4\pi$ | MC statistics, variation in tunes |
| Weak decay removal (optional) | udcs check evt record for weak decays | Compare to other Pythia settings |
| ISR | ISR on vs ISR off in Pythia | Variatons in tunes |

# PID correction

Using Martin Leitgab's 5x5 PID matrices in fine 17 x 9 $P_{lab}$ x $\cos\theta_{lab}$ binning for both hadrons:
-confirmed unitarity
-confirmed $\pi\pi$, $\pi K$, KK yields by comparing to $D^0$ BRs

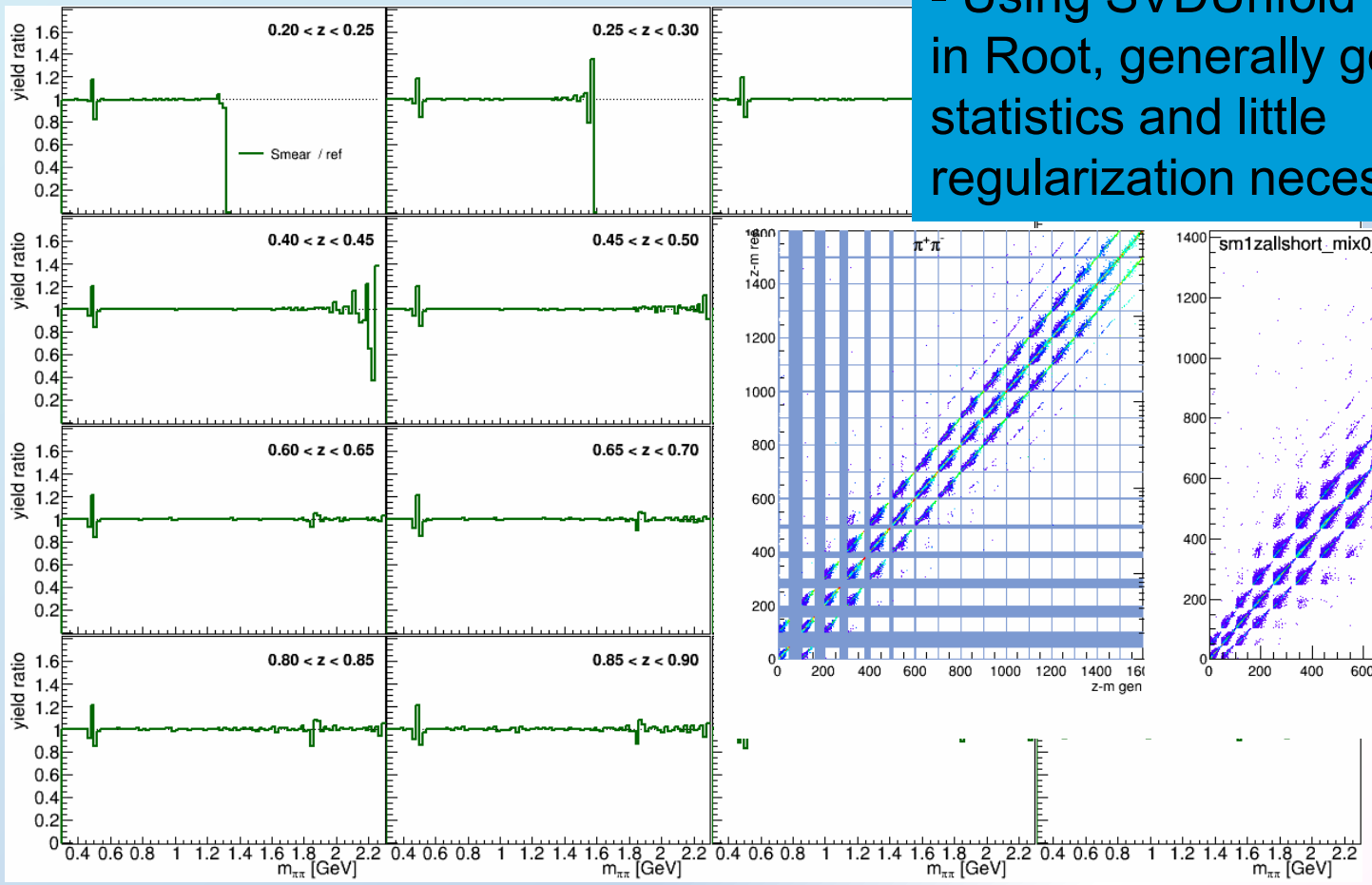



$$\frac{d^2\sigma}{\mathrm{d}z\mathrm{d}m_{\pi\pi}}$$

$m_{\pi\pi}$

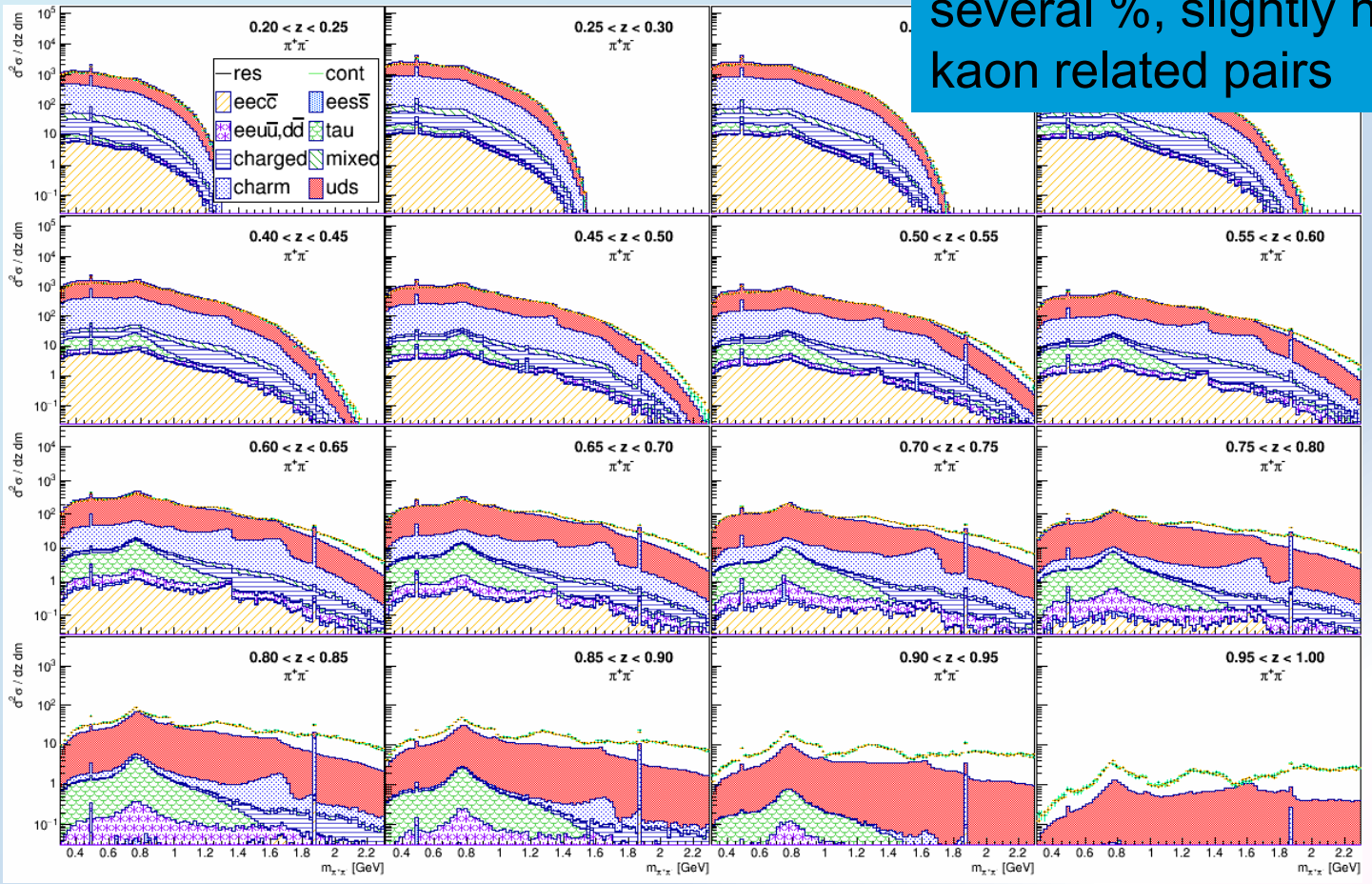
# Smearing

- Reduced smearing matrices from 1600 x 1600 to filled (ie kinematically reachable bins)
- Using SVDUnfold Method in Root, generally good statistics and little regularization necessary

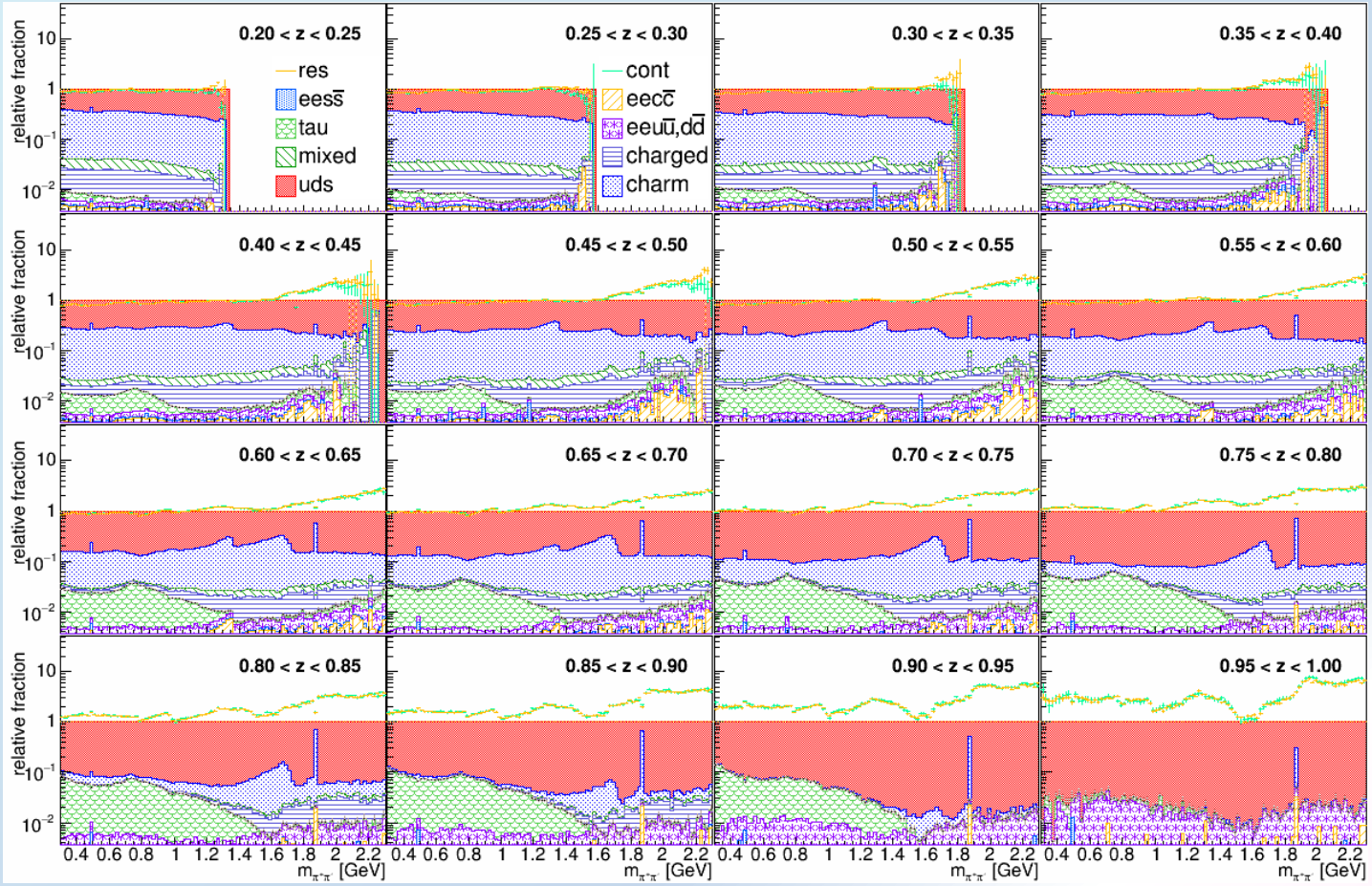


# Non-qqbar removal:

Remove all two-photon and tau events from yields, contributions generally up to several %, slightly higher for kaon related pairs



# Stacked, relative contributions

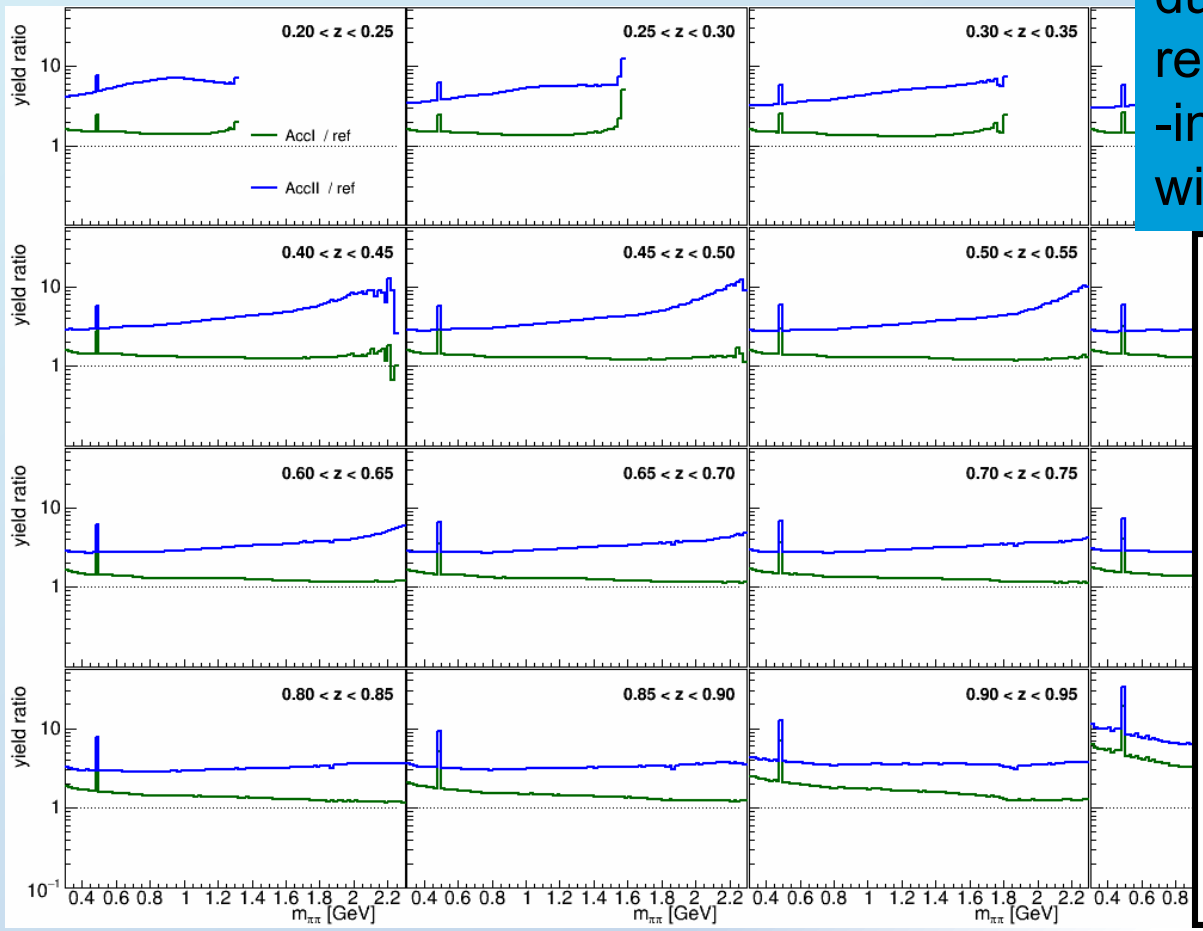


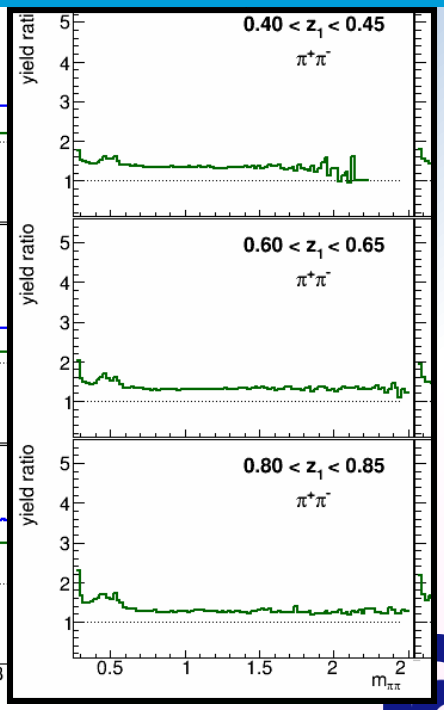
# Acceptance correction

ACCI:Reconstruction and efficiency correction in Barrel acceptance

ACCII: Barrel to 4π correction

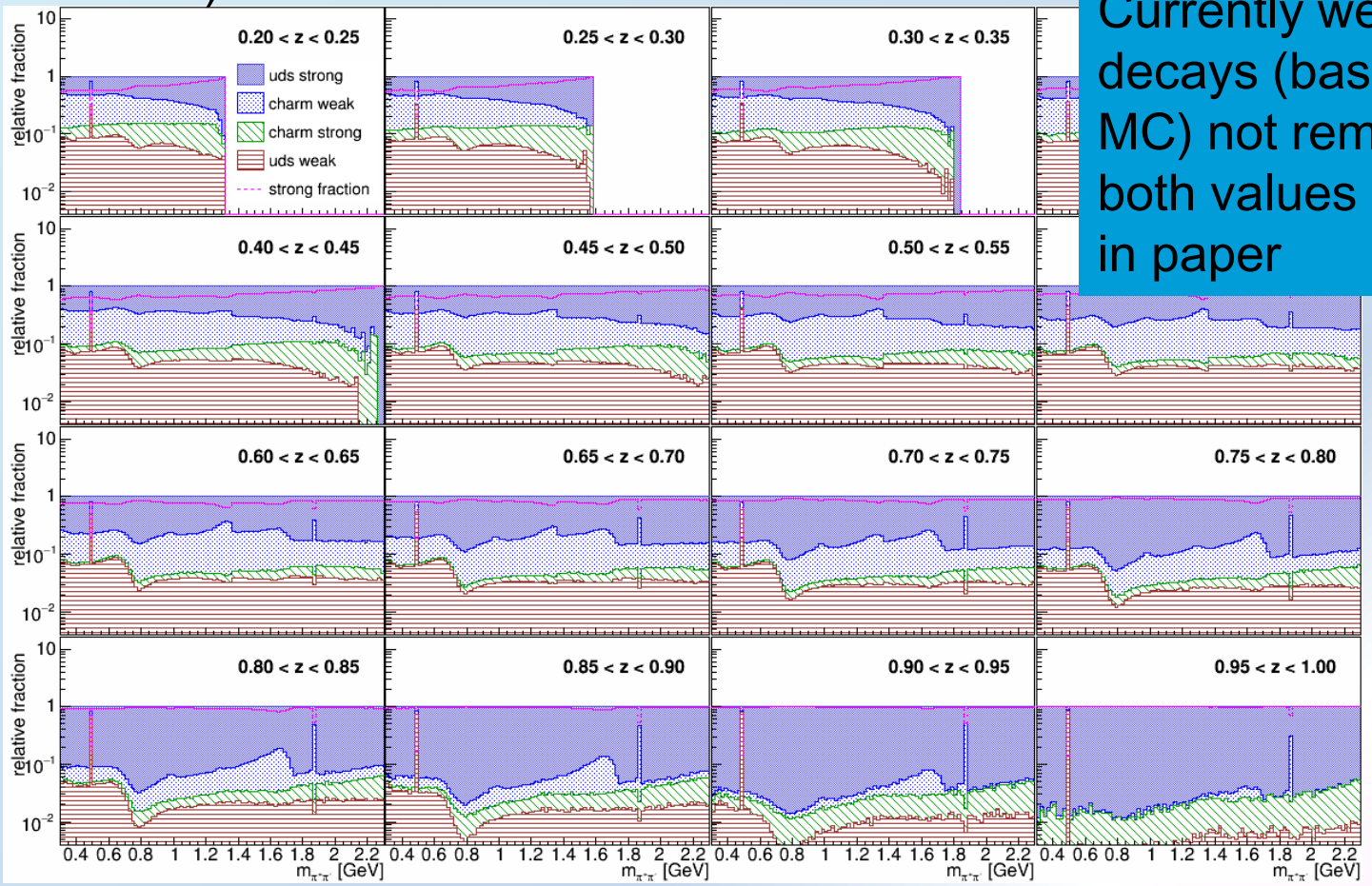
$K_S$ drop in efficiency due to SVD 3 hit requirement -increasing effect with z due to boost



# Weak correction(optional)

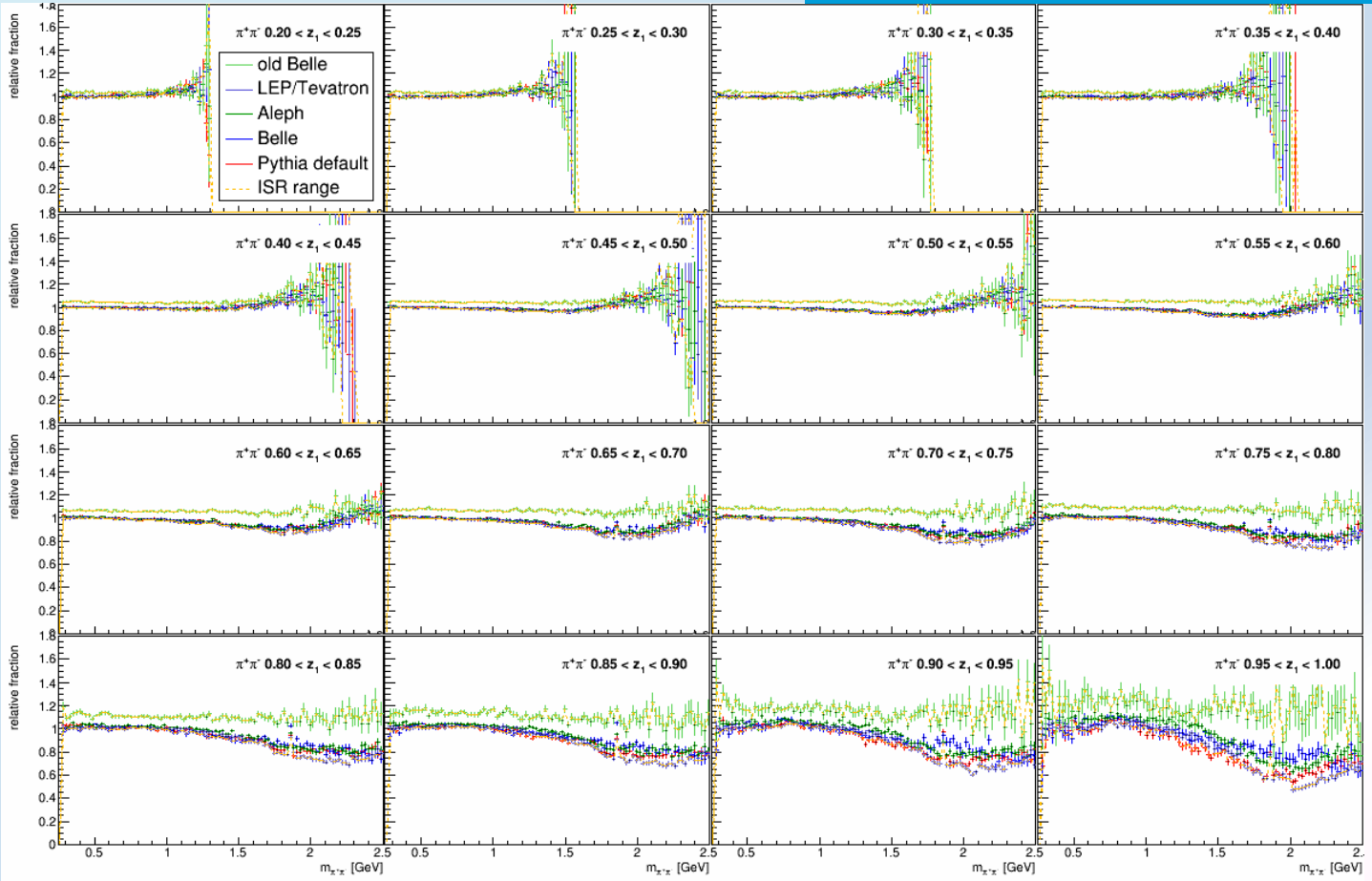
Traced in gen MC hadrons back to mothers with non ud content → if not vetoed ( K*, ssbar, ccbar resonances, some hyperons and excited states) → Weak




Currently weak decays (based on MC) not removed: both values provided in paper

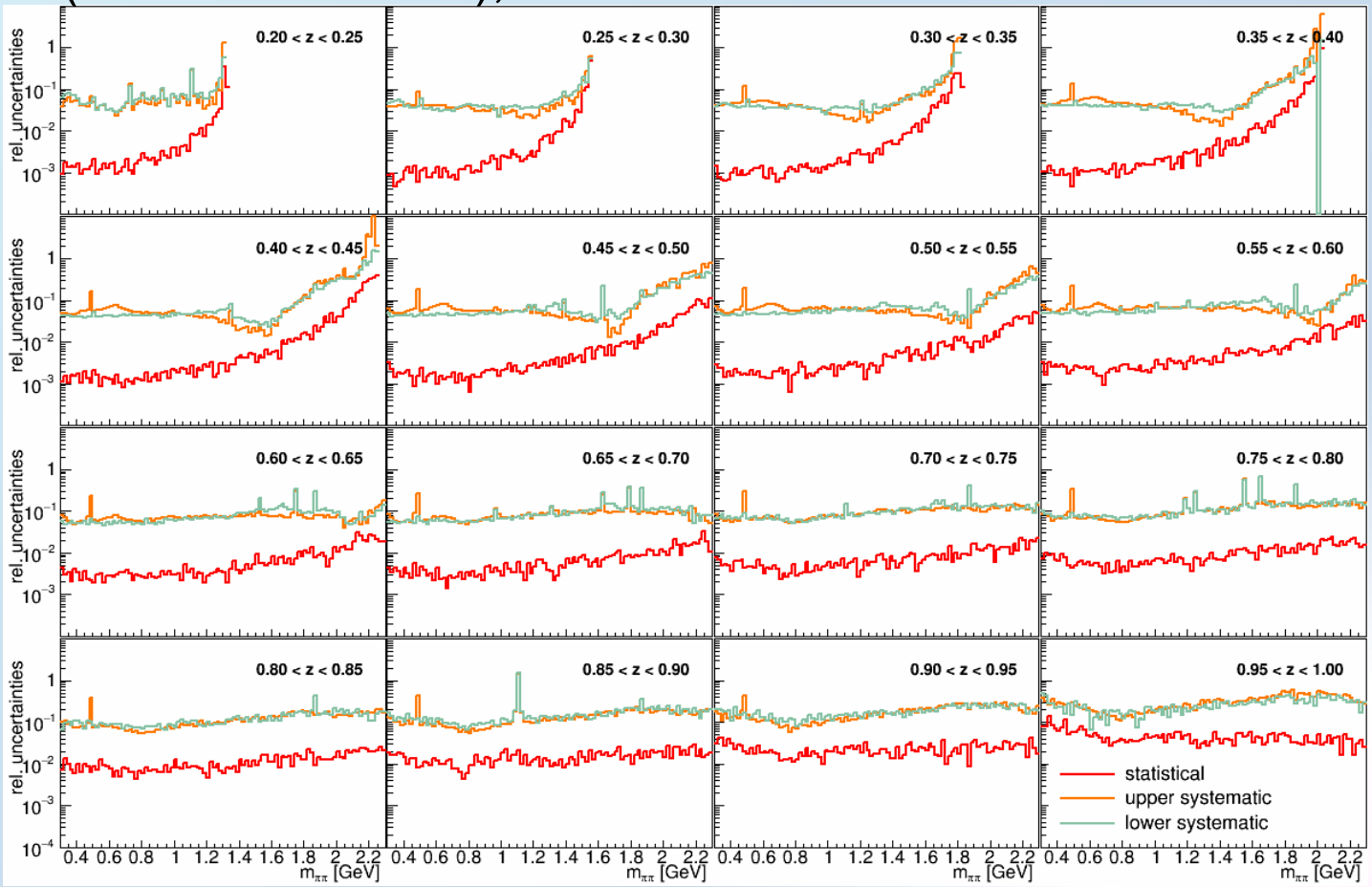
# ISR correction

All different tunes very simmilar except old Belle tune → assigned as systematics
-high mass drop of ratio due to boost



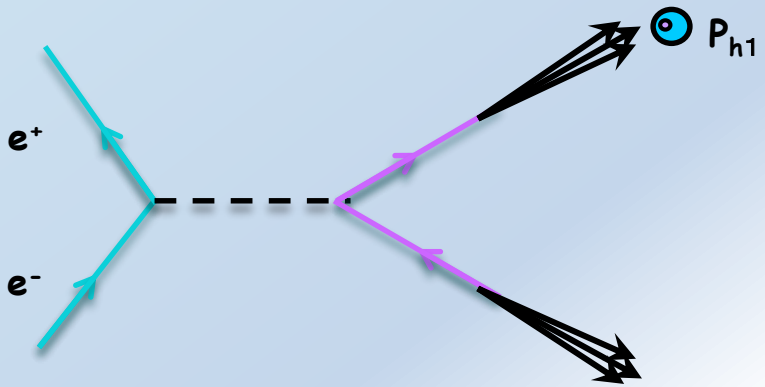
# Overall systematic uncertainties

Systematic uncertainties dominated by acceptance correction (for different tunes), PID uncertainties and ISR correction



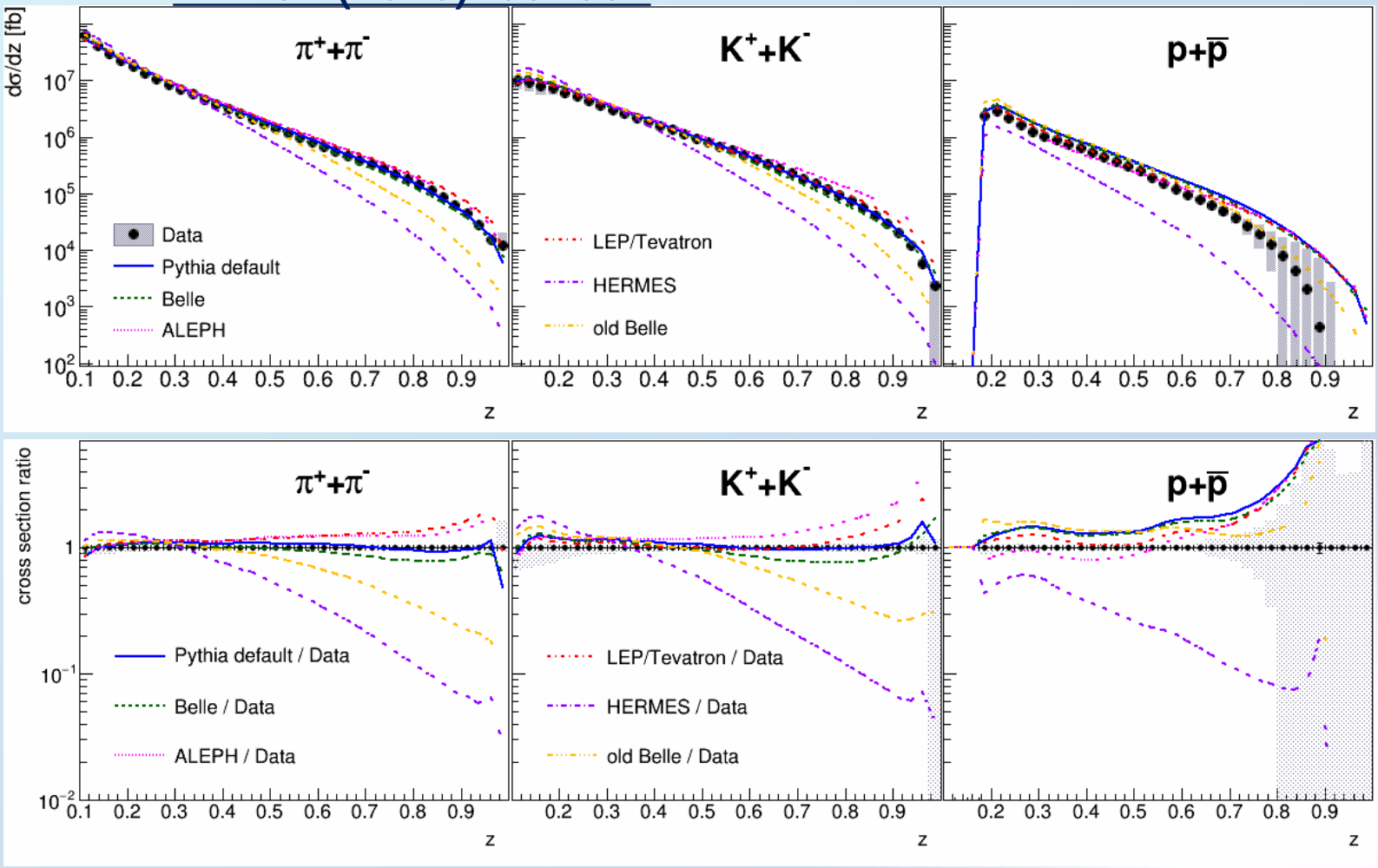
# Unpolarized fragmentation functions

$$D_{1,q}^{h}(z, Q^2)$$



# New addition: single protons

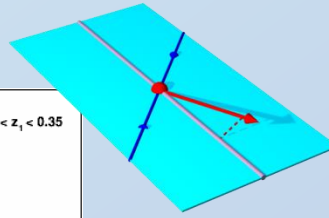
PRD92 (2015) 092007

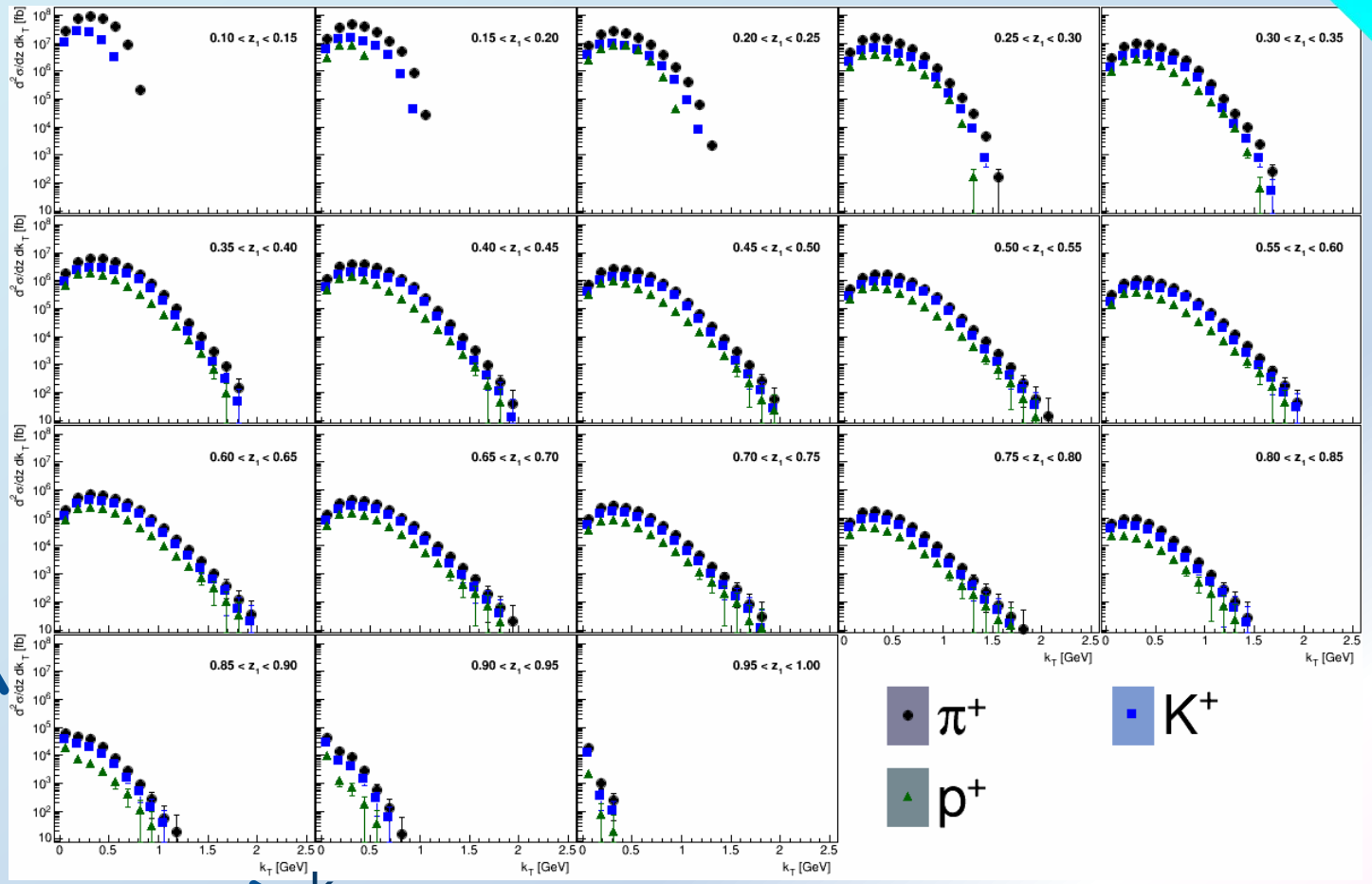



- Default Pythia and current Belle in good agreement with pions and kaons
- Protons not well described by any tune

# MC sample for various hadrons

## MC simulation



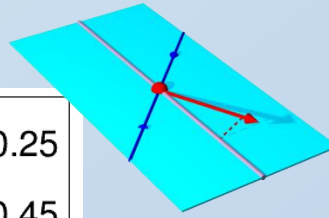


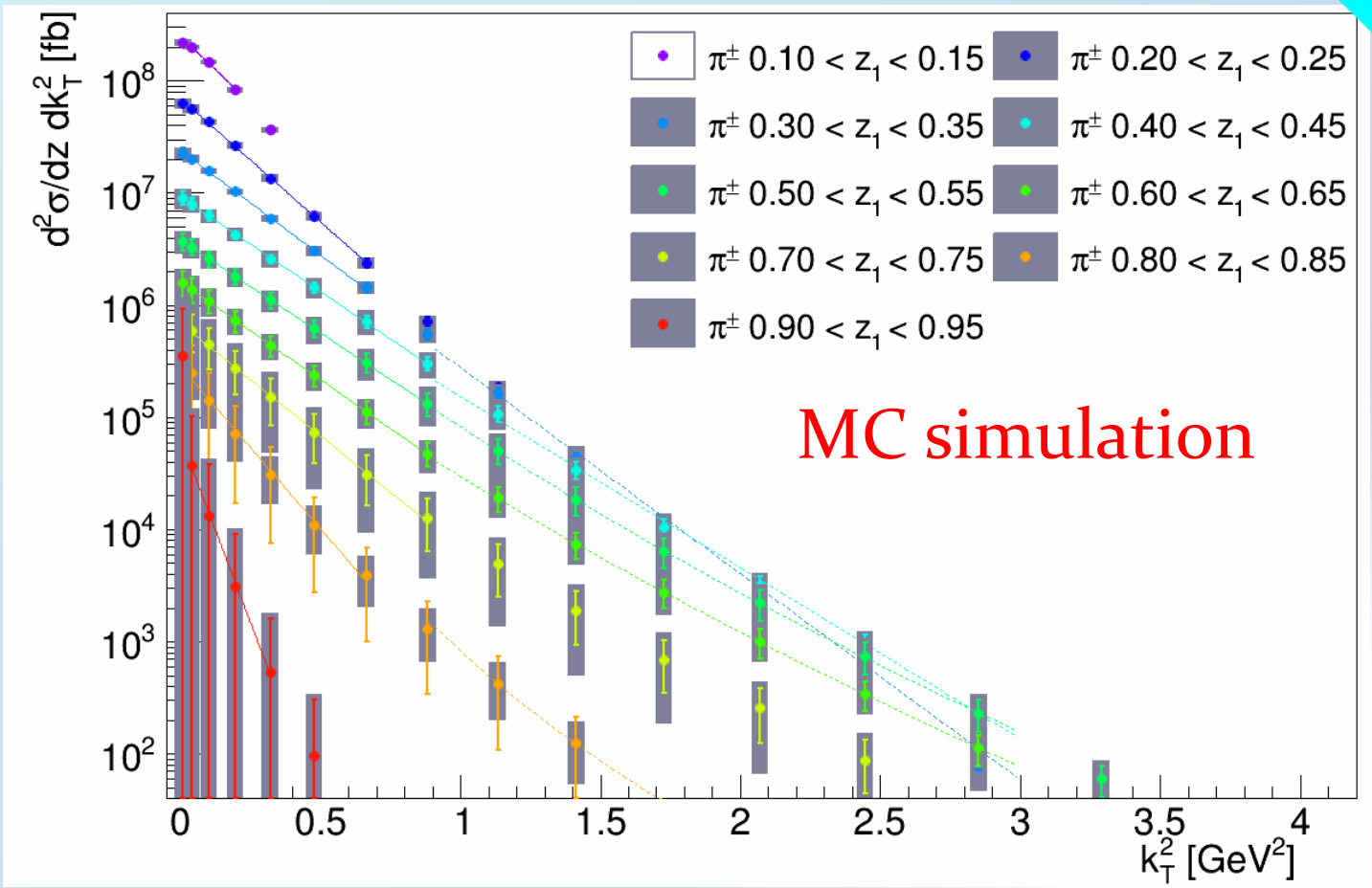

$\frac{d^2\sigma}{dzdk_T}$

$k_T$

# MC examples vs $k_T^2$

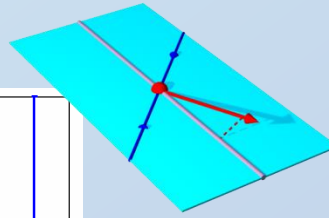
Fit exponential to smaller transverse momenta for Gaussian $k_T$ dependence and power low at higher $k_T$

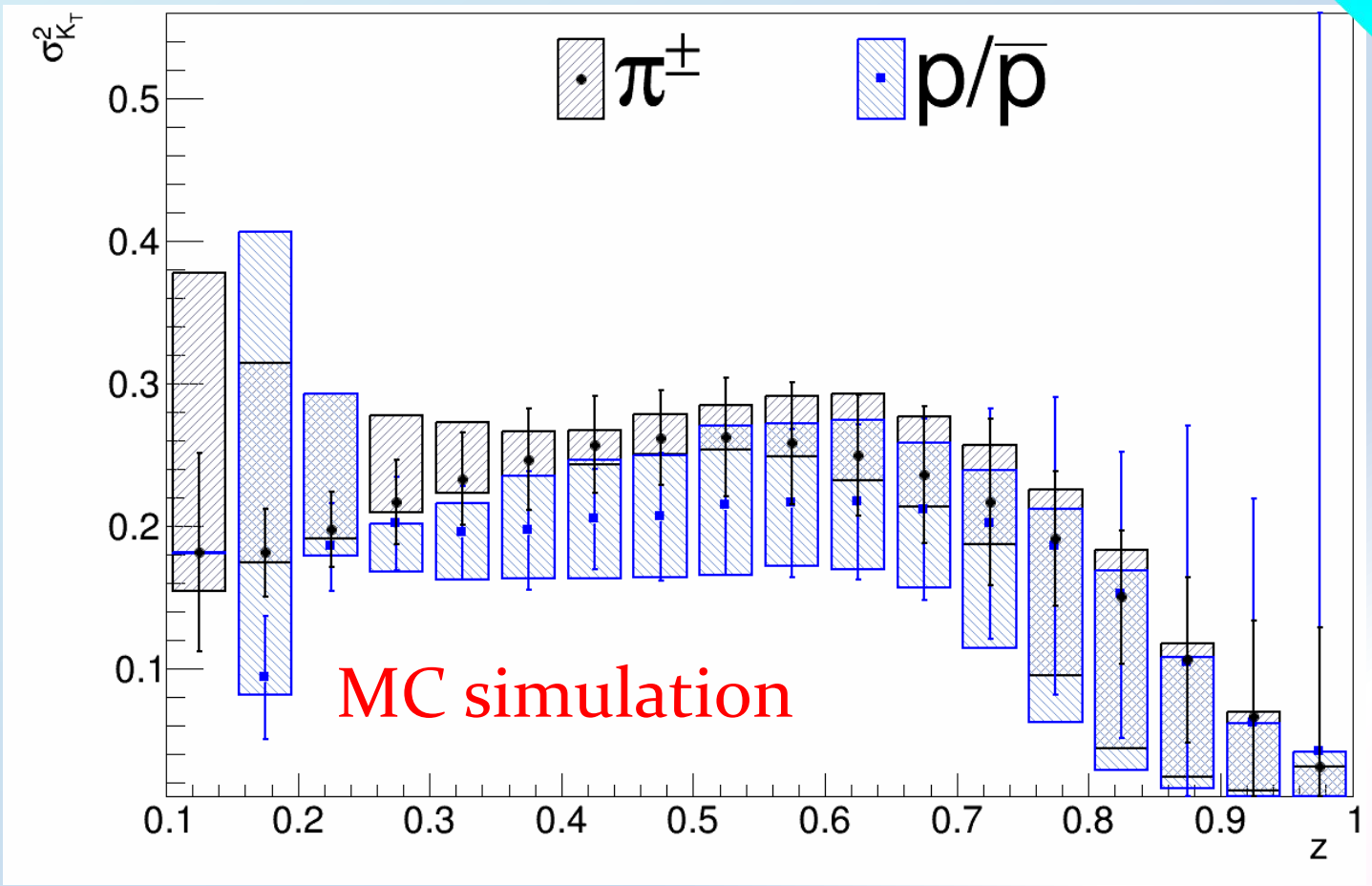




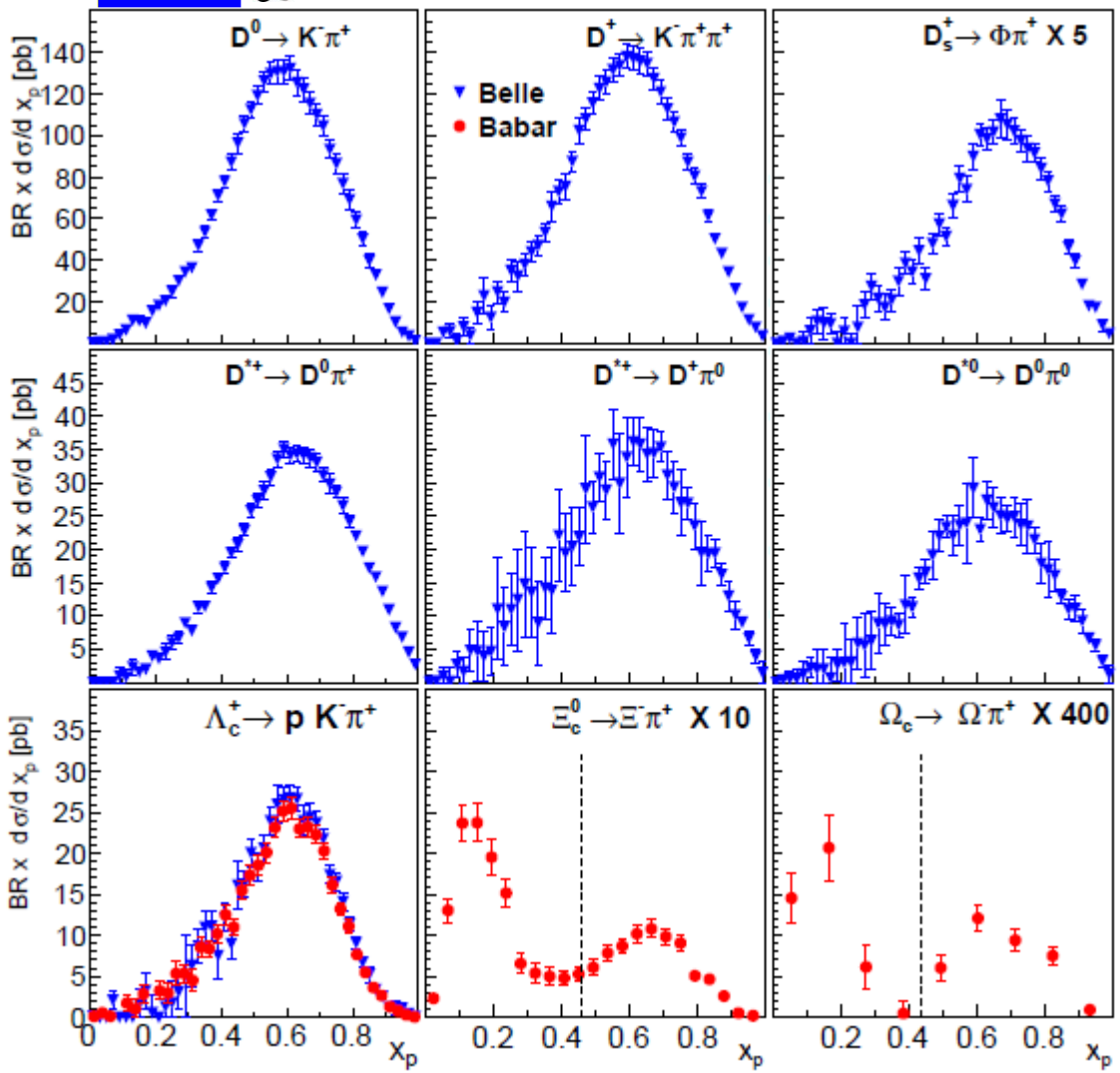
# MC Gaussian widths

Once available for data this will be the first direct (no convolutions) measurement of z dependence of Gaussian widths





# Charmed Fragmentation



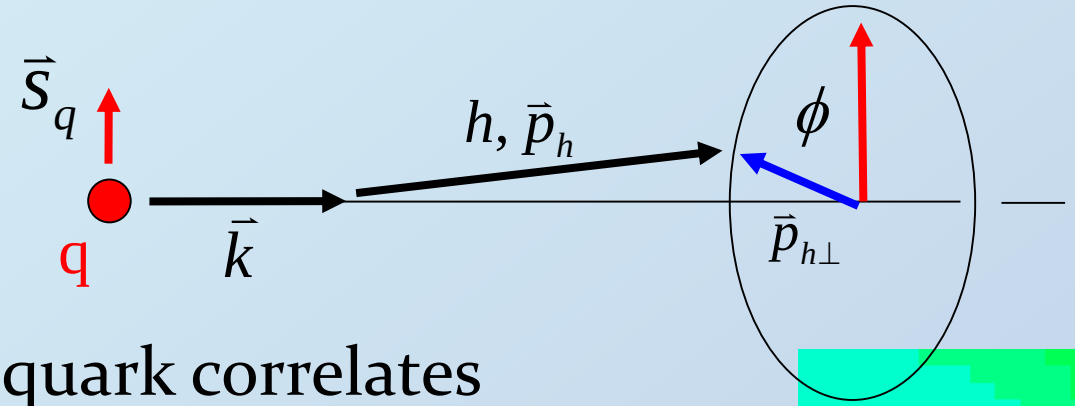

PRL.95, 142003 (2005)(Babar)
PRD73, 032002 (2006) (Belle)
PRD75, 012003 (2007)(Babar)
PRL 99, 062001 (2007)(Babar)

- Heavier particles generally plotted vs normalized momentum $x_p = \frac{P^h}{P^h_{max}}$
- Unlike light hadrons charmed hadrons contain large fraction of charm quark momentum

# Collins fragmentation function

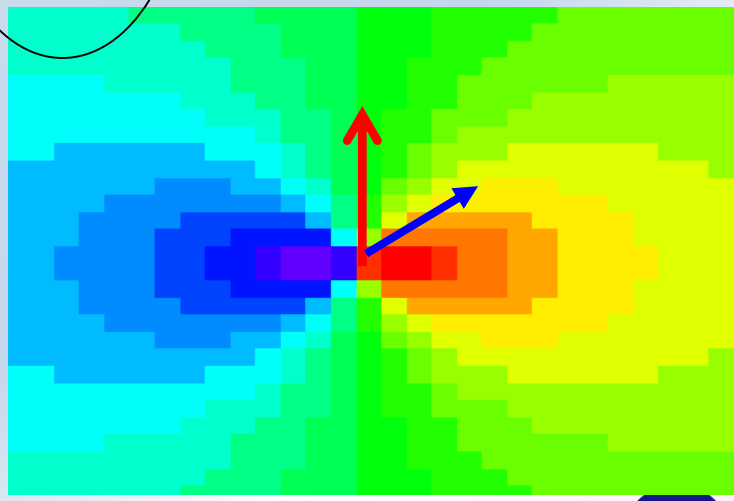
J. Collins, Nucl. Phys. B396, (1993) 161

$$D_{q^\uparrow}^{h}(z, P_{h\perp}) = D_{1,q}^{h}(z, P_{h\perp}^2) + H_{1,q}^{\perp h}(z, P_{h\perp}^2)\frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{z M_h}$$



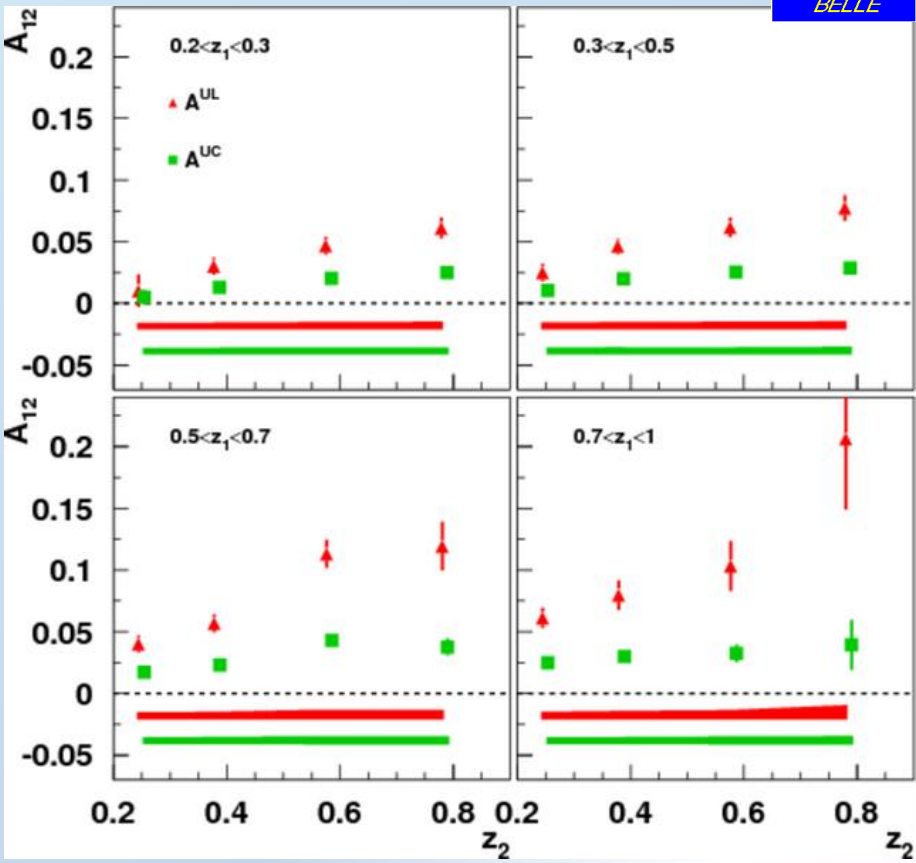

- Spin of quark correlates with hadron transverse momentum

➔translates into azimuthal anisotropy of final state hadrons



# Belle Collins asymmetries

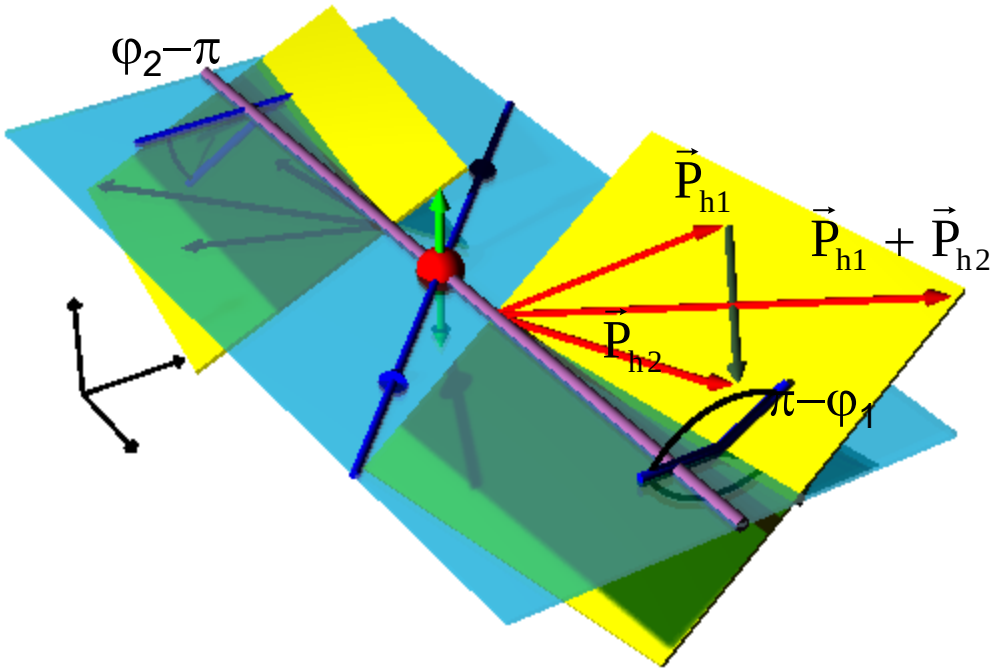
- Red points : $\cos(\phi_1 + \phi_2)$ moment of Unlike sign pion pairs over like sign pion pair ratio : $A^{UL}$
- Green points : $\cos(\phi_1 + \phi_2)$ moment of Unlike sign pion pairs over any charged pion pair ratio : $A^{UC}$
- Collins fragmentation is large effect
- Consistent with SIDIS indication of sign change between favored and disfavored Collins FF




RS et. Al. (Belle), PRL96: 232002
PRD 78:032011, Erratum D86:039905

# Interference Fragmentation (IFF) in $e^+e^-$

- $e^+e^- \rightarrow (\pi^+\pi^-)_{jet1}(\pi^+\pi^-)_{jet2}X$
- Theoretical guidance by papers of Boer,Jakob,Radici[PRD 67,(2003)] and Artru,Collins[ZPhysC69(1996)]
- Early work by Collins, Heppelmann, Ladinsky [NPB420(1994)]




Model predictions by:

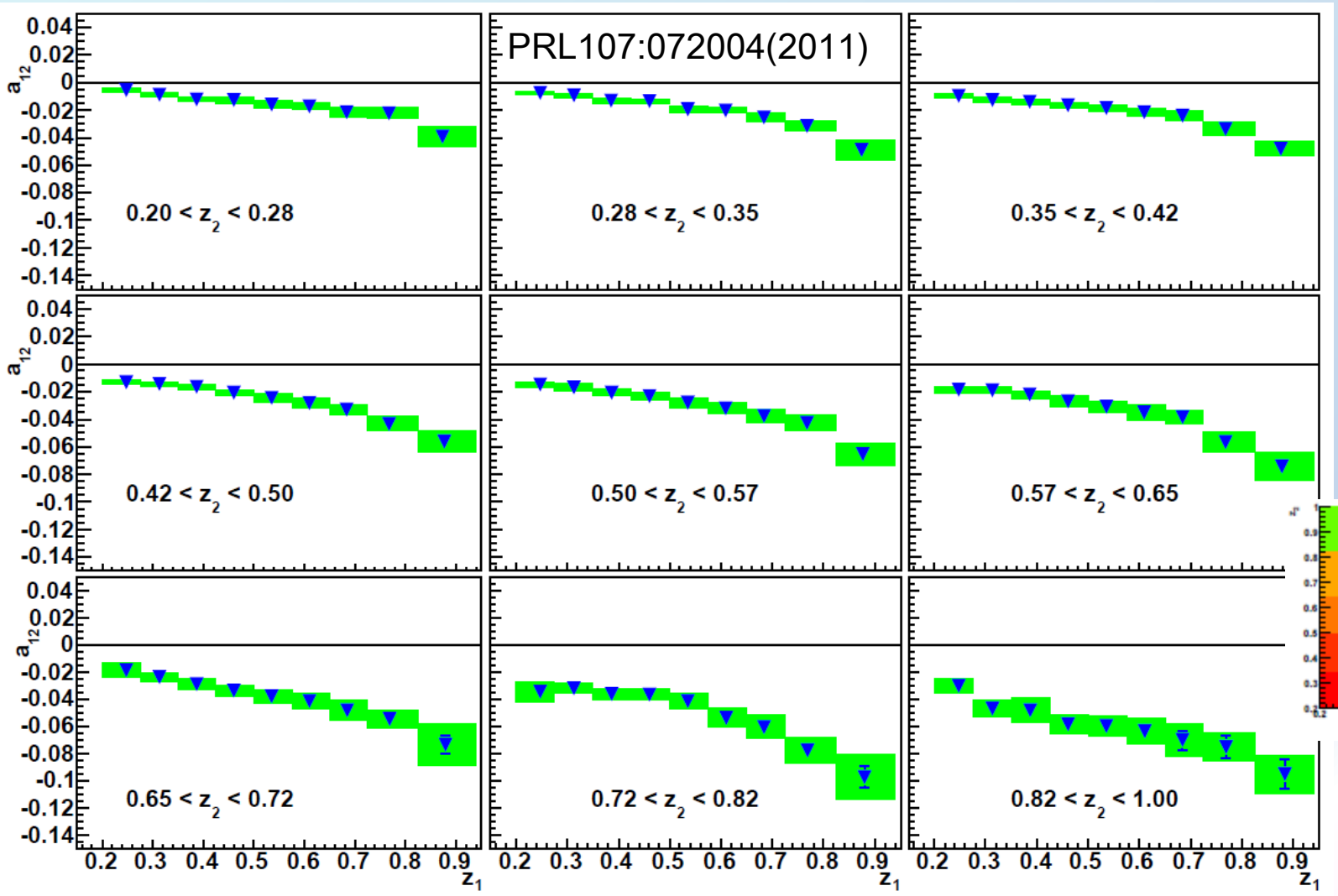
- Jaffe et al. [PRL **80,**(1998)]
- Radici et al. [PR**D 65,**(2002)]

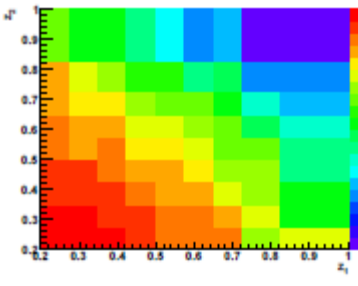
$$A \propto H_1^{\angle}(z_1, m_1)\overline{H}_1^{\angle}(z_2, m_2)\cos(\varphi_1 + \varphi_2)$$



RIKEN

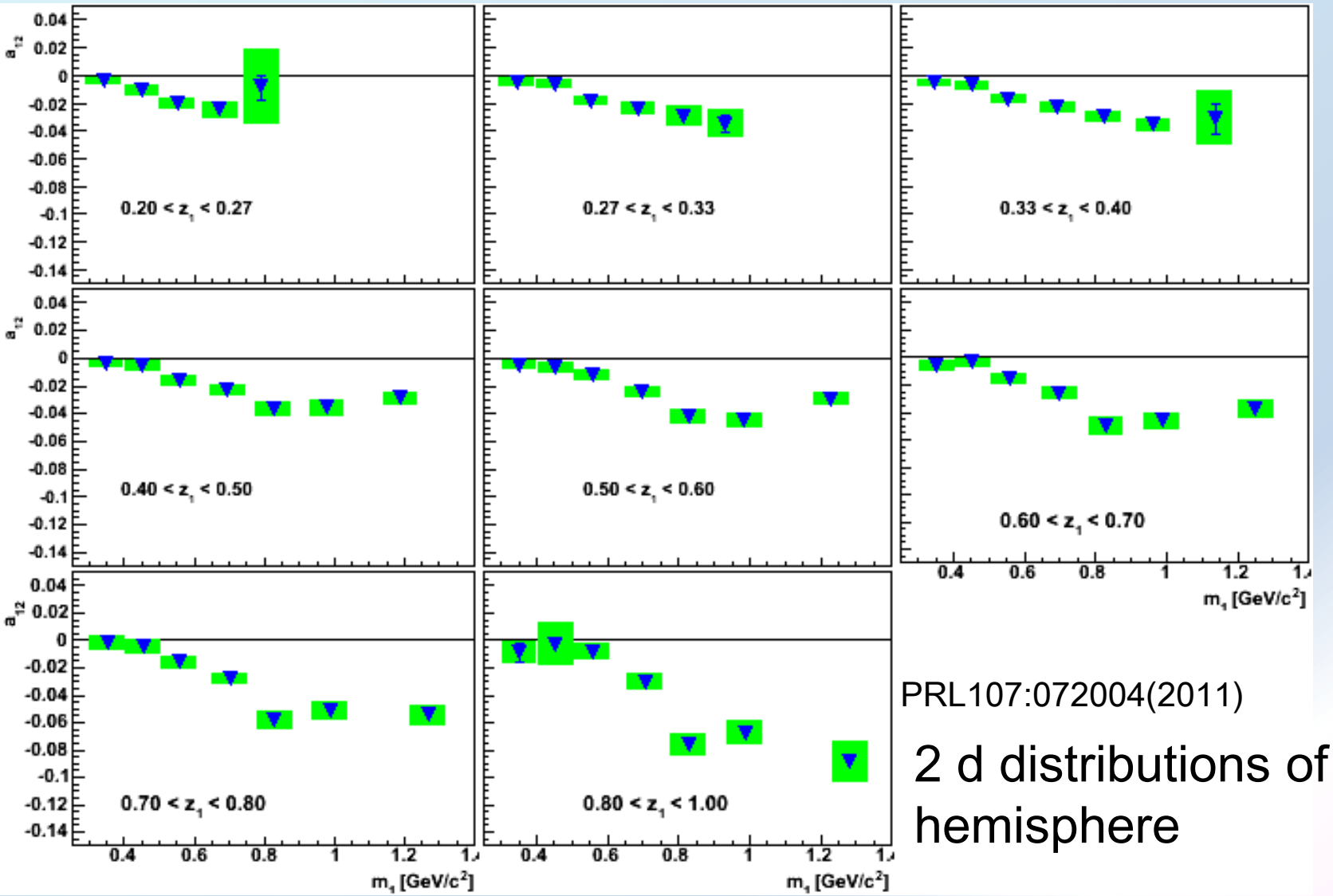
# Belle IFF asymmetries: ($z_1$ x $z_2$) Binning



Magnitude increasing with z

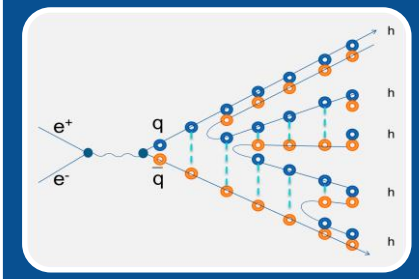
# Belle IFF asymmetries: $(z_1 \times m_1)$ Binning




PRL107:072004(2011)

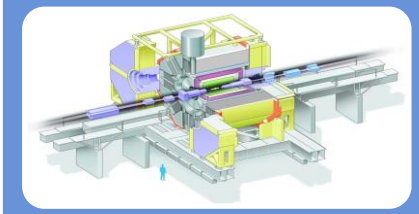
2 d distributions of one hemisphere

# What are fragmentation functions?



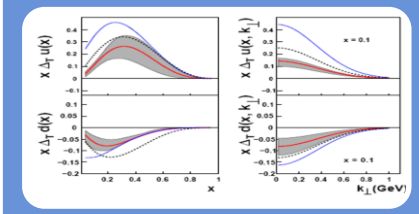

## How do quasi-free partons fragment into confined hadrons ?

- Does spin play a role ? Flavor dependence?
- What about transverse momentum (and its Evolution) ?



## What experiments measure :

- Normalized hadron momentum in CMS : $e^+e^- \rightarrow h(z)\ X$ ; $z = 2E_h/\sqrt{s}$
- Hadron pairs' azimuthal distributions : $e^+e^- \rightarrow h_1\ h_2\ X$ ; $\langle\cos(\phi_1+\phi_2)\rangle$ ; Collins FF、Interference （IFF）
- Cross sections or multiplicities differential in z: ep->hX, pp->hX



## Additional benefits of the FF measurements :

- Pol FFs necessary input to transverse spin SIDIS und pp measurements to extract Transversity distributions function
- Flavor separation of all Parton distribution functions (PDFs) via FFs (including unpolarized PDFs)
- Baseline for any Heavy Ion measurement
- Access to exotics?

# Fragmentation functions and spin structure of the nucleon

- Unpolarized fragmentation functions:
  - Provide flavor information in nucleon
  - Most apparent in SIDIS measurements related to $\Delta q(x)$
  - But also required for all RHIC hadron asymmetries (especially pion $A_{LL}$ charge ordering)
  - Transverse momentum dependence needed for Sivers and other TMDs
- Polarized fragmentation functions:
  - For transverse spin almost unique access (require two chiral-odd functions):
    - DY: $\delta q \times \delta q$ or
    - SIDIS/RHIC: $\delta q \times$ Collins or $\delta q \times$ IFF
  - FFs from Belle/Babar