Study of the exotic charmonium-like states from Lattice QCD

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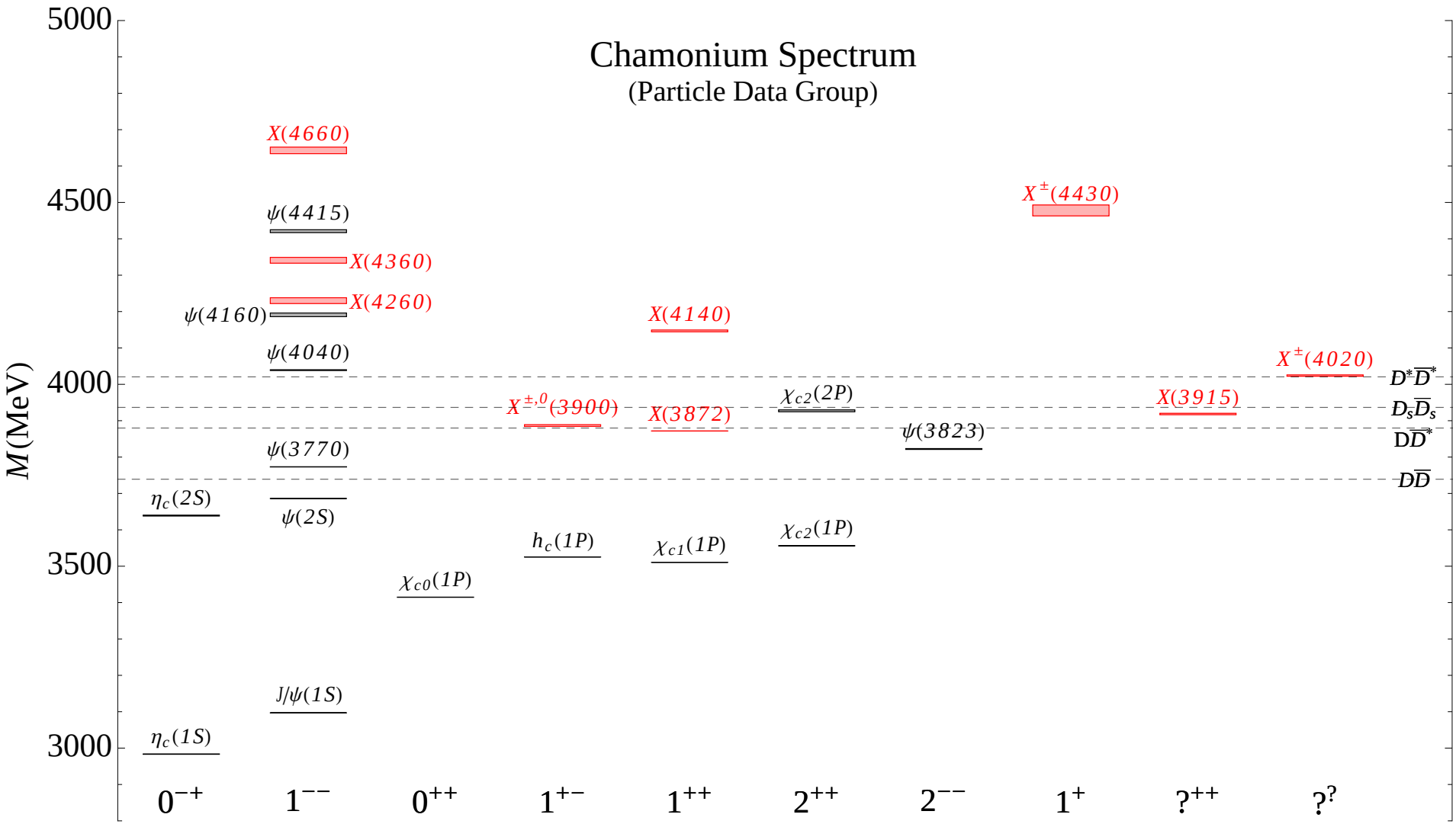
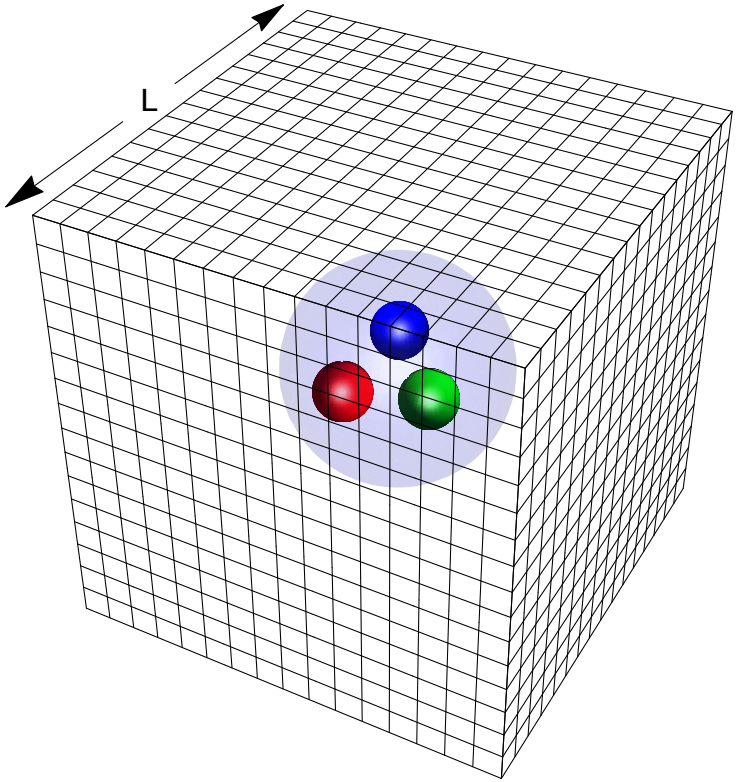
Experimental overview

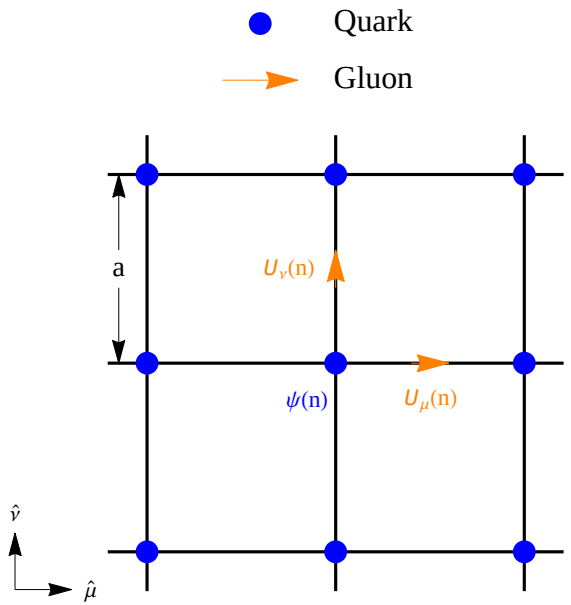


Outline

- Lattice QCD introduction.
  - Hadron spectrum in lattice QCD.
  - Hadron interactions in lattice QCD.
- Results;
  - Charmonium spectrum.
  - $Z_c^{\pm}(3900)$: $D\bar{D}^*$ scattering.

Lattice QCD





- A renormalization of QCD.
  – Input parameters: quark masses, gauge coupling, CP violating phase θ(is set to zero). The scale is set to $1/a$.
- A computational tool to solve QCD in the non-perturbative regime.

Lattice QCD

- Energy from correlation function:

$$\langle 0|\mathcal{O}(t)\mathcal{O}(0)^\dagger|0\rangle = \sum_n \frac{\langle 0|\mathcal{O}|n\rangle\langle n|\mathcal{O}^\dagger|0\rangle}{2E_n} e^{-E_n t} \longrightarrow \quad A e^{-E_0 t}$$

  $\mathcal{O}$: interpolating operator. e.g. pion operator $\bar{d}\gamma_5 u$.

- Exited and exotic spectrum
  - Build large basis of interpolating operators $\{\mathcal{O}_i\}$ with definite J^{PC}.
  - Construct the matrix of correlators

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}.$$

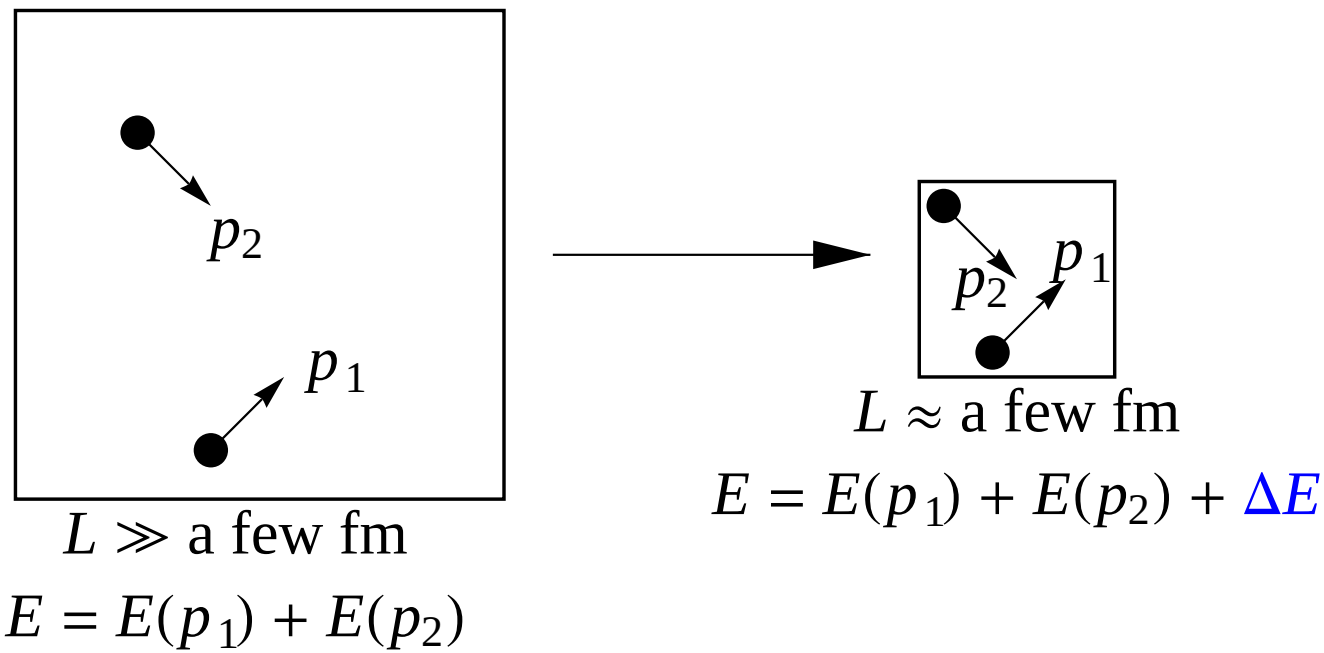
  - Solve the generalized eigenvalue problem

$$C_{ij}(t)v_j^n = \lambda_n(t) C_{ij}(t_0) v_j^n$$

    - ★ Eigenvalues: $\lambda_n(t) \to e^{-E_n t}(1 + O(e^{-\Delta E\ t}))$
    - ★ Eigenvectors related to the overlaps: $Z_i^n = e^{E_n t_0/2} v_j^{n*} C_{ji}(t_0)$.

Scattering in lattice QCD

- Maiani-Testa no-go theorem: Matrix elements related to scattering information is not directly accessible in Euclidean formulations of QCD in infinite volume.
- Lüscher's finite volume technique.

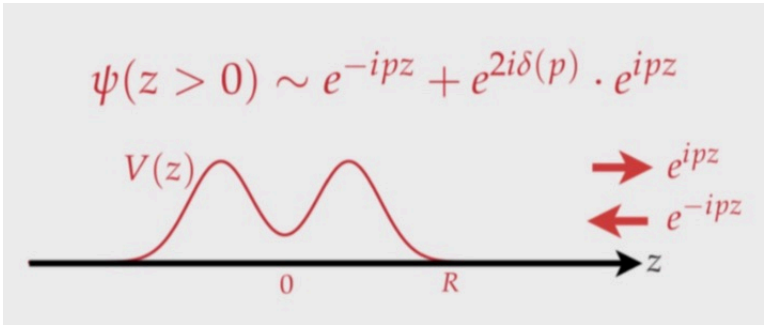


Scattering in lattice QCD

- One dimensional case

  Outside the range of potential:

  $$\psi(z) \sim \cos[p|z| + \delta(p)]$$



- Periodic boundary condition

$$\left.\begin{array}{rcl} \psi\left(-\frac{L}{2}\right) & = & \psi\left(\frac{L}{2}\right) \\ \frac{d\psi}{dz}\left(-\frac{L}{2}\right) & = & \frac{d\psi}{dz}\left(\frac{L}{2}\right) \end{array}\right\} \Longrightarrow pL + 2\delta(p) = 2n\pi$$

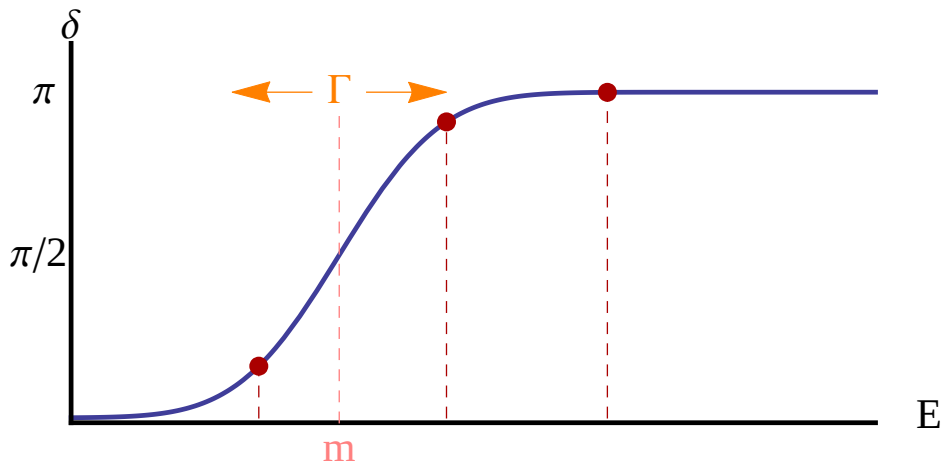
discrete energies $\Longleftrightarrow$ discrete values of phase shift

Scattering in Lattice QCD

In a periodic cubic box, the s-wave elastic scattering:

$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} \mathcal{Z}_{00}(1; q^2) \qquad \text{Lüsher's formula}$$

- Experimentally: determine the amplitudes as a function of energy E. e.g. a single elastic resonance:



- On lattice: discretized energy levels.

Charmonium spectrum

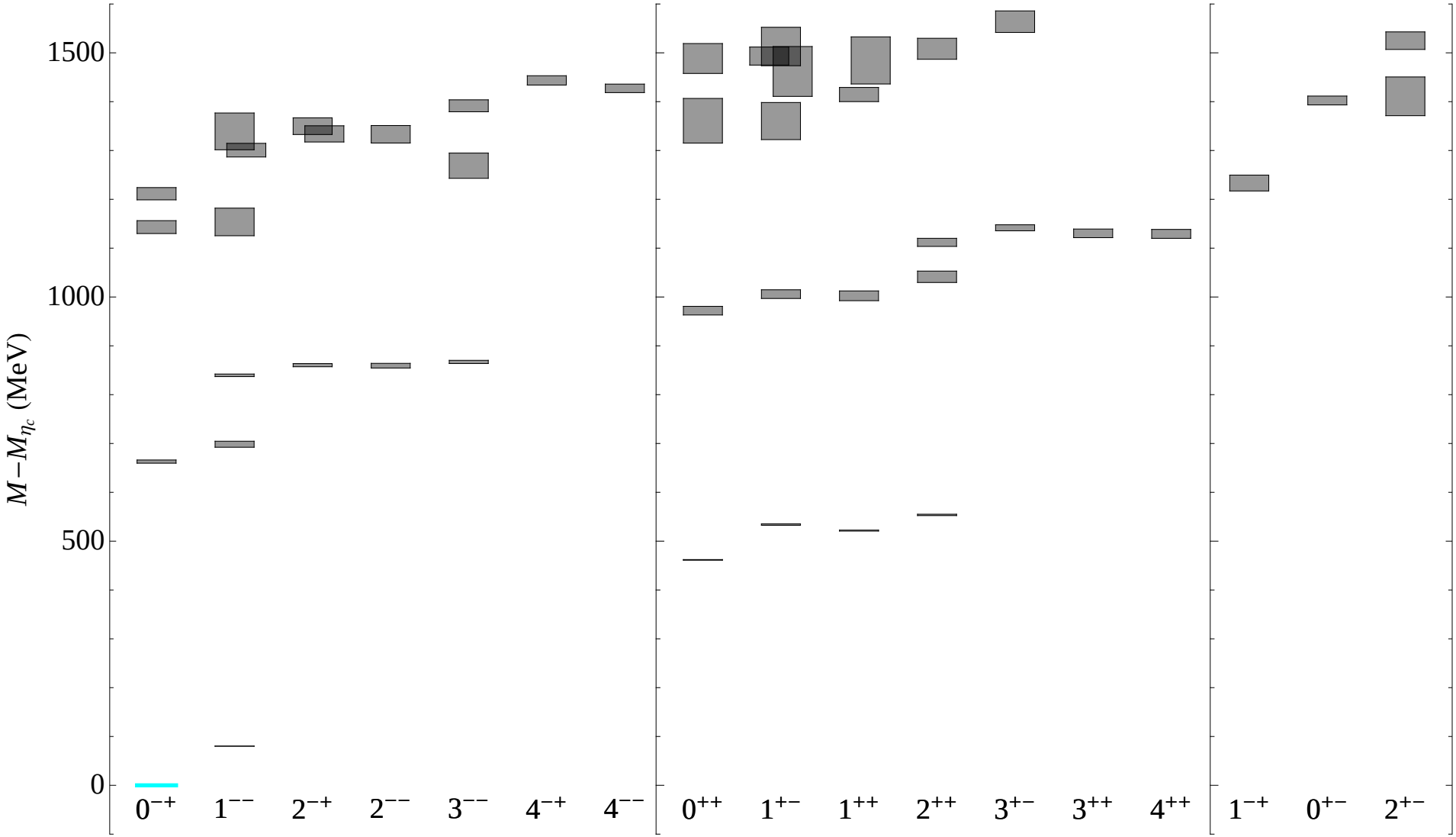
Interpolating operators:

- Simplest meson interpolation operators: local fermion bilinears. $\bar{\Psi}_{i\alpha}(\mathbf{x}, t)\Gamma_{\alpha,\beta}\Psi_{i,\beta}(\mathbf{x}, t)$, $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}$
- Non-local operators with definite (J, M),

$$\mathcal{O}^{J,M} \sim \bar{\Psi}(x)\Gamma_i \overleftrightarrow{D_j}\overleftrightarrow{D_k}\cdots\Psi(x)$$

  We use up to 3 derivative operators.

Charmonium spectrum: Results



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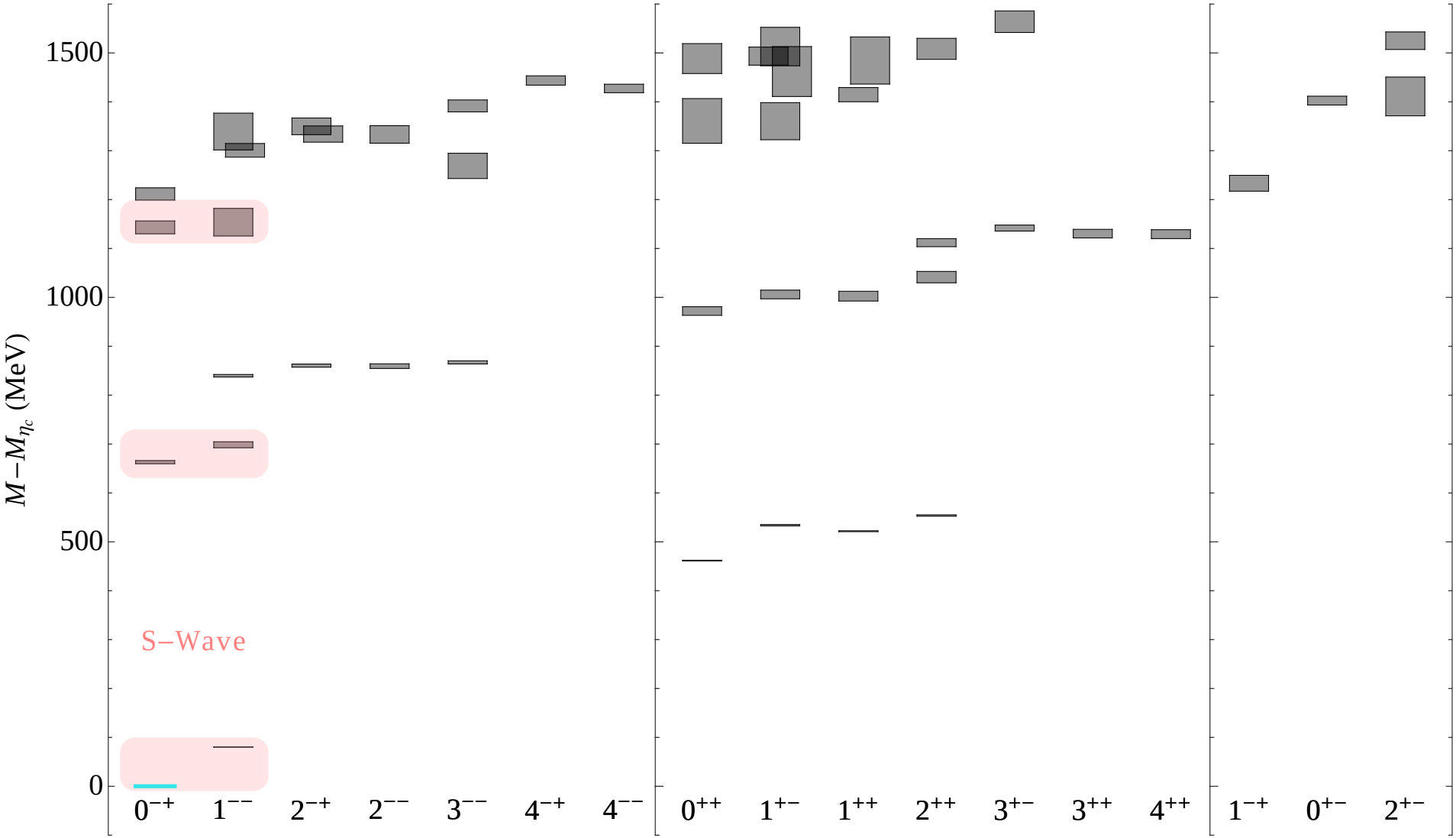

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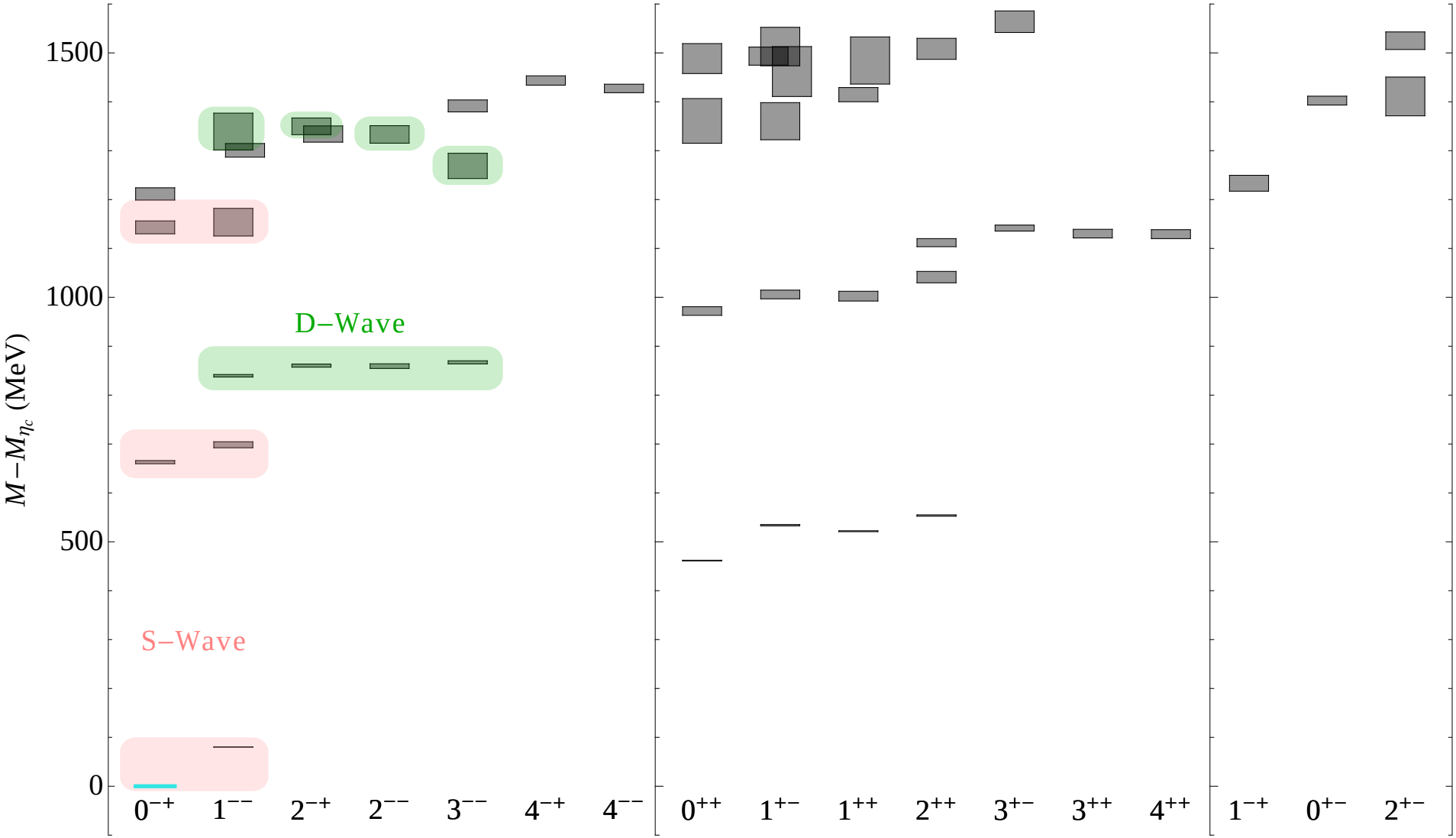

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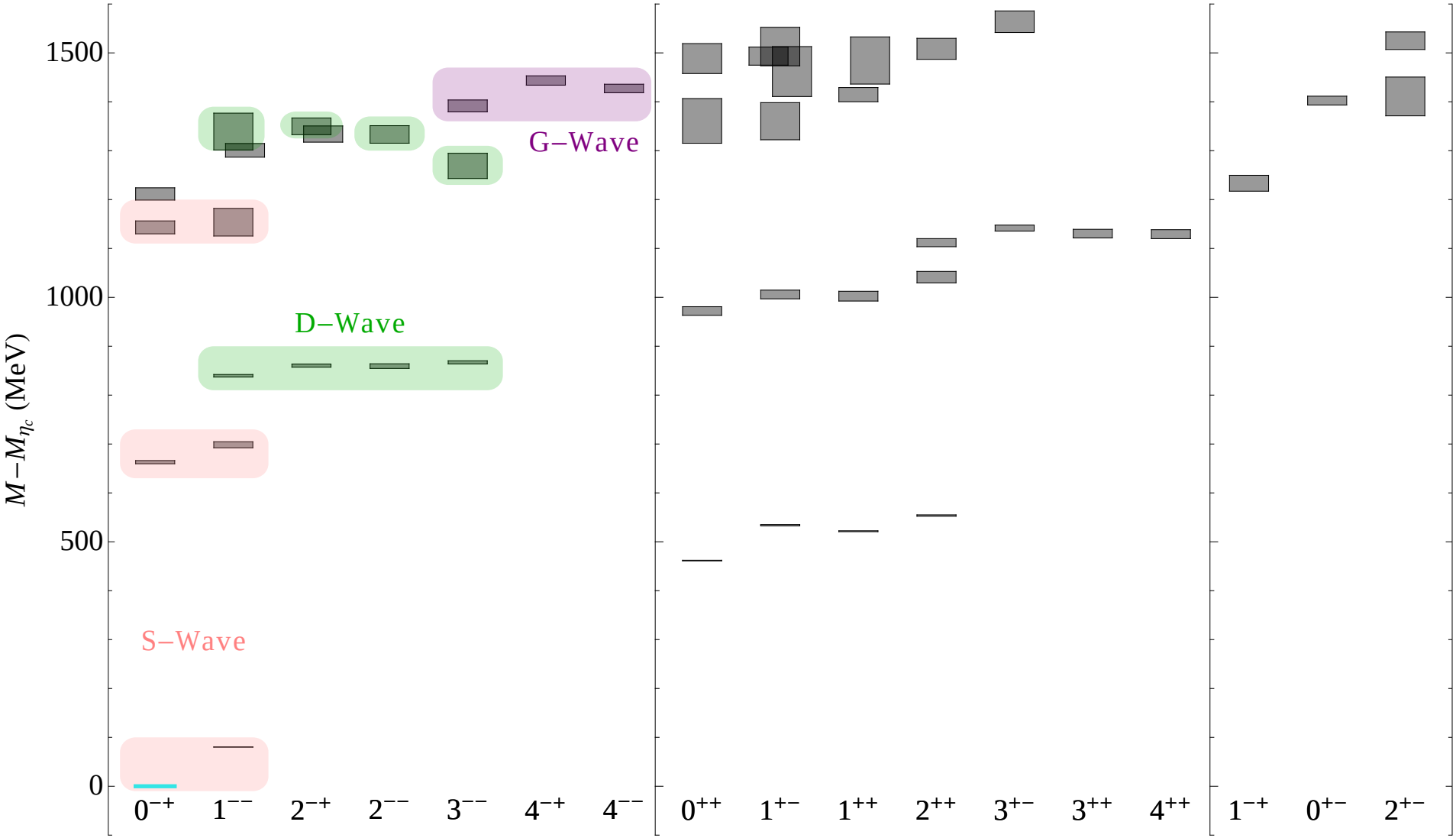
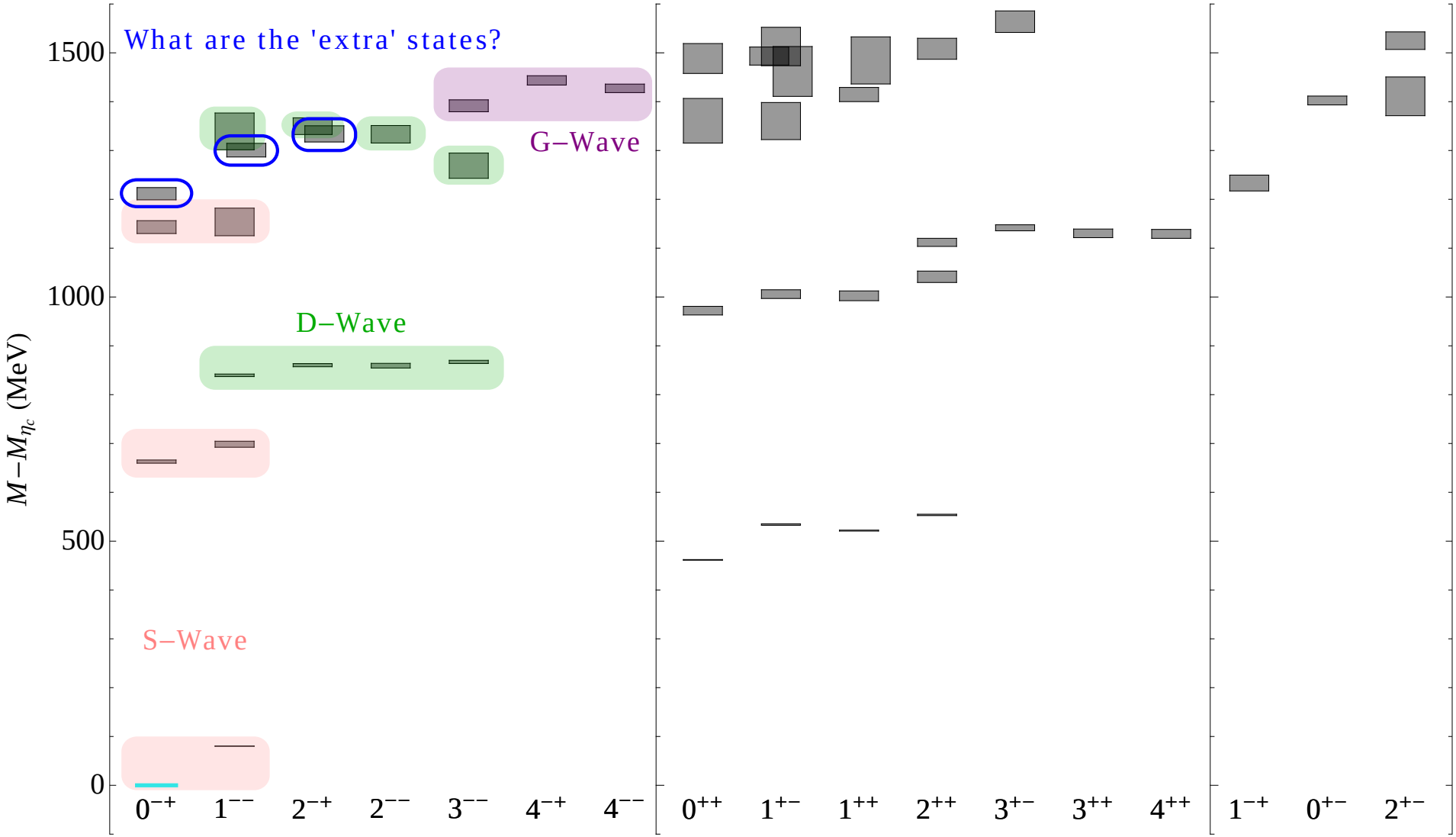


Charmonium spectrum: Results



Some model-dependent interpretation

Consider the following 3 operators in $J^{PC} = 1^{--}$ channel

$$\bar{\tilde{\psi}}\gamma_i \overset{\wedge}{\tilde{\psi}} \longrightarrow {}^3S_1$$

$\frac{1}{2}[1-\gamma_0]$

upper component projector
"non-relativistic"

two-derivative constructions : $D^{[2]}_{J,m} = \langle 1m_1; 1m_2|Jm\rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}$

$$\langle 1m_1; 2m_2|1m\rangle \, \bar{\tilde{\psi}}\gamma_{m_1} D^{[2]}_{J=2,m_2} \overset{\wedge}{\tilde{\psi}} \longrightarrow {}^3D_1$$

$\frac{1}{2}[1-\gamma_0]$

ignoring the gauge-field $Y_2^m(\overleftrightarrow{\partial})$

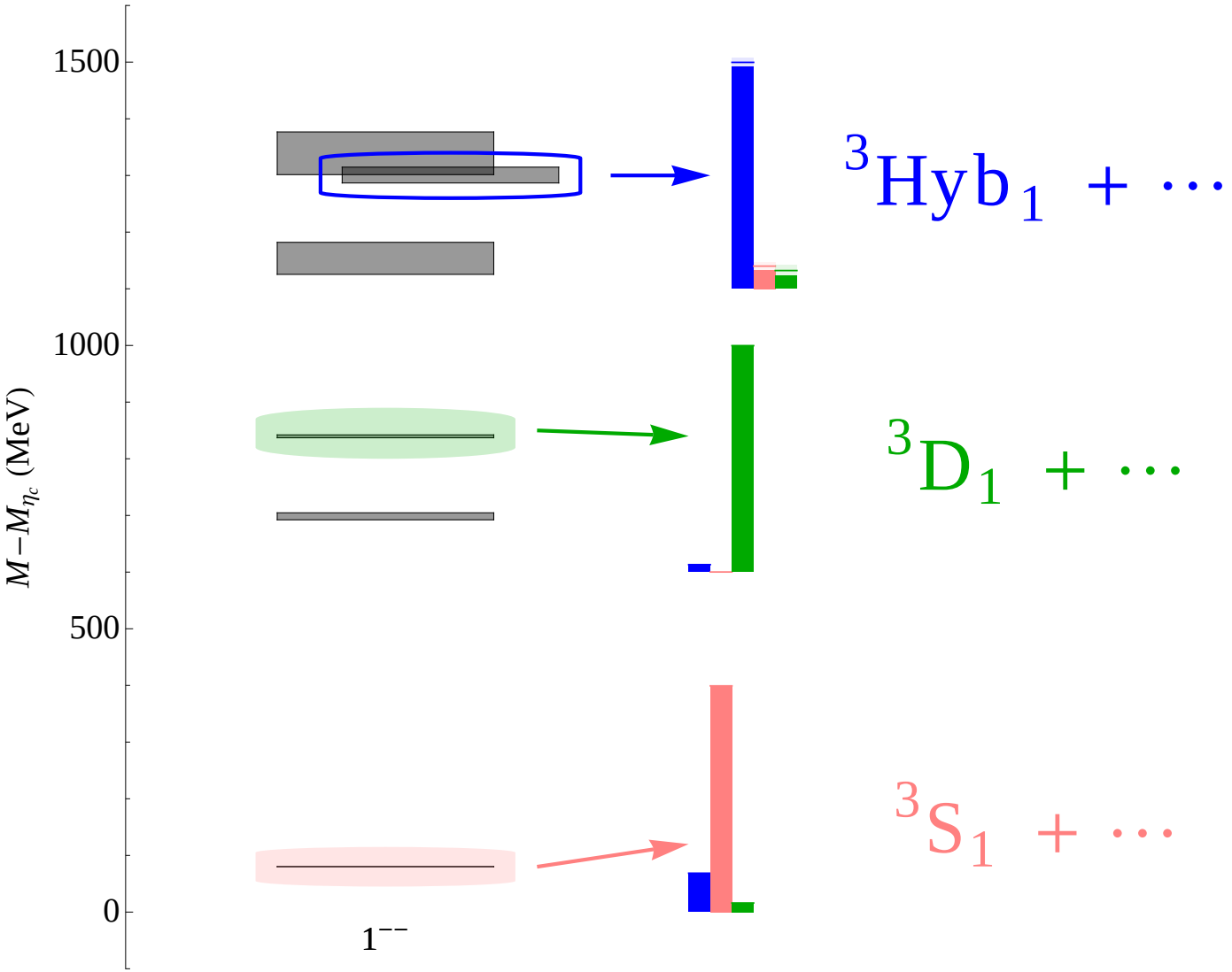
gauge-invariant
version of a D-wave ?

$$\bar{\tilde{\psi}}\gamma_5 D^{[2]}_{J=1,m} \overset{\wedge}{\tilde{\psi}} \longrightarrow {}^1\mathrm{hyb}_1$$

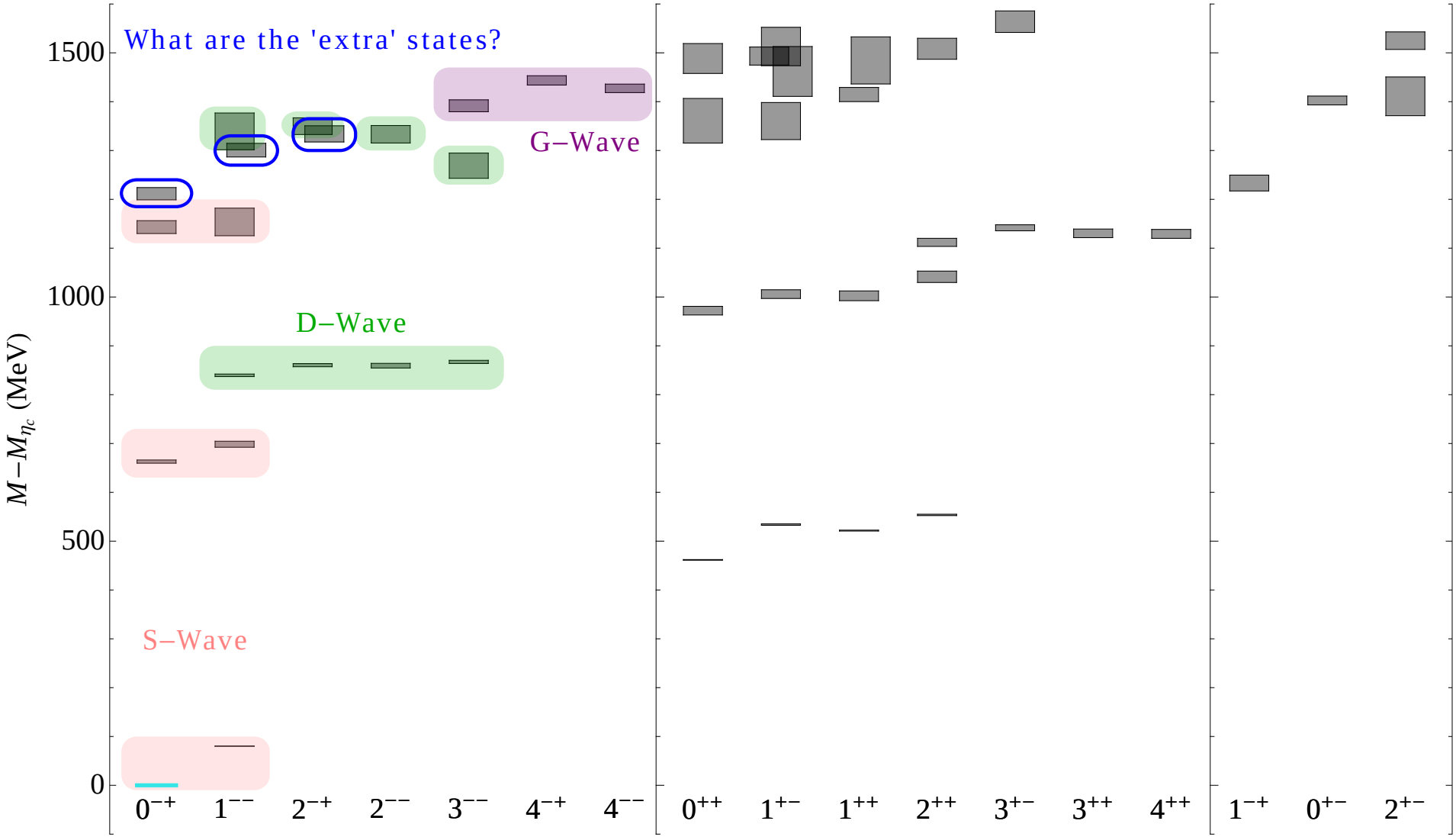
$\frac{1}{2}[1-\gamma_0]$

ignoring the gauge-field
$[D, D] \to [\partial, \partial] = 0$

Some model-dependent interpretation



Charmonium spectrum: Results



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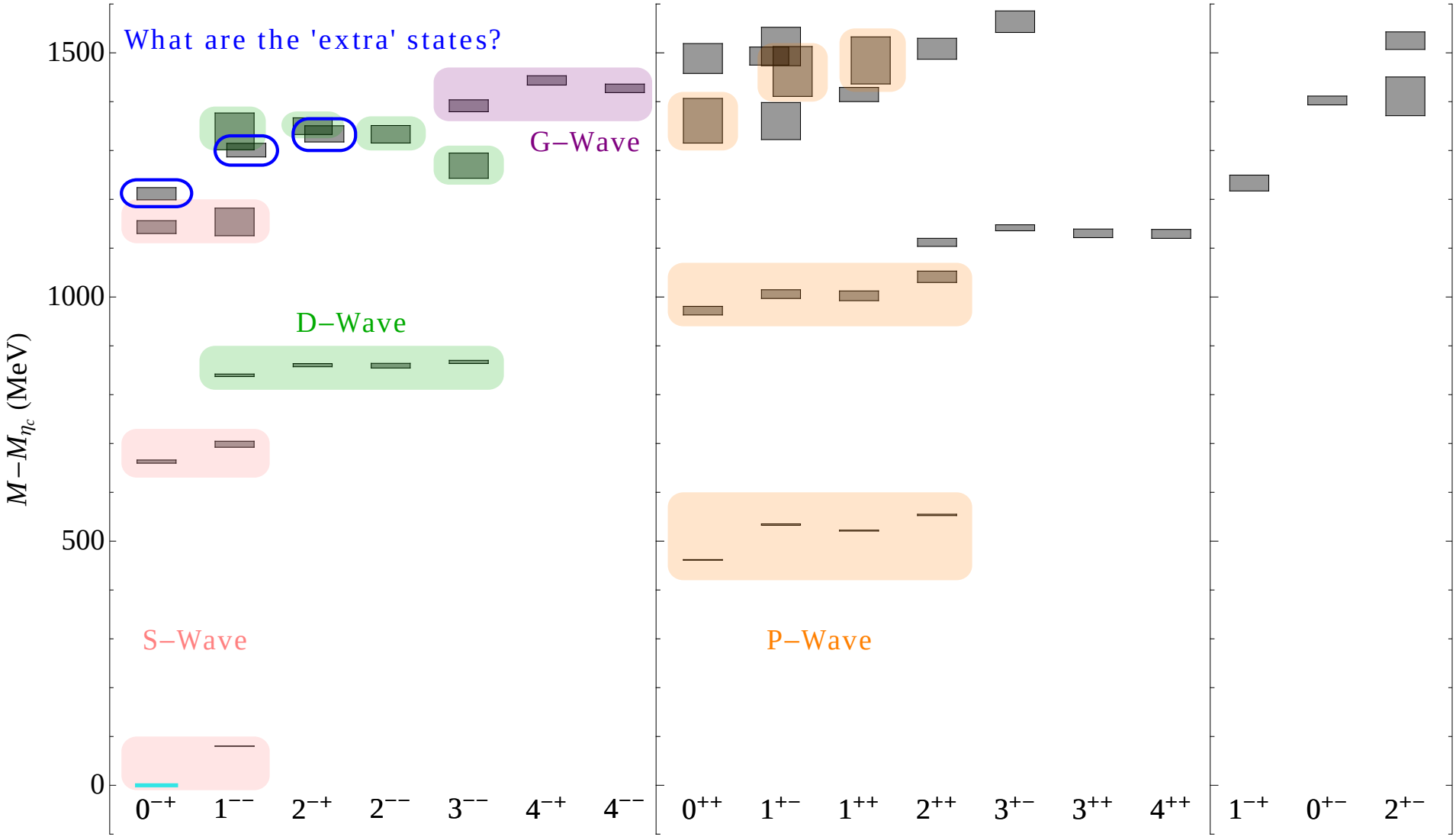

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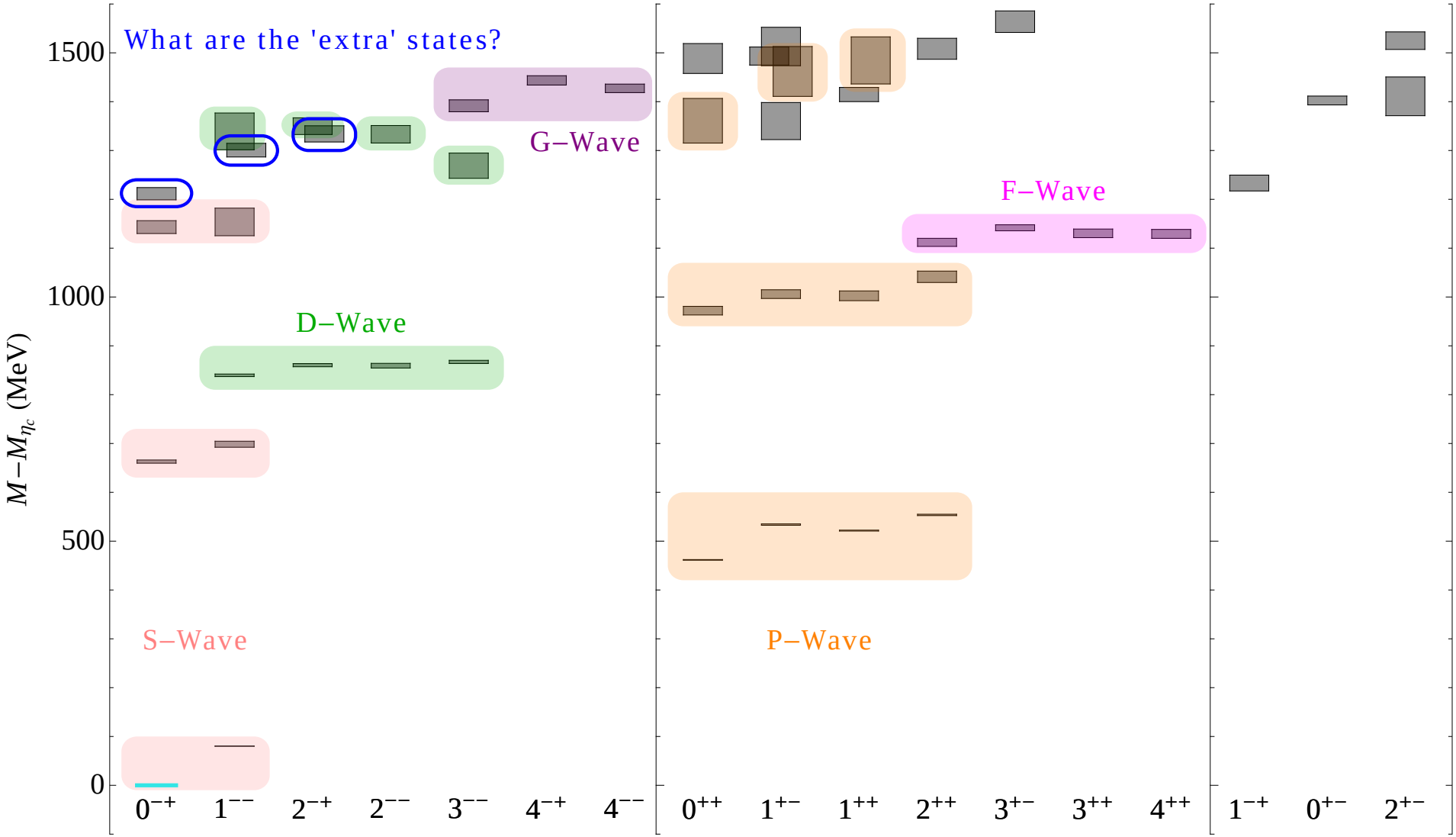

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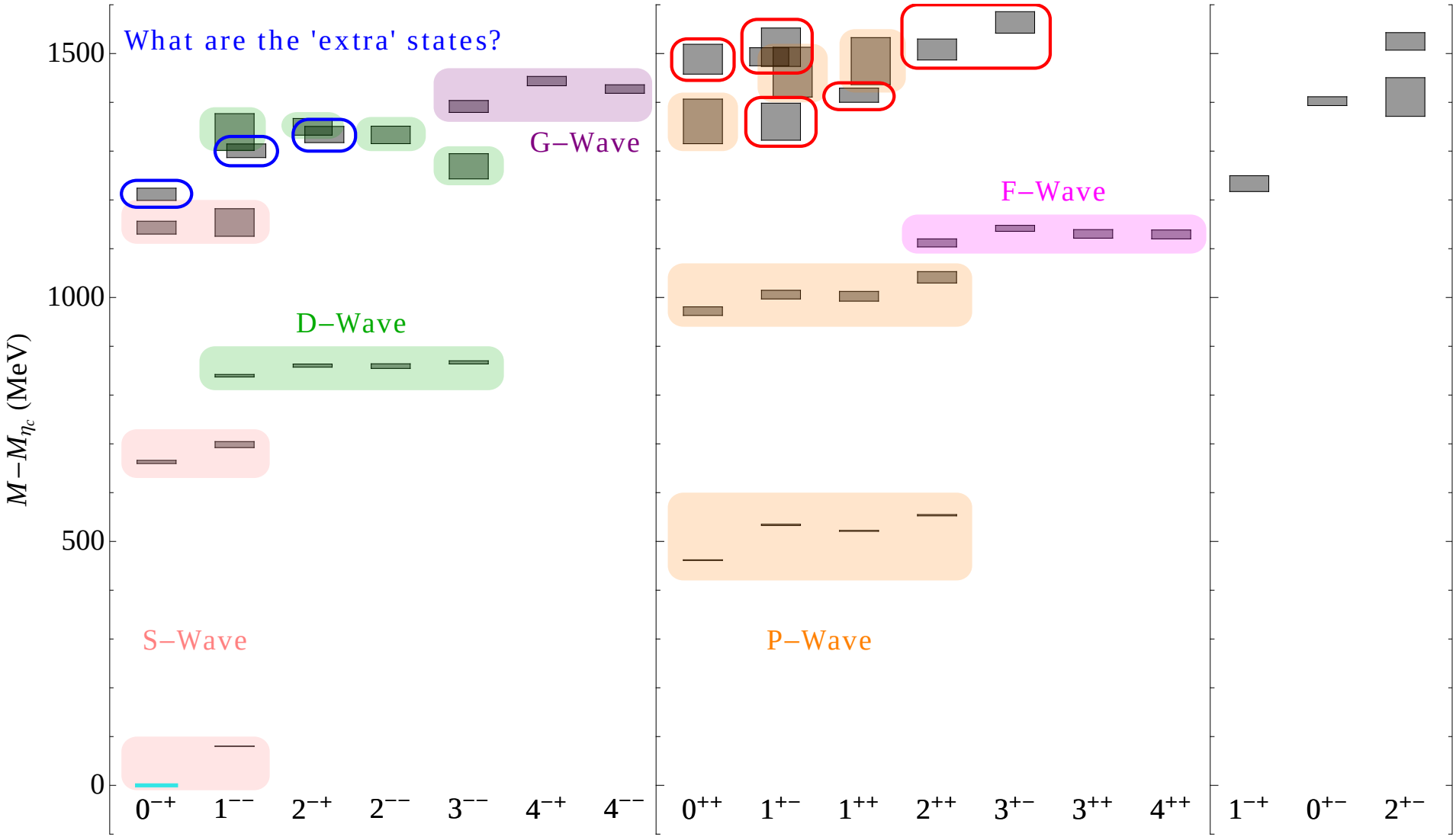

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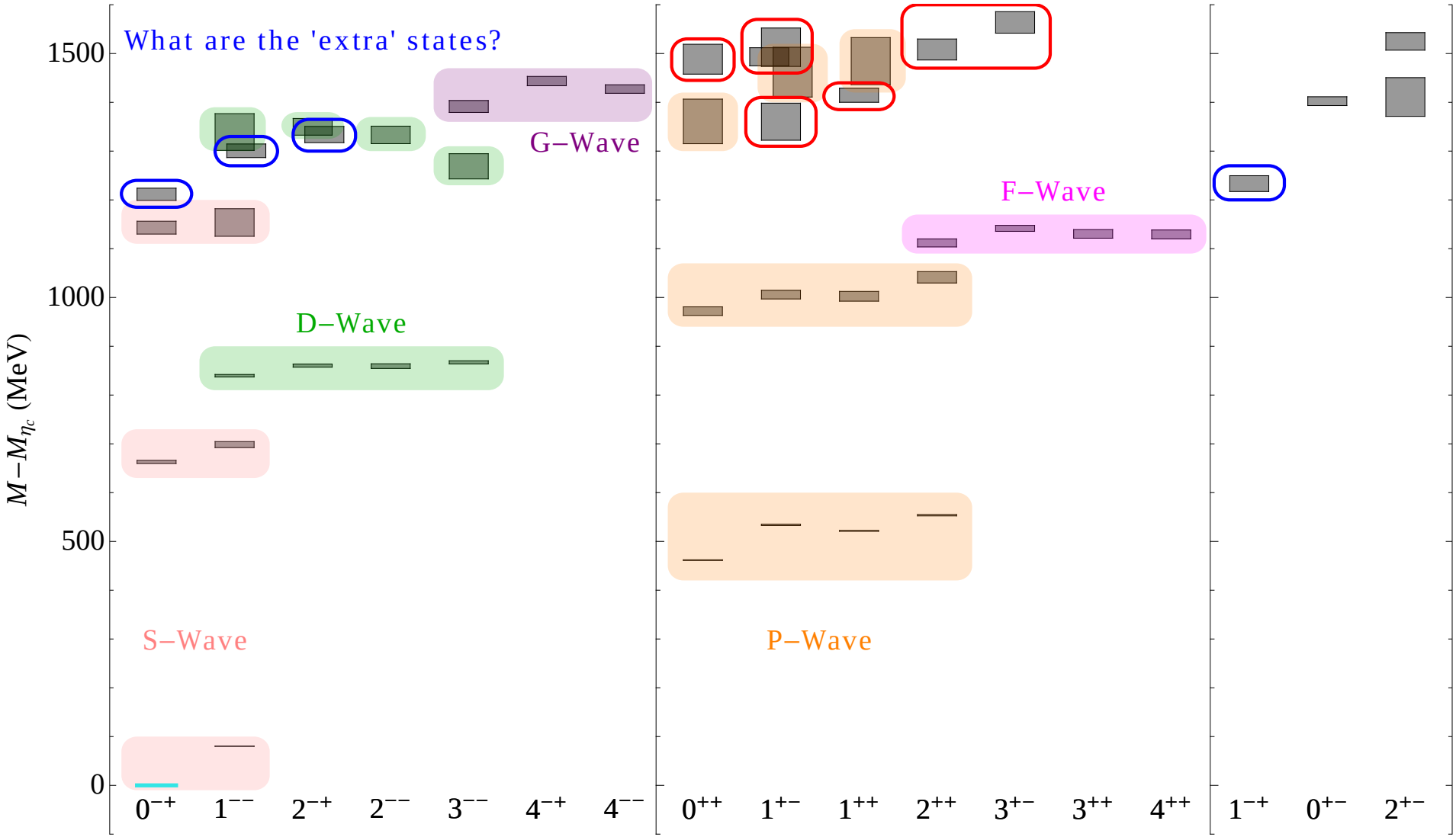

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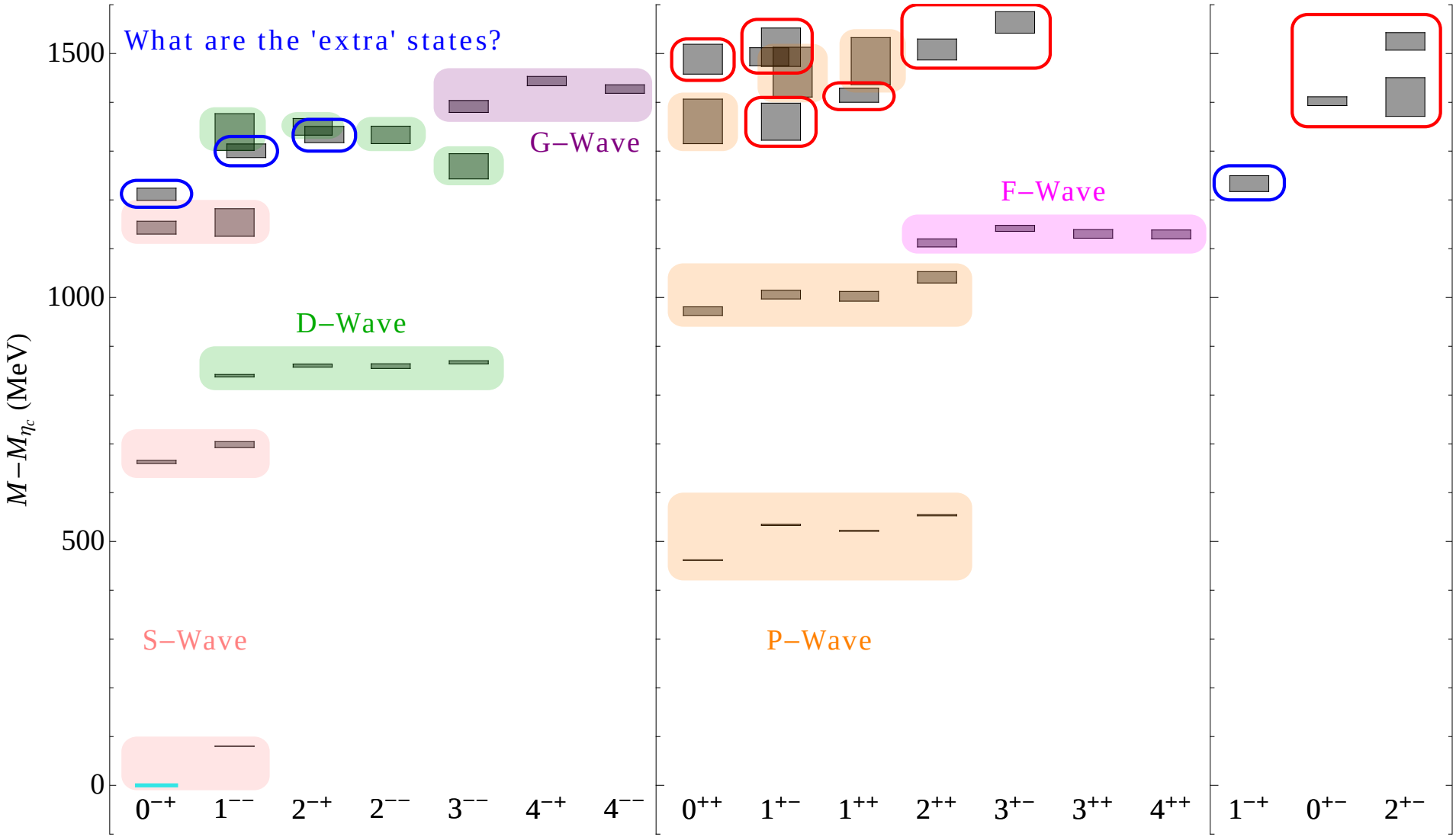


Discussion about hybrid mesons

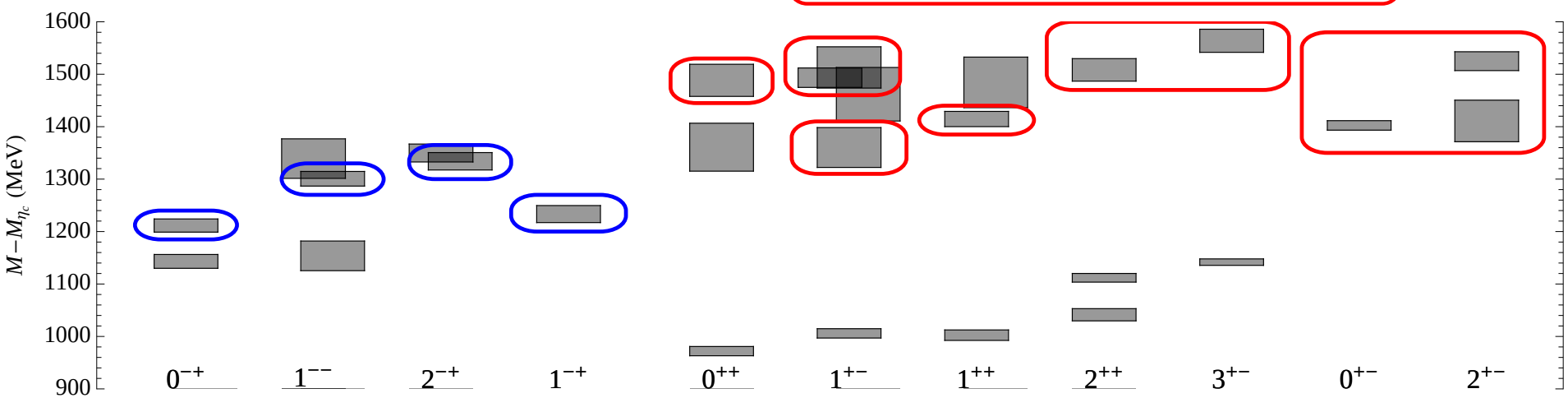
Consider the operator:

$$D^{[2]}_{J=1,m} = \langle 1m_1; 1m_2 | 1m \rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2} \sim B^a$$

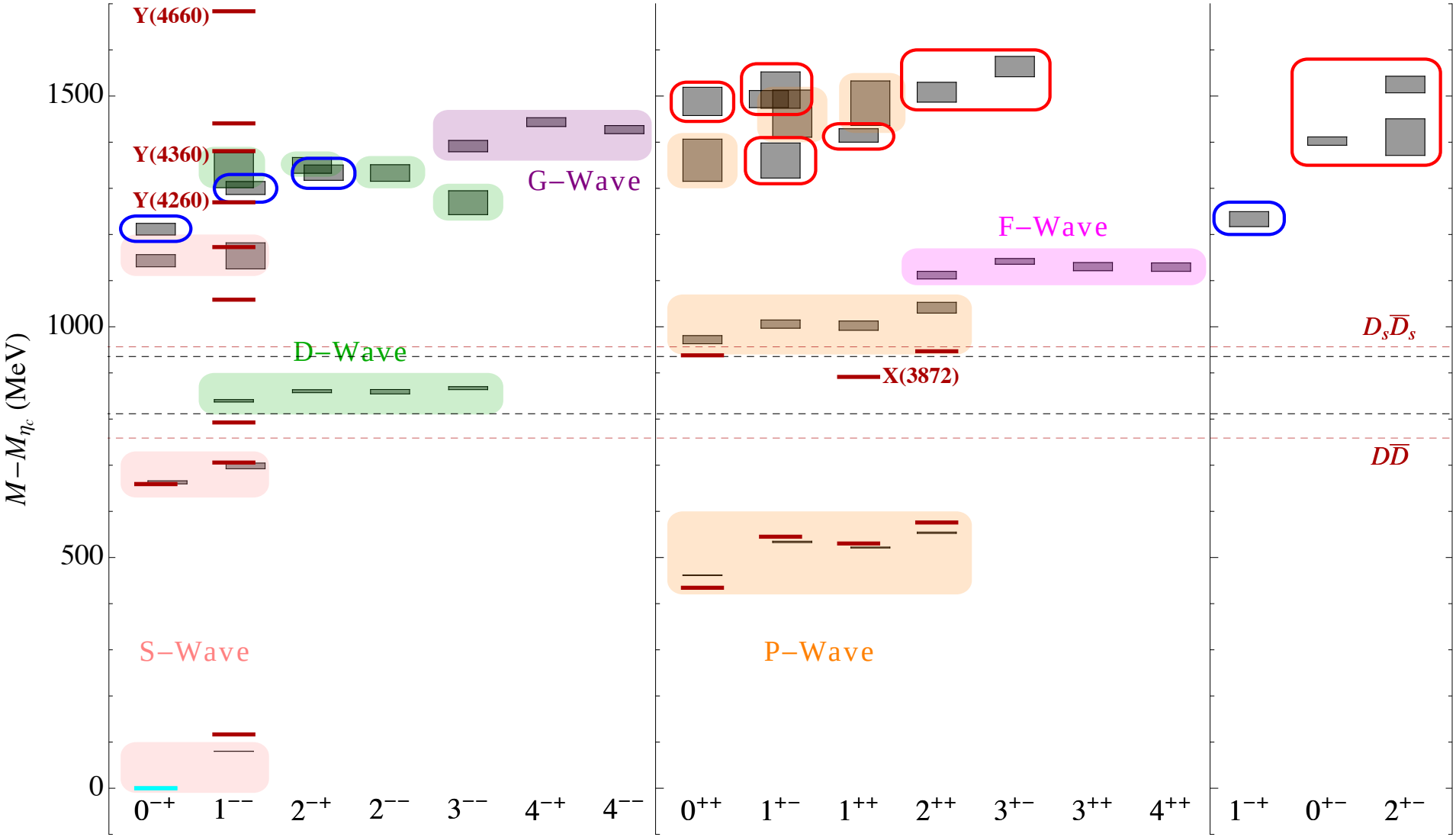
a 1^{+-} chromomagnetic gluonic excitation.

$q\bar{q}(^1S_0)B \sim 0^{-+} \otimes 1^{+-} = 1^{--}$
$q\bar{q}(^3S_0)B \sim 1^{--} \otimes 1^{+-} = (0,1,2)^{-+}$

$q\bar{q}(^1P_1)B \sim 1^{+-} \otimes 1^{+-} = (0,1,2)^{++}$
$q\bar{q}(^3P_0)B \sim 0^{++} \otimes 1^{+-} = 1^{+-}$
$q\bar{q}(^3P_1)B \sim 1^{++} \otimes 1^{+-} = (0,1,2)^{+-}$
$q\bar{q}(^3P_2)B \sim 2^{++} \otimes 1^{+-} = (1,2,3)^{+-}$

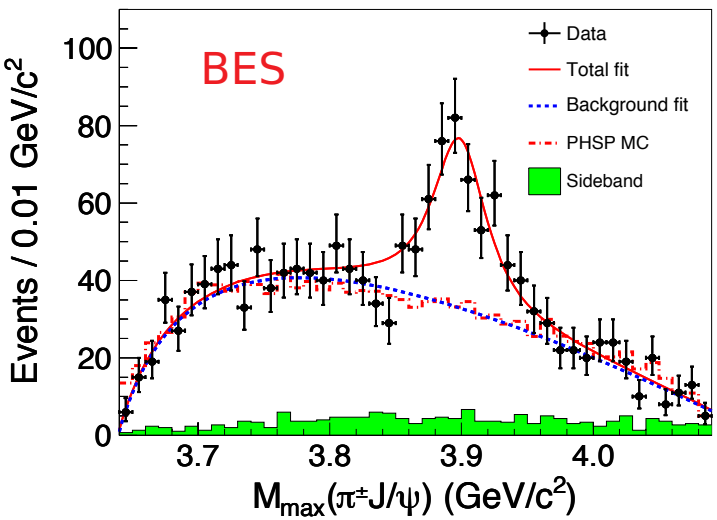


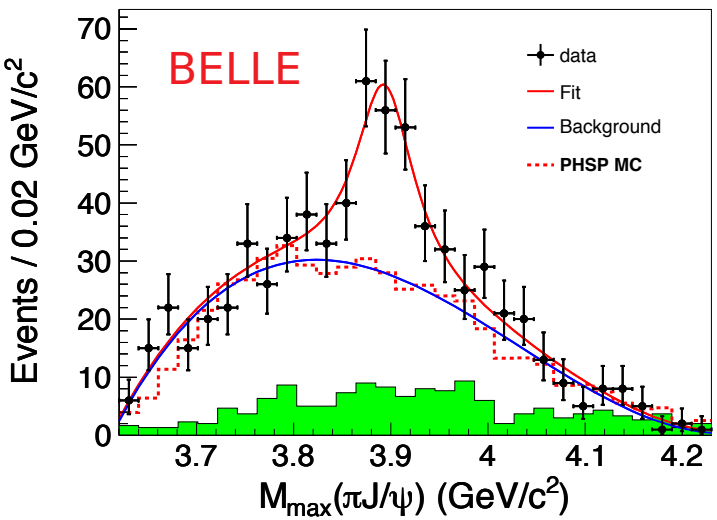
Charmonium spectrum: Compare with experiments



Search for $Z_c^{\pm}(3900)$

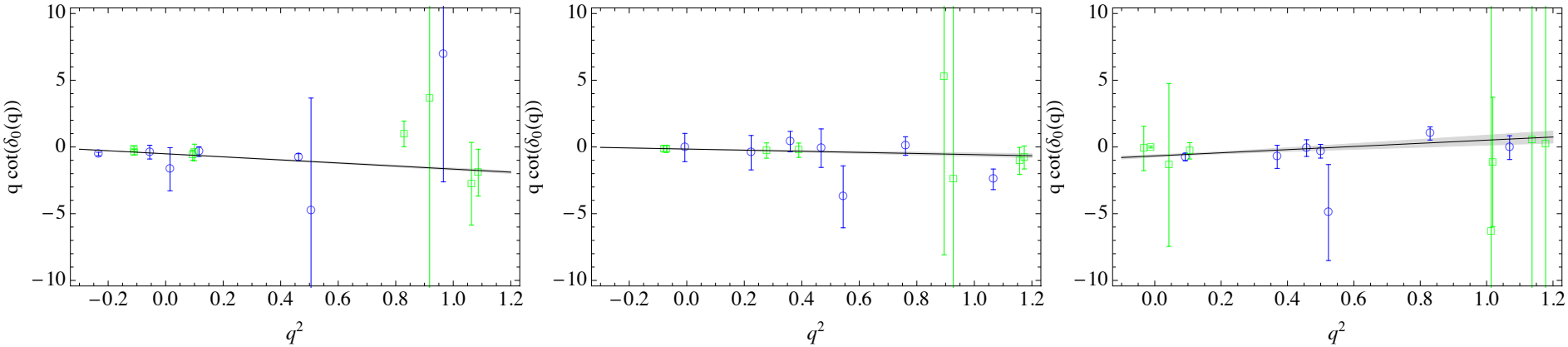
- The first confirmed charged charmonium-like state $Z_c^{\pm}(3900)$:





- The mass of this state is close to the DD^* threshold.
- Is $Z_c^{\pm}(3900)$ a molecular bound state formed by the D and $\bar{D}^*$ mesons?

Search for $Z_c^{\pm}(3900)$



$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} \mathcal{Z}_{00}(1; q^2)$$

$$q \cot \delta_0(q) = \frac{1}{a_0} + \frac{1}{2} r_0 q^2 + \cdots$$

	a_0(fm)
$\mu = 0.003$	-0.67(1)
$\mu = 0.006$	-2.1(1)
$\mu = 0.008$	-0.51(7)

- Computed $D\bar{D}^*$ scattering in Lattice QCD, found weakly repulsive interaction.

- *Conclusion*: *NO* evidence for a $D\bar{D}^*$ bound state.

- *Questions*:
  - Will a bound state appear at physical pion mass?
  - Will the results change if the coupled channel effects are taken into account?
    - ⋆ Study of the coupled channels $J/\Psi\pi - D\bar{D}^*$ still indicates repulsive interaction.
  - How significant are the effects of finite volume and finite lattice spacing?
  - Other channels?

S. Prelovsek, C. B. Lang, L. Leskovec, D. Mohler, Phys.Rev. D91 (2015) 014504.

Yoichi Ikeda et al. (HAL QCD), Phys. Rev. Lett. 117, 242001 (2016).

Thank you!