

XXVI International Workshop On Deep Inelastic Scattering and Related Subjects (2018)

Ad Lucem

The Photon in the MMHT PDFs

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Motivation

$$\sigma_{pp \rightarrow X}^{(N^k LO)} = \sum_{a,b} \int_{x_1 x_2} f_a(x_1)^{(N^k LO)} \hat{\sigma}_{ab \rightarrow k}^{(N^k LO)} f_b(x_2)^{(N^k LO)} D(k \rightarrow X)$$

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Calculate hard processes to a given order in pQCD and match PDFs accordingly.

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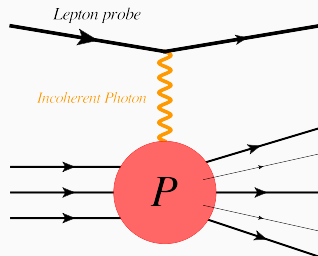
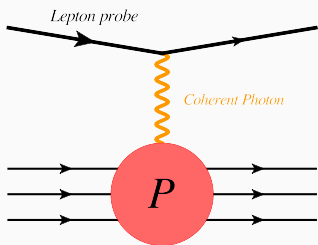
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$\alpha_S^2 \simeq \alpha_{EM} \rightarrow$ Expect QED to become relevant
Introduces the photon as an interacting parton: $\gamma(x, Q^2)$

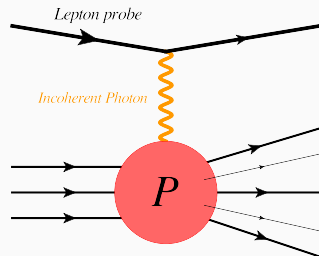
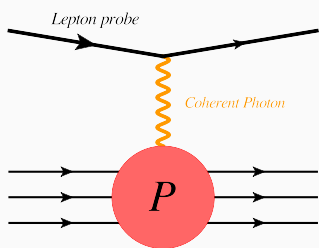
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$\gamma(x, Q_0 = 1\text{GeV}^2)$ expressed in terms of experimentally determined **structure functions**. Prior work, e.g. Martin et al (ArXiv:1406.2118), Harland Lang et al (ArXiv:1607.04635). Reformulated on stronger quantitative footing by LUXqed (ArXiv:1607.04266). Coherent photon from elastic scattering, incoherent photon from inelastic scattering.



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Errors ($\lesssim 5\%$) are then propagated from measurements of F_2 and F_L structure functions.

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Differences in the integral limits compared to LUXqed (since we evolve in **DGLAP** from $Q_0 = 1 \text{ GeV}^2$); higher twist/**proton** mass dependent terms more important, especially at high x .

- Coupled **DGLAP** evolution in x -space:

$$\mu_F \frac{d}{d\mu_F} f_i(x, \mu) = \frac{\alpha_S}{2\pi} \sum_j P_{ij}(\alpha_S(\mu)) \otimes f_j$$

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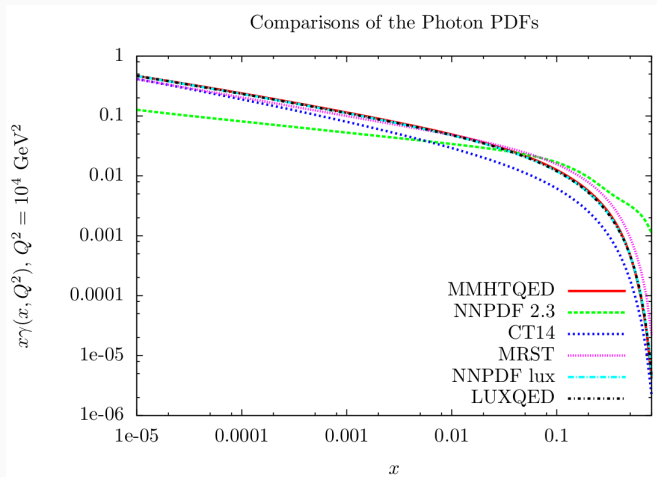
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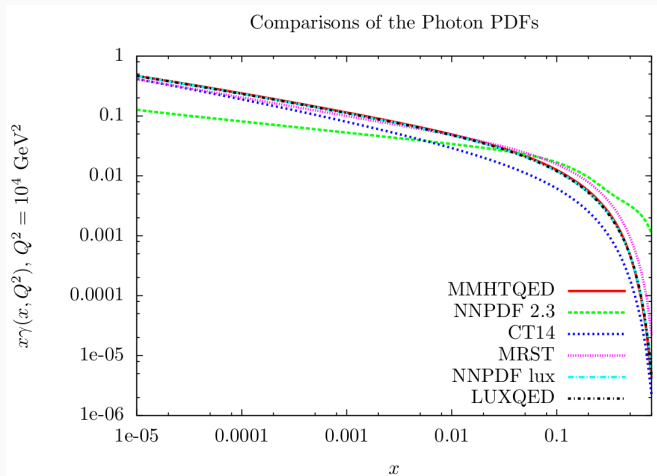
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- **Non**-iterative process: single directional evolution in **DGLAP** from input.

We have developed an equivalent **photon PDF** with full **QED DGLAP** evolution of all partons.



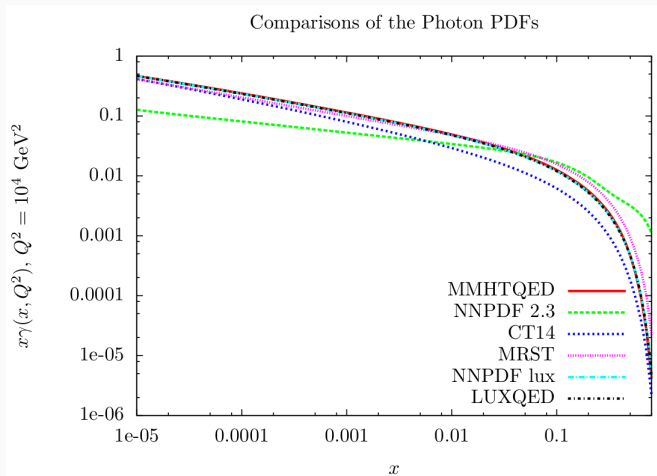
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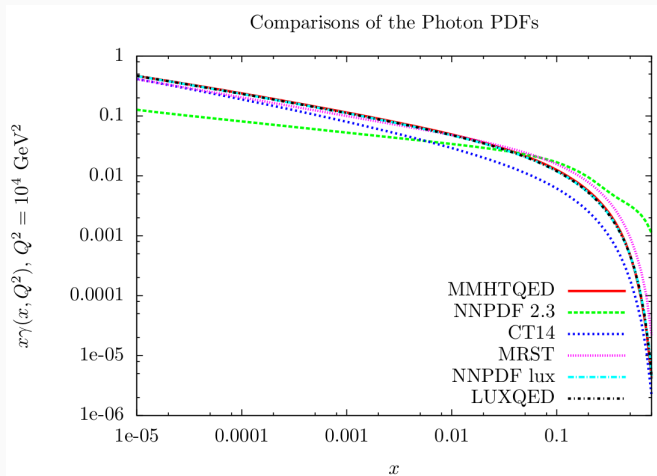
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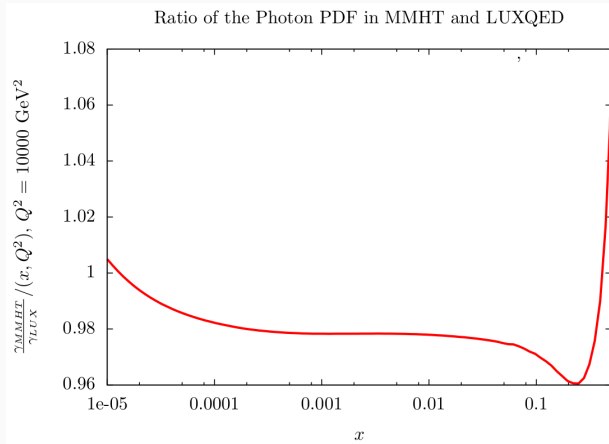
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Comparison with LUX



Differences ascribable mainly to difference in the quark and gluon PDFs that contribute to *$\gamma_{INCOHERENT}$* .

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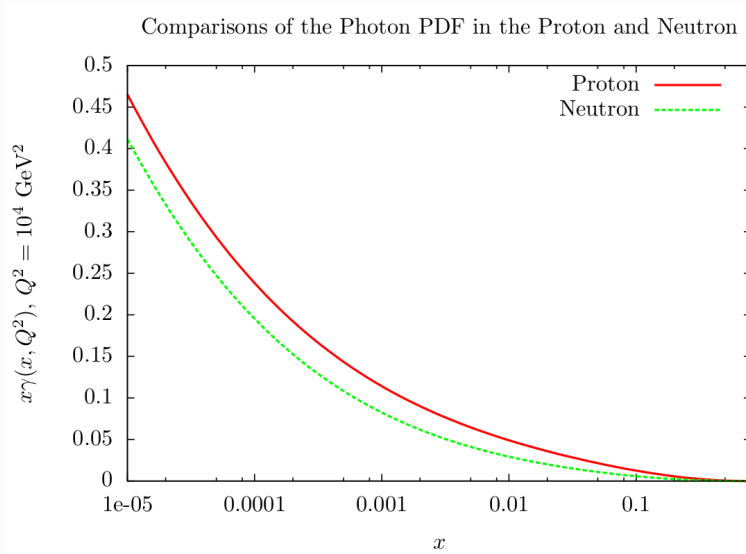
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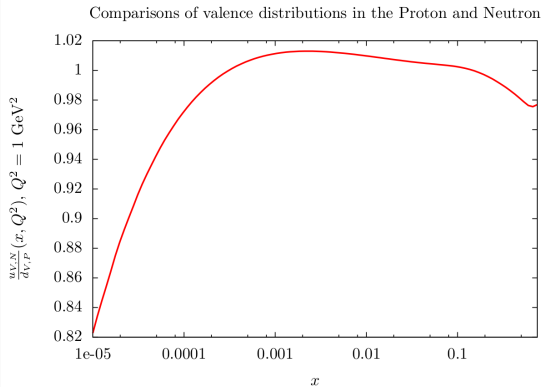
- Isospin violating terms in the neutron valence quark distributions taken proportional to QED evolved terms:

$$\Delta d_{V,n} = \epsilon \left(1 - \frac{e_d^2}{e_u^2}\right) \Delta u_{V,p}^{QED} \qquad \Delta u_{V,n} = \epsilon \left(1 - \frac{e_u^2}{e_d^2}\right) \Delta d_{V,p}^{QED}$$

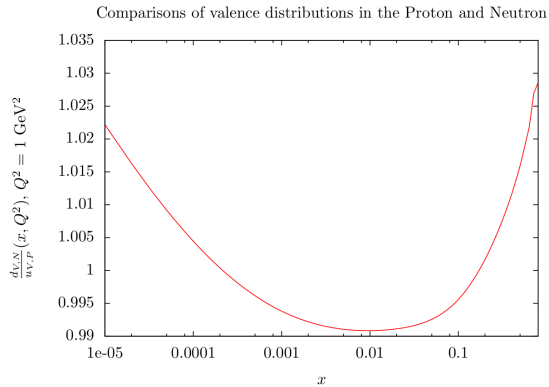


QED Isospin Violation

$$\frac{u_{V,n}}{d_{V,p}}, Q^2 = 1 \text{ GeV}^2$$

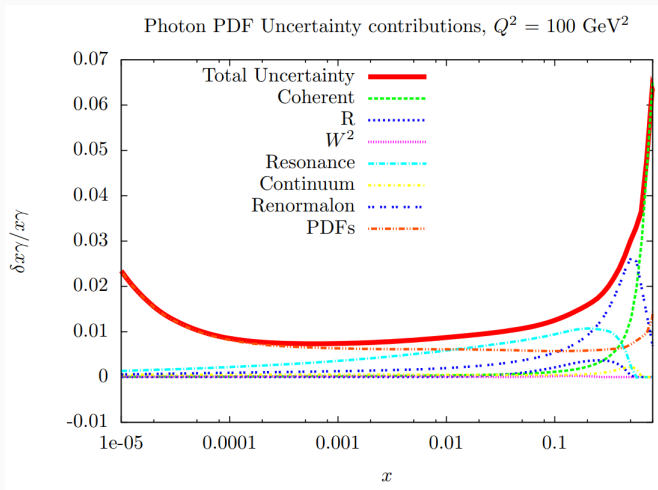


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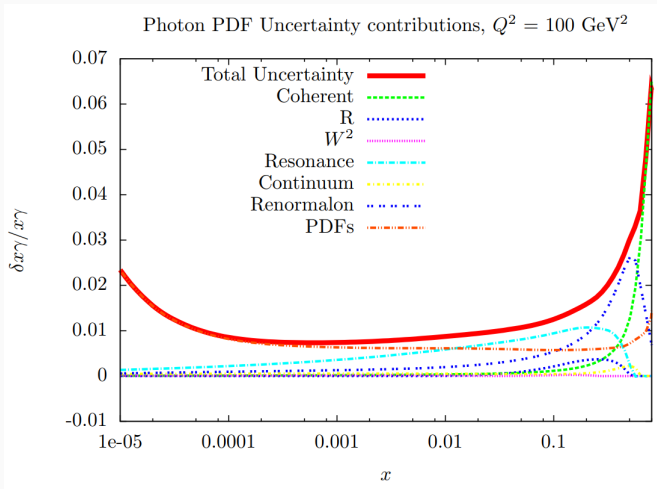
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Relative contributions to the **proton photon** PDF uncertainty well controlled.



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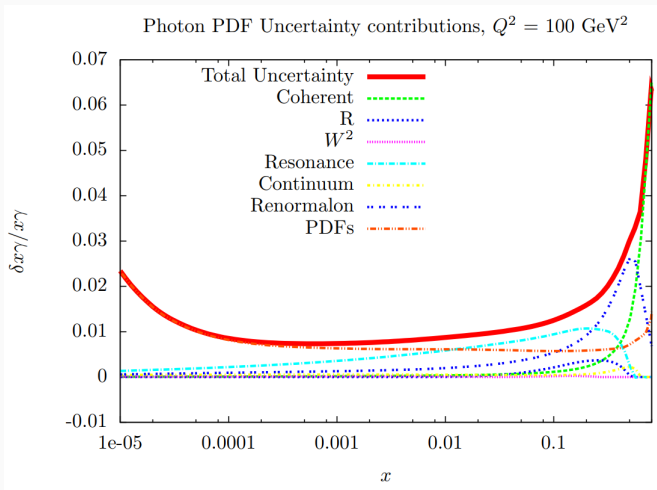
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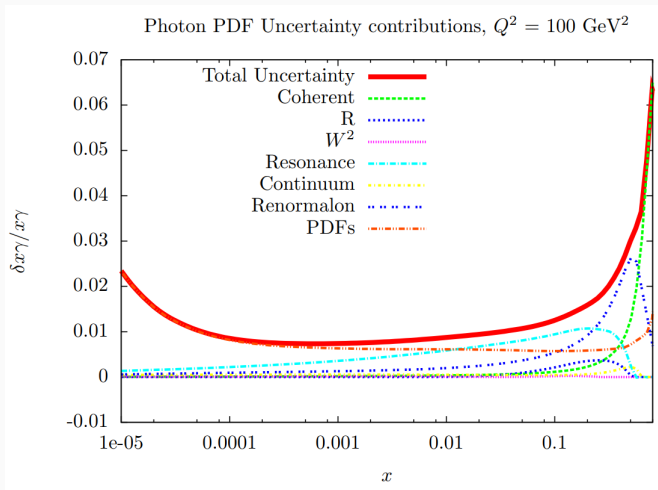
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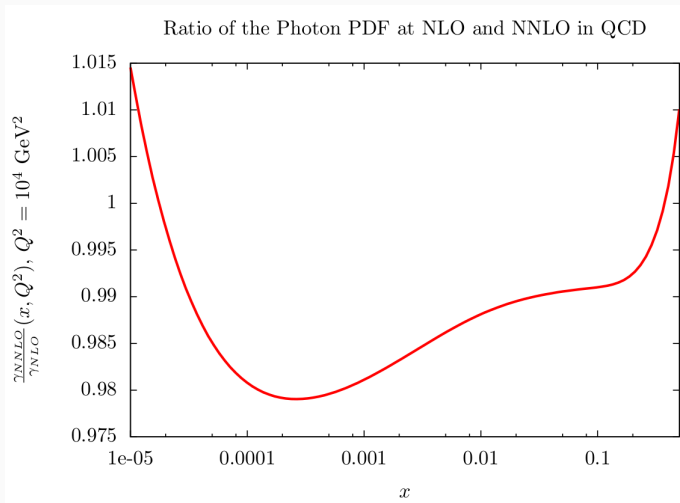
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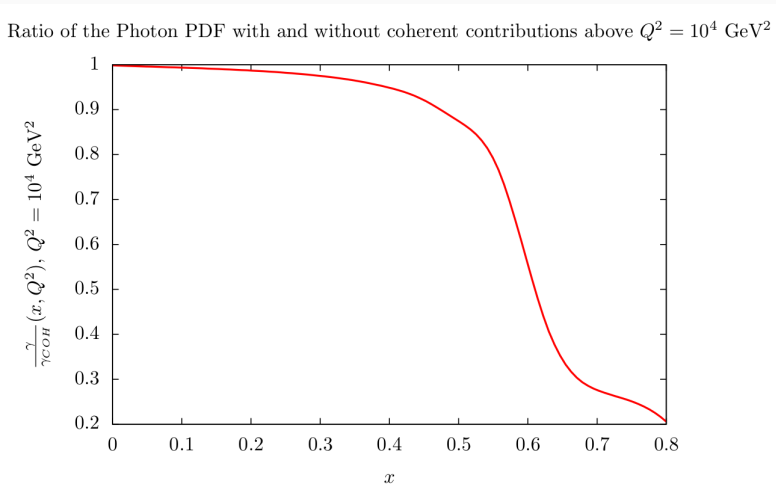
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- Model parameters in the Renormalon contributions given a conservative uncertainty estimate.

NNLO vs NLO

Percent level differences when using different orders of QCD in **DGLAP**.

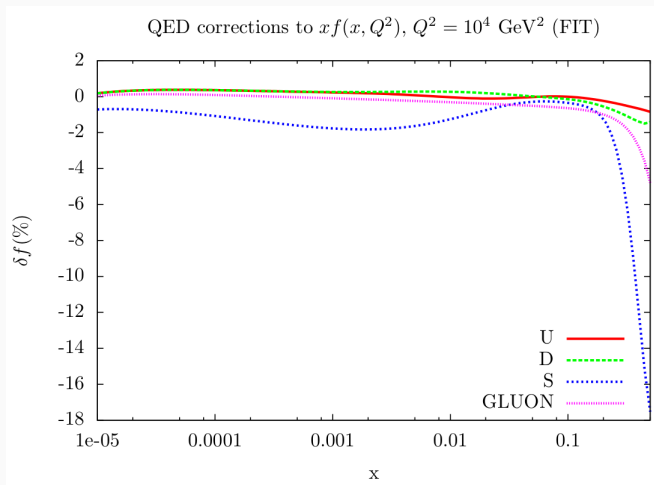


Significant contributions at high x from $\gamma_{COHERENT}$



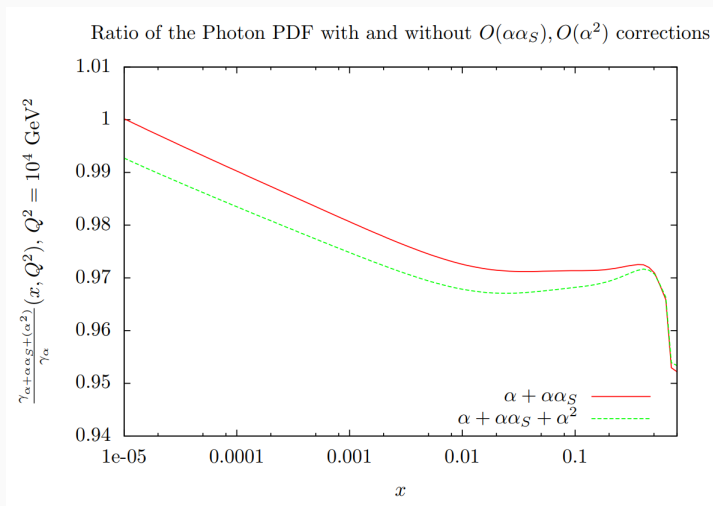
Measured effects on quarks

Calculated the effects of QED on **parton** momenta within the proton.



Higher orders

Included mixed order $\mathcal{O}(\alpha_S\alpha)$ and $\mathcal{O}(\alpha^2)$ corrections.



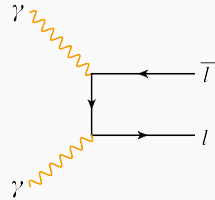
Phenomenological Importance

Anticipated experimental sensitivity to Electroweak correction

Process	Observable(s)	Estimated % difference(s)
Low mass W/Z production	Charge asymmetry, dilepton mass uncertainty	$\sim 1\%, \sim 3\%$
High mass VV production	WW pair production rate	$\sim 2\%$
Higgs + W	Differential P_T Higgs distribution	$\sim \mathbf{10\%}$
High mass Drell-Yan	Dilepton mass spectrum	$\sim \mathbf{1-16\%}$
Higgs production via VBF	γ induced cross section	$\sim 1\%$
Top pair production	Total, differential cross sections	$\sim 2\%, \sim 10\%$

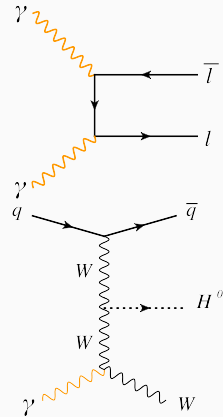
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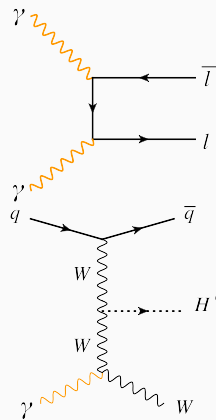
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Inclusion of **QED** can give up to 1 - 2% reduction in **(QCD) PDFs**.

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- To be released later this year.

**Thank you for your attention.
Any questions?**

Backup: Neutron details

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What do we mean by QED evolved?

$$\gamma(x, \mu^2)_{q,p} = \int_{Q_0^2}^{\mu^2} \frac{\alpha(Q^2)}{2\pi} \frac{dQ^2}{Q^2} \int_x^1 \frac{dz}{z} \left(e_q^2 \tilde{P}_{\gamma,q}(z) \mathbf{q}\left(\frac{x}{z}, Q^2\right) \right)$$

Contribution to the evolved photon from a specific quark flavour. Isospin assumed but inherently broken by QED, accuracy assured only to $\mathcal{O}(\alpha)$.

Strange, Charm and Bottom contributions taken as identical.

Backup: Neutron details

Elastic contributions to the neutron photon input use neutron structure function data from A1 collaboration.

Inelastic contributions approximated by the ratio of neutron to proton quarks at input.

$$rF_2 = \frac{4(d + \bar{d}) + (u + \bar{u}) + (s + \bar{s})}{4(u + \bar{u}) + (d + \bar{d}) + (s + \bar{s})}$$

Backup: Isospin violation

QED naturally introduces isospin violation: $\Delta d_V^{(n)} = d_V^{(n)} - u_V^{(p)}$

Assume isospin violating terms are proportional to quark contributions from QED driven DGLAP:

$$\Delta u_{V,n} = \epsilon \left(1 - \frac{e_u^2}{e_d^2}\right) \Delta d_{V,p}^{\text{QED}}$$

Constant of proportionality taken to preserve conservation of momentum:

$$\epsilon = \frac{\int_0^1 dx x (\gamma^p - \gamma^n)}{\int_0^1 dx x \left(\frac{3}{4} \Delta u_V^{\text{QED}} - 3 \Delta d_V^{\text{QED}}\right)} \quad (1)$$