Small-x Contributions of the Quark and Gluon Helicity to the Proton Spin

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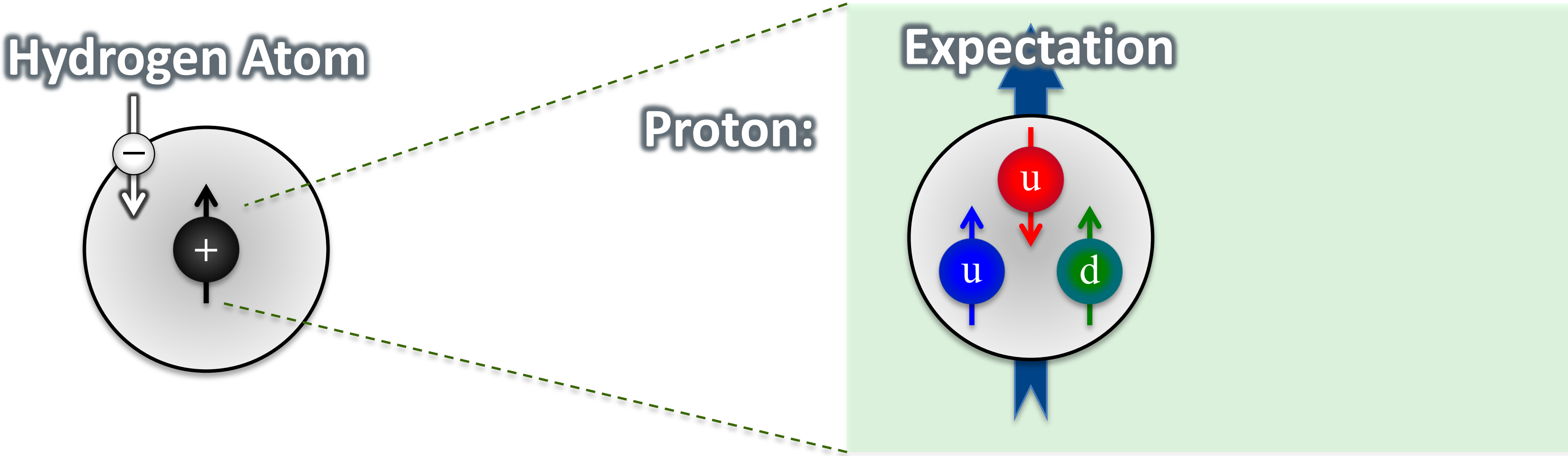
The Proton Spin: A Window to QCD



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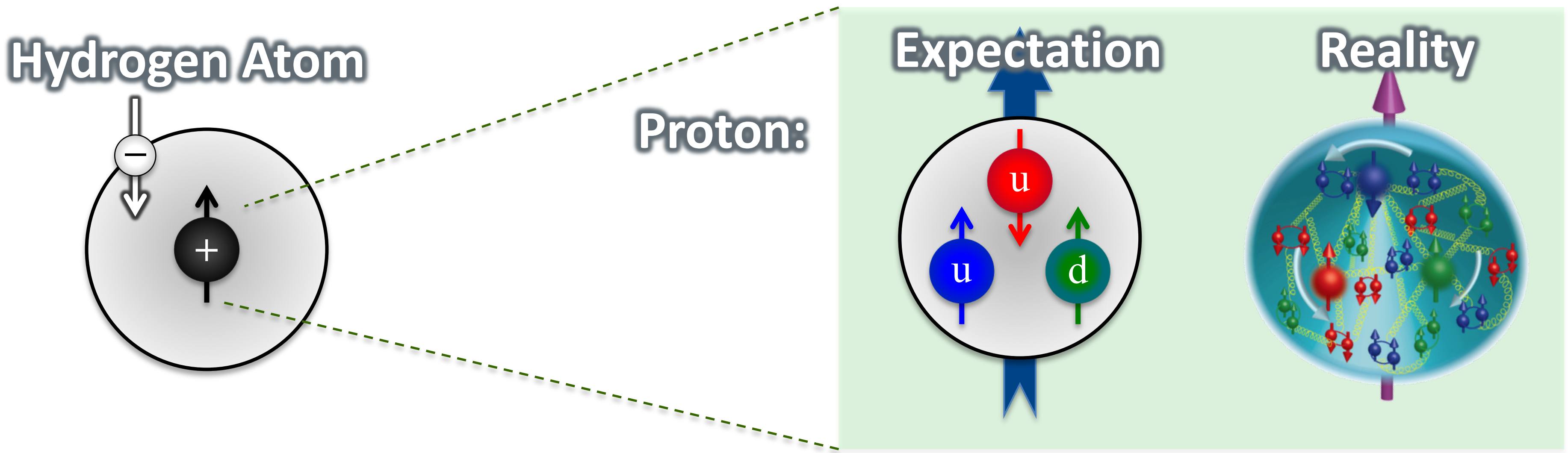

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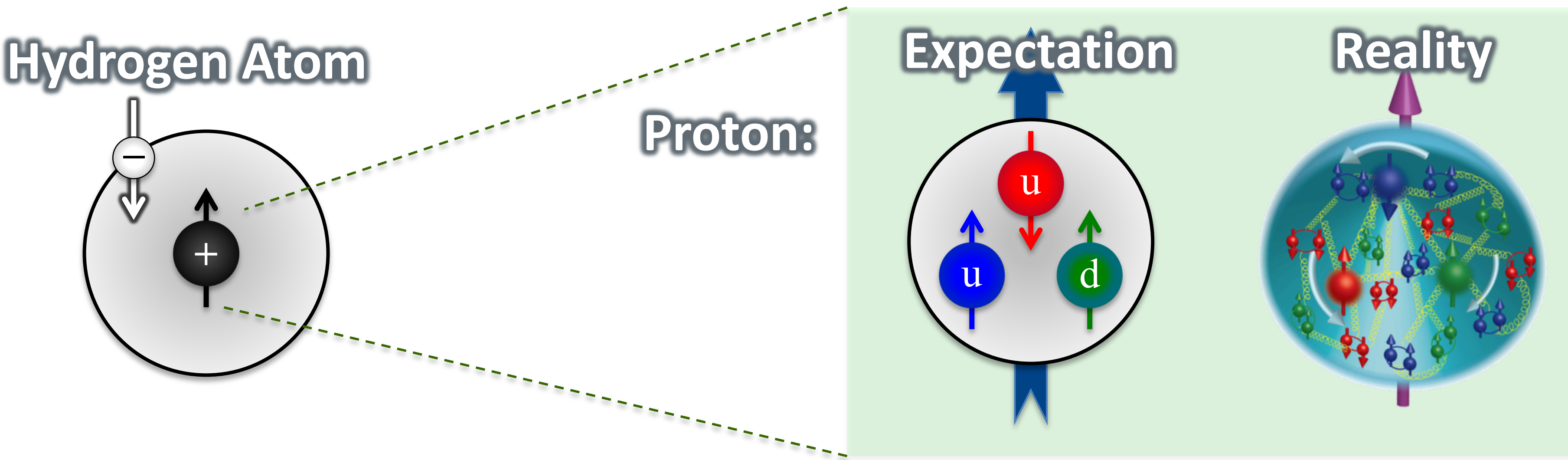

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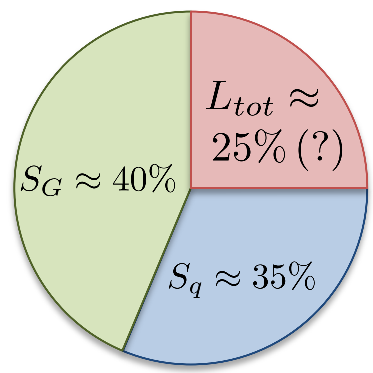

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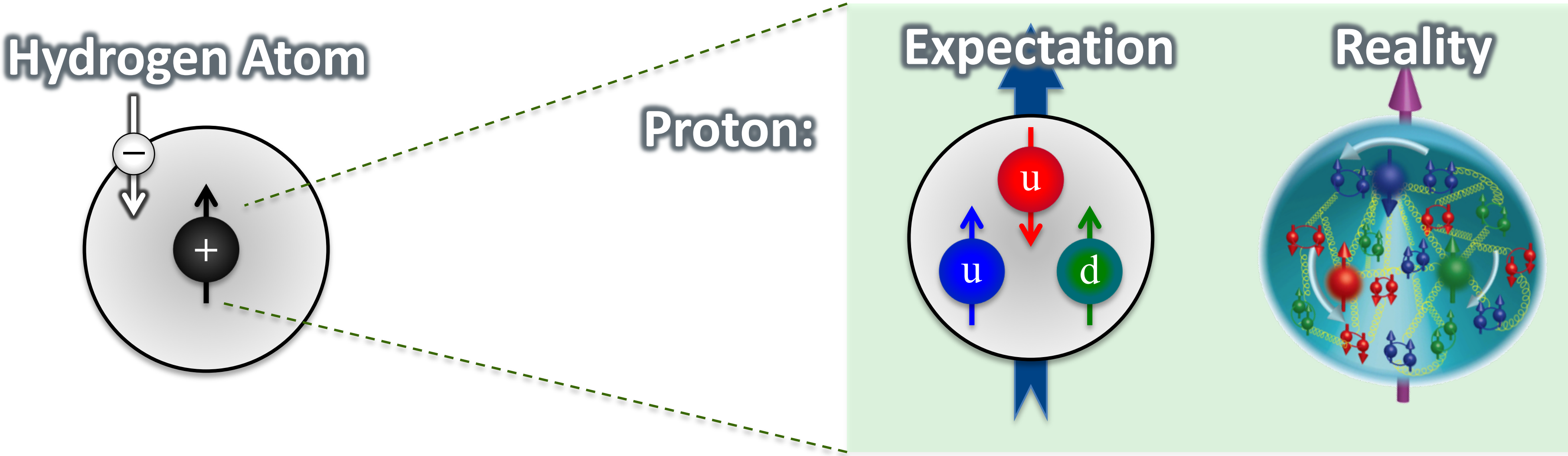

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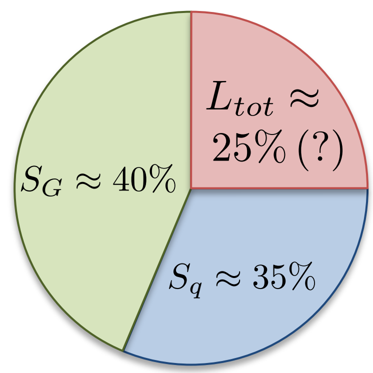

- These differences reflect the **richness of QCD**

The Proton Spin Budget in QCD

Jaffe and Manohar, Nucl. Phys. **B337** 509 (1990)

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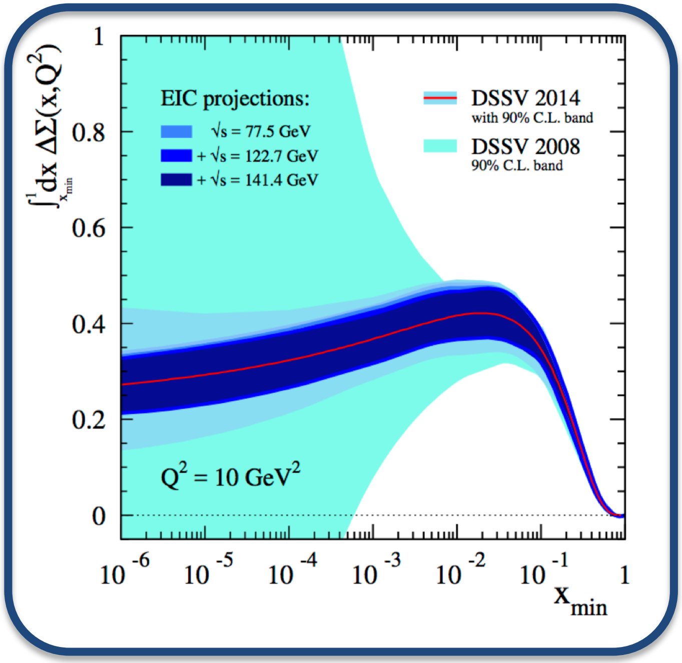
- Gluon Polarization: $S_G(Q^2) = \int_0^1 dx \, \Delta G(x, Q^2)$

$$\Delta G(x, Q^2) = \frac{-2i}{xp^+} \int \frac{dr^-}{2\pi} e^{ixp^+ r^-} \left\langle p S_L \right| \epsilon_T^{ij} \, \mathrm{tr} \left[F^{+i}(0) \, \mathcal{U}[0, r] \, F^{+j}(r) \, \mathcal{U}'[r, 0] \right] \left| p S_L \right\rangle$$

What Do We Know?

Quarks



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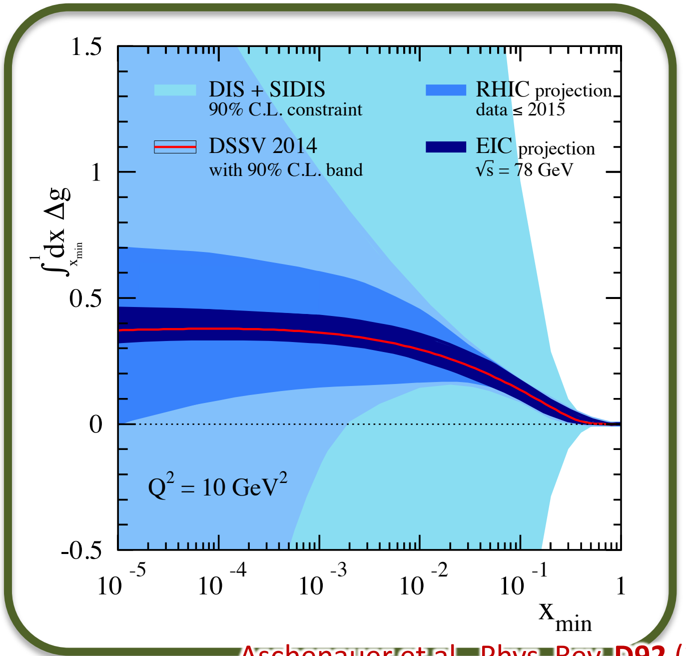

Aschenauer et al., Phys. Rev. **D92** (2015) no. 9 094030

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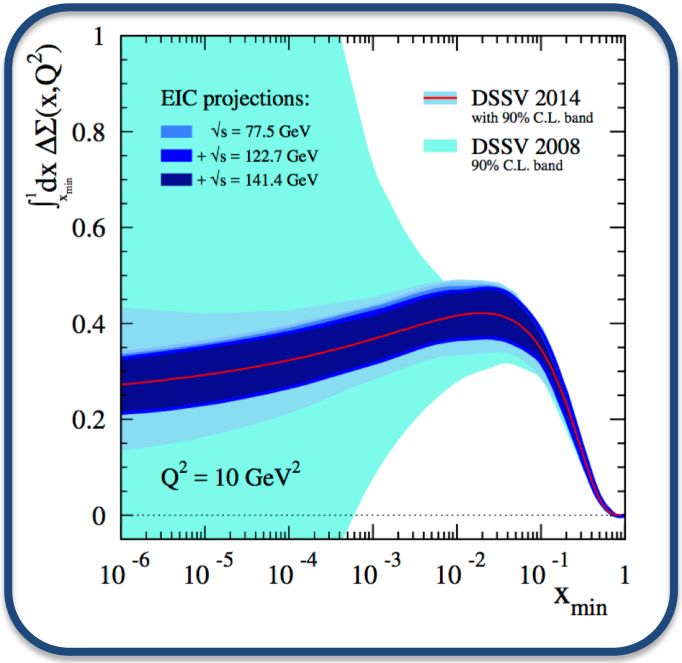

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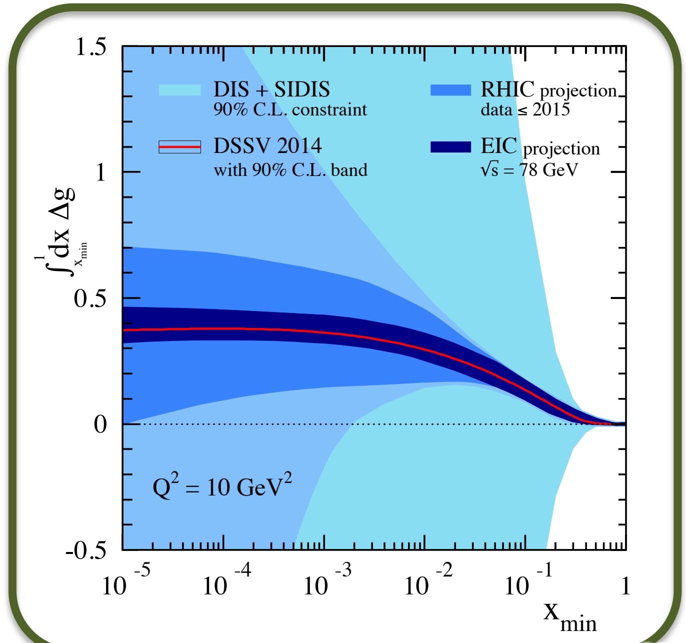

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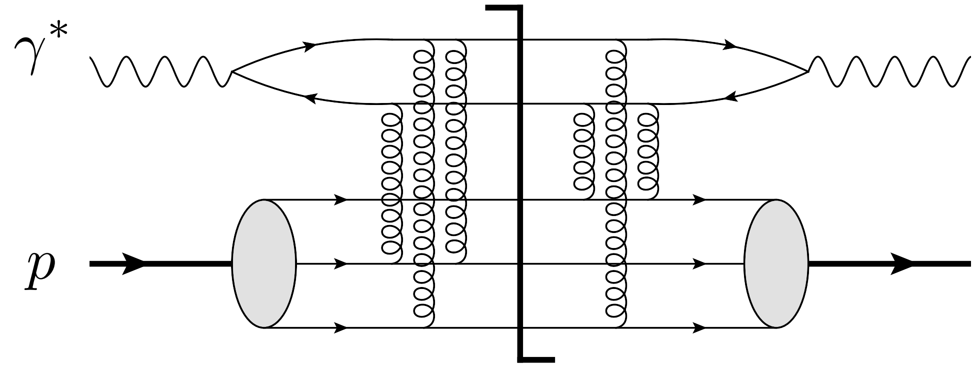
- Data constrains the polarization at **large x**
- Access to **low x** is always limited by **finite energy**
  - ➢ But need to integrate down to **x = 0**…
  - ➢ Need a theoretical basis to **extrapolate**

$$x \approx \frac{Q^2}{s}$$

PDFs and DIS at Small x

$$V_{\underline{x}} = \mathcal{P} \exp \left[ig \int dz^- \hat{A}^+(0^+, z^-, \underline{x}) \right]$$



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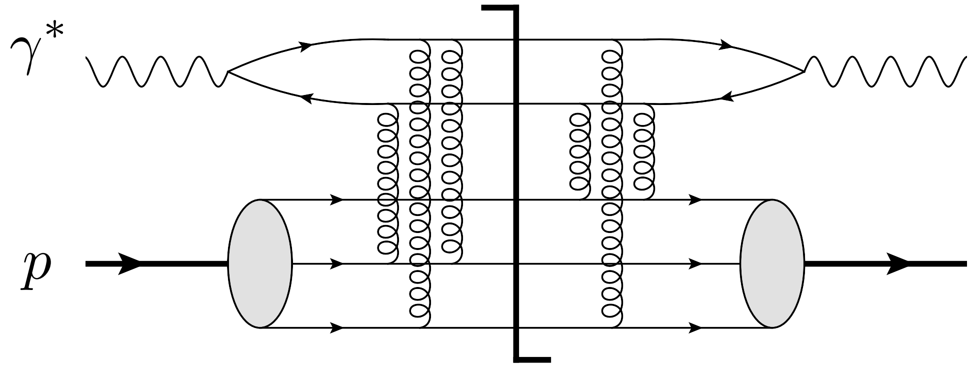
- Scattering described by **Wilson lines**: eikonal color rotations

$$V_{\underline{x}} = \mathcal{P} \exp \left[ig \int dz^- \, \hat{A}^+(0^+, z^-, \underline{x}) \right]$$

$$S_{10}(zs) \equiv \left\langle \frac{1}{N_c} \mathrm{tr}[V_{\underline{x}_0} V^\dagger_{\underline{x}_1}]_{(zs)} \right\rangle$$



$$x q_f(x, Q^2) \stackrel{L.O.}{=} \frac{Q^2 N_c}{2\pi^2 \alpha_{EM}} \int \frac{d^2 x_{10} \, dz}{4\pi z(1-z)} \sum_{L,T} \left| \Psi_f(x_{10}^2, z) \right|^2 \int d^2 b_{10} \, (1 - S_{10}(zs))$$

- At small-x kinematics, the proton is highly **Lorentz-contracted**
- Scattering described by **Wilson lines**: eikonal color rotations
- Natural degrees of freedom: **color dipoles** / quadrupoles / etc.

QCD Predicts the Small x Spectrum

- High energies open a **large logarithmic phase space** for soft radiation

$$\alpha_s \ll 1$$

$$\Delta Y \sim \ln \frac{s}{Q^2} \sim \ln \frac{1}{x} \gg 1$$

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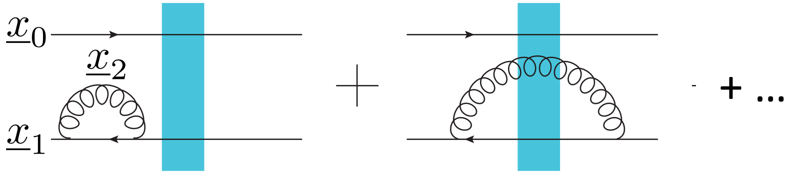

$$S_{10}(zs) = S_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2x_2 \left(\frac{1}{x_{21}^2} - 2\frac{\underline{x}_{21} \cdot \underline{x}_{20}}{x_{21}^2\, x_{20}^2} + \frac{1}{x_{20}^2} \right)$$

$$\times \left[\frac{1}{N_c^2} \left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^\dagger]\, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^\dagger] \right\rangle_{(z's)} - S_{10}(z's) \right]$$

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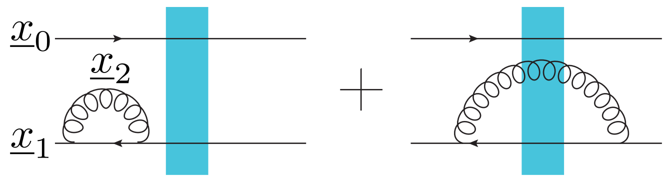

+ ...

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- Leading-log **resummation** leads to **power-law growth**

$$\alpha_s \ln \frac{1}{x} \sim 1$$

$$xq(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_P - 1}$$

  - ➢ Onset of the **high-density** regime

Polarization at Small x

- The **leading small-x behavior** is the radiation of soft **unpolarized gluons**
  - ➢ What is the spectrum of **polarized** radiation enhanced by a large logarithmic phase space?

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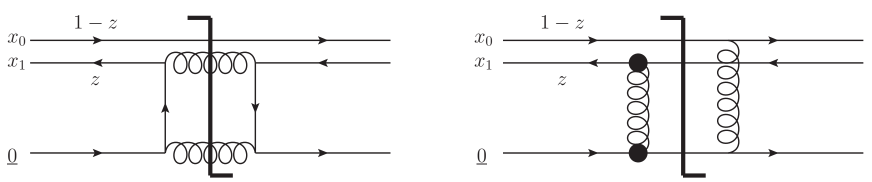
- Polarization transfer is **suppressed by one power** at tree level.

$$\frac{d\,\Delta\sigma^{Born}}{d^2b} \sim x \sim \frac{1}{s}$$

  - ➢ Sensitive to **quark and gluon** exchange



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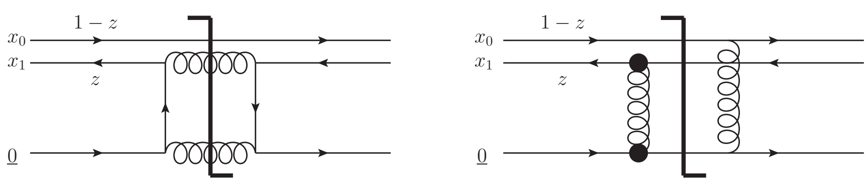
  - ➢ Resummation can lead to **enhancement**, not growth



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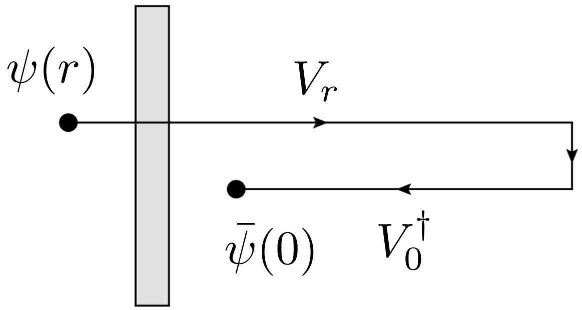
Dipole Degrees of Freedom

$$g_{1L}^{q}(x,k_T^2) = \frac{1}{(2\pi)^3}\,\frac{1}{2}\sum_{S_L} S_L \int d^2r\,dr^-\; e^{ixP^+r^-}\; e^{-i\underline{k}\cdot\underline{r}}\,\langle P,S_L|\,\bar{\psi}(0)\,\mathcal{U}[0,r]\,\frac{\gamma^+\gamma^5}{2}\,\psi(r)\,|P,S_L\rangle_{r^+=0}\,,$$



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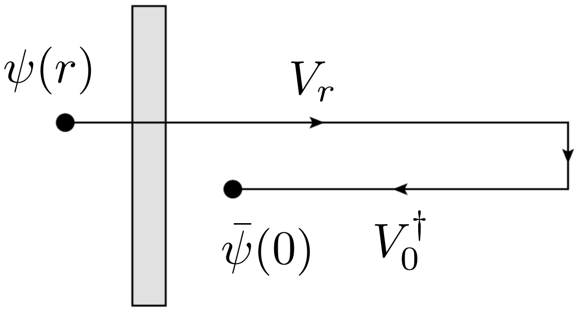
$$A^- = 0$$



$$\mathcal{U}[0, r] = V_{0_\perp}[0^-, \infty^-] \, V_{r_\perp}[\infty^-, r^-]$$

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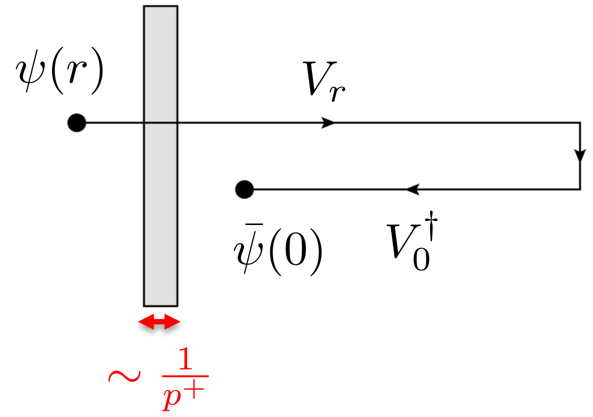

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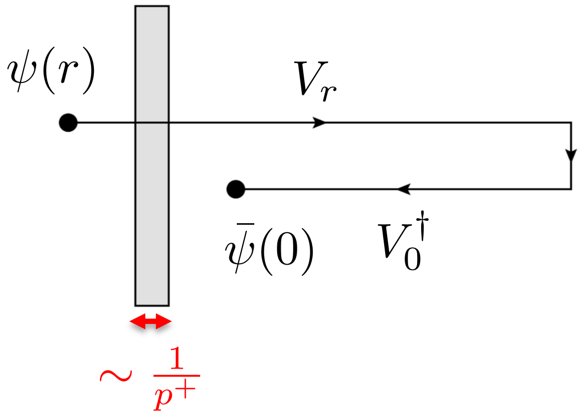

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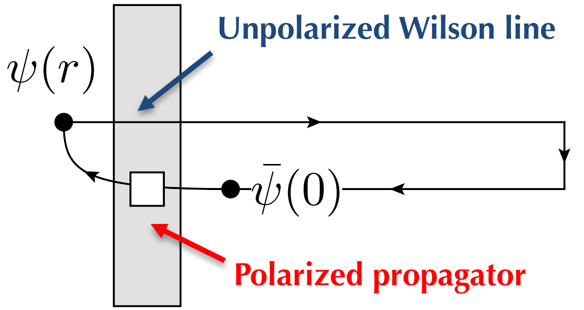
- Dominant contribution: **polarized propagator** in the background fields

  ➢ Dipole DIS



"Polarized Wilson Lines"

$$g_1^S(x,k_T^2) = \frac{8N_c}{(2\pi)^6}\sum_f \int \frac{dz}{z}\int d^2x_0\, d^2x_{0'}\, d^2x_1\, e^{-i\vec{k}_\perp\cdot(\vec{x}_{0\perp}-\vec{x}_{0'\perp})}$$

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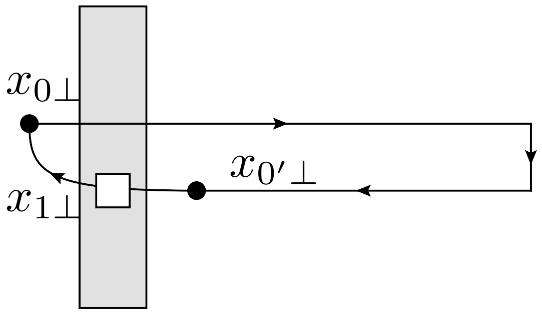

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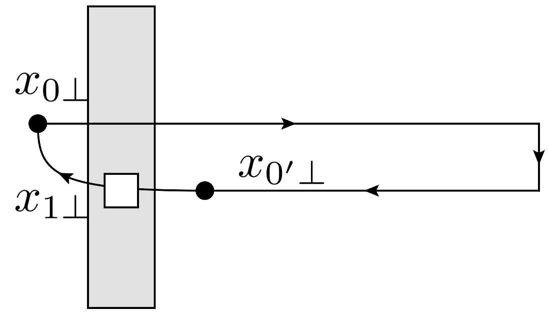

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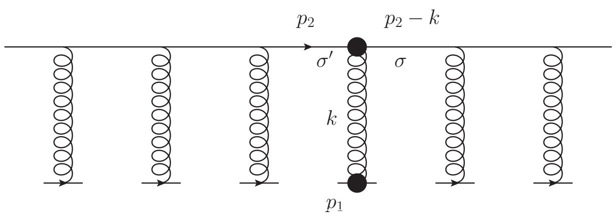
  - Polarized **gluon** exchange:

$$V_{\underline{x}}^{pol} = \frac{igp_1^+}{s} \int\limits_{-\infty}^{\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, F^{12}(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$



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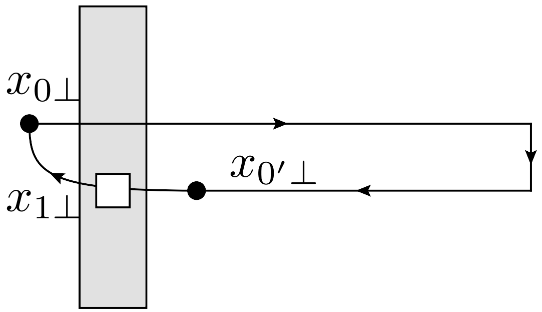

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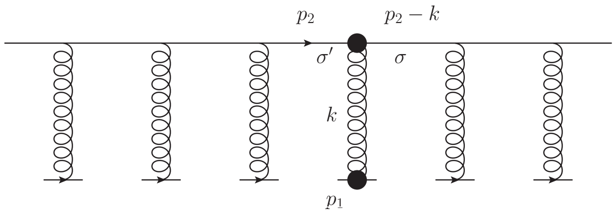
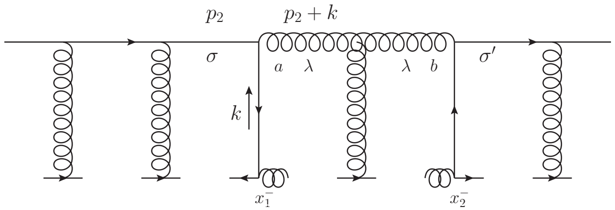
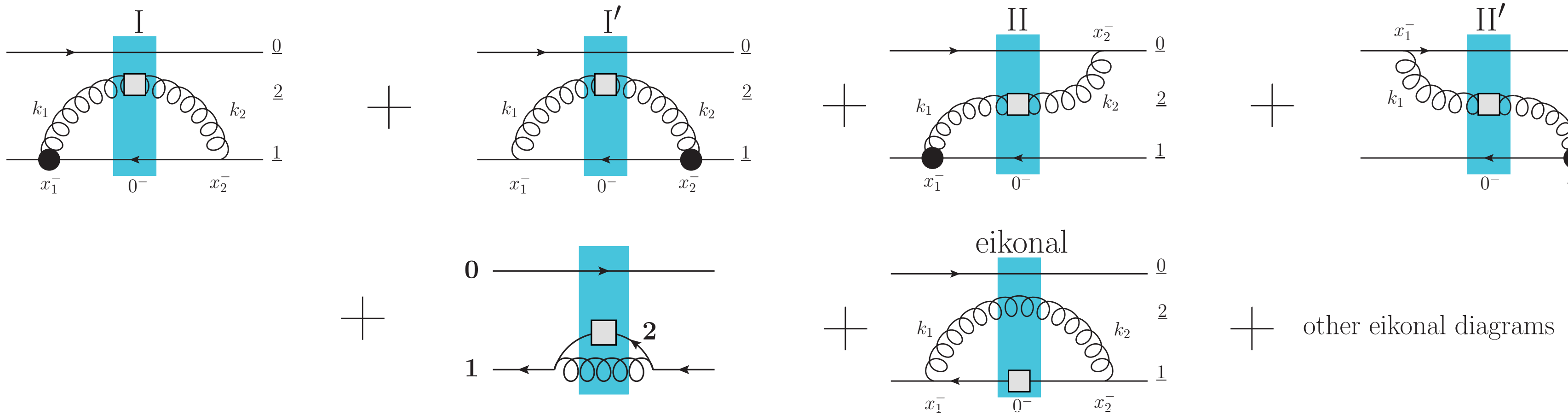


  - ➢ Polarized **quark** exchange (new):



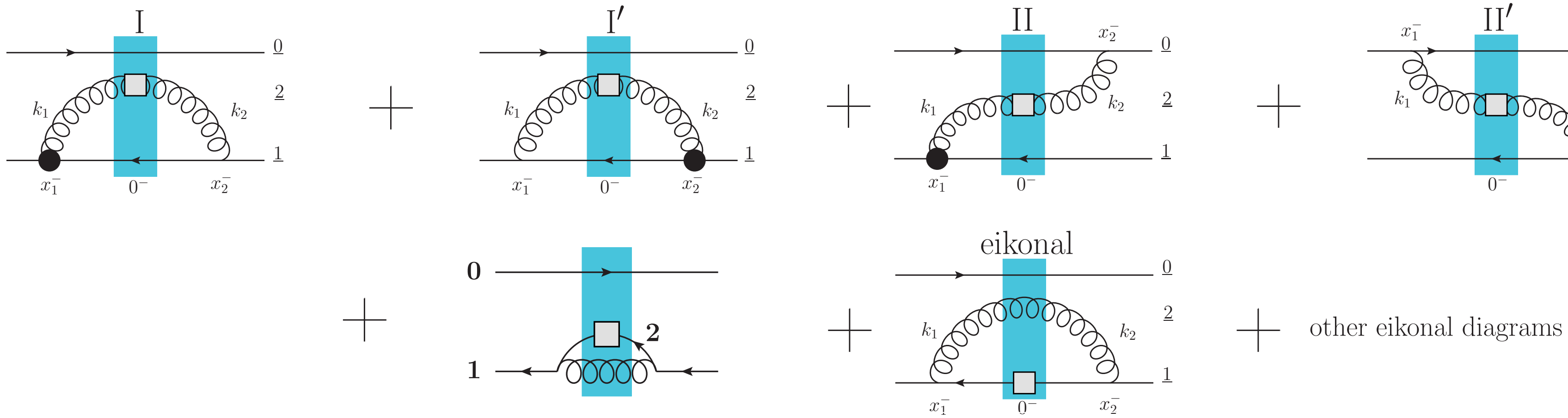
$$-\frac{g^2\,p_1^+}{s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^-\ \ V_{\underline{x}}[+\infty, x_2^-]\ t^b\ \psi_\beta(x_2^-, \underline{x})\ U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\frac{1}{2}\gamma^+\gamma^5\right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x})\ t^a\ V_{\underline{x}}[x_1^-, -\infty].$$



- **Arbitrary** division between **"fast quantum"** modes and **"slow classical"** modes
  - ➢ **RG evolution** in rapidity regulator

$$A^\mu(x) = A^\mu_{cl}(x) + a^\mu(x)$$

Evolution: the Background Field Method



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  - ➢ **RG evolution** in rapidity regulator
  - ➢ **Expand to lowest order** in quantum fields

$$\text{I}: \quad \mathrm{tr}\left[V_{\underline{0}}\ V_{\underline{1}}[-\infty, x_1^-]\ \underline{\nabla} \times \underline{a}(x_1^-, \underline{x}_1)\ V_{\underline{1}}[x_1^-, \infty]\right]$$

$$\text{II} + \text{II}': \quad \mathrm{tr}\left[V_{\underline{0}}\ V_{\underline{1}}[-\infty, x_1^-]\ \underline{\nabla} \times \underline{a}(x_1^-, \underline{x}_1)\ V_{\underline{1}}[x_1^-, \infty]\right]$$

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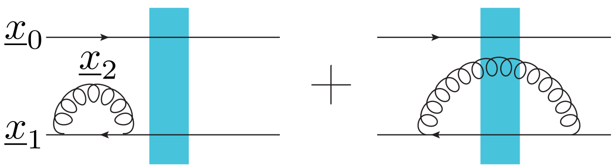
+ …

- Compare eikonal gluon radiation for **unpolarized PDFs**



+ "c.c."

$$\int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \left(\frac{1}{x_{21}^2} \right) \left\langle \frac{1}{N_c^2} \mathrm{tr}[V_{\underline{0}} V_{\underline{2}}^\dagger] \mathrm{tr}[V_{\underline{2}} V_{\underline{1}}^\dagger] - \frac{1}{N_c} \mathrm{tr}[V_{\underline{0}} V_{\underline{1}}^\dagger] \right\rangle$$

Emergence of Double Logarithms

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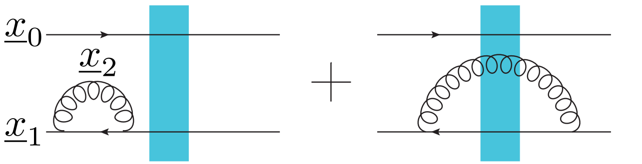

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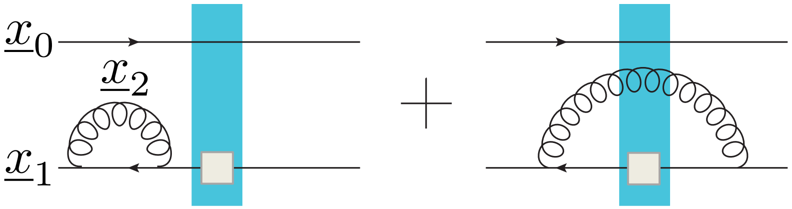
➢ Versus **polarized PDFs:**



+ "c.c."

$$\int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \left(\frac{1}{x_{21}^2}\right) \left[\frac{1}{N_c^2} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^{pol\,\dagger}]\, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^{\dagger}] \right\rangle\!\!\right\rangle_{(z's)} - \frac{1}{N_c} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^{pol\,\dagger}] \right\rangle\!\!\right\rangle_{(z's)} \right]$$

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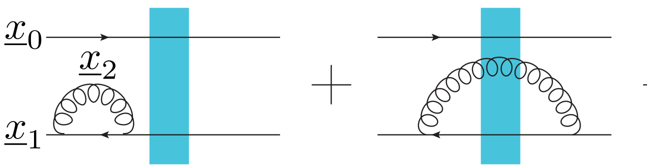

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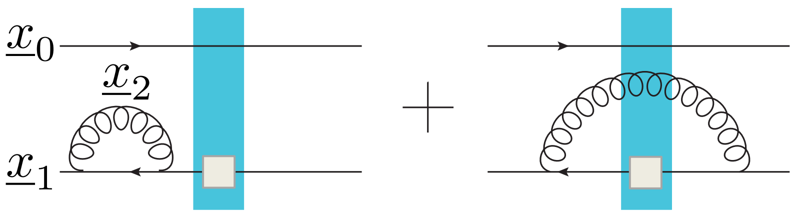

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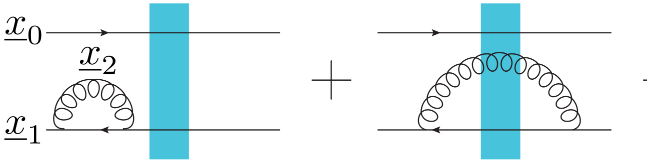

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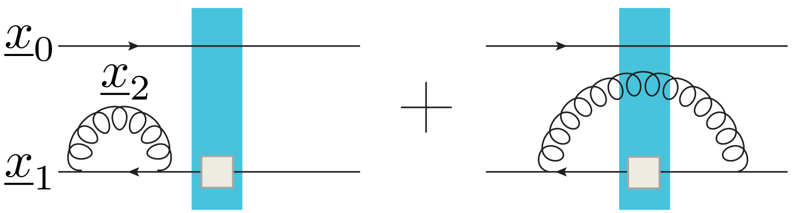

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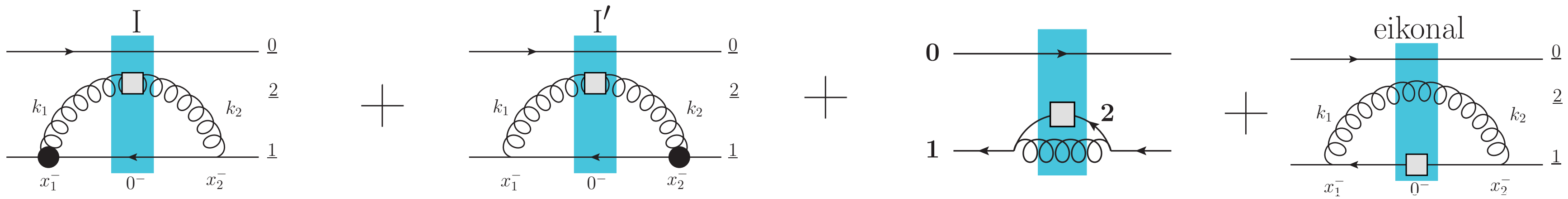
- Generates **double** logarithms: $\alpha_s \ln^2 \frac{1}{x}$

The Double-Logarithmic Phase Space



- **Double logarithmic phase space** is more sensitive to **details of the transverse plane**

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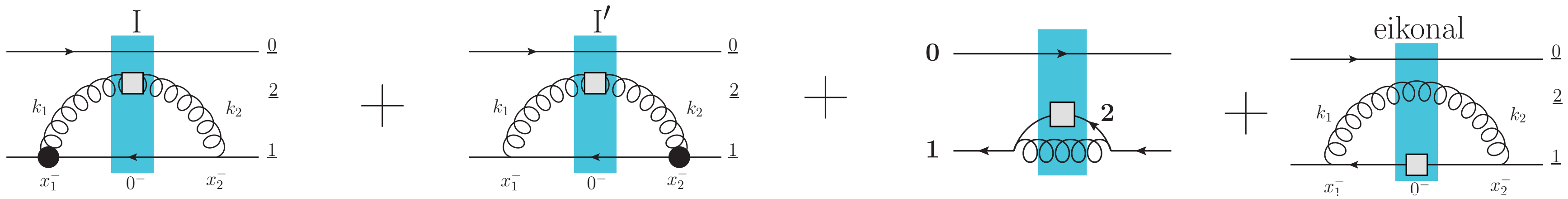

- **Double logarithmic phase space** is more sensitive to **details of the transverse plane**

$$\tau_i \propto z_i \Delta x_i^2 \qquad\qquad \tau_i \gg \tau_{i+1} \quad \Rightarrow \quad z_i \Delta x_i^2 \gg z_{i+1} \Delta x_{i+1}^2$$

  - **Lifetime ordering**, normally NLO, is a leading effect

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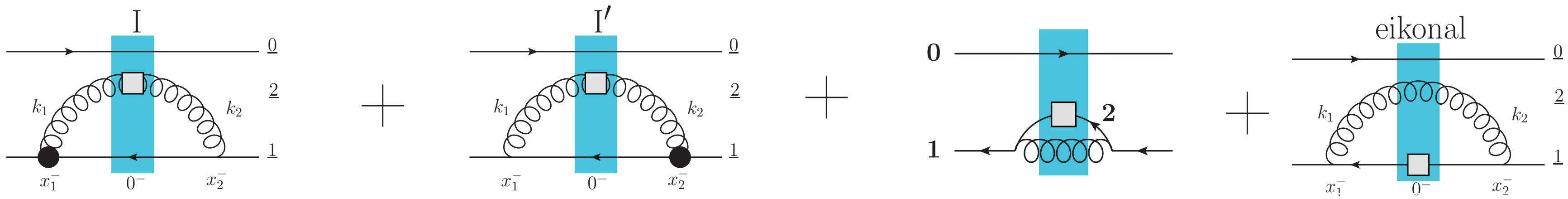

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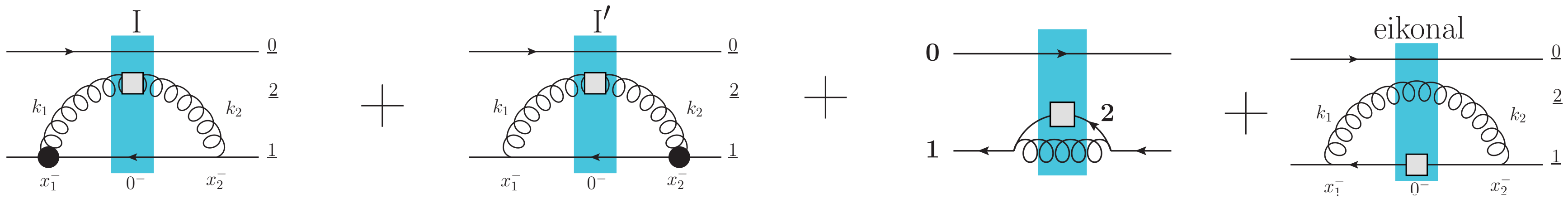

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  - ➢ **Lifetime ordering**, normally NLO, is a leading effect
  - ➢ Evolves toward **DGLAP-like phase space**: $\Delta x_{i+1}^2 \ll \Delta x_i^2$

- Also restricts leading region to **linear evolution**
  - ➢ Drive toward **small distances**
  - ➢ **Nonlinear (saturation) effects** destroy transverse logarithm
  - ➢ Double-log evolution is analogous to **polarized BFKL**

Closed Equations at Large Nc

- **Operator hierarchy**: closes with **large Nc** or **large Nc + Nf**

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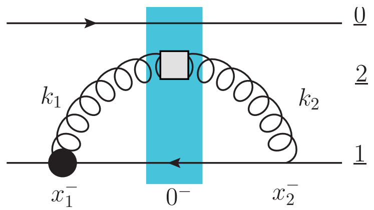

$\Rightarrow$

Constrained by lifetime of x_{21}

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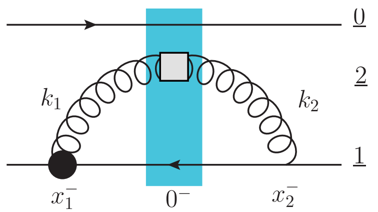

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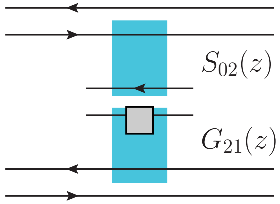

Constrained by lifetime of x_{21}

Further evolution of x_{21} dipole

Closed Equations at Large Nc

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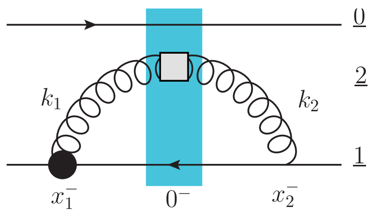

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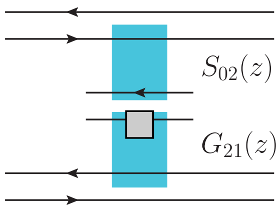

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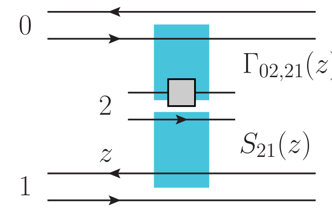
+



Evolution of x_{02} dipole is constrained by x_{21}

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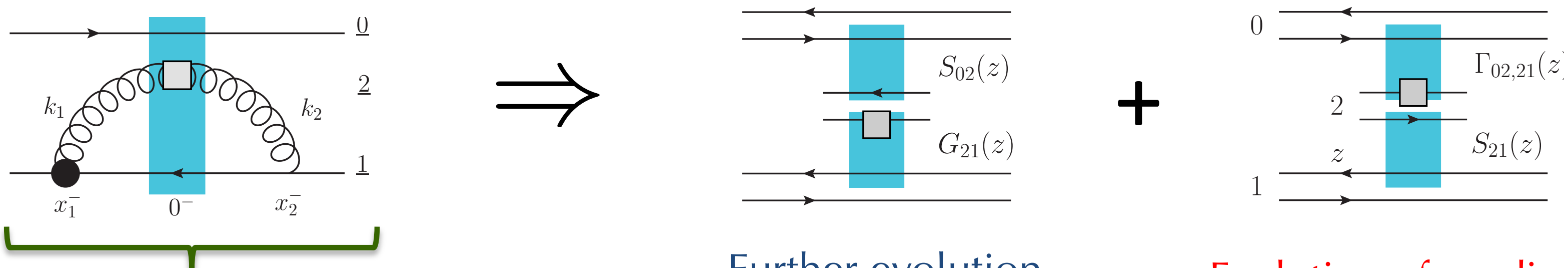


Constrained by lifetime of x_{21}

Further evolution of x_{21} dipole

Evolution of x_{02} dipole is constrained by x_{21}

- Even at large-Nc, leads to a **system of coupled equations** with **auxiliary "neighbor dipole" functions**

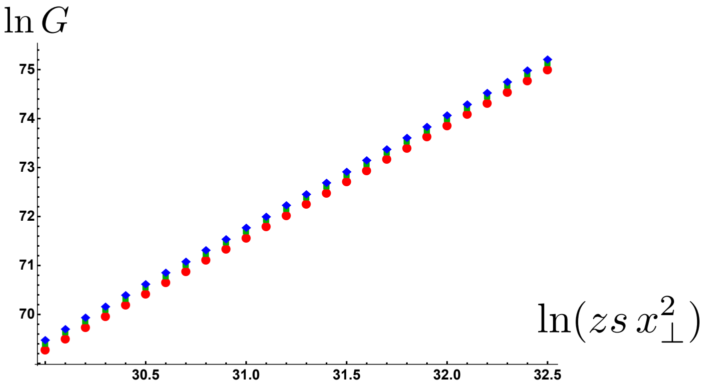
$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3 G(x_{21}^2, z') \right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = G^{(0)}(x_{10}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min[x_{10}^2, x_{21}^2 \frac{z'}{z''}]} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3 G(x_{32}^2, z'') \right]$$

Solution: the Quark Helicity Intercept

- After a few units of evolution, an **emergent scaling** sets in:

$$G(x_\perp^2, zs) \sim G(zs\, x_\perp^2)$$



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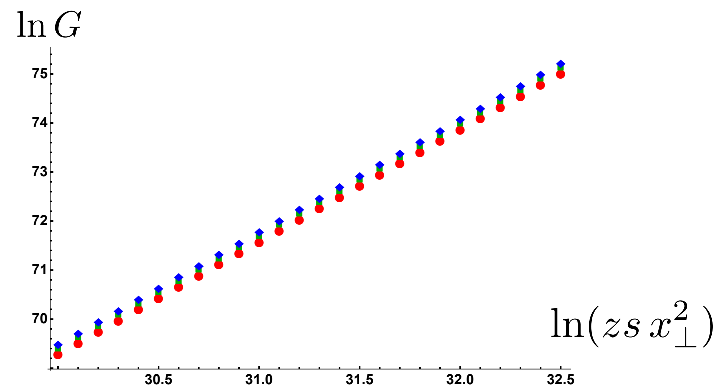

➢ Observation: characteristic of a **collinear regime**

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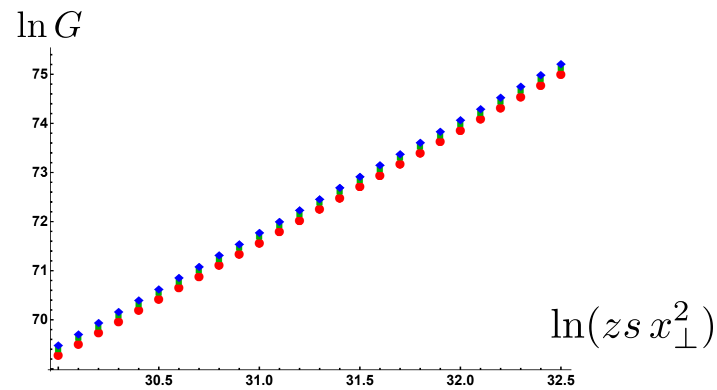

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➢ With scaling, the large-Nc evolution can be solved analytically using **Laplace-Mellin techniques:**

$$\left.\begin{aligned} G(x_\perp^2, zs) &\sim (zs)^{\alpha_h^q} \\ g_1(x, k_T^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^q} \\ \Delta q(x, Q^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^q} \end{aligned}\right\}$$

$$\boxed{\alpha_h^{q,S} = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}}} \qquad \boxed{\alpha_h^{q,NS} = \sqrt{2} \sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

Gluon Helicity Operators

$$g_{1L}^G(x, k_T^2) = \frac{-2i}{x\,P^+} \int \frac{d\xi^- \, d^2\xi}{(2\pi)^3} \; e^{ik\cdot\xi} \; \langle P, S_L | \, \epsilon_T^{ij} \, \mathrm{tr} \left[F^{+i}(0) \, \mathcal{U}[0,\xi] \, F^{+j}(\xi) \, \mathcal{U}'[\xi, 0] \right] | P, S_L \rangle_{\xi^+=0}$$

- There are **multiple gluon distributions**, corresponding to different choices of **gauge links**

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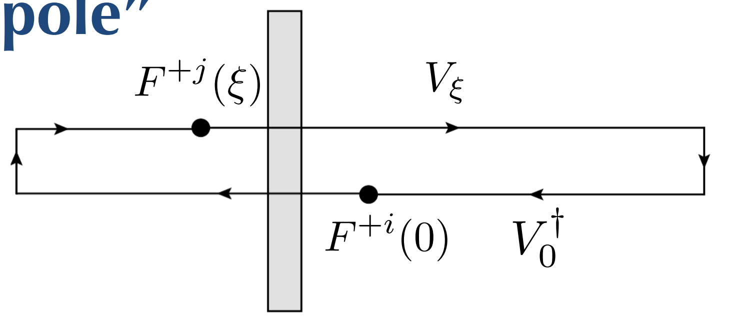
"Dipole"



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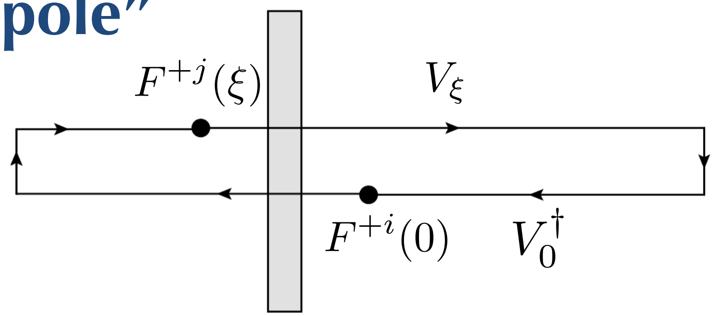

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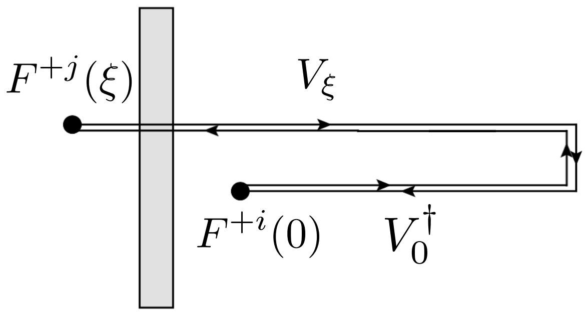

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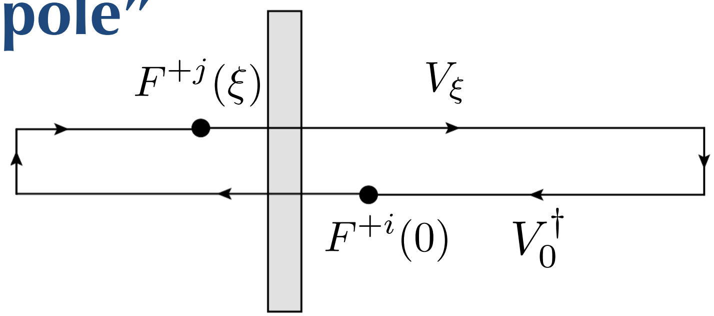

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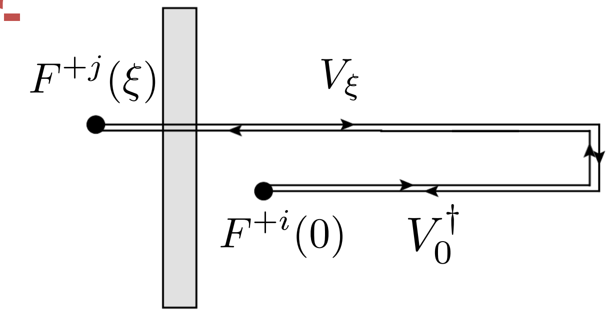

$(A^- = 0)$

- **Expand** the field-strength operators to **lowest nonvanishing order**:

$$F^{+i} = \underbrace{\partial^+ A_\perp^i}_{\text{Sub-Eikonal}} - \underbrace{\partial^i A^+}_{\text{Eikonal}} - \underbrace{ig[A^+, A_\perp^i]}_{\text{Sub-Eikonal}}$$

➢ Leads to **different operators** for dipole vs. WW gluon helicity

Gluon Helicity Operators at Small x

- **Dipole** gluon helicity:

$$g_{1L}^{G\,dip}(x,k_T^2) = \frac{-4i}{g^2(2\pi)^3} \int d^2x_{10}\, d^2b_{10}\;\; e^{+i\underline{k}\cdot\underline{x}_{10}}\; k_\perp^i \epsilon_T^{ij} \left\{ \left\langle \mathrm{tr}\left[V_{\underline{0}}\, (V_{\underline{1}}^{pol\,\dagger})_\perp^j \right] \right\rangle + \mathrm{c.c.} \right\}$$

$$(V_{\underline{x}}^{pol})_\perp^i \equiv \int\limits_{-\infty}^{+\infty} dx^-\, V_{\underline{x}}[+\infty, x^-]\; \big(ig\, P^+\, A_\perp^i(x) \big)\; V_{\underline{x}}[x^-, -\infty]$$

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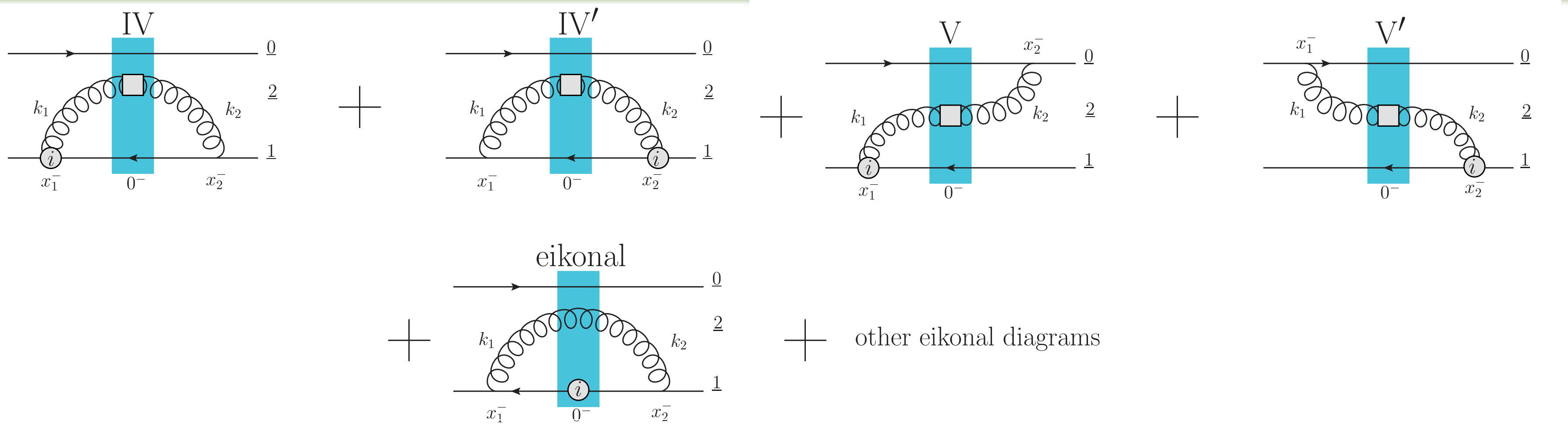
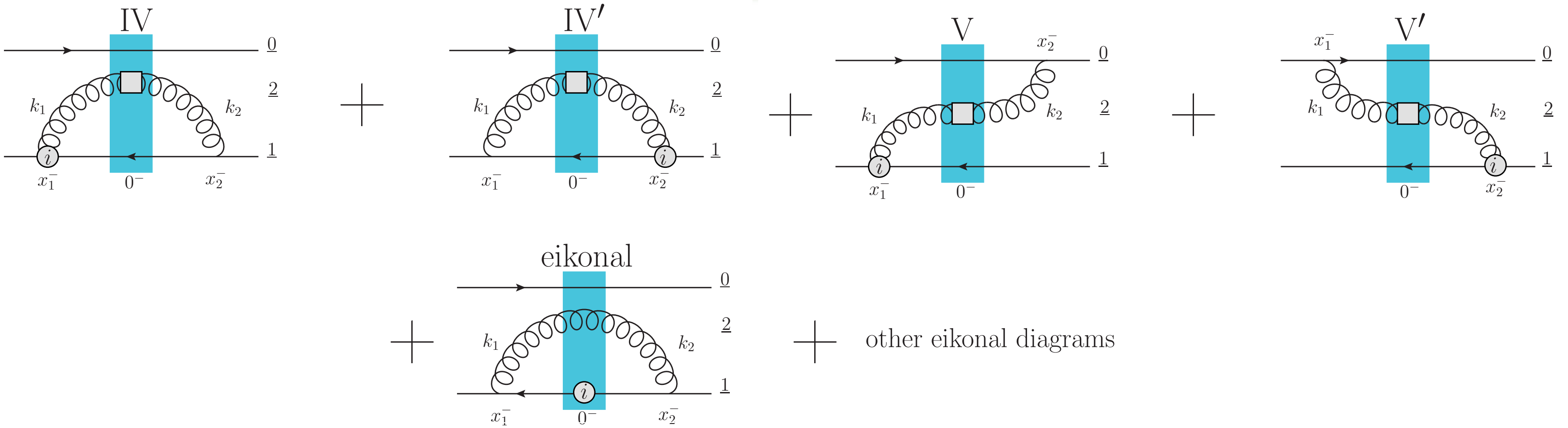
- Requires azimuthal correlations to **survive multiple scattering**
  - ➢ C.F. – **Axial vector current** for quark helicity
  - ➢ It is possible for gluon polarization to get **washed out**

Evolution: Dipole Gluon Helicity



- In this evolution, there is an **external direction** present

Evolution: Dipole Gluon Helicity



- In this evolution, there is an **external direction** present

$$(\delta G^i_{10})_{\mathrm{IV}}(zs) = (\delta G^i_{10})_{\mathrm{IV}'}(zs) = \frac{\alpha_s N_c}{4\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij} (x_{21})^j_\perp}{x_{21}^2} \left\langle\!\!\left\langle \frac{1}{N_c^2} \mathrm{tr} \left[V_{\underline{0}} t^a V_{\underline{1}}^\dagger t^b \right] \, (U_{\underline{2}}^{pol})^{ba} + \mathrm{c.c.} \right\rangle\!\!\right\rangle (z's)$$

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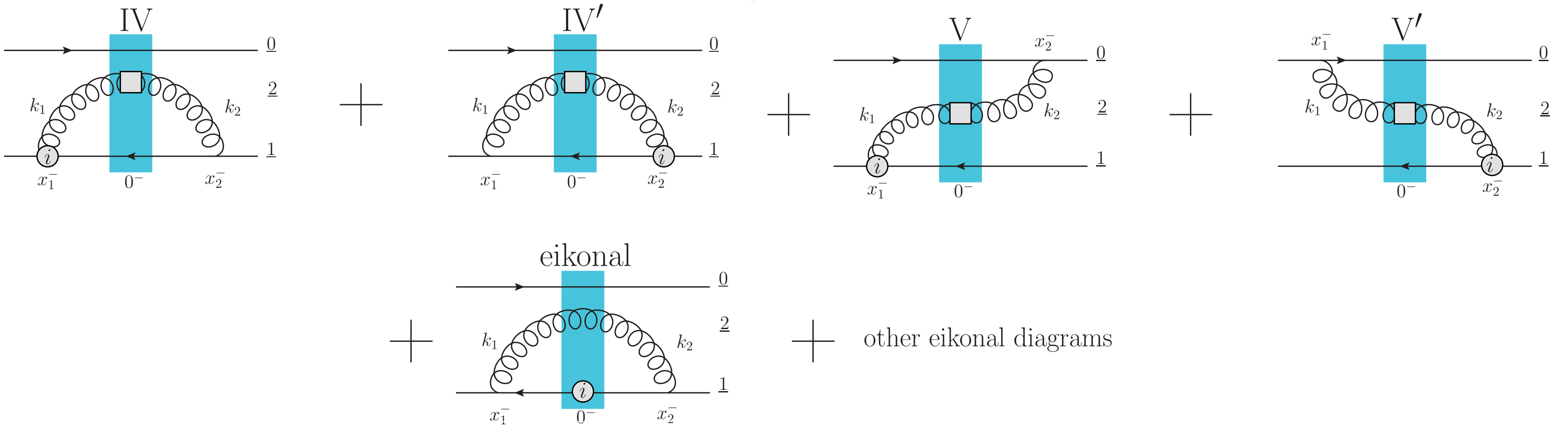

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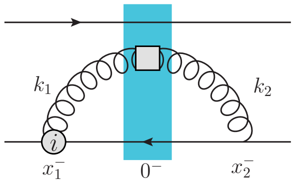
- ➢ **Quark helicity evolution** mixes into gluon helicity, but the transition is **not double-logarithmic**.

Leading Regions for Dipole Gluon Helicity



- Mixes in **quark helicity evolution**
- **Single-logarithmic** transition

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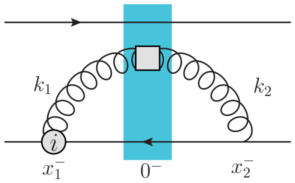

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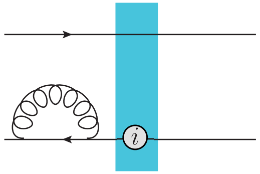
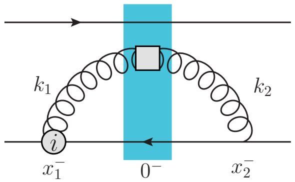


- Preserves **dipole gluon helicity** evolution operator
- **Double-logarithmic** evolution

Leading Regions for Dipole Gluon Helicity



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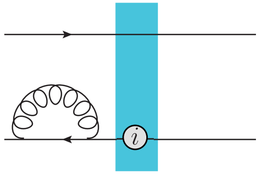

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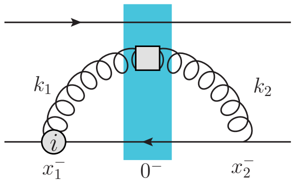

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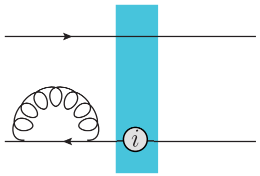

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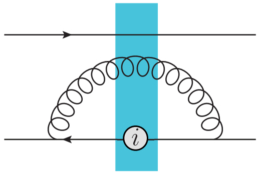

- **Unpolarized** small-x emissions are **isotropic**
- Real emissions **wash out directional correlations**
- **Depletes gluon helicity** during evolution

Dipole Gluon Helicity at Large Nc

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) - \left(\frac{\alpha_s N_c}{3\pi} \frac{1}{\alpha_h^q} G_0 \right) \left(zs \, x_{10}^2 \right)^{\alpha_h^q} \ln \frac{1}{x_{10}\Lambda}$$

$$- \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \, \Gamma_2(x_{10}^2, x_{21}^2, z's),$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) - \left(\frac{\alpha_s N_c}{3\pi} \frac{1}{\alpha_h^q} G_0 \right) \left(z's \, x_{10}^2 \right)^{\alpha_h^q} \ln \frac{1}{x_{10}\Lambda}$$

$$- \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left[x_{10}^2 \, , \, x_{21}^2 \frac{z'}{z''} \right]} \frac{dx_{31}^2}{x_{31}^2} \, \Gamma_2(x_{10}^2, x_{31}^2, z''s).$$

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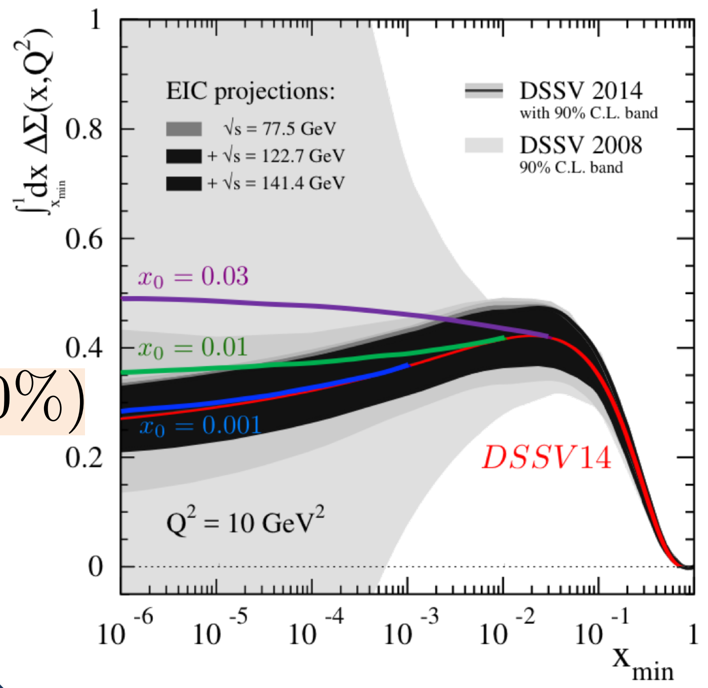
- Can be **solved analytically** using **Laplace-Mellin** techniques

$$\left.\begin{aligned} G_2(x_\perp^2, zs) &\sim (zs)^{\alpha_h^G} \\ g_1^{G\,,\,dip}(x, k_T^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^G} \\ \Delta G(x, Q^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^G} \\ g_1^{G\,,\,WW}(x, k_T^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^G} \end{aligned}\right\}$$

$$\boxed{\alpha_h^G = \frac{13}{4\sqrt{3}}\sqrt{\frac{\alpha_s\,N_c}{2\pi}}}$$

Some Crude Phenomenology

Quarks



$\mathcal{O}(20\%)$

adapted from Aschenauer et al., Phys. Rev. **D92** (2015) no. 9 094030

- Predict **enhancement** in quark/gluon spins from small x

Some Crude Phenomenology

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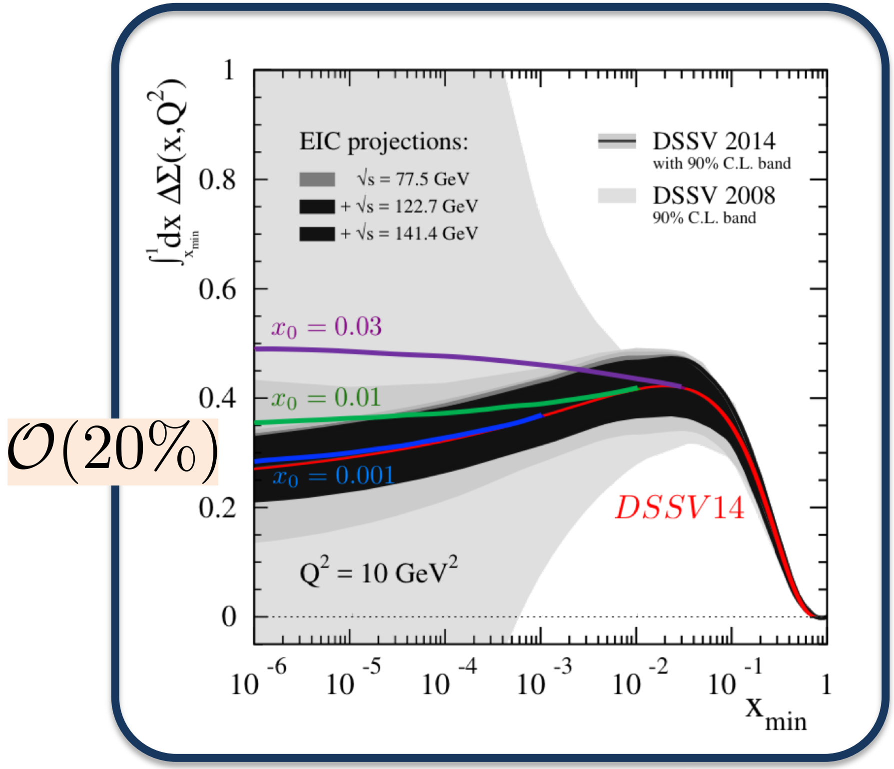
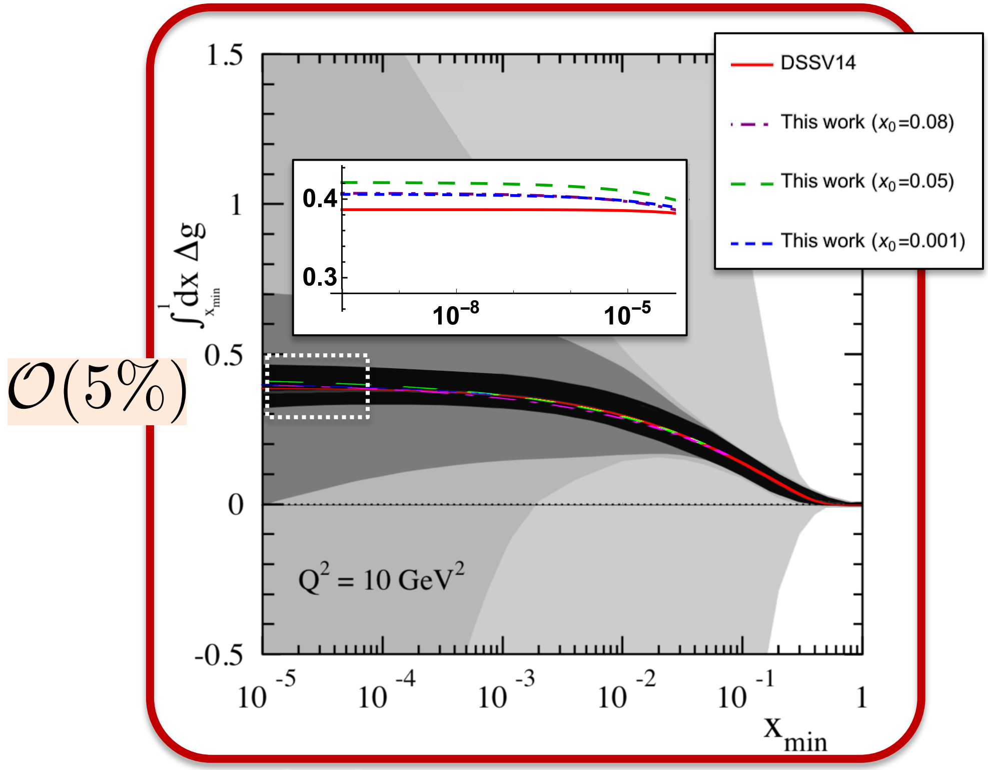


Gluons



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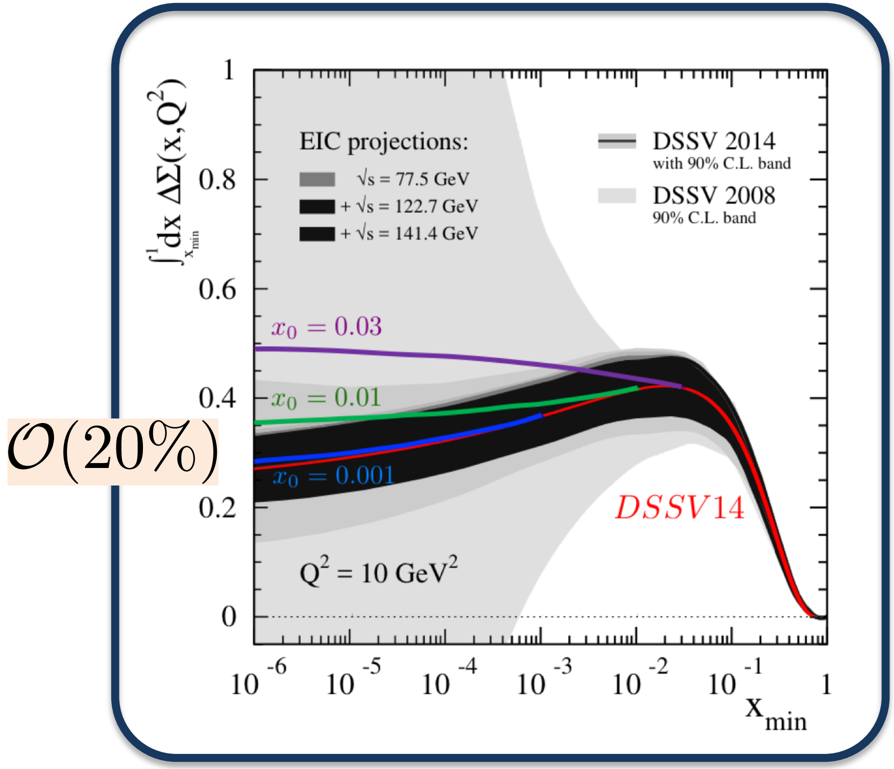
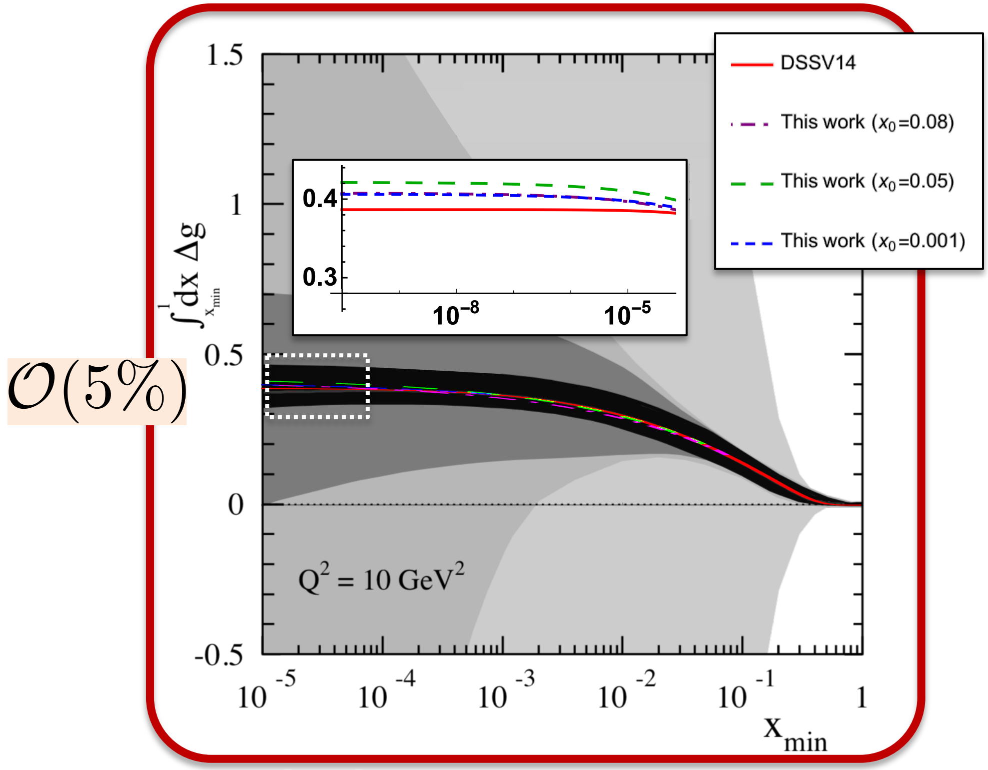
Quarks / **Gluons**





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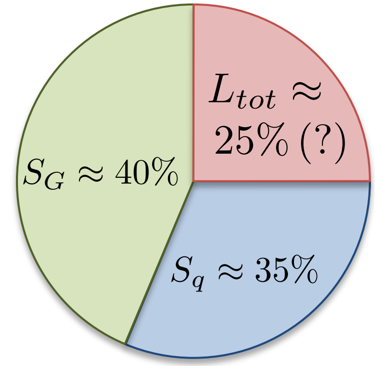
- Predict **enhancement** in quark/gluon spins from small x
- Inputs to **phenomenology**: priors, independent parameters, etc.
- **Caveats**: leading order, large Nc. Corrections may be important.

One Small Piece of a Big Picture

$x > 10^{-3}$ $\quad$ $x > 5 \times 10^{-2}$ $\quad$???

$$\frac{1}{2} = \boxed{S_q} + \boxed{S_G} + \boxed{L_q + L_G}$$



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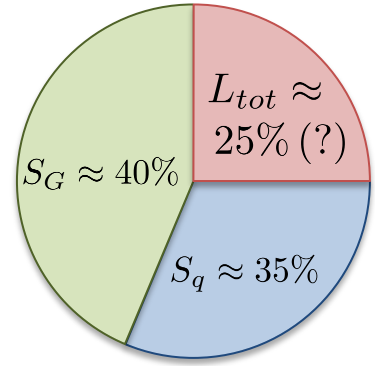

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JHEP **1210** 080 (2012)

Phys. Rev. **D95** 114032 (2017)

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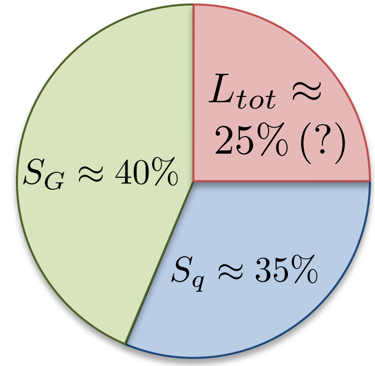

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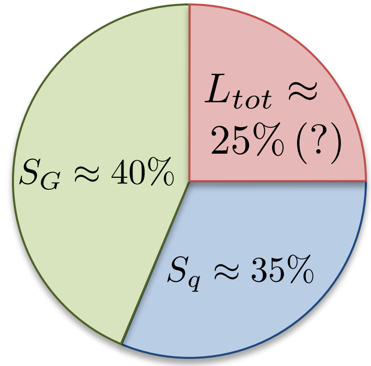

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JHEP **1210** 080 (2012)

Phys. Rev. **D95** 114032 (2017)

- The intersection of spin and small-x **challenges both paradigms**
  - ➢ **Significant effects** from small-x evolution
  - ➢ Gluons don't dominate, and dipoles aren't independent