Suppression of maximal linear gluon polarization in angular asymmetries

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collaborate with Daniel Boer, Piet Mulders and Jian Zhou,
JHEP10(2017)196

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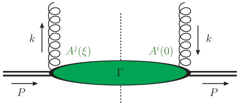
Outline

Gluon TMDs

TMDs: Transverse momentum dependent functions.

$$\Gamma^{[U,U']\,\mu\nu;\rho\sigma}(x,\boldsymbol{k}_T;P,n) \equiv \int \frac{d\xi{\cdot}P\, d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P|\, F^{\mu\nu}(0) U_{[0,\xi]} F^{\rho\sigma}(\xi) U'_{[\xi,0]}\, |P\rangle \Big|_{\xi\cdot n=0} .$$



$$\begin{aligned}
\Gamma^{ij}(x,\boldsymbol{k}_T^2) = {} & \frac{x}{2}\left[-g_T^{ij} f_1^g(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^{\perp g}(x,\boldsymbol{k}_T^2)\right] \\
& + \frac{x}{2}\left[i\epsilon_T^{ij} S_L\, g_1^g(x,\boldsymbol{k}_T^2) + \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha} S_L}{2M^2}\, h_{1L}^{\perp g}(x,\boldsymbol{k}_T^2)\right] \\
& + \frac{x}{2}\left[-\frac{g_T^{ij}\epsilon_T^{S_T k_T}}{M}\, f_{1T}^{\perp g}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij}\boldsymbol{k}_T{\cdot}\boldsymbol{S}_T}{M}\, g_{1T}^g(x,\boldsymbol{k}_T^2)\right. \\
& \left. - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M}\, h_1^g(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^{\perp g}(x,\boldsymbol{k}_T^2)\right]
\end{aligned}$$

process dependent

Gluons	$-g_T^{ij}$	$i\epsilon_T^{ij}$	k_T^i, k_T^{ij}, etc.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$
LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
LT	f_{1LT}^g	g_{1LT}^g	h_{1LT}^g, $h_{1LT}^{\perp g}$
TT	f_{1TT}^g	g_{1TT}^g	h_{1TT}^g, $h_{1TT}^{\perp g}$, $h_{1TT}^{\perp\perp g}$

Mulders, Rodrigues, PRD63(01); Meissner, Metz, Goeke, PRD76(07), renaming

Boer, Cotogno, Daal, Mulders, Signori, Zhou, JHEP 1610(16)

Gluon TMDs

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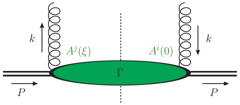

$$\Gamma^{ij}(x,\boldsymbol{k}_T^2) = \frac{x}{2}\left[-g_T^{ij} f_1^g(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2} h_1^{\perp g}(x,\boldsymbol{k}_T^2)\right]$$
$$+ \frac{x}{2}\left[i\epsilon_T^{ij} S_L\, g_1^g(x,\boldsymbol{k}_T^2) + \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^{\perp g}(x,\boldsymbol{k}_T^2)\right]$$
$$+ \frac{x}{2}\left[-\frac{g_T^{ij}\epsilon_T^{S_T k_T}}{M} f_{1T}^{\perp g}(x,\boldsymbol{k}_T^2) + \frac{i\epsilon_T^{ij} \boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M} g_{1T}^g(x,\boldsymbol{k}_T^2) \right.$$
$$\left. - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M} h_1^g(x,\boldsymbol{k}_T^2) - \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha S_T}}{2M^3} h_{1T}^{\perp g}(x,\boldsymbol{k}_T^2)\right]$$

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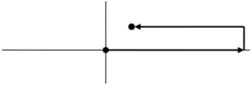
Boer, Cotogno, Daal, Mulders, Signori, Zhou, JHEP 1610(16).

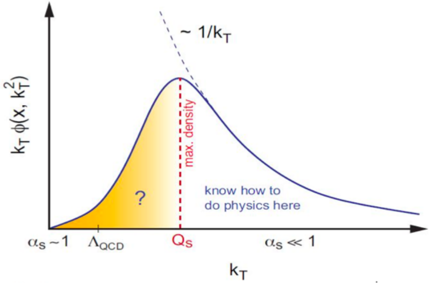
Unpolarized gluon distribution functions at small x

Using the McLerran-Venugopalan (MV) model , unpolarized distribution function is

$$x f^g_{1,WW}(x,k_\perp) = \frac{N_c^2-1}{N_c}\frac{S_\perp}{4\pi^4\alpha_s}\int d^2\xi_\perp\, e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp}\frac{1}{\xi_\perp^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$



$$x f^g_{1,DP}(x,k_\perp) = \frac{k_\perp^2 N_c}{2\pi^2\alpha_s}S_\perp\int\frac{d^2\xi_\perp}{(2\pi)^2}\, e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp}e^{-\frac{\xi_\perp^2 Q_s^2}{4}}$$



Kovchegov, PRD54(96)

Marian, Kovner, Mclerran, Weigert, RPD55 (97)

Linearly polarized gluon distribution functions at small x

Also in the MV model

$$x\, h_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{4\pi^3} S_\perp \int d^2\xi_\perp \frac{J_2(k_\perp \xi_\perp)}{\frac{1}{4\mu_A}\xi_\perp Q_s^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$

$$x\, h_{1,DP}^{\perp g}(x,k_\perp) = 2x\, f_{1,DP}^{g}(x,k_\perp) = \frac{k_\perp^2 N_c}{\pi^2 \alpha_s} S_\perp \int \frac{d^2\xi_\perp}{(2\pi)^2}\; e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp} e^{-\frac{\xi_\perp^2 Q_s^2}{4}}$$

A. Metz, J. Zhou, PRD84(11)

- ♠ At large $k_\perp, k_\perp \gg Q_s$

$$x\, f_{1,WW}^{g}(x,k_\perp),\; x\, h_{1,WW}^{\perp g}(x,k_\perp),\; x\, f_{1,DP}^{g}(x,k_\perp) = x\, h_{1,DP}^{\perp g}(x,k_\perp) \propto \frac{1}{k_\perp^2}$$

- ♠ At small $k_\perp, \Lambda_{QCD} \ll k_\perp \ll Q_s$

$$x\, f_{1,WW}^{g}(x,k_\perp) \propto ln\frac{Q_s^2}{k_\perp^2},\; x\, h_{1,WW}^{\perp g}(x,k_\perp) \propto \frac{1}{Q_s^2}$$

$$x\, f_{1,DP}^{g}(x,k_\perp) = x\, h_{1,DP}^{\perp g}(x,k_\perp) \propto k_\perp^2 e^{-k_\perp^2/Q_s^2}$$

Dominguez, Qiu, Xiao, Yuan PRD85 (12)

TMD evolution

When $k_T << Q$, standard perturbative QCD calculations generate large logarithms $\alpha_s^n \ln^{2n} \frac{Q^2}{k_T^2} + ...$, which can be attributed to the energy dependence of the TMDs.

The energy evolution of TMD is given by the Collins-Soper (CS) equation

$$\frac{\partial \ln \tilde{F}(x, \mathbf{b}_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(\mathbf{b}_T; \mu), \quad \text{with} \quad \tilde{K}(\mathbf{b}_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_s} \ln \left(\frac{\tilde{S}(\mathbf{b}_T; y_s, -\infty)}{\tilde{S}(\mathbf{b}_T; +\infty, y_s)} \right).$$

and Renormalization Group equations,

$$\frac{d\tilde{K}(\mathbf{b}_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu)), \qquad \frac{d \ln \tilde{F}(x, \mathbf{b}_T; \mu, \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2).$$

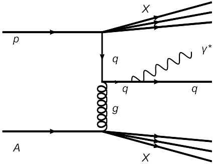
By solving these equations, the large logarithms can be resumed, result in the Sudakov factor in exponential.

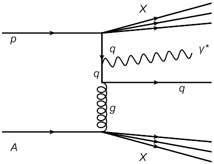
Collins, Foundations of perturbative QCD, (11); Aybat, Rogers, PRD83 (11)

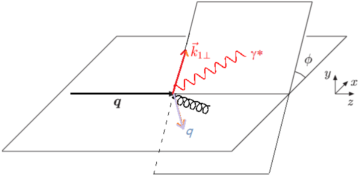
Azimuthal asymmetry in $pA \to \gamma^*\, jet\, X$

$h_1^{\perp g}$ have been probed in a lot of processes, such as $pp \to \gamma\gamma X, HX, Q\bar{Q}X...$
$pA \to \gamma\, jet\, X$ is power suppressed, but $\gamma^*\, jet\, X$ production is not

we study: $p + A \to q(p) + g(n) + X \to \gamma^*(k_1) + q(k_2) + X$







$$p = (p^+, 0, 0_\perp),\ n = (0, n^-, \vec{k}_\perp),$$
$$\vec{P}_\perp = \frac{\vec{k}_{1\perp} - \vec{k}_{2\perp}}{2} \simeq \vec{k}_{1\perp} \simeq -\vec{k}_{2\perp},$$
$$\vec{k}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp}, \quad k_1^+ = zp^+$$
$$\phi = \vec{P}_\perp \wedge \vec{k}_\perp$$

Observable: azimuthal asymmetry

Using hybrid approach[Dominguez, Marquet, Xiao, Yuan, PRD83(11), Mueller, Xiao, Yuan, PRD88(13)], the differential cross section is [A. Metz, J. Zhou, PRD84(11)] ,

$$
\begin{aligned}
\frac{\sigma^{pA\to\gamma^*\ q\ X}}{dP.S} &= \sum_q x_p f_1^q(x_p)\Big\{x f_{1,DP}^g(x,k_\perp)\ H_{Born} + \cos(2\phi)\ x\ h_{1,DP}^{\perp g}(x,k_\perp) H_{Born}^{\cos(2\phi)}\Big\} \\
&= \sum_q x_p f_1^q(x_p)\ x f_{1,DP}^g(x,k_\perp)\Big\{\ 1 + \cos(2\phi)\frac{2Q^2\hat{t}}{\hat{s}^2+\hat{u}^2+2Q^2\hat{t}}\Big\}
\end{aligned}
$$

- Azimuthal asymmetry disappears as Q^2 goes to zero.
- An important (only clean) process to measure $h_{1,DP}^{\perp g}$.

$$
< \cos(2\phi) >= \frac{\int \frac{d\sigma}{dP.S} d\phi \cos(2\phi)}{\int \frac{d\sigma}{dP.S} d\phi} = \frac{H_{Born}^{\cos(2\phi)}}{H_{Born}}, \qquad \text{[no evolution]}
$$

$pA \to \gamma^*$ jet X, resummation

differential cross section in k space

$$\frac{\sigma^{pA\to\gamma^*\ q\ X}}{dP.S} = \sum_q x_p f_1^q(x_p) \times \left\{x\, f_{1,DP}^g(x,k_\perp)\, H_{Born} + x\, h_{1,DP}^{\perp g}(x,k_\perp)\left[2(\hat{k}_\perp\cdot\hat{P}_\perp)^2-1\right] H_{Born}^{\cos(2\phi)}\right\}$$

in b space

$$\frac{\sigma^{pA\to\gamma^*\ q\ X}}{dP.S} = \sum_q \int d^2b_\perp\, e^{i\vec{k}_\perp\cdot\vec{b}_\perp}\, x_p f_1^q(x_p) \times \left\{x\, f_{1,DP}^g(x,b_\perp)\, H_{Born} + x\, h_{1,DP}^{\perp g}(x,b_\perp)\left[2(\hat{b}_\perp\cdot\hat{P}_\perp)^2-1\right] H_{Born}^{\cos(2\phi)}\right\}$$

$pA \to \gamma^*$ *jet* X, resummation

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Gluon TMDs in b space

$$x f^g_{1,DP}(x, b_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^2} \, e^{i\vec{k}_\perp \cdot \vec{b}_\perp} x f^g_{1,DP}(x, k_\perp)$$

$$\left[2(\hat{b}_\perp \cdot \hat{P}_\perp)^2 - 1\right] x \, h^{\perp g}_{1,DP}(x, b_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^2} \, e^{i\vec{k}_\perp \cdot \vec{b}_\perp} \, x f^g_{1,DP}(x, k_\perp) \left[2(\hat{k}_\perp \cdot \hat{P}_\perp)^2 - 1\right]$$

$$\Rightarrow \quad x \, h^{\perp g}_{1,DP}(x, b_\perp) = -\int \frac{dk_\perp}{2\pi} J_2(b_\perp k_\perp) \, x f^g_{1,DP}(x, k_\perp)$$

rewrite unpolarized gluon TMD in k space (using $S_\perp = 2\pi^2 \alpha_s A x G_p(x)/N_c Q_s^2$)

$$x f^g_{1,DP}(x, k_\perp) = \frac{k_\perp^2 A \, x \, G_p(x)}{Q_s^2} \int \frac{d^2 b_\perp}{(2\pi)^2} \, e^{-i\vec{k}_\perp \cdot \vec{b}_\perp} e^{-\frac{Q_s^2 b_\perp^2}{4}}$$

in b space (suppose Q_s is independent on $b_\perp$),

$$x f^g_{1,DP}(x, b_\perp) = A x G_p(x) \frac{1}{2\pi^2} \left[1 - \frac{Q_s^2 b_\perp^2}{4}\right] e^{-\frac{Q_s^2 b_\perp^2}{4}}$$

$$x \, h^{\perp g}_{1,DP}(x, b_\perp) = -A x G_p(x) \frac{1}{2\pi^2} \left[\frac{Q_s^2 b_\perp^2}{4}\right] e^{-\frac{Q_s^2 b_\perp^2}{4}}$$

$pA \to \gamma^*$ jet X, resummation

$$\frac{\sigma^{pA\to\gamma^*\ q\ X}}{dP.S} = \sum_q \int d^2b_\perp e^{i\vec{k}_\perp\cdot\vec{b}_\perp} x_p f_1^q(x_p,\mu_b) e^{-S(\mu_b,P_\perp)} \left\{ x f_{1,DP}^g(x,b_\perp,\mu_b^2,\mu_b) H_{Born} \right.$$
$$\left. + x\, h_{1,DP}^{\perp g}(x,b_\perp,\mu_b^2,\mu_b) \left[2(\hat{b}_\perp\cdot\hat{P}_\perp)^2 - 1\right] H_{Born}^{\cos(2\phi)} \right\}$$

$$S(\mu_b,P_\perp) = \int_{\mu_b}^{P_\perp} \frac{d\mu}{\mu}\alpha_s(\mu)\left(\frac{C_F+C_A}{\pi} ln\frac{P_\perp^2}{\mu^2} - \frac{C_F}{\pi}\frac{3}{2} - \frac{C_A}{\pi}\frac{11-2n_f/C_A}{6}\right)$$

where $\mu_b = 2e^{-\gamma_E}/b_\perp \equiv b_0/b_\perp$.

Consider $Q \sim P_\perp$ to avoid a three-scale problem

Mueller, Xiao, Yuan, PRD88(13)

- At large $b_\perp$ (small $k_\perp$), b_* method

$$b_\perp \Rightarrow b_{\perp *} = \frac{b_\perp}{\sqrt{1 + b_\perp^2/b_{max}^2}}, \qquad \mu_b \Rightarrow \mu_{b_*} \tag{1}$$

$$S_{NP} = \left(g_1 + g_2 \ln \frac{Q}{Q_0} + 2 g_1 g_3 \ln \frac{10 x x_0}{x_0 + x} \right) b_\perp^2$$

$$b_{max} = 1.5\text{GeV}^{-1},\ g_1 = 0.201\text{GeV}^2,\ g_2 = 0.184\text{GeV}^2,$$
$$g_3 = -0.129,\ x_0 = 0.009,\ Q_0 = 1.6\text{GeV}$$

Collins, Foundations of perturbative QCD, (11); Aybat, Rogers, PRD83 (11)

- At small $b_\perp$ (large $k_\perp$), μ_b' method:

$$\mu_b \Rightarrow \mu_b' \equiv \frac{1}{\sqrt{b_\perp^2/b_0^2 + 1/Q^2}} \tag{2}$$

D Boer, QCD Evolution 2015 proceeding;

Collins, Gamberg, Prokudin, Rogers, Sato, Wang, PRD94(16)

$$(1)+(2) \ \Rightarrow \mu_{b*}'$$

Numerical Results

RHIC : $\sqrt{s} = 200$ GeV, $Q_s^2(x) = 1GeV^2 A^{1/3} (\frac{x_0}{x})^{0.3}$ with $x_0 = 3 \times 10^{-4}$

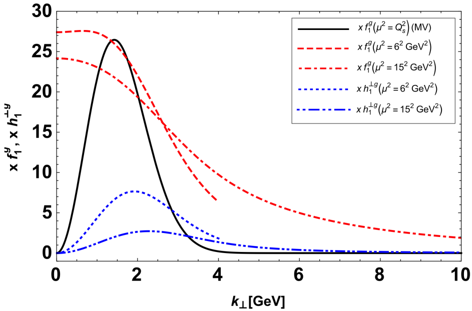


FIG. 1: The unpolarized and linearly polarized gluon TMDs as function of $k_\perp$ at different scales, at x=0.01

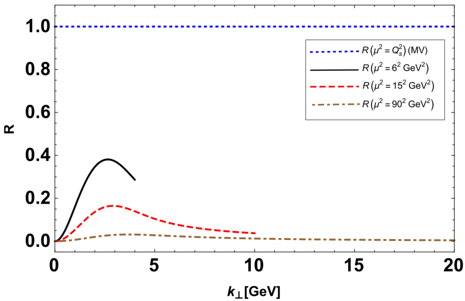
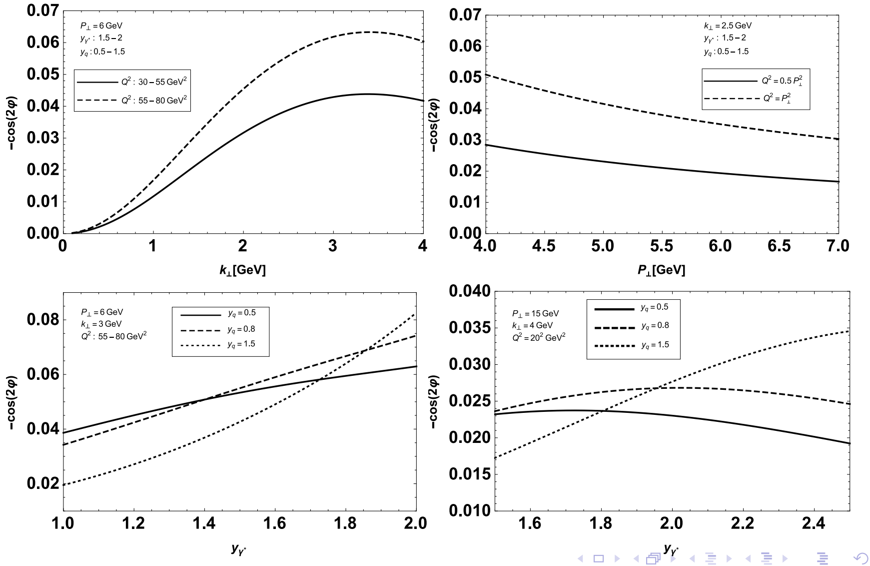


FIG. 2: The ratio $R = h_1^{\perp g}/f_1^g$ as function of $k_\perp$, at $x = 0.01$ for $\mu = 6$, 15 and 90 GeV.

$$x \, f_{1,DP}^g(x, k_\perp) = x \, h_{1,DP}^{\perp g}(x, k_\perp)$$

evolve from $P_\perp = Q_s \simeq 1.4$GeV to $P_\perp = 6$GeV, 15GeV

Numerical Results



Summary

- The effect of the linear gluon polarization is strongly suppressed due to TMD evolution effects.
- The experimental study of the $\cos(2\phi)$ asymmetry for $pA \to \gamma^*\ jet\ X$ at RHIC seems the most promising option, and may allow to test the k_t-resummation formalism in the small-x regime and the theoretical expectation that the Color Glass Condensate state is in fact polarized.

Thanks!

backups

parameterization of gluon correlator: definite rank TMDs

we use symmetric traceless tensors:

$$
\begin{aligned}
& k_T^i \\
k_T^{ij} &\equiv\; k_T^i k_T^j + \frac{1}{2}\boldsymbol{k}_T^2 g_T^{ij}, \\
k_T^{ijk} &\equiv\; k_T^i k_T^j k_T^k + \frac{1}{4}\boldsymbol{k}_T^2 \left(g_T^{ij} k_T^k + g_T^{ik} k_T^j + g_T^{jk} k_T^i\right), \\
& \ldots
\end{aligned}
$$

only has two independent components:

$$
k_T^{i_1 \ldots i_n} \to \frac{|\boldsymbol{k}_T|^n}{2^{n-1}}\, e^{\pm i n \varphi},
$$

and simple Bessel transform to b space

$$
F_m(x, k_T) = \int_0^\infty b db\, J_m(k_T b) F_m(x, b)
$$