Deep Inelastic Scattering in the Dipole Picture at Next-to-Leading Order

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B. Ducloué, H. Hänninen, T. Lappi, and Y. Zhu,
Deep inelastic scattering in the dipole picture at next-to-leading order,
Phys. Rev. D 96, 094017 (2017).

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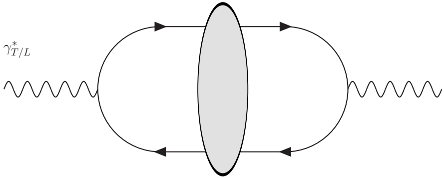
DIS in the Dipole Picture at Leading Order



Leading Order virtual photon fluctuation.

In Dipole Picture at Leading Order $\gamma^* p$ cross section is of the form

$$\sigma_{L,T}^{\mathrm{LO}}(x_{Bj}, Q^2) = 4N_{\mathrm{c}}\alpha_{em} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} \mathcal{K}_{L,T}^{\mathrm{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, x_{Bj}),$$

where

$$\mathcal{K}_L^{\mathrm{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, X) = 4Q^2 z_1^2 (1-z_1)^2 K_0^2(QX_2) \left(1 - S_{01}(X)\right),$$

$$\mathcal{K}_T^{\mathrm{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, X) = Q^2 z_1 (1-z_1) \left(z_1^2 + (1-z_1)^2\right) K_1^2(QX_2) \left(1 - S_{01}(X)\right)$$

with $\int_{\mathbf{x}_0} := \int \frac{\mathrm{d}^2\mathbf{x}_0}{2\pi}, \quad X_2^2 := z_1(1-z_1)\mathbf{x}_{01}^2,$
$S_{01}(X) := S(\mathbf{x}_{01} = \mathbf{x}_0 - \mathbf{x}_1, X) = 1/N_{\mathrm{c}} \left\langle \operatorname{Tr} U(\mathbf{x}_0) U^\dagger(\mathbf{x}_1) \right\rangle_X$

Target evolution and LO Dipole Picture results

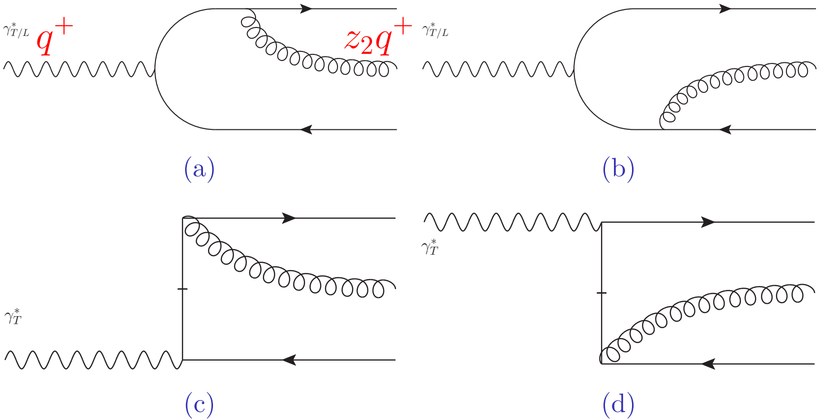
Target evolution is described by the B-JIMWLK equation, or approximatively by the BK equation:

$$\partial_y \langle S_{01} \rangle_y = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2 \mathbf{x}_2 \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{02}^2 \mathbf{x}_{21^2}} \left[\langle S_{02} \rangle_y \langle S_{21} \rangle_y - \langle S_{01} \rangle_y \right].$$

LO Dipole Picture DIS results with BK have been used to describe HERA data, e.g.:

- BK with running coupling corrections
  - J. L. Albacete et al., Eur. Phys. J. C71 (2011) 1705 [arXiv:1012.4408]
  - T. Lappi, H. Mäntysaari, Phys. Rev. D88 (2013) 114020 [arXiv:1309.6963]
- BK with resummation
  - J. L. Albacete, Nucl.Phys. A957 (2017) 71-84 [arXiv:1507.07120]
  - E. Iancu et al., Phys.Lett. B750 (2015) 643-652 [arXiv: 1507.03651]

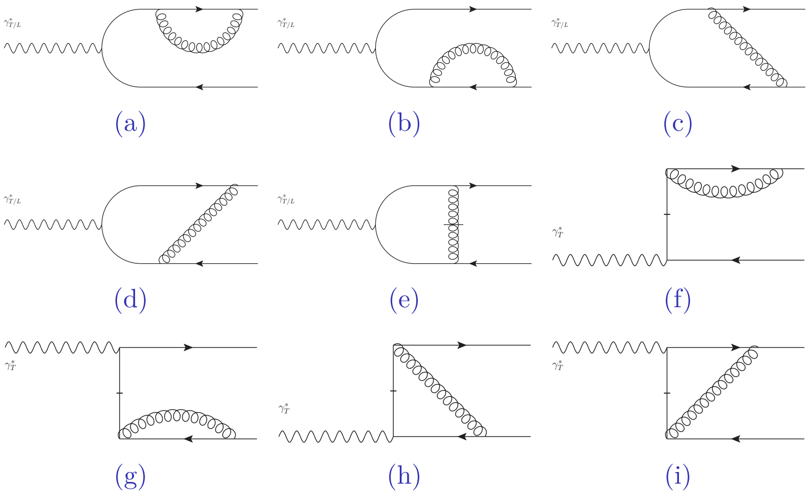
Next-to-Leading Order: Tree-level diagrams



Virtual photon fluctuation diagrams relevant to the scattering at next-to-leading order. Cause a logarithmic divergence for $\sigma^{\mathrm{NLO}}_{L,T}$ as $z_2 \to 0$.

Calculated by G. Beuf, Phys. Rev. D **96**, 074033 (2017).

Next-to-Leading Order: One-gluon-loop diagrams



Loop diagrams relevant at next-to-leading order.

Calculated by G. Beuf, Phys. Rev. D **94**, 054016 (2016).

Full NLO DIS cross section in the Dipole Picture

Next-to-Leading Order $\gamma^* p$ cross section can be partitioned as

$$\sigma^{\mathrm{NLO}}_{L,T} = \sigma^{\mathrm{IC}}_{L,T} + \sigma^{qg}_{L,T} + \sigma^{\mathrm{dip}}_{L,T},$$

where the NLO contributions are[1]:

$$\sigma^{qg}_{L,T} = 8 N_{\mathrm{c}} \alpha_{em} \frac{\alpha_s C_{\mathrm{F}}}{\pi} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int^{1-z_1} \frac{\mathrm{d}z_2}{z_2}$$

$$\times \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathcal{K}^{\mathrm{NLO}}_{L,T}\left(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)\right),$$

$$\sigma^{\mathrm{dip}}_{L,T} = 4 N_{\mathrm{c}} \alpha_{em} \frac{\alpha_s C_{\mathrm{F}}}{\pi} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1$$

$$\times \int_{\mathbf{x}_0,\mathbf{x}_1} \mathcal{K}^{\mathrm{LO}}_{L,T}(z_1, \mathbf{x}_0, \mathbf{x}_1, X^{\mathrm{dip}}) \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right].$$

z_2 = gluon momentum fraction. $X(z_2) = ?$

[1] G. Beuf, Phys. Rev. D **96**, 074033 (2017).

Subtraction of the soft gluon divergence

- Find a natural lower limit for gluon fractional momentum z_2.
- BK evolution of the target can be considered using two distinct variables: probe longitudinal momentum and target momentum fraction $X = \Delta k^- / P^-$.
- $\Delta k^- \gtrsim k_\perp^2/(2 z_2 q^+)$, $W^2 = 2q^+ P^- \quad \Longrightarrow \; X \equiv X(z_2) \approx k_\perp^2/(z_2 W^2)$
- With k^- ordering one can recover a lower limit: $z_2 \gtrsim (x_{Bj}/x_0)/(k_\perp^2/Q^2)$, where $k_\perp$ is gluon transverse momentum.
- For single inclusive pA dominant $k_\perp \sim pA$ hard scale.[2]

We assume the same for DIS, i.e. $k_\perp \sim Q$, and so $X(z_2) \equiv x_{Bj}/z_2$, with kinematical limit $X(z_2) < x_0$, i.e. $z_2 > x_{Bj}/x_0$.

[2] E. Iancu et al. , JHEP **12** (2016) 041; B. Ducloué et al., Phys. Rev. D **95** 114007 (2017).

First subtraction scheme: the 'unsubtracted' form

With the above assumptions for the subtraction scheme we get

$$\sigma_{L,T}^{\mathrm{NLO}} = \sigma_{L,T}^{\mathrm{IC}} + \sigma_{L,T}^{qg,\mathrm{unsub.}} + \sigma_{L,T}^{\mathrm{dip}}\,,$$

where $\sigma_{L,T}^{\mathrm{IC}}$ is the LO result with non-evolved $S_{01}(X = x_0)$ and

$$\begin{aligned}\sigma_{L,T}^{qg,\mathrm{unsub.}} &= 8N_\mathrm{c}\alpha_{em}\frac{\alpha_s C_\mathrm{F}}{\pi}\sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{x_{Bj}/x_0}^{1-z_1} \frac{\mathrm{d}z_2}{z_2} \\ &\quad \times \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathcal{K}_{L,T}^{\mathrm{NLO}}\left(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)\right), \\ X(z_2) &= x_{Bj}/z_2, \\ z_2 &= \text{gluon momentum fraction.}\end{aligned}$$

Second subtraction scheme: the 'subtracted' form

$$\sigma_{L,T}^{\mathrm{NLO}} = \sigma_{L,T}^{\mathrm{IC}} + \sigma_{L,T}^{qg,\mathrm{unsub.}} + \sigma_{L,T}^{\mathrm{dip}}$$

can be rewritten in an equivalent form:

$$\sigma_{L,T}^{\mathrm{NLO}} = \sigma_{L,T}^{\mathrm{LO}} + \sigma_{L,T}^{qg,\mathrm{sub.}} + \sigma_{L,T}^{\mathrm{dip}},$$

where $\sigma_{L,T}^{\mathrm{LO}}$ is the LO result with evolved $S_{01}(X = x_{Bj})$ and

$$\sigma_{L,T}^{qg,\mathrm{sub.}} = 8 N_{\mathrm{c}} \alpha_{em} \frac{\alpha_s C_{\mathrm{F}}}{\pi} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{x_{Bj}/x_0}^1 \frac{\mathrm{d}z_2}{z_2} \times \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \Big[\theta(1-z_1-z_2) \mathcal{K}_{L,T}^{\mathrm{NLO}}\left(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)\right) - \mathcal{K}_{L,T}^{\mathrm{NLO}}\left(z_1, 0, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)\right) \Big].$$

Numerical results

Few details about the published numerical results:

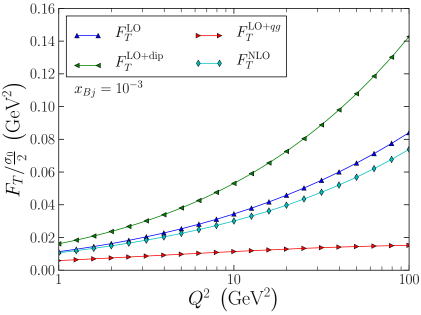
- Impact parameter dependence neglected $\rightarrow$ plotting $F_{L,T}/(\sigma_0/2)$.
- Leading order BK, with McLerran-Venugopalan initial condition.

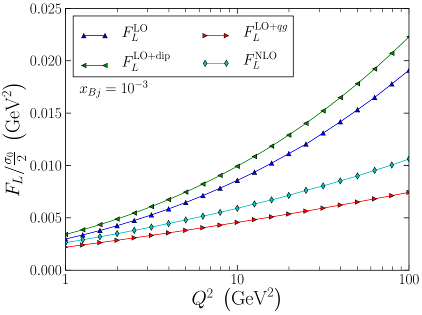
Both fixed ($\alpha_s = 0.2$) and parent dipole running coupling were used:

$$\alpha_{\mathrm{s}}(\mathbf{x}_{01}^2) = \frac{12\pi}{(11N_{\mathrm{c}} - 2n_{\mathrm{f}}) \ln\left(\frac{4e^{-2\gamma_e}}{\mathbf{x}_{01}^2 \Lambda_{\mathrm{QCD}}^2}\right)},$$

with $\Lambda_{\mathrm{QCD}} = 0.241$ GeV.

Results: 'unsubtracted' scheme, fixed coupling





LO and NLO contributions to F_T (left) and F_L (right) as a function of Q^2 at $x_{Bj} = 10^{-3}$ with $\alpha_{\mathrm{s}} = 0.2$.

- Full NLO corrections are moderate.
- In this scheme there are large cancellations between the NLO contributions.

A possible approximative scheme: 'x_{Bj}-subtracted'

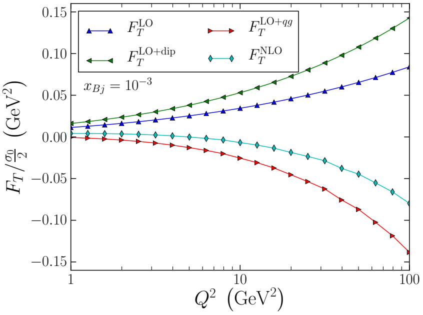
As an approximation set $X(z_2) \equiv x_{Bj}$ and $x_{Bj}/x_0 \to 0$.
Motivation: S_{01} is evaluated only at x_{Bj}.

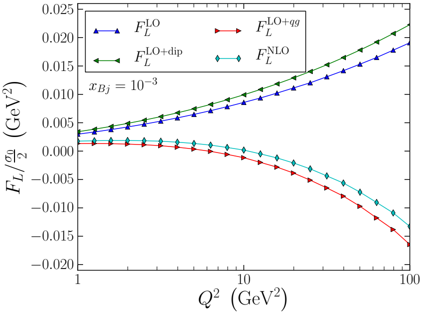
$$\begin{aligned}
\sigma_{L,T}^{\mathrm{NLO},x_{Bj}-\mathrm{sub.}} &= \sigma_{L,T}^{\mathrm{LO}} + \sigma_{L,T}^{qg,\mathrm{sub.}*} + \sigma_{L,T}^{\mathrm{dip}}\,, \\
\sigma_{L,T}^{qg,\mathrm{sub.}*} &= 8N_\mathrm{c}\alpha_{em}\frac{\alpha_s C_\mathrm{F}}{\pi}\sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{\color{red}0}^1 \frac{\mathrm{d}z_2}{z_2} \\
&\quad \times \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \Big[\theta(1-z_1-z_2)\mathcal{K}_{L,T}^{\mathrm{NLO}}(z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,{\color{magenta}x_{Bj}}) \\
&\qquad - \mathcal{K}_{L,T}^{\mathrm{NLO}}(z_1,0,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,{\color{magenta}x_{Bj}})\Big].
\end{aligned}$$

Analogous to a subtraction scheme used for single inclusive particle production in pA.[3]

[3] G. A. Chirilli, B.-W. Xiao and F. Yuan, Phys. Rev. Lett. **108** (2012) 122301

Results: approximative scheme, fixed coupling

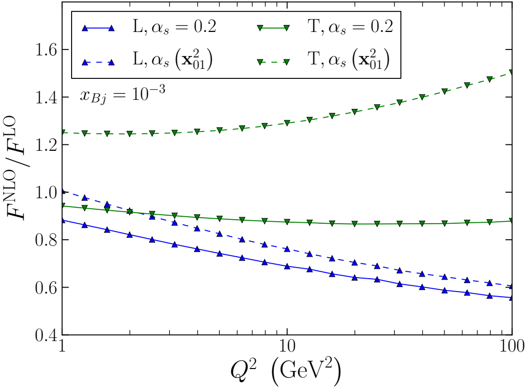




LO and NLO contributions to F_T (left) and F_L (right) as a function of Q^2 at $x_{Bj} = 10^{-3}$ with $\alpha_s = 0.2$ and using the x_{Bj}-subtraction procedure.

- The approximations break the scheme and lead to negative NLO structure functions.
- Similar to the negativity issue seen with single inclusive pA.

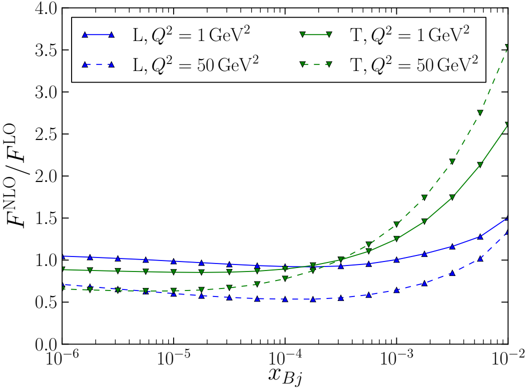
Results: 'unsubtracted' scheme, fixed v. runn. coupling



NLO/LO ratio for F_L and F_T as a function of Q^2 at $x_{Bj} = 10^{-3}$ with fixed (solid) and running (dashed) coupling.

- NLO corrections sensitive to subtraction scheme details e.g. running coupling due to the large cancellations.

Results: 'unsubtracted' scheme, running coupling



NLO/LO ratio for F_L and F_T as a function of x_{Bj} at $Q^2 = 1$ GeV^2 (solid) and $Q^2 = 50$ GeV^2 (dashed) with running coupling.

- By scheme construction $\sigma^{qg} \xrightarrow[x_{Bj}\to x_0]{} 0 \implies$ large transient effect at large x_{Bj} especially for F_T.

Conclusions

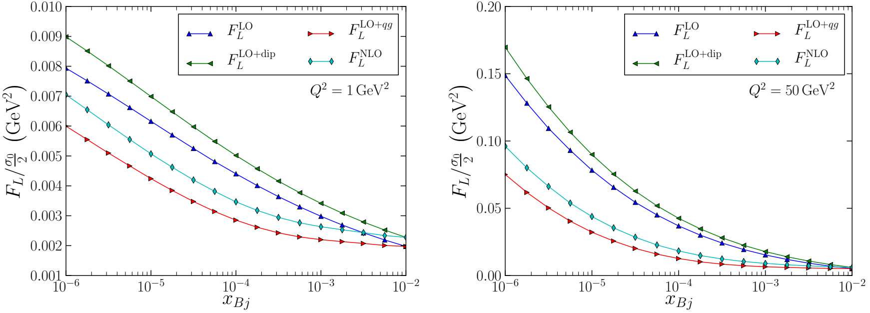
What was done:

- Successfully evaluated NLO corrections to DIS structure functions for the first time.
- Choice of subtraction scheme can have strong effects.

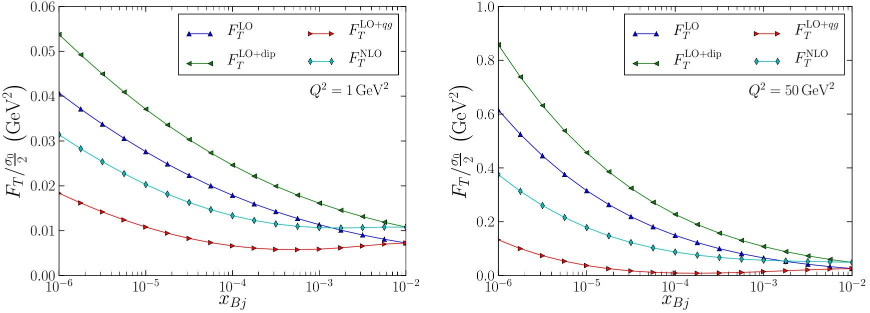
To be done:

- Evaluation of NLO structure functions using both NLO impact factors and resummed/NLO BK equation.
- Careful treatment of the transient effect; choice of subtraction scheme.
- Fit to HERA data.
- Extension of impact factor results for massive quarks.

Backup slides



LO and NLO contributions to F_L as a function of x_{Bj} at $Q^2 = 1$ GeV^2 (left) and $Q^2 = 50$ GeV^2 (right) with $\alpha_s = 0.2$.



LO and NLO contributions to F_T as a function of x_{Bj} at $Q^2 = 1$ GeV^2 (left) and $Q^2 = 50$ GeV^2 (right) with $\alpha_{\rm s} = 0.2$.