$\gamma \longrightarrow \rho^0$ impact factor in QCD

light-cone sum rule calculation

Kazuhiro Tanaka (Juntendo U/KEK)

Outline:

1. $\gamma \to \rho^0$ forward transition amplitude

   impact factor

2. QCD calculation of impact factor

   for $\gamma \to \rho^0{}_L$, $\gamma \to \rho^0{}_T$

   QCD factorization

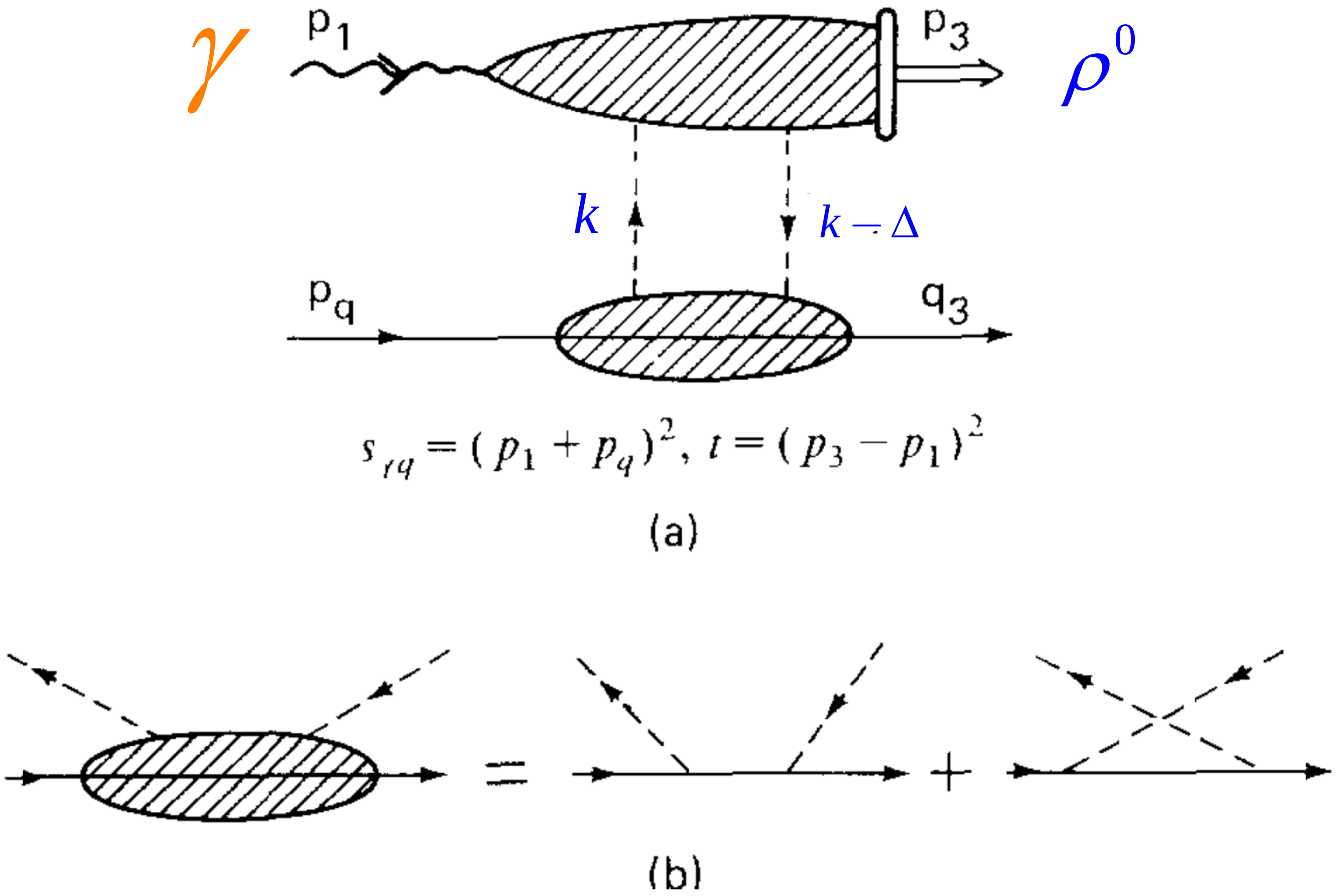
   light-cone sum rule (LCSR)

3. Application to two-photon process

   $$\gamma\gamma \to \rho^0{}_L\, \rho^0{}_T$$

$$s \gg -t \gg \Lambda_{QCD}^2$$

$$t = \Delta^2 = -\frac{s}{2}(1 - \cos\theta)$$



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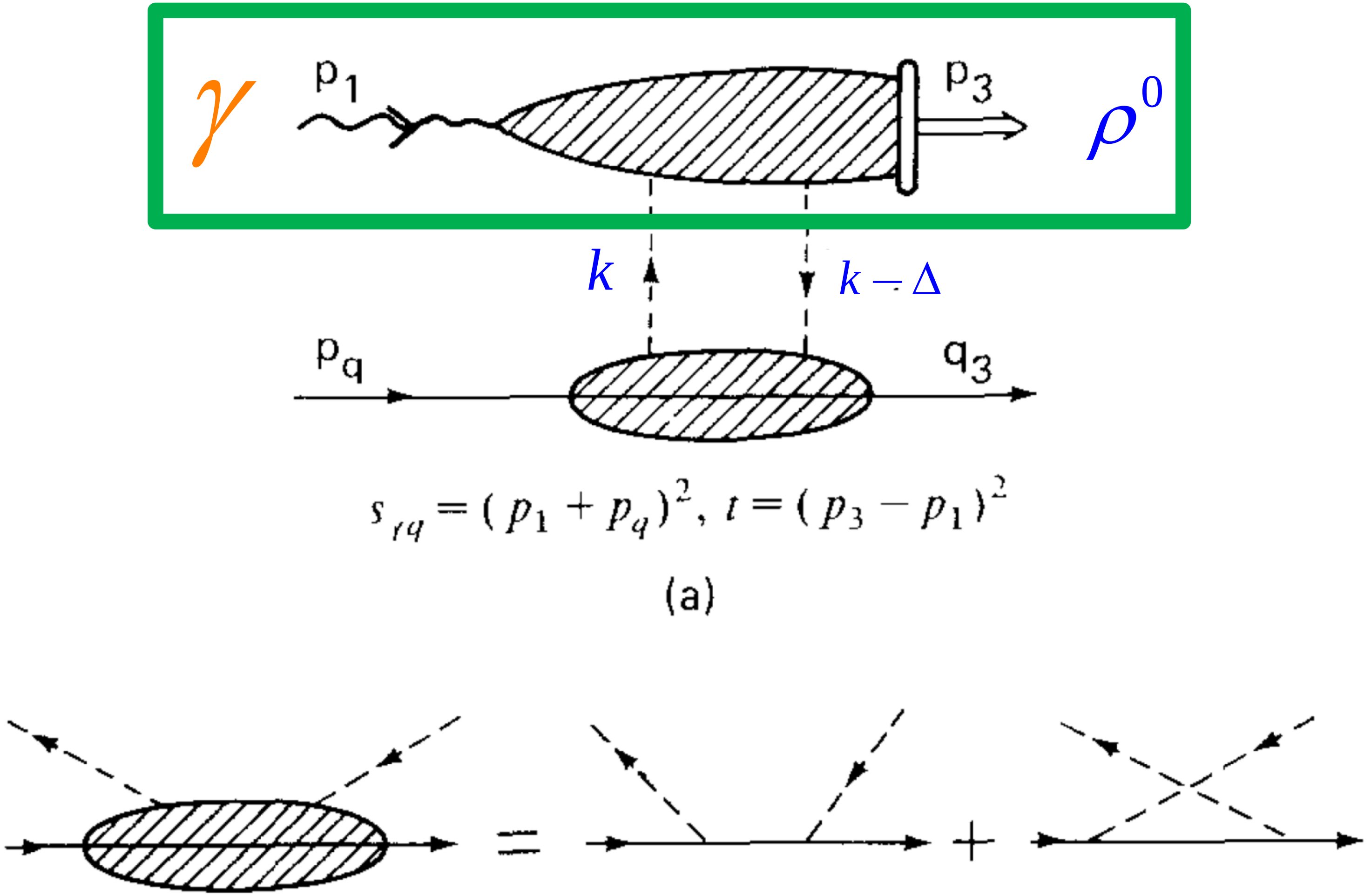
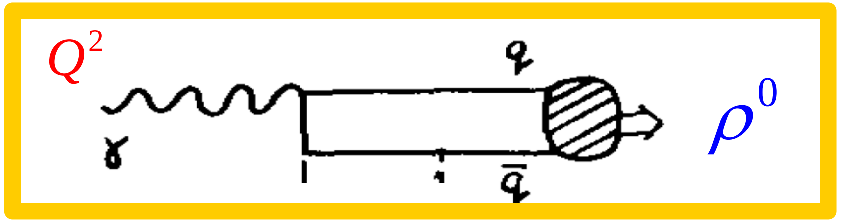
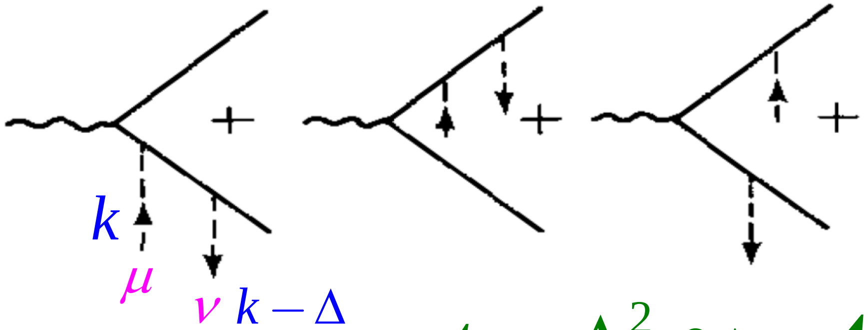


(a)

(b)

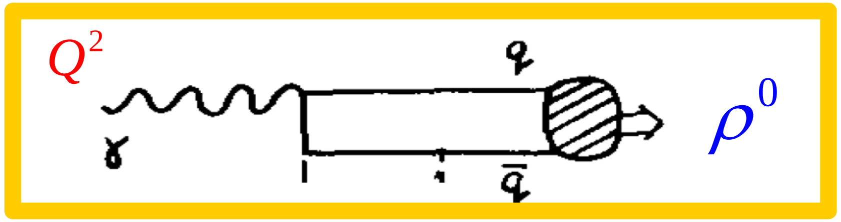


"Impact factor"



$$(k,\ \mu \to k-\Delta,\ \nu)$$

$$t = \Delta^2 \simeq -\boldsymbol{\Delta}_\perp^2$$



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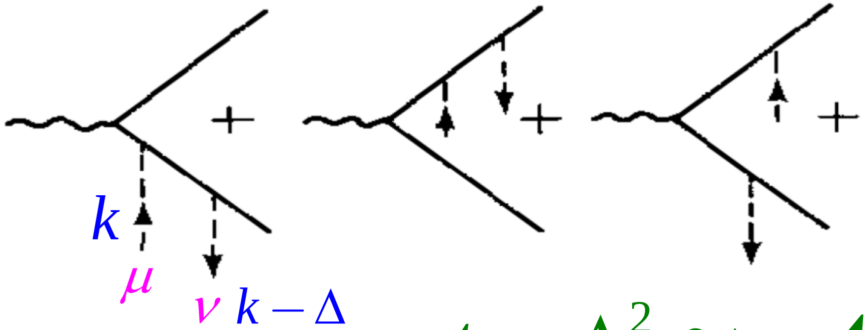

$$(k,\ \mu \rightarrow k-\Delta,\ \nu) \otimes \Psi^{\rho^0}$$

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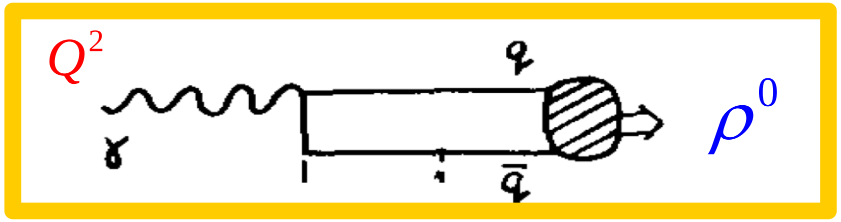

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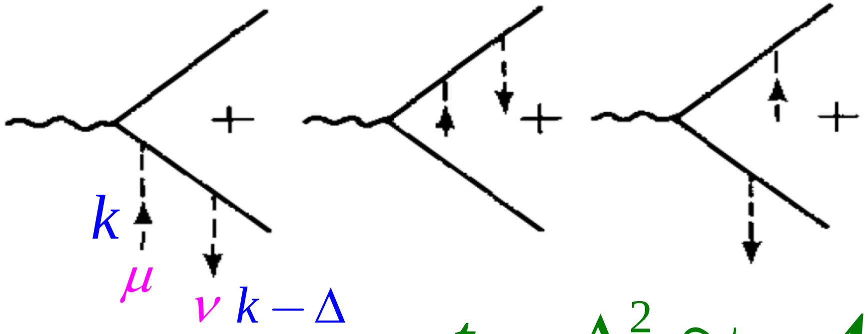


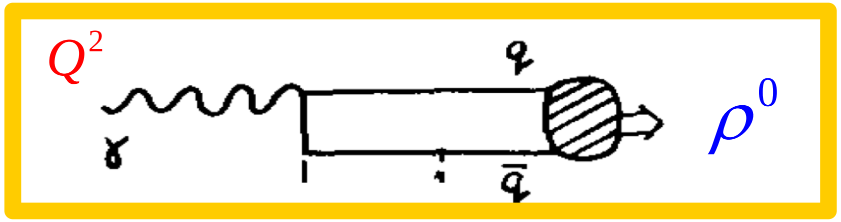
$$(k,\ \mu \to k-\Delta,\ \nu) \quad \otimes \Psi^{\rho^0}$$

$$t = \Delta^2 \simeq -\Delta_\perp^2$$

$$k_0^2 - k_3^2 \ll k_\perp^2$$

$$s \gg -t \gg \Lambda_{\mathrm{QCD}}^2$$

light-cone expansion



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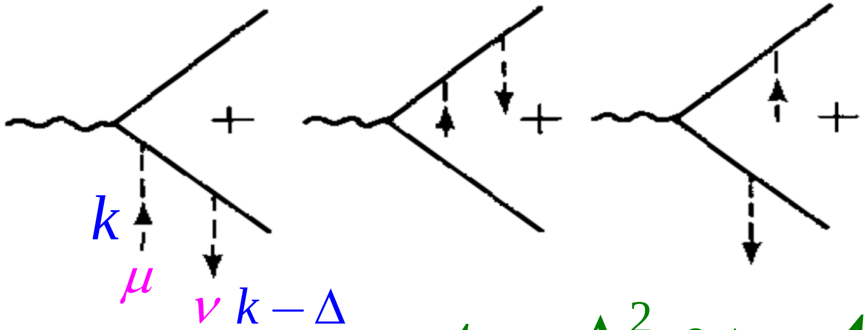

$$(k,\ \mu \rightarrow k-\Delta,\ \nu) \otimes \begin{cases} \phi_L(u) \\ g_T(u) \end{cases}$$

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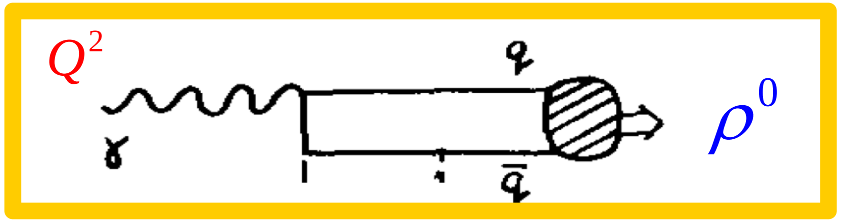

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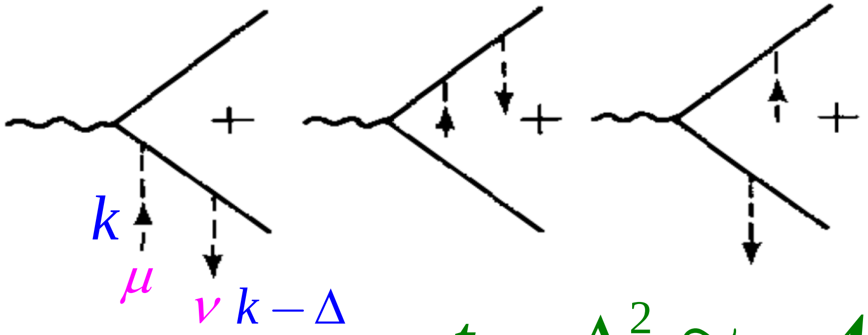

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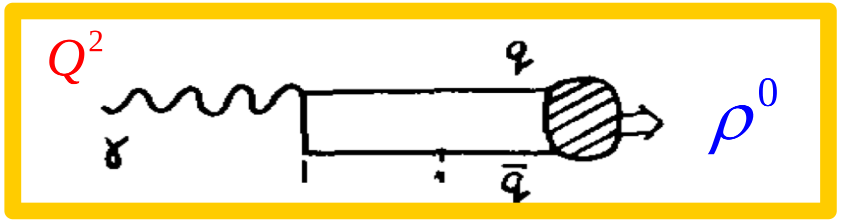
$$g_+^\mu g_+^\nu \sqrt{\alpha_{em}}\,\alpha_s \frac{(2\pi)^{3/2}}{N_c} f_V \int_0^1 du\, \phi_L(u) \frac{2u-1}{2} \left\{ \frac{\boldsymbol{e}_\gamma \cdot \boldsymbol{\Delta}_\perp}{u\boldsymbol{\Delta}_\perp^2 + (1-u)Q^2} - \frac{\boldsymbol{e}_\gamma \cdot \boldsymbol{\Delta}_\perp}{(1-u)\boldsymbol{\Delta}_\perp^2 + uQ^2} - \frac{\boldsymbol{e}_\gamma \cdot (u\boldsymbol{\Delta}_\perp - \boldsymbol{k}_\perp)}{(u\boldsymbol{\Delta}_\perp - \boldsymbol{k}_\perp)^2 + u(1-u)Q^2} + \frac{\boldsymbol{e}_\gamma \cdot [(1-u)\boldsymbol{\Delta}_\perp - \boldsymbol{k}_\perp]}{[(1-u)\boldsymbol{\Delta}_\perp - \boldsymbol{k}_\perp]^2 + u(1-u)Q^2} \right\}$$

for ρ_L^0

Ginzburg, Ivanov, PRD54 ('96) 5523

Segond, Szymanowski, Wallon, EPJC52 ('07) 93



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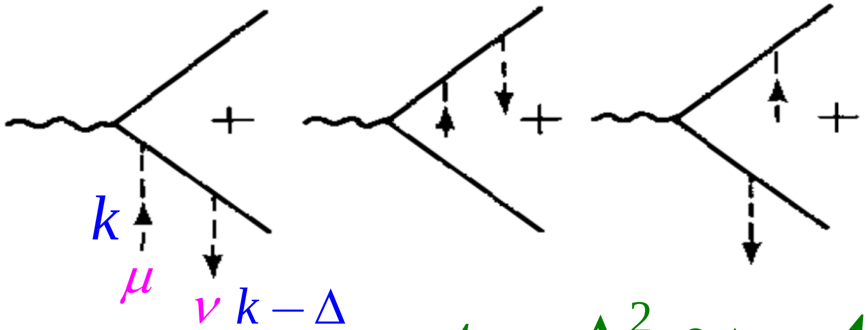
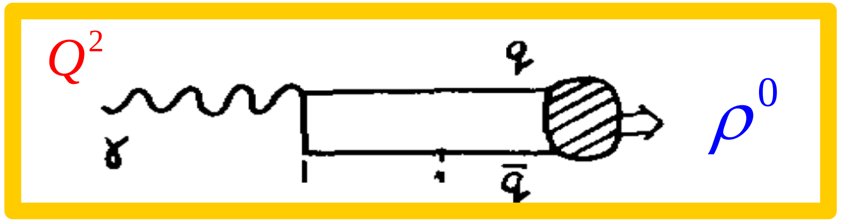


$$+ \; (k,\ \mu \to k-\Delta,\ \nu) \otimes \begin{cases} \phi_L(u) \\ g_T(u) \end{cases}$$

$$t = \Delta^2 \simeq -\Delta_\perp^2$$

$$\sim \log \frac{Q^2}{-t} \text{ for } \rho_T^0$$



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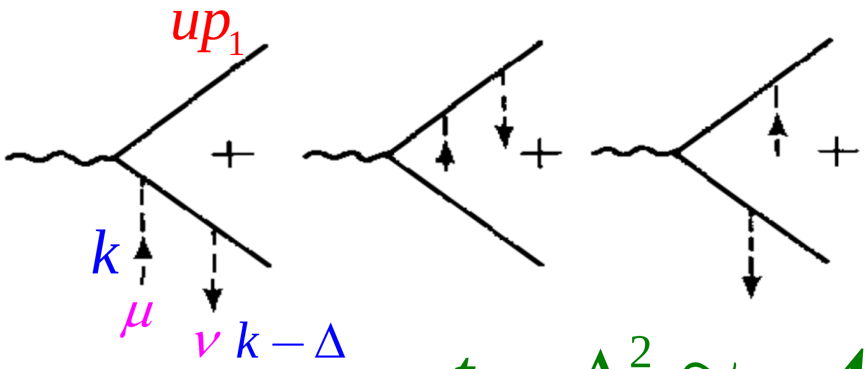

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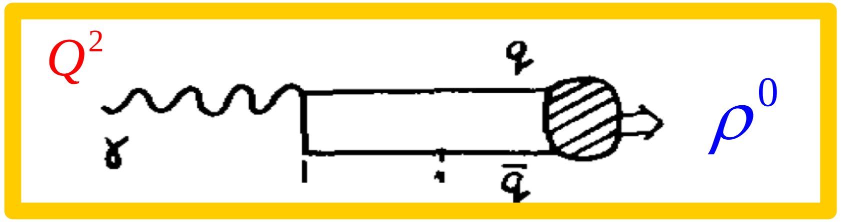
IR divergent! $\sim \log \frac{Q^2}{-t}$

nonfactorizable for ρ_T^0



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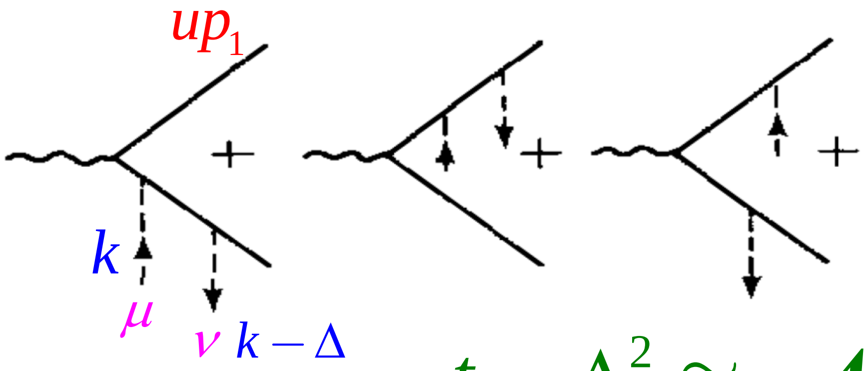

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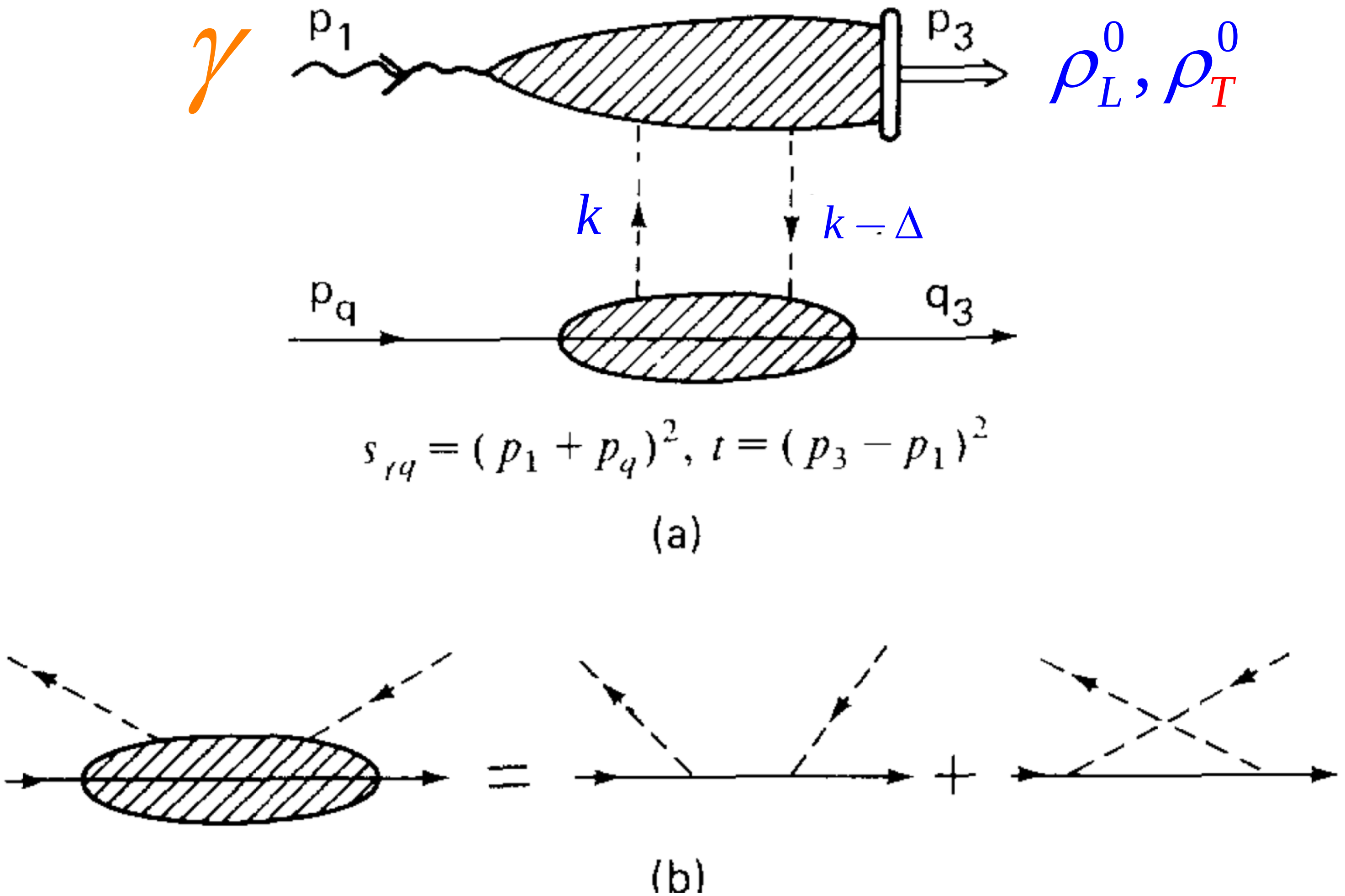
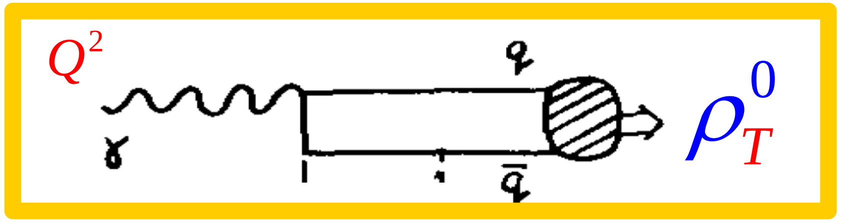
end-point $(u \to 0,\ 1)$ **behaviors:**

$$\phi_L(u) \sim u(1-u) \qquad g_T(u) \sim 1$$

$$s \gg -t \gg \Lambda^2_{\rm QCD}$$

$$t = -\frac{s}{2}(1-\cos\theta)$$





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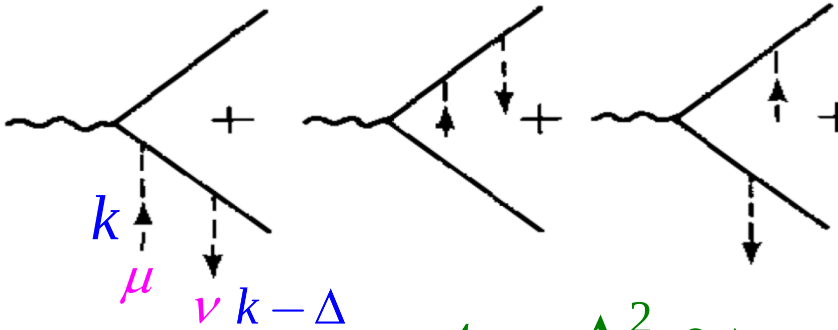

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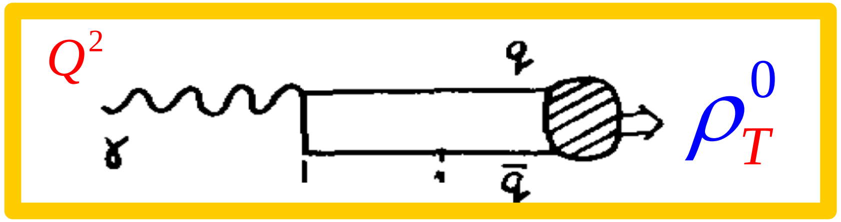

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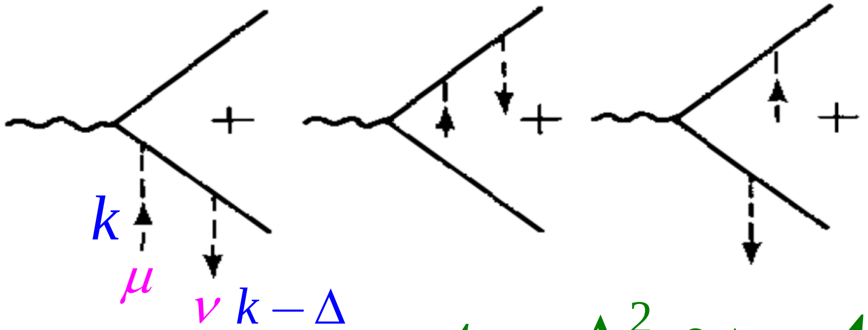

$$(k,\ \mu \rightarrow k-\Delta,\ \nu) \otimes g_T(u)$$

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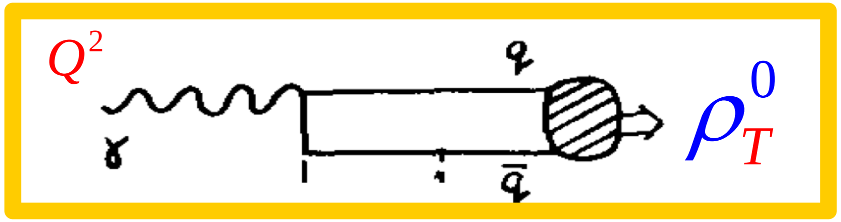
$$Q^2 \gg \Lambda_{\mathrm{QCD}}^2$$



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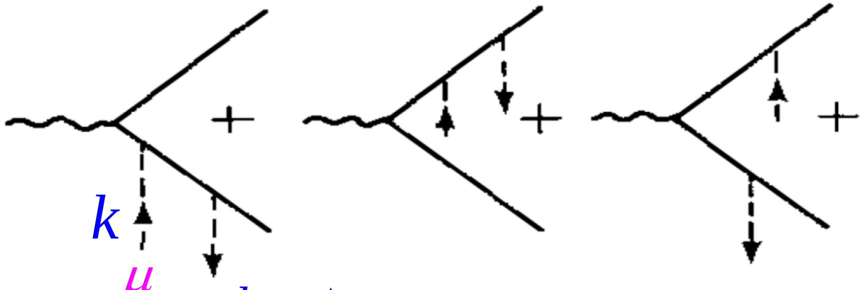

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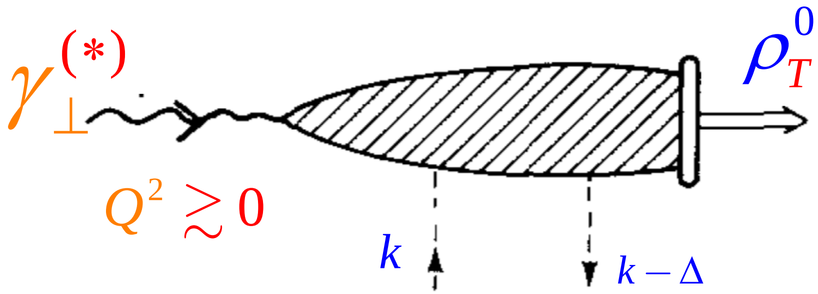
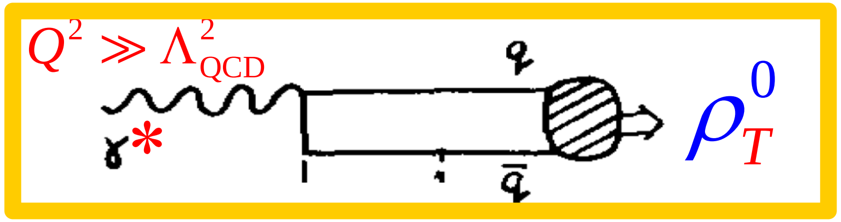


$$Q^2 \gtrsim 0$$

$$\int_0^\infty dm^2 \, \frac{\chi(m^2)}{Q^2 + m^2} = \frac{a}{Q^2 + m_V^2} + \int_{m_{th}^2}^\infty dm^2 \, \frac{\chi(m^2)}{Q^2 + m^2}$$



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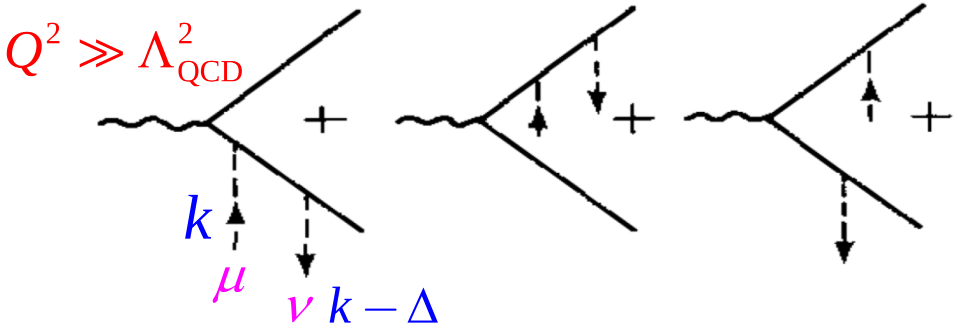
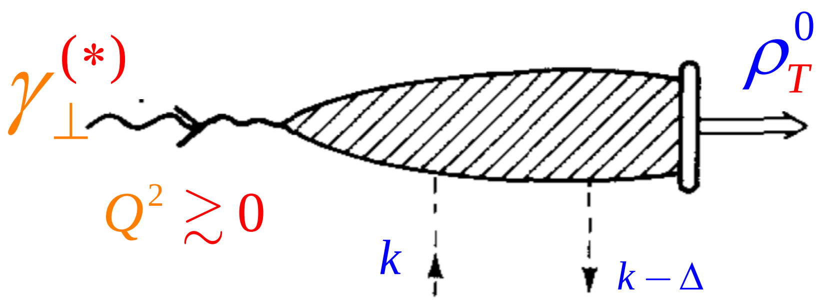


$$(k,\ \mu \to k-\Delta,\ \nu) \quad \otimes g_T(u)$$

LCSR (quark-hadron duality)



$$\int_0^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2}$$

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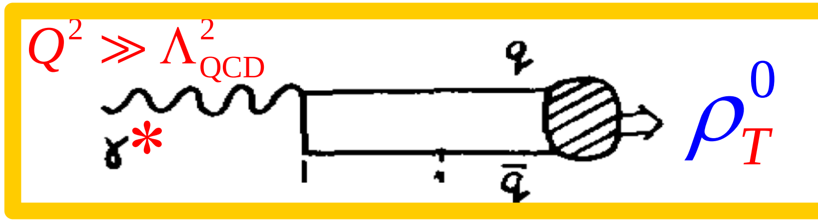

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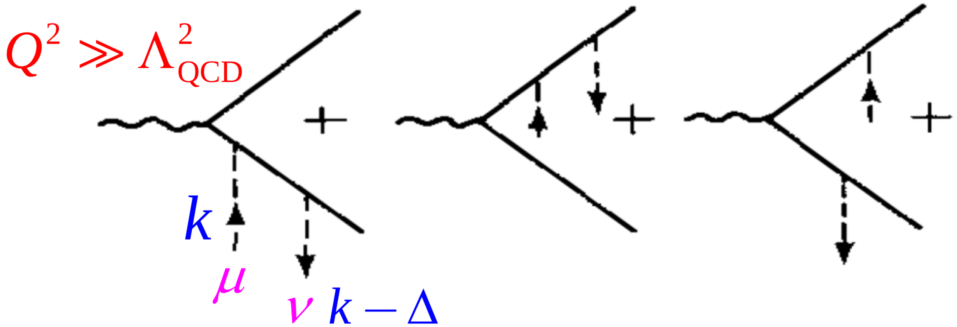
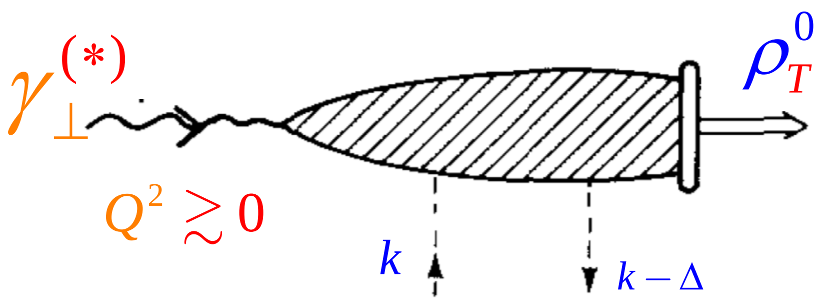


$$+ \left(k,\ \mu \to k-\Delta,\ \nu\right) \quad \otimes g_T(u)$$

$$a \propto e^{\frac{m_V^2}{M_B^2}} \int_{u_0}^{1} du \left(2 g_T^{(v)}(u) - \frac{1}{2} \frac{\partial g_T^{(a)}(u)}{\partial u} \right) e^{-\frac{(1-u)\Delta_\perp^2}{u M_B^2}}$$

LCSR (quark-hadron duality)



$$\int_0^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2}$$

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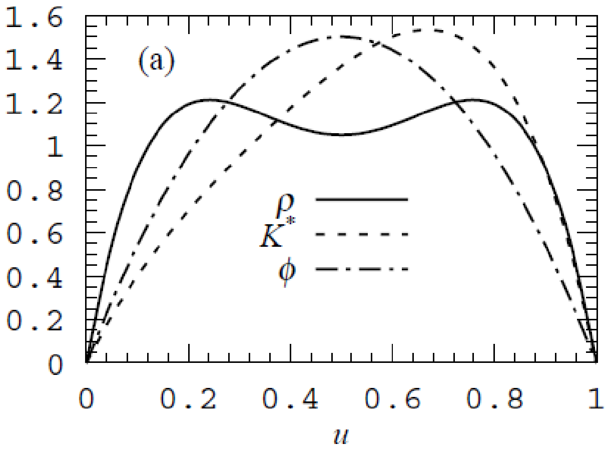
Ball, Braun, Koike, KT, NPB529 ('98) 323

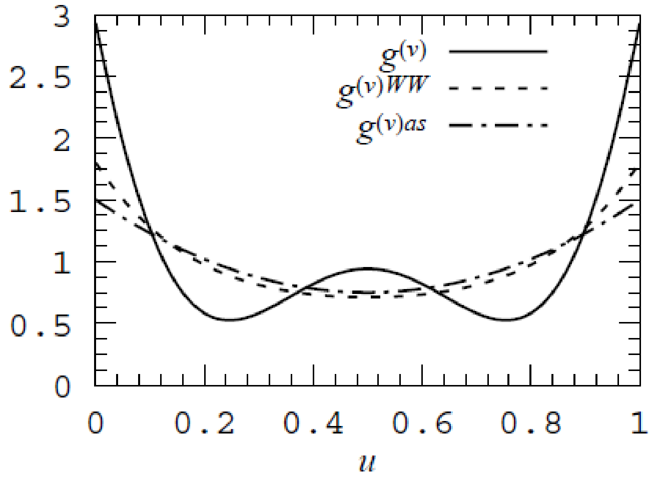
ρ meson WFs

$$\xi = u - (1-u) = 2u - 1$$

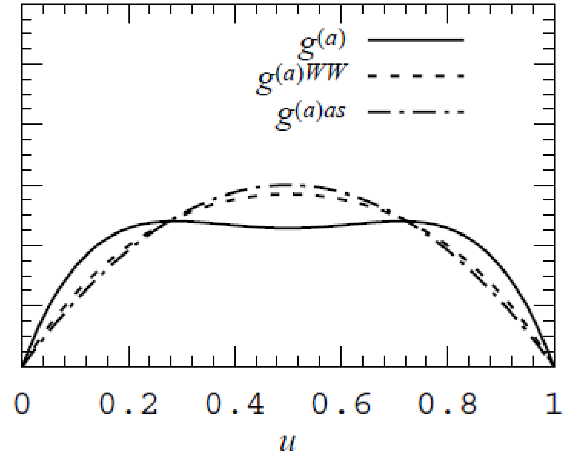
$\phi_L(u)$



$g_T^{(v)}(u)$



$g_T^{(a)}(u)$



$$\phi_L(u) = 6u(1-u)\sum_{n=0,2,4,\cdots} b_n C_n^{3/2}(2u-1) = 6u(1-u)\left(1 + b_2\frac{3}{2}(5\xi^2 - 1) + \cdots\right)$$

$$b_2 = 0.18 \pm 0.10$$

$$g_T^{(v)}(u) = \sum_{n=0,2,4,\cdots}(G_n - G_{n-1})C_n^{1/2}(2u-1)$$

$$g_T^{(a)}(u) = 8u(1-u)\sum_{n=0,2,4,\cdots}\frac{G_n - G_{n+1}}{(n+1)(n+2)}C_n^{3/2}(2u-1)$$

ρ meson WFs

$z^2 = 0$

$\xi = u - (1-u) = 2u - 1$

Ball, Braun, Koike, KT, NPB529 ('98) 323

$$\langle 0 | \bar{q}(z)\gamma_\mu q(-z) | \rho^0(p,e)\rangle = f_\rho m_\rho \, p_\mu \frac{e\cdot z}{p\cdot z}\int_0^1 du\, e^{i\xi p\cdot z}\phi_L(u)$$

$$+ f_\rho m_\rho e_{T\mu}\int_0^1 du\, e^{i\xi p\cdot z} g_T^{(v)}(u)$$

$$\mathrm{P}\exp\left[ig\int_{-1}^{1} dt z_\mu A^\mu(tz)\right]$$

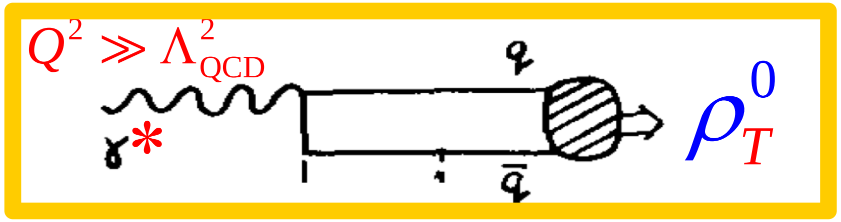
$$\langle 0 | \bar{q}(z)\gamma_\mu\gamma_5 q(-z) | \rho^0(p,e)\rangle = \frac{1}{2} f_\rho m_\rho \epsilon_{\mu\nu\alpha\beta} e_T^\nu p^\alpha z^\beta \int_0^1 du\, e^{i\xi p\cdot z} g_T^{(a)}(u)$$

$$\phi_L(u) = 6u(1-u)\left[1 + \sum_{n=1}^{\infty} b_{2n} C_{2n}^{3/2}(2u-1)\right] = 6u(1-u)\left[1 + b_2 \frac{3}{2}(5\xi^2 - 1) + \cdots\right]$$

$$g_T^{(v)}(u) = \frac{3}{4}\left(1+\xi^2\right) + b_2 \frac{3}{7}(3\xi^2 - 1) + \cdots$$

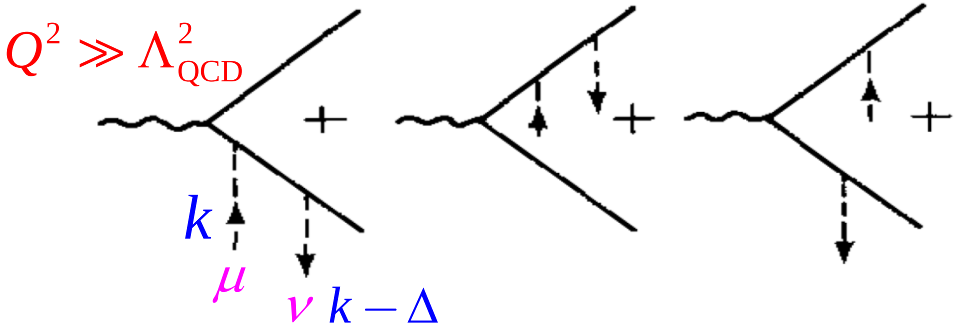
$$g_T^{(a)}(u) = 6u(1-u)\left[1 + b_2 \frac{1}{4}(5\xi^2 - 1) + \cdots\right]$$

$$b_2 = 0.18 \pm 0.10$$



"Impact factor"

$$Q^2 \gg \Lambda^2_{QCD}$$



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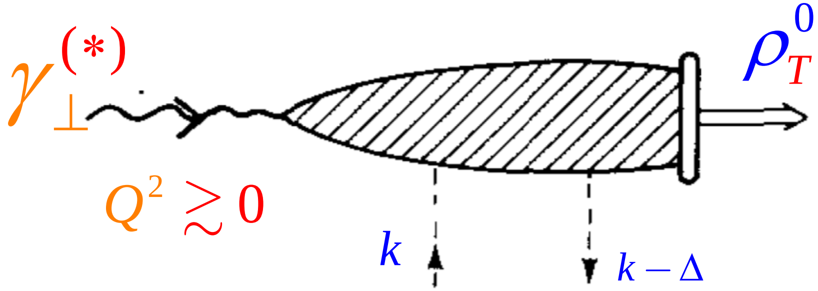
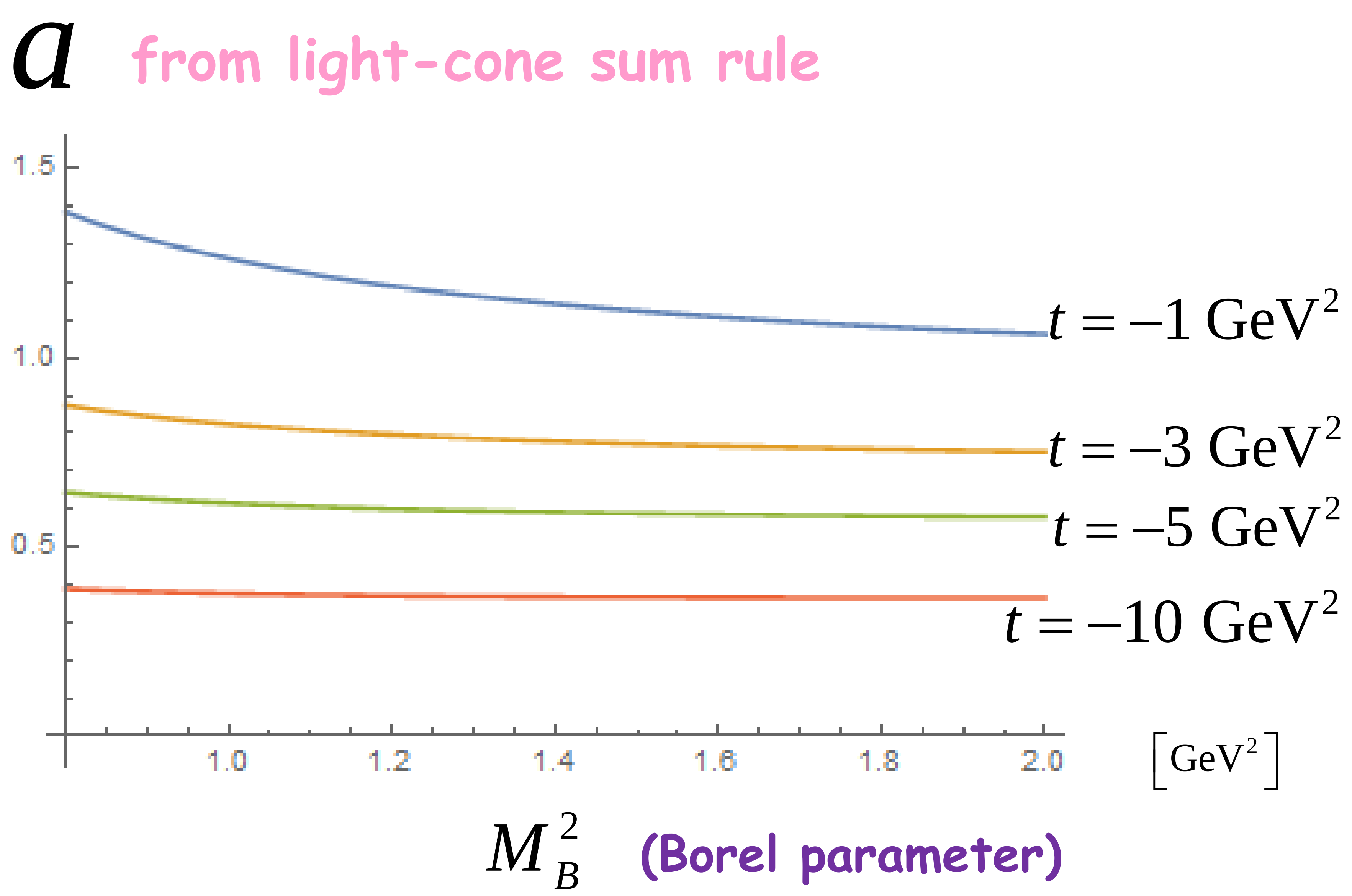
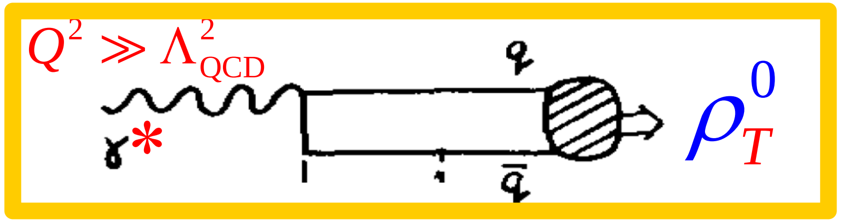
LCSR (quark-hadron duality)



$$\int_0^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2} = \frac{a}{Q^2+m_V^2} + \int_{m_{th}^2}^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2}$$



a
from light-cone sum rule
1.5
1.0
0.5
$t = -1\ \mathrm{GeV}^2$
$t = -3\ \mathrm{GeV}^2$
$t = -5\ \mathrm{GeV}^2$
$t = -10\ \mathrm{GeV}^2$
1.0
1.2
1.4
1.6
1.8
2.0
$[\mathrm{GeV}^2]$
M_B^2 (Borel parameter)



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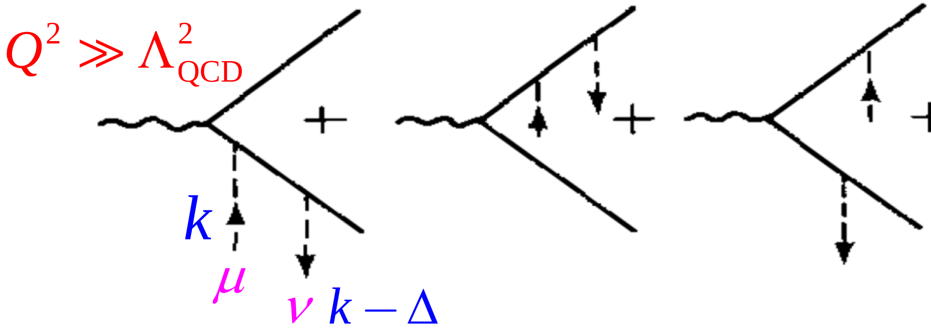

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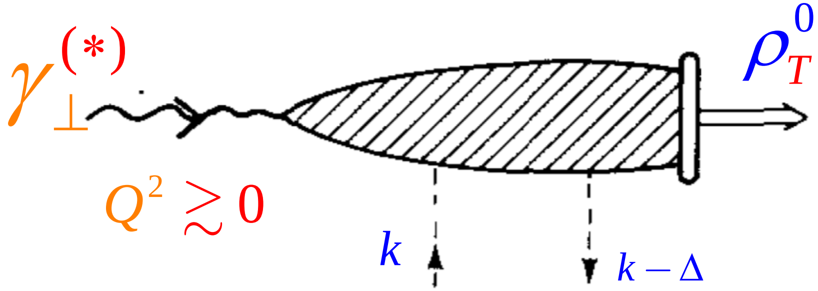
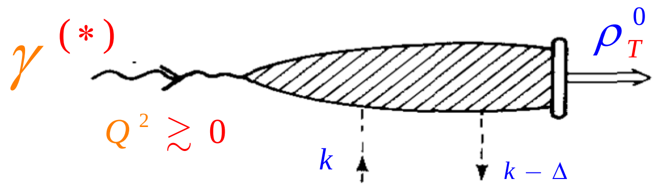
LCSR (quark-hadron duality)



$$\int_0^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2}$$

$$= \frac{a}{Q^2+m_V^2} + \int_{m_{th}^2}^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2}$$



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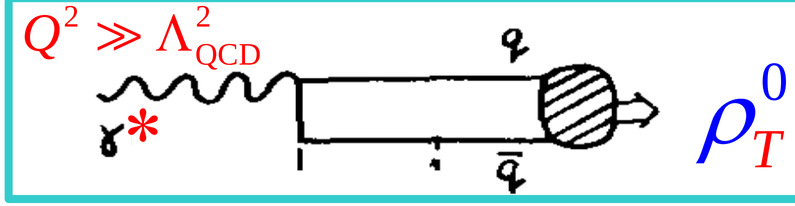
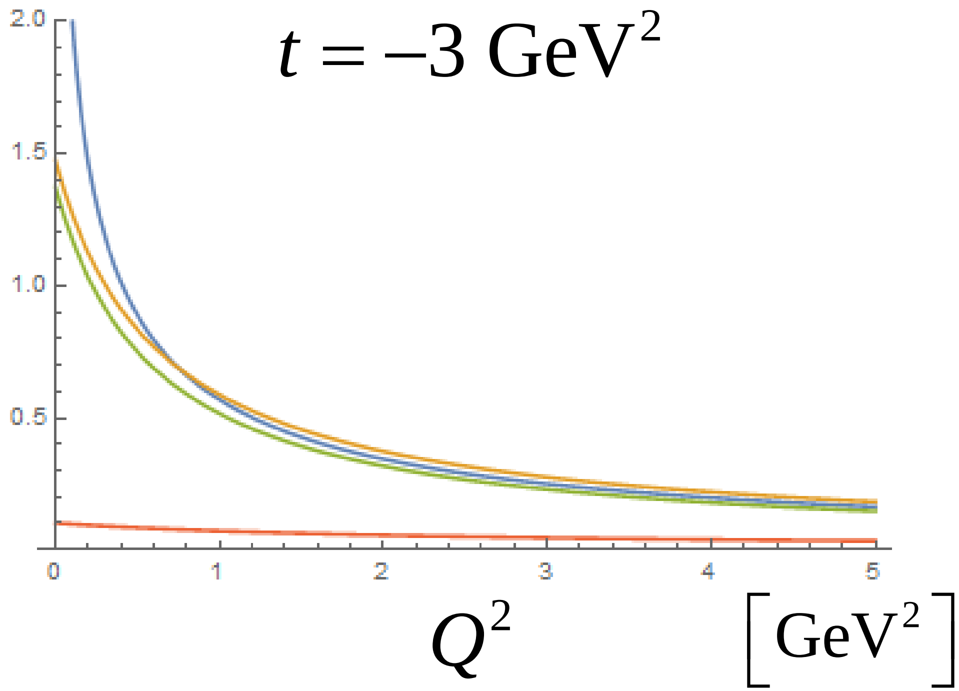
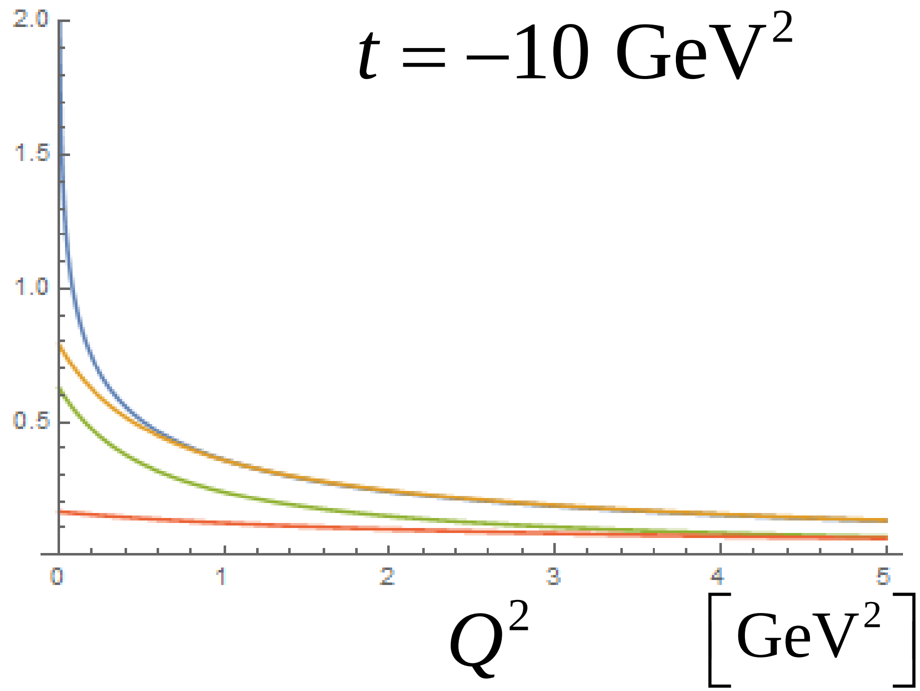
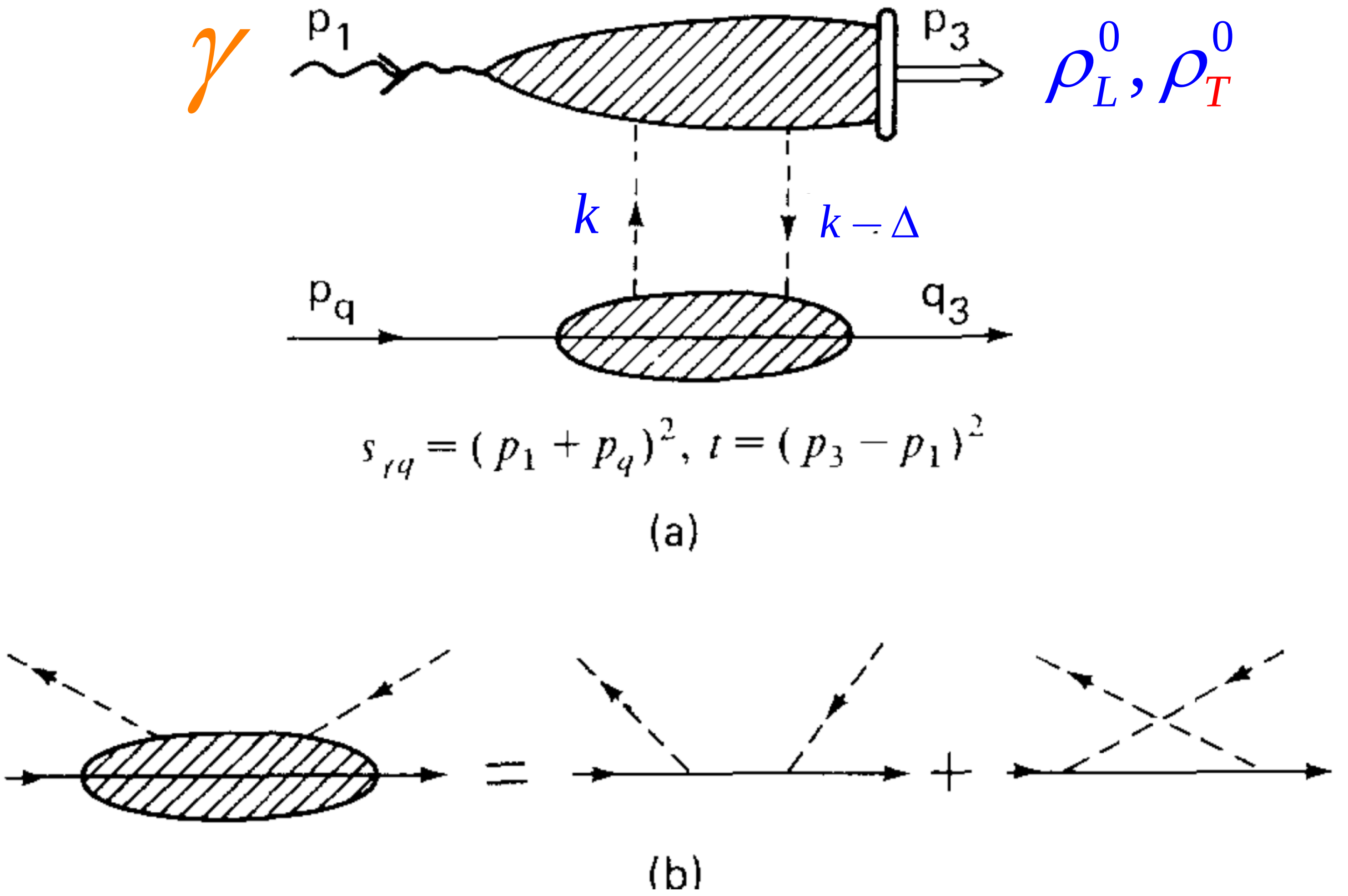






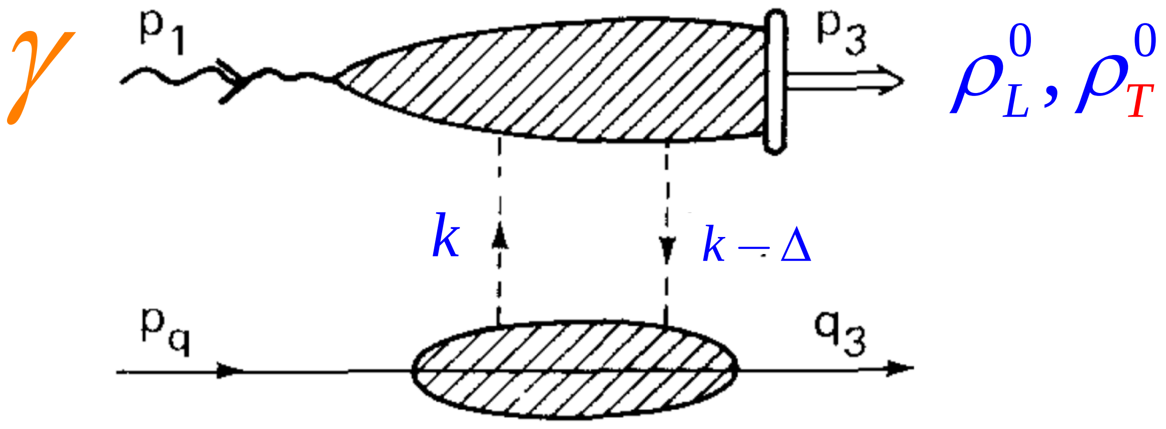
$$s \gg -t \gg \Lambda_{\mathrm{QCD}}^2$$

$$t = -\frac{s}{2}\left(1-\cos\theta\right)$$

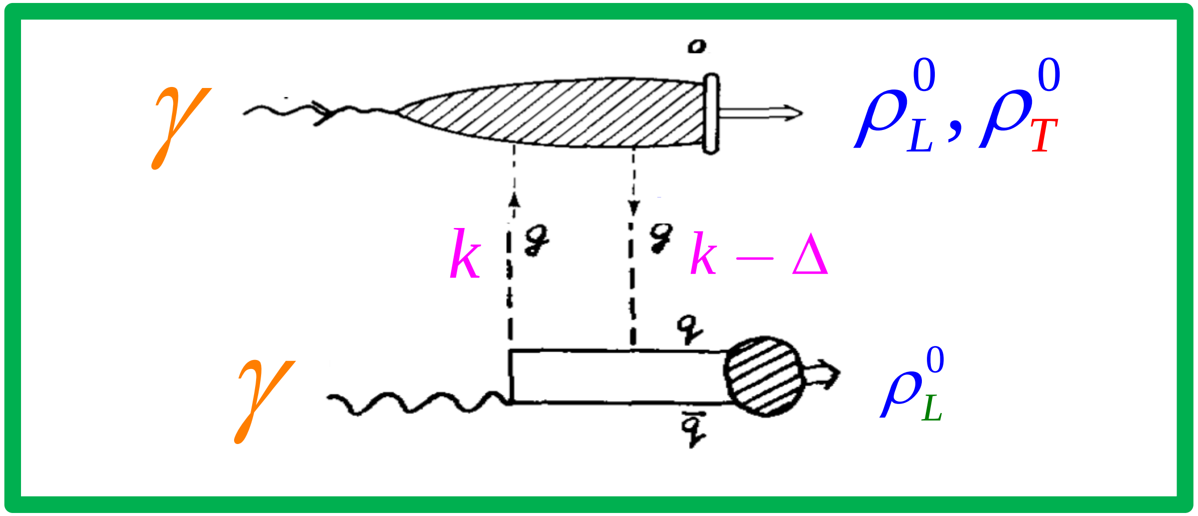


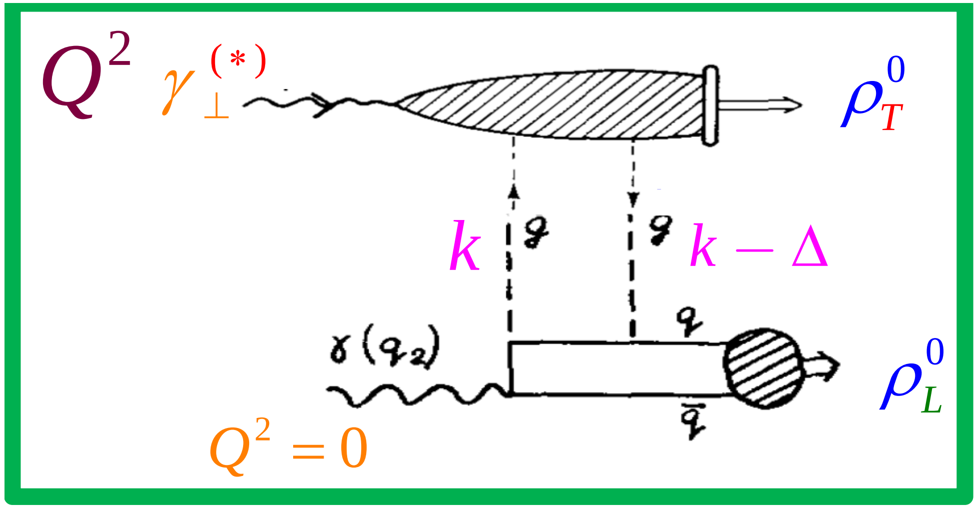
$$s \gg -t \gg \Lambda^2_{\text{QCD}}$$

$$t = -\frac{s}{2}(1-\cos\theta)$$



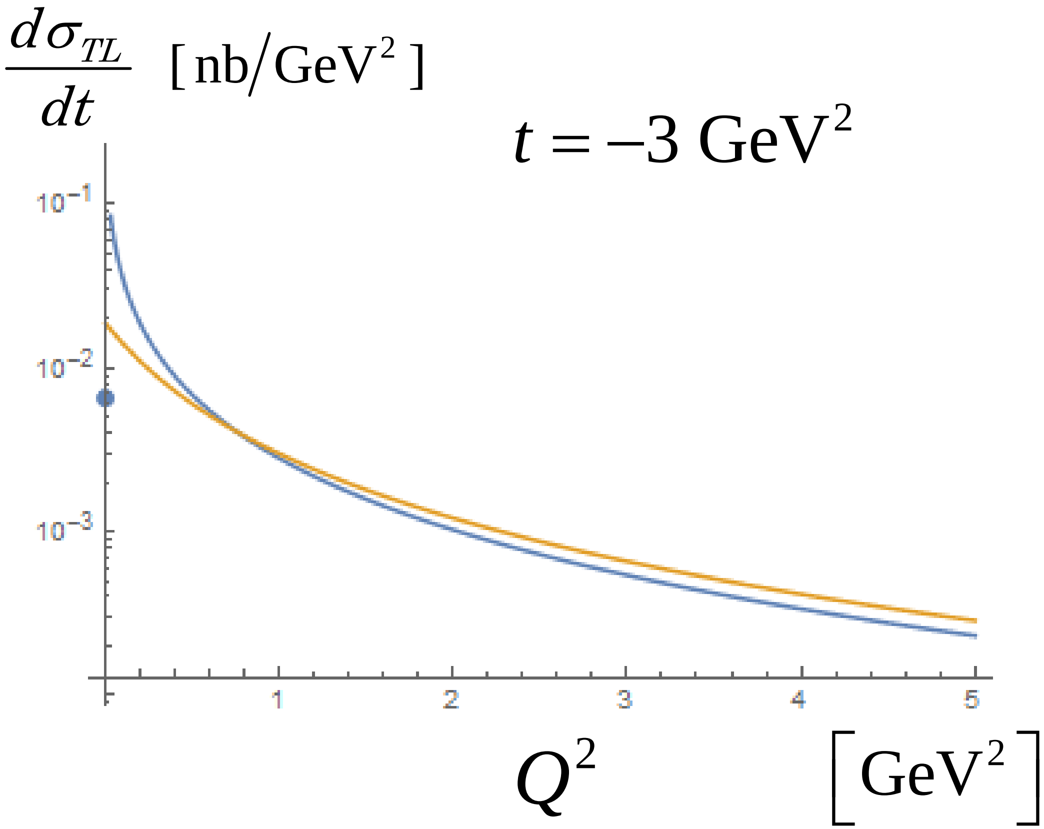
$$s_{\gamma q} = (p_1 + p_q)^2,\ t = (p_3 - p_1)^2$$

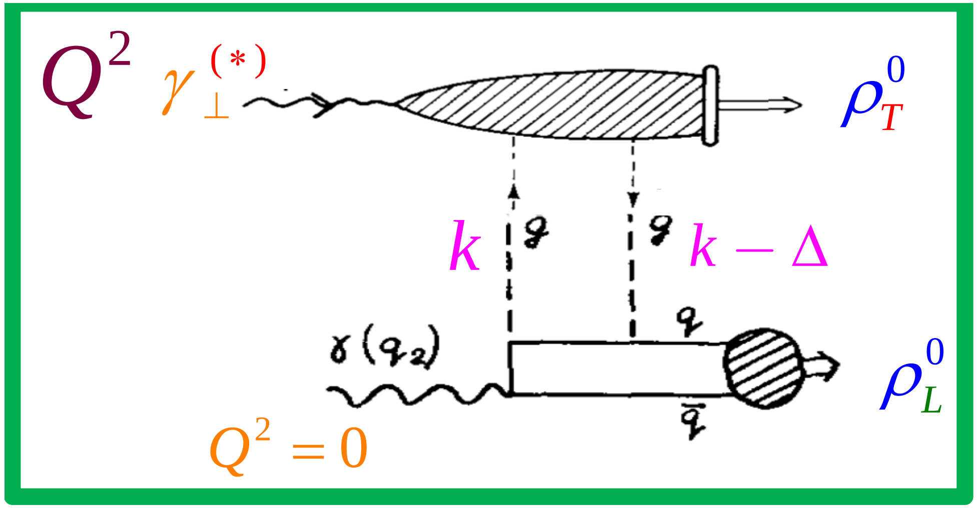




$$s \gg -t \gg \Lambda^2_{\mathrm{QCD}}$$

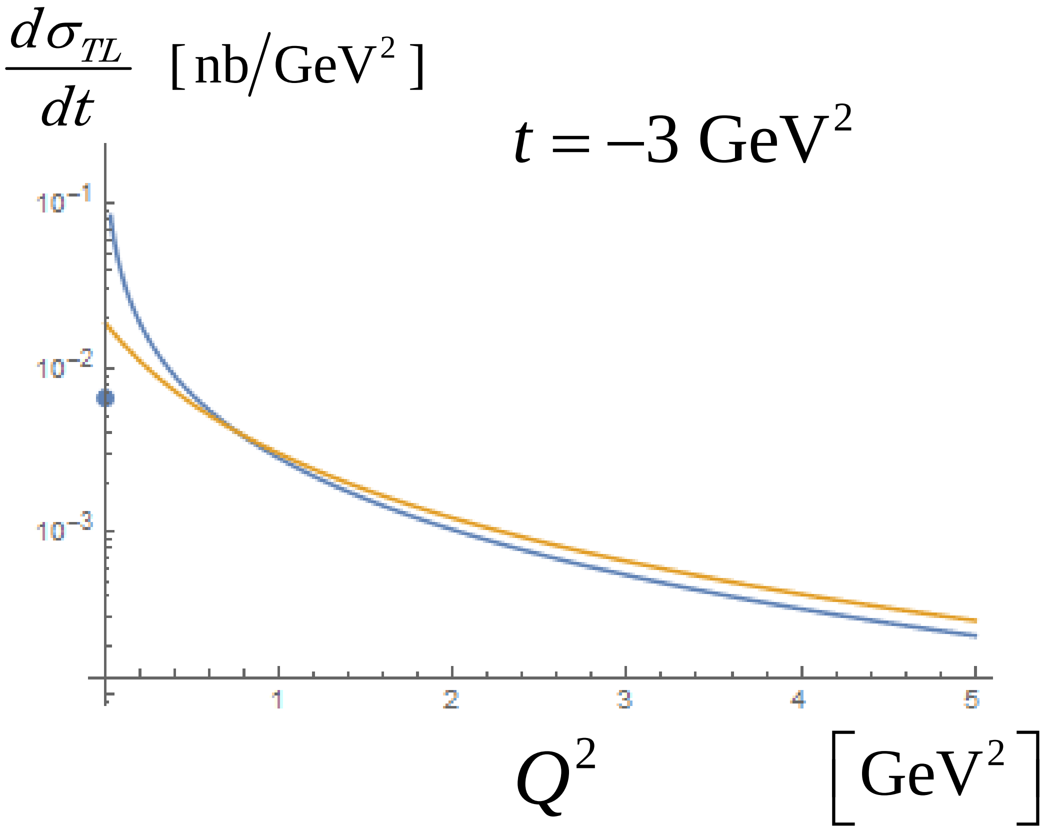
$$s = 100\ \mathrm{GeV}^2$$

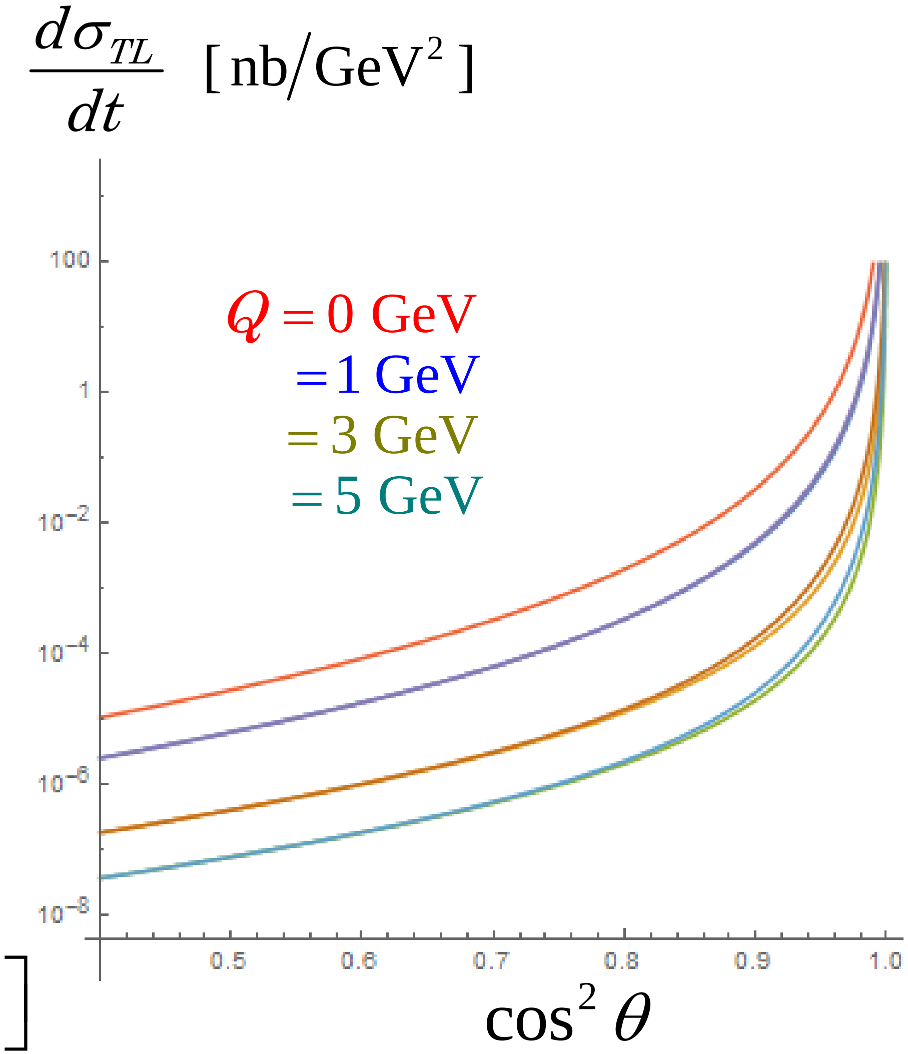


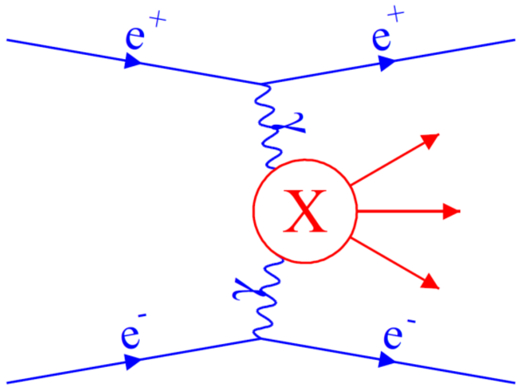


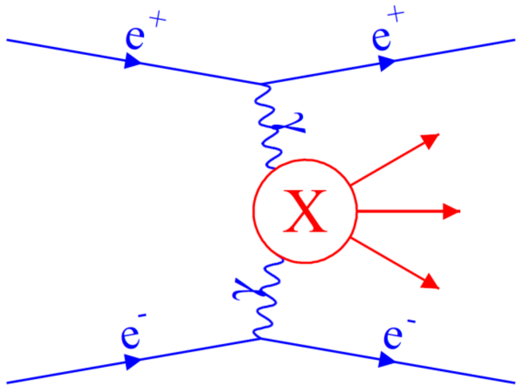
$$s \gg -t \gg \Lambda_{QCD}^2$$

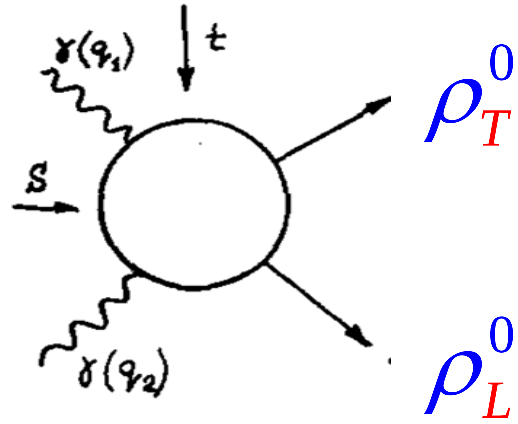
$$s = 100\ \text{GeV}^2$$

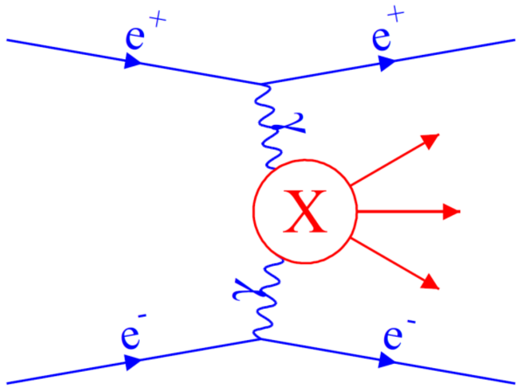





e+
e+
X
e-
e-

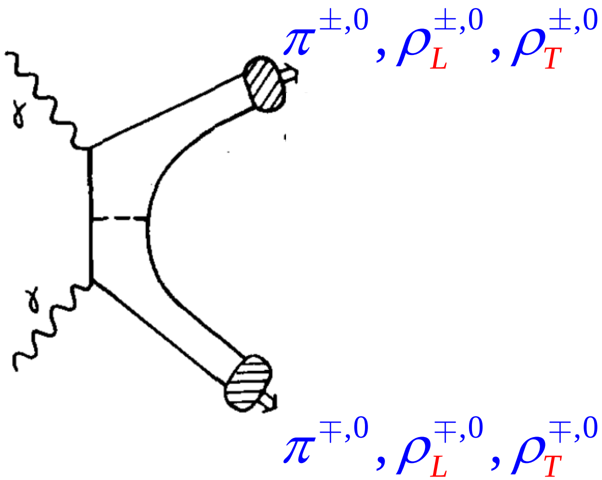

e+
e+
X
e-
e-

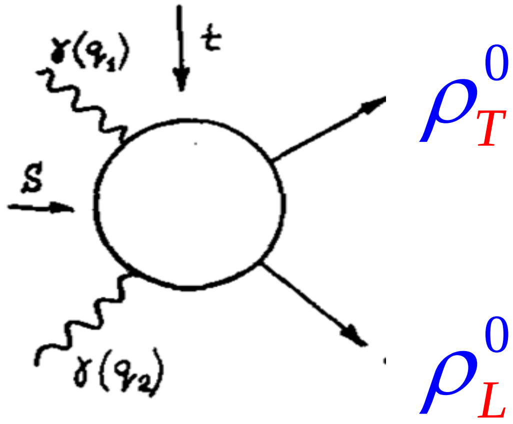

γ(q1)
t
s
γ(q2)
ρ0T
ρ0L



$$s \sim -t \gg \Lambda^2_{\rm QCD}$$

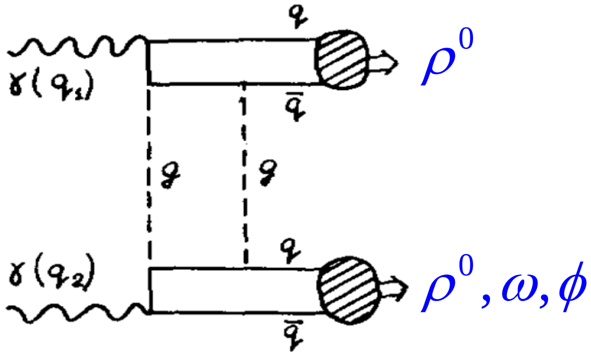
$q\bar{q}$ **exchange**

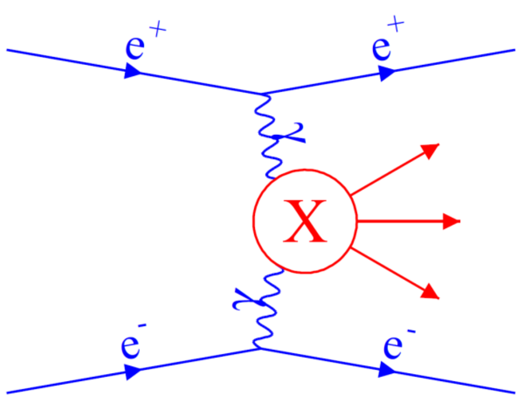


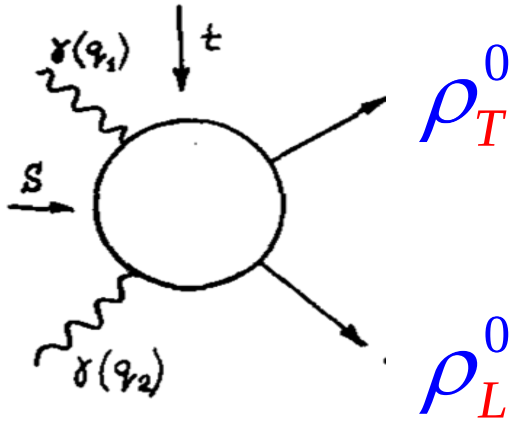


$$s \gg -t \gg \Lambda^2_{\rm QCD}$$

gg **exchange**





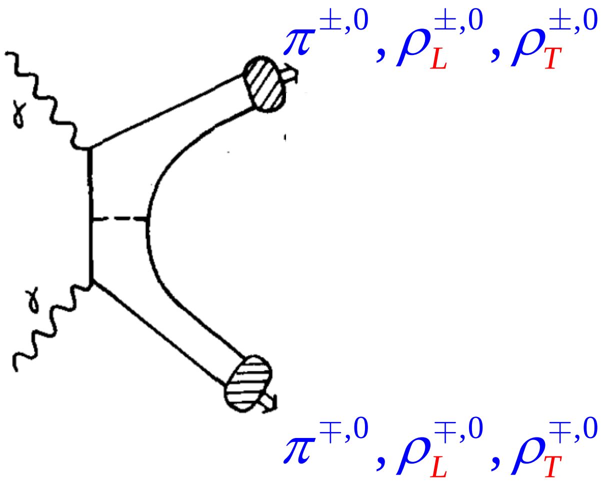


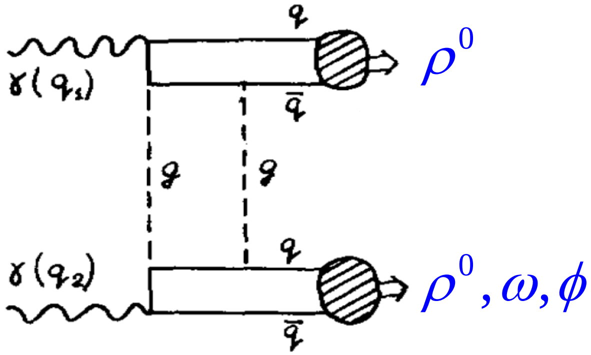
$$s \sim -t \gg \Lambda^2_{\rm QCD}$$

$q\bar{q}$ exchange

$$s \gg -t \gg \Lambda^2_{\rm QCD}$$

gg exchange



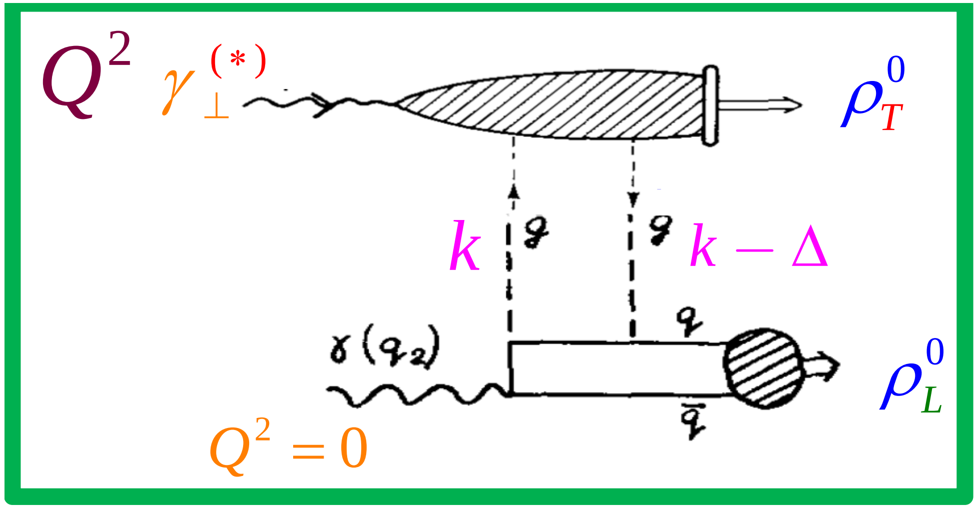


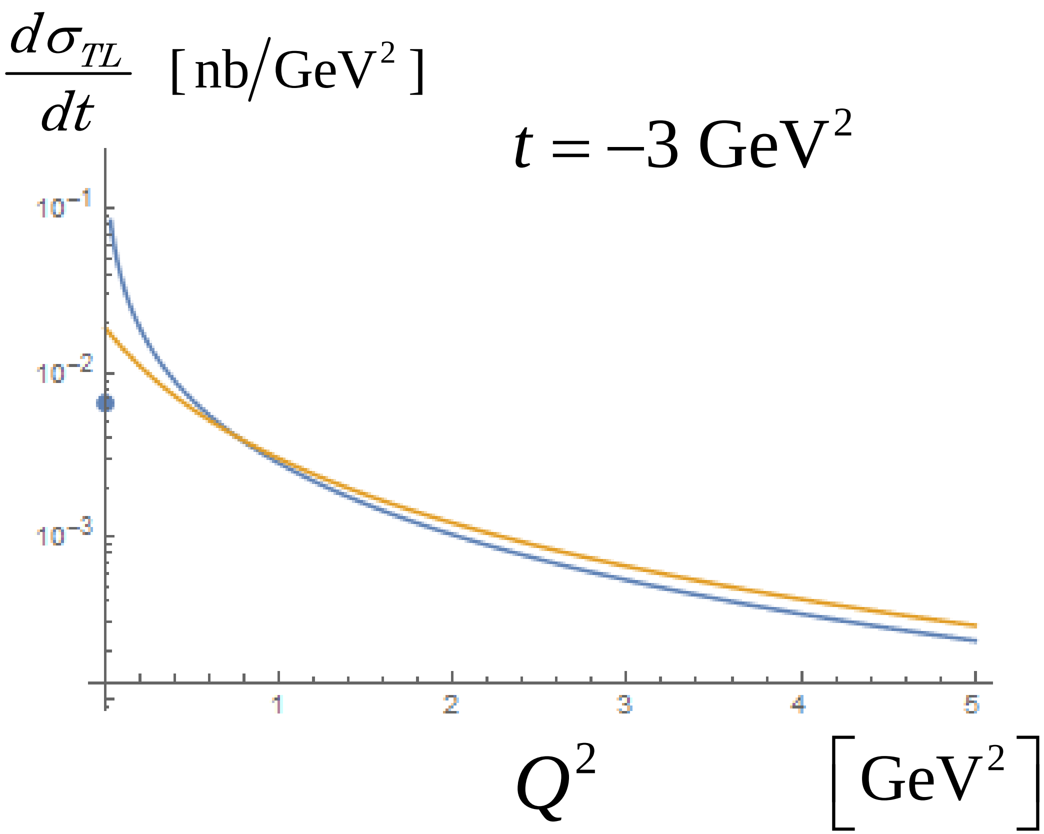
first QCD calculation for

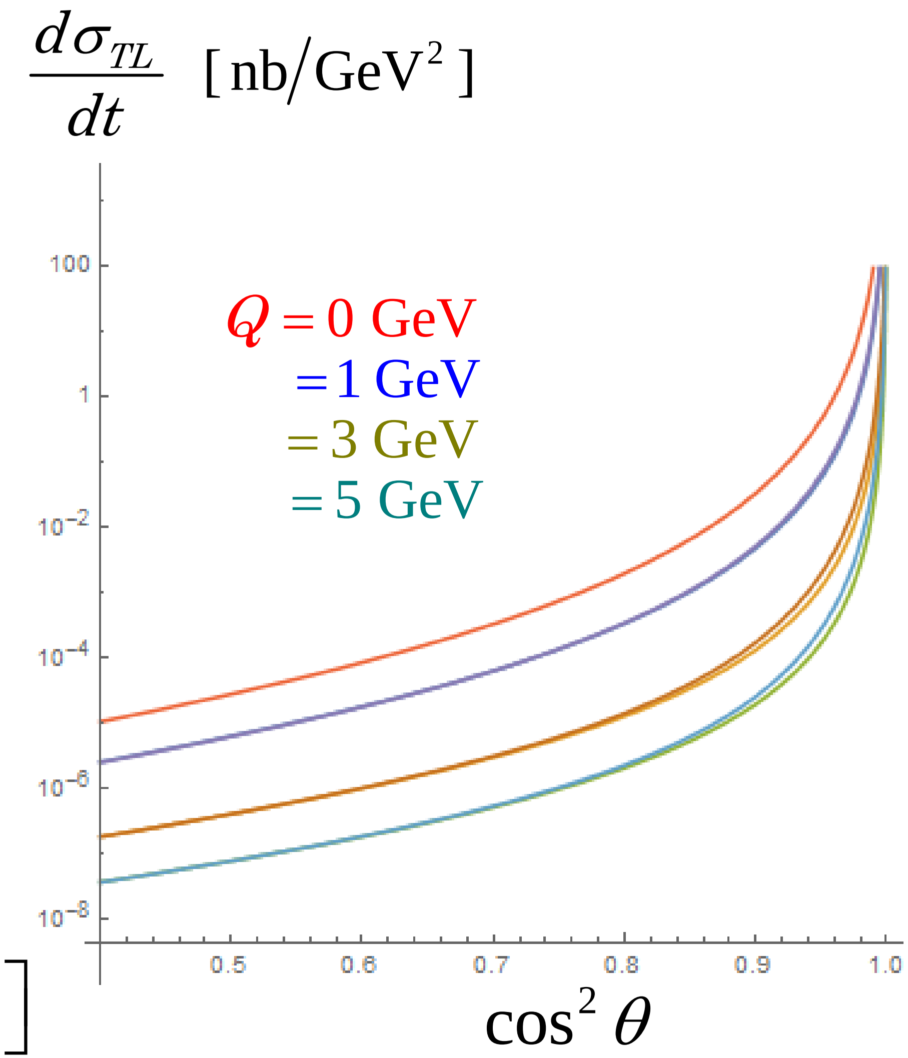
$$\gamma\gamma \longrightarrow \rho_L^0 \rho_T^0$$

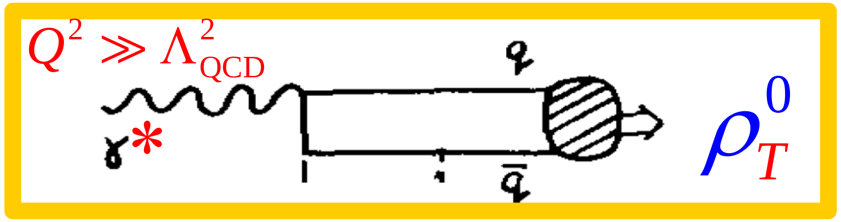
$s \gg -t \gg \Lambda^2_{QCD}$

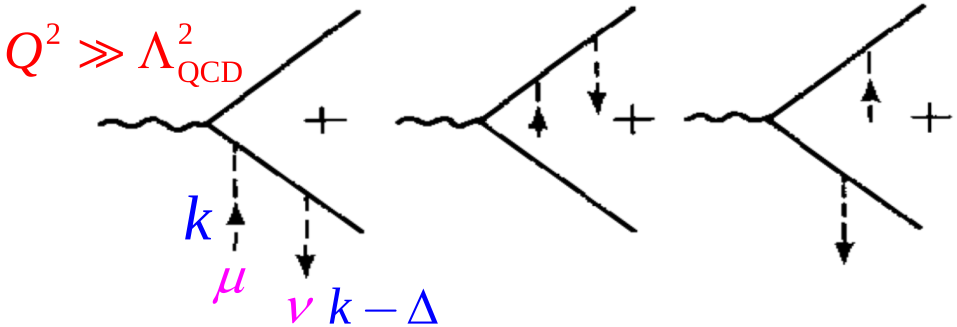
$s = 100\ \text{GeV}^2$





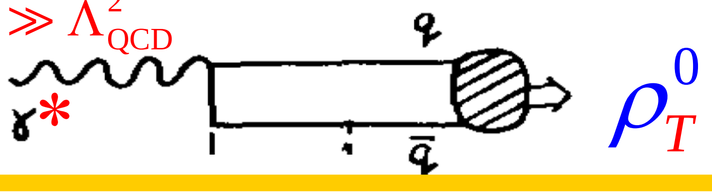




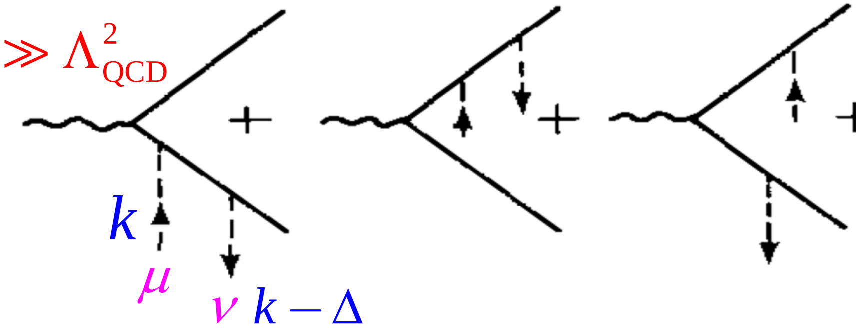


$$+ \; (k,\ \mu \to k-\Delta,\ \nu) \quad \otimes g_T(u)$$

$Q^2 \gg \Lambda^2_{\text{QCD}}$

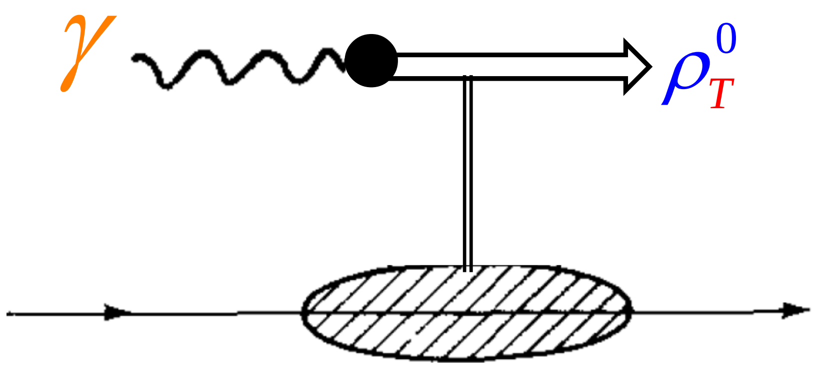


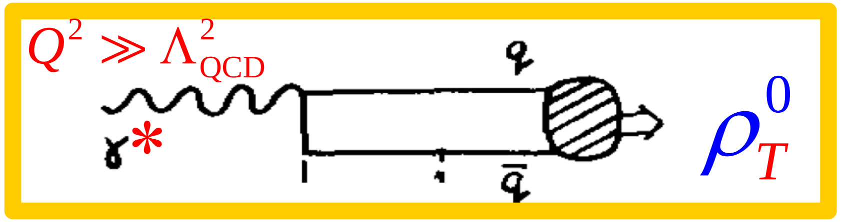
$Q^2 \gg \Lambda^2_{\text{QCD}}$



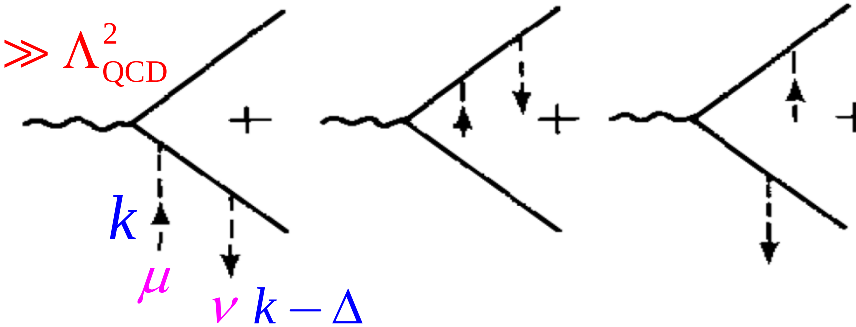
$$(k,\ \mu \rightarrow k-\Delta,\ \nu) \quad \otimes g_T(u)$$

$Q^2 \sim 0$ VMD $\oplus$ pomeron

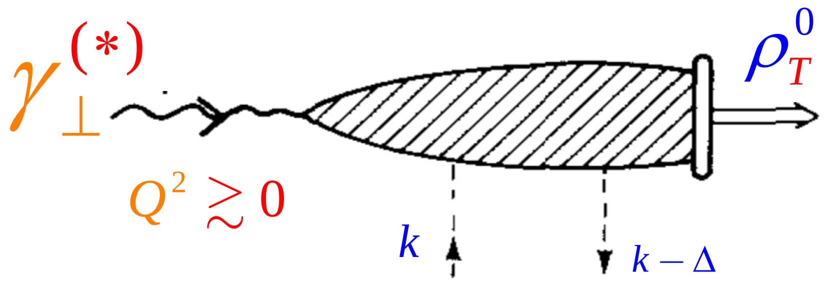




$Q^2 \gg \Lambda^2_{QCD}$



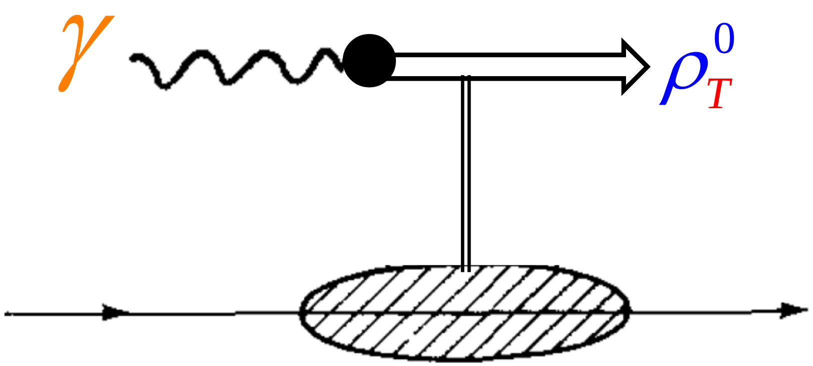
$$\left(k,\ \mu \to k-\Delta,\ \nu\right) \quad \otimes g_T(u)$$

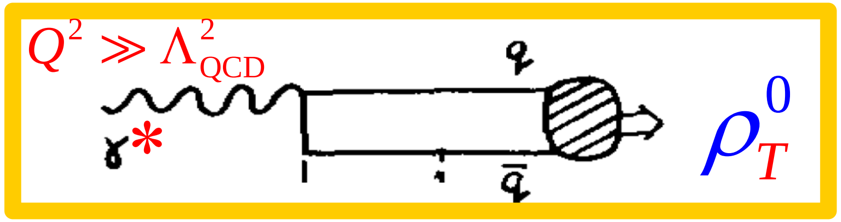


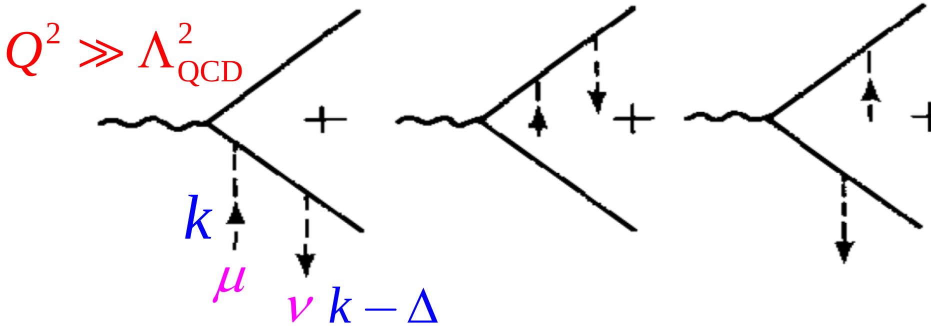
$$= \frac{a}{Q^2+m_V^2} + \int_{m_{th}^2}^{\infty} dm^2 \, \frac{\chi(m^2)}{Q^2+m^2}$$

$$a \propto e^{\frac{m_V^2}{M^2}} \int_{u_0}^{1} du \left(2 g_T^{(v)}(u) - \frac{1}{2} \frac{\partial g_T^{(a)}(u)}{\partial u} \right) e^{-\frac{(1-u)\Delta_\perp^2}{uM^2}}$$

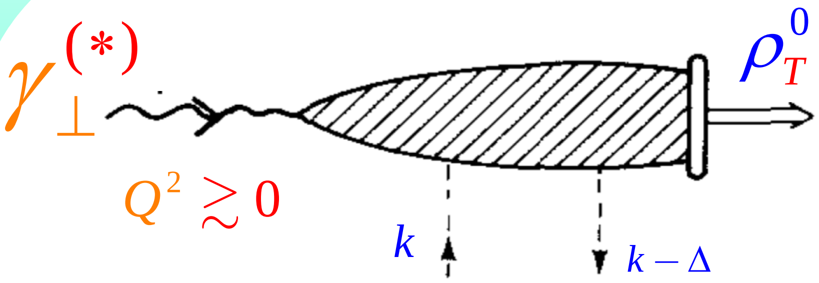
$Q^2 \sim 0$ VMD ⊕ pomeron







$$+\ (k,\ \mu \to k-\Delta,\ \nu) \ \otimes g_T(u)$$



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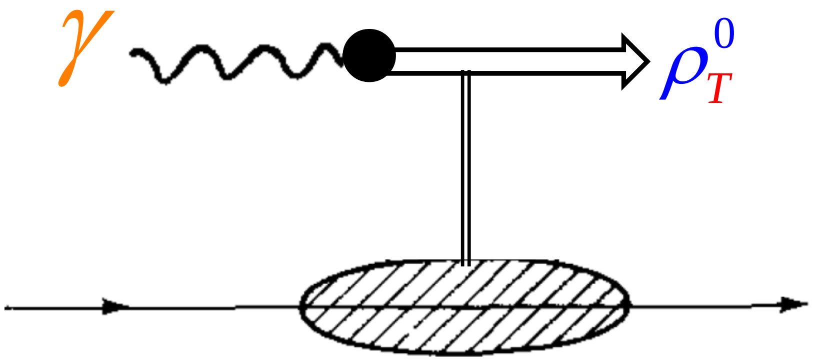
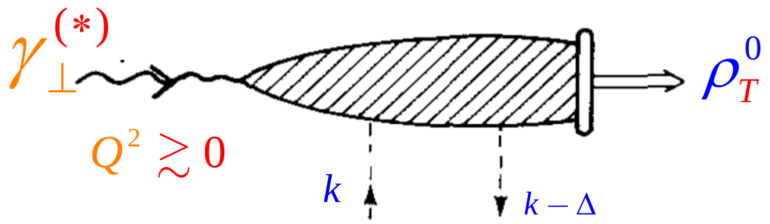
$Q^2 \sim 0$ VMD $\oplus$ pomeron



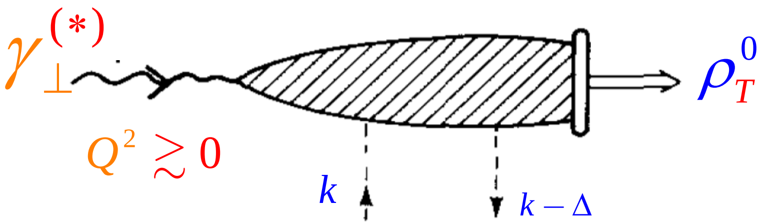
Summary:



LCSR calculation for the $\gamma \to \rho^0$ impact factor

allows us to obtain ``interpolating formula"
between pQCD for $Q^2 \gg \Lambda_{\rm QCD}^2$ **and** VMD $\oplus$ pomeron for $Q^2 \sim 0$

Summary:



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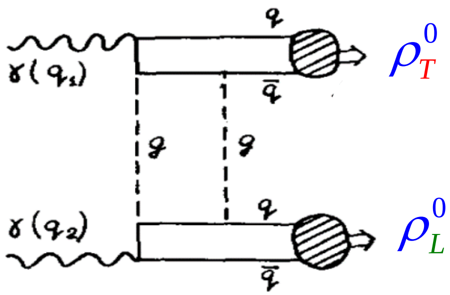
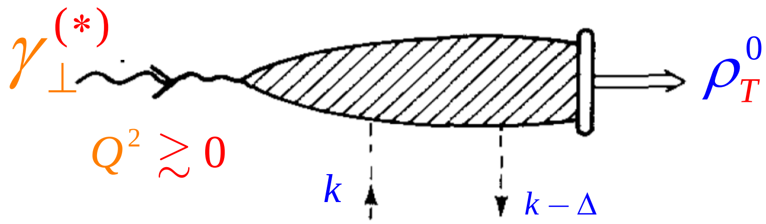


first QCD calculation for $\frac{d\sigma_{TL}}{dt}$

$\gamma\gamma \to \rho_L^0 \rho_T^0$

Summary:



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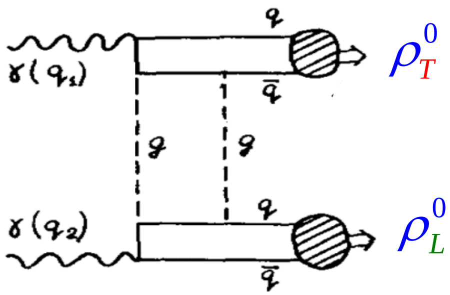

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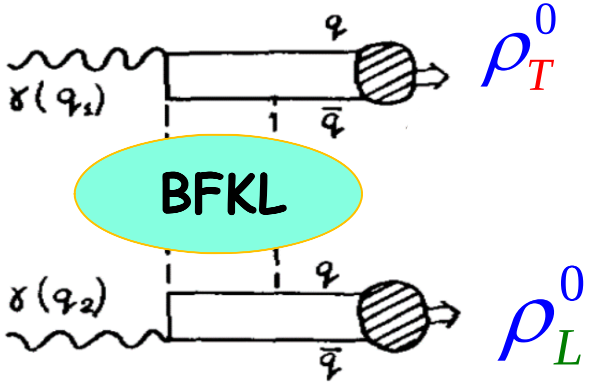
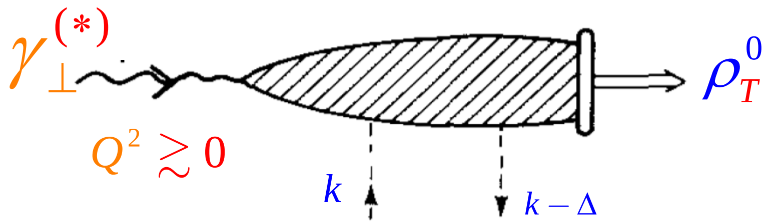
$\gamma\gamma \to \rho_L^0 \rho_T^0$

ToDo 1:



Summary:



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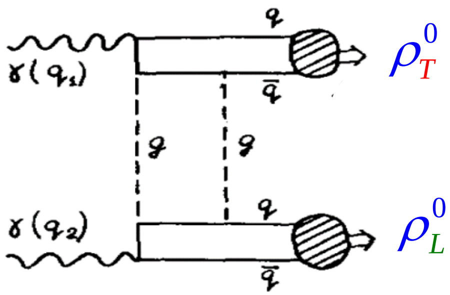

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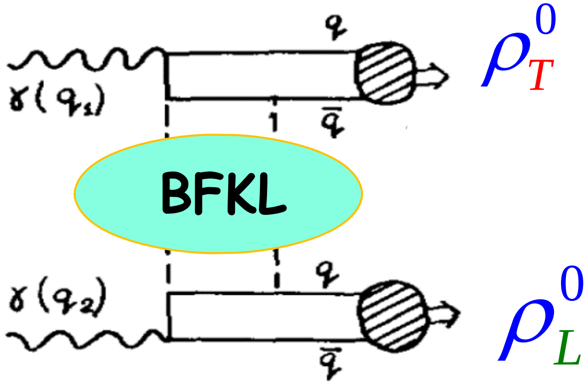

ToDo 2: