Towards the N^{3}LO evolution of parton distributions

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Introduction

This talk is about 4-loop splitting functions governing evolution of N^{3}LO parton distribution functions

DGLAP equation:

$$\frac{d}{d\ln\mu^2} f_i(x,\mu^2) = \sum_j \Big[P_{ij}(a_s(\mu^2)) \otimes f_j(\mu^2) \Big](x)$$

$$P_{ij} = a_s P_{ij}^{(0)} + a_s^2 P_{ij}^{(1)} + a_s^3 P_{ij}^{(2)} + a_s^4 P_{ij}^{(3)} + \ldots$$

Do we need this order?

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Do we need this order? Yes!

Precision physics at the LHC

No signals beyond the SM
Breakthrough would come from precision physics(?)

NNLO QCD corrections calculated for many processes

Even N^3LO, e.g., inclusive $gg \to H$

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger '16

N^3LO inclusive DIS: Moch, Vermaseren, Vogt '05; for F_3 '08
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Missing N^{3}LO PDFs

N^{3}LO $gg \to H$ computed with NNLO PDFs

δ(scale)	δ(trunc)	δ(PDF-TH)	δ(EW)	$\delta(t, b, c)$	$\delta(1/m_t)$
$^{+0.10\ \mathrm{pb}}_{-1.15\ \mathrm{pb}}$	$\pm$0.18 pb	$\pm$0.56 pb	$\pm$0.49 pb	$\pm$0.40 pb	$\pm$0.49 pb
$^{+0.21\%}_{-2.37\%}$	$\pm$0.37%	$\pm$1.16%	$\pm$1%	$\pm$0.83%	$\pm$1%

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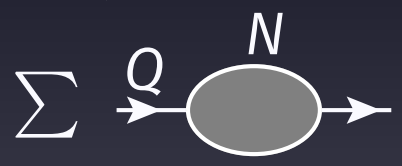
$$\gamma_{ab}(N) = -\int_0^1 dx\, x^{N-1} P_{ab}(x)$$

can be computed by (poles of) massless propagator-type integrals with N-dependence (details later)



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3-loop full-N (hence full-x): Moch, Vermaseren, Vogt '04; for polarized '14

4-loop full N-dependence is too difficult...

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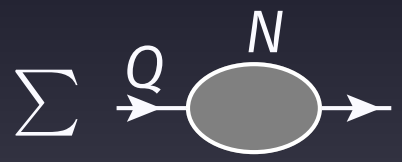

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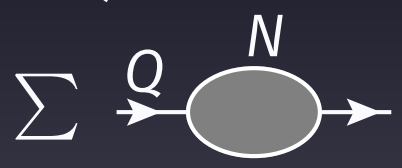

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Fix $N = 2, 4, 6, \ldots$
$\rightarrow$ Just massless propagator-type integrals

$\sum$ 

(fixed N)

Computable at 4-loops

From values at fixed N
we could get approximation
or reconstruction

3-loop: exact vs. approx.
(non-singlet, n_f-independent part)
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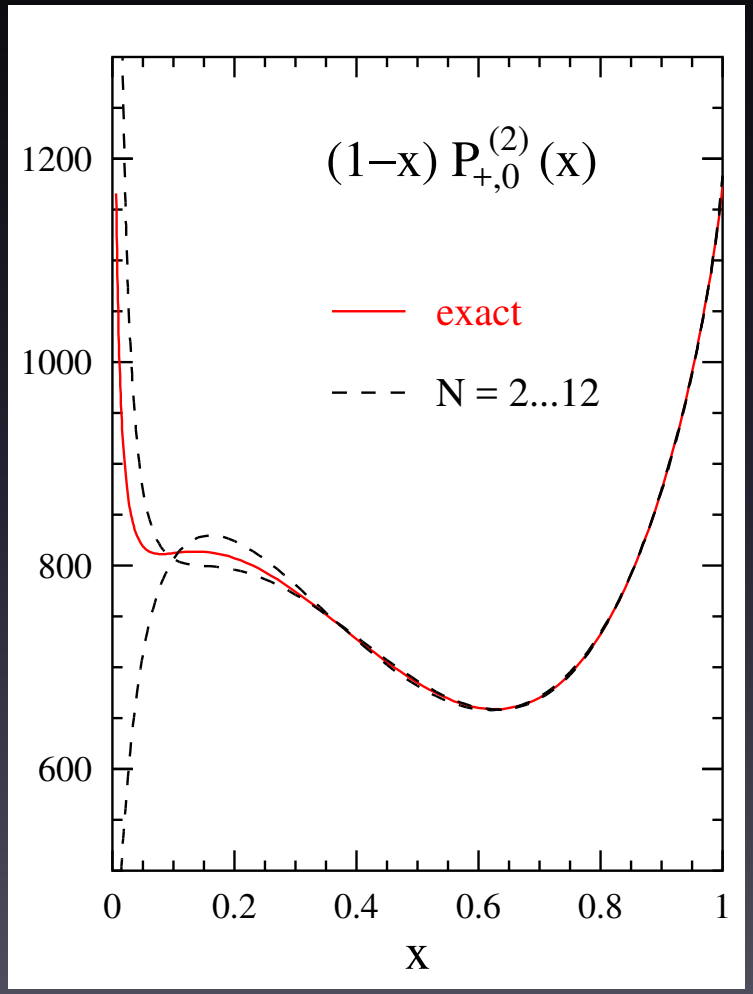

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The Forcer program

Analytically performs 4-loop massless propagator-type integrals

Ruijl, TU, Vermaseren '17; `https://github.com/benruijl/forcer`
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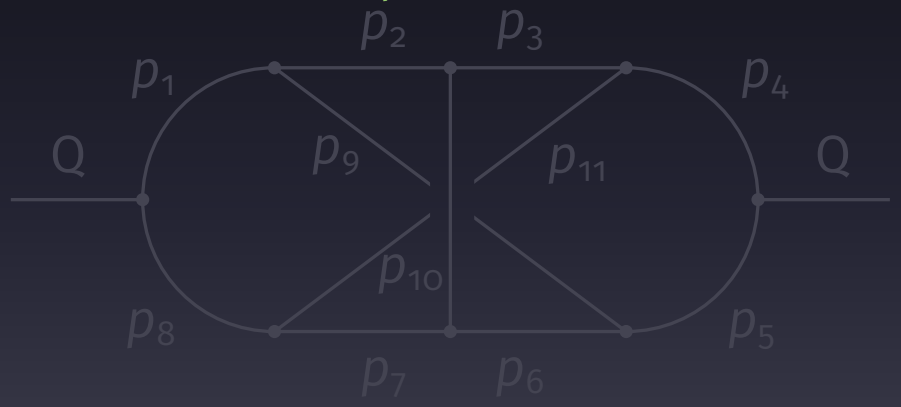

$Q^2 \neq 0$

$$\int d^D p_1 d^D p_2 d^D p_3 d^D p_4 \frac{(2Q\cdot p_2)^{-n_{12}}(2p_1\cdot p_4)^{-n_{13}}(2Q\cdot p_3)^{-n_{14}}}{(p_1)^{n_1}\cdots(p_{10})^{n_{10}}}$$

Performance: 4-loop QCD β-function < 3 min.

(background field method, $\xi = 0$, modern 32 core machine)

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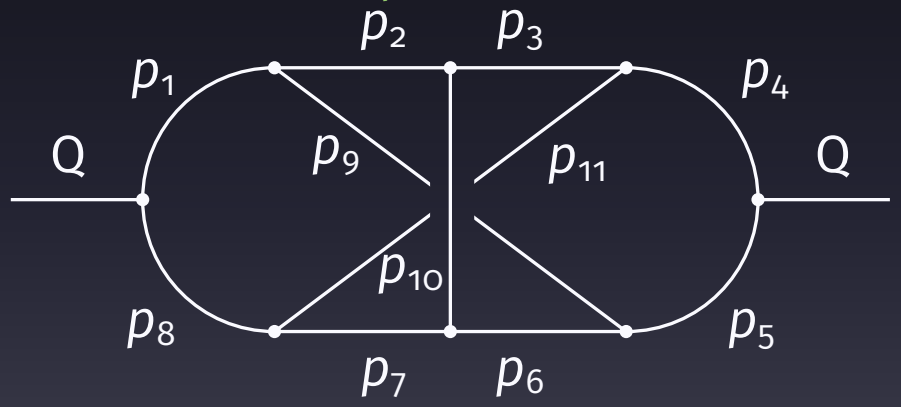

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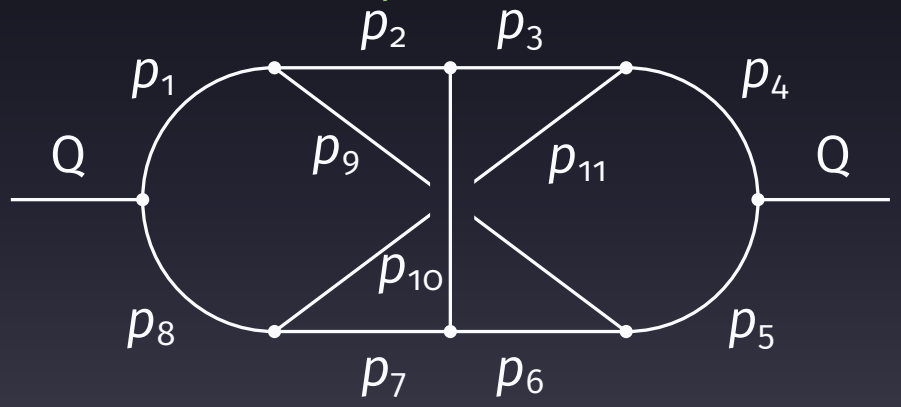

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Harmonic projection to probe-parton forward scattering

Gorishnii, Larin, Tkachev '83; Gorishnii, Larin '87

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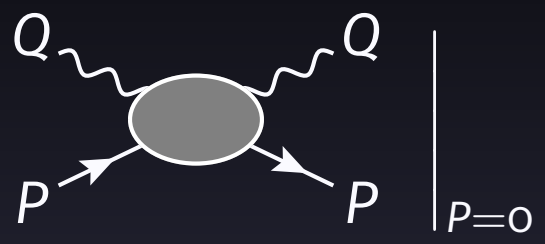

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Quickly becomes hard as N increases

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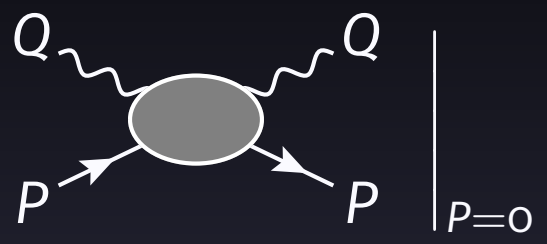

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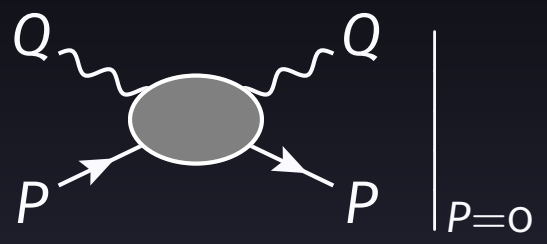

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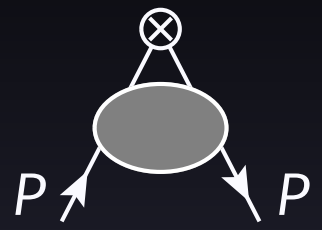

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Complicated Feynman rules, $(L+2)$-point vertices

$G_\alpha^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_{N-1}} G^{\mu_N\}\alpha} \sim$

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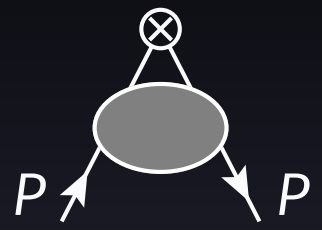

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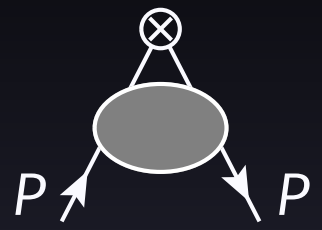

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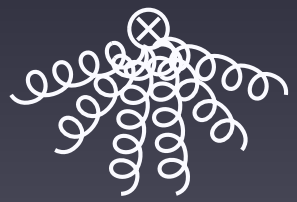
$G_\alpha^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_{N-1}} G^{\mu_N\}\alpha} \sim$ 

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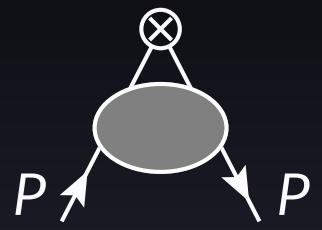

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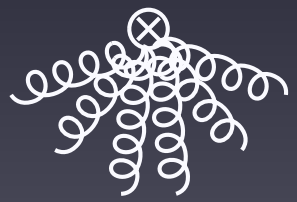

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Strategy

Compute N-th moments by Method I (forward scattering) as much as possible (Easy to debug)

Computed up to $N \leq 6$ at 4-loops in general
Confirmed known low-N non-singlet results ($N \leq 4$)

Velizanin '11 '14; Baikov, Chetyrkin, Kühn, Rittinger

Up to $N > 40$ for high-n_f parts (DIS 2017 talk by Josha Davies)

Enough to reconstruct high-n_f parts

Davies, Ruijl, TU, Vermaseren, Vogt NPB915 (2017) 335 [arXiv:1610.07477]

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Reconstructing full-N expression

Up to $N = 20$ for non-singlet, large-n_c
$\gamma_{NS}^{\pm}(N)$: $(q_i \pm \bar{q}_i) - (q_k \pm \bar{q}_k)$

Ansatz: if analogous to lower orders

$$\gamma_{NS}^{(n)}(N) = \sum_{w=0}^{2n+1} c_{00w} S_w(N) + \sum_a \sum_{k=1}^{2n+1} \sum_{w=0}^{2n+1-k} c_{akw} \frac{S_w(N)}{(N+a)^k}$$

γ_{NS}: constrained by 'self-tuning' (conjecture, conformal symmetry)
$\gamma_{NS}^{+} = \gamma_{NS}^{-}$ for large-n_c
Large-N and small-x limits

Small-x ressumation: known coefficients

$N \leq 18$ Diophantine eqs. to fix remaining coefficients
Checked by $N = 19, 20$

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Ansatz: if analogous to lower orders

$$\gamma_{NS}^{(n)}(N) = \sum_{w=0}^{2n+1} c_{00w} S_w(N) + \sum_a \sum_{k=1}^{2n+1} \sum_{w=0}^{2n+1-k} c_{akw} \frac{S_w(N)}{(N+a)^k}$$

γ_{NS}: constrained by 'self-tuning' (conjecture, conformal symmetry)
$\gamma_{NS}^{+} = \gamma_{NS}^{-}$ for large-n_c
Large-N and small-x limits

Small-x ressumation: known coefficients

$N \leq 18$ Diophantine eqs. to fix remaining coefficients
Checked by $N = 19, 20$

Reconstructing full-N expression

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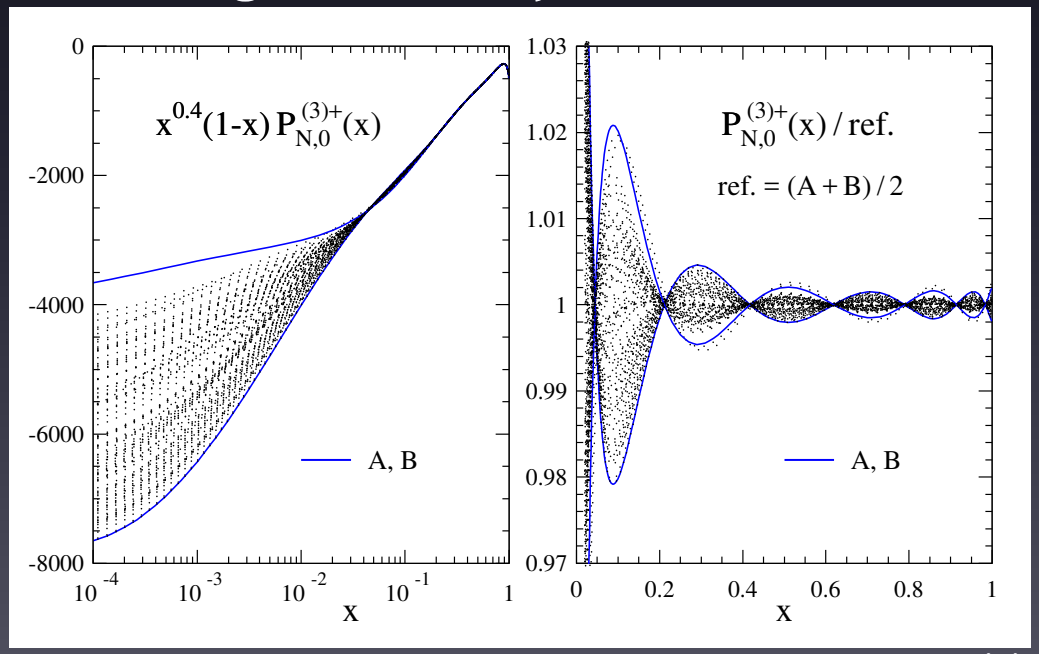
Approximation for large-n_c suppressed terms

Remaining large-n_c suppressed terms (non-singlet)

90 resulting trial functions, parameters fixed from the first 8 moments, two representatives chosen that indicate the remaining uncertainty

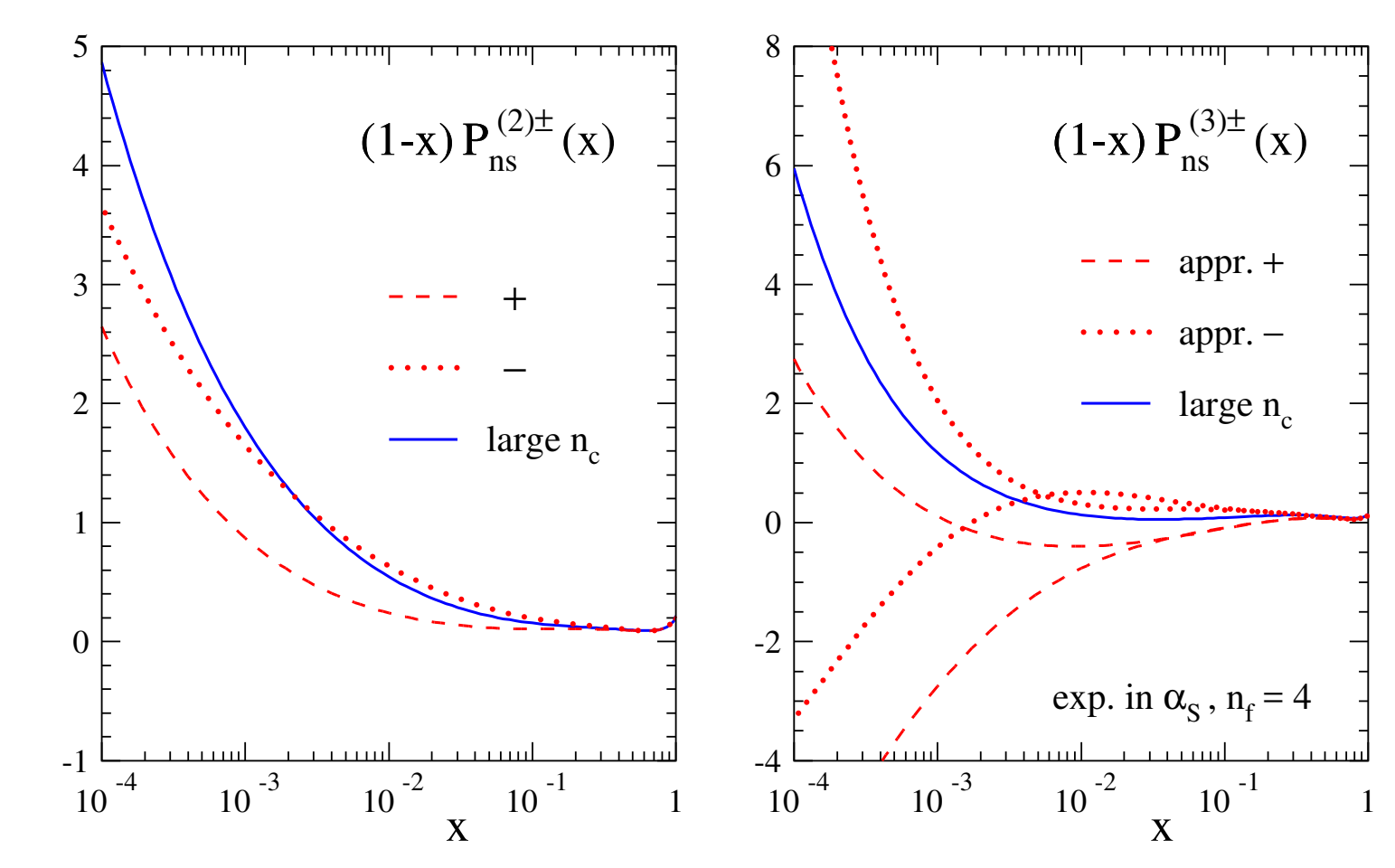


Checked by the 9th moment, e.g., $P^{+(3)}_{N,1}(N=18)$:

$195.8888792_B < 195.8888857\ldots_{\text{exact}} < 195.8888968_A$

NS splitting functions: NNLO vs N^{3}LO

$$P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \alpha_s^4 P^{(3)} + \ldots$$

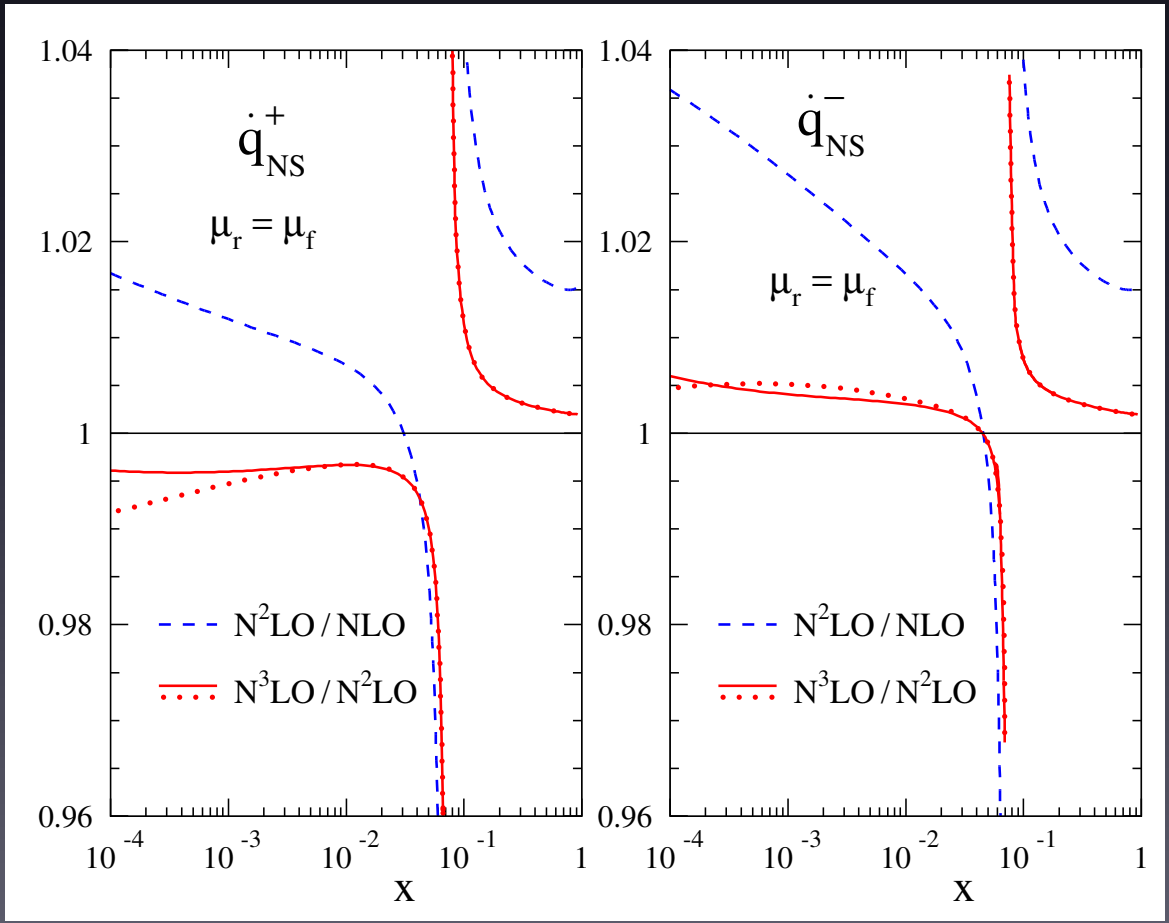


$q_{NS}^{\pm}$ evolution: NNLO vs N^{3}LO

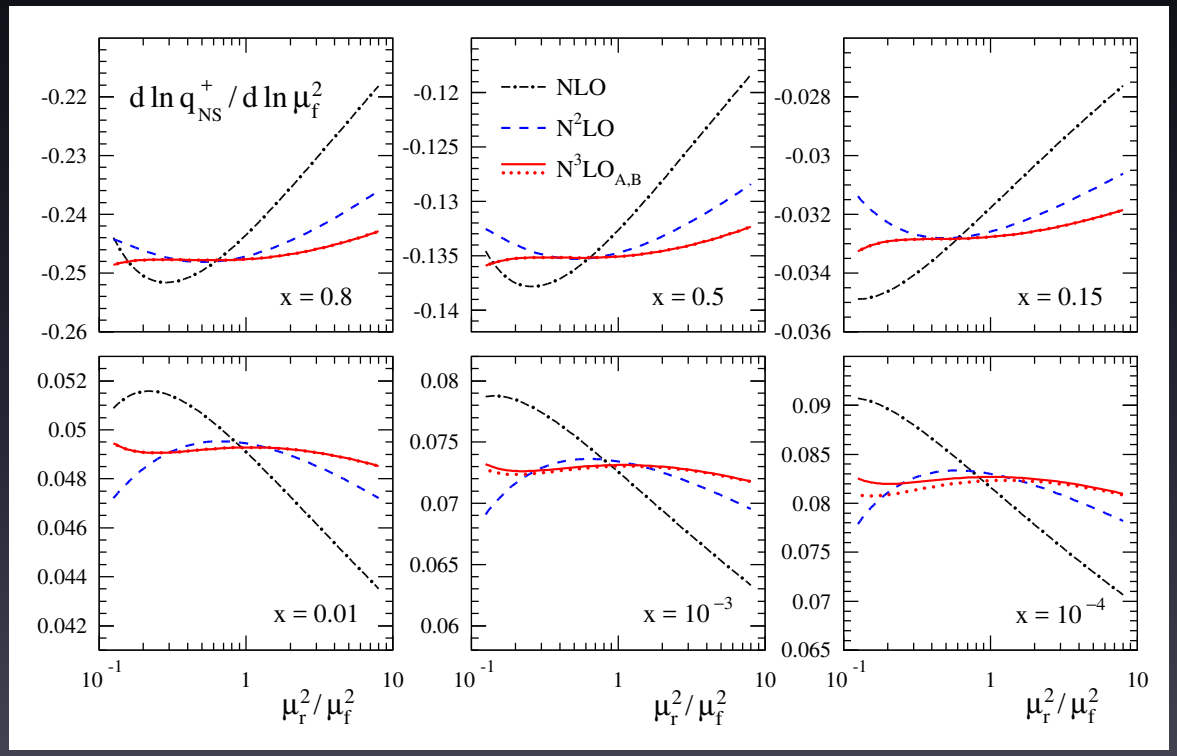
Logarithmic derivatives w.r.t. the factorization scale

$$\dot{q} \equiv \frac{d \ln q}{d \ln \mu_f^2}$$

Reference point: $xq(x, \mu_0^2) = x^{0.5}(1-x)^3$ with $\alpha_s(\mu_0^2) = 0.2$



q_{NS}^{+} evolution: renormalization scale dependence



$$\dot{q} \equiv \frac{d \ln q}{d \ln \mu_f^2}$$

N^{3}LO: stable

scale uncertainty below 1% for $\mu_f/2 \leq \mu_r \leq 2\mu_f$

Status of singlet case

So far, up to $N = 8$ for γ_{qq}, γ_{gq} (yesterday),
up to $N = 6$ for γ_{qg}, γ_{gg} by forward scattering

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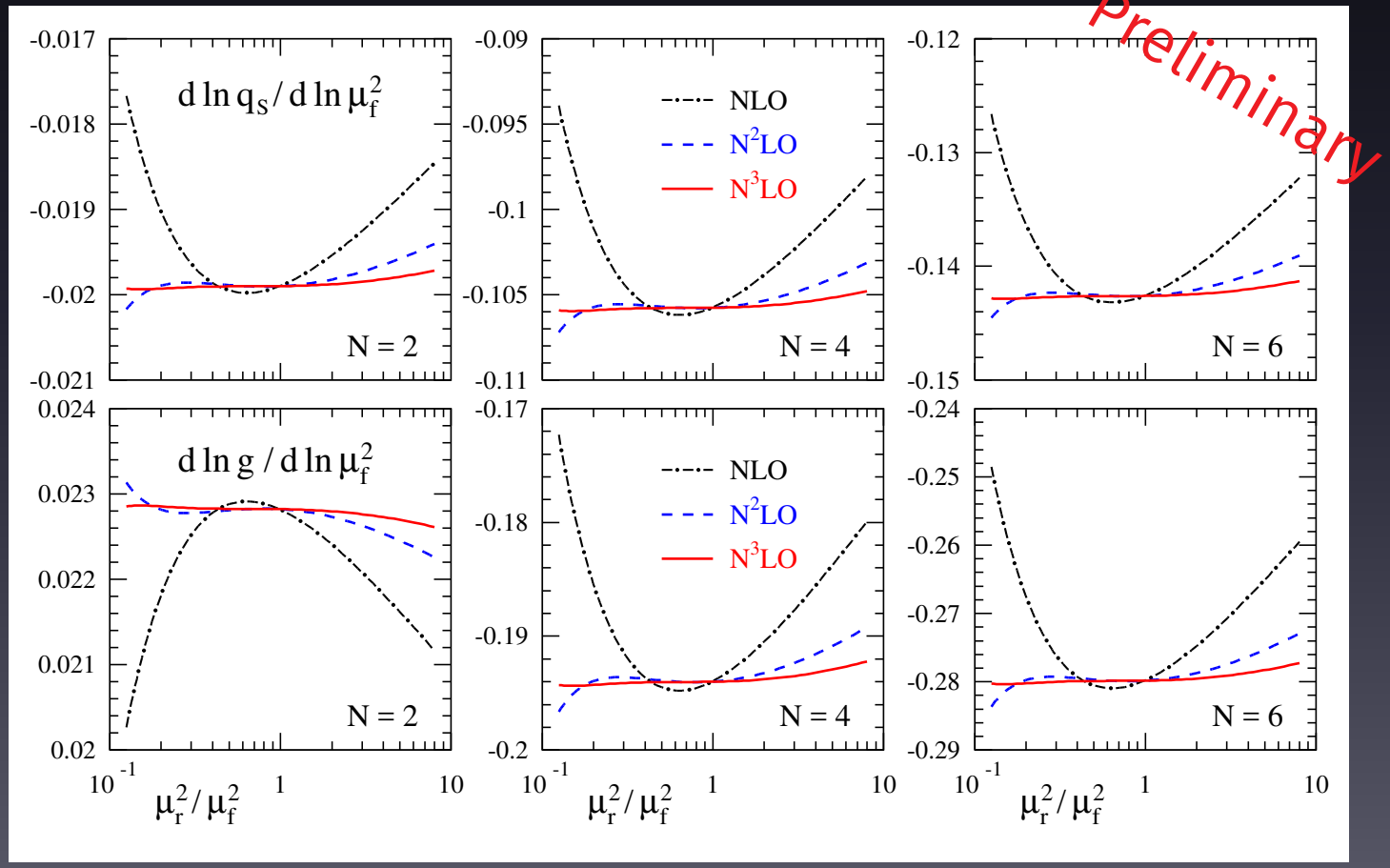

Summary and outlook

Precision physics at the LHC requires 4-loop splitting functions

$\gamma_{NS}^{(3)}(N)$ reconstructed at large-n_c limit
Remaining large-n_c suppressed terms approximated
Approximate results for $P_{NS}^{(3)}(x)$ enough for phenomenology

For more details (valence dist., fragmentation funcs., etc) see JHEP 1710 (2017) 041 [arXiv:1707.08315]

Singlet part: work in progress

Promissing renormalization scale stability

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