DIS2018

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Study of the nucleon structure through a global fit of partonic TMDs

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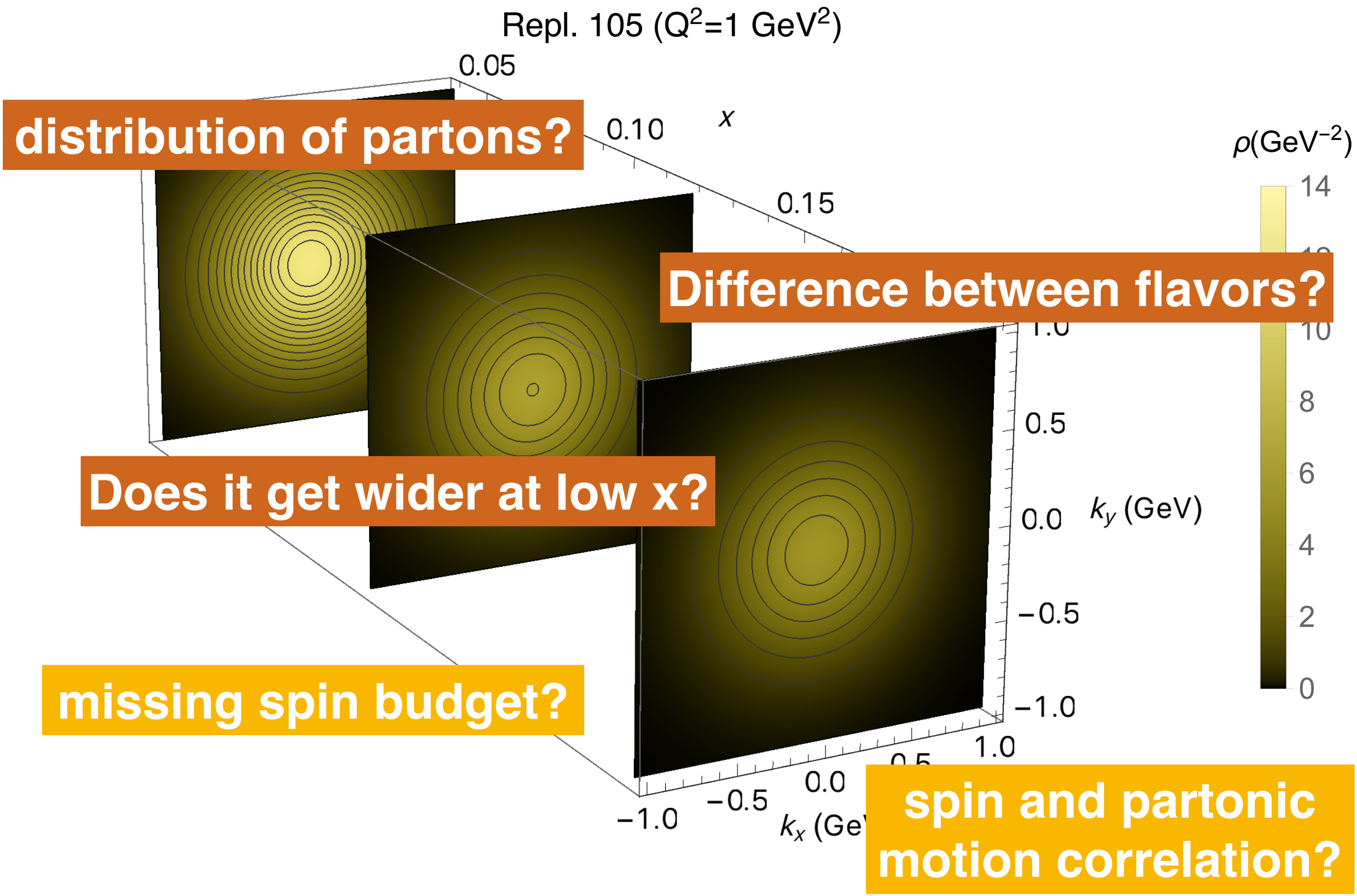
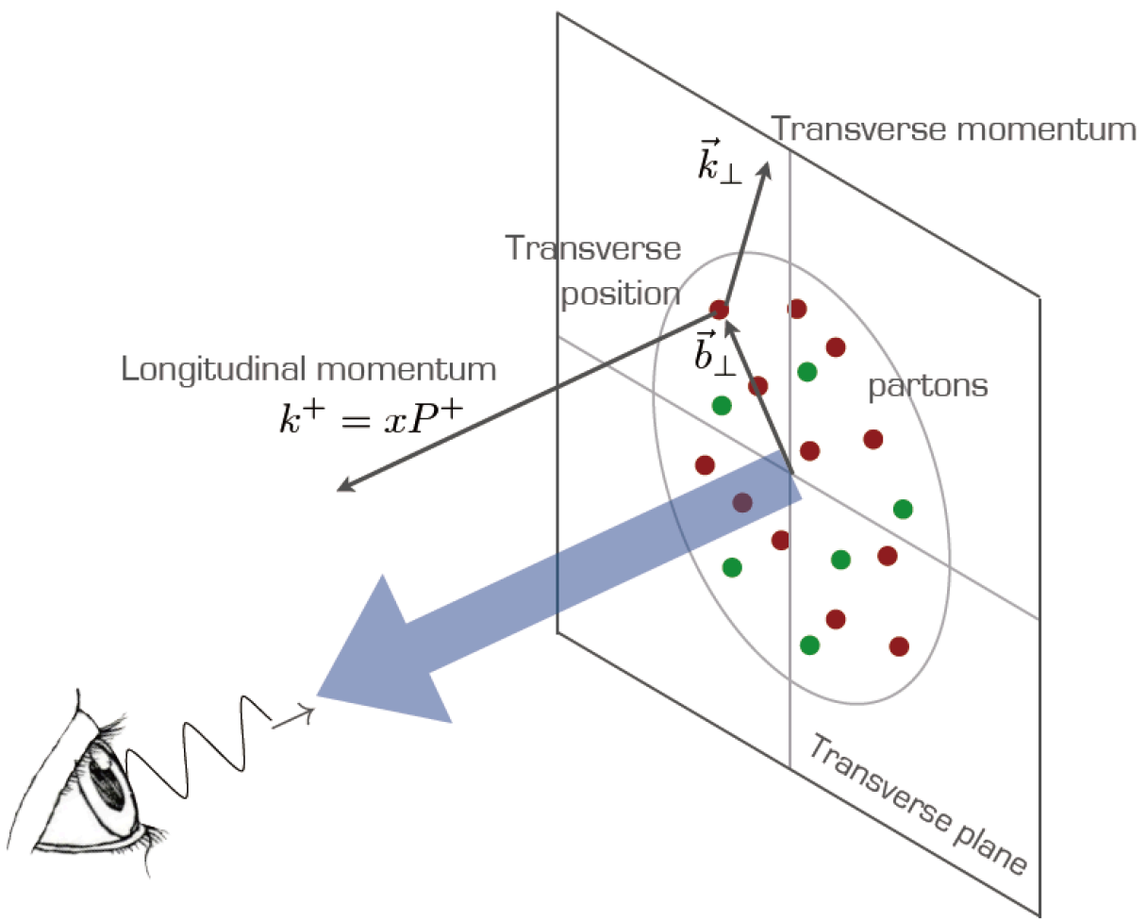
3DSPIN MAPPING THE PROTON IN 3D

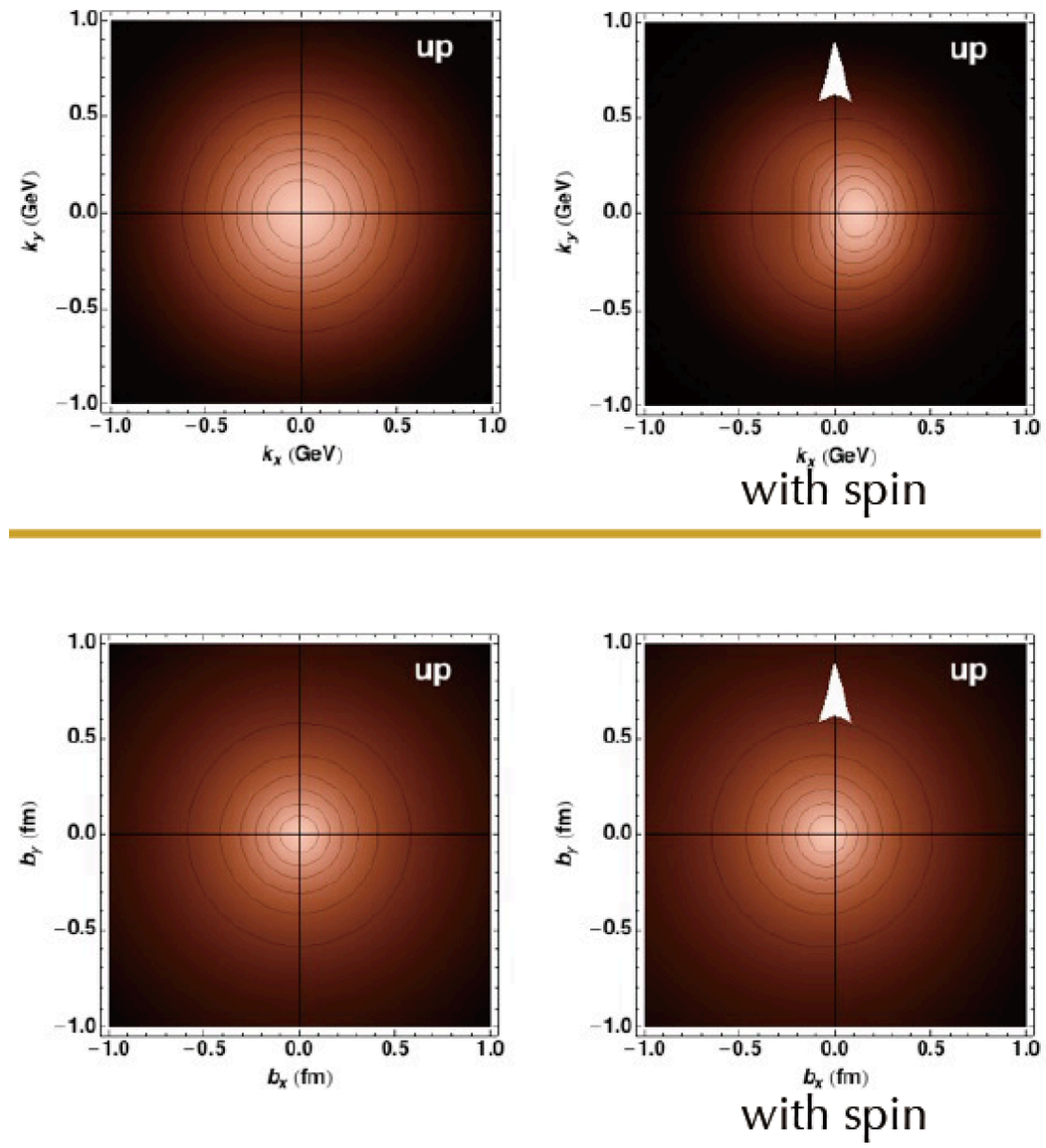
erc

3DSPIN: structure of the nucleon



Orbital motion - Nucleon Structure from 1D to 3D





Generalized parton distribution (GPD)
Transverse momentum dependent parton distribution (TMD)

[Bacchetta's talk (2016)]

Transverse Momentum Distributions: TMD PDF

Unpolarized

quark pol.

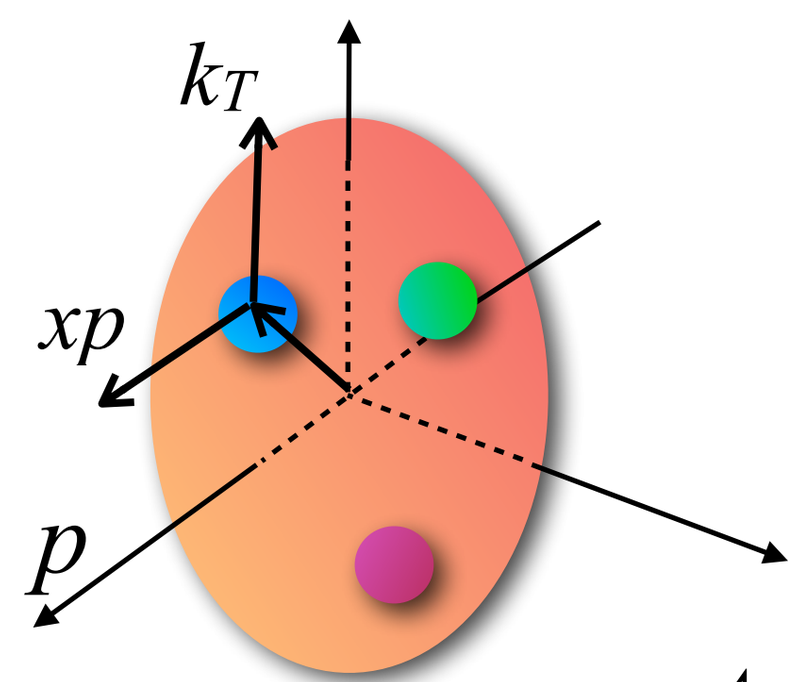
nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

dependence on:

longitudinal momentum fraction x

transverse momentum $\boldsymbol{k}_\perp$

energy scale



Why studying unpolarized TMDs?

nucleon tomography

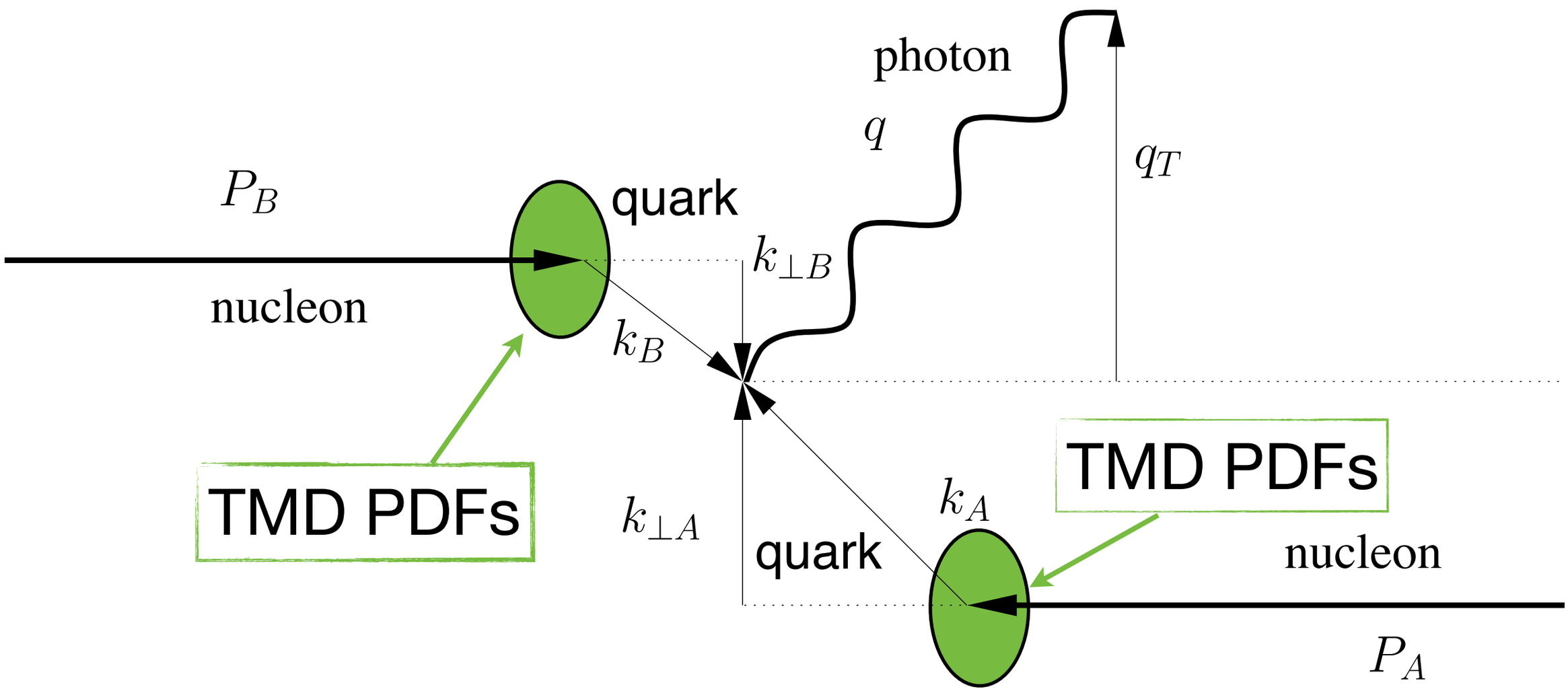
High-energy phenomenology

Open questions :

1) what is the functional form of TMDs at low transverse momentum ?
2) what is its kinematic and flavor dependence ?
3) how can we separate the descriptions at low and high transverse momenta ?
4) how can we match TMD and collinear factorization ?
5) can we test the generalized universality of TMDs ?
6) can we perform a global fit of TMDs ?

Extraction from SIDIS & Drell-Yan

Drell-Yan \ Z production

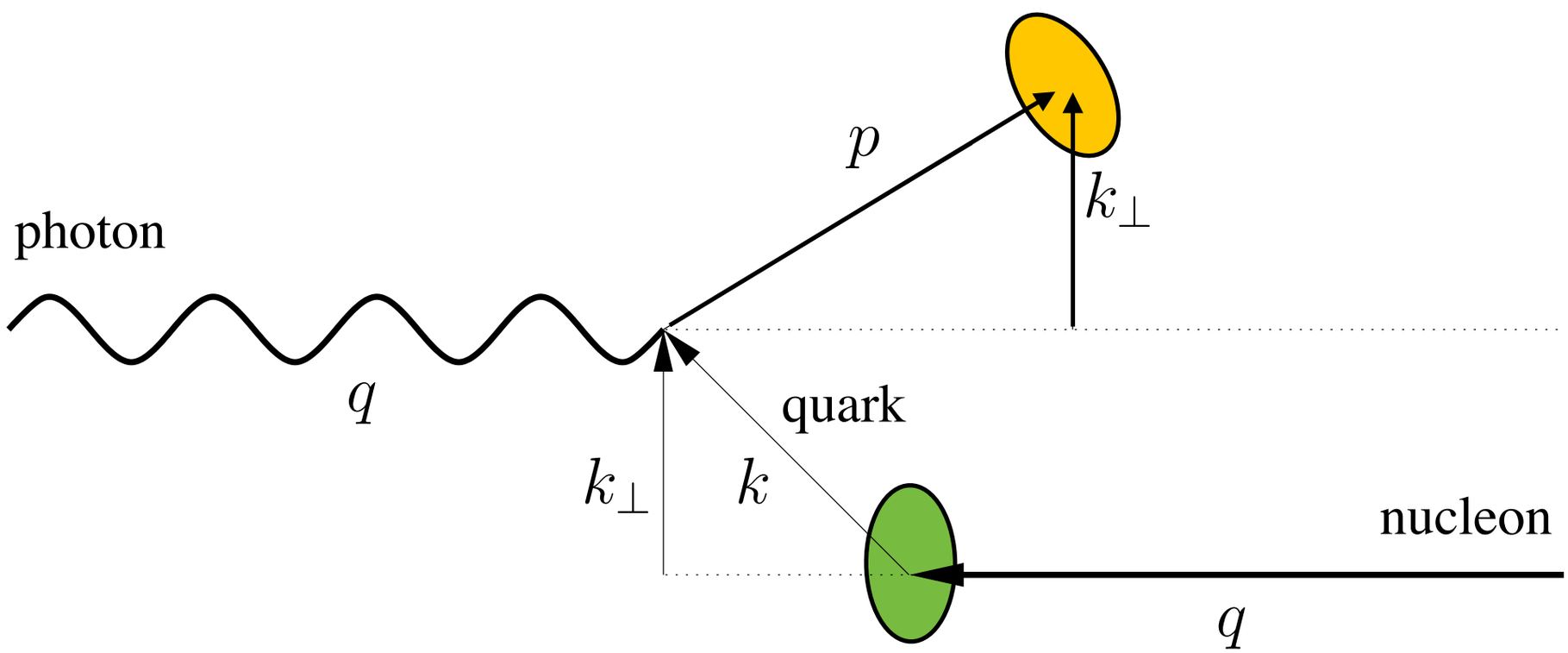


$$A + B \to \gamma^* \to l^+ l^-$$

$$A + B \to Z \to l^+ l^-$$

Extraction from SIDIS & Drell-Yan

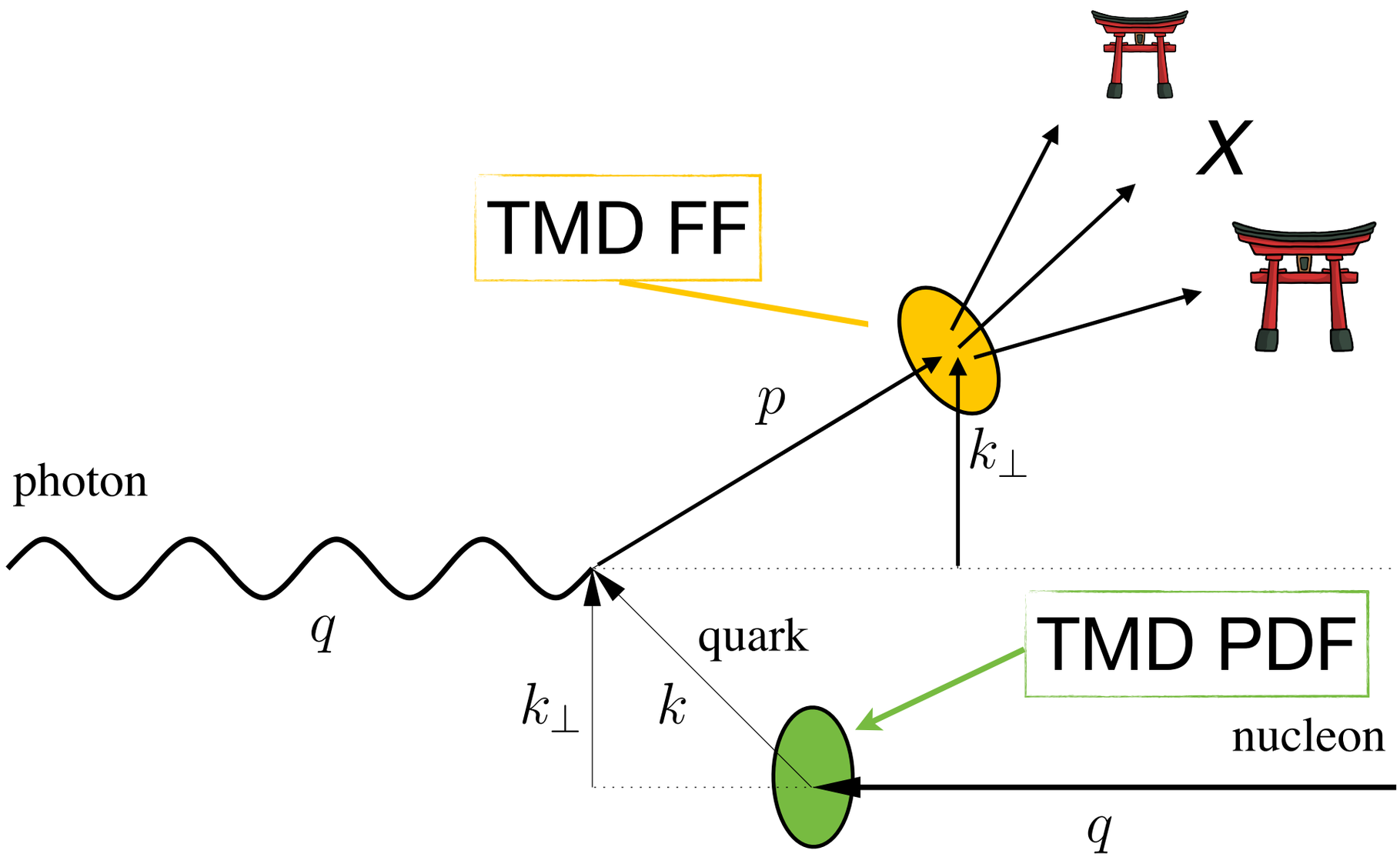
Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

Extraction from SIDIS & Drell-Yan

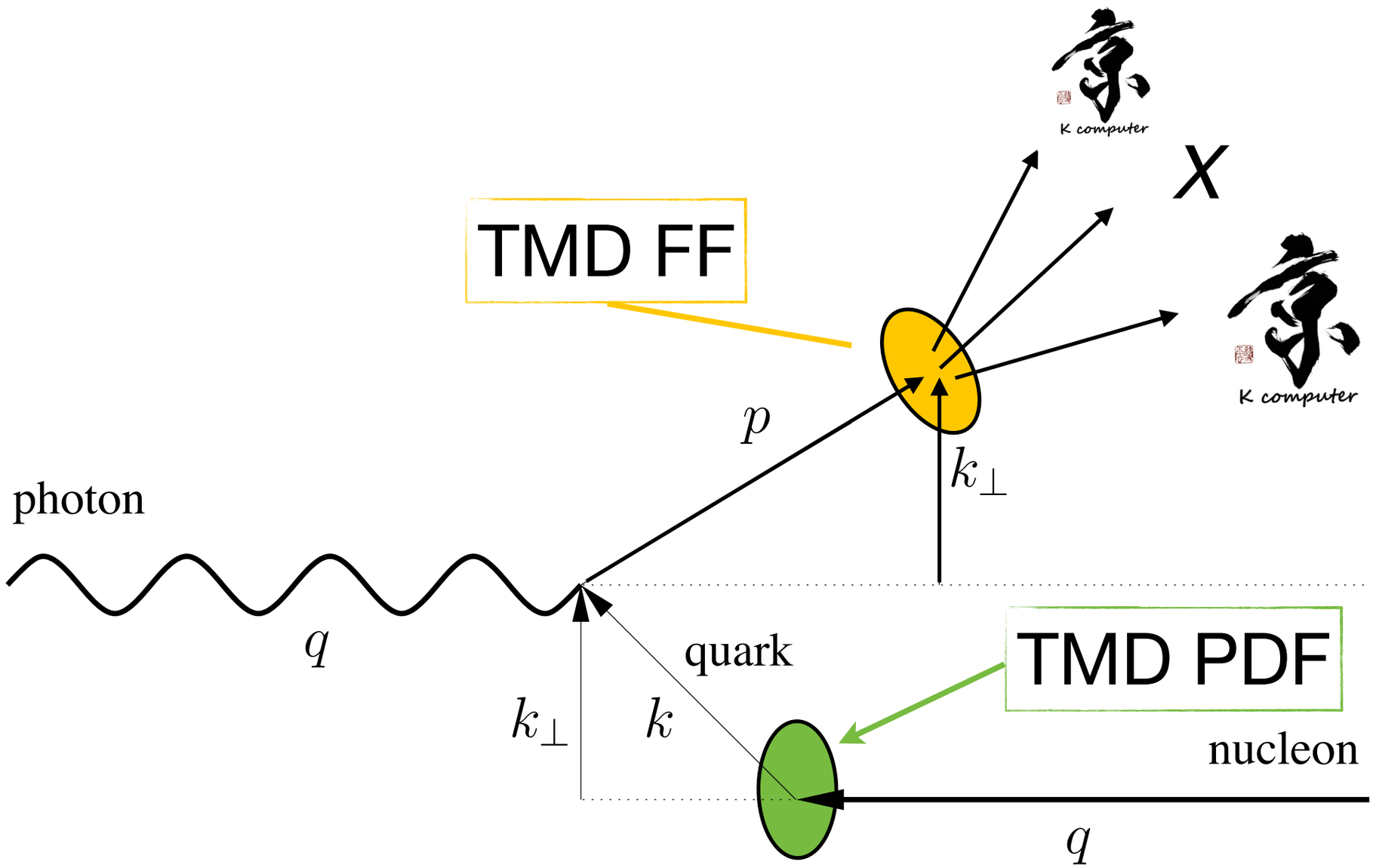
Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

Extraction from SIDIS & Drell-Yan

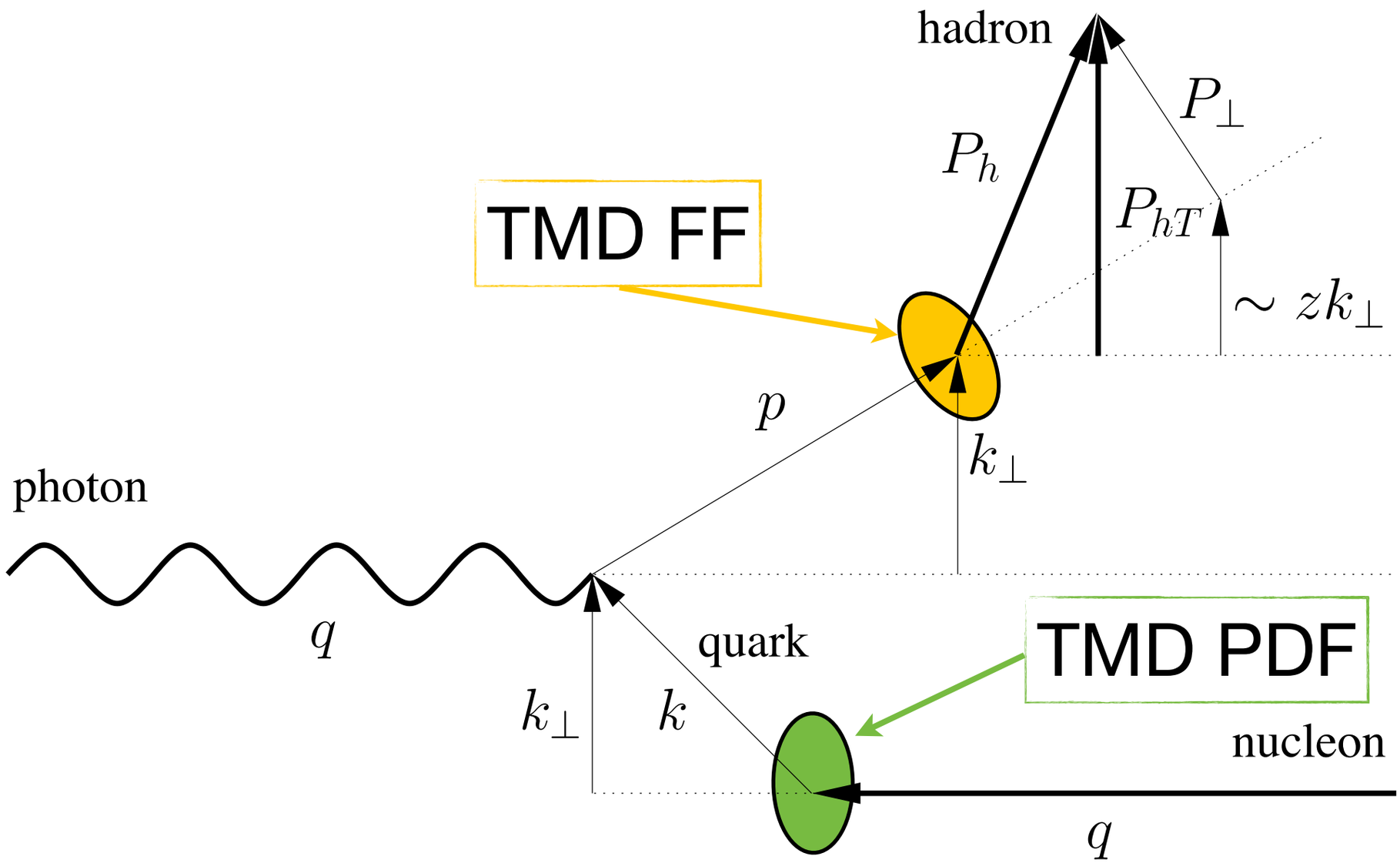
Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

Extraction from SIDIS & Drell-Yan

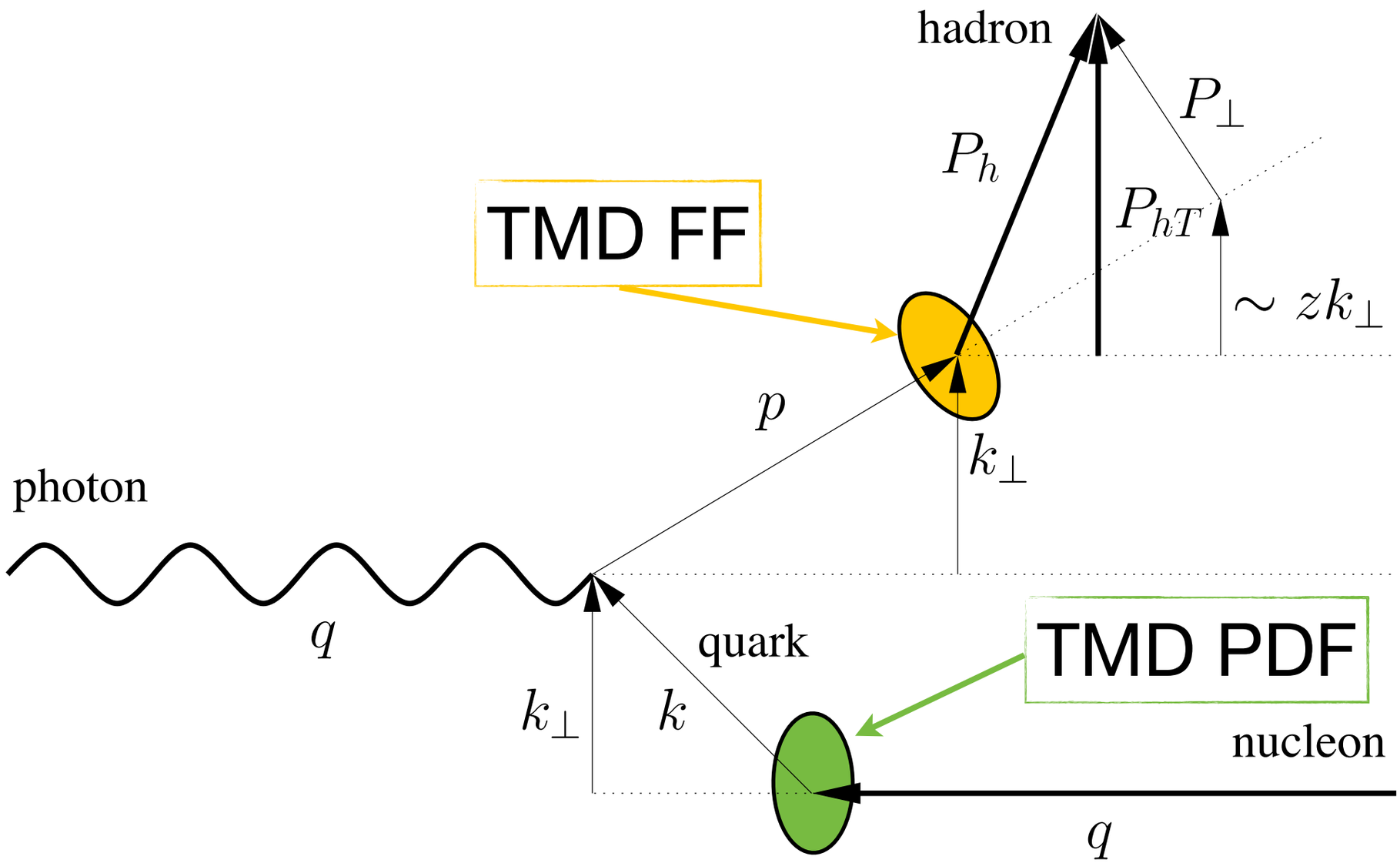
Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

Extraction from SIDIS & Drell-Yan

Semi-inclusive Deep Inelastic Scattering



universality

$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

TMDs: Fragmentation Function

quark pol.

Unpolarized

U	L	T
D_1		$H_1^\perp$

TMD Fragmentation Functions
(TMD FFs)

dependence on:

longitudinal momentum fraction **z**

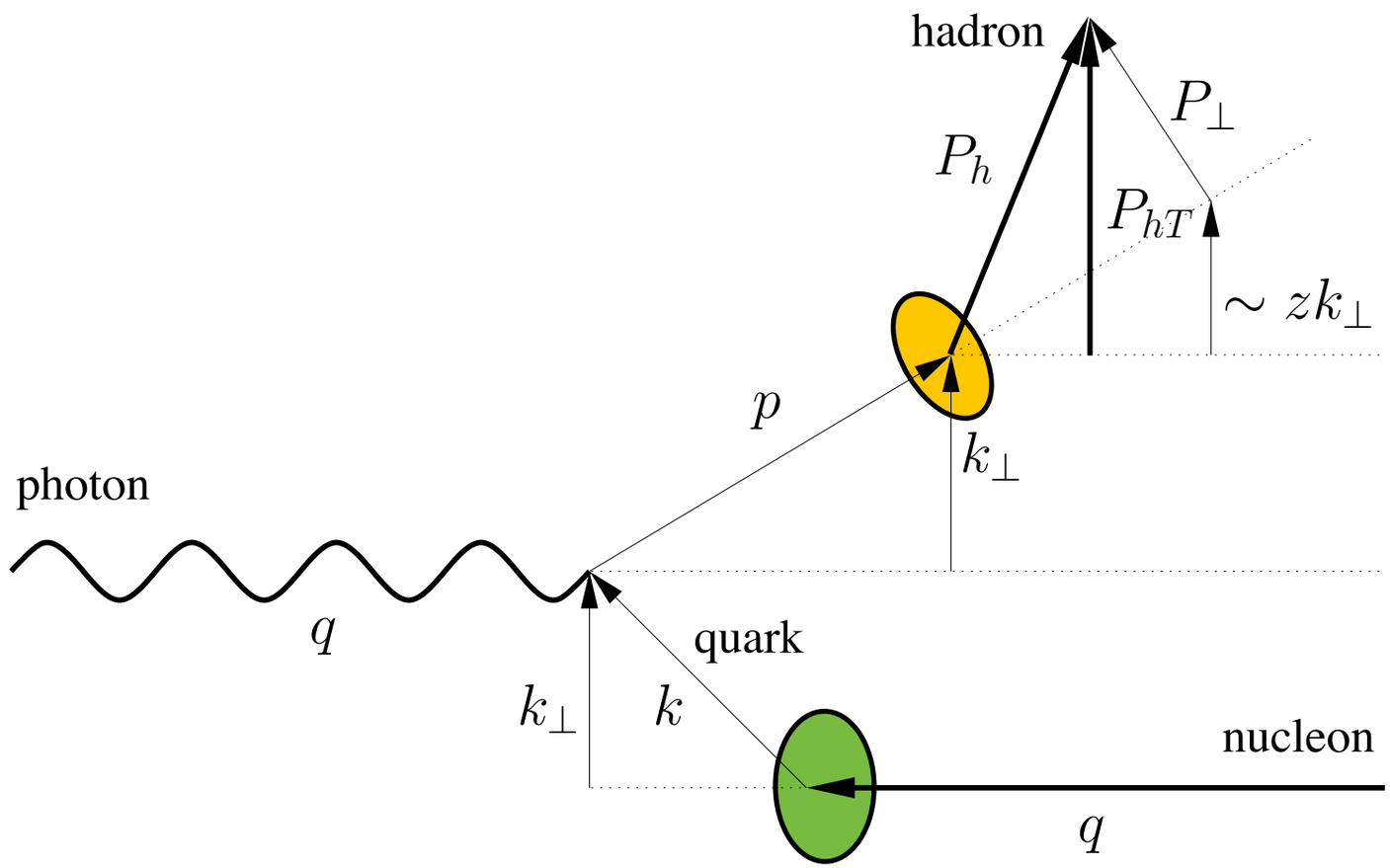
transverse momentum $\boldsymbol{P}_\perp$

energy scale

Structure functions and TMDs

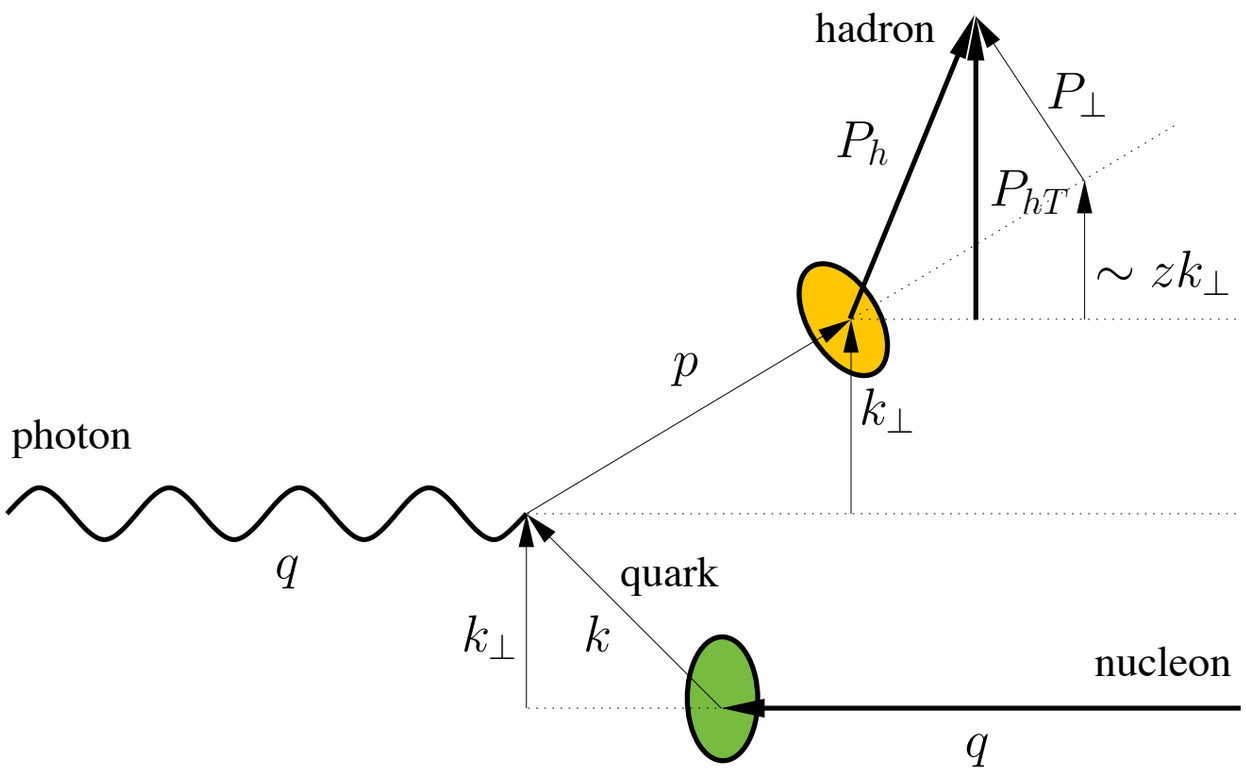
multiplicities

$$m_N^h\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)=\frac{d\sigma_N^h/\left(dxdzd\boldsymbol{P}_{hT}^2dQ^2\right)}{d\sigma_{DIS}/\left(dxdQ^2\right)}\approx\frac{\pi F_{UU,T}\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)}{F_T(x,Q^2)}$$



$$F_{UU,T}\left(x,z,P_{hT}^2,Q^2\right)=\sum_a\mathcal{H}_{UU,T}^a\left(Q^2;\mu^2\right)\int d^2k_T\,d^2P_T\,f_1^a\left(x,k_T^2;\mu^2\right)D_1^{h/a}\left(z,P_T^2;\mu^2\right)$$
$$\cdot\delta^2(zk_T-P_{hT}+P_T)+Y_{UU,T}(Q^2,P_{hT}^2)+\mathcal{O}\left(M^2/Q^2\right)$$

Structure functions and TMDs



At our accuracy level
(LO-NLL)

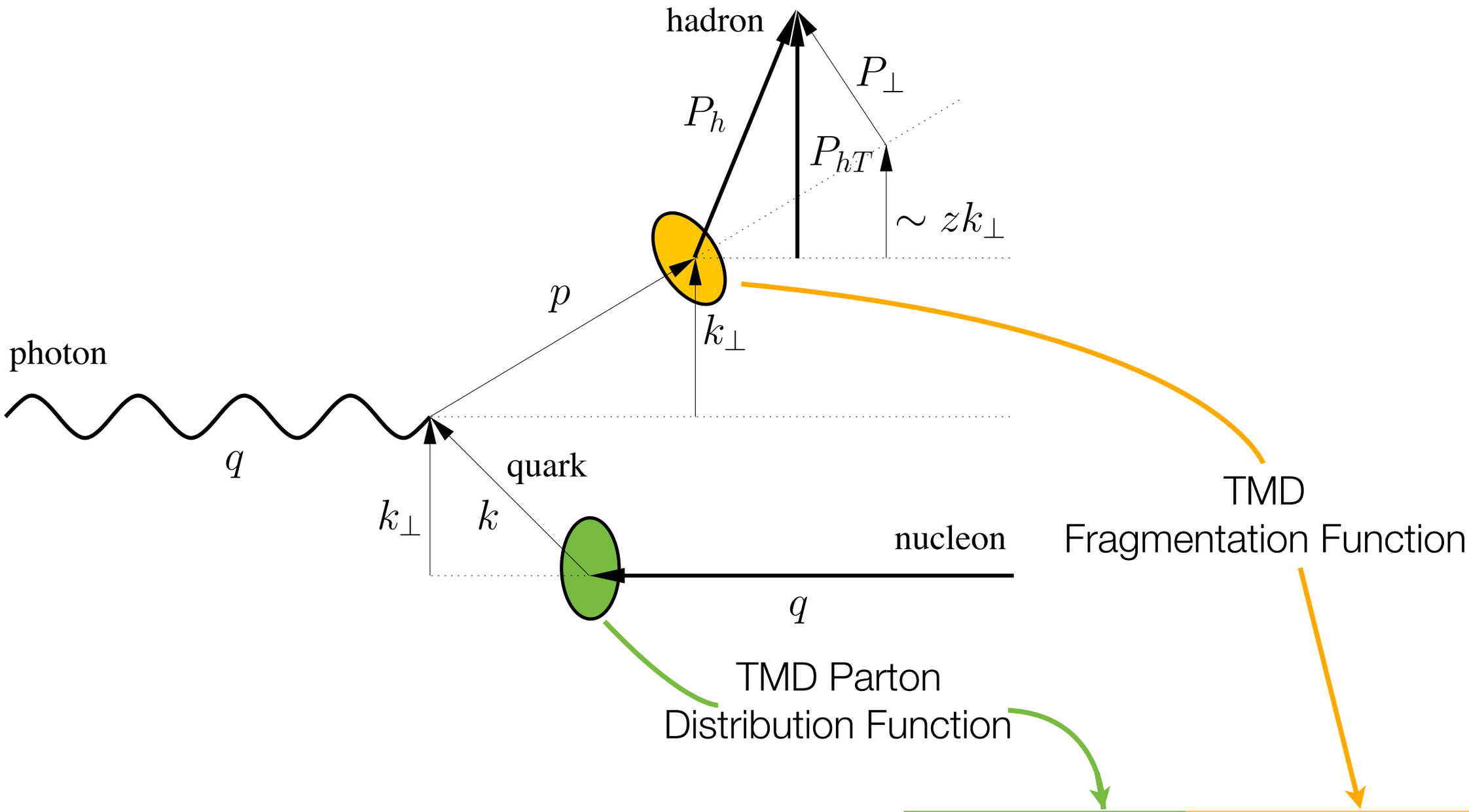
$$\mathcal{H}_{UU,T} \simeq \mathcal{O}\left(\alpha_s^0\right)$$

$$Y_{UU,T}\left(Q^2, P_h^2 T\right) \simeq 0$$

$$F_{UU,T}\left(x, z, P_{hT}^2, Q^2\right) = \sum_a \mathcal{H}_{UU,T}^a\left(Q^2;\mu^2\right) \int d^2k_T\, d^2P_T\, f_1^a\left(x, k_T^2;\mu^2\right) D_1^{h/a}\left(z, P_T^2;\mu^2\right)$$

$$\cdot\delta^2(zk_T - P_{hT} + P_T) + Y_{UU,T}(Q^2, P_{hT}^2) + \mathcal{O}\left(M^2/Q^2\right)$$

Structure functions and TMDs

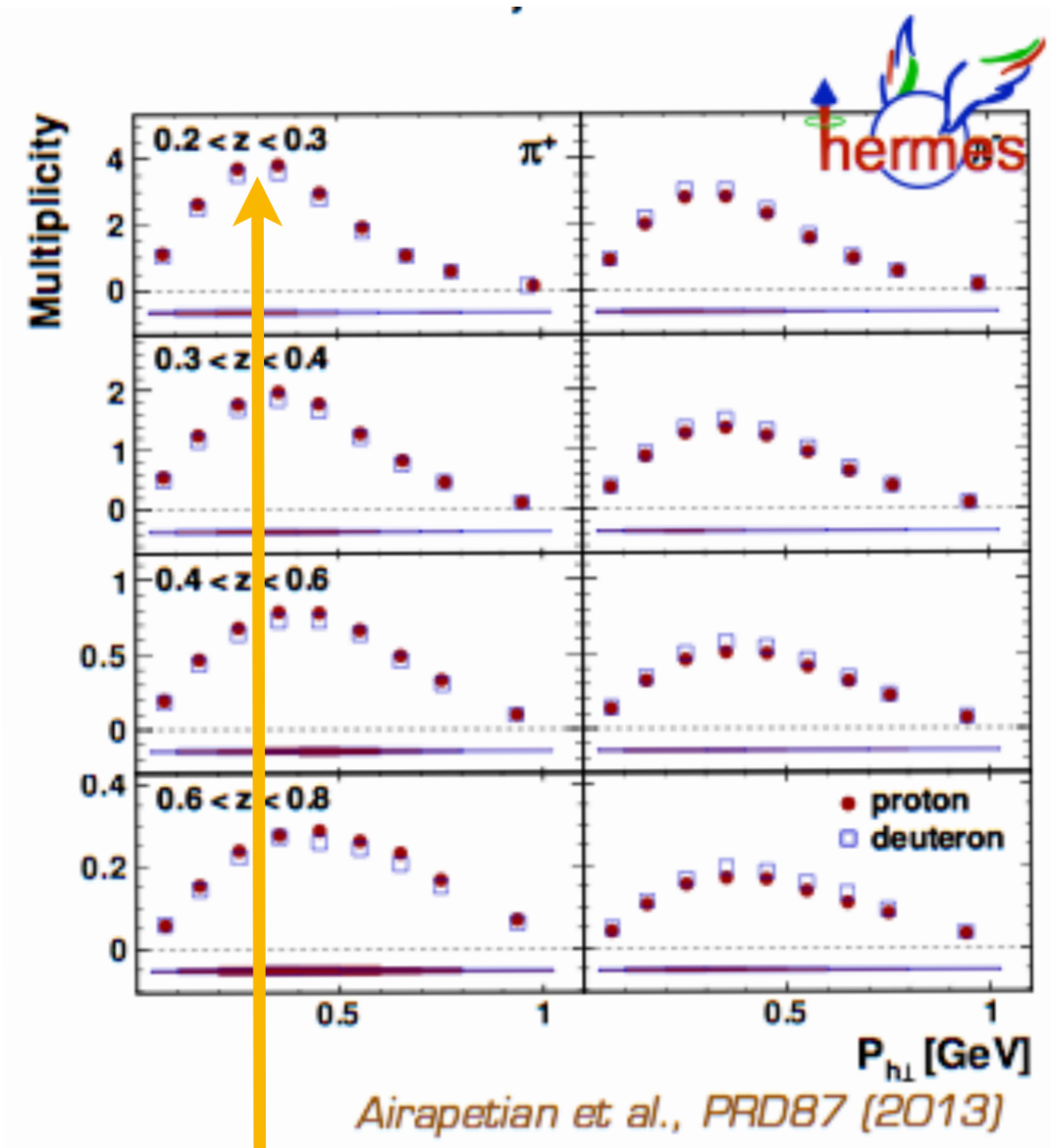


$$F_{UU,T}\left(x, z, P_{hT}^2, Q^2\right) \simeq \sum_a \int d^2k_T\, d^2P_T\, f_1^a\left(x, k_T^2; \mu^2\right) D_1^{h/a}\left(z, P_T^2; \mu^2\right)$$

$$\cdot \delta^2(zk_T - P_{hT} + P_T)$$

TMD Evolution

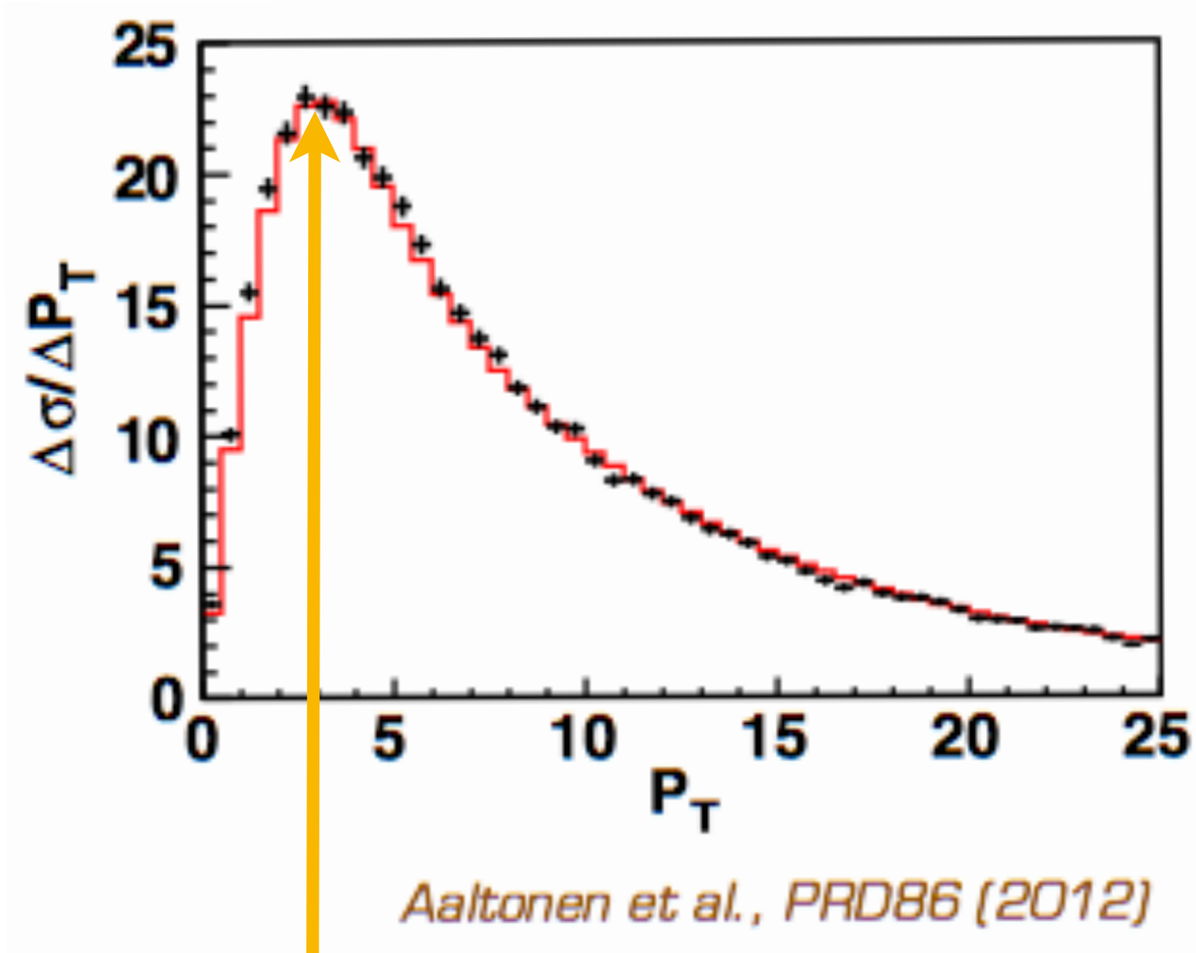
HERMES, Q ≈ 1.5 GeV



Airapetian et al., PRD87 (2013)

to reproduce shift of TMD peak with energy scale

CDF, Q ≈ 91 GeV



Aaltonen et al., PRD86 (2012)

Width of TMDs changes of one order of magnitude → EVOLUTION

Evolved TMDs

Fourier transform: ξ_T space

Discussed also in the talk of Y.Zhou

$$\tilde{f}_1^a\left(x,\xi_T;\mu^2\right) =$$

$$= \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)\left(x,\bar{\xi}_*;\mu_b\right) e^{\tilde{S}(\bar{\xi}_*;\mu_b,\mu)} e^{g_K(\xi_T)\ln(\mu/\mu_0)} \hat{f}_{NP}^a\left(x,\xi_T\right)$$

nonperturbative part of evolution

(Wilson Coefficient)

(Sudakov form factor)

collinear PDF

pQCD

non-perturbative part of TMD

small kT $\xrightarrow{\text{pQCD}}$ large log. $\alpha_S^n \ln^{2n}\frac{Q^2}{k_T^2} + \ldots$ $\rightarrow$ resummed in CSS formalism

(energy dependence)

Evolved TMDs

Fourier transform: ξ_T space

Discussed also in the talk of Y.Zhou

$$\tilde{f}_1^a\left(x,\xi_T;\mu^2\right) =$$

$$= \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)\left(x,\bar{\xi}_*;\mu_b\right) e^{\tilde{S}(\bar{\xi}_*;\mu_b,\mu)} e^{g_K(\xi_T)\ln(\mu/\mu_0)} \hat{f}_{NP}^a\left(x,\xi_T\right)$$

nonperturbative part of evolution

(Wilson Coefficient)

(Sudakov form factor)

collinear PDF

pQCD

non-perturbative part of TMD

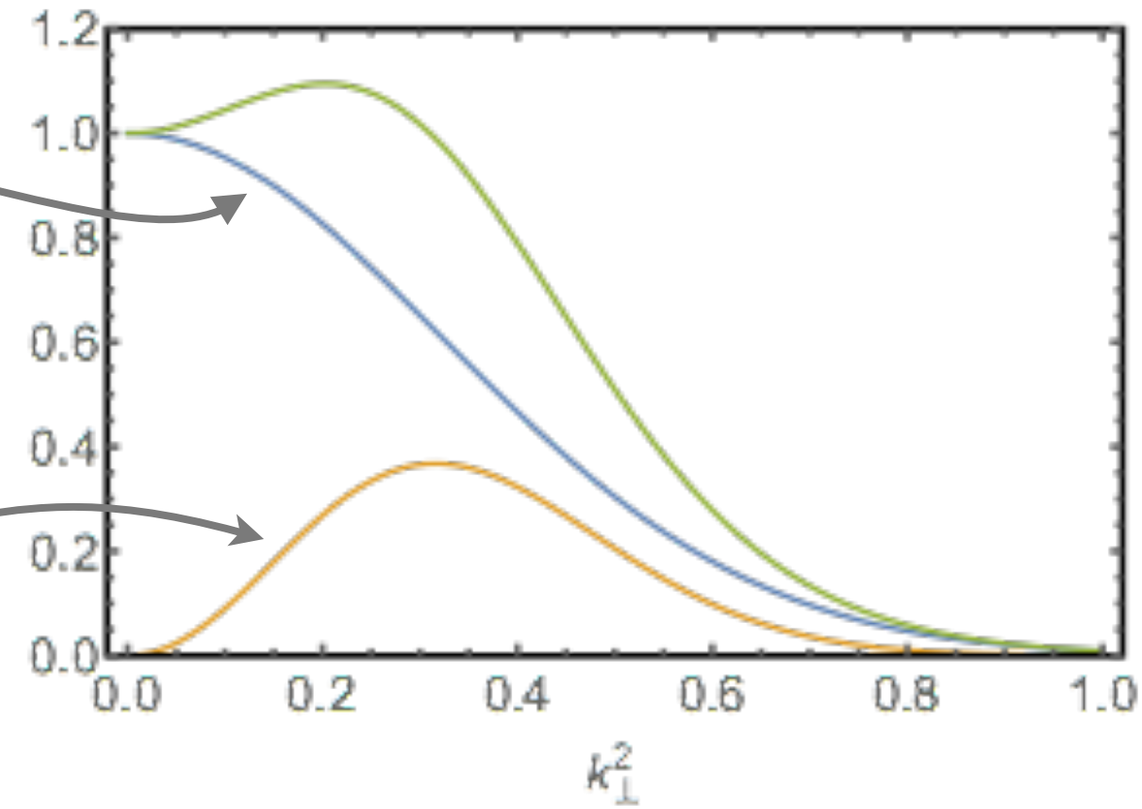
Non-perturbative contributions have to be extracted from experimental data, after parametrization

Model: non perturbative elements

input TMD PDF (Q^2=1GeV^2)

$$\hat{f}^a_{NP} = \mathcal{F}.\mathcal{T}. \text{ of}$$

$$\left(e^{-\frac{k_T^2}{g1a}} + \lambda k_T^2 e^{-\frac{k_T^2}{g1a}} \right)$$



sum of two different gaussians
with kinematic dependence on transverse momenta

width x-dependence

$$g_1(x) = N_1 \frac{(1-x)^\alpha \, x^\sigma}{(1-\hat{x})^\alpha \, \hat{x}^\sigma}$$

where

$$N_1 \equiv g_1(\hat{x})$$
$$\hat{x} = 0.1$$

Model: non perturbative elements

Free parameters

$$N_1, \alpha, \sigma, \lambda$$ 4 for TMD PDF

$$N_3, N_4, \beta, \delta, \gamma, \lambda_F$$ 6 for TMD FF

$$g_K = -g_2 \frac{b_T^2}{2}$$ 1 for NP contribution to TMD evolution

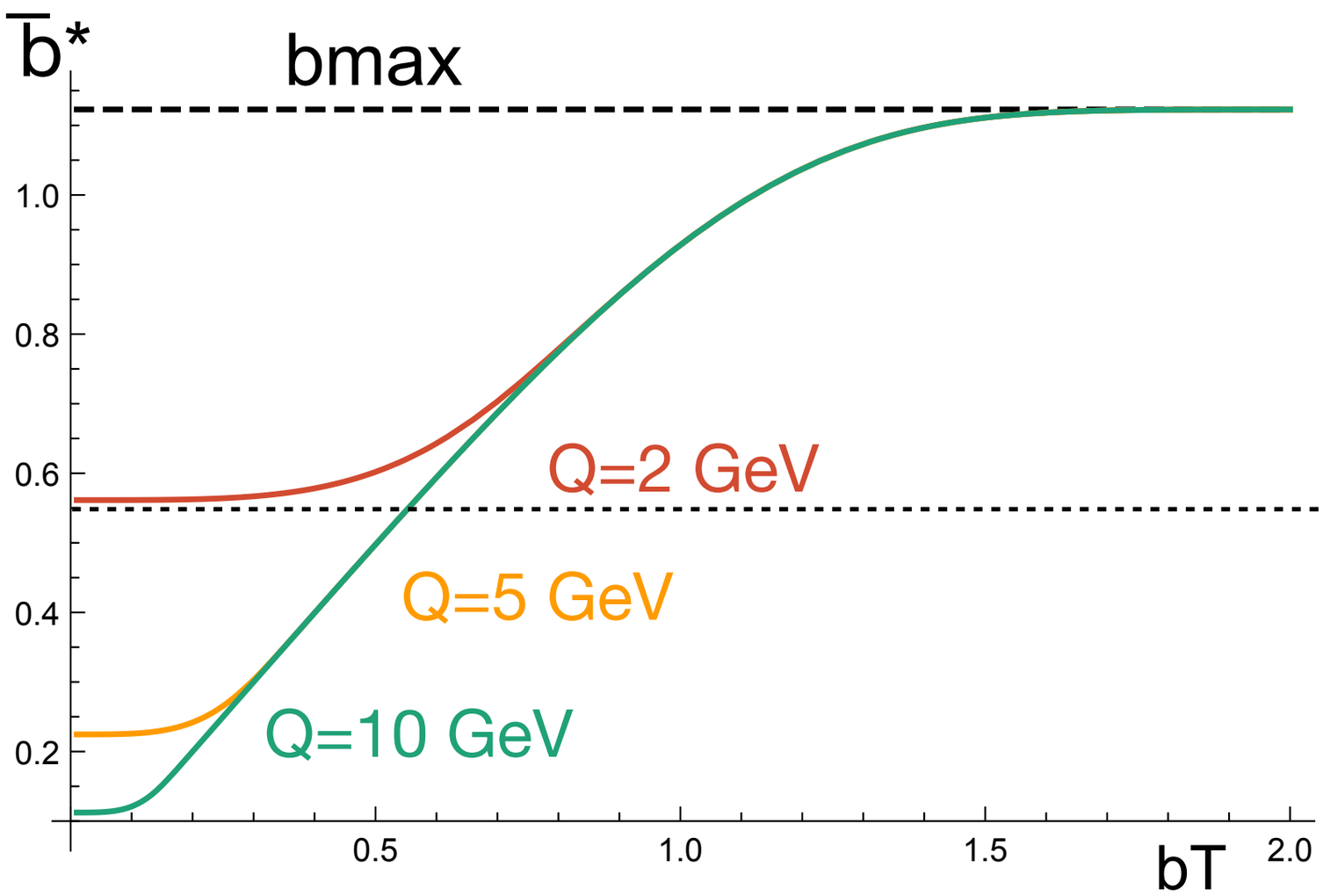
In total we have 11 parameters, for intrinsic transverse momentum (4 PDFs, 6 FFs) and evolution (g2)

Evolution and b_T regions

alternative notation: ξ_T

$$\mu_b = 2e^{-\gamma_E}/b_*$$

$$\bar{b}_\star(b_T; b_{\min}, b_{\max}) = b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right)^{1/4}$$



$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$

The importance of bmin is a signal that we are exiting the proper TMD region and approaching the region of collinear factorization, especially in SIDIS data at low Q

Experimental data

COMPASS

SIDIS μN

6252
data points

hermes

SIDIS eN

1514
data points

E288
E605

Drell-Yan

203
data points

DØ
CDF

Z Production

90
data points

Experimental data

COMPASS

SIDIS μN

6252
data points

hermes

SIDIS eN

1514
data points

NEW: [Phys.Rev. D97 (2018) no.3, 032006]

E288
E605

Drell-Yan

203
data points

Z Production

90
data points

Data selection and analysis

$Q2 > 1.4\ GeV^2$

$0.2 < z < 0.7$

$P_{hT}\ ,\ q_T < Min[0.2Q\ ,\ 0.7Qz] + 0.5\ GeV$

Motivations behind kinematical cuts

TMD factorization ($Ph_T/z << Q^2$)

Avoid target fragmentation (low z)

and exclusive contributions (high z)

Experimental data

SIDIS μN

6252
data points

SIDIS eN

1514
data points

Total: **8059** data

E288
E605

Drell-Yan

203
data points

Z Production

90
data points

Data to be included

7 TeV
8 TeV

$$q\bar{q} \to Z_0/\gamma^* + X$$

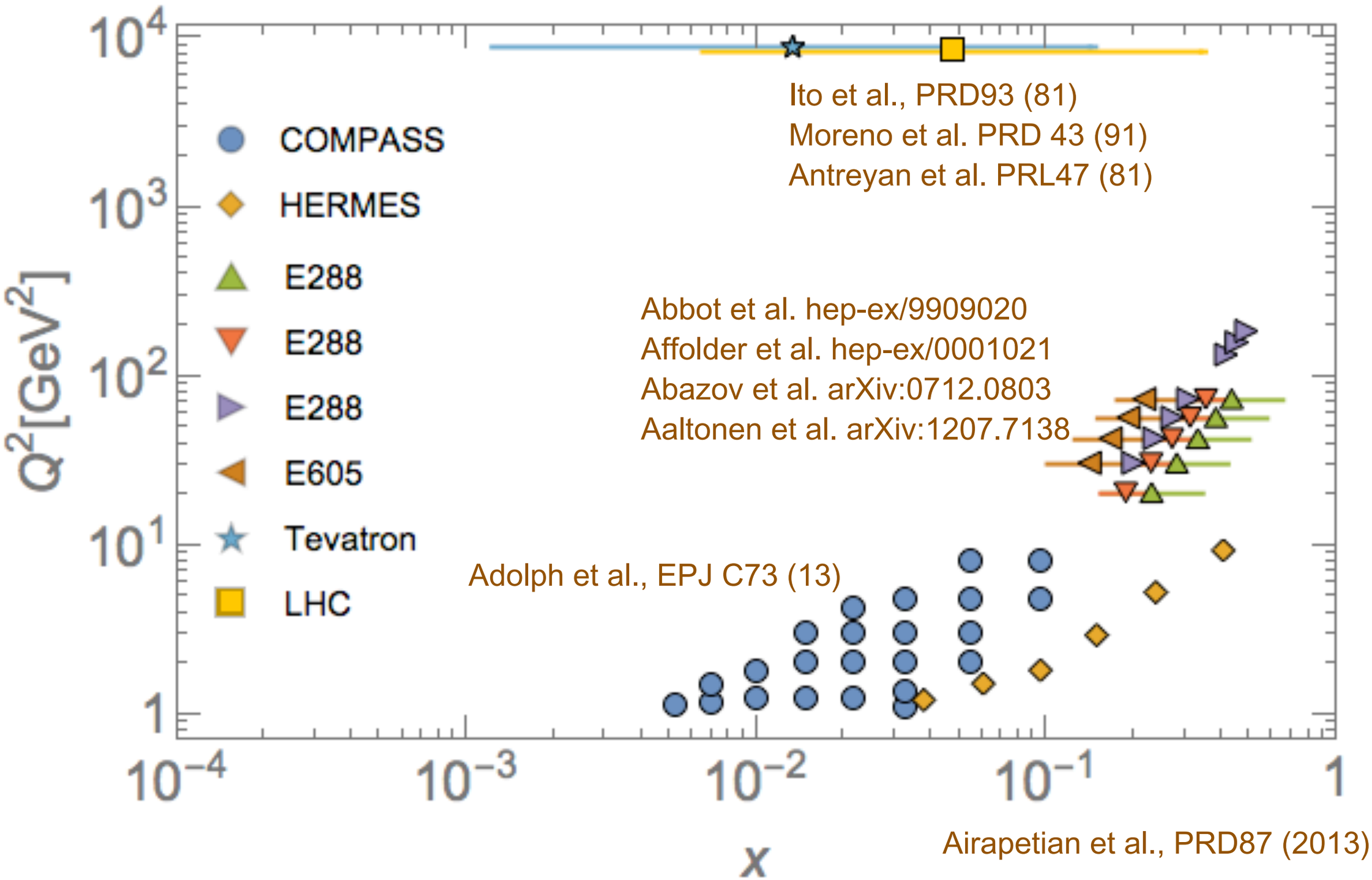
7 TeV
8 TeV

$$pp \to Z_0/\gamma^* \to (\mu^+ + \mu^-/e^+ + e^-)$$

7 TeV
8 TeV
13 TeV

$$pp \to Z_0 \to \mu^+ + \mu^-$$

Data region


10^4
10^3
10^2
10^1
1
Q^2[GeV^2]
COMPASS
HERMES
E288
E288
E288
E605
Tevatron
LHC
Ito et al., PRD93 (81)
Moreno et al. PRD 43 (91)
Antreyan et al. PRL47 (81)
Abbot et al. hep-ex/9909020
Affolder et al. hep-ex/0001021
Abazov et al. arXiv:0712.0803
Aaltonen et al. arXiv:1207.7138
Adolph et al., EPJ C73 (13)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
Airapetian et al., PRD87 (2013)

An almost global fit

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

[JHEP06(2017)081]

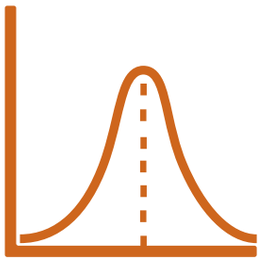
Summary of results

Total number of data points: 8059

Total number of free parameters: 11

➛ 4 for TMD PDFs ➛ 6 for TMD FFs

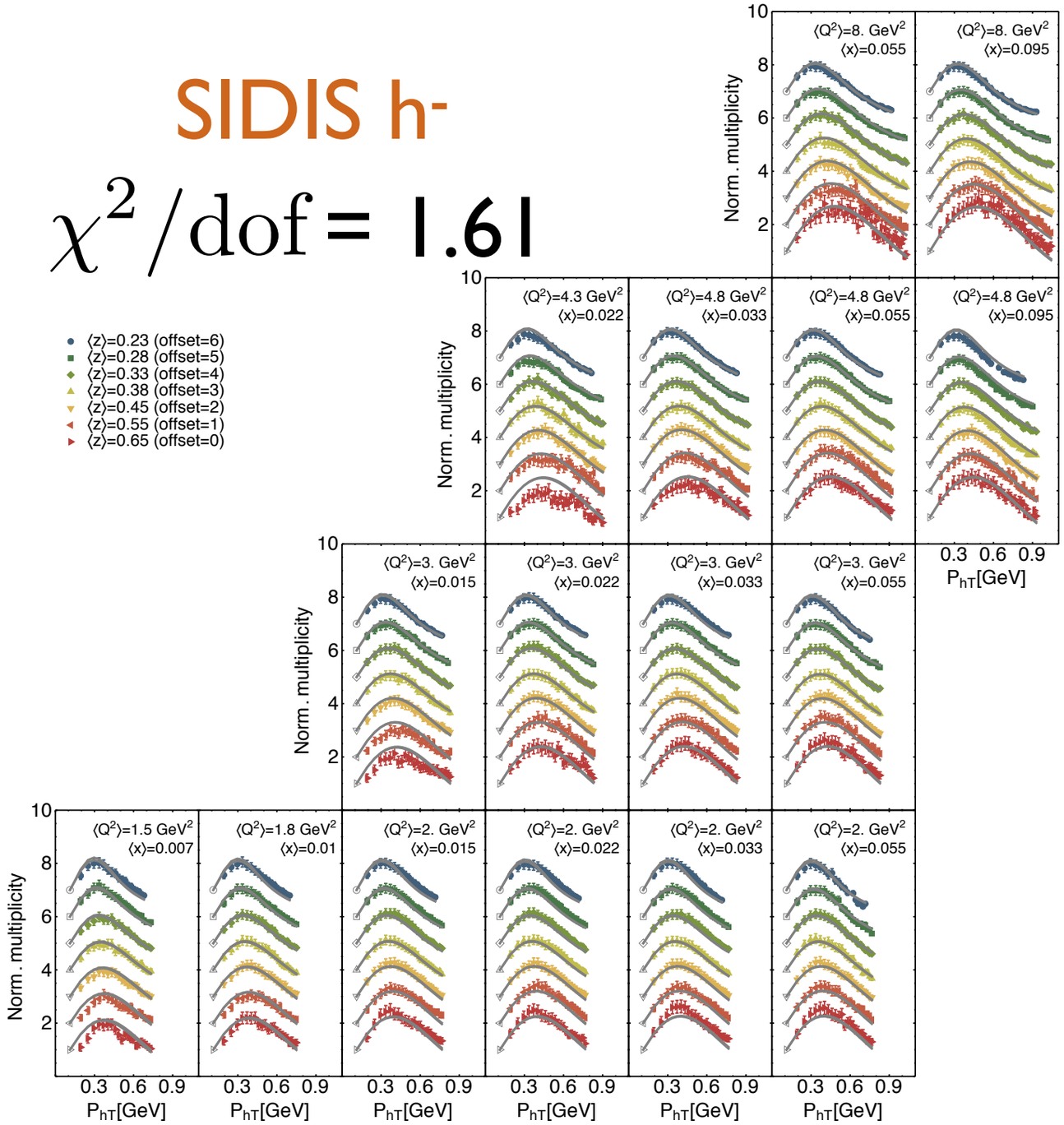
➛ 1 for TMD evolution



$$\chi^2/d.of. = 1.55 \pm 0.05$$

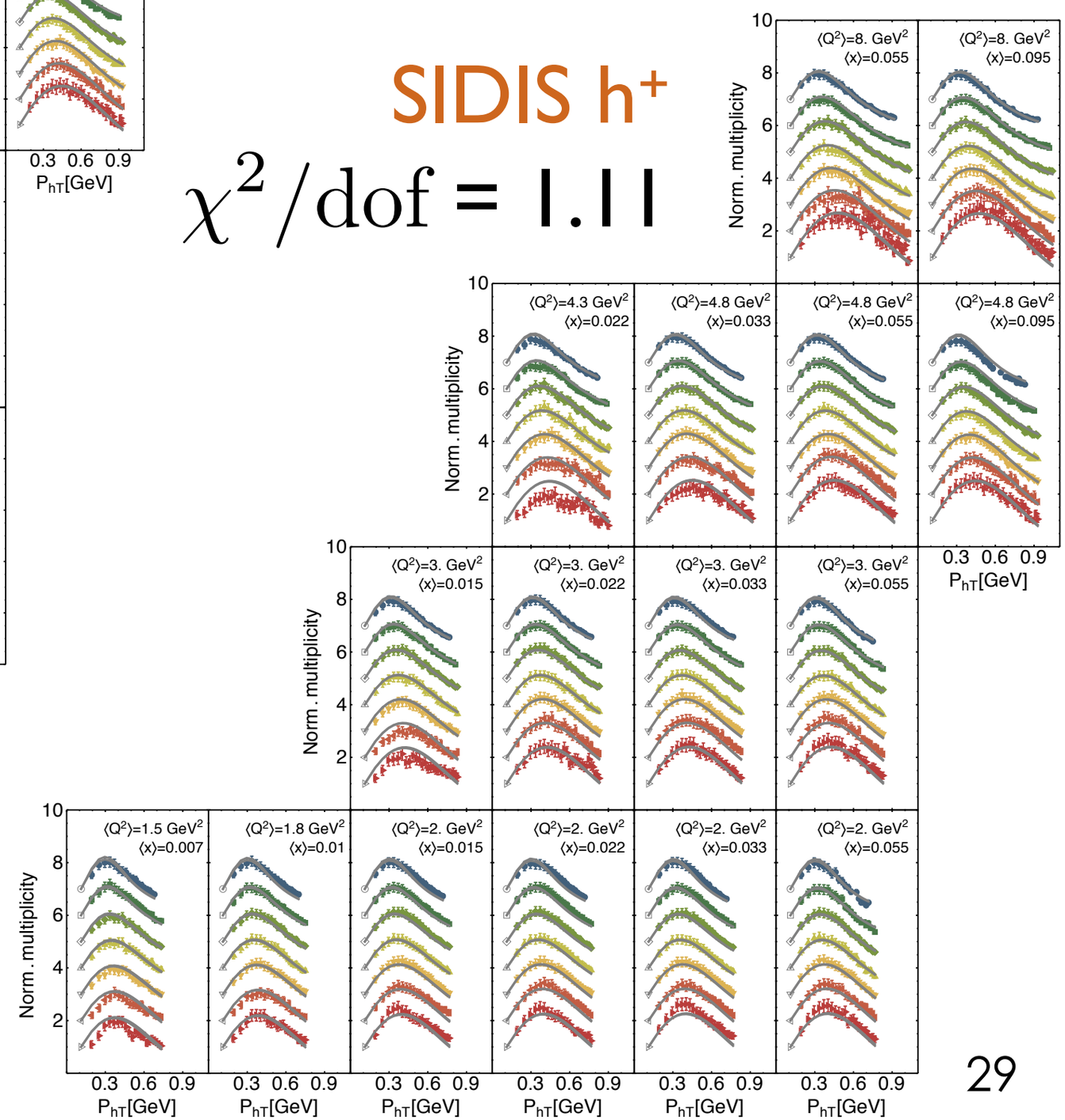
SIDIS h-

$$\chi^2/\mathrm{dof} = 1.61$$

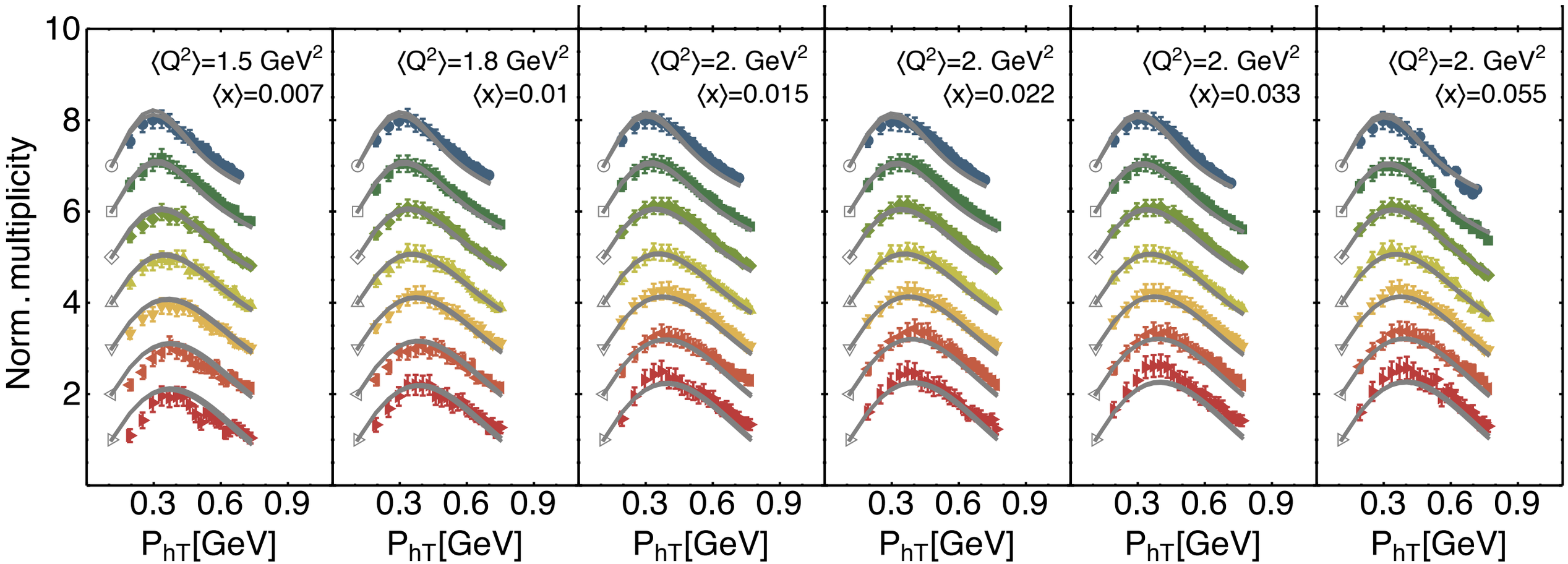


SIDIS h+

$$\chi^2/\mathrm{dof} = 1.11$$



COMPASS data
SIDIS h^+

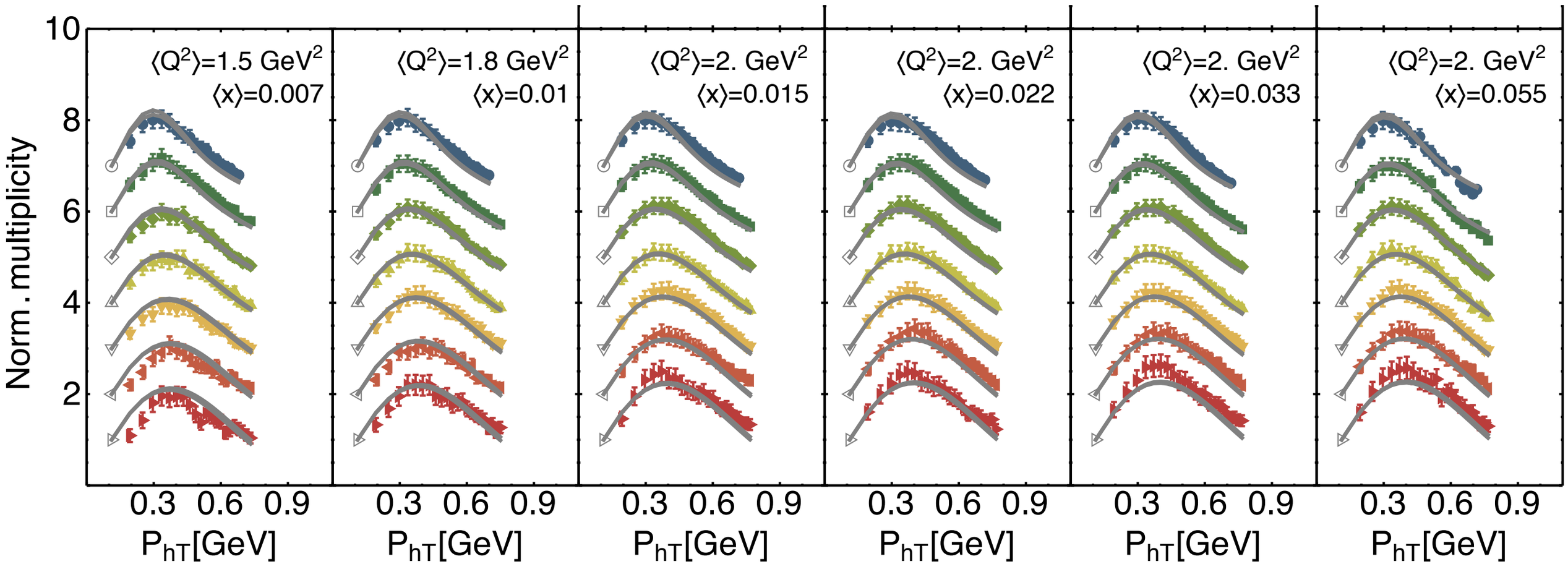


to avoid known problems
with Compass data normalization:

Observable

$$\frac{m_N^h\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right)}{m_N^h\left(x, z, \min[\boldsymbol{P}_{hT}^2], Q^2\right)}$$

COMPASS data
SIDIS h^+

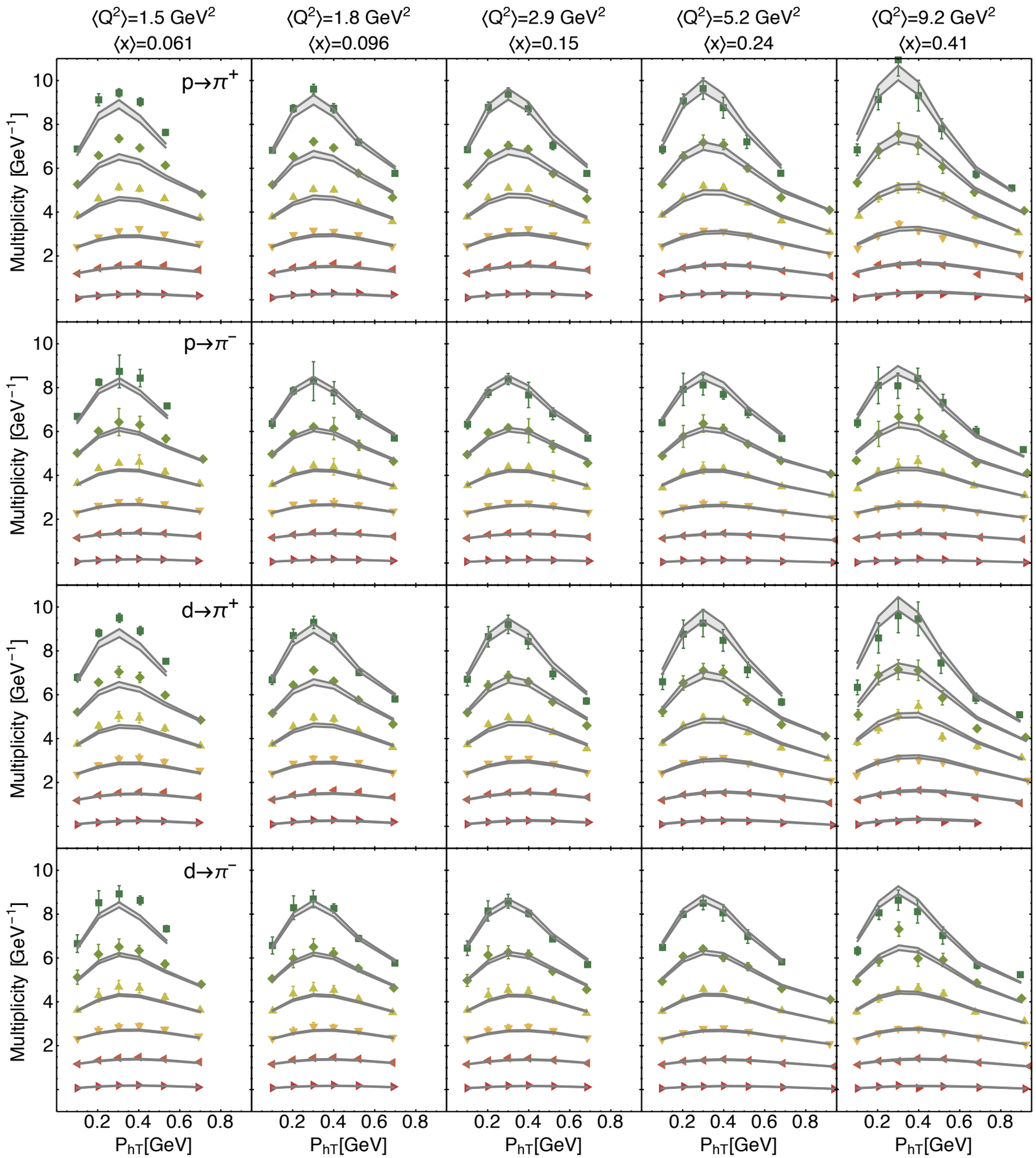


NEW recent Data:
[Phys.Rev. D97 (2018) no.3, 032006]

Observable:

$$\frac{m_N^h\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right)}{m_N^h\left(x, z, \min[\boldsymbol{P}_{hT}^2], Q^2\right)}$$

Hermes data pion production



χ^2/dof

4.83

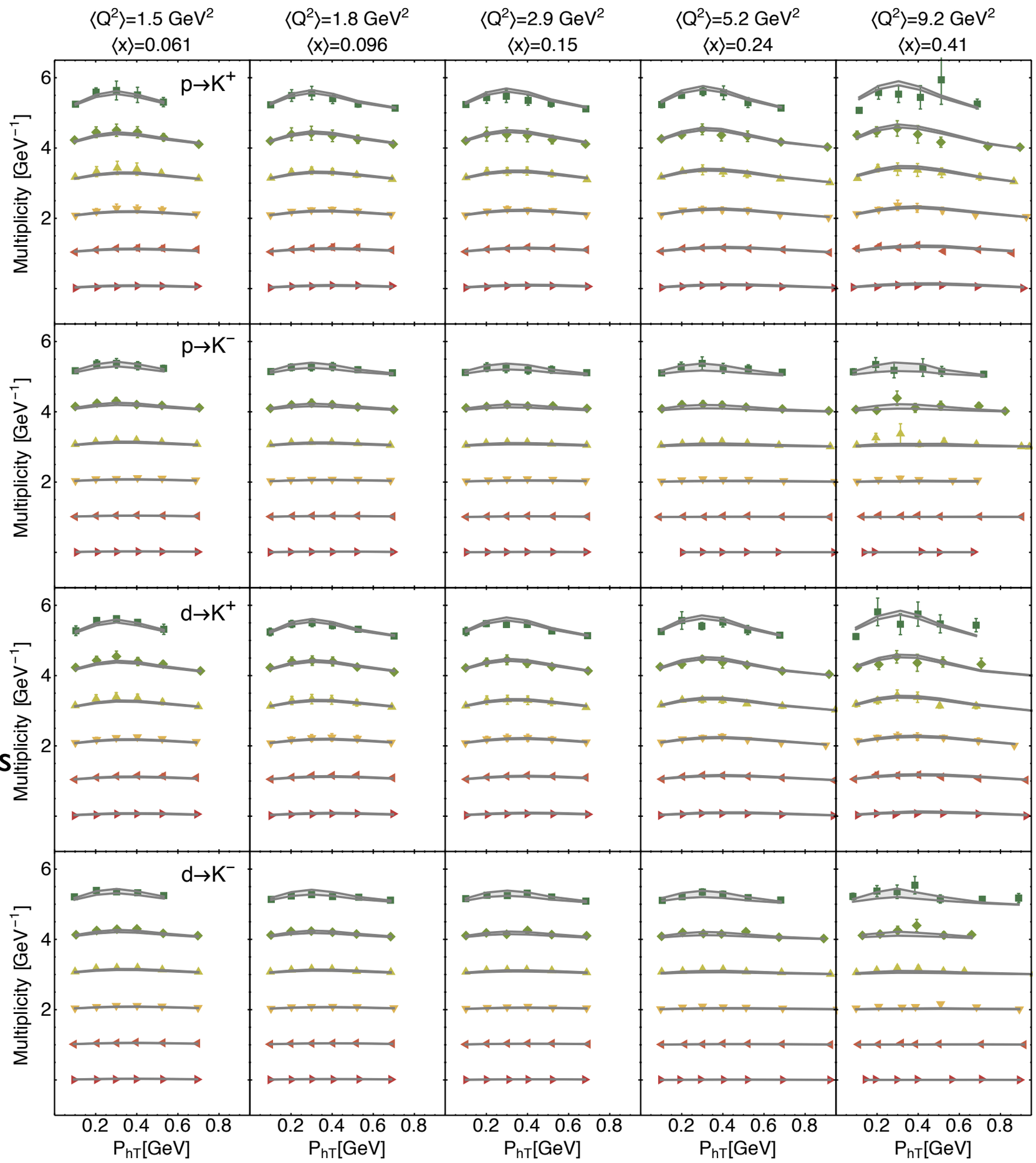
2.47

3.46

2.00

π

Hermes data pion production



χ^2/dof

0.91

0.82

1.31

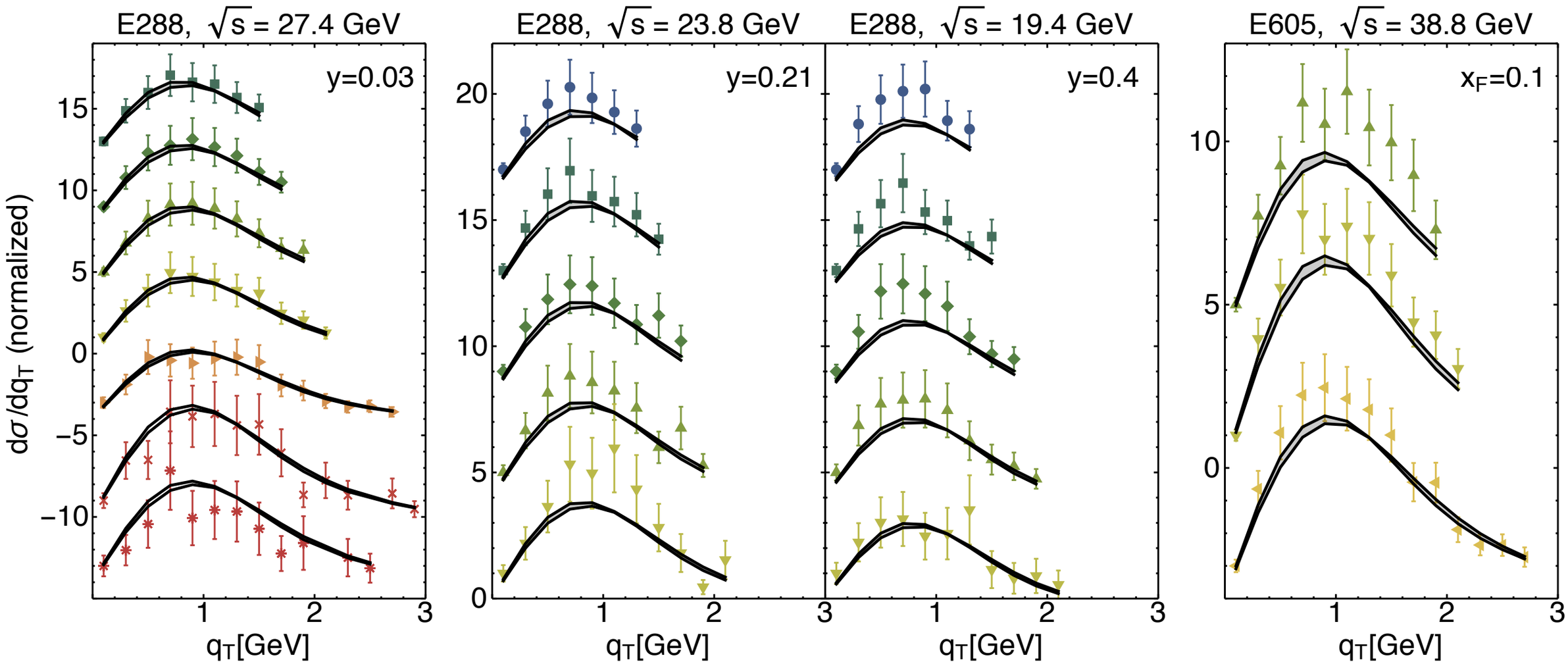
2.54

K

better agreement than pions: larger uncertainties form FFs

Drell-Yan data

χ^2/dof 0.32 0.84 0.99 1.12



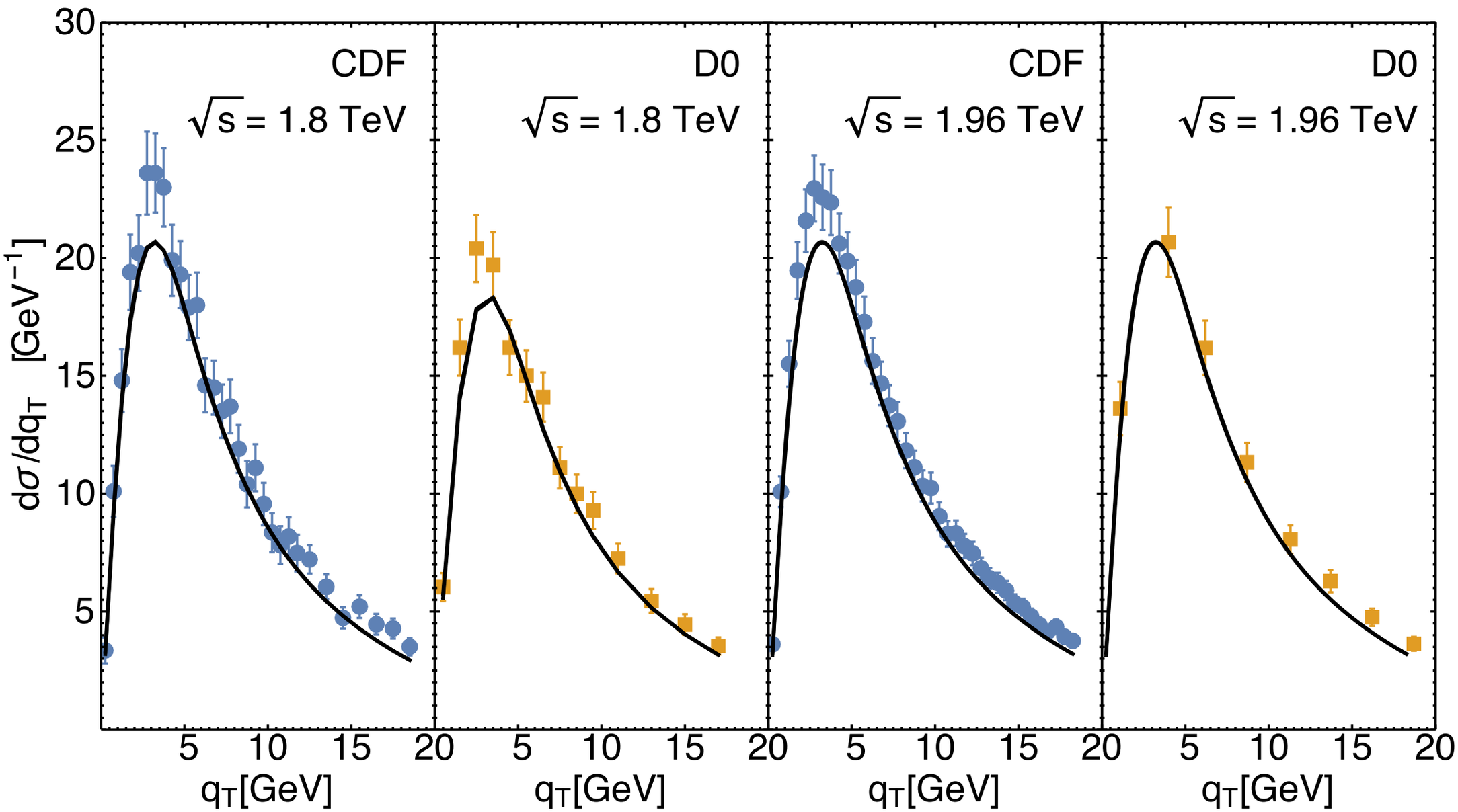
Fermilab

Q^2 Evolution: The peak is now at about 1 GeV, it was at 0.4 GeV for SIDIS

Z-boson production data

normalization : fixed from DEMS fit, different from exp.
(not really relevant for TMD parametrizations)

χ^2/dof 1.36 1.11 2.00 1.73



Q^2 Evolution: The peak is now at about 4 GeV

Best fit values

TMD PDFs	N_1 [GeV^2]	α	σ		λ [GeV^{-2}]	
All replicas	0.28 ± 0.06	2.95 ± 0.05	0.17 ± 0.02		0.86 ± 0.78	
Replica 105	0.285	2.98	0.173		0.39	
TMD FFs	**N_3 [GeV^2]**	**β**	**γ**	**δ**	**λ_F [GeV^{-2}]**	**N_4 [GeV^2]**
All replicas	0.21 ± 0.02	1.65 ± 0.49	2.28 ± 0.46	0.14 ± 0.07	5.50 ± 1.23	0.04 ± 0.01
Replica 105	0.212	2.10	2.52	0.094	5.29	0.04

TABLE XI: 68% confidence intervals of best-fit values for parametrizations of TMDs at $Q = 1$ GeV.

Flavor independent scenario:

$$N_1 = 0.28 \pm 0.06 \text{ GeV}^2$$

$$N_3 = 0.21 \pm 0.02 \text{ GeV}^2$$

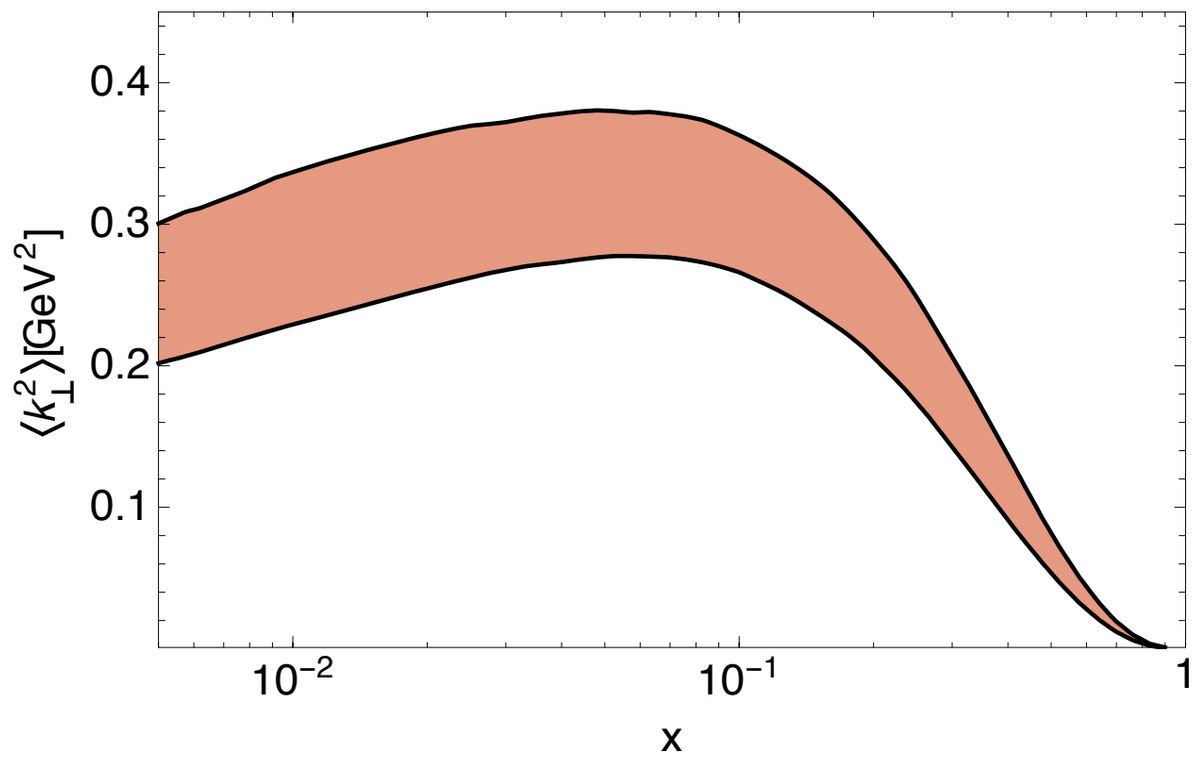
$$N_4 = 0.04 \pm 0.01 \text{ GeV}^2$$

$$g_2 = 0.13 \pm 0.01 \text{ GeV}^2$$

best value from 200 replicas

compatible with other extractions

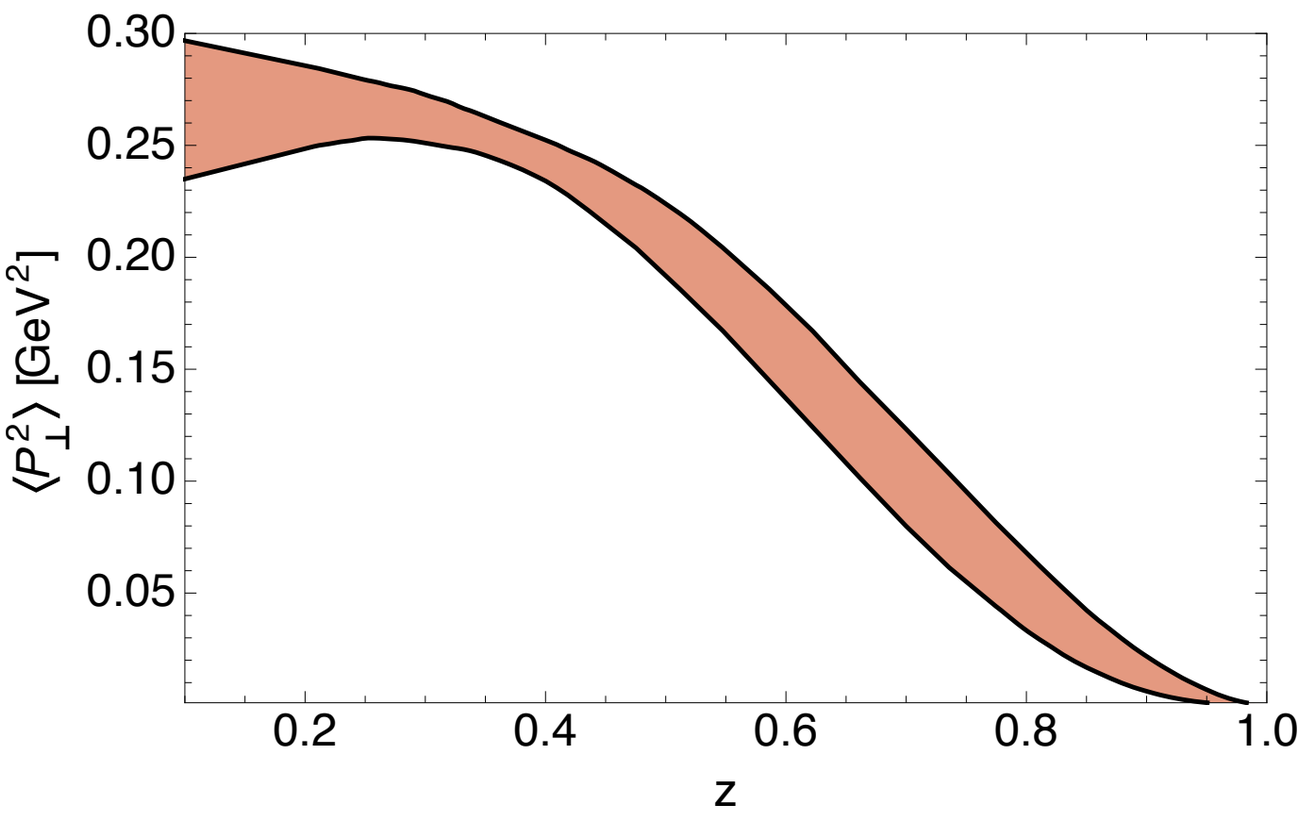
Mean transverse momentum



Change in TMD width
x-dependence

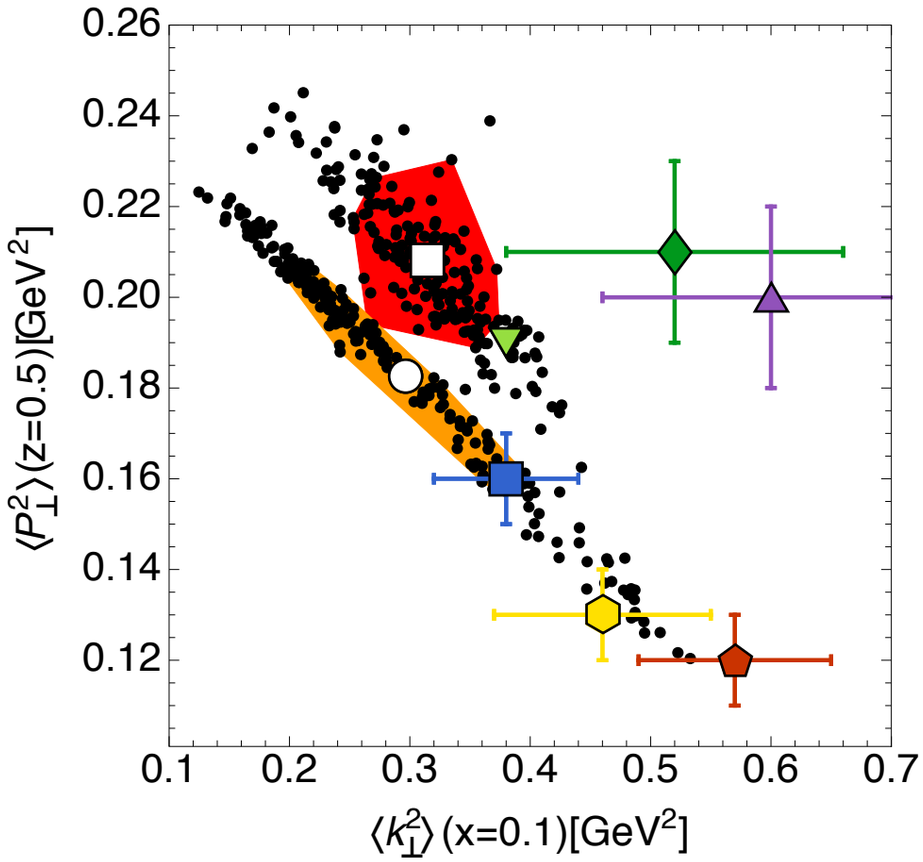
in TMD PDF

Q^2=1GeV^2



in TMD FF

Best fit value: transverse momenta



Bacchetta, Delcarro, Pisano, Radici, Signori JHEP06(2017)081

Signori, Bacchetta, Radici, Schnell arXiv:1309.3507

Schweitzer, Teckentrup, Metz, arXiv:1003.2190

Anselmino et al. arXiv:1312.6261 [HERMES]

Anselmino et al. arXiv:1312.6261 [HERMES, high z]

Anselmino et al. arXiv:1312.6261 [COMPASS, norm.]

Anselmino et al. arXiv:1312.6261 [COMPASS, high z, norm.]

Echevarria, Idilbi, Kang, Vitev arXiv:1401.5078 (Q = 1.5 GeV)

Red/orange regions: **68% CL** from replica method

Inclusion of DY/Z diminishes the correlation

Inclusion of Compass increases the $\langle P_\perp^2 \rangle$ and reduces its spread

e+e- would further reduce the correlation

Stability of our results

Test of our default choices

How does the χ^2 of a single replica change if we modify them?

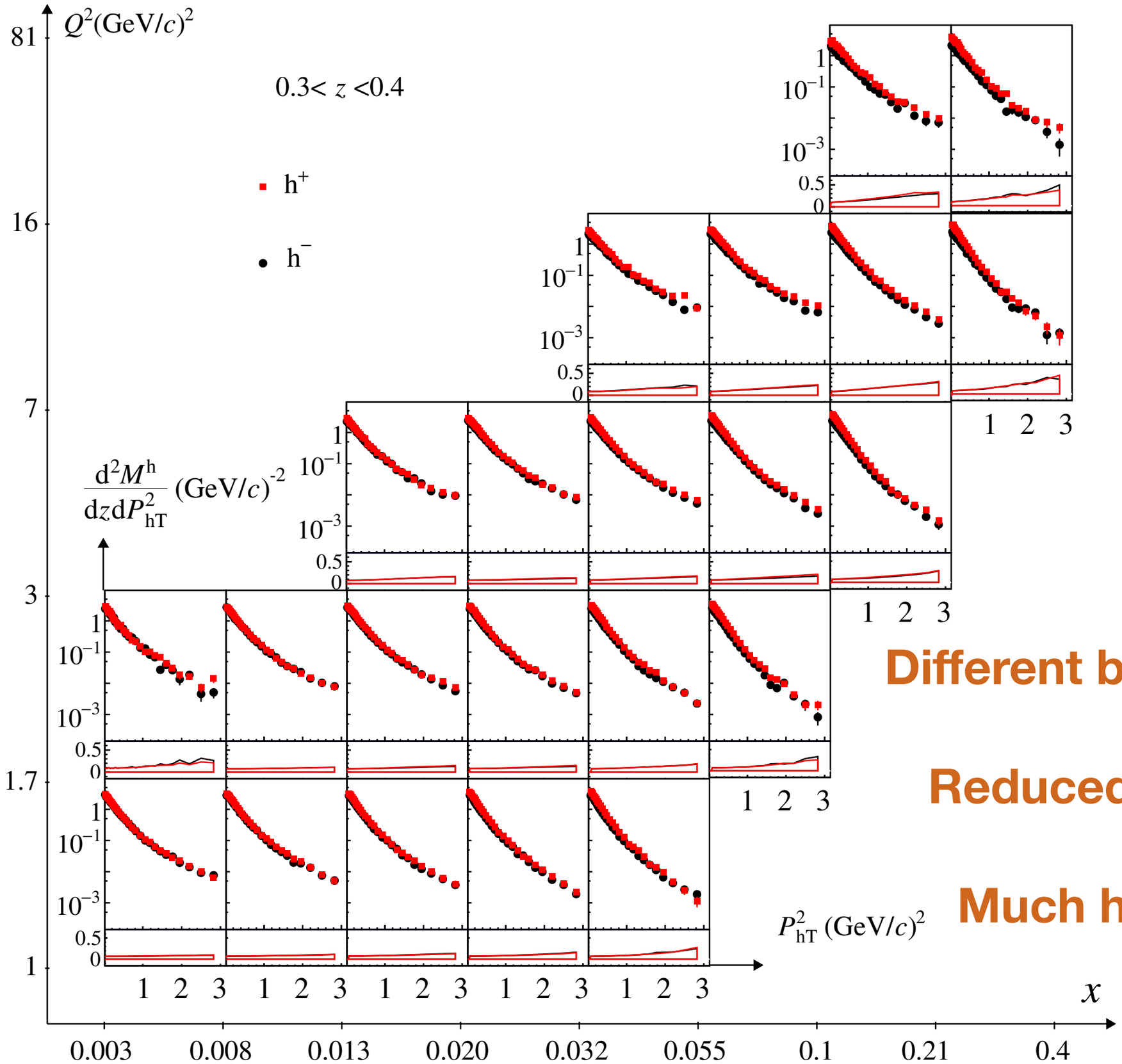
Original χ^2/dof = **1.51**

Normalization of HERMES data as done for COMPASS:
χ^2/dof = 1.27

Parametrizations for collinear PDFs
(NLO GJR 2008 default choice):
NLO MSTW 2008 (1.84), NLO CJ12 (1.85)

More stringent cuts
(TMD factorization better under control) χ^2/dof $\rightarrow$ 1
Ex: Q2 > 1.5 GeV^2; 0.25 < z < 0.6; PhT < 0.2Qz $\Rightarrow$ χ^2/dof = 1.02 (477 bins)

Analysis of New Compass Data



Different binning in Z (larger)

Reduced number of data

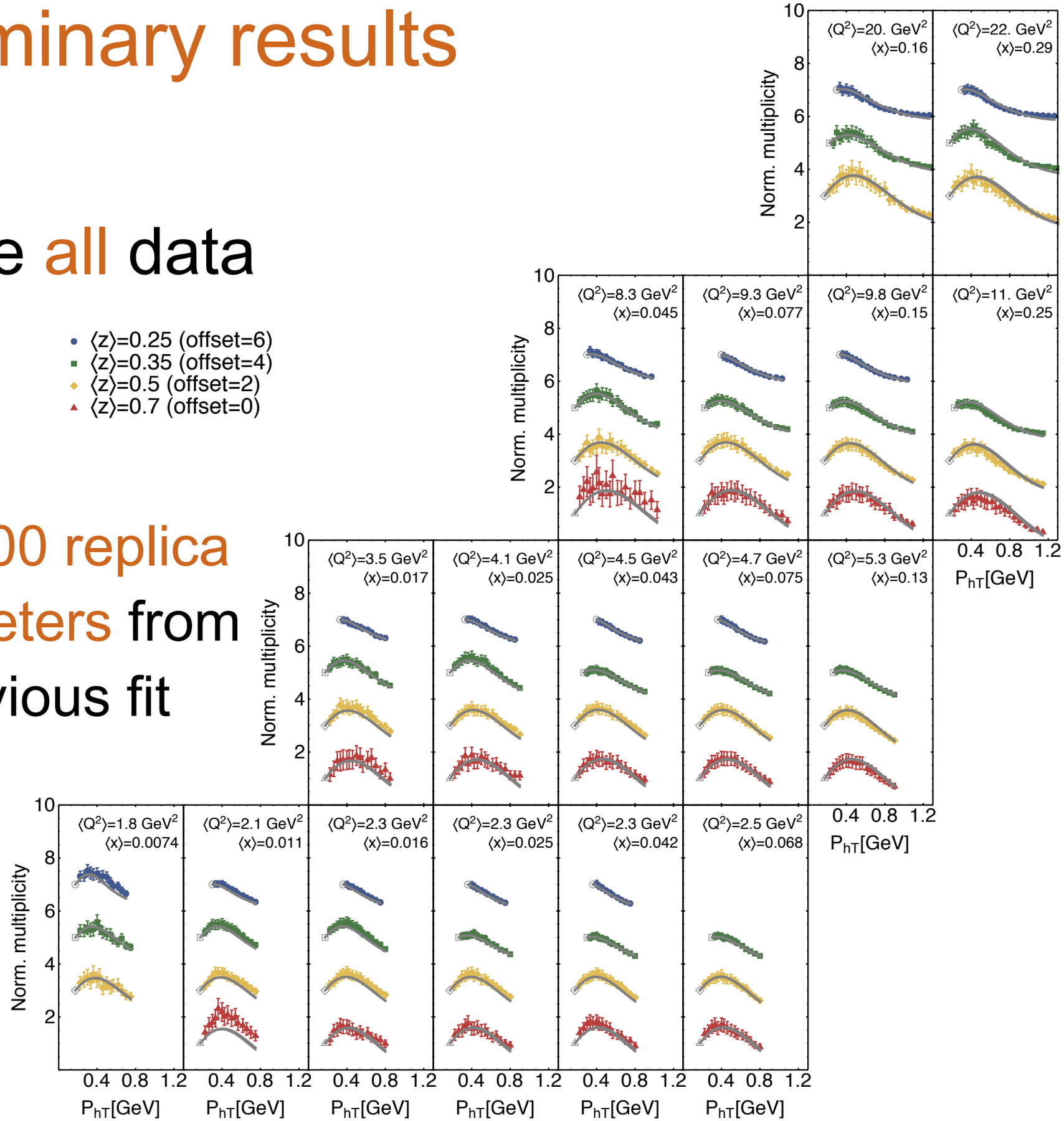
Much higher statistics

Preliminary results

SIDIS h^+

Include all data

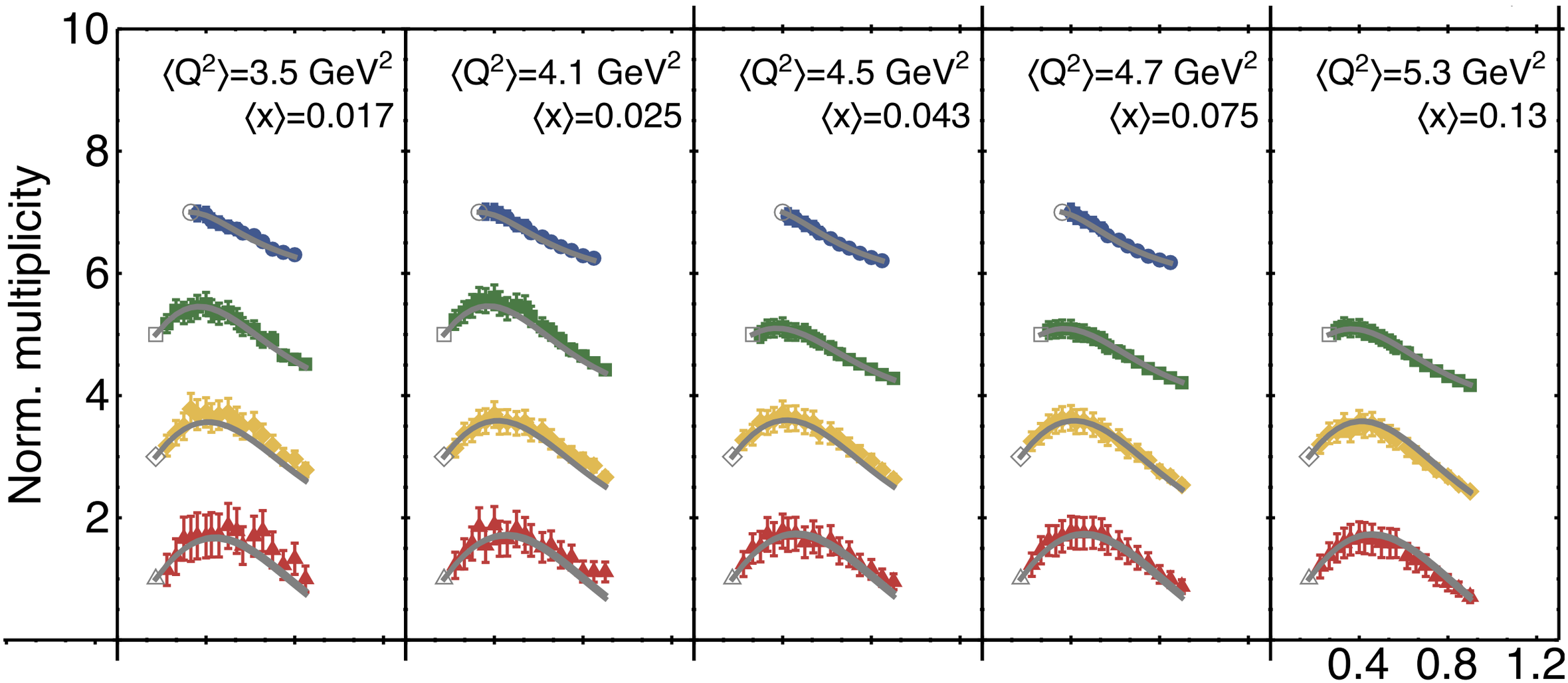
Use 200 replica parameters from previous fit



Normalized at 1st data point of bin

Include all data

SIDIS h^+

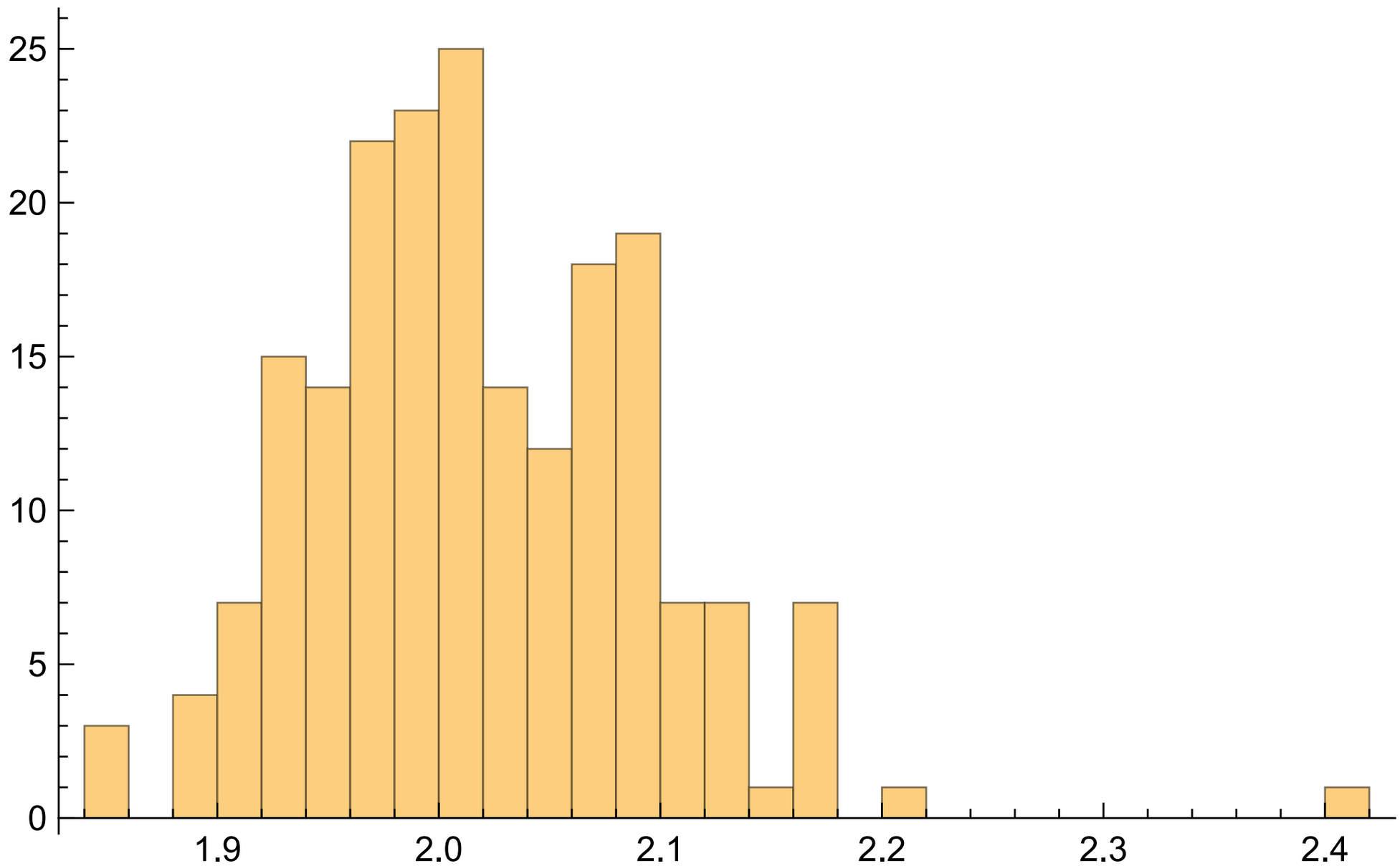


Use 200 replica parameters from previous fit

Normalized at 1st data point of bin

Include all data

SIDIS h^+



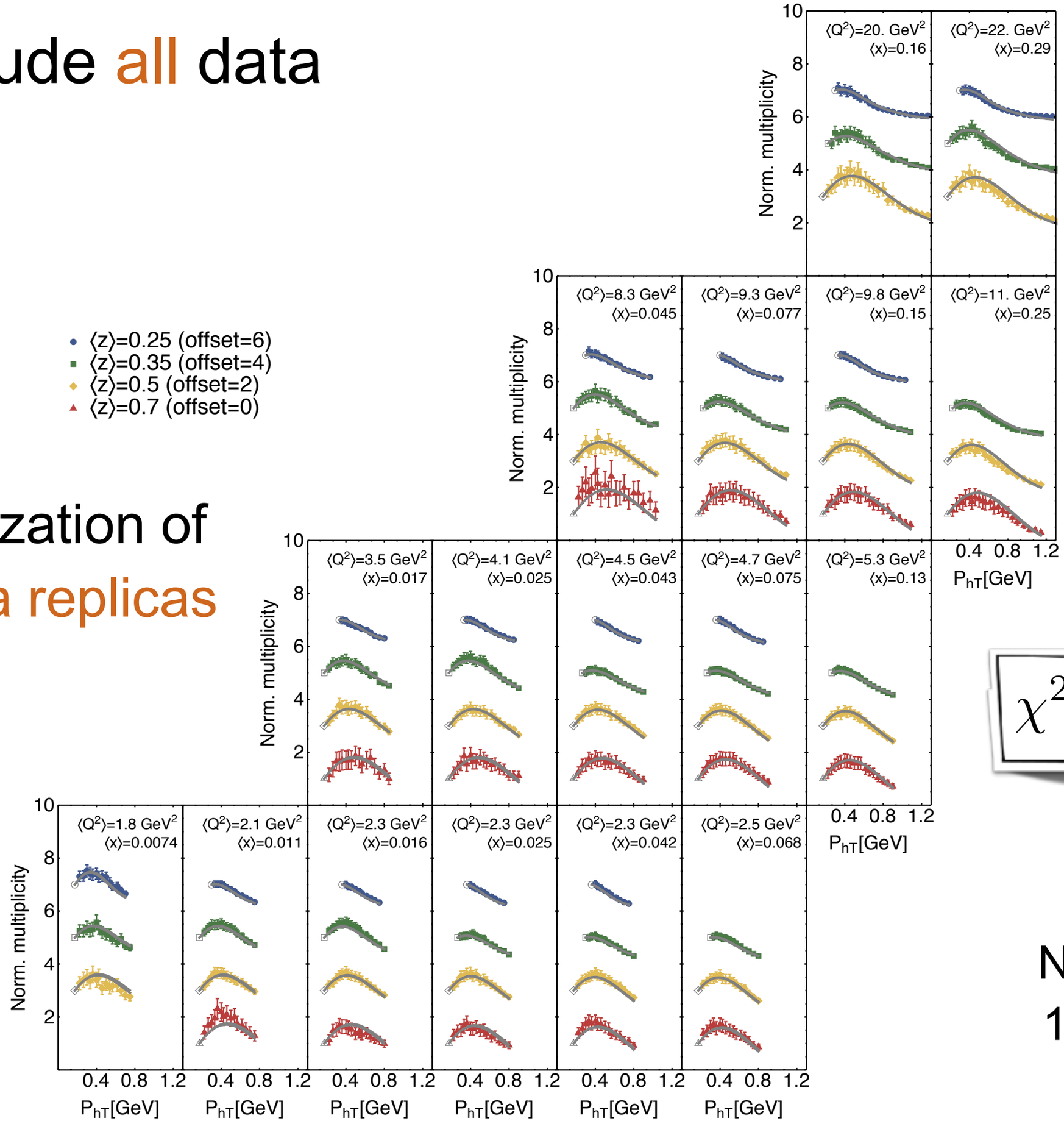
Use 200 replica parameters from previous fit

$$\chi^2/\mathrm{dof} = 2.01$$

Normalized at 1st data point of bin

Include all data

SIDIS h^+



Minimization of

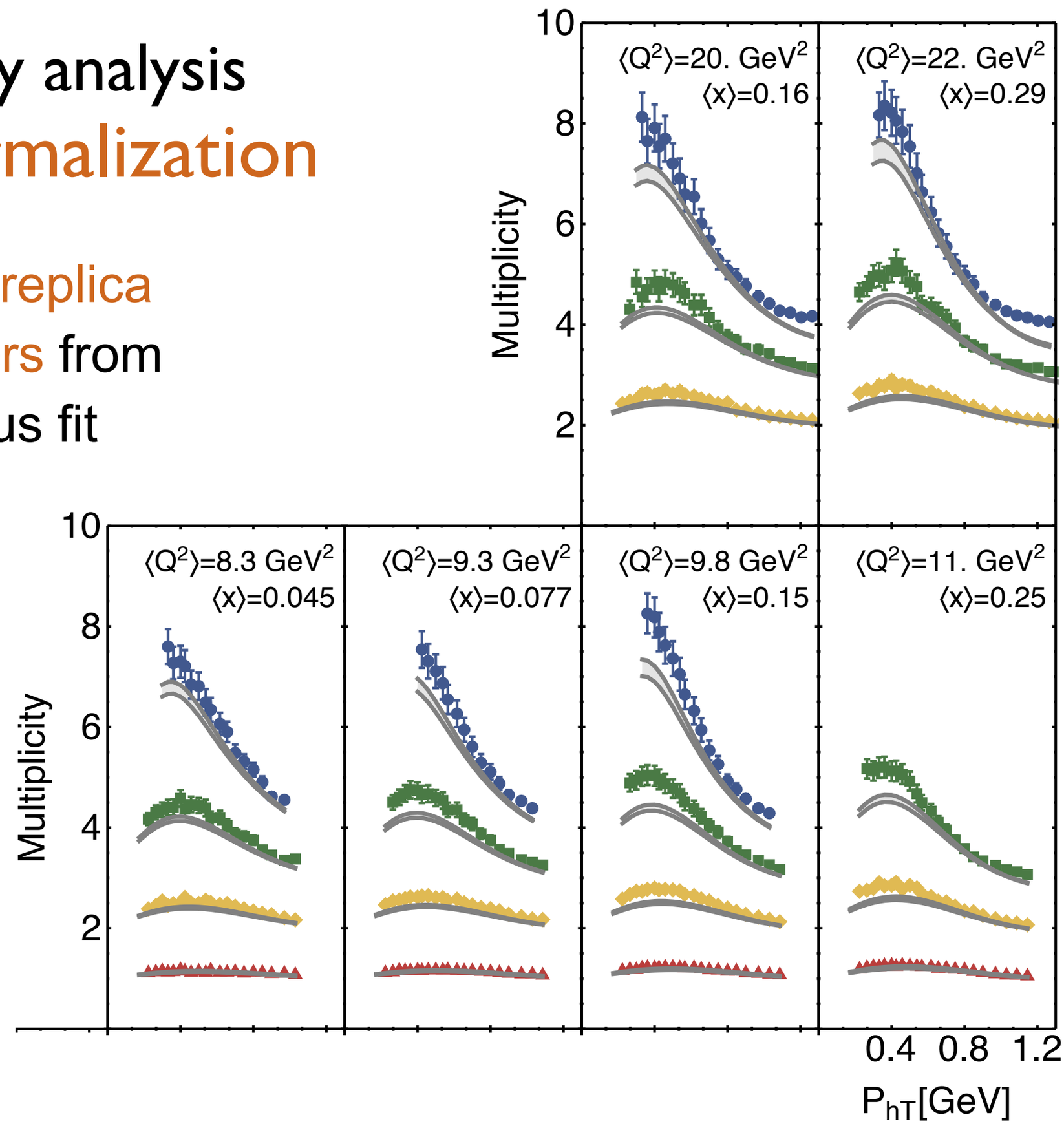
50 data replicas

$$\chi^2/\mathrm{dof} = 1.71$$

Normalized at 1st data point of bin

Exploratory analysis without normalization

Use 200 replica parameters from previous fit



- ⟨z⟩=0.25 (offset=4)
- ⟨z⟩=0.35 (offset=3)
- ⟨z⟩=0.5 (offset=2)
- ⟨z⟩=0.7 (offset=1)

SIDIS h^+

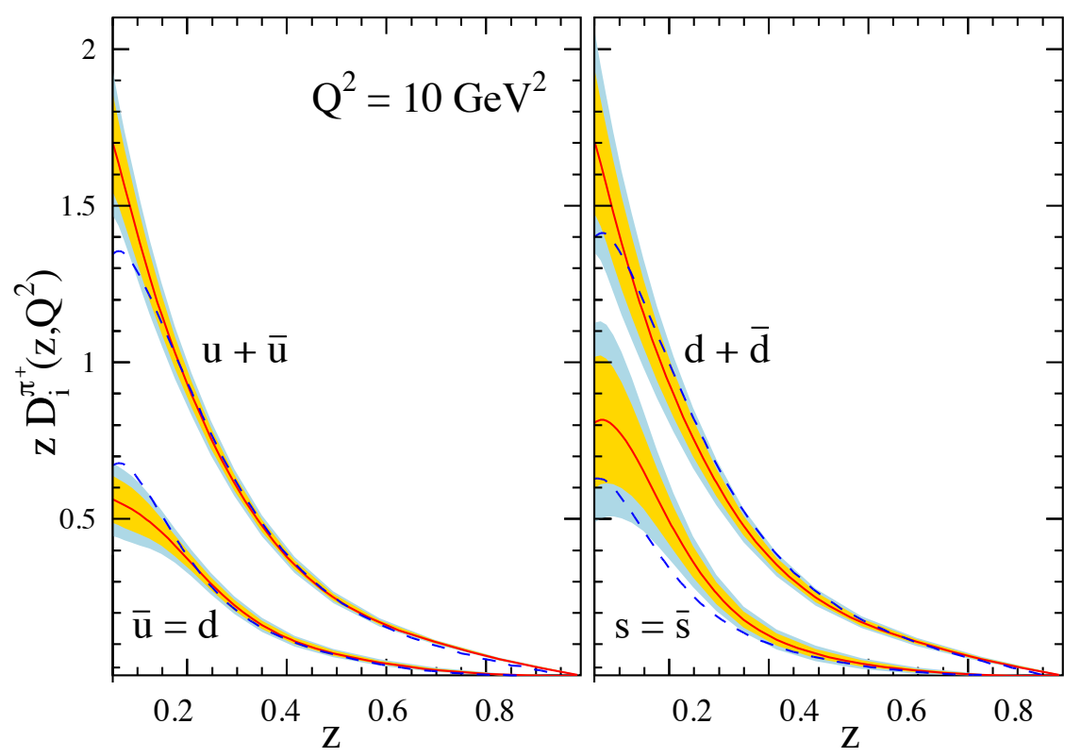
Exploratory analysis
without normalization

Use 200 replica parameters from previous fit $\longrightarrow$ χ^2/dof **> 4**

Sensitive to z value

Less stable with regards to kinematical cuts

...



FF DSS

SIDIS h^+

Preliminary analysis of Z data at LHC

8 TeV

7 TeV

7 TeV 8 TeV

To be included in future fit

Preliminary analysis using current parameters

Next step: minimisation of replicated data

Conclusions

First global extraction of TMDs from SIDIS, DY and Z boson

Test of the universality and evolution formalism of partonic TMDs

Definition of a parametrization of TMDs able to describe more than 8000 data points

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Conclusions and open issues

First global extraction of TMDs from SIDIS, DY and Z boson

Test of the universality and evolution formalism of partonic TMDs

Definition of a parametrization of TMDs from 8000 data points

New Data

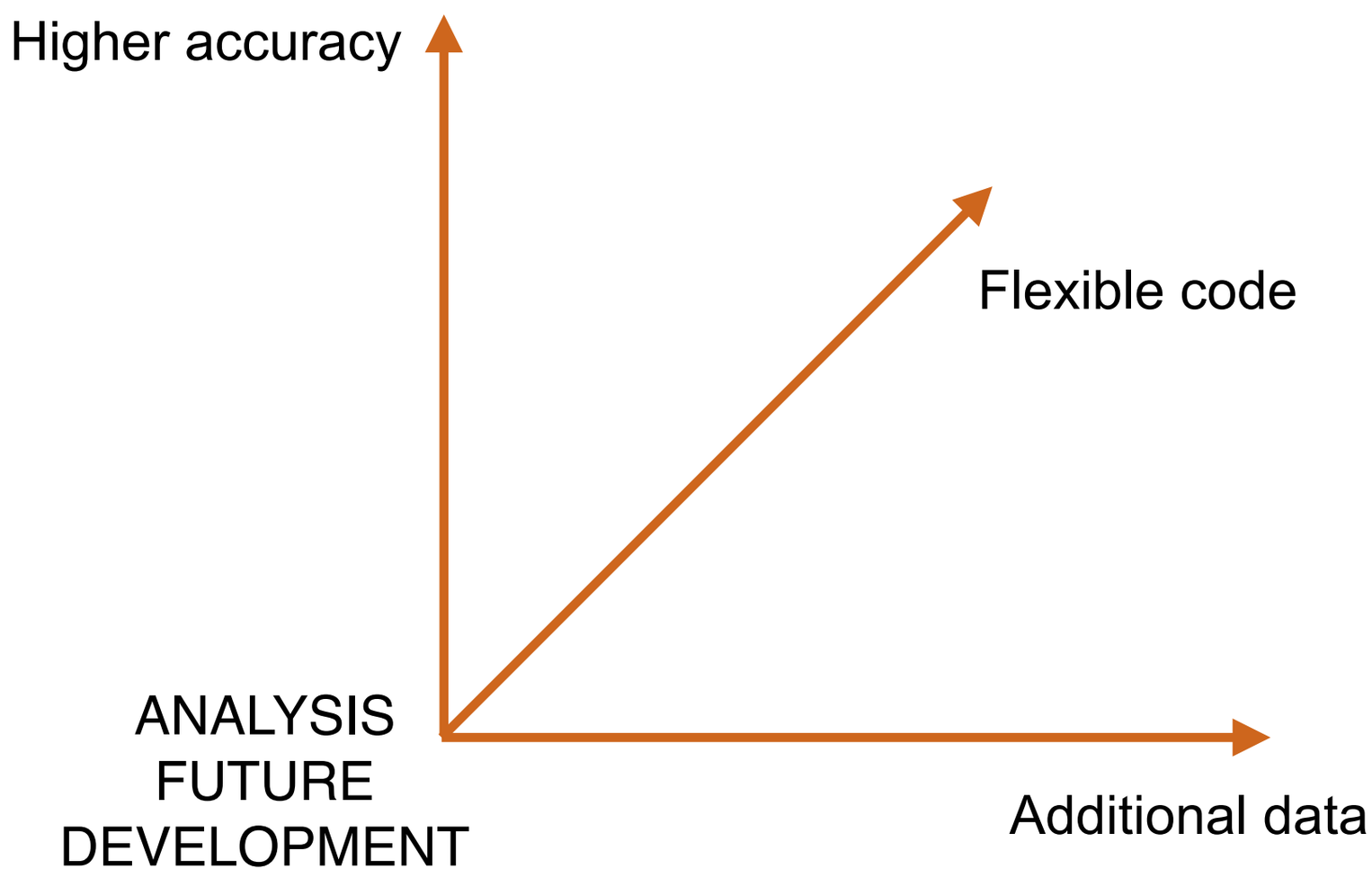
- compatible with parameters obtained from previous analysis
- requires further considerations on normalisation

Conclusions and open issues

First global extraction of TMDs from SIDIS, DY and Z boson

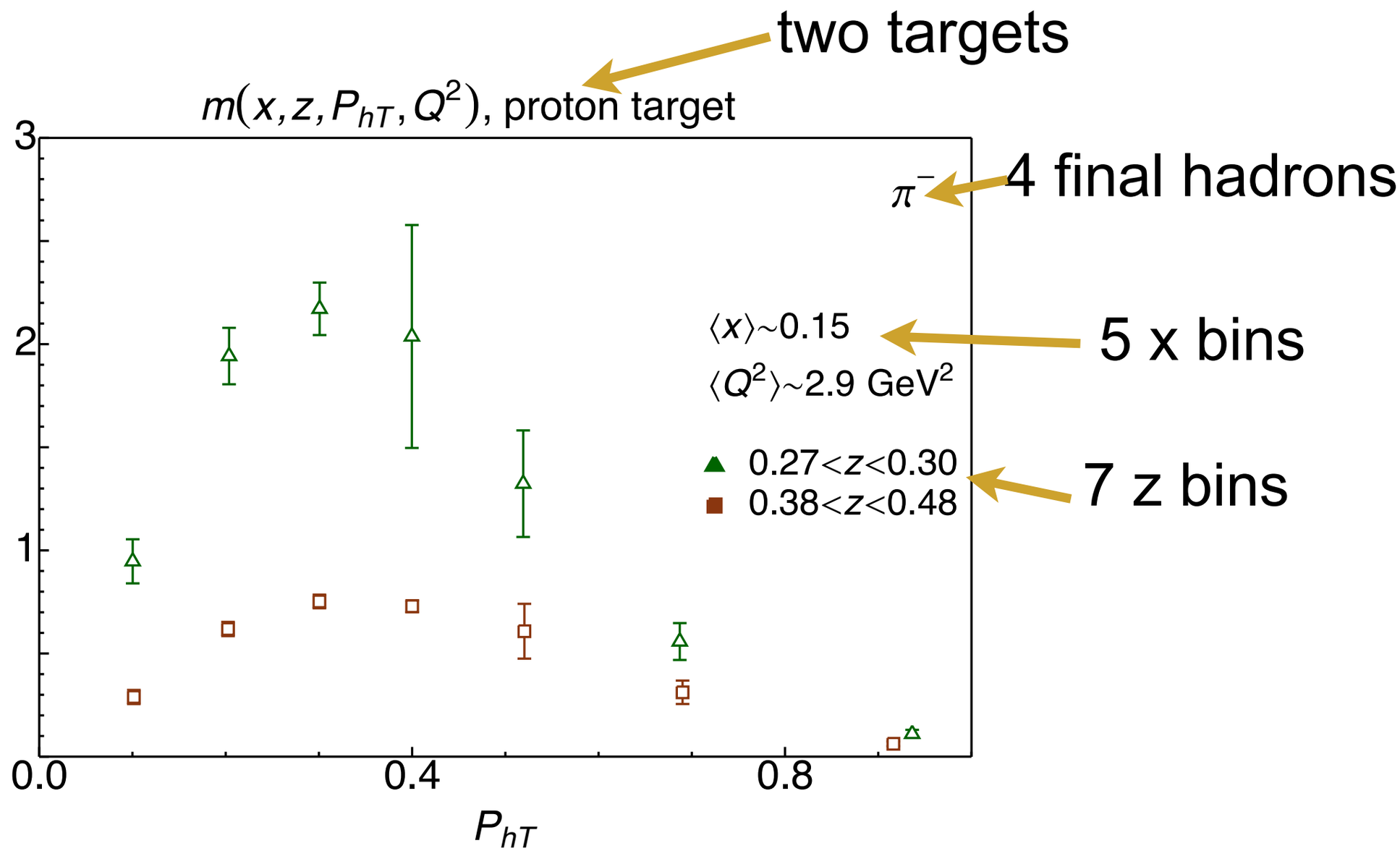
Test of the universality and evolution formalism of partonic TMDs

Definition of a parametrization of TMDs from 8000 data points



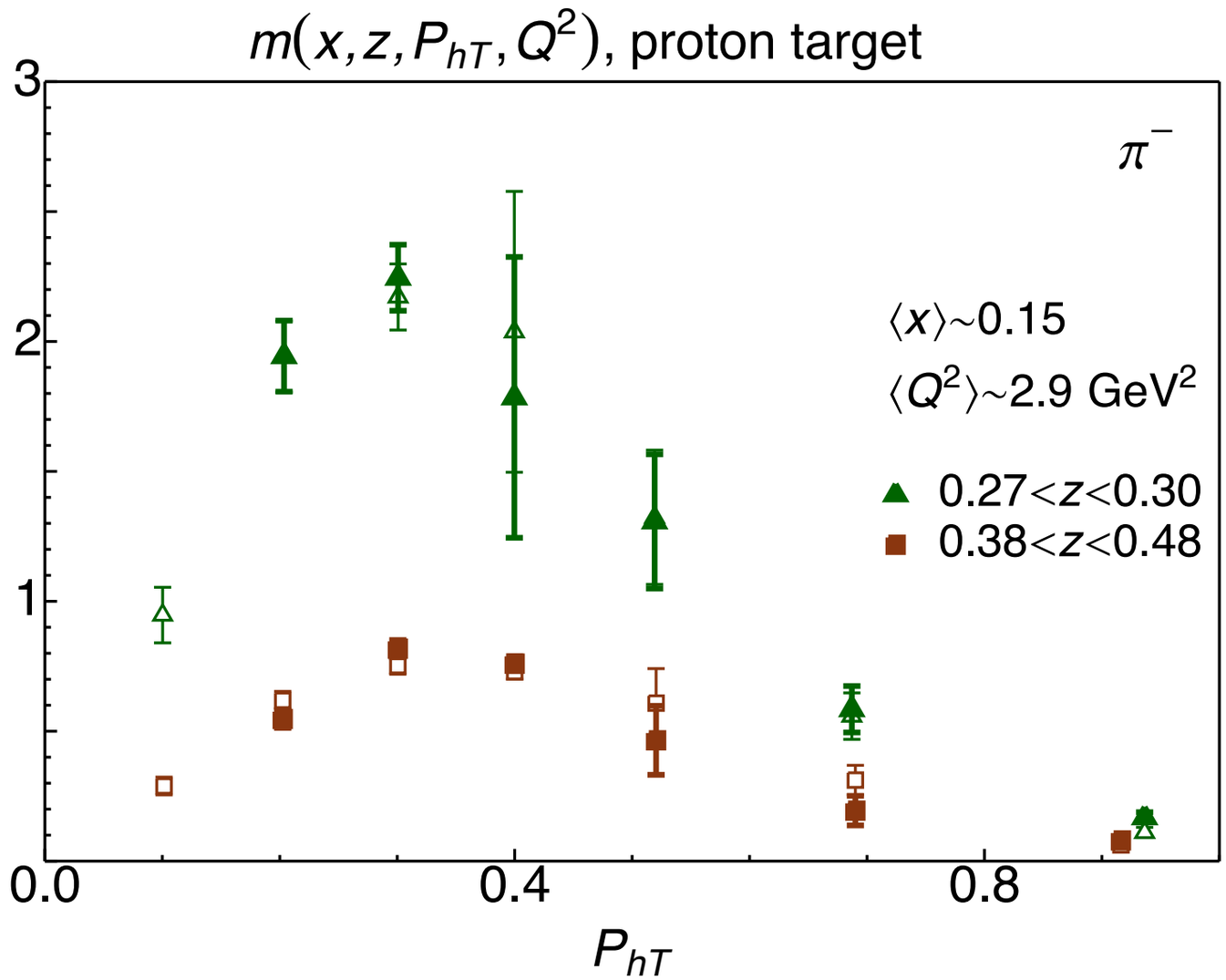
BACKUP

The replica method



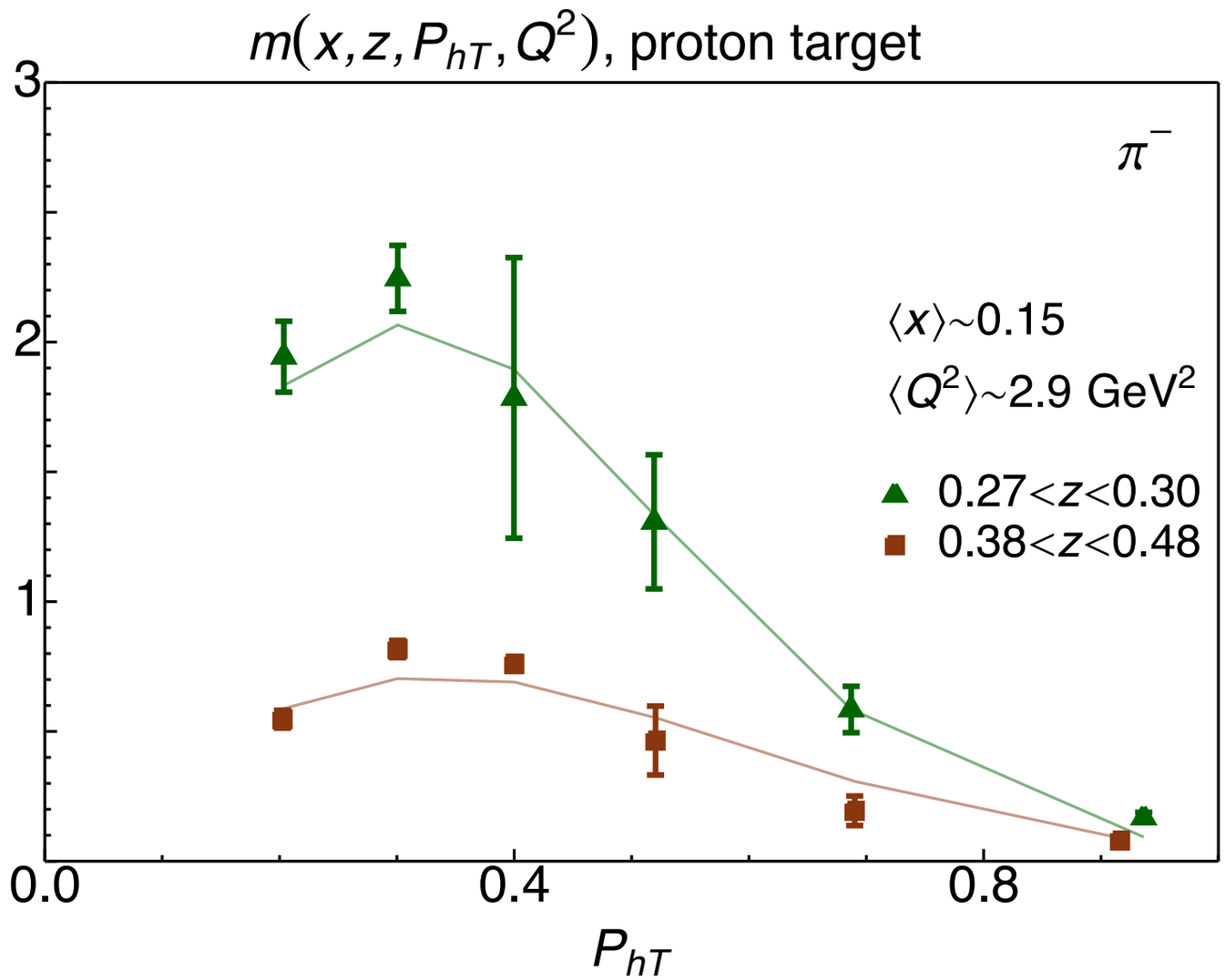
Example of original data

The replica method



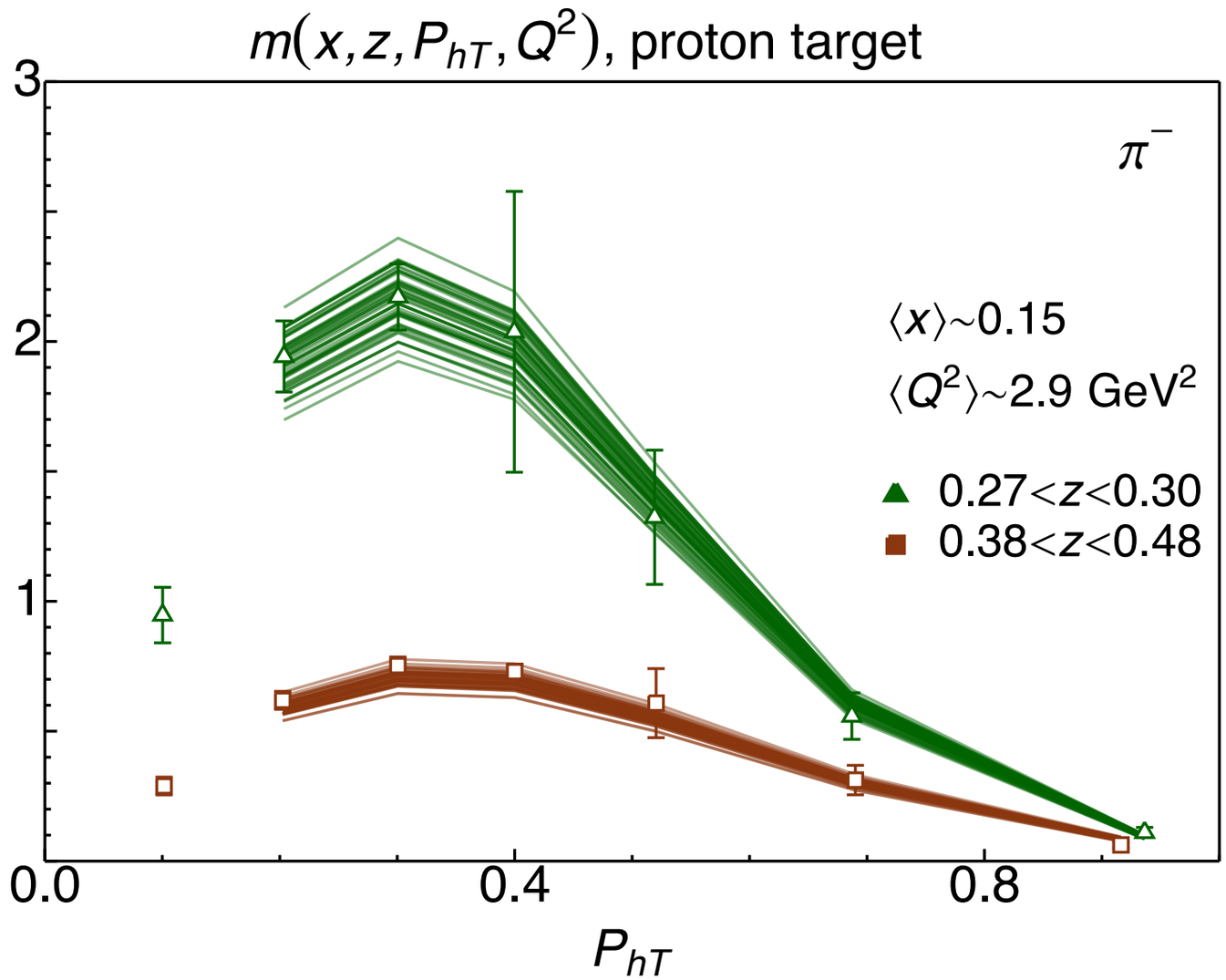
Data are replicated (with Gaussian distribution)

The replica method



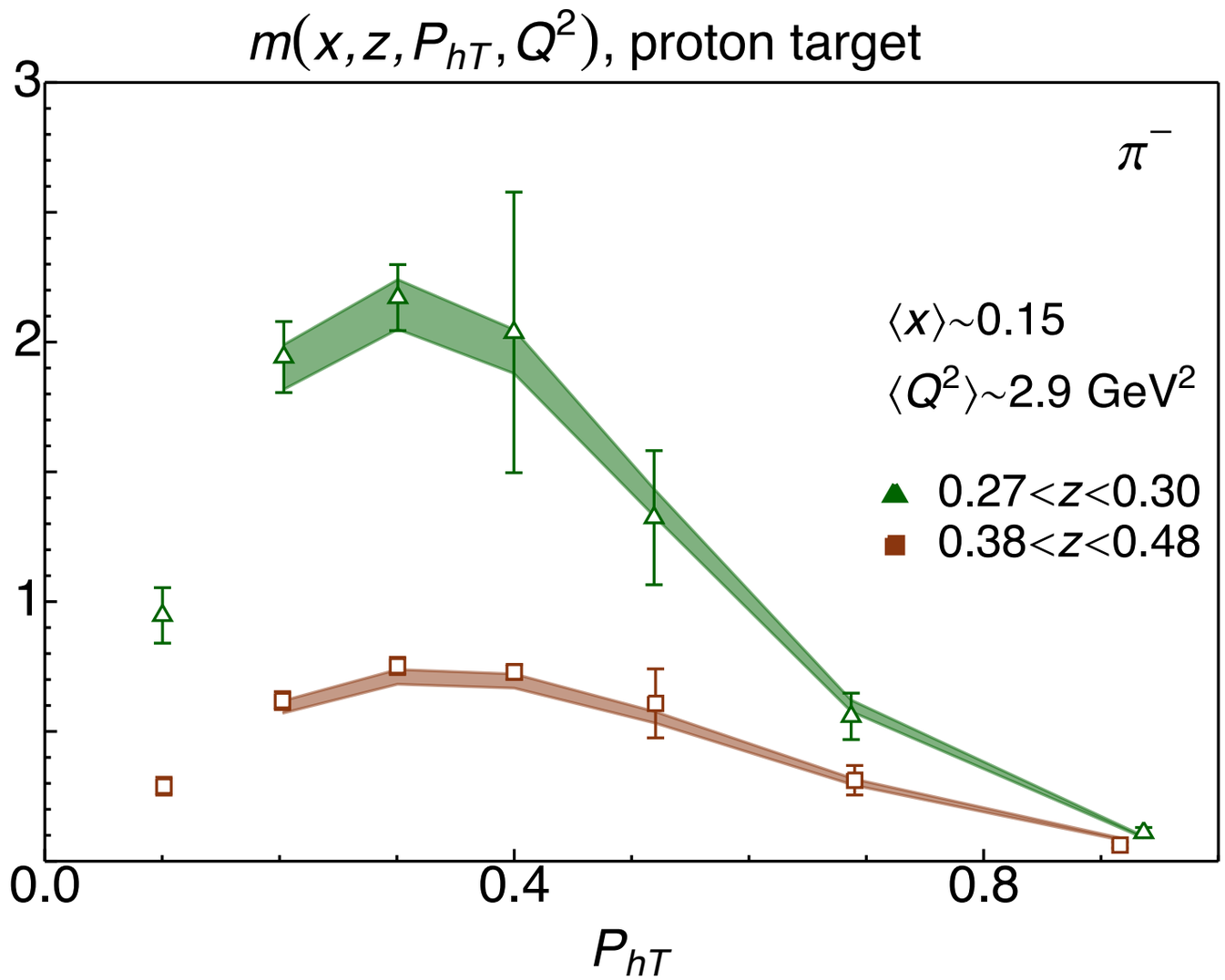
The fit is performed on the replicated data

The replica method



The procedure is repeated 200 times

The replica method



For each point, a central 68% confidence interval is identified

Previous fit studies

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	✘	✘	✔	✔	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evo (QPM)	✔	✘	✘	✘	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	**8059**

μ and b_* prescriptions

Choice Choice

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$\mu_b = 2e^{-\gamma_E}/b_*$ $\quad b_* \equiv \dfrac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$ *Collins, Soper, Sterman, NPB250 (85)*

$\mu_b = 2e^{-\gamma_E}/b_*$ $\quad b_* \equiv b_{\rm max}\left(1 - e^{-\frac{b_T^4}{b_{\rm max}^4}}\right)^{1/4}$ *Bacchetta, Echevarria, Mulders, Radici, Signori arXiv:1508.00402*

$\mu_b = Q_0 + q_T$ $\quad b_* = b_T$ *DEMS 2014*

Complex-b prescription *Laenen, Sterman, Vogelsang, PRL 84 (00)*

Pavia 2017 perturbative ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$A_1(\mathcal{O}(\alpha_S^1))$ ✔	$A_2(\mathcal{O}(\alpha_S^2))$ ✔	$A_3(\mathcal{O}(\alpha_S^3))$	$\dots$
	$B_1(\mathcal{O}(\alpha_S^1))$ ✔	$B_2(\mathcal{O}(\alpha_S^2))$	$\dots$
$C_0(\mathcal{O}(\alpha_S^0))$ ✔	$C_1(\mathcal{O}(\alpha_S^1))$	$C_2(\mathcal{O}(\alpha_S^2))$	$\dots$
$H_0(\mathcal{O}(\alpha_S^0))$ ✔	$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	$\dots$
	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	$\dots$

Model: non perturbative elements

input TMD FF (Q^2=1 GeV^2)

$$\hat{D}_{1NP}^{a\to h} = \text{ F.T. of } \frac{1}{g_{3a\to h} + (\lambda_F/z^2) g_{4a\to h}^2} \left(e^{-\frac{\boldsymbol{P}_\perp^2}{g_{3a\to h}}} + \lambda_F \frac{\boldsymbol{P}_\perp^2}{z^2} \, e^{-\frac{\boldsymbol{P}_\perp^2}{g_{4a\to h}}} \right)$$

sum of two different gaussians
with different variance
with kinematic dependence on transverse momenta

width z-dependence

$$g_{3,4}(z) = N_{3,4} \, \frac{(z^\beta + \delta) \, (1-z)^\gamma}{(\hat{z}^\beta + \delta) \, (1-\hat{z})^\gamma}$$

where

$$N_{3,4} \equiv g_{3,4}(\hat{z})$$

$$\hat{z} = 0.5$$

Average transverse momenta

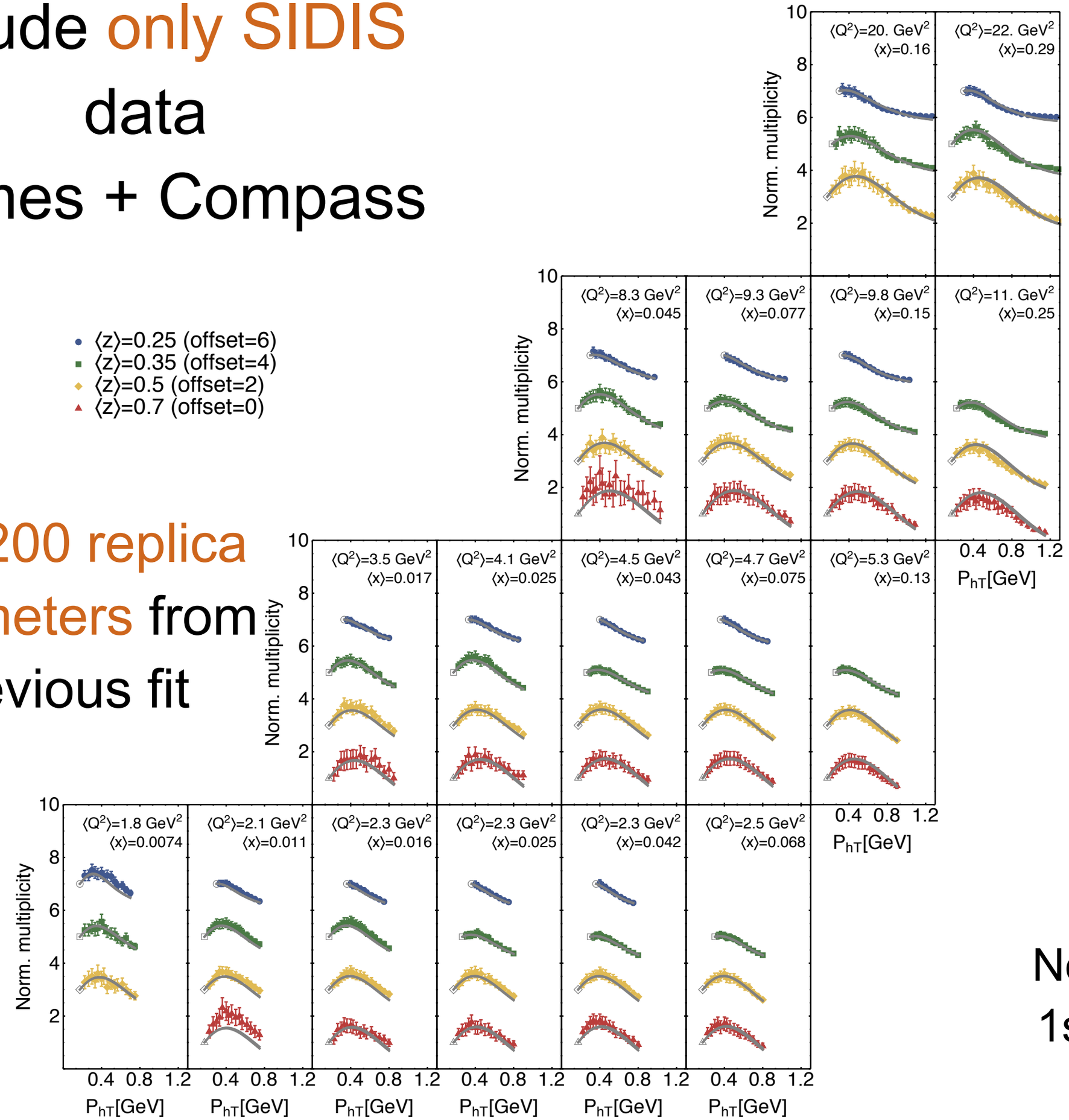
$$\langle \boldsymbol{k}_\perp^2 \rangle (x) = \frac{g_1(x) + 2\lambda g_1^2(x)}{1 + \lambda g_1(x)}$$

$$\langle \boldsymbol{P}_\perp^2 \rangle (z) = \frac{g_3^2(z) + 2\lambda_F g_4^3(z)}{g_3(z) + \lambda_F g_4^2(z)}$$

Include only SIDIS data

Hermes + Compass

SIDIS h^+

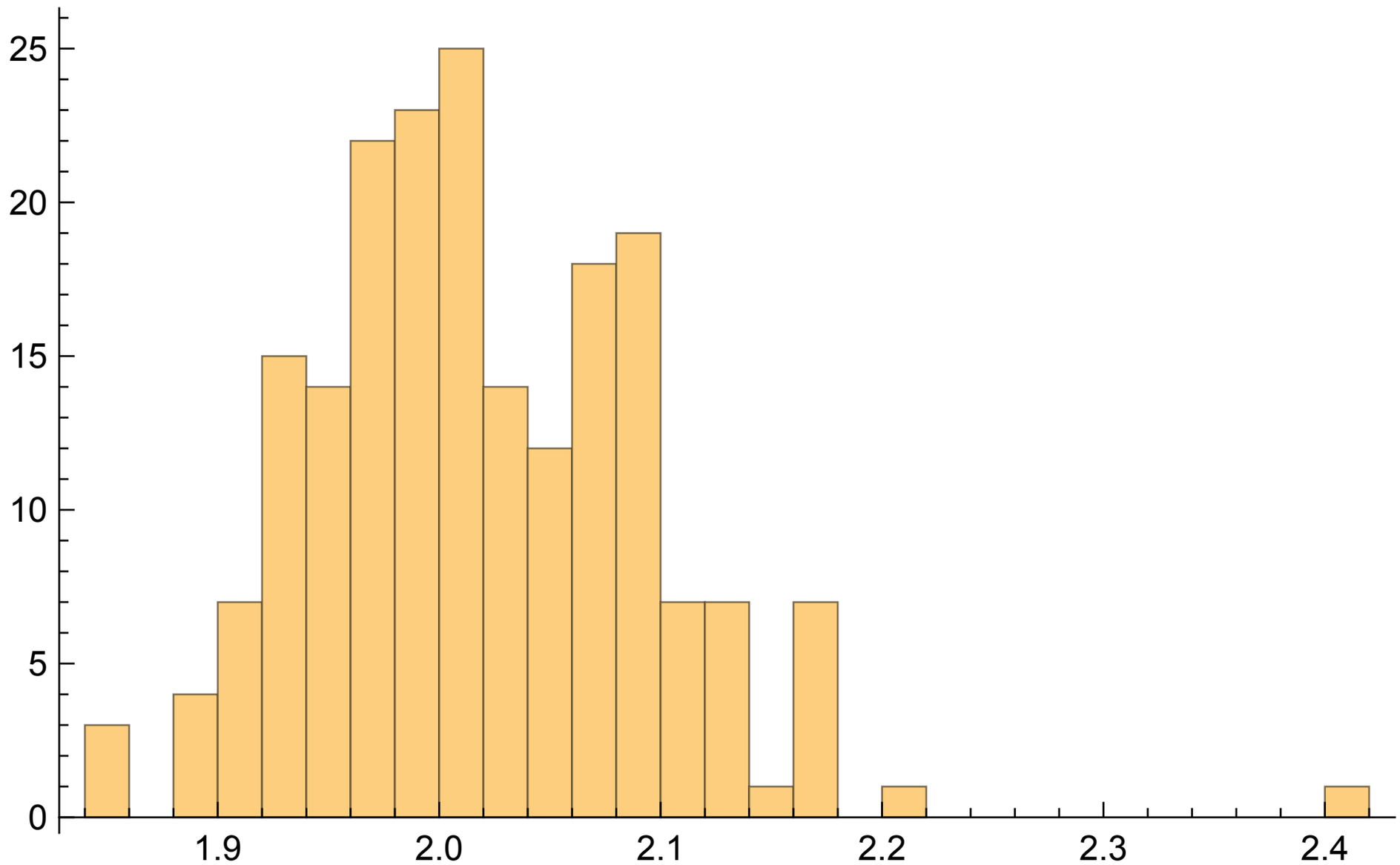


Use 200 replica parameters from previous fit

Normalized at 1st data point of bin

Include only SIDIS data

SIDIS h^+



Use 200 replica parameters from previous fit

$\chi^2/\mathrm{dof} = 2.07$

Normalized at 1st data point of bin

Include only SIDIS data

Hermes + Compass

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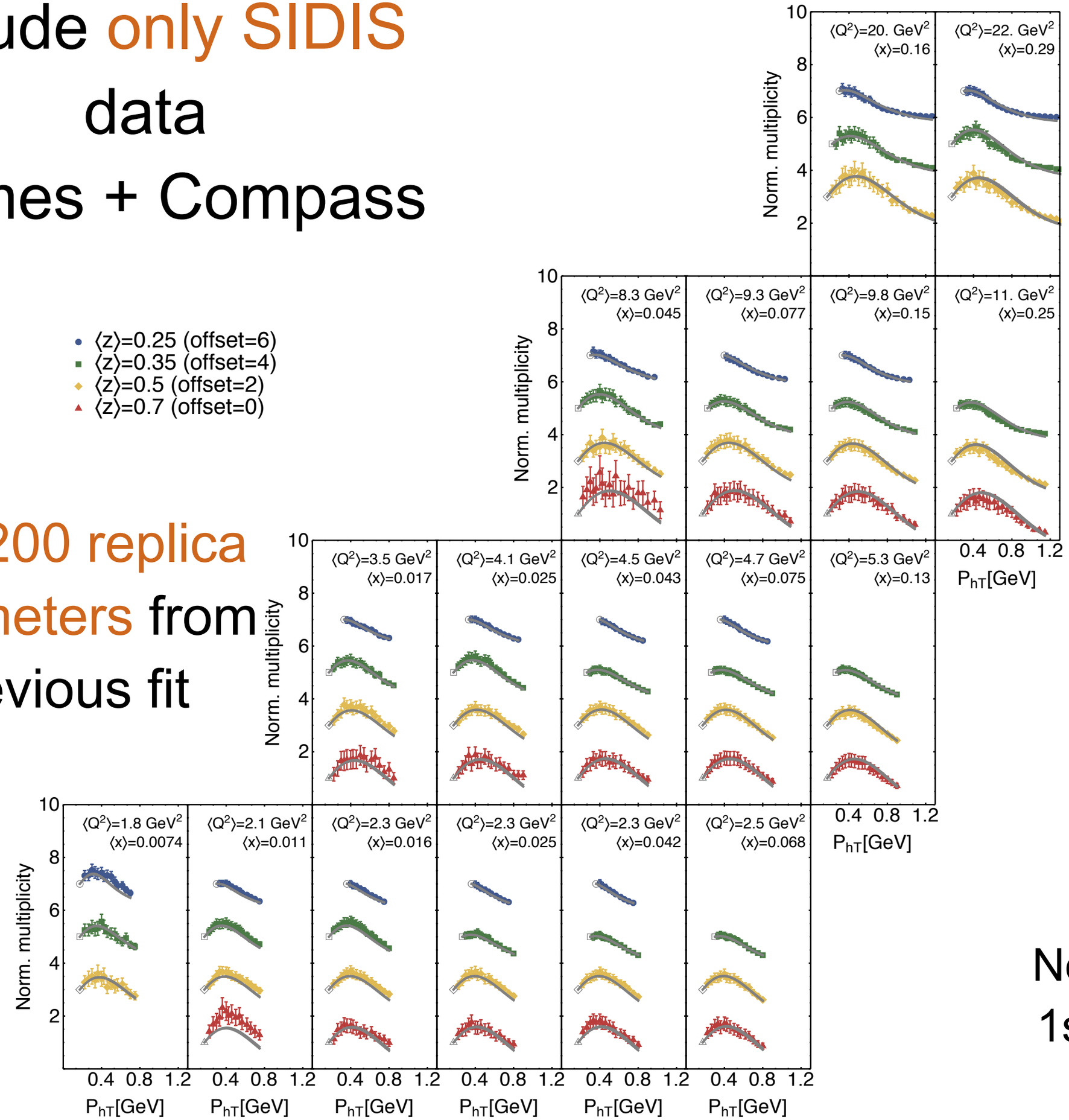

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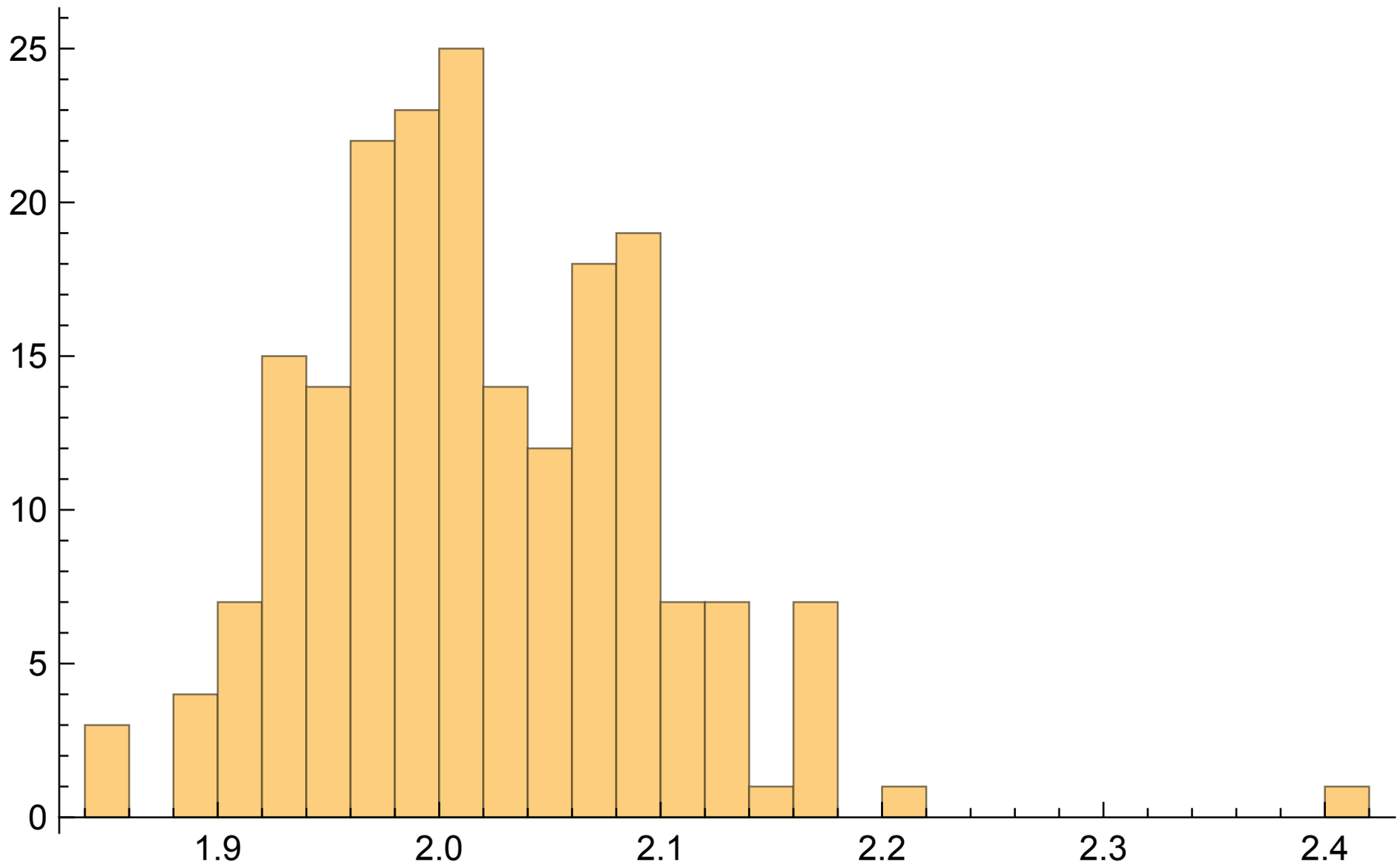
Include only SIDIS data

SIDIS h^+



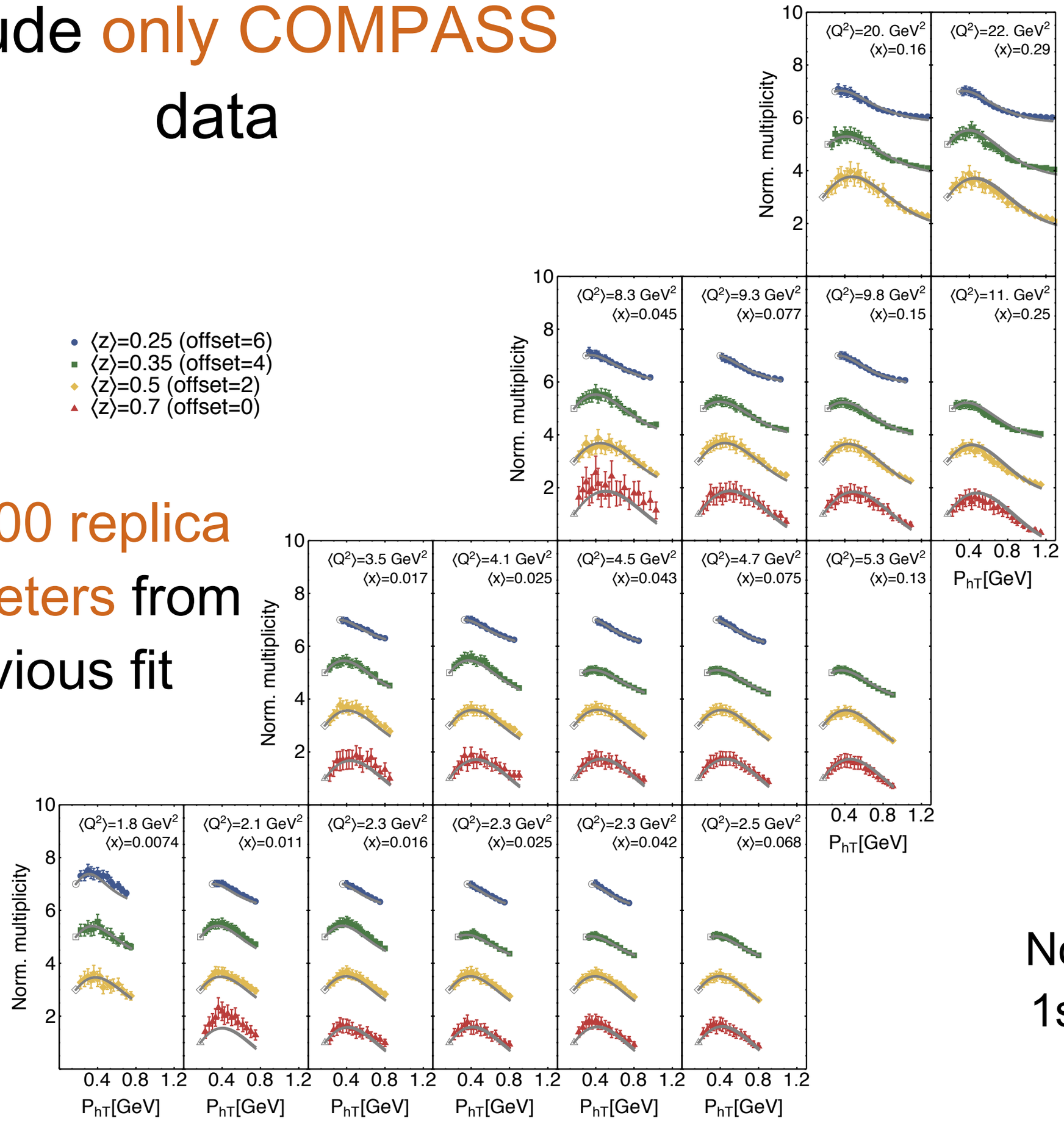
Use 200 replica parameters from previous fit

$\chi^2/\mathrm{dof} = 2.07$

Normalized at 1st data point of bin

Include only COMPASS data

SIDIS h^+

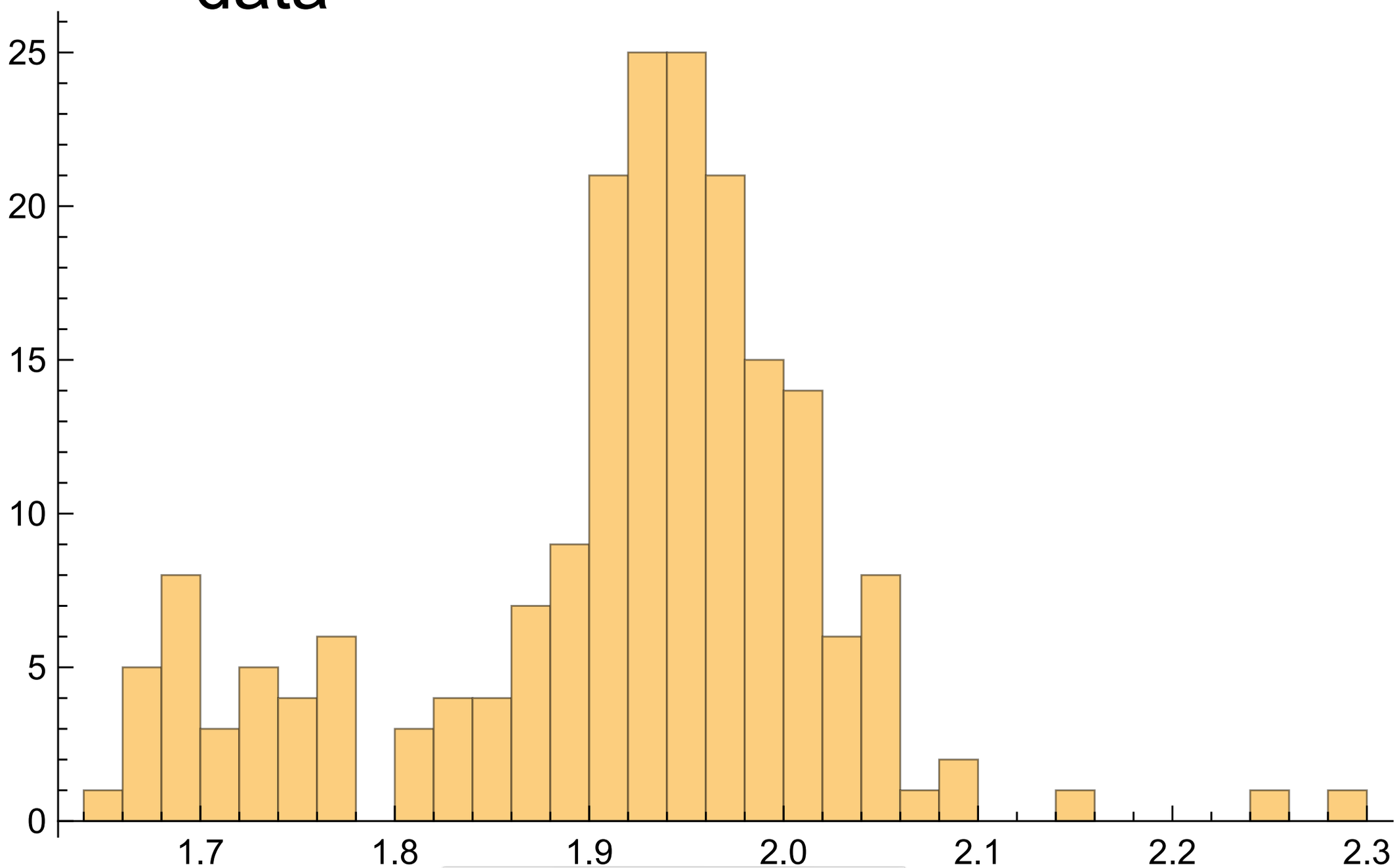


Use 200 replica parameters from previous fit

Normalized at 1st data point of bin

Include only COMPASS data

SIDIS h^+



Use 200 replica parameters from previous fit

$\chi^2/\mathrm{dof} = 1.91$

Normalized at 1st data point of bin